Point and line duality

– A line **l** is a homogeneous 3-vector

– It is ⊥ to every point (ray) **p** on the line: **l p**=0
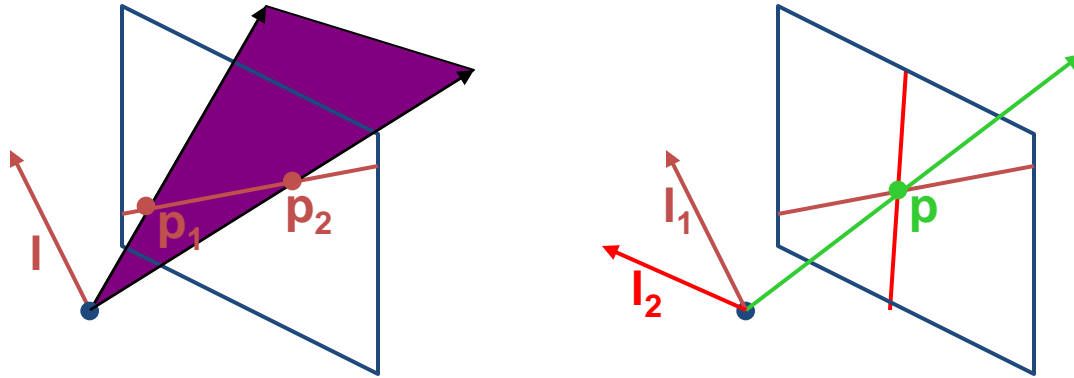
What is the line **l** spanned by rays $\mathbf{p_1}$ and $\mathbf{p_2}$?

- **l** is ⊥ to $\mathbf{p_1}$ and $\mathbf{p_2}$ ⇒ $\mathbf{l} = \mathbf{p_1} \times \mathbf{p_2}$
- **l** is the plane normal

What is the intersection of two lines $\mathbf{l_1}$ and $\mathbf{l_2}$?

- **p** is ⊥ to $\mathbf{l_1}$ and $\mathbf{l_2}$ ⇒ $\mathbf{p} = \mathbf{l_1} \times \mathbf{l_2}$

Points and lines are *dual* in projective space

- given any formula, can switch the meanings of points and lines to get another formula

Points from two lines
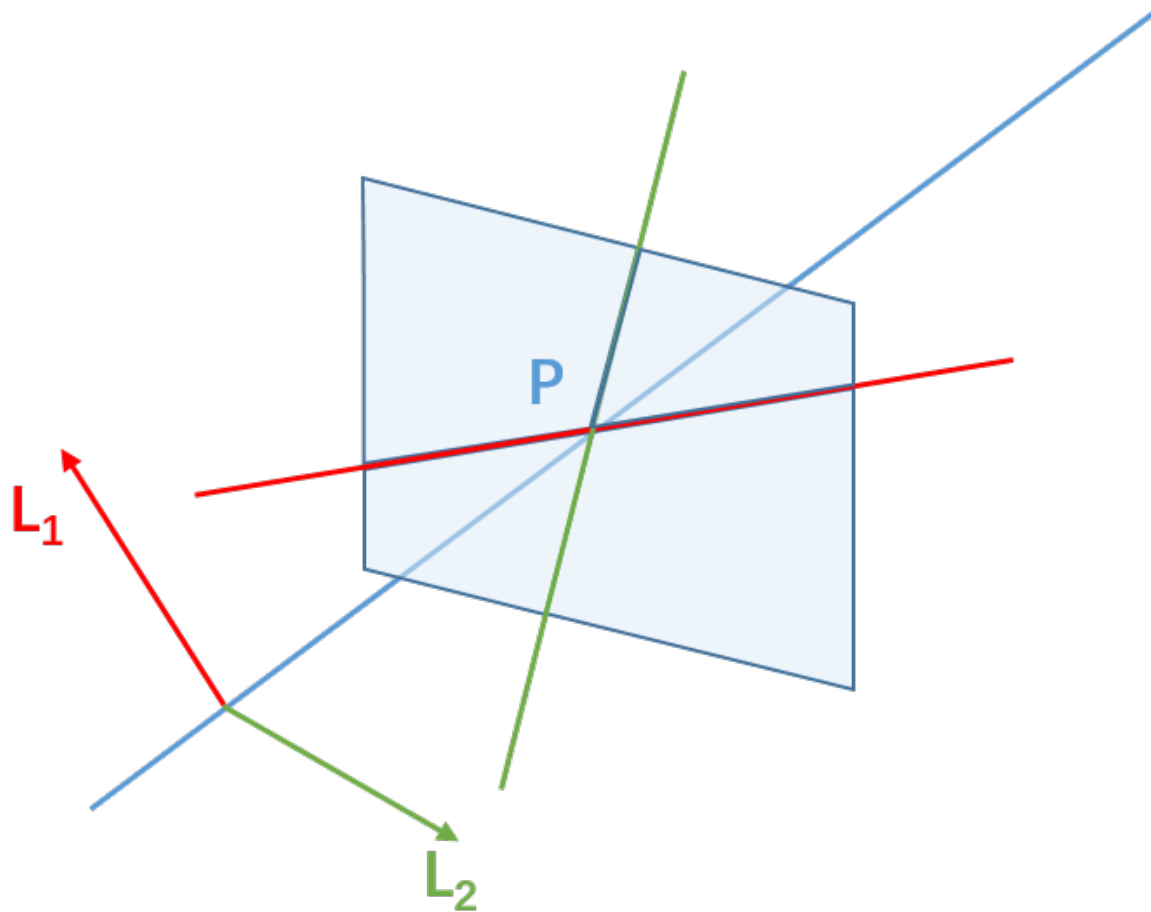
When P has the form (x,y,0)?

