Central Washington University

CS471 Optimization

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Genetic Algorithm, Differential Evolution, Particle Swarm Optimization, Sine Cosine Algorithm, Bat Algorithm, Directional Bat Algorithm, Ant Lion Optimizer

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1 Introduction

For this project, many population-based optimization algorithms will be tested. These are Genetic Algorithm (GA) and Differential Evolution Algorithm (DE), Particle Swarm Optimization (PSO) and Sine Cosine Algorithm (SCA), Bat Algorithm(BA), Directional Bat Algorithm (DBA) and the Ant Lion Optimizer (ALO).

1.1 Genetic Algorithm

The GA is a heuristic search and optimization technique that simulates the process of natural evolution. The main operations of GA are Selection, Crossover, Mutation, and Elitism.

1.2 Differential Evolution

The DE algorithm employs the difference of two randomly selected parameter vectors as the source fo random variations for a third parameter vector. Some advantages of DE are: few numbers of control parameters which make it easier to calibrate or tune, it is inherently parallel, and it has a faster convergence. There are many strategies of DE, in this project 10 different strategies will be tested.

1.3 Particle Swarm Optimization

The PSO is inspired by the flocking and schooling patterns of birds and fish, the key point of PSO is that it keeps tracking the personal best and global best variables, and make the particles move toward those best values.

1.4 Sine Cosine Algorithm

The SCA created in 2015 is very similar to PSO. However, it only keeps track of only the global optimal value, and in each iteration, it introduces sine and cosine formula which multiply with a decreasing range value.

1.5 Bat Algorithm

The BA proposed by Yang in 2010 is based on swarm intelligence algorithm, it is inspired by the echolocation behavior of microbats, as all the population-based algorithms the BA tries to find the optimal value by exploration and exploitation. At the beginning of the iterations, the algorithm focuses more on exploration by selecting in each iteration one of the best solutions (local minimums) randomly, then it adds a random walk step to it when this is better than the global best it will be accepted as a new solution. At the end part of the iteration the algorithm focus more in exploitation by moving the bats toward the best global best.

1.6 Directional Bat Algorithm

The DBA proposed by Chakri in 2016 is based on BA, it claims to be an improved version of BA adding 4 modifications that enhance the exploitation and exploration. The 1st modification consists of directional echolocation which makes the bat move in 2 combined directions (global best and random selected) instead of only one direction to the global best. The 2nd modification consists in decrease the search space through the iterations. The 3rd modification changed the formula about how the pulse rate and loudness increase and increase respectively. The 4th modification makes the algorithm to accept the new solution more often (increase exploration) and update the global best whenever the bat's random walk produces a better solution than the global optimal value.

1.7 Ant Lion Optimizer

The ALO algorithm simulates the population of antlions and the population of ants together, it mimics the interaction between antlions and ants in the trap. First, the antlion population is initialized randomly, then the elite antlion is located based on the fitness (cost), then each ant select an antlion using roulette wheel and random walk around it, also the same ant random walk around the elite ant, after that the new position of ant is calculated based on the two random walks, at the end, the antlions will update their positions to the best fitness ants position, and the elite antlion is also updated.

2 Method

The GA, DE, PSO, SCA, BA, DBA, ALO algorithms are coded using C++ object-oriented programming. The implemented classes are Population, PopulationBenchmark, Functions, Runner, GeneticAlgorithm, DifferentialEvolution, ParticleSwarm, SineCosine, BatAlgorithm, DirectionalBatAlgorithm, AntLionOptimizer class. Additionally, python script with jupyter notebook is implemented to read the configuration parameters and call the C++ executable, then it collects the result, displays it in table and figures, and prints the output in latex format.

The obtained results are from **50** runs of each algorithm with the population size equal to **200**, and the generations or iterations equal to **100**, the number of dimensions is set to 30.

The computer used to run the project has the following specification: Intel Core i7-9750H 2.6GHZ up to 4.5GHZ with 16 GB of RAM

2.1 Configuration Parameters

All evolution-based algorithms are very defendant of its parameters. To ensure that the optimization algorithm works correctly, the parameters had to be calibrated manually, the following tables show the result of the calibration:

The calibrated GA parameters are:

crossover rate	mutation rate	mutation value range	mutation precision	elitism rate
0.9	0.05	0.1	1	0.3

Table 1: Configuration parameters of GA obtained by manual tuning

The calibrated DE parameters are:

DE strategies	crossover rate	scaling factor F	scaling factor lambda
DE_best_1_exp	0.8	0.3	-
DE_rand_1_exp	0.9	0.1	-
DE_randbest_1_exp	0.9	0.4	0.4
DE_best_2_exp	0.9	0.25	-
DE_rand_2_exp	0.9	0.05	-
DE_best_1_bin	0.6	0.5	-
DE_rand_1_bin	0.8	0.2	-
DE_randbest_1_bin	0.7	0.5	0.5
DE_best_2_bin	0.8	0.4	-
DE_rand_2_bin	0.7	0.1	-

Table 2: Configuration parameters of DE obtained by manual tuning

The calibrated PSO parameters are:

c1	c2	w
1	1	0.7

Table 3: Configuration parameters of PSO obtained by manual tuning

SCA parameters from the research paper are:

a	r3
2	2

Table 4: Configuration parameters of SCA from the research paper

The calibrated BA parameters are:

r0	A0 min	A0 max (exploration)	alfa	gamma	f min	f max (exploitation)	ratio best
0.1	1	11	0.7	0.9	0	2	0.2

Table 5: Configuration parameters of BA obtained by manual tuning, the A0 Max was increased to enhance the exploration

The calibrated DBA parameters are:

r0	rinf	A0	Ainf	fmin	fmax
0.1	0.7	0.9	0.6	0	2

Table 6: Configuration parameters of DBA obtained by manual tuning

The ALO algorithm does not have any configuration parameter.

3 Results

The results of applying different optimization algorithms with 18 benchmarking functions are displayed in the following pages:

3.1 Function 1: Schwefel

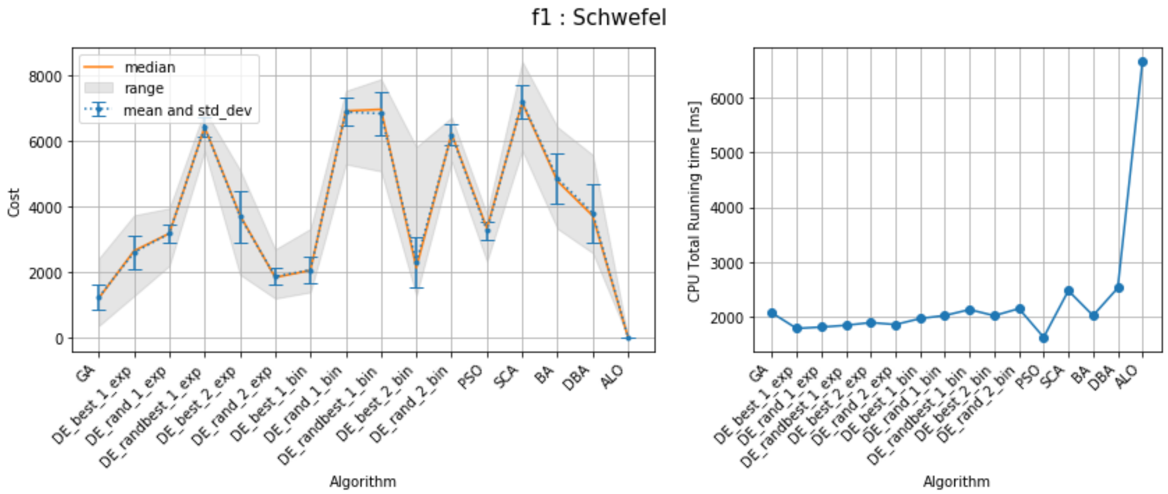

Figure 1: Cost and CPU total running time of Function 1: Schwefel

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	1227.950	376.908	1193.970	349.859	2415.230	2072.570
DE_best_1_exp	2594.260	506.957	2644.100	1266.590	3732.340	1790.220
DE_rand_1_exp	3176.050	273.965	3175.680	2189.160	3947.180	1815.360
DE_randbest_1_exp	6421.590	300.963	6427.380	5696.810	6941.060	1847.510
DE_best_2_exp	3698.620	785.839	3736.930	1918.250	5127.440	1897.470
DE_rand_2_exp	1861.060	262.467	1831.280	1188.600	2691.680	1861.120
DE_best_1_bin	2061.100	395.574	2047.320	1380.740	3321.640	1970.680
DE_rand_1_bin	6891.470	427.318	6922.800	5294.120	7537.300	2026.100
DE_randbest_1_bin	6841.440	652.654	6970.580	5078.730	7899.160	2133.580
DE_best_2_bin	2294.620	751.674	2123.900	1274.250	5827.550	2022.660
DE_rand_2_bin	6191.390	308.229	6226.270	5379.710	6725.530	2151.730
PSO	3266.090	284.030	3331.530	2320.860	3771.680	1629.050
SCA	7207.630	500.074	7175.650	5709.280	8434.870	2478.020
BA	4858.750	777.198	4775.090	3320.900	6432.790	2025.140
DBA	3800.850	897.100	3716.040	2578.980	5579.770	2527.020
ALO	0.044	0.099	0.009	0.000	0.547	6652.570

