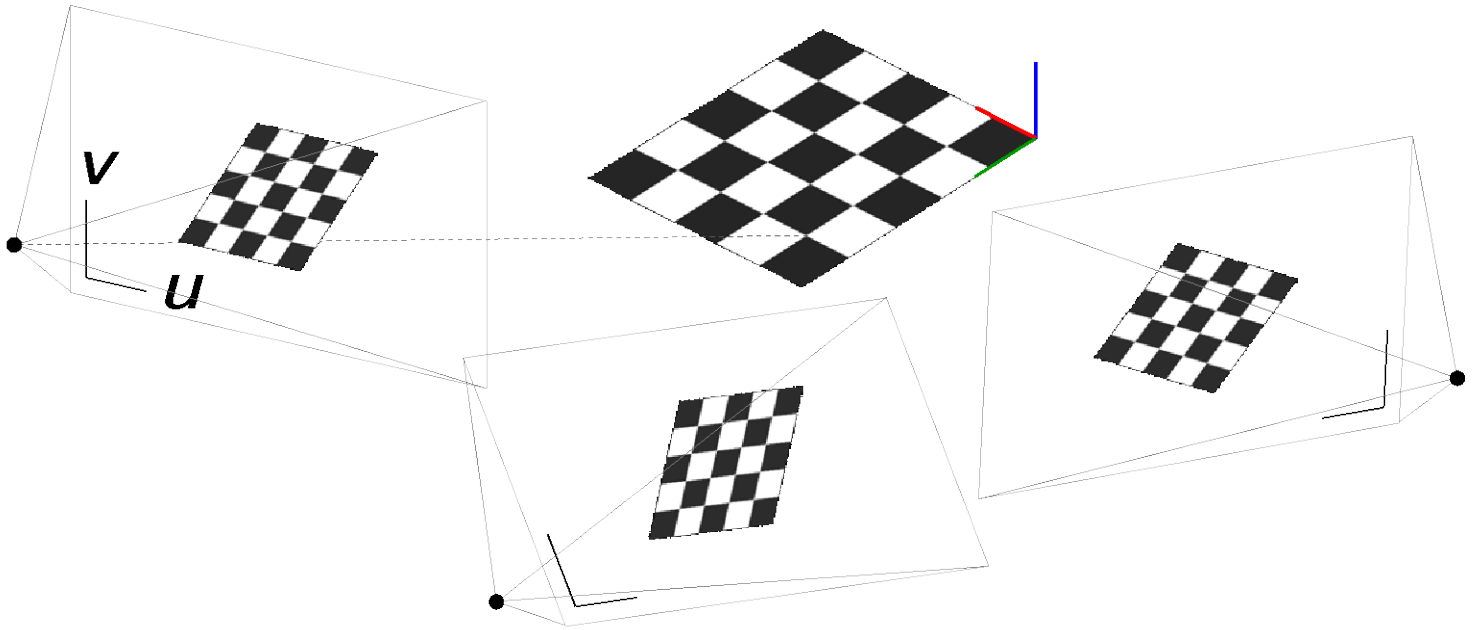
Multiple View Geometry: 3rd person view measurement

