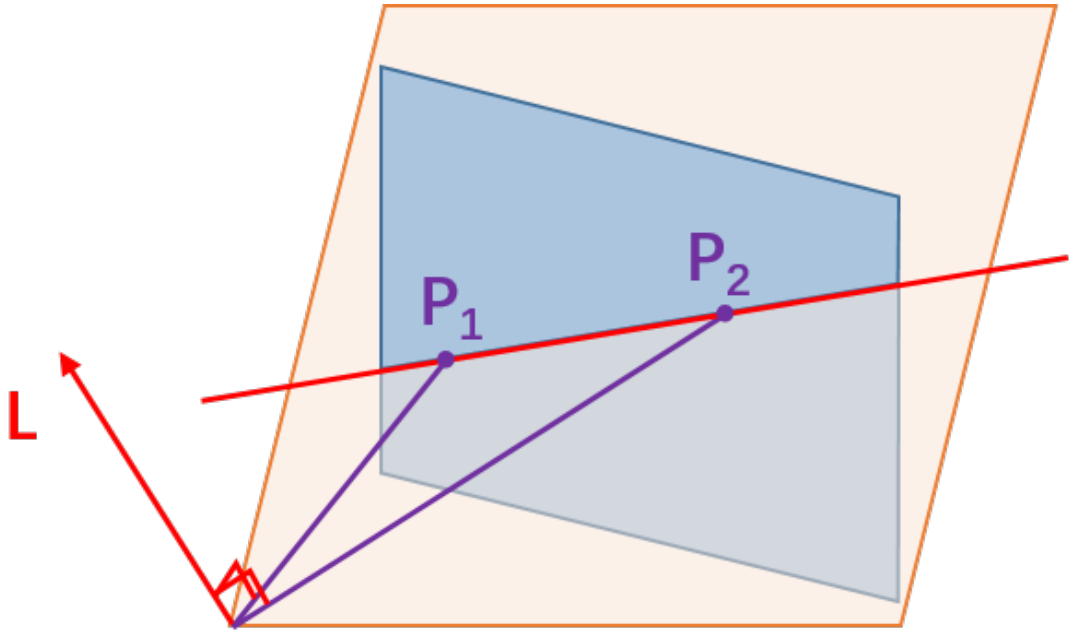
Projective lines from two points



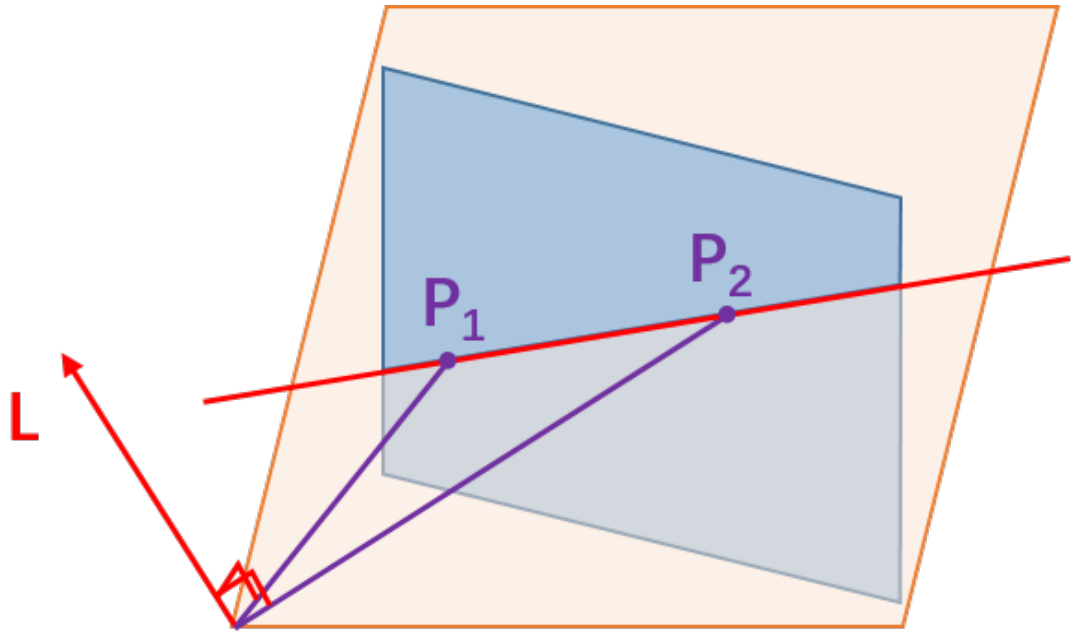
Line passing through two points

Two points: x x'