Point at infinity

Example: Consider two *parallel* horizontal lines:

$$x = 1; x = 2;$$

Intersection =

$$det[(i, j, k); (-1, 0, 1); (-1, 0, 2)]$$

$$= (0, 1, 0)$$

Point at infinity in the direction of y

Point at infinity, Ideal points

$$l = (a, b, c) \qquad l' = (a, b, c')$$

Intersection:

$$
\begin{aligned}
l \times l' &= l \times l' \\
&= \begin{vmatrix} i & j & k \\ a & b & c \\ a & b & c' \end{vmatrix} \\
&= (bc' - bc, ca - c'a, ab - ab)^T \\
&= (c' - c)(b, -a, 0)^T
\end{aligned}
$$

Any point $(x_1, x_2, 0)$ is intersection of lines at infinity

Points at infinity

- ## Under projective transformation,
  - All parallel lines intersects at the point at infinity

  line $l = (a, b, c)^T$ intersects at $(b, -a, 0)^T$

  - One point at infinity $\Leftrightarrow$ one parallel line direction

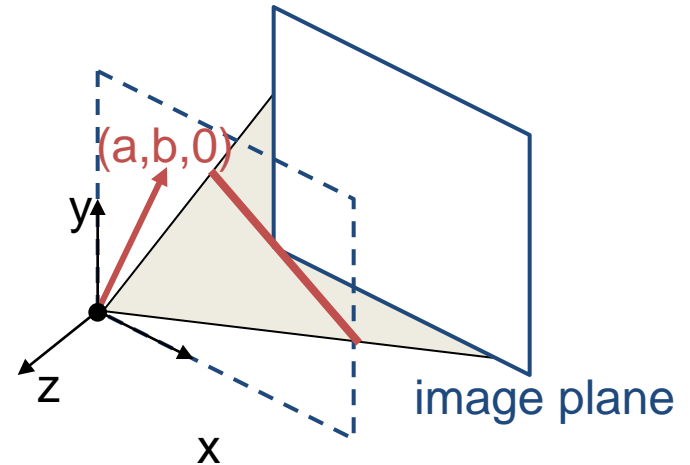- ## Where are the points at infinity in the image plane?

Line at infinity

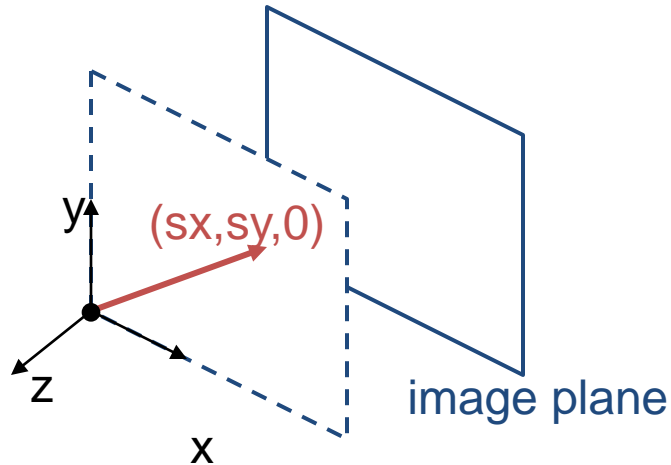- A line passing all points at infinity:

$$l_\infty = (0, 0, 1)^T$$

- Because :

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 0 \end{bmatrix} = 0$$

Ideal points and lines

- Ideal point ("point at infinity")
  - p ≅ (x, y, 0) – parallel to image plane
  - It has infinite image coordinates

Ideal line
  - l ≅ (a, b, 0) – parallel to image plane

  - Corresponds to a line in the image (finite coordinates)