Author	AP/SD/AS
Date	17.1.013
Code	

UNIVERSITY OF ABERDEEN
DEPARTMENT OF MATHEMATICAL SCIENCES

EG1504 ENGINEERING MATHEMATICS 1
EXERCISES 4 (DIFFERENTIATION)

1. The volume of a sphere is increasing at the rate of 0.1 cubic metres per second. How rapidly is its radius increasing when the radius is 0.3 metres?

[The information given to you concerns dV/dt, the information you have to work out concerns dr/dt. You need the formula which gives the volume of a sphere in terms of its radius. (It is in the Engineering Mathematics Handbook if you have forgotten it.) Differentiate both sides of this equation with respect to t. (Remember to use the chain rule when you are doing this.) Then plug the numbers in, and you should have the answer.]

2. Oil is leaking from a sinking tanker at the rate of 40 cubic metres per hour. Assume that the slick formed is circular and has a uniform thickness of 3 mm at all times. How quickly is the radius of the slick increasing when the radius is 100 metres ?

3. A ladder of length 5 m is propped up against a vertical wall. The foot of the ladder, which rests on a horizontal floor, is moving away from the base of the wall at a constant rate of 20 cm/sec. How fast is the top of the ladder sliding down the wall when the foot of the ladder is at a distance 4 m from the base of the wall?

4. The distances u, v of a small object and its image from a lens of focal length f are related by

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

If the object is 4 cm. from a lens of focal length 2.5 cm and is moving away from the lens at a rate of 9 cm/sec find the rate at which the image is moving. In which direction does it move?

The following questions are designed to give you further practice in calculating derivatives (in preparation for the class test which will soon be upon us).

5. Differentiate the following with respect to x:

$$x^2 + 3x^{-1} - \sqrt{x}, \qquad x^3 - 2x^{-2} + 3\sqrt{x}, \qquad x^5 + 2x^{-1} + 3x^{\frac{4}{3}},$$
$$x^3 - x^{-1} + 2x^{-\frac{1}{2}}, \qquad x^4 + 3x^{-3} - 2\sqrt{x}, \qquad x^2 + 3x^{-1} - 2x^{\frac{1}{3}}.$$

6. Differentiate the following with respect to x:

$$x^2 \sin x, \qquad x^5 \cos x, \qquad x^3 \tan x,$$
$$x^3 \cos x, \qquad x^4 \tan x, \qquad x^4 \sin x.$$

7. Differentiate the following with respect to x:

$$(x-1)^3(x+2)^2, \qquad (x+4)^2(x-3)^4, \qquad (x+2)^3(x-2)^2,$$
$$(x-2)^4(x-3)^3, \qquad (x+3)^3(x-2)^2, \qquad (x+2)^2(x-3)^4.$$

8. Differentiate the following with respect to x:

$$(x^2 - x - 1)^3, \qquad (x^2 + 2x - 3)^3, \qquad (2x^2 + 3x - 1)^4,$$

$$(2x^2 - x + 1)^3, \qquad (2x^3 + x^2 + 1)^3, \qquad (x^3 - 2x^2 + 1)^4.$$

9. Differentiate the following with respect to x:

$$\frac{x + \cos x}{x^2 + 1}, \qquad \frac{x + \tan x}{x^3 - 2}, \qquad \frac{x - \sin x}{x^4 + 1},$$

$$\frac{x - \tan x}{x^3 + 1}, \qquad \frac{x + \sin x}{x^3 - 1}, \qquad \frac{x + \cos x}{x^4 + 1}.$$

10. Differentiate the following functions:

$$\sin(2x^2 + 1), \qquad \cos(x^3 - x^2), \qquad \tan(x + \sqrt{x}),$$

$$\sin(2 + \cos x), \qquad \sqrt{x^3 + x}, \qquad \sqrt{x + \sqrt{x}}.$$

Author	AP/SD/AS
Date	17.1.013
Code	

UNIVERSITY OF ABERDEEN
DEPARTMENT OF MATHEMATICAL SCIENCES

EG1504 ENGINEERING MATHEMATICS 1
SOLUTIONS TO EXERCISES 4 (DIFFERENTIATION)

1. From the handbook

$$V = \frac{4}{3}\pi r^3 \quad \text{and so} \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

Now put $dV/dt = 0.1$, $r = 0.3$ and do the sums. You should get $dr/dt = .088$. The units are metres per second, and so if we change these to centimetres per sec we see that the radius is increasing at the rate of 8.8 cm/sec at the instant referred to.

2. Let V be the volume in cubic metres, r the radius in metres and d the depth in metres of the slick. Then $V = \pi r^2 d$. (Handbook page 21). d is constant and equal to 0.003 metres. So $V = 0.003\,\pi r^2$. Differentiate with respect to time to get

$$\frac{dV}{dt} = 0.003\,\pi 2r\frac{dr}{dt}.$$

Now put in the numbers. When $dV/dt = 40$ and $r = 100$, we get (after some arithmetic) $dr/dt = 21.22$ metres per hour.