Define a line

l is the line passing two points

$$l = x \times x'$$



Line passing through two points

Two points: $x \quad x'$

Define a line

$$l = x \times x'$$

l is the line passing two points

Proof:

$$x \cdot (x \times x') = 0$$
$$x' \cdot (x \times x') = 0$$

$$x \cdot l = 0$$
$$x' \cdot l = 0$$

Line passing through two points

- N

$$
\begin{aligned}
l &= x \times x' \\
&= \begin{vmatrix} i & j & k \\ x_1 & x_2 & x_3 \\ x'_1 & x'_2 & x'_3 \end{vmatrix} \\
&= (x_2x'_3 - x_3x'_2)i \\
&\quad +(x_3x'_1 - x_1x'_3)j \\
&\quad +(x_1x'_2 - x_2x'_1)k \\
&= (x_2x'_3 - x_3x'_2, x_3x'_1 - x_1x'_3, x_1x'_2 - x_2x'_1)^T
\end{aligned}
$$

Matlab codes

- function l = get_line_by_two_points(x, y)

- x1 = [x(1), y(1), 1]';
- x2 = [x(2), y(2), 1]';
- l = cross(x1, x2);
- l = l / sqrt(l(1)*l(1)+l(2)*l(2));

Example of Line

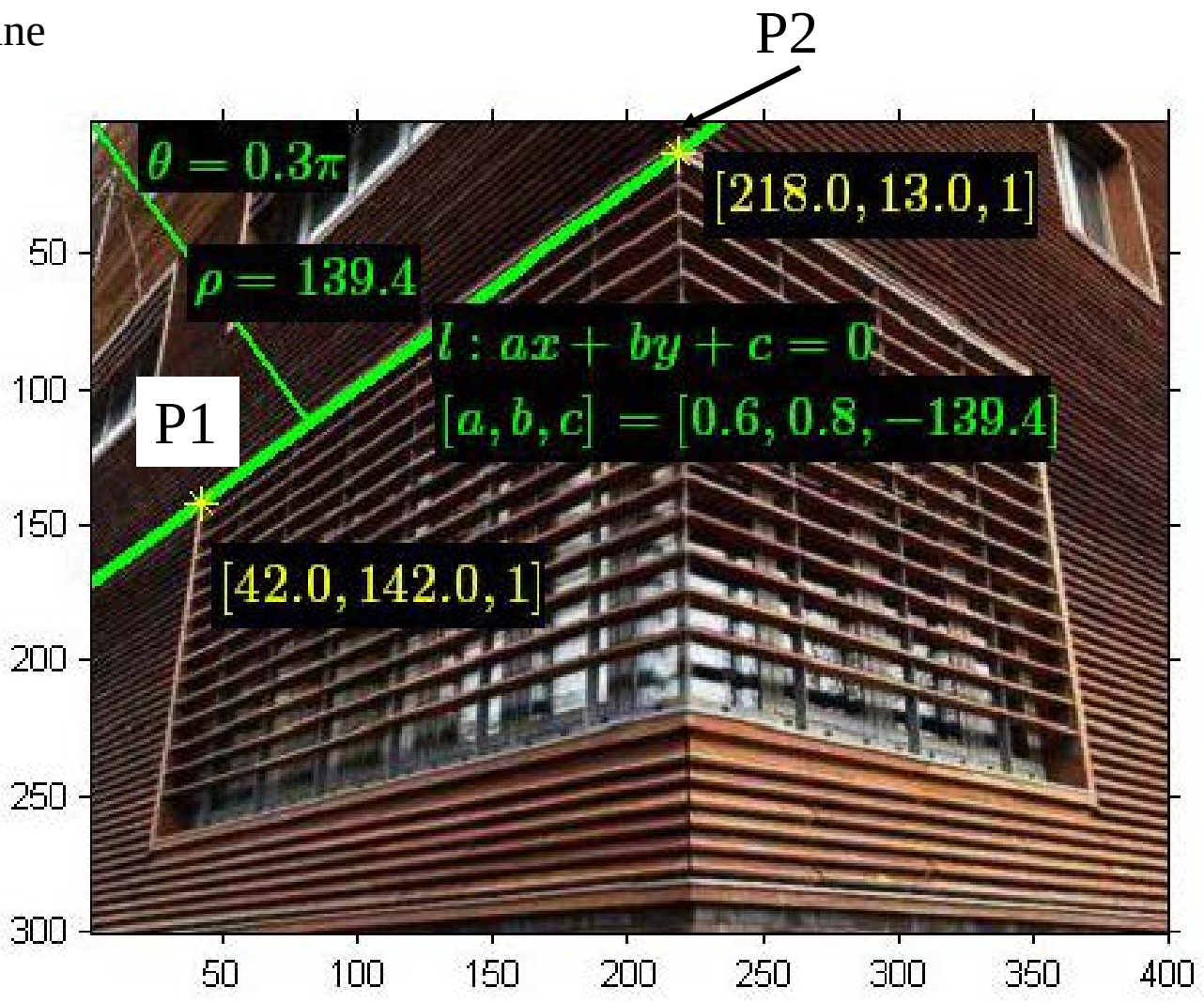
Verify p2 lies on the line

```
>> 218*0.6+13*0.8

ans =

  141.2000
```