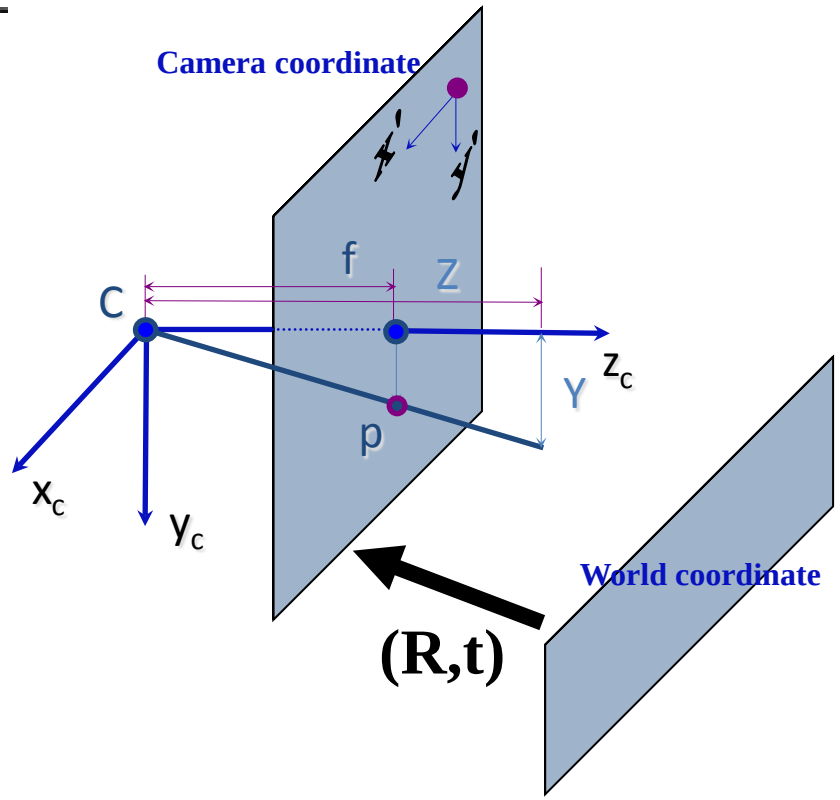


Step 3: 3rd person to 1st person 3D mapping: the world to camera coordinates

$$\mathbf{X_c} = \begin{bmatrix} R_{3x3} & t_{3x1} \\ \mathbf{0} & 1 \end{bmatrix} \mathbf{X}$$

3rd person 3D point

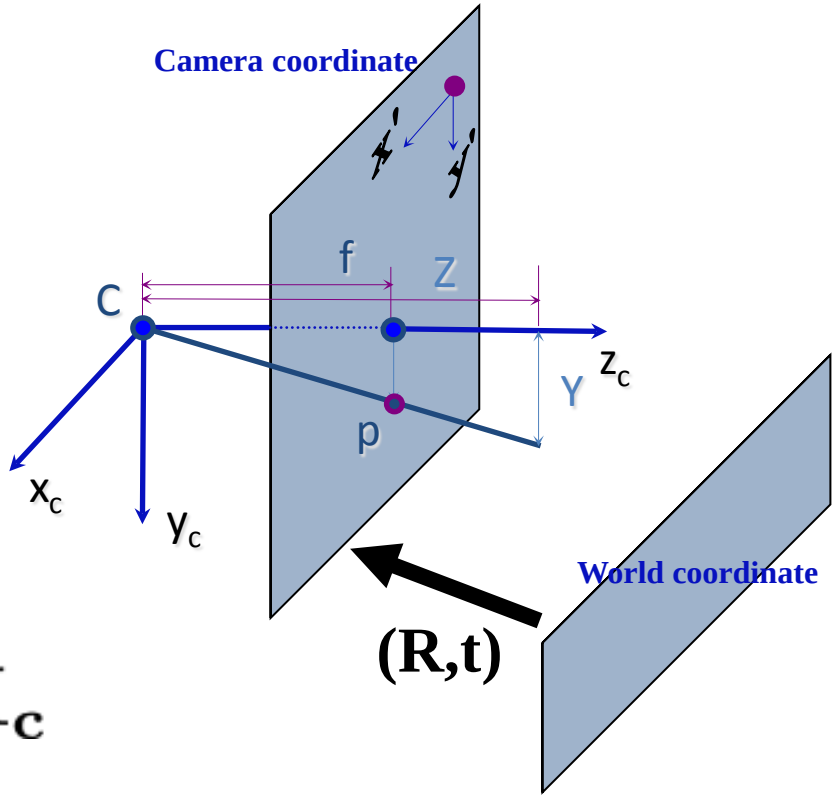
1st person 3D point



Combining Internal and External parameters

1) $$\mathbf{Xc} = \begin{bmatrix} \mathrm{R} & \mathrm{t} \\ \mathbf{0} & 1 \end{bmatrix} \mathbf{X}$$