Table 7: Function 1: Statistical Analysis of the Cost

Best Algorithm:
ALO, Cost (mean): 0.043685

3.2 Function 2: De Jong 1

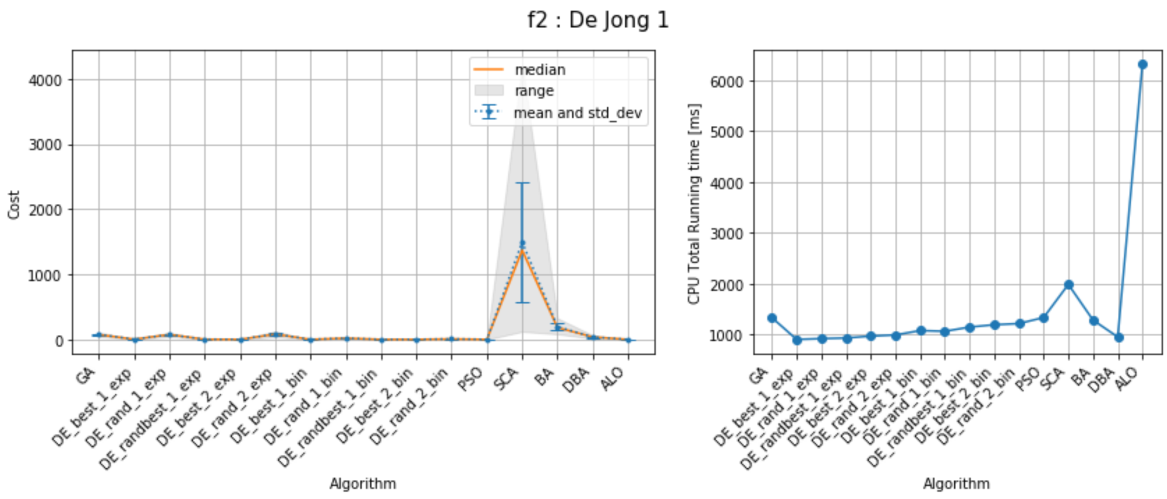

Figure 2: Cost and CPU total running time of Function 2: De Jong 1

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	76.080	15.059	73.447	40.136	108.911	1328.930
DE_best_1_exp	3.423	1.641	2.907	1.441	11.133	900.488
DE_rand_1_exp	73.259	15.934	75.316	40.990	102.826	917.260
DE_randbest_1_exp	2.555	0.998	2.322	1.046	5.322	929.018
DE_best_2_exp	0.452	0.562	0.245	0.016	2.981	967.223
DE_rand_2_exp	83.211	20.792	80.720	33.725	126.054	989.034
DE_best_1_bin	0.003	0.002	0.002	0.001	0.009	1075.570
DE_rand_1_bin	19.215	3.416	19.479	12.869	27.767	1061.240
DE_randbest_1_bin	0.002	0.001	0.001	0.001	0.005	1143.720
DE_best_2_bin	0.007	0.004	0.007	0.002	0.019	1190.380
DE_rand_2_bin	8.040	1.648	7.548	5.268	13.677	1213.190
PSO	0.001	0.001	0.000	0.000	0.004	1332.550
SCA	1495.250	923.455	1369.840	121.011	4229.320	1984.370
BA	196.477	59.316	182.925	82.012	326.640	1281.490
DBA	36.711	11.181	38.088	16.488	64.476	949.907
ALO	0.008	0.018	0.003	0.000	0.120	6326.930

Table 8: Function 2: Statistical Analysis of the Cost

Best Algorithm:
PSO, Cost (mean): 0.000594

3.3 Function 3: Rosenbrock's Saddle

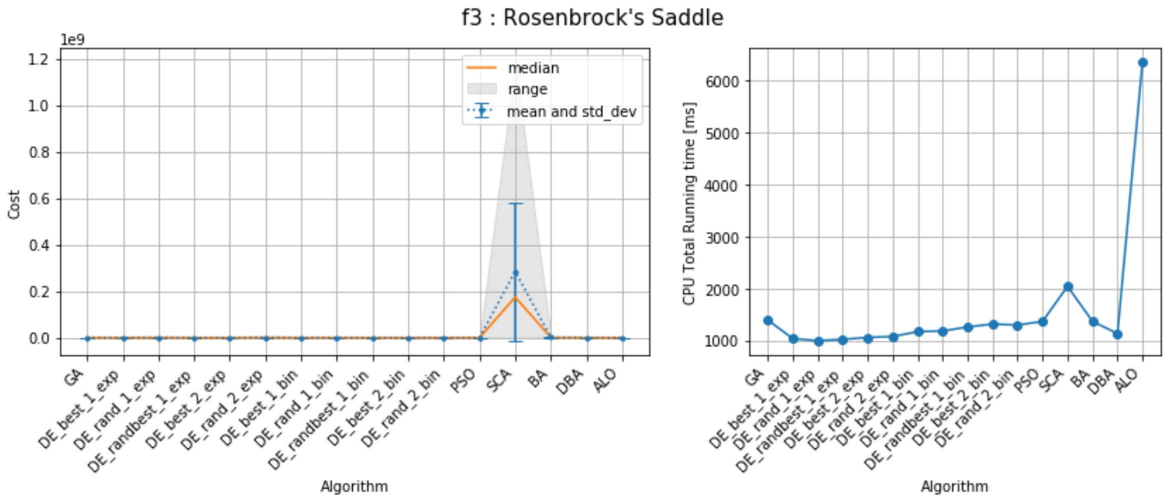

Figure 3: Cost and CPU total running time of Function 3: Rosenbrock's Saddle

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	327876.000	193268.000	273707.000	39384.100	826084.000	1394.160
DE_best_1_exp	13272.100	18675.300	6240.760	812.307	105860.000	1038.630
DE_rand_1_exp	587827.000	244150.000	577216.000	193550.000	1213960.000	998.366
DE_randbest_1_exp	1902.470	1779.390	1192.520	311.325	7486.780	1023.540
DE_best_2_exp	3945.820	5922.150	1148.390	229.942	28348.400	1060.810
DE_rand_2_exp	838744.000	464531.000	809726.000	160200.000	3414370.000	1078.390
DE_best_1_bin	608.995	1877.080	145.437	29.976	10137.500	1171.100
DE_rand_1_bin	72522.400	45622.200	64170.500	32422.700	343226.000	1184.670
DE_randbest_1_bin	625.385	2338.530	102.556	23.043	16303.700	1261.130
DE_best_2_bin	751.848	1690.870	156.577	30.614	8310.250	1318.590
DE_rand_2_bin	34467.400	13728.600	35458.700	12362.800	64474.200	1298.300
PSO	20887.400	139901.000	85.231	24.667	1000030.000	1368.990
SCA	284742000.000	297691000.000	174375000.000	5325270.000	1185420000.000	2042.450
BA	1168380.000	695395.000	915598.000	334772.000	3431950.000	1370.260
DBA	48239.000	68139.000	26502.000	1280.890	400122.000	1134.390
ALO	2.579	3.368	1.530	0.000	14.067	6349.710

Table 9: Function 3: Statistical Analysis of the Cost

Best Algorithm:
ALO, Cost (mean): 2.579130
Two-Sample Z-Test Hypothesis Testing: confidence interval = 95%

Null hypothesis: The best algorithm and the tested one are equal
DE_randbest_1_bin, Cost (mean): 625.385000 , P value: 0.059700
PSO, Cost (mean): 20887.400000 , P value: 0.291200

3.4 Function 4: Rastrigin

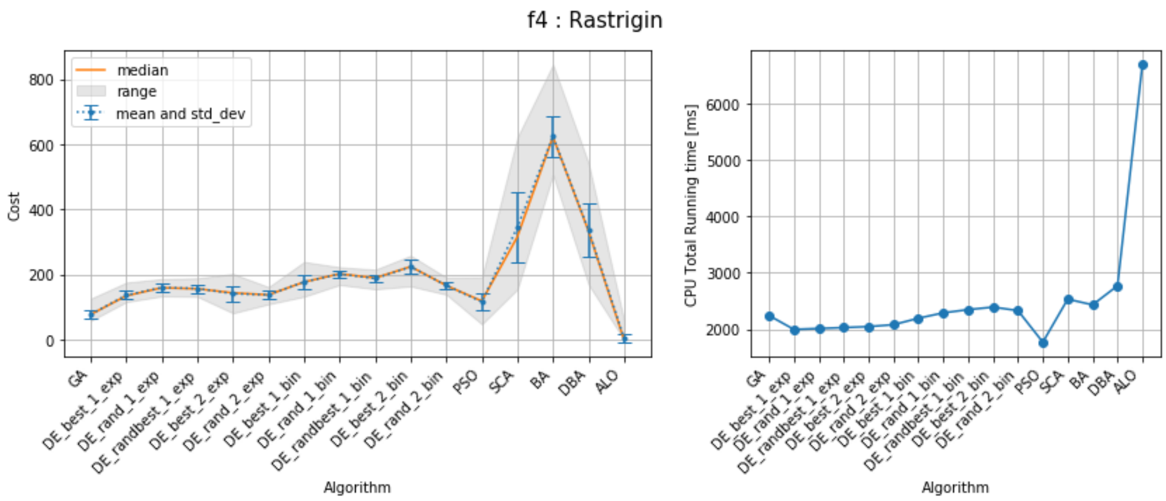

Figure 4: Cost and CPU total running time of Function 4: Rastrigin

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	78.319	12.938	77.636	58.474	126.663	2240.370
DE_best_1_exp	136.788	12.600	135.141	116.298	176.007	1994.340
DE_rand_1_exp	160.392	12.252	159.635	133.816	186.928	2011.130
DE_randbest_1_exp	157.514	13.193	156.599	132.396	189.072	2026.750
DE_best_2_exp	141.617	22.523	143.871	81.862	202.623	2044.490
DE_rand_2_exp	138.059	13.107	137.958	109.360	162.163	2078.800
DE_best_1_bin	177.708	19.888	177.356	132.649	239.429	2193.280
DE_rand_1_bin	201.957	11.553	202.602	168.646	223.115	2288.740
DE_randbest_1_bin	188.274	12.445	189.753	155.486	215.544	2345.570
DE_best_2_bin	223.900	20.874	224.576	164.271	258.584	2390.460
DE_rand_2_bin	166.760	10.923	166.589	138.901	191.543	2328.860
PSO	116.462	26.355	118.929	47.772	192.216	1768.780
SCA	345.389	108.363	316.188	155.007	622.249	2531.900
BA	624.340	63.733	623.268	506.487	845.286	2430.460
DBA	337.212	81.701	334.151	169.461	542.711	2768.970
ALO	5.612	12.956	0.006	0.000	60.127	6693.500

Table 10: Function 4: Statistical Analysis of the Cost

Best Algorithm:
ALO, Cost (mean): 5.612020

3.5 Function 5: Griewangk

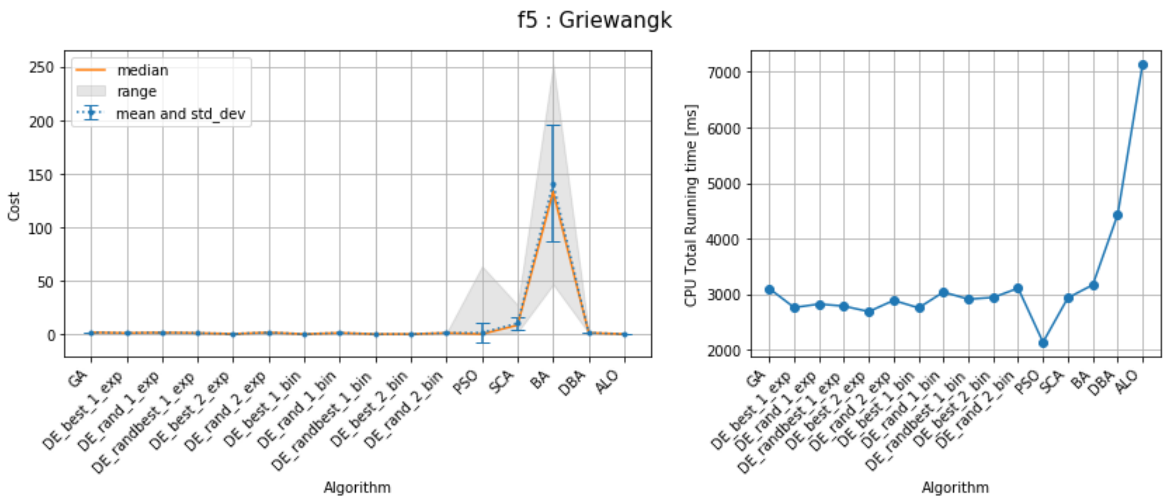

Figure 5: Cost and CPU total running time of Function 5: Griewangk

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	1.597	0.148	1.581	1.336	1.995	3101.980
DE_best_1_exp	0.993	0.036	1.003	0.894	1.042	2760.570
DE_rand_1_exp	1.471	0.115	1.459	1.243	1.896	2822.620
DE_randbest_1_exp	0.963	0.064	0.975	0.761	1.054	2785.770
DE_best_2_exp	0.243	0.178	0.213	0.033	0.849	2687.190
DE_rand_2_exp	1.496	0.137	1.497	1.226	1.911	2888.700
DE_best_1_bin	0.014	0.014	0.011	0.001	0.060	2753.980
DE_rand_1_bin	1.120	0.028	1.119	1.047	1.191	3038.580
DE_randbest_1_bin	0.016	0.018	0.012	0.001	0.109	2910.310
DE_best_2_bin	0.028	0.020	0.022	0.004	0.088	2941.890
DE_rand_2_bin	1.049	0.011	1.045	1.029	1.079	3107.410
PSO	1.283	8.874	0.011	0.000	63.397	2136.850
SCA	10.016	6.436	8.308	1.873	27.210	2934.730
BA	140.993	54.319	133.177	46.431	252.059	3167.760
DBA	1.216	0.078	1.210	1.091	1.380	4431.220
ALO	0.037	0.082	0.001	0.000	0.436	7133.400