(close)

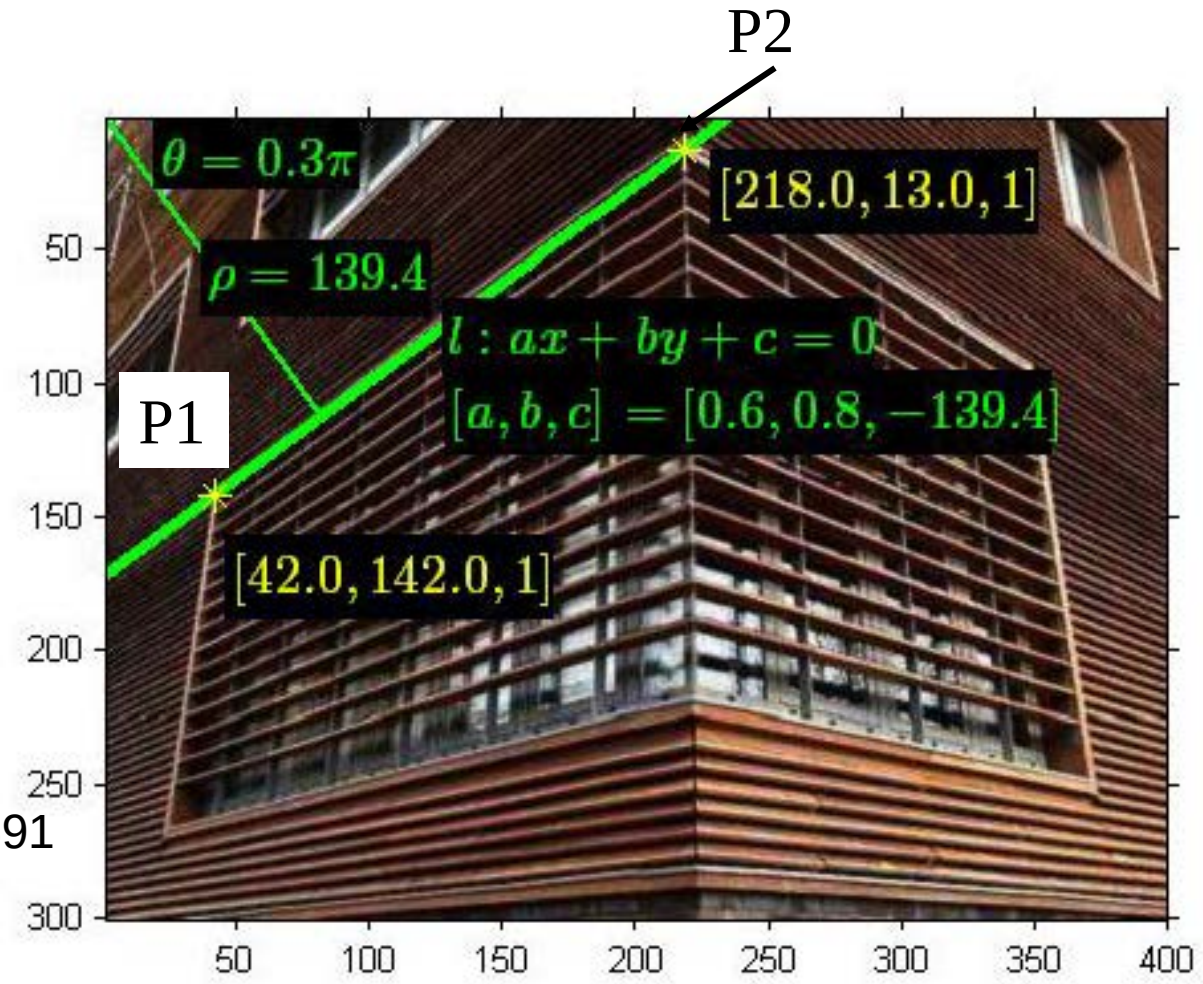


Example of Line

Test line:

```
>> p1 = [42,142,1];
>> p2=[218,13,1];
>> l = cross(p1,p2)
