Perception: Rotations and Translations

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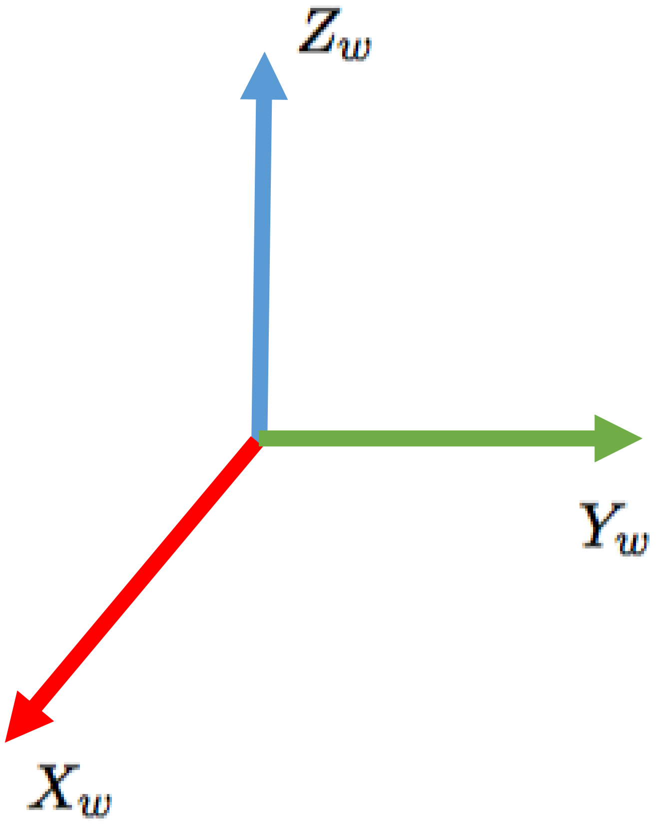
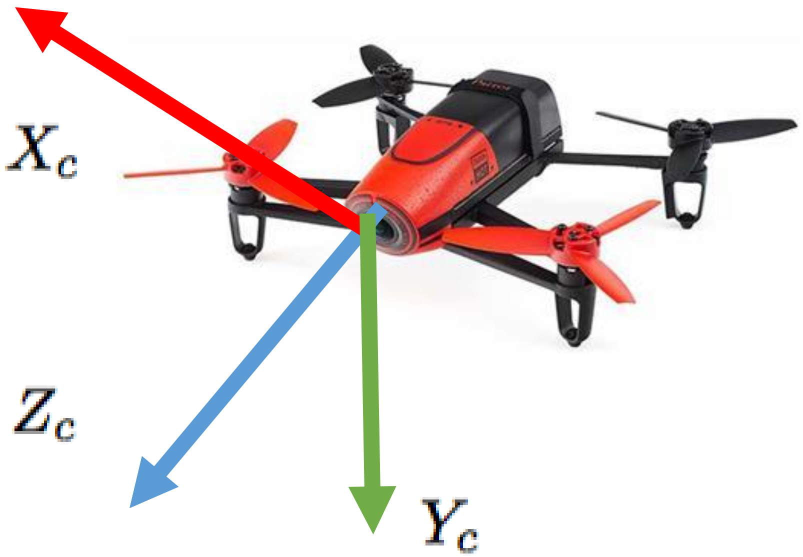
Transformation between camera and world coordinate systems