Table 11: Function 5: Statistical Analysis of the Cost

Best Algorithm:
DE_best_1_bin, Cost (mean): 0.014306
Two-Sample Z-Test Hypothesis Testing: confidence interval = 95%

Null hypothesis: The best algorithm and the tested one are equal
DE_randbest_1_bin, Cost (mean): 0.015799 , P value: 0.649800
PSO, Cost (mean): 1.282560 , P value: 0.312200
ALO, Cost (mean): 0.036574 , P value: 0.058700

3.6 Function 6: Sine Envelope Sine Wave

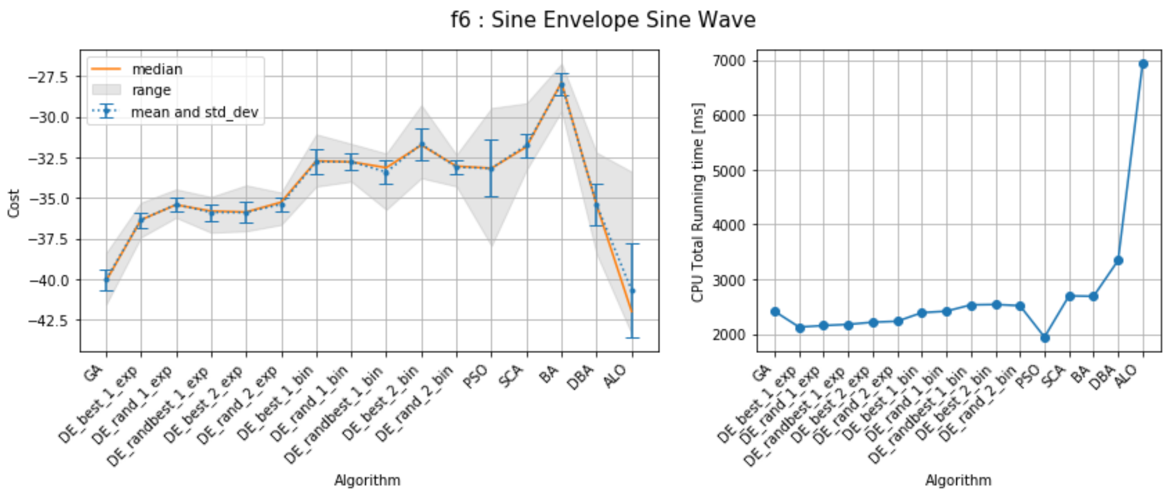

Figure 6: Cost and CPU total running time of Function 6: Sine Envelope Sine Wave

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	-40.040	0.632	-40.134	-41.591	-38.404	2414.860
DE_best_1_exp	-36.360	0.470	-36.356	-37.427	-35.285	2127.630
DE_rand_1_exp	-35.411	0.411	-35.423	-36.217	-34.466	2155.490
DE_randbest_1_exp	-35.890	0.521	-35.816	-37.136	-34.934	2176.320
DE_best_2_exp	-35.896	0.628	-35.873	-37.051	-34.218	2216.660
DE_rand_2_exp	-35.393	0.441	-35.268	-36.656	-34.665	2233.480
DE_best_1_bin	-32.782	0.754	-32.722	-34.297	-31.074	2390.570
DE_rand_1_bin	-32.770	0.508	-32.780	-33.986	-31.659	2417.020
DE_randbest_1_bin	-33.406	0.749	-33.138	-35.723	-32.257	2532.230
DE_best_2_bin	-31.684	1.004	-31.743	-33.778	-29.277	2539.310
DE_rand_2_bin	-33.105	0.463	-33.044	-34.285	-32.331	2518.100
PSO	-33.175	1.747	-33.173	-37.954	-29.456	1945.080
SCA	-31.745	0.716	-31.855	-33.273	-29.159	2696.900
BA	-27.969	0.697	-27.873	-29.657	-26.713	2688.380
DBA	-35.407	1.316	-35.426	-38.460	-32.202	3342.620
ALO	-40.692	2.889	-42.007	-43.378	-33.381	6934.210

Table 12: Function 6: Statistical Analysis of the Cost

Best Algorithm:
ALO, Cost (mean): -40.692200
Two-Sample Z-Test Hypothesis Testing: confidence interval = 95%

Null hypothesis: The best algorithm and the tested one are equal
GA, Cost (mean): -40.040300 , P value: 0.119100

3.7 Function 7: Stretch V Sine Wave

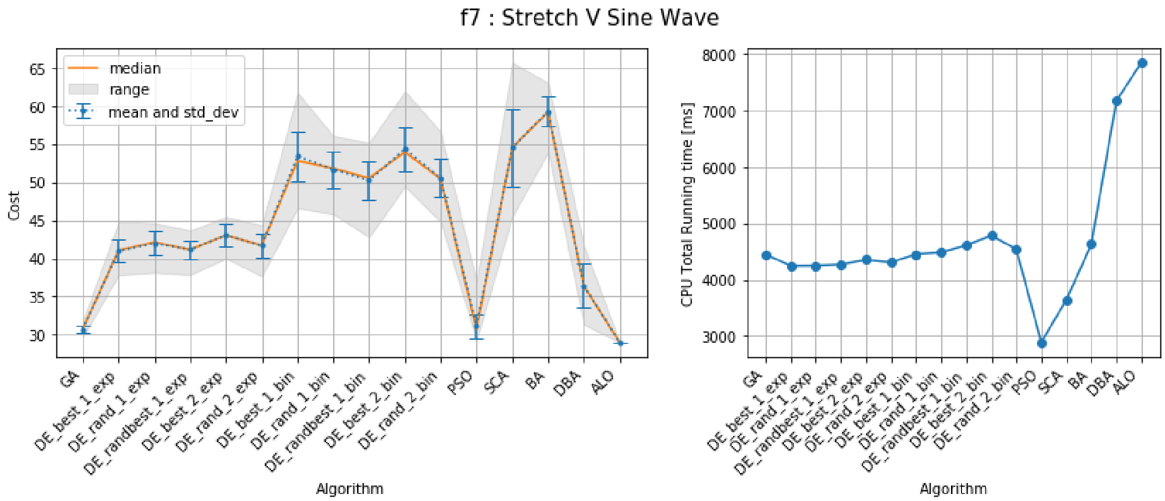

Figure 7: Cost and CPU total running time of Function 7: Stretch V Sine Wave

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	30.725	0.480	30.732	29.672	31.823	4445.970
DE_best_1_exp	40.988	1.459	41.054	37.850	44.852	4246.020
DE_rand_1_exp	42.010	1.621	42.124	38.145	44.662	4249.330
DE_randbest_1_exp	41.171	1.217	41.167	37.871	43.679	4268.700
DE_best_2_exp	43.031	1.491	43.035	40.008	45.454	4352.520
DE_rand_2_exp	41.704	1.596	41.722	37.668	44.371	4305.120
DE_best_1_bin	53.395	3.311	52.803	46.592	61.756	4450.770
DE_rand_1_bin	51.687	2.404	51.818	45.803	56.073	4481.830
DE_randbest_1_bin	50.244	2.544	50.535	42.806	55.193	4609.430
DE_best_2_bin	54.359	2.894	53.953	49.449	61.945	4779.410
DE_rand_2_bin	50.529	2.501	50.428	44.736	56.592	4544.370
PSO	31.156	1.613	30.718	29.252	36.986	2886.280
SCA	54.498	5.046	54.564	45.505	65.696	3640.540
BA	59.303	1.982	59.181	53.928	63.024	4628.420
DBA	36.421	2.899	36.396	31.316	41.690	7174.750
ALO	29.001	0.003	29.000	29.000	29.014	7853.840

