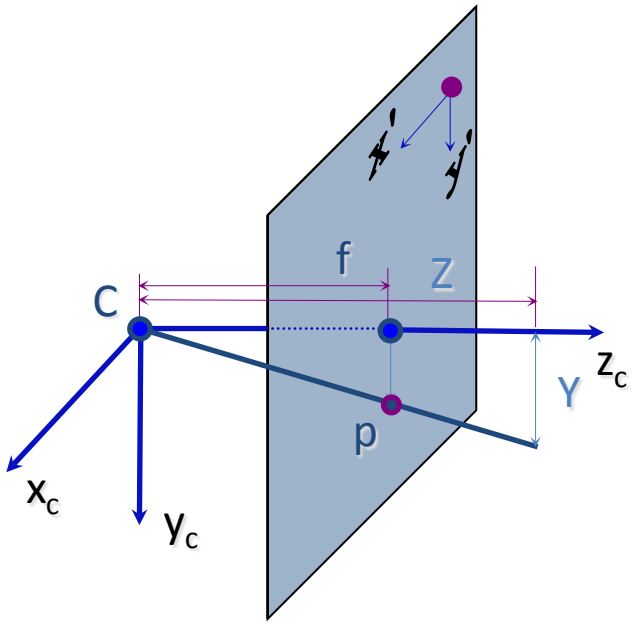
1st Person Camera world



3D to 2D image: $x' = f\frac{X}{Z}$ $y' = f\frac{Y}{Z}$

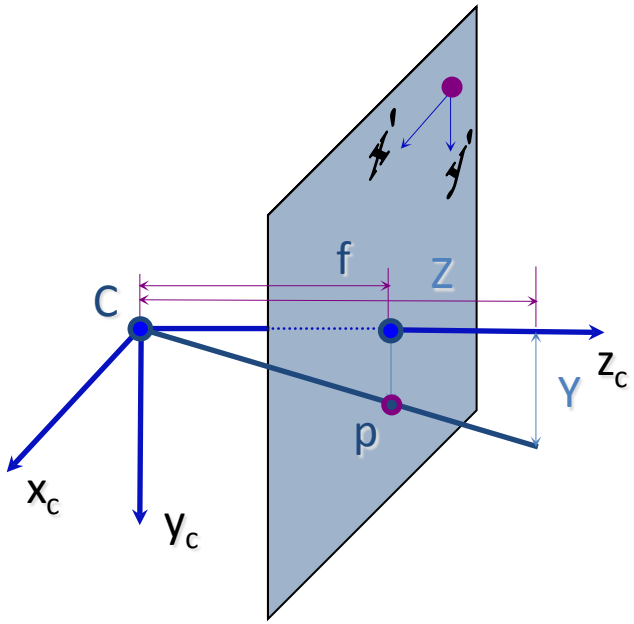
Projection equation:

Zx' = f X
Zy' = f Y
Z = Z

Step 1: Camera projection matrix

$$Zx' = fX$$
$$Zy' = fY$$
$$Z = Z$$

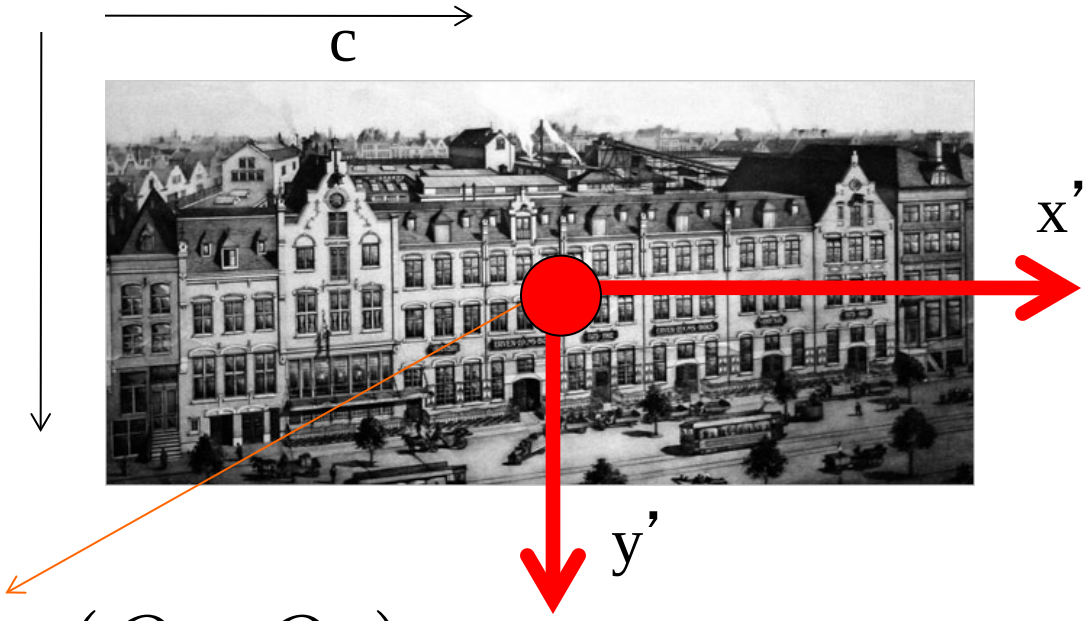
$$z_c \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \left[\begin{array}{ccc|c} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ \hline 1 \end{bmatrix}$$



$$\mathbf{x} = \mathbf{P}_0 \mathbf{X}$$

Conversion from mm to pixels

2D image to 2D pixel:



Optical Center= (O_c, O_r)

$$(c - O_c) = \frac{x'}{s_x} \qquad (r - O_r) = \frac{y'}{s_y}$$

$s_x =$ size of pixel width $\qquad s_y =$ size of pixel height

Step 2: Intrinsic camera parameters: map <u>camera</u> coordinate to *<u>pixel</u>* coordinate

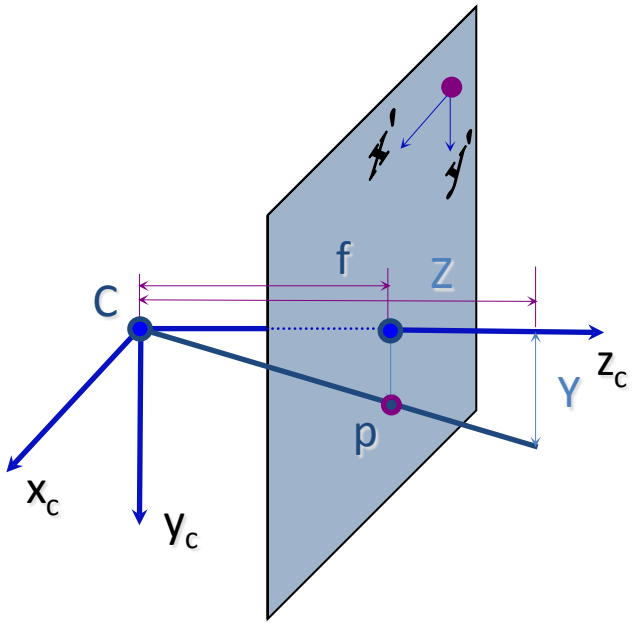
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \alpha_x & s & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

Pixel world

K

(3x3 submatrix)

Optical world



α_x, α_y is pixel scaling factor

p_x, p_y is the principle point (where optical axis hits image plane)

s is the slant factor, when the image plane is not normal to the optical axis

Combine the Intrinsic camera parameters

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \alpha_x & s & p_x \\ 0 & \alpha_y & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc|c} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \left[\begin{array}{ccc|c} \alpha_x f & sf & p_x & 0 \\ 0 & \alpha_y f & p_y & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{bmatrix} x_c \\ y_c \\ z_c \\ 1 \end{bmatrix}$$

K

(Calibration matrix)

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \alpha_x f & sf & p_x \\ 0 & \alpha_y f & p_y \\ 0 & 0 & 1 \end{bmatrix} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{bmatrix} X_c \\ Y_c \\ Z_c \\ 1 \end{bmatrix}$$

1st Person Camera projection

$$Z\begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{0} \end{bmatrix} \begin{bmatrix} X_{\text{cam}} \\ Y_{\text{cam}} \\ Z_{\text{cam}} \\ 1 \end{bmatrix}$$

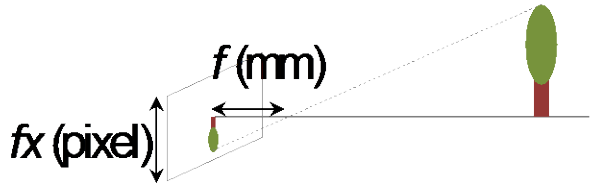
2D pixel in 1st person view

3D World in 1st person view

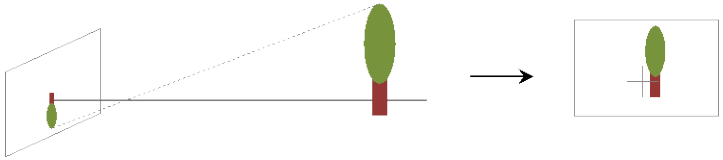
guration (internal parameter)

$$z\begin{bmatrix} u_{\text{img}} \\ v_{\text{img}} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{3\times 3} & \mathbf{0} \end{bmatrix} \begin{bmatrix} X_{\text{cam}} \\ Y_{\text{cam}} \\ Z_{\text{cam}} \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & s & p_x \\ & f_y & p_y \\ & & 1 \end{bmatrix} \begin{bmatrix} X_{\text{cam}} \\ Y_{\text{cam}} \\ Z_{\text{cam}} \end{bmatrix}$$

- A scale factor that converts physical focal length to pixel unit, i.e., f (mm) → f_x (pixel).



- Position of image center (principal point), i.e., p_x, p_y



- A skew factor between x and y axis of the image, i.e., $u_{img} = f_x x + sy + p_x$.