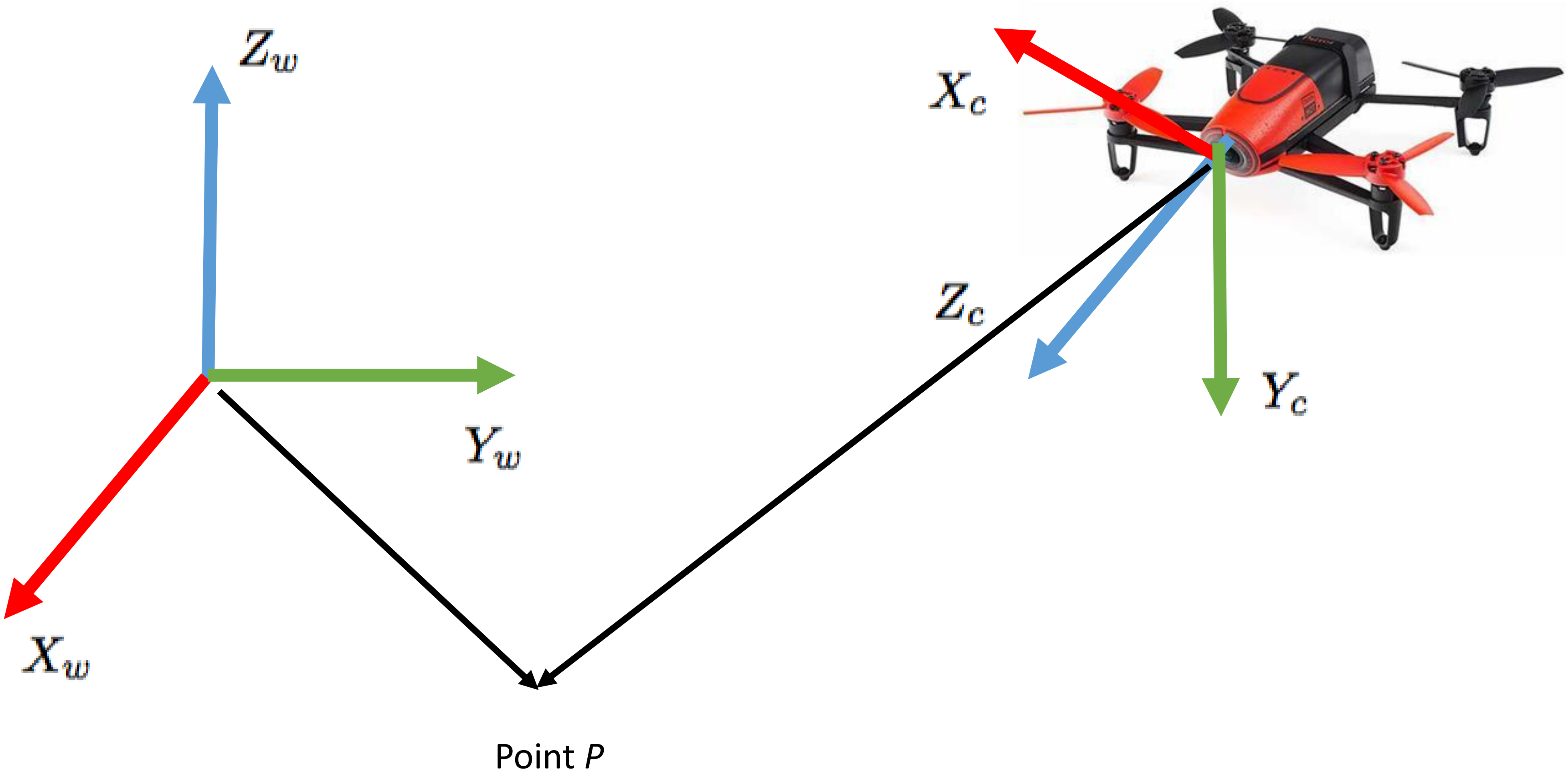


Red for X-Axis
Green for Y-Axis
Blue for Z-Axis

Remember
RGB is XYZ

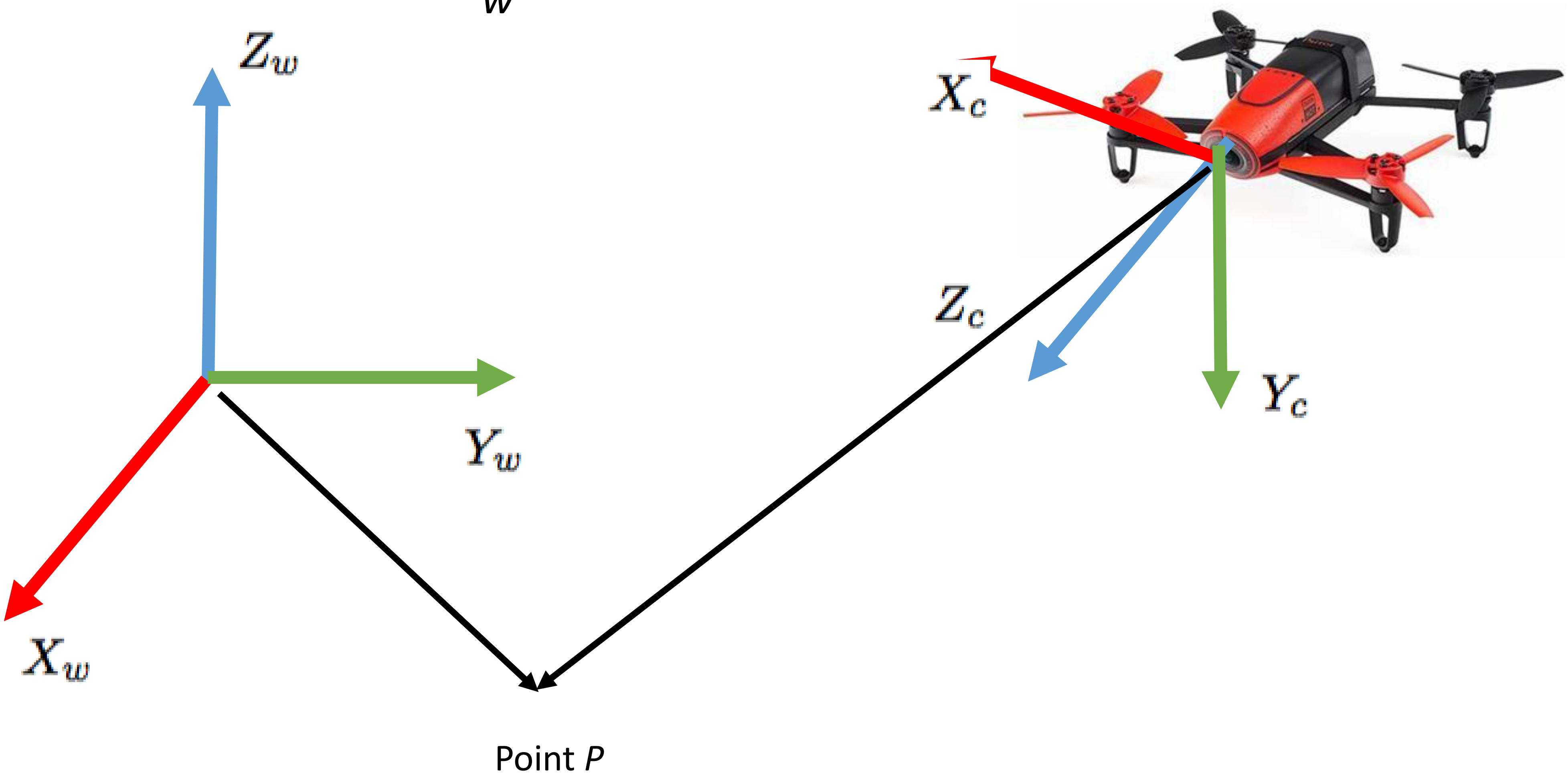
$$ {}^{c}P = {}^{c}R_{w}\,{}^{w}P + {}^{c}T_{w} $$

Point *P* can be expressed with respect to “w” or “c” coordinate frames



$${}^{c}P = {}^{c}R_{w}\,{}^{w}P + {}^{c}T_{w}$$

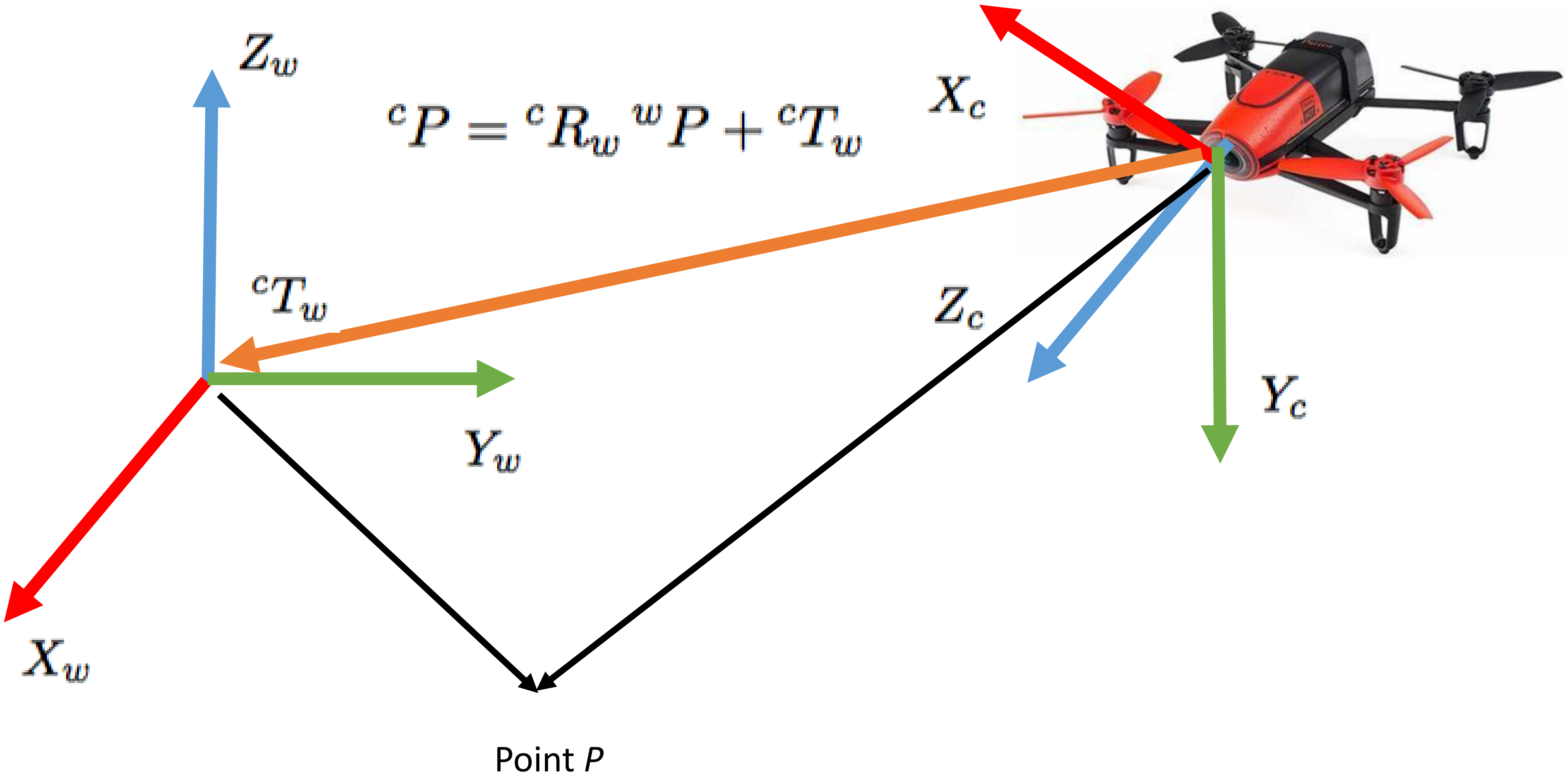
What is the geometric meaning of the rotation ${}^{c}R_{w}$ and the translation ${}^{c}T_{w}$?