Table 13: Function 7: Statistical Analysis of the Cost

Best Algorithm:
ALO, Cost (mean): 29.001000

3.8 Function 8: Ackley One

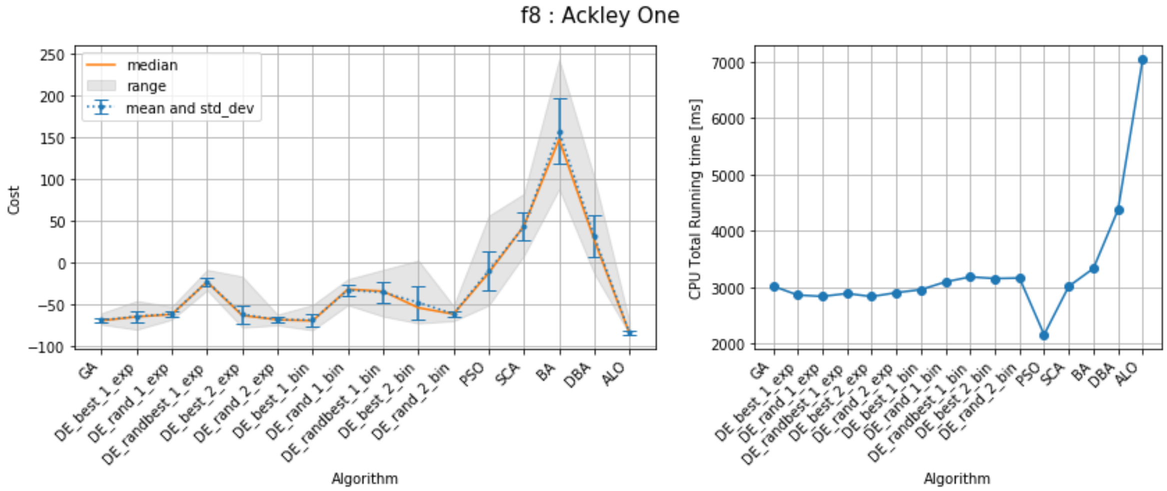

Figure 8: Cost and CPU total running time of Function 8: Ackley One

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	-68.310	2.866	-68.841	-73.409	-59.809	3023.300
DE_best_1_exp	-63.738	6.795	-64.212	-79.510	-45.220	2864.830
DE_rand_1_exp	-60.805	3.664	-61.318	-67.539	-51.961	2840.750
DE_randbest_1_exp	-22.559	5.518	-22.506	-32.439	-7.954	2892.530
DE_best_2_exp	-61.269	11.156	-62.670	-77.042	-15.970	2837.860
DE_rand_2_exp	-67.769	2.943	-67.742	-74.377	-61.532	2907.380
DE_best_1_bin	-68.010	7.053	-68.962	-80.134	-49.972	2961.000
DE_rand_1_bin	-32.317	6.338	-31.277	-50.263	-19.241	3098.500
DE_randbest_1_bin	-35.068	11.964	-33.732	-63.685	-7.987	3187.070
DE_best_2_bin	-47.107	20.246	-53.419	-71.912	3.193	3157.750
DE_rand_2_bin	-60.806	3.521	-60.985	-69.328	-51.036	3166.910
PSO	-9.022	23.102	-11.826	-50.599	56.589	2163.080
SCA	43.560	16.424	44.209	8.777	82.984	3018.800
BA	157.836	39.611	147.827	88.467	243.619	3336.570
DBA	31.779	25.138	27.697	-12.694	102.399	4377.200
ALO	-83.278	2.285	-84.013	-85.032	-70.118	7044.210

Table 14: Function 8: Statistical Analysis of the Cost

Best Algorithm:
ALO, Cost (mean): -83.277600

3.9 Function 9: Ackley Two

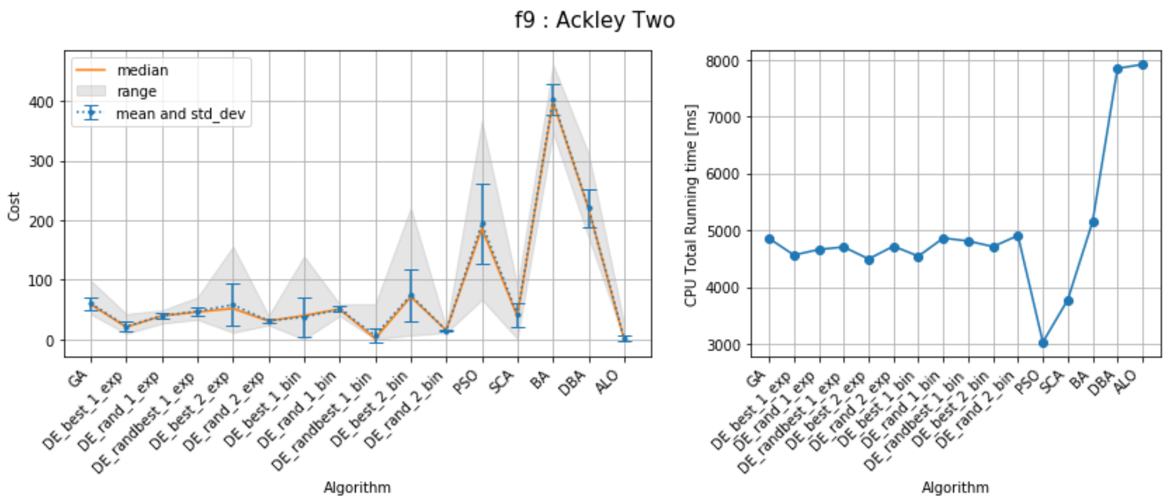

Figure 9: Cost and CPU total running time of Function 9: Ackley Two

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	60.544	10.325	58.879	42.370	98.786	4854.400
DE_best_1_exp	22.074	7.606	20.809	9.750	42.903	4561.730
DE_rand_1_exp	39.602	4.888	39.607	26.848	49.058	4660.250
DE_randbest_1_exp	47.446	7.534	46.223	33.274	70.883	4703.410
DE_best_2_exp	58.112	35.814	51.937	11.665	156.192	4493.600
DE_rand_2_exp	31.213	4.138	30.978	23.721	39.898	4718.220
DE_best_1_bin	37.730	33.429	40.066	0.514	139.746	4541.870
DE_rand_1_bin	50.575	4.836	50.998	39.607	59.232	4861.140
DE_randbest_1_bin	6.924	11.481	1.560	0.470	59.343	4809.270
DE_best_2_bin	74.527	44.138	71.563	6.883	221.156	4710.260
DE_rand_2_bin	15.052	2.129	15.461	11.166	19.563	4906.100
PSO	194.587	66.520	186.522	66.782	367.321	3028.180
SCA	41.552	20.662	38.865	1.321	89.197	3773.180
BA	402.363	26.340	399.424	347.437	461.734	5152.000
DBA	220.732	31.440	220.125	171.381	312.239	7848.350
ALO	2.561	4.434	0.633	-0.000	26.558	7918.470

Table 15: Function 9: Statistical Analysis of the Cost

Best Algorithm:
ALO, Cost (mean): 2.561230

3.10 Function 10: Egg Holder

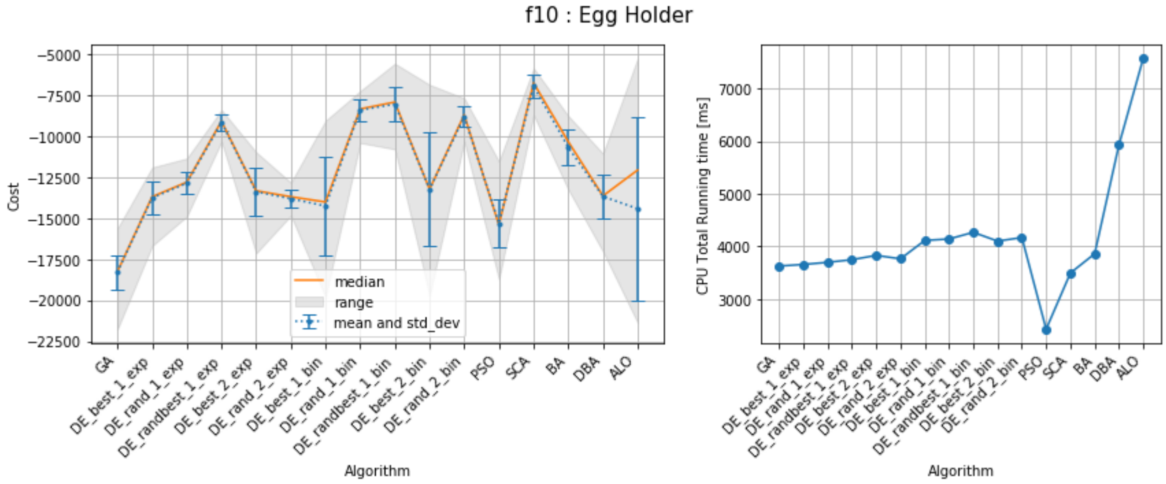

Figure 10: Cost and CPU total running time of Function 10: Egg Holder

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	-18289.500	1065.430	-18252.300	-21763.400	-15602.600	3629.530
DE_best_1_exp	-13753.900	1036.030	-13685.700	-16680.900	-11893.400	3659.750
DE_rand_1_exp	-12855.500	667.577	-12794.400	-14880.200	-11341.000	3700.050
DE_randbest_1_exp	-9134.640	502.491	-9077.200	-10320.300	-8387.130	3750.190
DE_best_2_exp	-13381.200	1482.760	-13318.400	-17129.000	-10961.900	3831.990
DE_rand_2_exp	-13802.800	536.610	-13684.100	-14828.600	-12812.300	3766.820
DE_best_1_bin	-14247.100	3019.220	-14004.900	-19783.300	-9040.200	4115.300
DE_rand_1_bin	-8399.290	678.009	-8331.020	-10394.700	-7231.850	4142.880
DE_randbest_1_bin	-8037.240	1061.310	-7909.960	-10781.700	-5557.910	4269.810
DE_best_2_bin	-13237.800	3458.150	-13231.700	-19690.000	-6848.170	4102.850
DE_rand_2_bin	-8796.180	638.176	-8686.570	-10312.000	-7672.990	4170.560
PSO	-15308.800	1469.980	-15401.800	-18721.100	-11522.900	2431.640
SCA	-6927.370	703.404	-6738.400	-8701.610	-5841.150	3497.470
BA	-10675.600	1102.080	-10322.900	-13246.100	-8750.400	3863.150
DBA	-13664.000	1345.500	-13634.000	-17024.600	-11046.600	5929.030
ALO	-14414.900	5595.070	-12056.100	-21364.700	-5242.670	7570.600

