Central Washington University

CS530 High Performance Computing

Spring 2019

Lab1: Matrix Power Operation

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February 11, 2020

Contents

1 Introduction

For this project, the power operation of the square matrix has been coded in C++ and Python, there are in total 4 different versions:

1) C++ dynamic allocated double pointer matrix.
2) C++ dynamic allocated linked list matrix.
3) python ijk for loop matrix.
4) python numpy library linalg.matrix_power.

The CPU running time of each version has been recorded, for the c++ versions we used four different methods:

1) g++ gprof profiler: get the CPU spent time of each function.
2) clock() function: compute the processor (CPU) time (linux).
3) time function: work with wall time in second
4) gettimeofday: work with wall time in microsecond

For the python versions, we obtained the time using time.clock and time.time function from the time library, this library is using c time library. the time.clock function means different from windows and linux, on Windows means wall clock time and in linux time.clock means CPU processor time [1]. In our case, the python codes have been run on a windows machine.

1.1 Time complexity

If we analyze the square matrix power expressed as m^n, where m denotes the size of mxm matrix, and n denotes the power of the matrix. We can obtain the time complexity equation illustrated in equations 1 and 2

$$[m^2 * [(m-1) + m)]] * (n-1) \tag{1}$$

The value $[(m-1) + m)]$ represents the operation times when we do matrix multiplication, where $(m-1)$ is the number of sum times, and m is the number of the multiplication times in each of the m^2 matrix elements. The $(n-1)$ represents the numbers of matrix multiplication required when it is elevated to the power of n. This equation 1 can be simplified to equation 2.

$$[2m^3 - m^2] * (n - 1) \tag{2}$$

2 Computer Specification

For this Lab, we used a Microsoft Surface Pro 3 which has the following specification: Core i7-4650U CPU 1.7GHz 2.3GHz with 8 GB of RAM.

The C++ codes have been run on docker where 2 CPUs and 2GB of RAM was assigned. The Docker image was from professor Szilard "szilard-vajda/ubuntu_cs470"

3 Method

We have implemented a specific class in c++ for each solution, the matrix double pointer solution is declared in matrix_double_pointer class displayed in the listing 1, and the matrix linked list solution is declared in the listing 2.

Listing 1: matrix_double_pointer class

```
class matrix_double_pointer
{
int m; // mxm matrix
long double **matrix;

public:
matrix_double_pointer(int m); // constructor
matrix_double_pointer(const matrix_double_pointer& a); // copy constructor for return
~matrix_double_pointer(); //destructor
void init_identity(); //init identity matrix
void init(long double x); //init matrix all with x
void init_rand(); //init matrix with pseudo random
void print_values();
void print_memory_address();
matrix_double_pointer operator* (const matrix_double_pointer& a);
matrix_double_pointer operator^ (int n);
matrix_double_pointer& operator= (const matrix_double_pointer& a); // assigment
long double** get_matrix();
};
```

To implement classes that work with pointers, we must take some special consideration [2]. These classes should have:

- a destructor (to delete the allocated memory).
- a copy constructor (to copy the object when is returned, passed as parameter or initialization from another object of the same class).
- an overloaded operator = (to assign or copy the data to another object).

From listing 1 and 2 we can observe that all these 3 member functions are implemented. In addition, we have overload the operator $\wedge$ to get the matrix power in a more intuitive way.

Listing 2: matrix_linked_list class

```
struct node
{
    long double data;
    int j;
    node *right;
};
struct head_node
{
    long double data;
    int i;
    int j;
    node *right;
    head_node *down_head_node;
};

class matrix_linked_list
{
    int m; // mxm matrix
    head_node *main_head_node;
    head_node *current_head_node; //for get_value_loop_opt()
    node *current_node; //for get_value_loop_opt()
    head_node *current_head_node_add; // add_value_loop_opt()
    node *current_node_add; // for add_value_loop_opt()

    void start_using_get_value_loop_opt();
    long double get_value_loop_opt(int i, int j); // optimized version of get_value
         (make the loop faster using temp node address)
    void add_value_loop_opt(long double value, int i, int j); // // optimized version
         of add_value (make the loop faster using temp node address)

    public:
    matrix_linked_list(); // constructor
    matrix_linked_list(int m); // constructor
    matrix_linked_list(const matrix_linked_list& a); // copy constructor for return
    ~matrix_linked_list(); //destructor
    void load(long double **matrix, int m); // receive square matrix and size mxm
    void delete_all_node();
    void print_values();
    void print_values_ij(); //print the data and also print i,j
    void set_main_head_node(head_node *main_head_node);
    long double get_value(int i, int j);
    void add_value(long double value, int i, int j); // add value to the existing i,j
        if not -> create new node
    matrix_linked_list operator^ (int n);
    matrix_linked_list& operator= (const matrix_linked_list& a); // assigment
};
```

To create a linked list matrix, we used two different data structure: a head_node struct for the elements which is located at the first column of the matrix, this head node contains the data, i, j values and two pointers: a node pointer to the next right node, and a head_node pointer to the next down head node. Another data structure is a node structure which contains the data, j value and a node pointer to the next right node.