l =
129   176   -30410
```

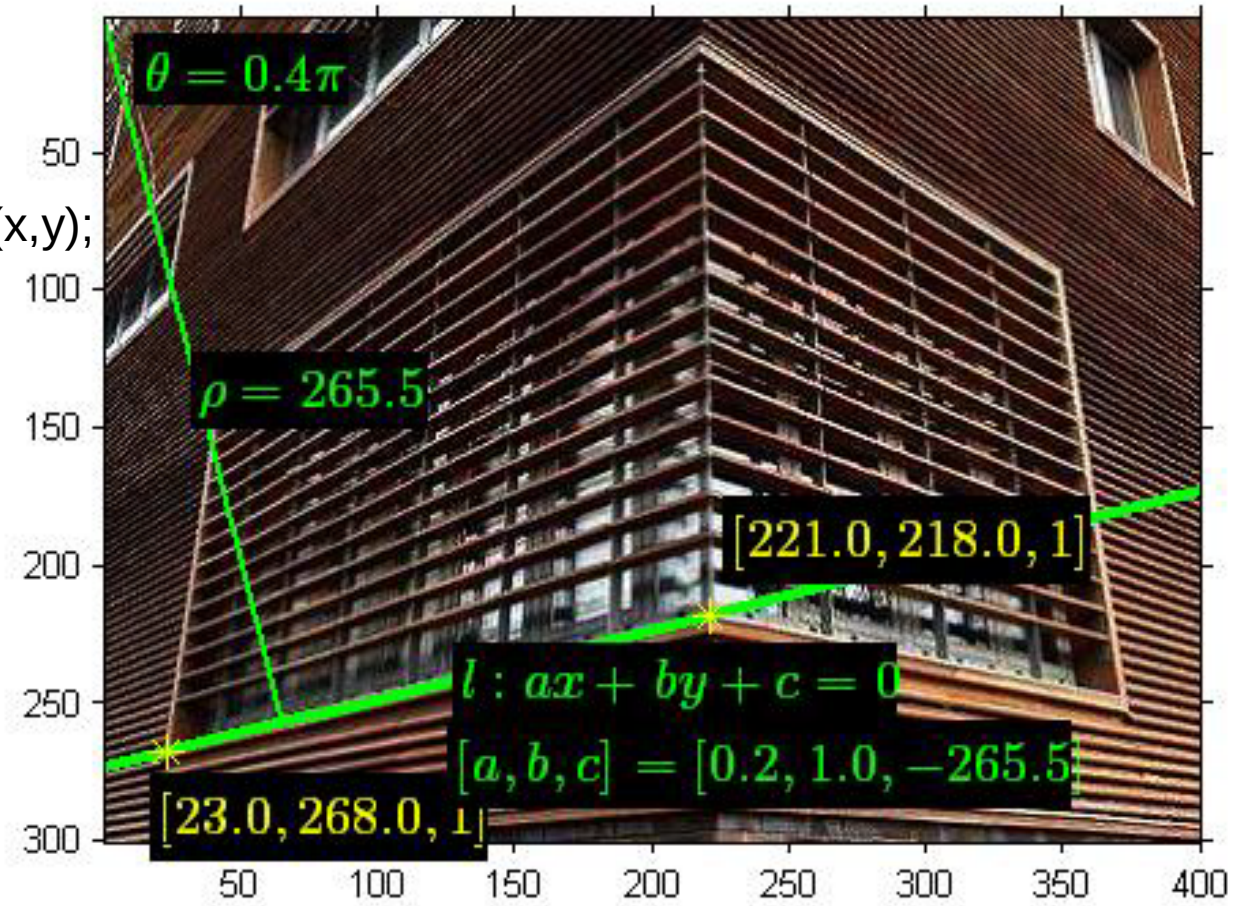
```
>> l = l/sqrt(l(1)^2+l(2)^2)
l =
 0.5912    0.8066 -139.3591
```