Table 16: Function 10: Statistical Analysis of the Cost

Best Algorithm:
GA, Cost (mean): -18289.500000

3.11 Function 11: Rana

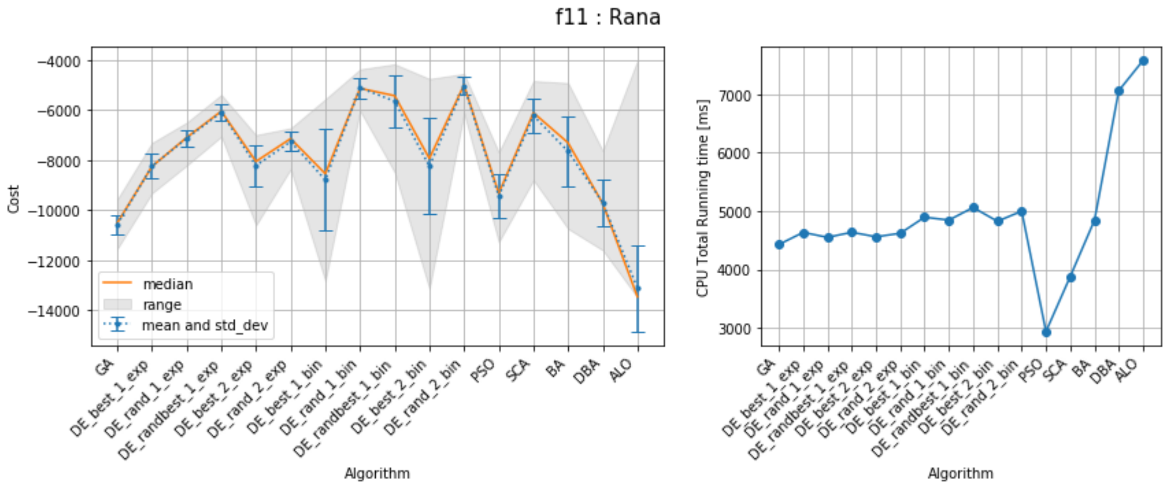

Figure 11: Cost and CPU total running time of Function 11: Rana

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	-10569.000	393.232	-10513.100	-11543.200	-9561.400	4430.570
DE_best_1_exp	-8245.300	487.518	-8255.980	-9322.990	-7311.790	4637.610
DE_rand_1_exp	-7134.590	332.694	-7101.360	-8217.160	-6500.540	4552.570
DE_randbest_1_exp	-6094.790	337.200	-6048.610	-7060.660	-5391.770	4639.810
DE_best_2_exp	-8221.190	803.864	-8054.260	-10599.000	-6998.940	4562.220
DE_rand_2_exp	-7264.640	378.668	-7143.100	-8363.740	-6723.810	4622.370
DE_best_1_bin	-8791.590	2030.280	-8547.050	-12827.700	-5564.650	4897.740
DE_rand_1_bin	-5116.550	408.855	-5134.810	-6031.840	-4374.390	4847.640
DE_randbest_1_bin	-5649.690	1023.720	-5431.090	-8490.210	-4169.940	5063.420
DE_best_2_bin	-8247.990	1911.010	-7936.330	-13136.300	-4755.360	4827.680
DE_rand_2_bin	-5045.660	348.447	-4996.840	-6130.120	-4554.520	5001.290
PSO	-9438.670	872.168	-9330.130	-11272.400	-7684.170	2941.300
SCA	-6228.130	673.403	-6109.160	-8801.870	-4840.150	3883.000
BA	-7652.450	1392.080	-7310.310	-10741.600	-4921.700	4847.390
DBA	-9698.780	939.215	-9736.200	-11599.800	-7627.440	7062.840
ALO	-13111.900	1733.270	-13463.900	-13480.000	-4020.970	7580.100

Table 17: Function 11: Statistical Analysis of the Cost

Best Algorithm:
ALO, Cost (mean): -13111.900000

3.12 Function 12: Pathological

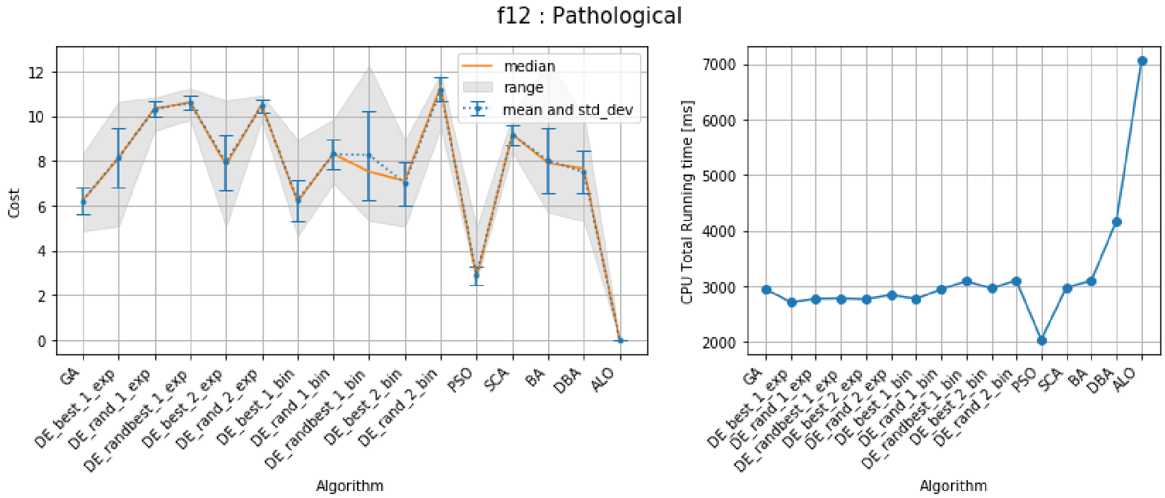

Figure 12: Cost and CPU total running time of Function 12: Pathological

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	6.217	0.586	6.230	4.848	8.322	2945.800
DE_best_1_exp	8.163	1.329	8.131	5.090	10.654	2712.270
DE_rand_1_exp	10.313	0.344	10.333	9.358	10.842	2774.290
DE_randbest_1_exp	10.602	0.307	10.628	9.849	11.247	2782.110
DE_best_2_exp	7.937	1.221	7.819	5.091	10.707	2766.020
DE_rand_2_exp	10.471	0.271	10.515	9.812	10.945	2843.880
DE_best_1_bin	6.244	0.913	6.180	4.642	8.942	2771.420
DE_rand_1_bin	8.309	0.656	8.315	7.006	9.855	2942.740
DE_randbest_1_bin	8.266	1.994	7.522	5.329	12.265	3089.480
DE_best_2_bin	7.006	0.980	7.106	5.069	8.920	2961.390
DE_rand_2_bin	11.206	0.546	11.346	9.491	11.911	3101.850
PSO	2.887	0.414	2.773	2.640	5.004	2036.560
SCA	9.168	0.418	9.200	8.410	10.031	2974.950
BA	8.020	1.479	7.903	5.704	12.505	3099.120
DBA	7.507	0.951	7.662	5.305	10.102	4175.760
ALO	0.000	0.000	0.000	0.000	0.000	7063.580

Table 18: Function 12: Statistical Analysis of the Cost

Best Algorithm:
ALO, Cost (mean): 0.000000

3.13 Function 13: Michalewicz

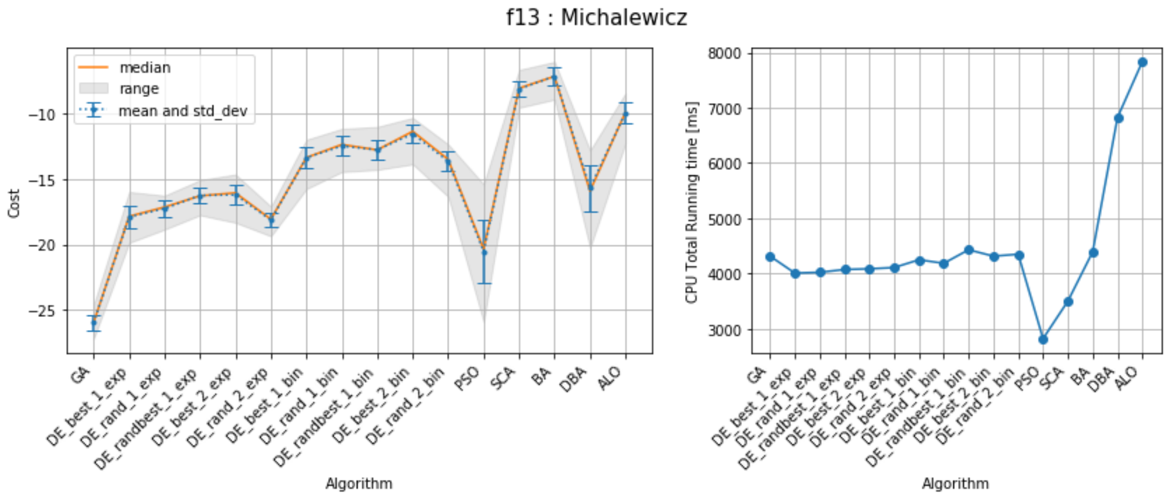

Figure 13: Cost and CPU total running time of Function 13: Michalewicz

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	-25.973	0.550	-25.951	-27.184	-24.584	4315.810
DE_best_1_exp	-17.900	0.897	-17.857	-19.866	-15.982	4009.570
DE_rand_1_exp	-17.257	0.639	-17.148	-18.853	-16.269	4023.420
DE_randbest_1_exp	-16.258	0.561	-16.291	-17.758	-15.084	4074.420
DE_best_2_exp	-16.215	0.768	-16.065	-18.332	-14.650	4086.760
DE_rand_2_exp	-18.131	0.513	-18.076	-19.363	-17.096	4109.900
DE_best_1_bin	-13.380	0.838	-13.372	-15.761	-12.002	4250.480
DE_rand_1_bin	-12.490	0.729	-12.378	-14.461	-11.170	4186.710
DE_randbest_1_bin	-12.781	0.713	-12.786	-14.298	-11.023	4431.400
DE_best_2_bin	-11.556	0.705	-11.374	-13.875	-10.335	4316.230
DE_rand_2_bin	-13.656	0.756	-13.510	-16.355	-12.357	4350.370
PSO	-20.529	2.371	-20.360	-25.914	-15.397	2823.570
SCA	-8.138	0.582	-8.082	-9.561	-6.677	3502.060
BA	-7.169	0.661	-7.175	-8.937	-6.061	4394.190
DBA	-15.695	1.779	-15.923	-20.261	-12.819	6825.380
ALO	-9.952	0.767	-9.920	-12.278	-8.455	7833.270

Table 19: Function 13: Statistical Analysis of the Cost

Best Algorithm:
GA, Cost (mean): -25.973400

3.14 Function 14: Masters' Cosine Wave

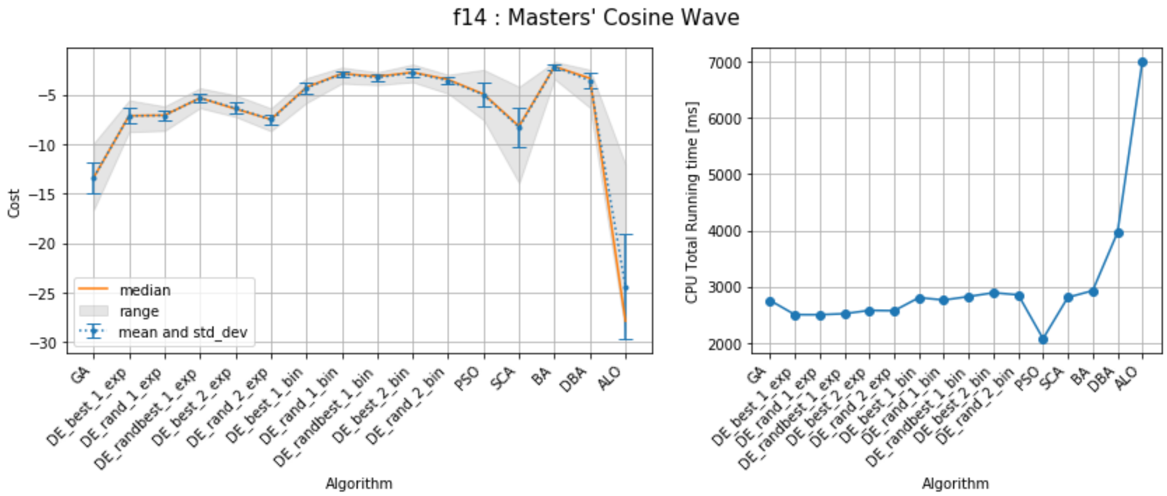

Figure 14: Cost and CPU total running time of Function 14: Masters' Cosine Wave

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	-13.381	1.599	-13.384	-16.633	-9.898	2755.500
DE_best_1_exp	-7.140	0.771	-7.127	-8.758	-5.558	2502.870
DE_rand_1_exp	-7.101	0.492	-7.080	-8.649	-6.173	2501.480
DE_randbest_1_exp	-5.333	0.454	-5.332	-6.343	-4.330	2519.360
DE_best_2_exp	-6.356	0.534	-6.436	-7.230	-5.045	2578.470
DE_rand_2_exp	-7.510	0.544	-7.499	-8.679	-6.369	2572.430
DE_best_1_bin	-4.322	0.612	-4.257	-5.828	-3.324	2805.490
DE_rand_1_bin	-2.910	0.335	-2.870	-3.858	-2.245	2762.950
DE_randbest_1_bin	-3.275	0.354	-3.167	-4.015	-2.732	2824.200
DE_best_2_bin	-2.789	0.425	-2.738	-3.732	-1.939	2892.490
DE_rand_2_bin	-3.575	0.382	-3.475	-4.884	-2.978	2855.320
PSO	-5.021	1.191	-4.948	-7.564	-2.476	2075.840
SCA	-8.302	2.002	-8.186	-13.939	-4.191	2809.800
BA	-2.191	0.299	-2.154	-3.429	-1.672	2929.150
DBA	-3.575	0.802	-3.349	-6.319	-2.469	3963.750
ALO	-24.384	5.268	-27.870	-29.000	-12.044	6999.430