In this class we implemented two extra functions: add_value_loop_opt() and get_value_loop_opt() that make the matrix multiplication faster, these function are optimized function of add_value() and get_value() that must be used inside a loop, more details will be explained in the subsection 4.2.

4 Results

Two solutions of C++ matrix power operation result are displayed in the listing 3, we can observe that the $m = 3$ and $n = 3$ operation was calculated correctly, also we can notice from the listing that in the linked list solution the memory is not allocated when the value equal to 0.

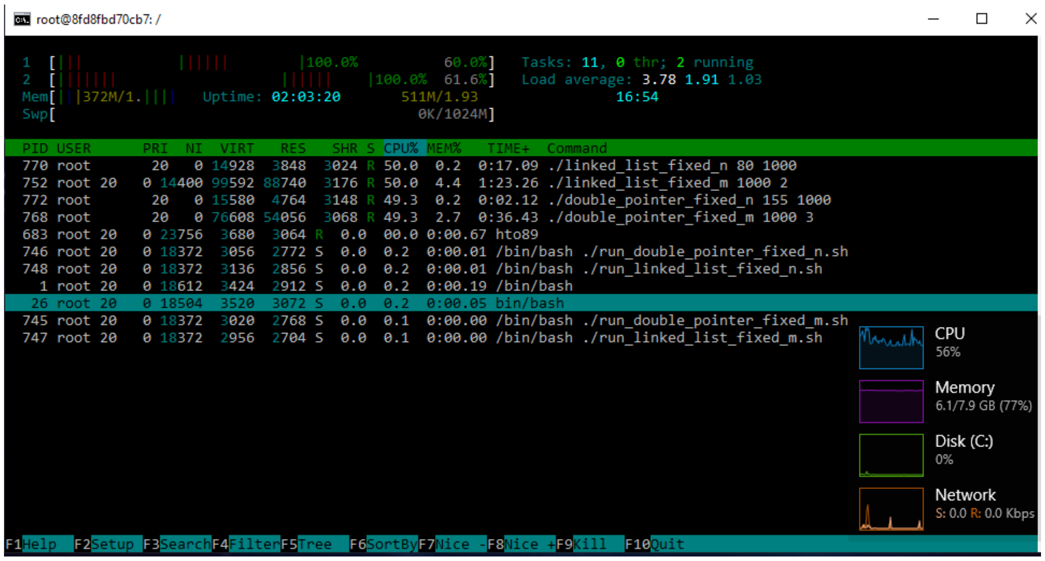
Listing 3: matrix power results

```
Input (m x m) matrix dimension m: 3
3
Input n from (m x m)^n: 3
3

matrix values // random double pointer matrix mxm
8:0:1:
0:3:1:
6:5:2:

matrix values (i,j) // random linked list matrix mxm
(0,0) 8 : ( ,2) 1 :
(1,1) 3 : ( ,2) 1 :
(2,0) 6 : ( ,1) 5 : ( ,2) 2 :

matrix values // double pointer matrix power result m^n
620:65:95:
78:67:30:
570:150:115:

matrix values (i,j) // linked list matrix power result m^n
```

```
(0,0) 620 : ( ,1) 65 : ( ,2) 95 :
(1,0) 78 : ( ,1) 67 : ( ,2) 30 :
(2,0) 570 : ( ,1) 150 : ( ,2) 115 :
```

4.1 Process Running Time

First, we run two different solutions of matrix power operation with c++ codes simultaneously in 4 different tasks (fig. 1) which each task consumes 50% of each core (detected by docker). Then, we obtained the processing time from four different methods of running time calculution: g++ gprof profiler, clock() function, time() function and the gettimeofday() function, the results are shown in figure 2.