What is the geometric meaning of the translation ${}^cT_w$?

This is easy to see if we set ${}^wP$ to zero.

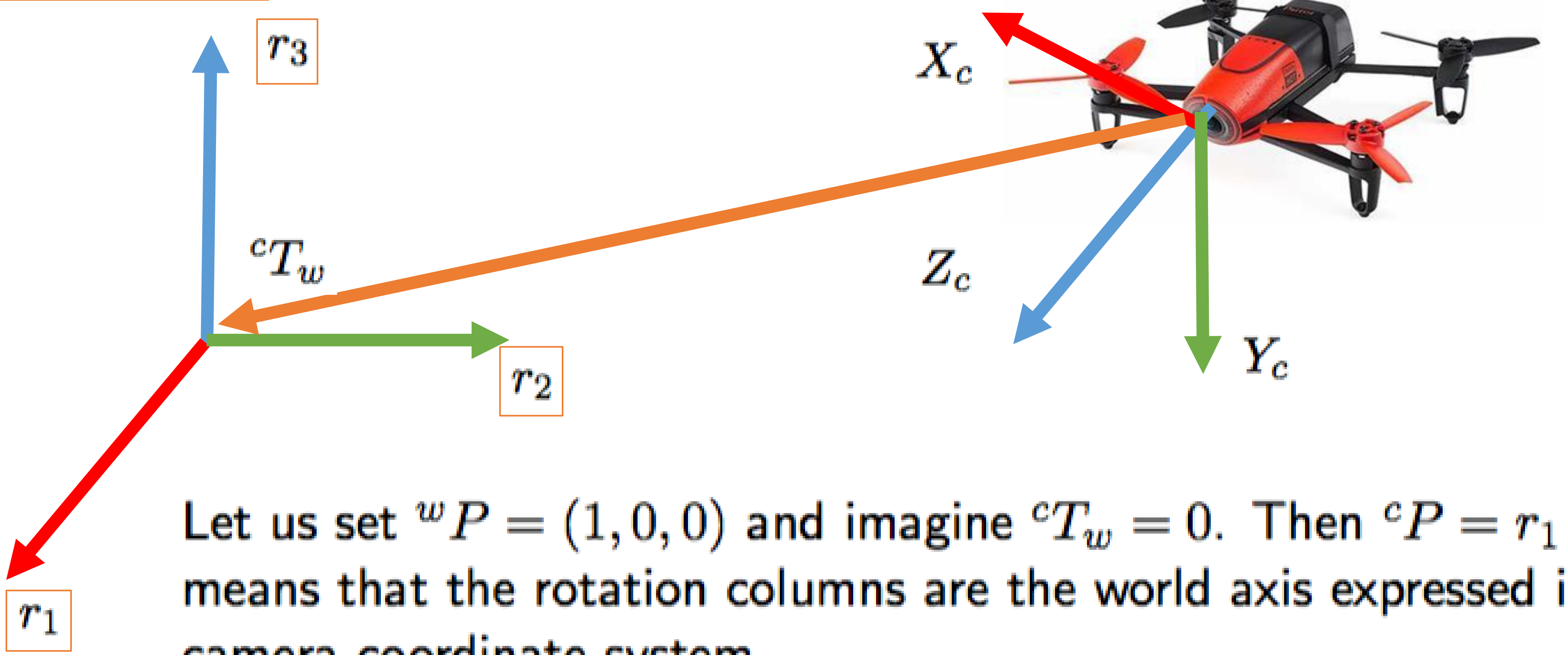
Then, ${}^cP = {}^cR_w 0 + {}^cT_w$ is the vector from camera origin to world origin:



What is the geometric meaning of the rotation ${}^{c}R_{w}$?
Let the rotation matrix be written as 3 orthogonal column vectors:

$$\boxed{{}^{c}R_{w} = \begin{pmatrix} r_1 & r_2 & r_3 \end{pmatrix}}$$

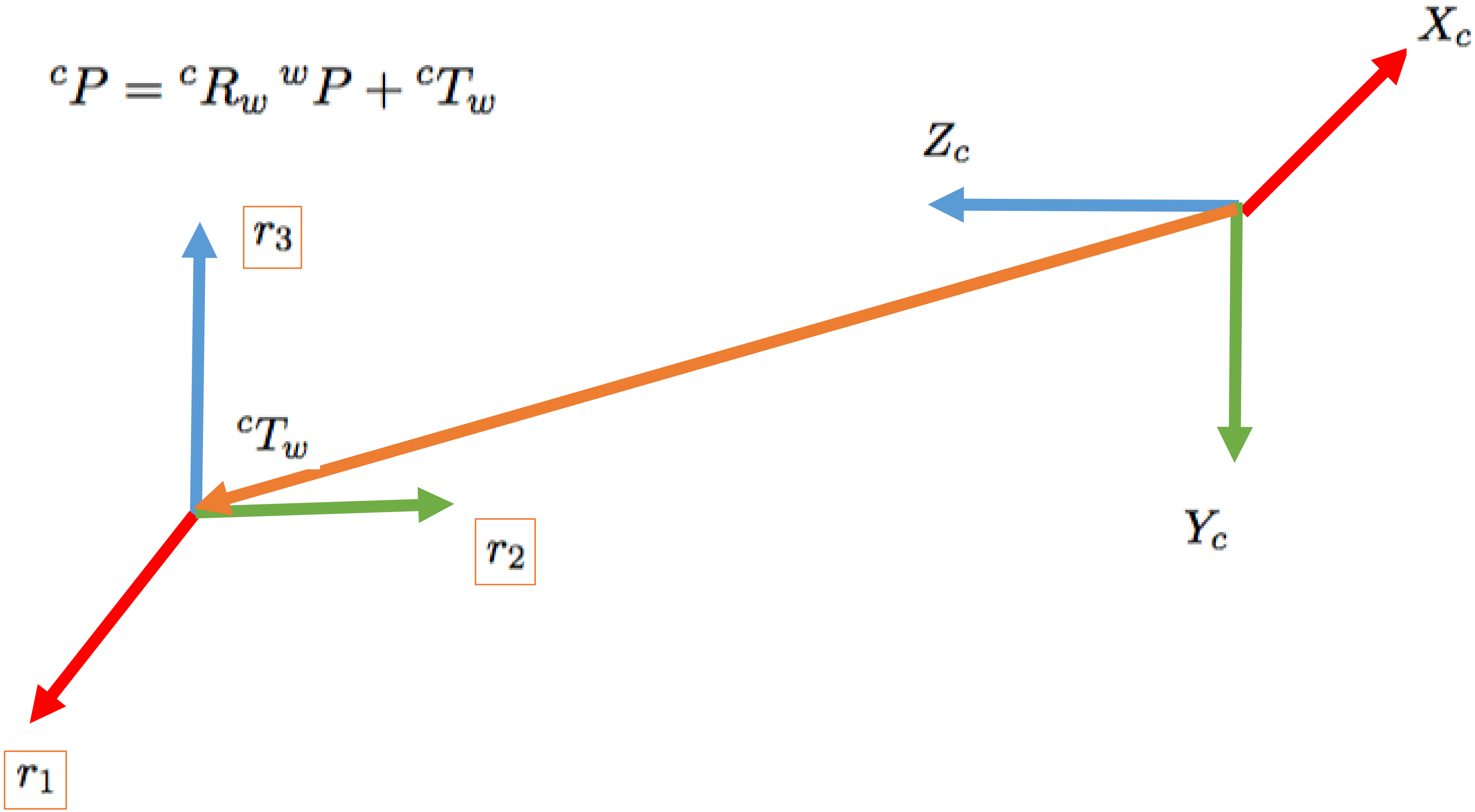
$${}^{c}P = {}^{c}R_{w}\,{}^{w}P + {}^{c}T_{w}$$



Let us set ${}^{w}P = (1, 0, 0)$ and imagine ${}^{c}T_{w} = 0$. Then ${}^{c}P = r_1$ which means that the rotation columns are the world axis expressed in the camera coordinate system.