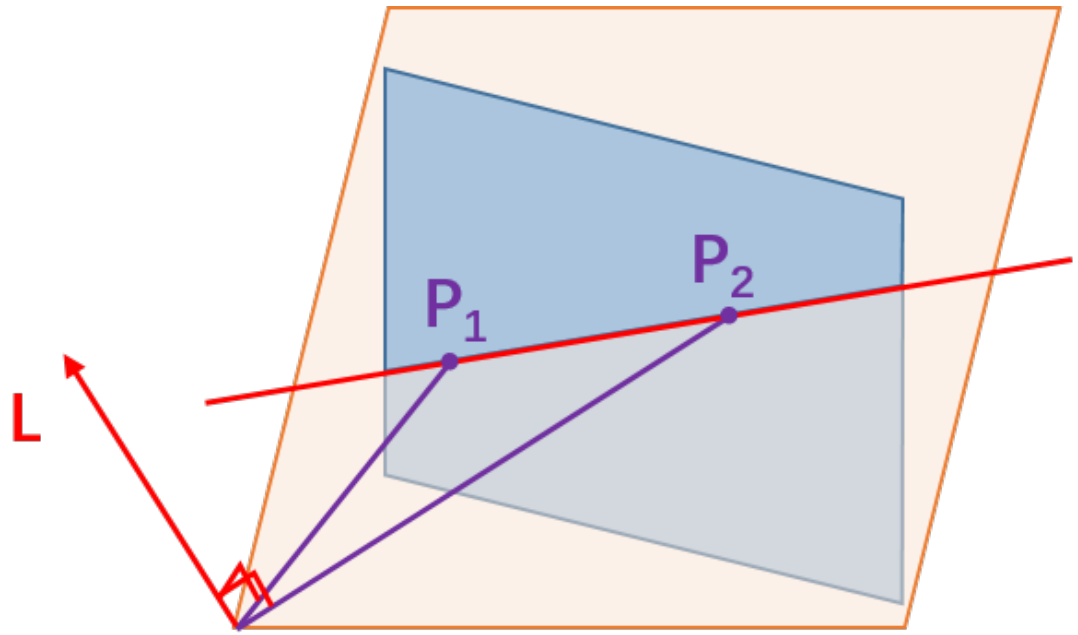
```
>> x = [221;23];
>> y = [218;268];
>> l = get_line_by_two_points(x,y);
>> l

l =


   -0.2448
   -0.9696
  265.4744
```