2) $$\mathbf{x} = \mathbf{K}_{3\times3}\,[\mathbf{I};0]_{3\times4}\,\mathbf{X_c}$$



1) Translate the world coordinate into the camera coordinate
2) Translate the camera coordinate into the pixel coordinate

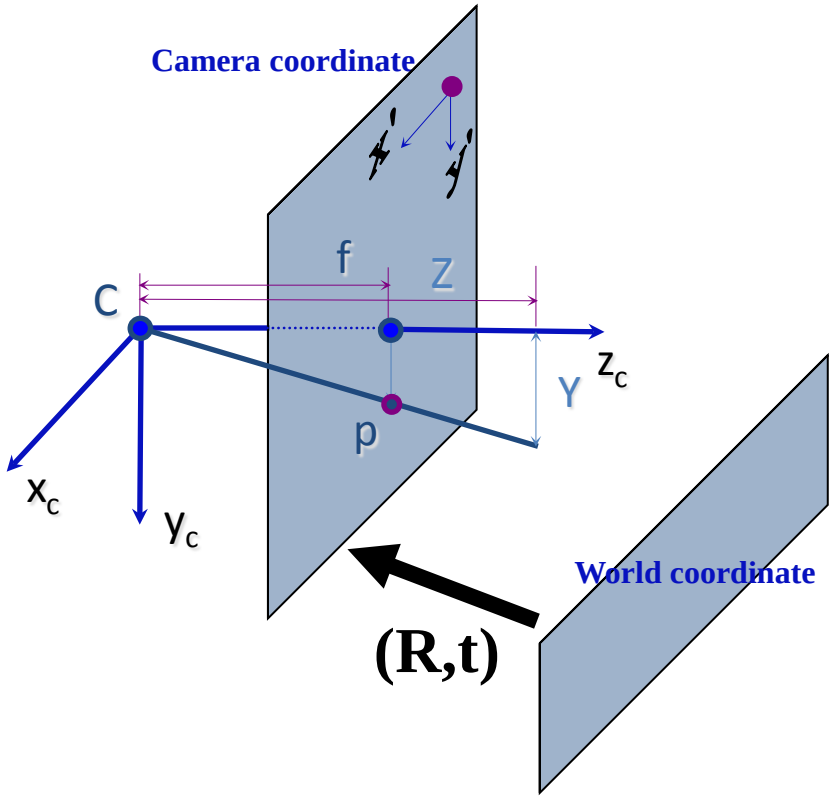
Combining Internal and External parameters

$$\mathbf{Xc} = \begin{bmatrix} \mathrm{R} & \mathrm{t} \\ \mathbf{0} & 1 \end{bmatrix} \mathbf{X}$$

$$\mathbf{x} = \mathbf{K}_{3\times3}\ [\mathbf{I}; 0]_{3\times4}\ \mathbf{X_c}$$

After simplication:

$$x = K\ [R,\ t]\ X$$

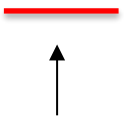


Ideal Perspective Projection

$$z\begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

Ideal Perspective Projection

$$z\begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$



3D World in 3rd person view

Ideal Perspective Projection

$$z\begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

3D World in 1st person view

Ideal Perspective Projection

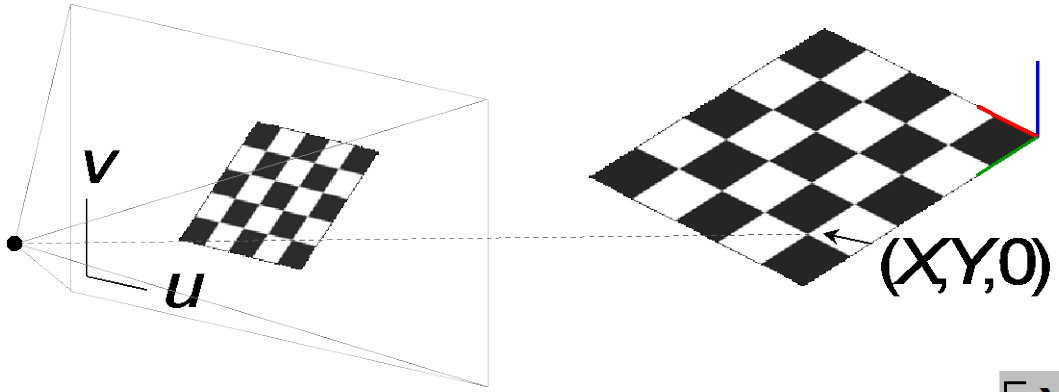
$$z\begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

2D pixels in 1st person view

Camera Projection

$$\begin{bmatrix} x \\ 1 \end{bmatrix} = L \left(K \begin{bmatrix} R & t \end{bmatrix} \begin{bmatrix} X \\ 1 \end{bmatrix} \right)$$

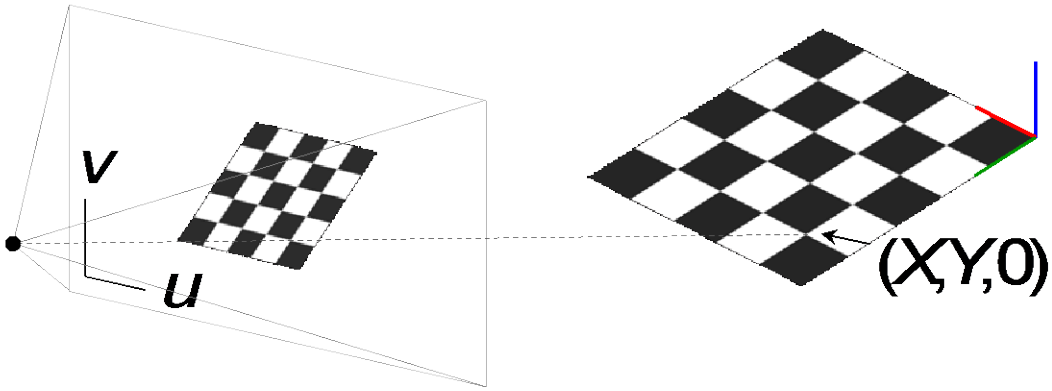
Special case, planar world, homograph



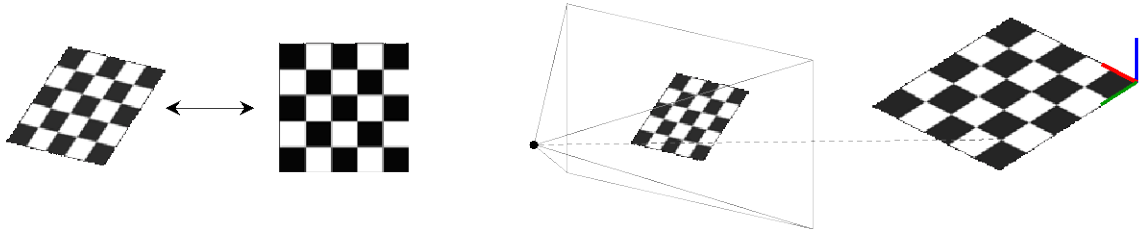
$$z\begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_1 \\ r_{21} & r_{22} & r_{23} & t_2 \\ r_{31} & r_{32} & r_{33} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 0 \\ 1 \end{bmatrix}$$