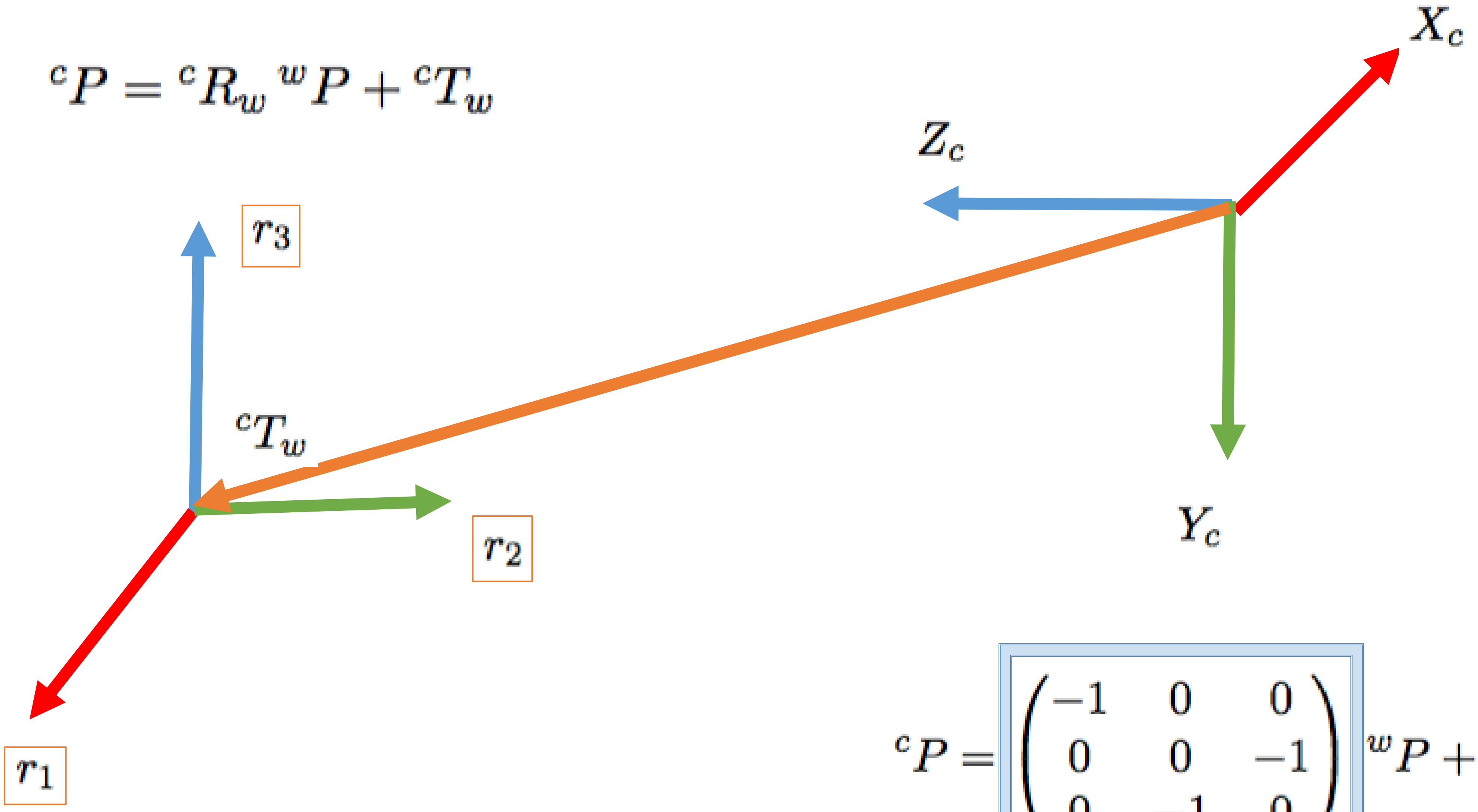
Let us look at the simple example:

$$ {}^{c}P = {}^{c}R_{w}\,{}^{w}P + {}^{c}T_{w} $$



How does the rotation matrix read?

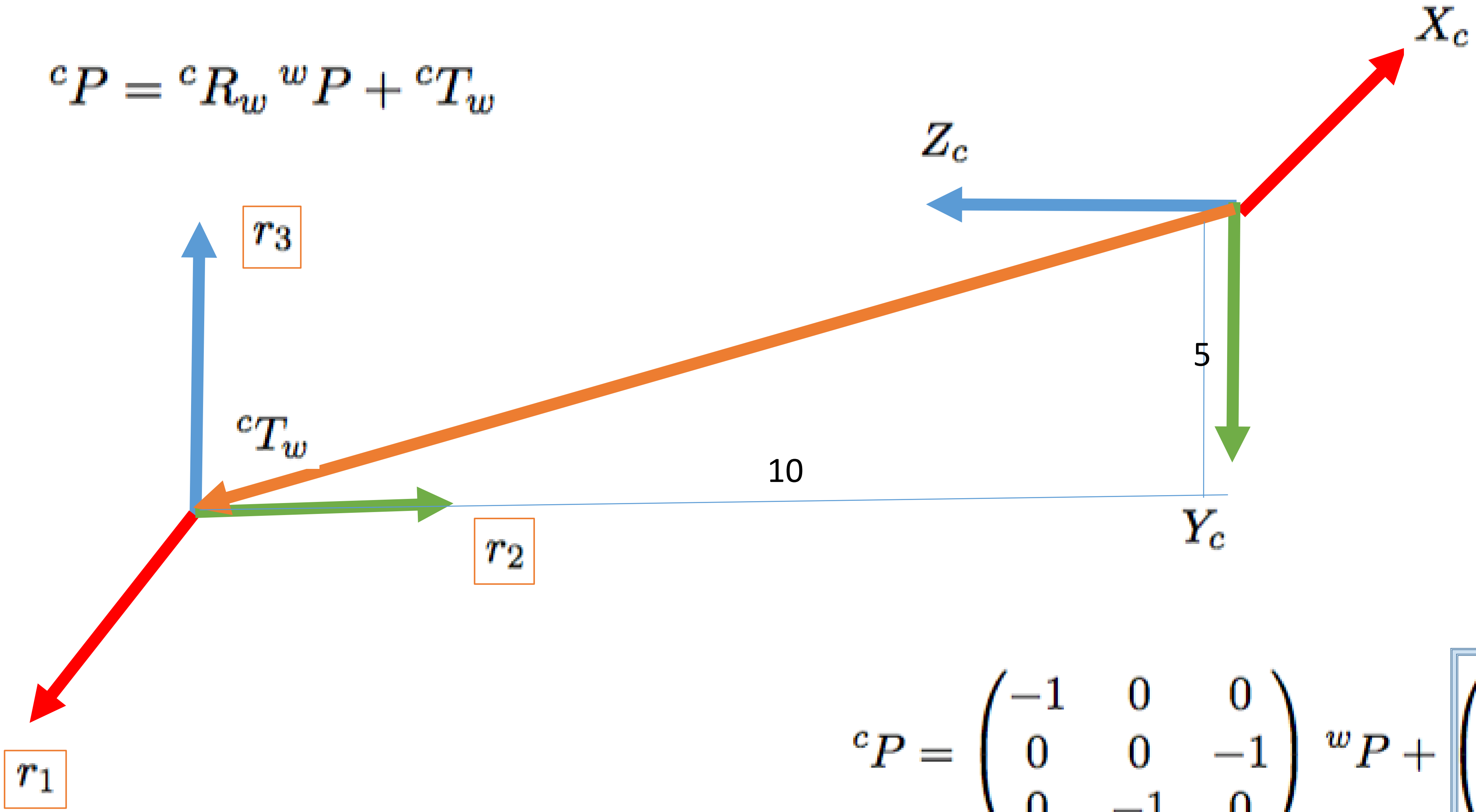
$$ {}^{c}P = {}^{c}R_{w}\, {}^{w}P + {}^{c}T_{w} $$