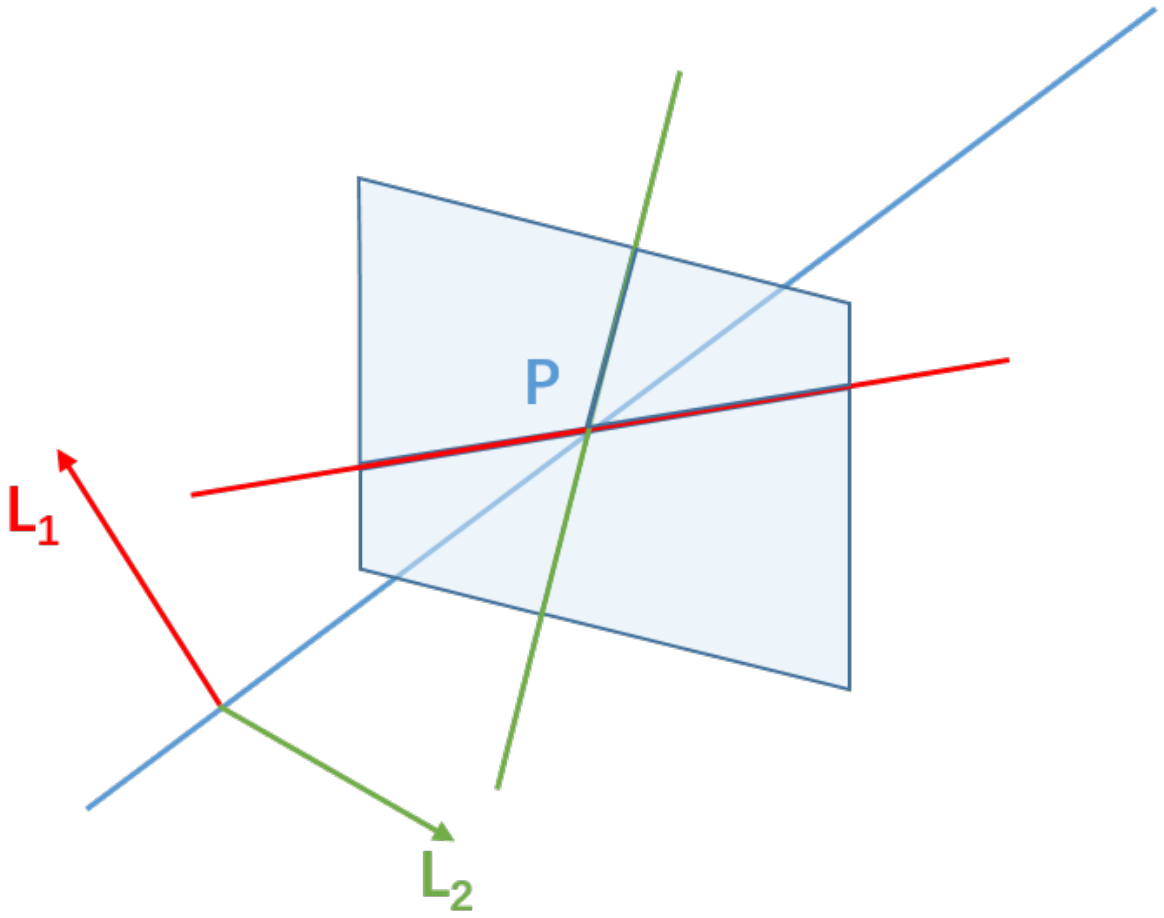


Projective lines from two points



When does the line has the form (a, b, 0)?
When does the line has the form (0, 0, 1)?

