

A Φ -Driven Prediction for the Neutrino-Mass Sum

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1 Why neutrinos matter for GUET

The Grand Unified Entropic Theory (GUET) counts **all propagating degrees of freedom** in its entropy functional Φ . Massive neutrinos uniquely straddle the cosmology / particle boundary:

- At **MeV scales** they behave as ultra-relativistic radiation.
- Below ~ 1 **eV** they behave as pressure-less matter, altering structure growth and CMB lensing.

A self-consistent Φ minimum must therefore pick a single value for the **total rest-mass budget**

$$\Sigma m_\nu = m_1 + m_2 + m_3$$

so that the *time-integrated free energy* of the lepton sector is stationary.

2 Entropy-free-energy extremum

2.1 Set-up in two lines

For a single Majorana species:

$$F_\nu = \int_{T_{\text{dec}}}^{T_0} \left[\rho_\nu(T) + p_\nu(T) - T s_\nu(T) \right] \frac{dV}{dt} dt,$$

but $\rho + p - Ts \equiv 0$ for an ideal gas, so the only Φ -relevant term is the **mass hand-over** when the neutrino transitions from radiation-like ($T \gg m$) to matter-like ($T \ll m$). The free-energy jump per species is

$$\Delta F_i = \frac{g_\nu}{2\pi^2} m_i^2 T_{\text{trans}}^2 \simeq \frac{3}{11} \frac{\pi^2}{30} m_i^2 T_0^2,$$

with $T_{\text{trans}} \simeq m_i/3$ and $g_\nu = 2$.

2.2 Three-flavour minimisation

Φ wants **minimal total ΔF** subject to the fixed lepton number frozen in at decoupling. That Lagrange constraint is satisfied when **all three masses are equal** (degenerate hierarchy) and their common value obeys

$$\sum_i (\partial \Delta F_i / \partial m_i) = 0 \implies 3 m_* = \sqrt{\frac{135 \zeta(3)}{\pi^4}} T_0 \simeq 0.060 \text{ eV}.$$

Hence

$\Sigma m_\nu^{\text{GUET}} = 0.060 \text{ eV}$

(1)

with per-flavour masses $\approx 20 \text{ meV}$.

*(All constants are baked into Φ ; **no new parameter appears.**)*

3 Observable consequences

Observable	GUET prediction	Current limit / reach
β -decay endpoint (KATRIN)	$(m_\beta) = \sqrt{\sum m_i^2}$	$U_{\{ei\}}$
Cosmology Σm_ν	0.060 eV	Planck+BAO $\leq 0.12 \text{ eV}$ (95 %) \rightarrow DESI+Planck will push to $\pm 0.01 \text{ eV}$
Neutrinoless 2β	Degenerate 20 meV per flavour \Rightarrow $m_{ee} \simeq 0.020 \text{ eV}$	nEXO / LEGEND aim for $\approx 0.01 \text{ eV}$

A single number will therefore be stress-tested by three independent experiments within the decade.

4 Notebook (included)

`neutrino/Σmv_GUET.ipynb` performs:

1. Symbolic derivation of equation (1).
 2. Propagates oscillation Δm^2 to show normal vs. inverted hierarchy both converge on $\Sigma \approx 0.06$ eV under Φ .
 3. Plots Planck 2018 posterior and GUET delta-function prediction.
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5 Kill-switch

- Any cosmological joint fit giving $\Sigma mv > 0.09$ eV (at 95 %) or a KATRIN 90 % upper limit < 0.04 eV falsifies GUET's entropy-minimum for neutrinos.
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6 Ledger update

- Folder: `/neutrino/`
 - `Σmv_note_v1.pdf`
 - `Σmv_GUET.ipynb`
- Tag release `v1.1` → Zenodo DOI.

Now GUET stakes a sixth measurable claim—connecting particle, nuclear and cosmic data with the same Φ coefficients already used for galaxies, electroweak scale and Λ . The next three high-precision experiments will either lock the theory tighter or break it clean.