$$ {}^{c}P = \boxed{\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}}\, {}^{w}P + \begin{pmatrix} ? \\ ? \\ ? \end{pmatrix} $$

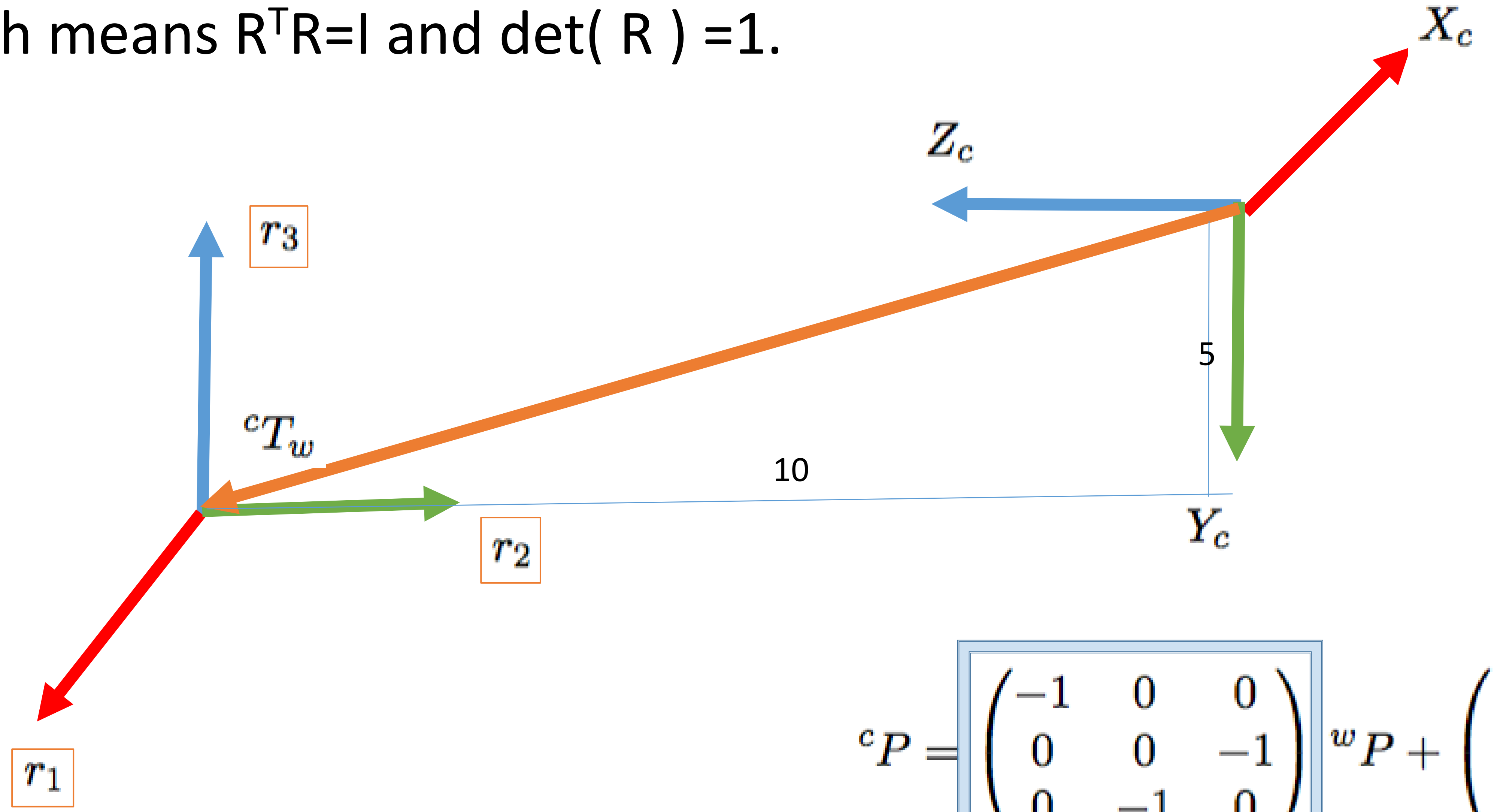
What about the translation:

$$^{c}P = {}^{c}R_w \, {}^{w}P + {}^{c}T_w$$



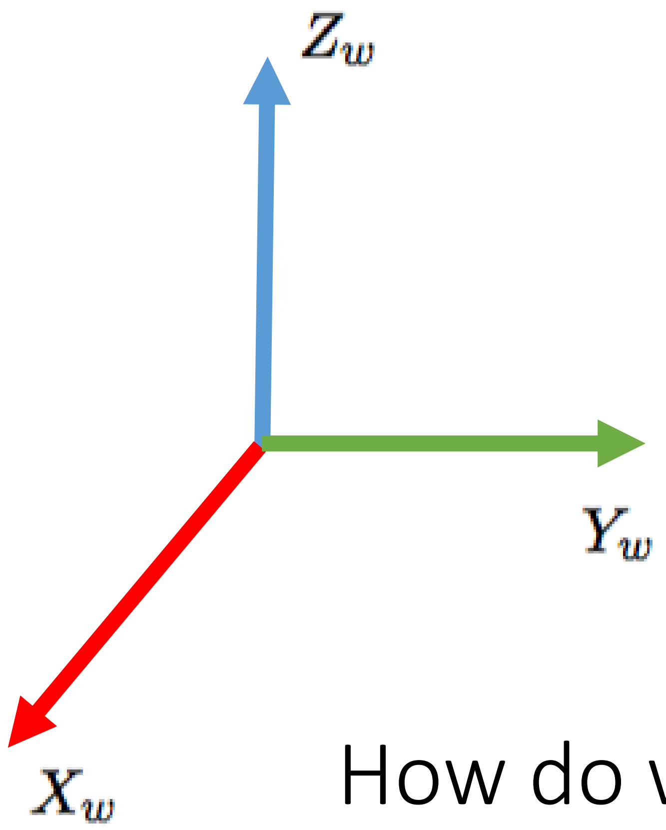
$$^{c}P = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix} {}^{w}P + \boxed{\begin{pmatrix} 0 \\ 5 \\ 10 \end{pmatrix}}$$