Table 20: Function 14: Statistical Analysis of the Cost

Best Algorithm:
ALO, Cost (mean): -24.383900

3.15 Function 15: Quartic

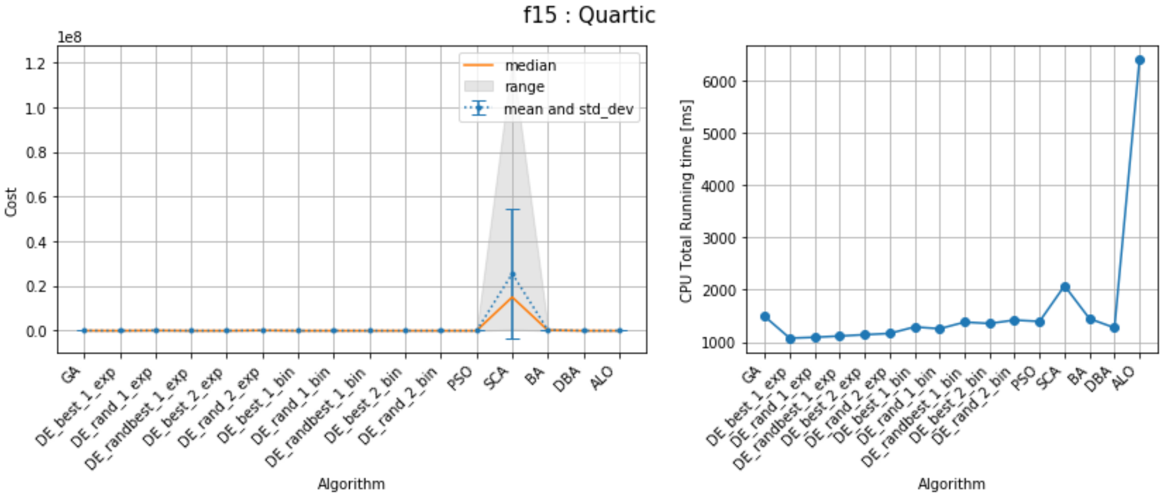

Figure 15: Cost and CPU total running time of Function 15: Quartic

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	44470.800	24400.800	40083.200	8031.960	116445.000	1485.830
DE_best_1_exp	947.130	889.073	692.658	32.479	4291.420	1072.870
DE_rand_1_exp	94613.300	37297.100	96982.200	34981.900	199166.000	1090.210
DE_randbest_1_exp	89.971	118.516	36.497	4.766	529.270	1113.890
DE_best_2_exp	244.925	646.618	65.635	1.216	4463.240	1139.650
DE_rand_2_exp	133879.000	68969.800	125159.000	43305.500	349768.000	1164.070
DE_best_1_bin	0.029	0.070	0.007	0.000	0.422	1288.130
DE_rand_1_bin	10670.600	5041.410	9569.610	1678.420	24112.200	1253.430
DE_randbest_1_bin	3.964	18.988	0.012	0.000	134.742	1380.030
DE_best_2_bin	0.040	0.159	0.006	0.001	1.137	1353.450
DE_rand_2_bin	3715.320	1960.350	3370.310	1470.040	11655.600	1419.340
PSO	0.000	0.000	0.000	0.000	0.000	1392.580
SCA	25492800.000	28849300.000	15001300.000	554160.000	121235000.000	2074.320
BA	274670.000	228659.000	179170.000	27804.200	1209450.000	1445.850
DBA	5871.970	4573.400	4895.840	901.039	26064.100	1271.940
ALO	0.013	0.088	0.000	0.000	0.631	6396.300

Table 21: Function 15: Statistical Analysis of the Cost

Best Algorithm:
PSO, Cost (mean): 0.000001
Two-Sample Z-Test Hypothesis Testing: confidence interval = 95%

Null hypothesis: The best algorithm and the tested one are equal
DE_randbest_1_bin, Cost (mean): 3.963670 , P value: 0.139900
DE_best_2_bin, Cost (mean): 0.039723 , P value: 0.077600
ALO, Cost (mean): 0.013024 , P value: 0.296900

3.16 Function 16: Levy

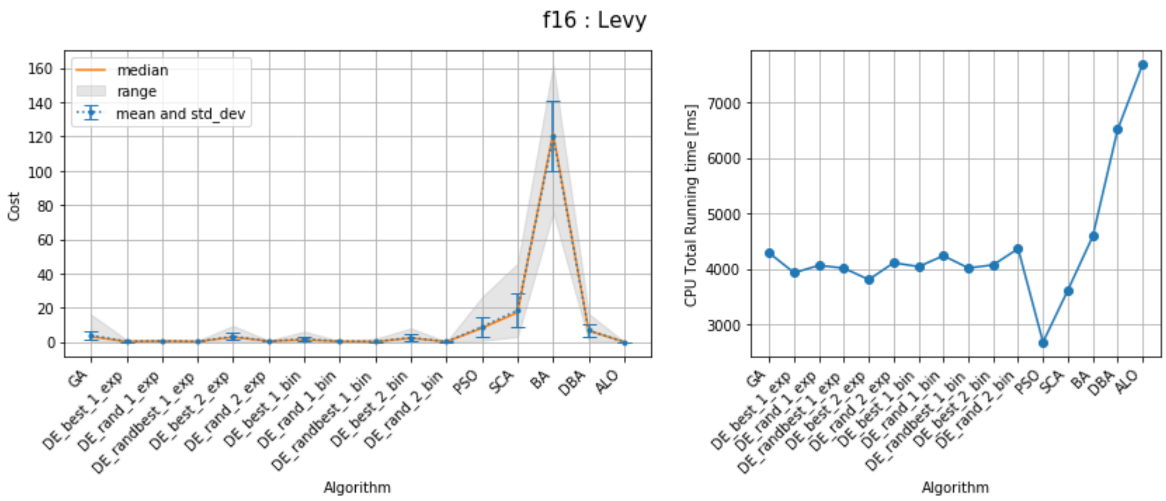

Figure 16: Cost and CPU total running time of Function 16: Levy

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	3.845	2.650	3.112	0.640	16.040	4286.270
DE_best_1_exp	0.293	0.286	0.190	0.035	1.299	3930.070
DE_rand_1_exp	0.578	0.112	0.552	0.406	0.859	4063.700
DE_randbest_1_exp	0.291	0.155	0.241	0.107	0.745	4012.540
DE_best_2_exp	3.158	2.056	2.773	0.361	9.424	3807.100
DE_rand_2_exp	0.523	0.118	0.503	0.293	0.821	4110.930
DE_best_1_bin	1.589	1.244	1.222	0.000	6.090	4040.910
DE_rand_1_bin	0.391	0.094	0.378	0.160	0.565	4236.930
DE_randbest_1_bin	0.208	0.361	0.090	0.000	1.907	4016.220
DE_best_2_bin	2.519	1.892	2.227	0.090	8.004	4072.410
DE_rand_2_bin	0.110	0.025	0.105	0.066	0.170	4364.090
PSO	8.790	5.504	8.089	0.723	26.274	2681.750
SCA	18.451	9.892	17.175	3.011	45.824	3618.720
BA	120.242	20.295	121.984	75.598	162.128	4588.260
DBA	6.721	3.728	6.576	0.684	16.474	6509.460
ALO	0.009	0.007	0.008	0.001	0.040	7679.500

Table 22: Function 16: Statistical Analysis of the Cost

Best Algorithm:
ALO, Cost (mean): 0.009452

3.17 Function 17: Step

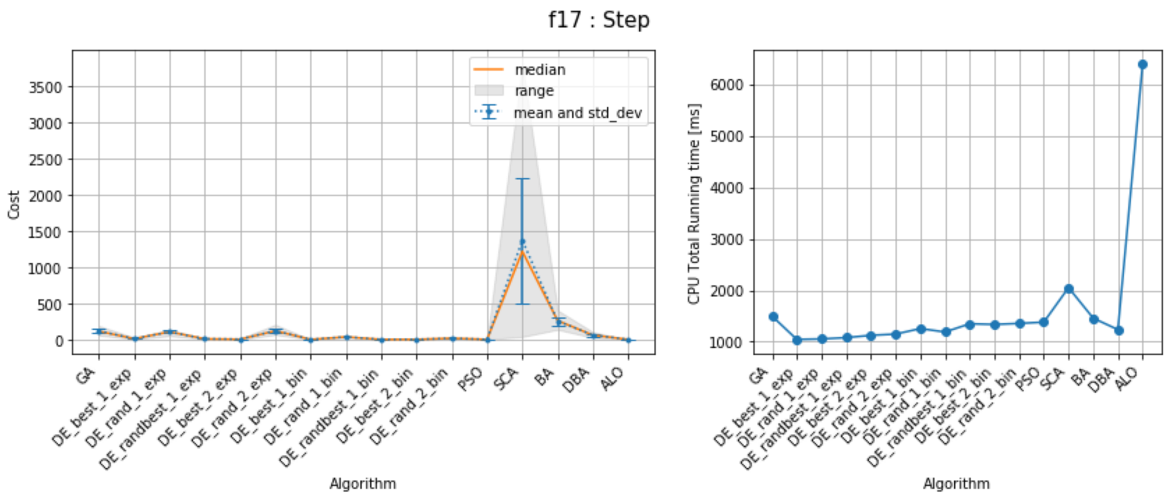

Figure 17: Cost and CPU total running time of Function 17: Step

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	125.421	28.308	120.986	64.643	195.547	1495.940
DE_best_1_exp	17.826	3.661	17.180	12.716	34.064	1044.910
DE_rand_1_exp	110.850	19.415	112.765	56.362	142.078	1054.840
DE_randbest_1_exp	16.331	1.566	16.019	13.849	20.439	1080.690
DE_best_2_exp	10.773	2.710	9.733	8.221	22.181	1124.630
DE_rand_2_exp	128.013	30.635	126.135	73.262	211.891	1148.730
DE_best_1_bin	7.705	0.072	7.688	7.595	7.966	1256.120
DE_rand_1_bin	43.111	6.135	42.627	29.990	59.466	1189.570
DE_randbest_1_bin	7.659	0.039	7.653	7.605	7.820	1349.140
DE_best_2_bin	7.854	0.113	7.836	7.652	8.139	1336.450
DE_rand_2_bin	24.323	2.694	24.445	19.570	32.268	1355.820
PSO	7.679	0.143	7.640	7.542	8.140	1381.340
SCA	1376.250	864.277	1223.120	45.231	3809.660	2049.890
BA	261.551	58.007	267.306	143.930	409.819	1458.380
DBA	67.753	16.406	66.787	32.294	110.711	1235.500
ALO	7.677	0.192	7.643	7.500	8.431	6390.780