Points from two lines

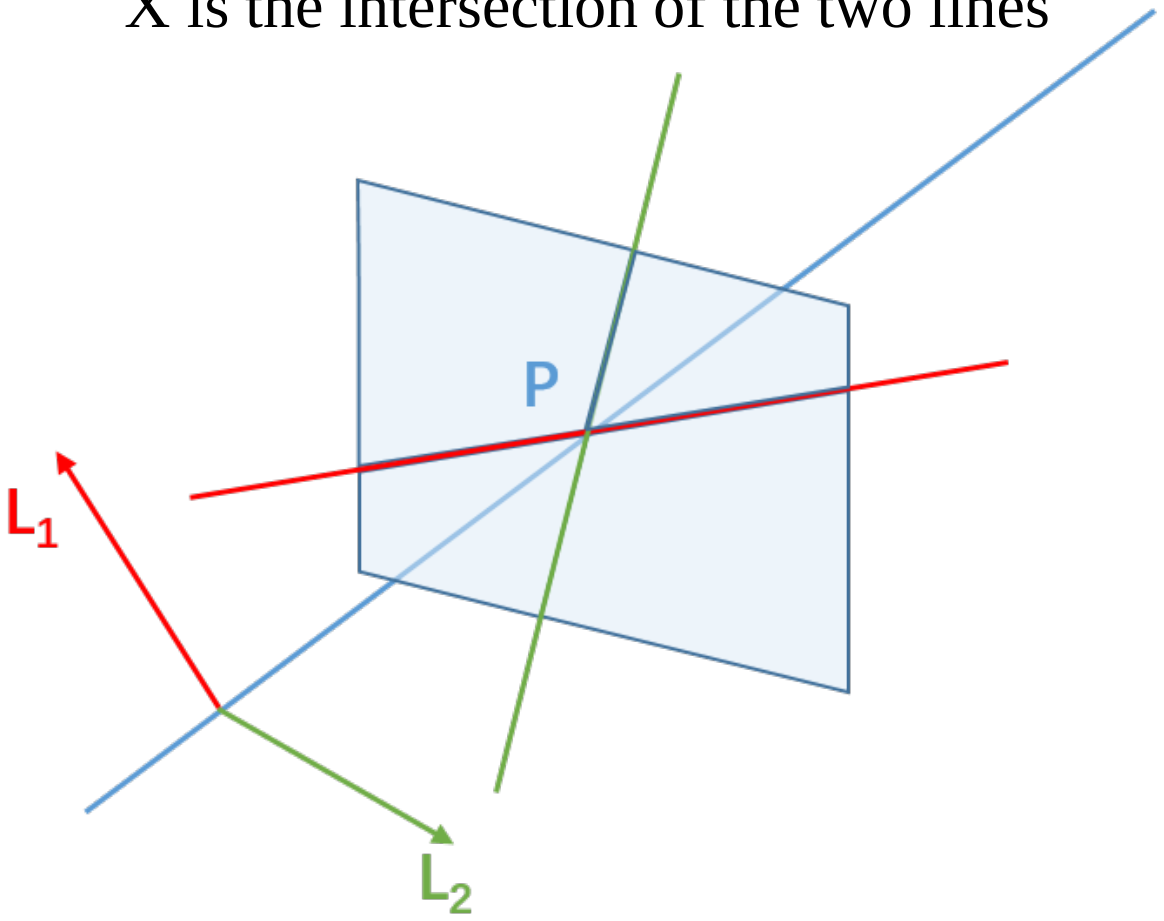


Intersection of lines

Given two lines: l , l'