3. The starting point in this type of problem is to define relevant variables. In this case let x be the distance of the foot of the ladder from the base of the wall, let y be the height of the top of the ladder from the floor (both in metres) and let t be the time in seconds from a particular instant. By Pythagoras's theorem we have

$$x^2 + y^2 = 25.$$

We are told that when $x = 4$, $dx/dt = 0.2$ (watch out for units!). We want to know dy/dt. We use the equation above to get dy/dt in terms of dx/dt. Differentiating with respect to t:

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0 \quad \text{so} \quad \frac{dy}{dt} = -\frac{x\frac{dx}{dt}}{y}$$

When $x = 4$ we have $y = 3$ and so $\dfrac{dy}{dt} = -\dfrac{4}{15} = -0.26667$ (since $dy/dt < 0$, y is decreasing).

The top of the ladder is sliding *down* the wall at a rate of $|-0.26667| = 0.26667\,\text{m/sec}$.

4. Differentiating:

$$-\frac{\dot{u}}{u^2} - \frac{\dot{v}}{v^2} = 0$$

(in honour of Newton we have used 'dot' for 'd/dt'). Note f is constant. So

$$\dot{v} = -\frac{v^2}{u^2}\,\dot{u}\ .$$

Now plug in the numbers. When $u = 4$, $v = 20/3$ (calculated from the given equation). So put u and v equal to these values and $\dot{u} = 9$ into the equation for $\dot{v}$ and you have your answer (-25). Since the answer is negative the image is moving towards the lens.

5. Answers:

$$2x - 3x^{-2} - \frac{1}{2\sqrt{x}}, \qquad 3x^2 + 4x^{-3} + \frac{3}{2\sqrt{x}}, \qquad 5x^4 - 2x^{-2} + 4x^{1/3}$$

$$3x^2 + x^{-2} - x^{-3/2}, \qquad 4x^3 - 9x^{-4} - \frac{1}{\sqrt{x}}, \qquad 2x - 3x^{-2} - \frac{2}{3}x^{-2/3}.$$

6. Answers:

$$2x\sin x + x^2\cos x,\qquad 5x^4\cos x - x^5\sin x,\qquad 3x^2\tan x + x^3\sec^2 x,$$
$$3x^2\cos x - x^3\sin x,\qquad 4x^3\tan x + x^4\sec^2 x,\qquad 4x^3\sin x + x^4\cos x.$$

7. Answers:

$$\begin{aligned}
3(x-1)^2(x+2)^2 + 2(x-1)^3(x+2) &= (x-1)^2(x+2)[3(x+2)+2(x-1)]\\
&= (x-1)^2(x+2)(5x+4)\\
2(x+4)(x-3)^4 + 4(x+4)^2(x-3)^3 &= (x+4)(x-3)^3[2(x-3)+4(x+4)]\\
&= (x+4)(x-3)^3(6x+10)\\
3(x+2)^2(x-2)^2 + 2(x+2)^3(x-2) &= (x+2)^2(x-2)[3(x-2)+2(x+2)]\\
&= (x+2)^2(x-2)(5x-2)\\
4(x-2)^3(x-3)^3 + 3(x-2)^4(x-3)^2 &= (x-2)^3(x-3)^2[4(x-3)+3(x-2)]\\
&= (x-2)^3(x-3)^2(7x-18)\\
3(x+3)^2(x-2)^2 + 2(x+3)^3(x-2) &= (x+3)^2(x-2)[3(x-2)+2(x+3)]\\
&= (x+3)^2(x-2)5x\\
2(x+2)(x-3)^4 + 4(x+2)^2(x-3)^3 &= (x+2)(x-3)^3[2(x-3)+4(x+2)]\\
&= (x+2)(x-3)^3(6x+2)
\end{aligned}$$

8. Answers:

$$3(x^2-x-1)^2(2x-1),\qquad 3(x^2+2x-3)^2(2x+2),\qquad 4(2x^2+3x-1)^3(4x+3),$$
$$3(2x^2-x+1)^2(4x-1),\qquad 3(2x^3+x^2+1)^2(6x^2+2x),\qquad 4(x^3-2x^2+1)^3(3x^2-4x).$$

9. Answers:

$$\frac{(1-\sin x)(x^2+1)-2x(x+\cos x)}{(x^2+1)^2},\qquad \frac{(1+\sec^2 x)(x^3-2)-3x^2(x+\tan x)}{(x^3-2)^2},$$
$$\frac{(1-\cos x)(x^4+1)-4x^3(x-\sin x)}{(x^4+1)^2},\qquad \frac{(1-\sec^2 x)(x^3+1)-3x^2(x-\tan x)}{(x^3+1)^2},$$
$$\frac{(1+\cos x)(x^3-1)-3x^2(x+\sin x)}{(x^3-1)^2},\qquad \frac{(1-\sin x)(x^4+1)-4x^3(x+\cos x)}{(x^4+1)^2}.$$

10. Answers:

$$4x\cos(2x^2+1),\qquad (2x-3x^2)\sin(x^3-x^2),\qquad \left(1+\frac{1}{2\sqrt{x}}\right)\sec^2(x+\sqrt{x}),$$
$$-\sin(x).\cos(2+\cos x),\qquad \frac{3x^2+1}{2\sqrt{x^3+x}},\qquad \frac{1+2\sqrt{x}}{4\sqrt{x}\sqrt{x+\sqrt{x}}}.$$