Table 23: Function 17: Statistical Analysis of the Cost

Best Algorithm:
DE_randbest_1_bin, Cost (mean): 7.659330
Two-Sample Z-Test Hypothesis Testing: confidence interval = 95%

Null hypothesis: The best algorithm and the tested one are equal

PSO, Cost (mean): 7.679030 , P value: 0.348300
ALO, Cost (mean): 7.677180 , P value: 0.518700

3.18 Function 18: Alphine

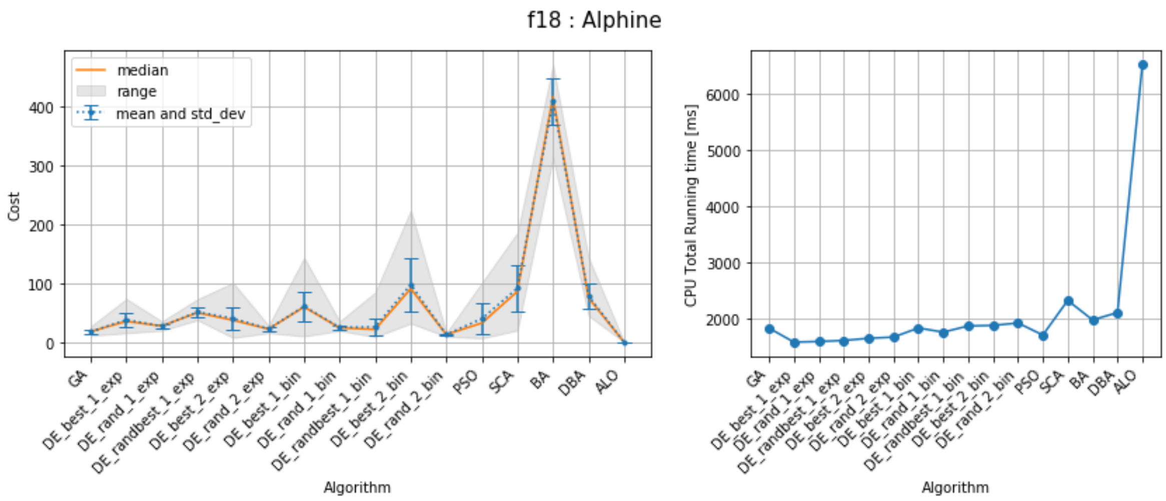

Figure 18: Cost and CPU total running time of Function 18: Alphine

algorithm	mean	std_dev	median	range_min	range_max	time_ms
GA	17.991	4.037	18.067	10.891	27.051	1826.850
DE_best_1_exp	37.864	12.668	35.370	15.437	73.569	1576.760
DE_rand_1_exp	27.112	3.996	27.426	19.596	35.474	1588.730
DE_randbest_1_exp	51.131	8.403	49.970	37.658	73.088	1607.030
DE_best_2_exp	40.449	19.763	37.293	7.506	100.949	1645.680
DE_rand_2_exp	22.361	3.222	22.231	15.525	28.308	1667.370
DE_best_1_bin	60.902	25.219	59.704	10.036	143.576	1829.760
DE_rand_1_bin	25.037	3.675	24.234	19.301	35.590	1755.460
DE_randbest_1_bin	25.796	13.889	21.654	9.023	84.681	1866.120
DE_best_2_bin	97.684	45.079	90.748	31.946	224.148	1875.420
DE_rand_2_bin	12.928	1.452	13.000	9.392	15.897	1919.240
PSO	39.814	25.583	32.434	6.663	101.126	1703.470
SCA	91.572	38.420	85.955	20.058	186.021	2321.580
BA	408.737	38.876	416.711	312.912	471.564	1973.070
DBA	78.506	21.623	75.316	44.873	143.701	2104.510
ALO	0.056	0.117	0.008	0.000	0.575	6530.740

Table 24: Function 18: Statistical Analysis of the Cost

Best Algorithm:
ALO, Cost (mean): 0.055910

3.19 Summary

The table 30 shows a summary of the best mean cost obtained for each function.

algorithm	f1	f2	f3	f4	f5	f6	f7	f8	f9
GA	1227.950	76.080	327876.000	78.319	1.597	-40.040	30.725	-68.310	60.544
DE_best_1_exp	2594.260	3.423	13272.100	136.788	0.993	-36.360	40.988	-63.738	22.074
DE_rand_1_exp	3176.050	73.259	587827.000	160.392	1.471	-35.411	42.010	-60.805	39.602
DE_randbest_1_exp	6421.590	2.555	1902.470	157.514	0.963	-35.890	41.171	-22.559	47.446
DE_best_2_exp	3698.620	0.452	3945.820	141.617	0.243	-35.896	43.031	-61.269	58.112
DE_rand_2_exp	1861.060	83.211	838744.000	138.059	1.496	-35.393	41.704	-67.769	31.213
DE_best_1_bin	2061.100	0.003	608.995	177.708	0.014	-32.782	53.395	-68.010	37.730
DE_rand_1_bin	6891.470	19.215	72522.400	201.957	1.120	-32.770	51.687	-32.317	50.575
DE_randbest_1_bin	6841.440	0.002	625.385	188.274	0.016	-33.406	50.244	-35.068	6.924
DE_best_2_bin	2294.620	0.007	751.848	223.900	0.028	-31.684	54.359	-47.107	74.527
DE_rand_2_bin	6191.390	8.040	34467.400	166.760	1.049	-33.105	50.529	-60.806	15.052
PSO	3266.090	0.001	20887.400	116.462	1.283	-33.175	31.156	-9.022	194.587
SCA	7207.630	1495.250	284742000.000	345.389	10.016	-31.745	54.498	43.560	41.552
BA	4858.750	196.477	1168380.000	624.340	140.993	-27.969	59.303	157.836	402.363
DBA	3800.850	36.711	48239.000	337.212	1.216	-35.407	36.421	31.779	220.732
ALO	0.044	0.008	2.579	5.612	0.037	-40.692	29.001	-83.278	2.561

algorithm	f10	f11	f12	f13	f14	f15	f16	f17	f18
GA	-18289.500	-10569.000	6.217	-25.973	-13.381	44470.800	3.845	125.421	17.991
DE_best_1_exp	-13753.900	-8245.300	8.163	-17.900	-7.140	947.130	0.293	17.826	37.864
DE_rand_1_exp	-12855.500	-7134.590	10.313	-17.257	-7.101	94613.300	0.578	110.850	27.112
DE_randbest_1_exp	-9134.640	-6094.790	10.602	-16.258	-5.333	89.971	0.291	16.331	51.131
DE_best_2_exp	-13381.200	-8221.190	7.937	-16.215	-6.356	244.925	3.158	10.773	40.449
DE_rand_2_exp	-13802.800	-7264.640	10.471	-18.131	-7.510	133879.000	0.523	128.013	22.361
DE_best_1_bin	-14247.100	-8791.590	6.244	-13.380	-4.322	0.029	1.589	7.705	60.902
DE_rand_1_bin	-8399.290	-5116.550	8.309	-12.490	-2.910	10670.600	0.391	43.111	25.037
DE_randbest_1_bin	-8037.240	-5649.690	8.266	-12.781	-3.275	3.964	0.208	7.659	25.796
DE_best_2_bin	-13237.800	-8247.990	7.006	-11.556	-2.789	0.040	2.519	7.854	97.684
DE_rand_2_bin	-8796.180	-5045.660	11.206	-13.656	-3.575	3715.320	0.110	24.323	12.928
PSO	-15308.800	-9438.670	2.887	-20.529	-5.021	0.000	8.790	7.679	39.814
SCA	-6927.370	-6228.130	9.168	-8.138	-8.302	25492800.000	18.451	1376.250	91.572
BA	-10675.600	-7652.450	8.020	-7.169	-2.191	274670.000	120.242	261.551	408.737
DBA	-13664.000	-9698.780	7.507	-15.695	-3.575	5871.970	6.721	67.753	78.506
ALO	-14414.900	-13111.900	0.000	-9.952	-24.384	0.013	0.009	7.677	0.056

Table 25: mean cost of each optimization algorithm with different bench-mark functions. iterations:100 population:200

function_id	best_algorithm	best_cost (mean)	similar_result
1	ALO	0.044	
2	PSO	0.001	
3	ALO	2.579	DE_randbest_1_bin, PSO
4	ALO	5.612	
5	DE_best_1_bin	0.014	DE_randbest_1_bin, PSO, ALO
6	ALO	-40.692	GA
7	ALO	29.001	
8	ALO	-83.278	
9	ALO	2.561	
10	GA	-18289.500	
11	ALO	-13111.900	
12	ALO	0.000	
13	GA	-25.973	
14	ALO	-24.384	
15	PSO	0.000	DE_randbest_1_bin, DE_best_2_bin, ALO
16	ALO	0.009	
17	DE_randbest_1_bin	7.659	PSO, ALO
18	ALO	0.056	