Define a point $x = l \times l'$

X is the intersection of the two lines

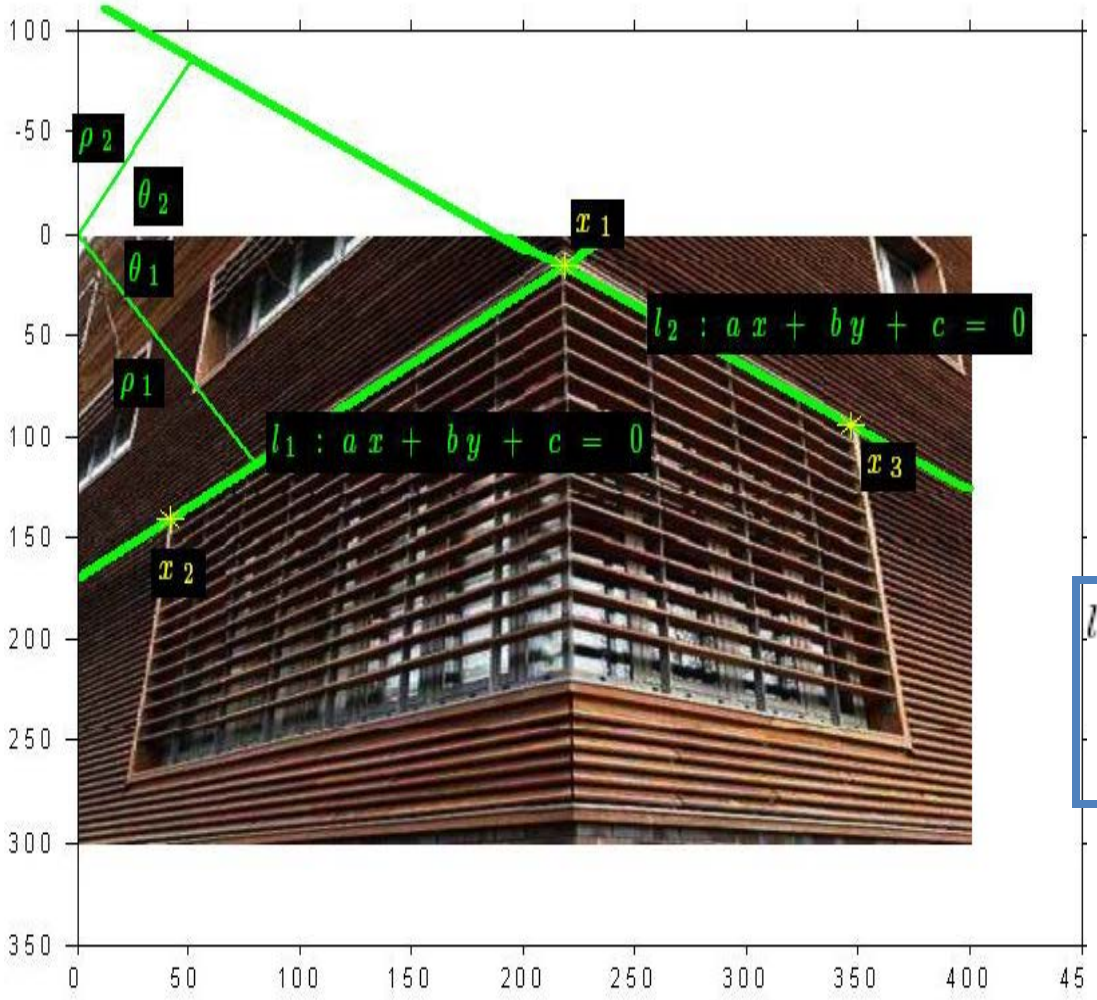


Matlab codes

- function x0 = get_point_by_two_line(l, l1)

- x0 = cross(l, l1);
- x0 = [x0(1)/x0(3); x0(2)/x0(3)];

Example of Lines Intersection



$$
\begin{aligned}
x_1 &= (218, 16, 1)^T \\
x_2 &= (42, 141, 1)^T \\
x_3 &= (347, 94, 1)^T
\end{aligned}
$$

$$
\begin{aligned}
l_1 &= x_1 \times x_2 \\
&= k(-0.58, -0.82, 139.28)^T \\
l_2 &= x_1 \times x_3 \\
&= k(0.52, -0.86, -99.11)^T
\end{aligned}
$$

$$
\begin{aligned}
l_1 \times l_2 &= (-199.98, -14.68, -0.92)^T \\
&= -0.92(218.0, 16.0, 1.0)^T \\
&= kx_1
\end{aligned}
$$