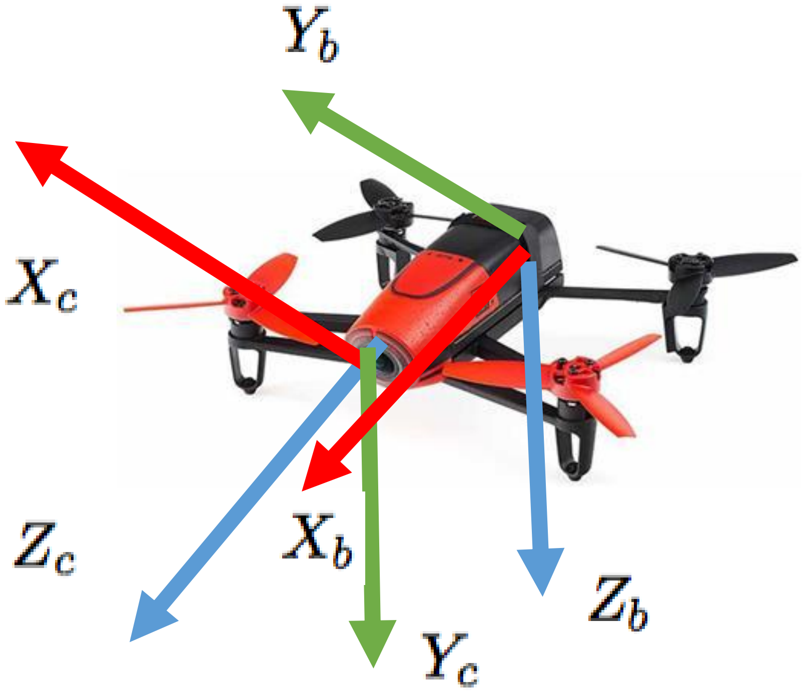
We have to make sure that the 3x3 matrix is a rotation matrix,
Which means $R^TR=I$ and $\det(R) =1$.



$$ {}^cP = \boxed{\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}} {}^wP + \begin{pmatrix} 0 \\ 5 \\ 10 \end{pmatrix} $$

Now imagine one more coordinate frame: a body frame with axes corresponding to roll (X_b), pitch (Y_b), yaw (Z_b) angles.

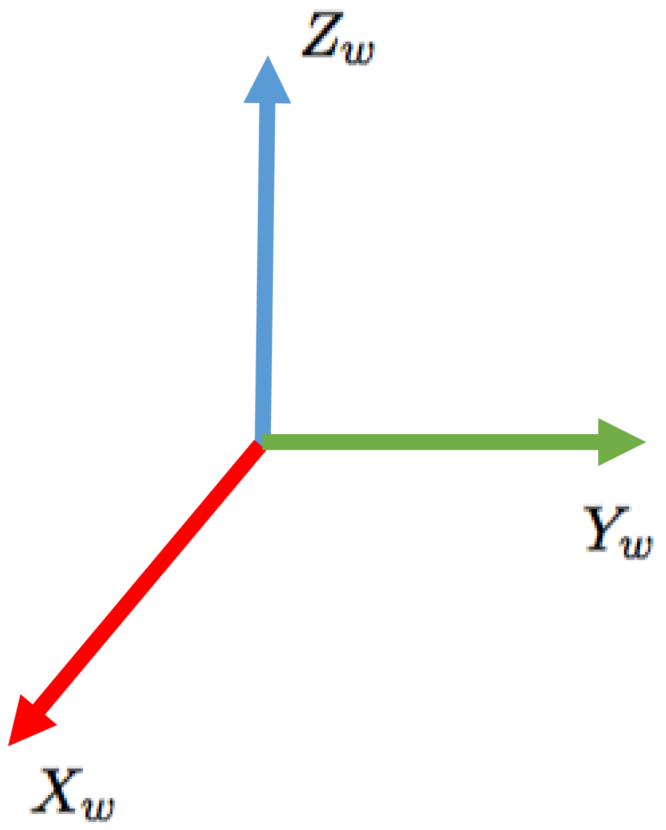


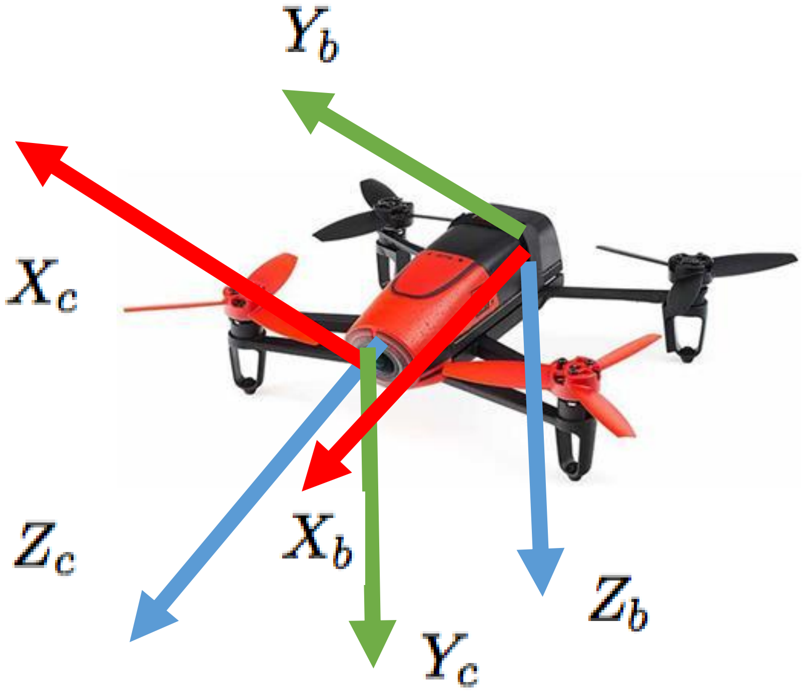


How do we compose transformations?

The easiest way to transform between coordinate systems is to use 4x4 matrices:

$$ {}^{c}M_{w} = \begin{pmatrix} & {}^{c}R_{w} & & {}^{c}T_{w} \\ 0 & 0 & 0 & 1 \end{pmatrix} $$