Table 26: best mean cost of each bench-mark functions

4 Testing with different iterations and populations

4.1 iteration: 200 population: 200

algorithm	f1	f2	f3	f4	f5	f6	f7	f8	f9
GA	1112.630	5.892	2793.560	23.149	1.067	-41.728	29.729	-82.148	18.947
DE_best_1_exp	818.031	0.000	226.672	45.388	0.009	-38.472	36.599	-79.549	2.980
DE_rand_1_exp	1257.490	0.105	714.642	45.913	0.131	-37.673	37.800	-86.060	0.363
DE_randbest_1_exp	5349.210	0.001	170.952	104.180	0.009	-37.636	37.282	-50.032	2.535
DE_best_2_exp	1696.720	0.002	851.988	69.924	0.025	-37.802	38.714	-63.594	48.809
DE_rand_2_exp	118.620	0.675	10713.600	21.054	0.263	-37.743	37.461	-85.751	0.298
DE_best_1_bin	1959.070	0.000	77.386	69.477	0.006	-33.524	51.691	-71.410	33.071
DE_rand_1_bin	6247.280	0.006	233.834	153.858	0.018	-33.813	45.837	-70.955	0.324
DE_randbest_1_bin	5538.660	0.000	218.849	164.571	0.004	-33.961	47.625	-76.861	5.529
DE_best_2_bin	1922.260	0.000	108.453	190.028	0.009	-32.482	53.403	-60.138	59.691
DE_rand_2_bin	5464.790	0.001	164.307	100.340	0.003	-34.381	44.513	-85.831	0.053
PSO	3253.400	0.000	160886.000	121.126	0.014	-35.867	30.194	-2.911	210.716
SCA	7080.270	117.846	11869300.000	169.699	1.735	-32.666	48.863	20.980	2.646
BA	4994.940	94.129	351464.000	543.358	133.951	-28.552	58.386	124.767	392.025
DBA	3810.520	17.037	33449.300	334.082	1.102	-36.391	34.169	29.500	222.807
ALO	71.152	0.000	1.745	0.008	0.000	-43.199	29.000	-83.978	0.604

algorithm	f10	f11	f12	f13	f14	f15	f16	f17	f18
GA	-19007.100	-11053.000	5.554	-27.065	-14.803	159.207	3.376	23.265	5.674
DE_best_1_exp	-17495.600	-10156.800	6.954	-22.294	-12.031	0.011	0.158	7.543	5.977
DE_rand_1_exp	-15427.400	-8315.950	8.408	-20.724	-13.628	7.311	0.001	8.348	0.240
DE_randbest_1_exp	-10818.400	-7055.290	9.695	-18.770	-7.712	0.049	0.015	7.575	25.308
DE_best_2_exp	-16930.100	-9722.580	7.090	-19.187	-9.579	9.977	1.950	7.931	17.305
DE_rand_2_exp	-16432.000	-8363.040	9.155	-21.644	-14.223	763.248	0.002	9.046	0.122
DE_best_1_bin	-18460.100	-10405.500	5.881	-14.908	-6.186	0.000	1.743	7.500	25.320
DE_rand_1_bin	-9844.840	-5697.990	5.100	-13.870	-3.519	0.685	0.000	7.707	0.131
DE_randbest_1_bin	-9614.670	-6410.850	7.571	-13.731	-3.899	3.869	0.246	7.500	3.769
DE_best_2_bin	-17654.300	-9353.810	6.988	-12.124	-3.824	0.002	2.104	7.500	69.942
DE_rand_2_bin	-10110.000	-5455.780	8.712	-15.269	-4.614	4.846	0.000	7.552	0.024
PSO	-15525.100	-9425.460	3.008	-21.109	-5.213	0.000	9.014	7.524	25.213
SCA	-7251.820	-6421.380	8.923	-8.258	-13.443	1517870.000	5.598	139.382	47.632
BA	-11386.500	-7863.540	7.933	-7.342	-2.297	26675.900	105.081	140.907	397.843
DBA	-13742.900	-10095.900	7.433	-17.348	-3.936	1227.830	7.065	44.402	71.843
ALO	-15082.400	-13270.400	0.000	-9.560	-28.172	0.000	0.001	7.511	0.003

Table 27: mean cost of each optimization algorithm with different bench-mark functions. iterations:200 population:200

function_id	best_algorithm	best_cost (mean)	similar_result
1	ALO	71.152	DE_rand_2_exp
2	PSO	0.000	DE_randbest_1_bin
3	ALO	1.745	DE_rand_2_exp
4	ALO	0.008	
5	ALO	0.000	
6	ALO	-43.199	
7	ALO	29.000	
8	DE_rand_1_exp	-86.060	
9	DE_rand_2_bin	0.053	
10	GA	-19007.100	DE_best_1_bin
11	ALO	-13270.400	
12	ALO	0.000	
13	GA	-27.065	
14	ALO	-28.172	
15	PSO	0.000	DE_best_2_exp, DE_best_1_bin, DE_rand_1_bin, DE_randbest_1_bin, DE_best_2_bin, DE_rand_2_bin, ALO
16	DE_rand_2_bin	0.000	
17	DE_randbest_1_bin	7.500	DE_best_1_bin, PSO
18	ALO	0.003	

Table 28: best mean cost of each bench-mark functions

4.2 iteration: 500 population: 500

algorithm	f1	f2	f3	f4	f5	f6	f7	f8	f9
GA	346.864	0.025	122.842	0.351	0.134	-43.046	29.106	-86.234	0.889
DE_best_1_exp	396.466	0.000	25.724	6.169	0.008	-40.980	32.104	-82.573	0.103
DE_rand_1_exp	0.001	0.000	31.544	0.001	0.000	-40.169	32.984	-86.333	0.000
DE_randbest_1_exp	3560.760	0.000	36.208	49.267	0.002	-39.930	33.171	-74.767	1.160
DE_best_2_exp	1087.850	0.000	24.237	23.521	0.015	-40.025	33.496	-70.173	25.029
DE_rand_2_exp	-0.002	0.000	39.264	0.000	0.000	-40.137	32.700	-86.333	0.000
DE_best_1_bin	1777.820	0.000	19.859	32.035	0.009	-34.937	45.604	-73.242	29.896
DE_rand_1_bin	5293.970	0.000	31.142	113.277	0.000	-35.227	38.805	-86.333	0.000
DE_randbest_1_bin	2393.760	0.000	29.024	135.499	0.004	-35.425	43.725	-79.566	0.381
DE_best_2_bin	1895.380	0.000	9.337	113.615	0.010	-33.917	48.660	-70.656	45.034
DE_rand_2_bin	4575.130	0.000	43.035	47.380	0.000	-35.809	36.782	-86.333	0.000
PSO	3204.070	0.000	20919.700	87.138	0.013	-37.427	30.005	-30.332	157.399
SCA	6517.380	0.011	687.530	29.613	0.127	-33.900	41.917	3.168	0.003
BA	4460.820	8.839	11416.500	320.595	50.833	-28.761	51.448	318.422	404.284
DBA	3690.000	0.391	1107.200	278.966	0.273	-38.525	32.054	9.072	181.164
ALO	71.062	0.000	0.098	0.000	0.000	-43.380	29.000	-84.851	0.000

algorithm	f10	f11	f12	f13	f14	f15	f16	f17	f18
GA	-21637.800	-12360.500	3.811	-29.191	-22.825	0.004	0.321	8.161	0.293
DE_best_1_exp	-22301.800	-13277.900	4.705	-28.003	-22.526	0.000	0.013	7.500	0.000
DE_rand_1_exp	-19611.600	-10095.300	1.890	-25.086	-22.496	0.000	0.000	7.500	0.003
DE_randbest_1_exp	-13425.000	-8621.130	8.034	-23.195	-14.014	0.000	0.000	7.500	6.940
DE_best_2_exp	-20798.500	-12037.600	5.630	-24.343	-15.538	0.000	0.996	7.500	0.125
DE_rand_2_exp	-20567.800	-10162.800	4.164	-26.053	-28.988	0.010	0.000	7.500	0.002
DE_best_1_bin	-18800.800	-11966.600	5.111	-18.748	-11.524	0.000	0.673	7.500	0.386
DE_rand_1_bin	-15014.500	-7984.760	3.136	-15.626	-4.756	0.000	0.000	7.500	0.017
DE_randbest_1_bin	-14260.400	-8166.830	6.450	-15.481	-5.208	0.000	0.023	7.500	0.000
DE_best_2_bin	-18263.100	-12214.200	6.295	-13.771	-5.766	0.000	1.792	7.500	6.555
DE_rand_2_bin	-14726.500	-6288.340	4.102	-17.209	-6.541	0.000	0.000	7.500	0.012
PSO	-15949.900	-9814.110	2.742	-21.960	-6.469	0.000	5.412	7.500	10.842
SCA	-8186.760	-7252.980	8.330	-8.778	-26.774	21.615	2.067	7.598	5.970
BA	-12123.300	-8260.450	7.541	-9.690	-2.860	125.619	63.409	29.474	342.043
DBA	-14240.600	-10597.200	7.174	-19.843	-4.471	6.227	4.783	12.422	37.666
ALO	-19142.200	-13232.900	0.000	-10.138	-29.000	0.000	0.000	7.500	0.000

Table 29: mean cost of each optimization algorithm with different bench-mark functions. iterations:500 population:500

function_id	best_algorithm	best_cost (mean)	similar_result
1	DE_rand_2_exp	-0.002	ALO
2	ALO	0.000	DE_best_2_exp
3	ALO	0.098	PSO
4	DE_rand_2_exp	0.000	
5	DE_rand_2_bin	0.000	DE_rand_2_exp
6	ALO	-43.380	
7	ALO	29.000	
8	DE_rand_1_exp	-86.333	DE_rand_2_exp, DE_rand_1_bin, DE_rand_2_bin
9	DE_rand_2_bin	0.000	DE_best_1_exp, DE_randbest_1_exp, DE_randbest_1_bin, ALO
10	DE_best_1_exp	-22301.800	
11	DE_best_1_exp	-13277.900	ALO
12	ALO	0.000	
13	GA	-29.191	
14	ALO	-29.000	
15	DE_best_1_bin	0.000	DE_best_1_exp, DE_randbest_1_exp, DE_best_2_exp, DE_rand_2_exp, DE_randbest_1_bin
16	DE_randbest_1_exp	0.000	DE_best_1_exp
17	DE_best_1_exp	7.500	DE_rand_1_exp, DE_randbest_1_exp, DE_rand_2_exp, DE_best_1_bin, DE_rand_1_bin, DE_randbest_1_bin, DE_best_2_bin, DE_rand_2_bin, PSO, ALO
18	ALO	0.000	DE_best_2_exp, DE_best_1_bin, DE_randbest_1_bin

Table 30: best mean cost of each bench-mark functions

5 Conclusion

In this project many optimization algorithms have been implemented and tested, they are Genetic Algorithm, Differential Evolution Algorithm, Particle Swarm Optimization, Sine Cosine Algorithm, Bat Algorithm, Directional Bat Algorithm, and Ant Lion Optimizer. Many configurations using different population sizes and iterations have been tested. From the results, it can be observed that ALO won most of the time. However, it is much slower than other algorithms. Additionally, it can also be noticed that PSO runs much faster than other algorithms in numerous times. In summary, we can conclude that almost all of the studied algorithms are pretty good since after 500 iterations with a population size of 500 in most of them could find results that are very close to the optimal value. Despite this, one inconvenient of the evolutionary-based optimization algorithm is that the calibration of its parameters requires very tedious work. Though ALO can be the exception, this algorithm has the beauty of solving the problem without the requirement of configuration parameters, but the reason for why ALO performs better than another algorithm is still unknown, perhaps there is a hidden mathematical explanation that we don't understand yet.