x K $R \in \mathbb{R}^{3\times3}$ t X

Special case, planar world, homograph



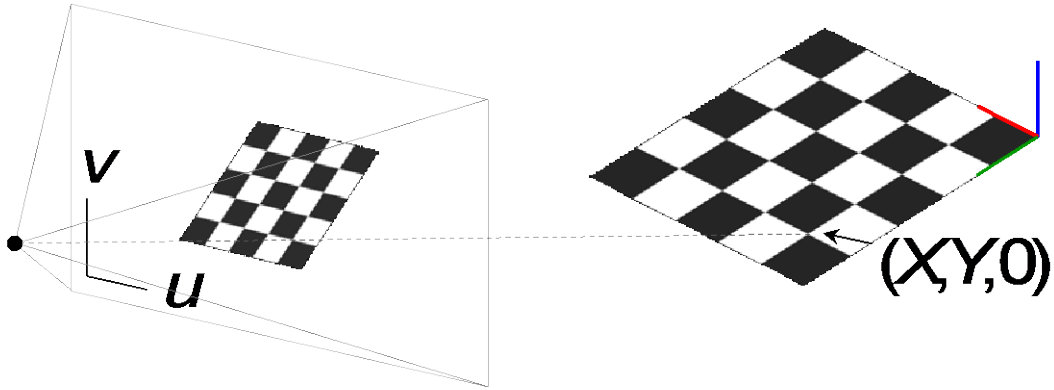
$$z\begin{bmatrix} u_{img} \\ v_{img} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

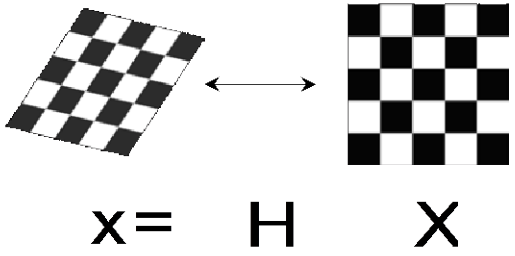


$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} = z' \begin{bmatrix} f_x & s & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} r_{11} & r_{12} & t_1 \\ r_{21} & r_{22} & t_2 \\ r_{31} & r_{32} & t_3 \end{bmatrix}$$

$\mathbf{h}_1 \quad \mathbf{h}_2 \quad \mathbf{h}_3$ $\qquad$ $\mathbf{K}$ $\qquad$ $\mathbf{r}_1 \quad \mathbf{r}_2 \quad \mathbf{t}_1$

Special case, planar world, homograph





x = H X

$$z\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ 1 \end{bmatrix}$$

H homography between 3D plane and 2D image plane.

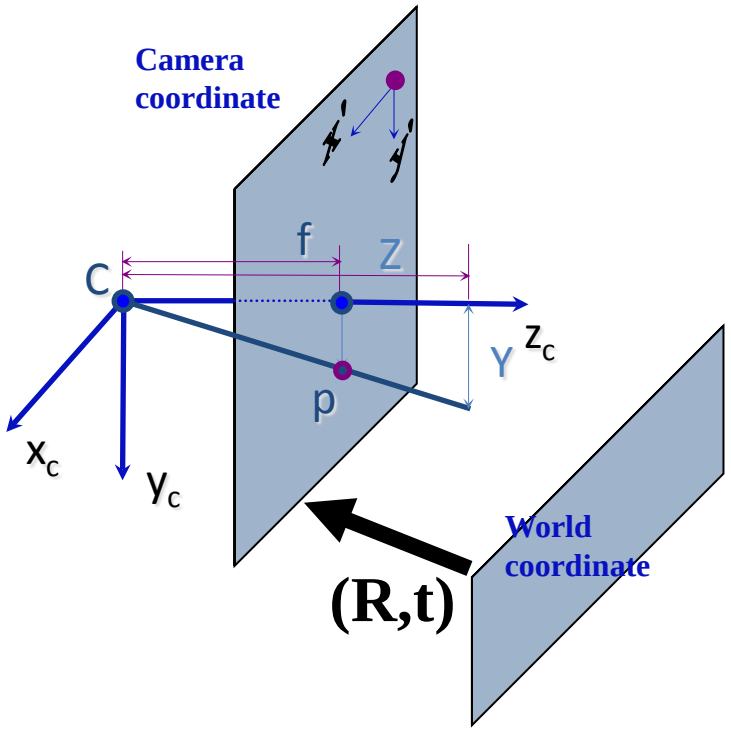
Special case: rotating camera

$$\mathbf{x} = K\,[R,\, t]\,\mathbf{X}$$

with $t = 0$

Expand:

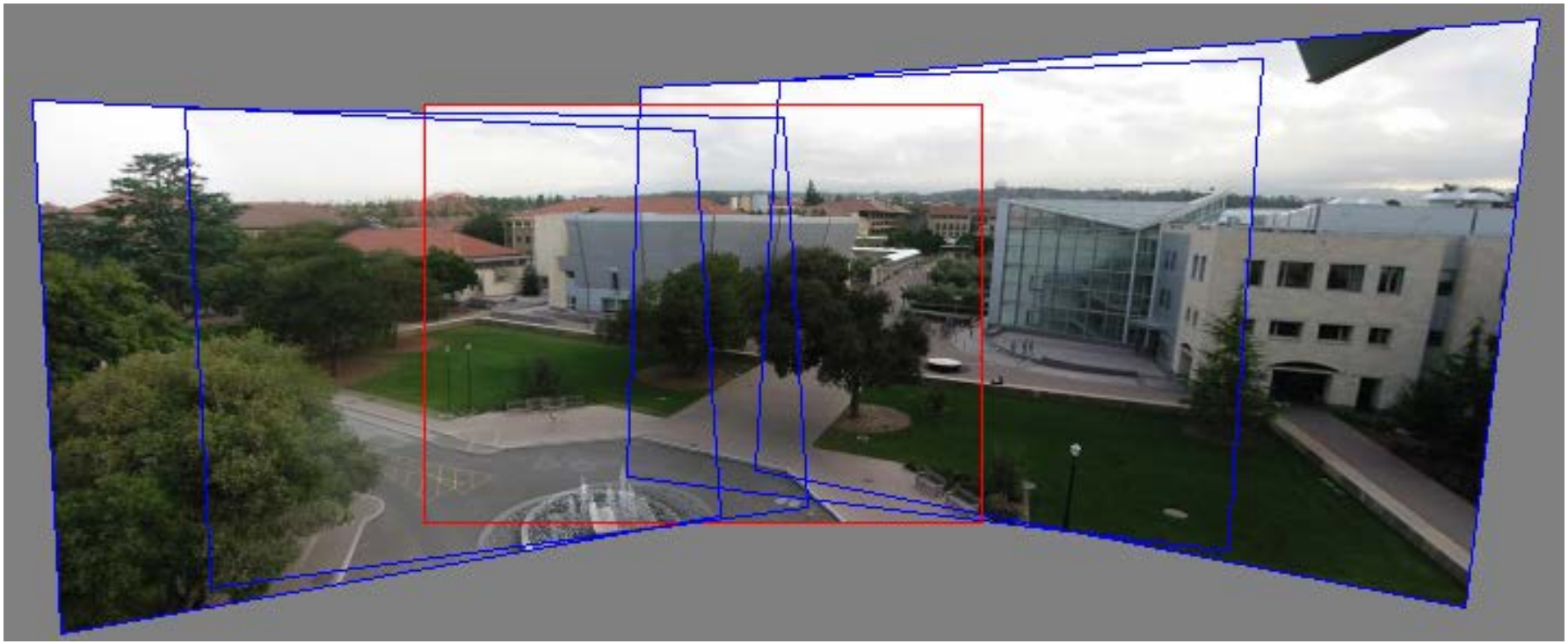
$$\mathbf{x} = K\,[R]\,\mathbf{X}$$



$$\mathbf{X} = \begin{pmatrix} \mathbf{x}_w \\ \mathbf{y}_w \\ \mathbf{z}_w \end{pmatrix}$$

This is also a homography with $H = R$

Panoramas



$$\mathbf{x} = K\,[R]\,\mathbf{X}$$