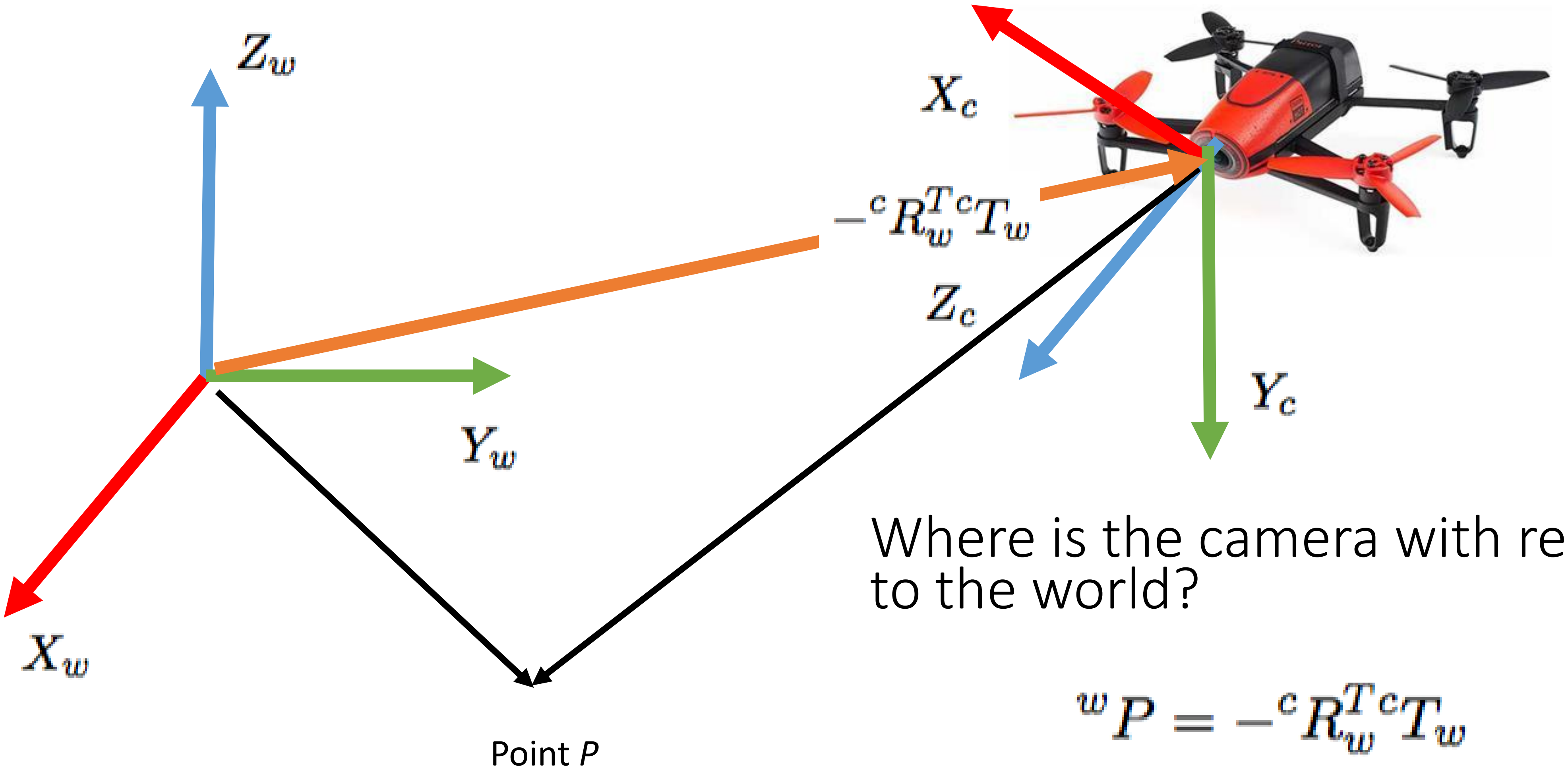


Then we just concatenate the 4x4 matrices

$$ {}^{w}M_{b} = {}^{w}M_{c}\, {}^{c}M_{b} $$

What about the inverse transformation?

$$ {}^{w}M_{c} = \begin{pmatrix} & {}^{c}R_{w}^{T} & & -{}^{c}R_{w}^{T}\,{}^{c}T_{w} \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

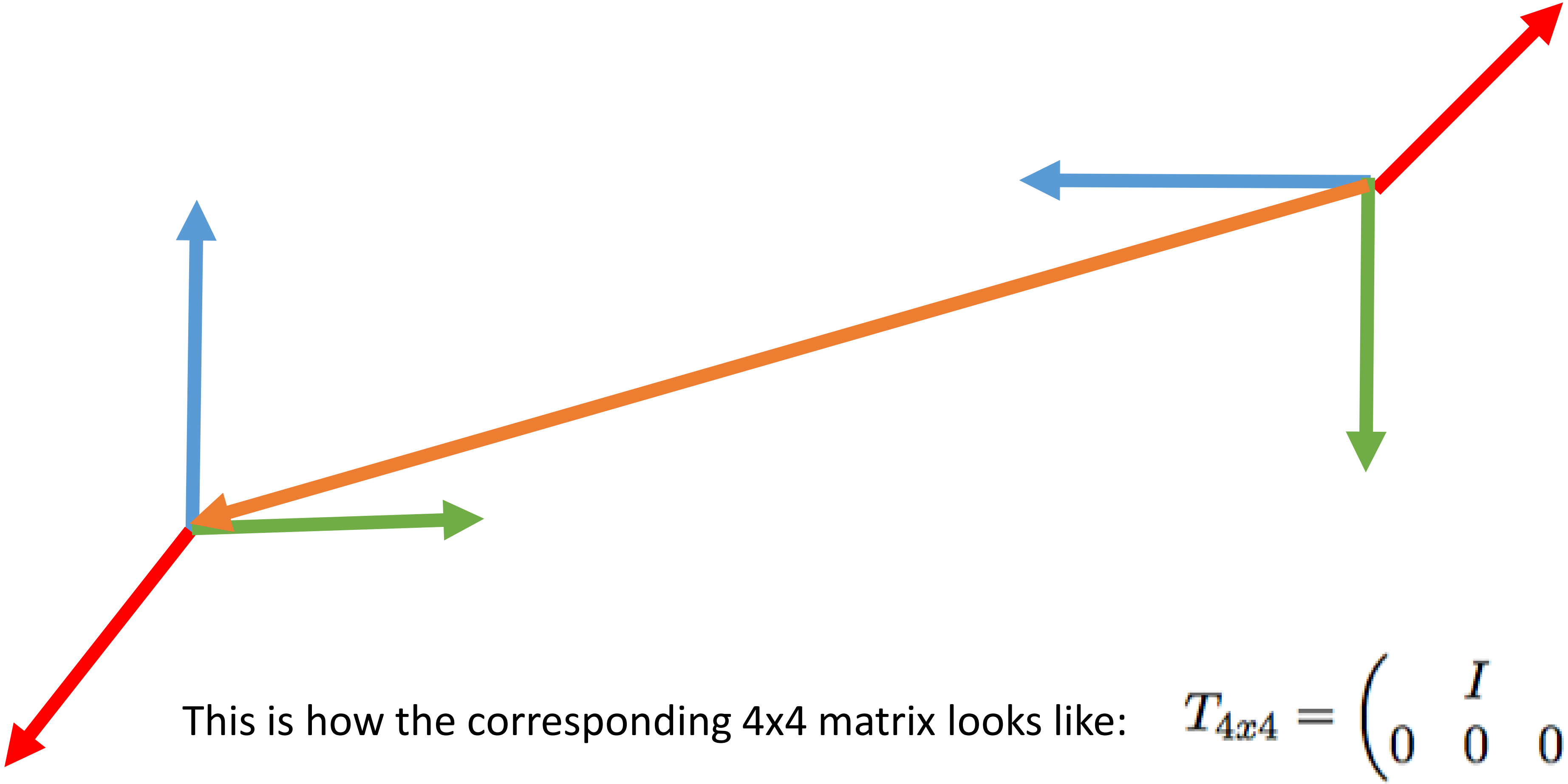


Where is the camera with respect to the world?

$$ {}^{w}P = -{}^{c}R_{w}^{T}\,{}^{c}T_{w} $$

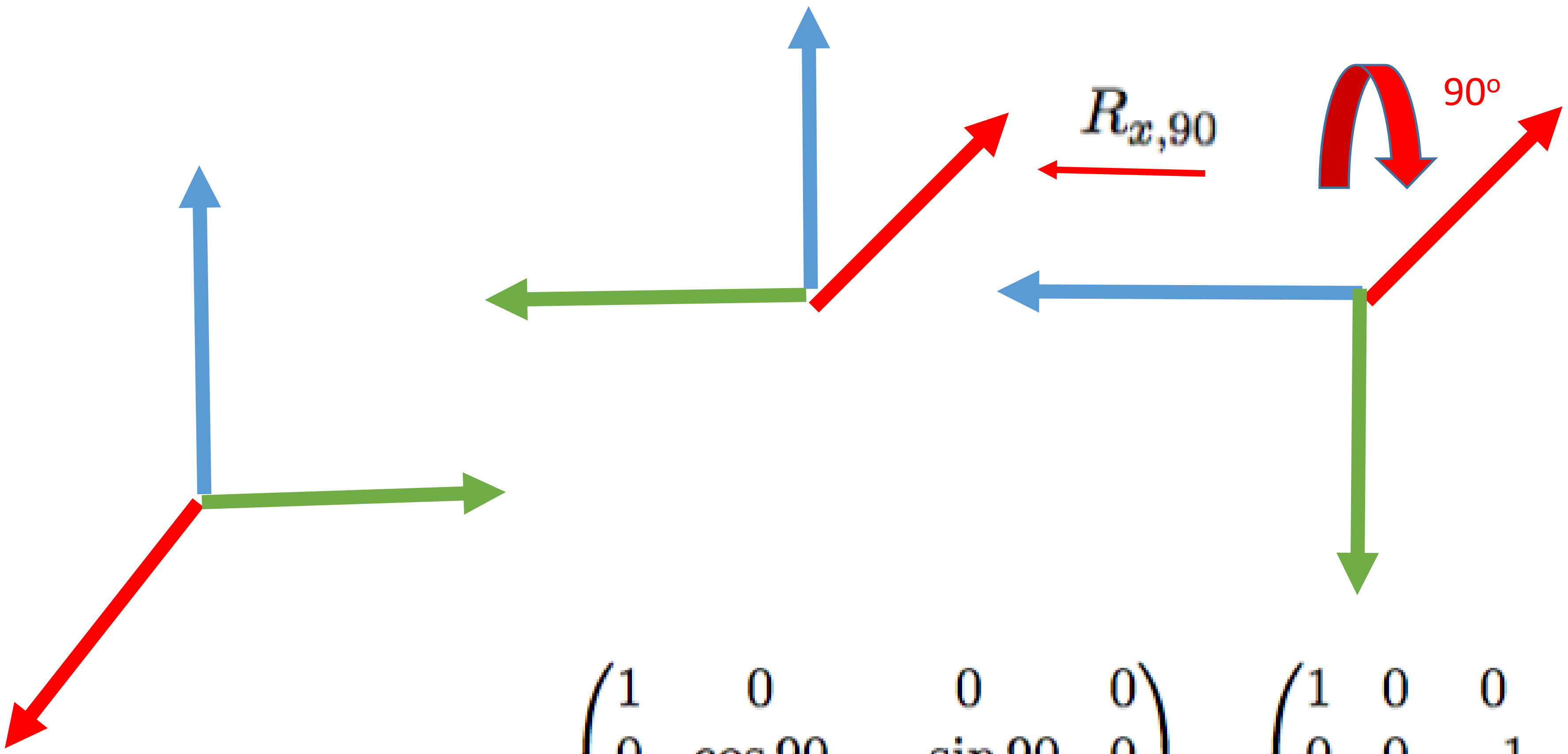
Alternative interpretation as a sequence of motions:

1. The camera frame first translates to the world



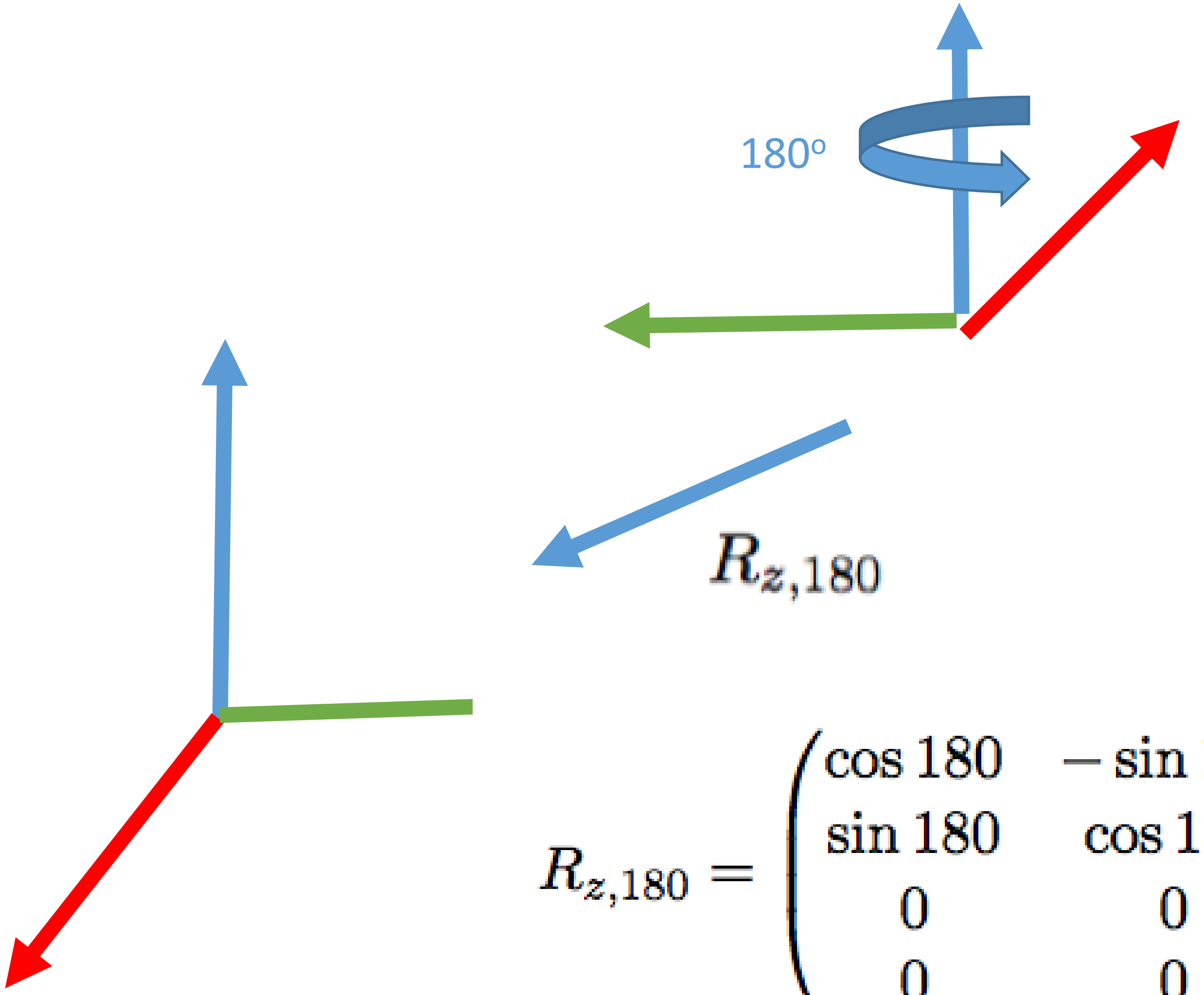
This is how the corresponding 4x4 matrix looks like: $T_{4x4} = \begin{pmatrix} & I & & {}^{c}T_{w} \\ 0 & 0 & 0 & 1 \end{pmatrix}$

2. The camera frame rotates 90 degrees around x



$$R_{x,90} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90 & -\sin 90 & 0 \\ 0 & \sin 90 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

A3. The camera frame rotates 180 degrees around z



$$R_{z,180} = \begin{pmatrix} \cos 180 & -\sin 180 & 0 & 0 \\ \sin 180 & \cos 180 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

How do we compose these motions? Golden rule: when we move coordinate frames and we refer to the most recent coordinate frame
we always postmultiply!

$$
\begin{aligned}
{}^{c}M_{w} &= T R_{x,90} R_{z,180} \\
&= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 5 \\ 0 & -1 & 0 & 10 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}
$$