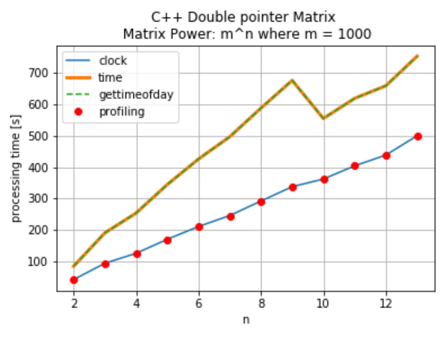


```
root@8fd8fbd70cb7: /

1  [|||           ||||||          |100.0%           60.0%]   Tasks: 11, 0 thr; 2 running
2  [|||||||                 ||||||     |100.0%  61.6%]   Load average: 3.78 1.91 1.03
Mem[|||372M/1.||||   Uptime: 02:03:20       511M/1.93                  16:54
Swp[                                          0K/1024M]

PID USER       PRI  NI  VIRT   RES   SHR S CPU% MEM%   TIME+  Command
770 root        20   0 14928  3848  3024 R 50.0  0.2  0:17.09 ./linked_list_fixed_n 80 1000
752 root 20    0 14400 99592 88740  3176 R 50.0  4.4  1:23.26 ./linked_list_fixed_m 1000 2
772 root        20   0 15580  4764  3148 R 49.3  0.2  0:02.12 ./double_pointer_fixed_n 155 1000
768 root        20   0 76608 54056  3068 R 49.3  2.7  0:36.43 ./double_pointer_fixed_m 1000 3
683 root 20    0 23756  3680  3064 R  0.0  00.0 0:00.67 hto89
746 root 20    0 18372  3056  2772 S  0.0  0.2  0:00.01 /bin/bash ./run_double_pointer_fixed_n.sh
748 root 20    0 18372  3136  2856 S  0.0  0.2  0:00.01 /bin/bash ./run_linked_list_fixed_n.sh
  1 root 20    0 18612  3424  2912 S  0.0  0.2  0:00.19 /bin/bash
 26 root 20    0 18504  3520  3072 S  0.0  0.2  0:00.05 bin/bash
745 root 20    0 18372  3020  2768 S  0.0  0.1  0:00.00 /bin/bash ./run_double_pointer_fixed_m.sh
747 root 20    0 18372  2956  2704 S  0.0  0.1  0:00.00 /bin/bash ./run_linked_list_fixed_m.sh

F1Help  F2Setup F3SearchF4FilterF5Tree  F6SortByF7Nice -F8Nice +F9Kill  F10Quit
```

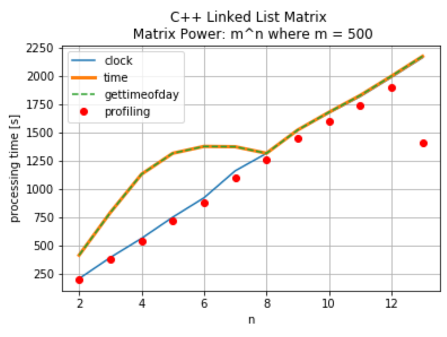
Figure 1: Running the code in different thread

As described above, the time() and getmeofday() work with wall time, this time may take longer than the processing time that the CPU assigned to a specific task because many tasks share the CPU resource. The clock() the time from profiling is actually CPU processing time, this gives us the time that the CPU used to run the program. For example, at the beginning of figure 2 (a) the wall time (time and gettimeof day) is twice bigger than the clock time, this is because we started the 4 tasks simultaneously and each task consume 50% of 1 core of the CPU, so the resource of the core is shared by two tasks. This behavior can be observed in every sub-figure from figure 2. However, after some tasks finished, the core resource is released, so another task will take the resource, and this will change the previous relation between CPU clock time and wall time. Additionally, we can observe that at the end of figure 2 (b) and (c) the CPU clock time is equal to wall time since at that moment each task was taking 100% of each core of the CPU.

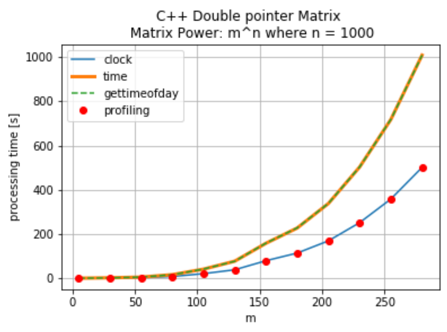
Another important aspect can be seen in figure 2 (b) and (d), is that clock() time was very similar to profiling time, but when the processing time is bigger than 2000 the time from the g++ profiler become strange, this seems to be that the time variable from g++ profile has some problem with overflow. Thus, in the following analysis, we will focus on the processing time calculated by the clock() function.



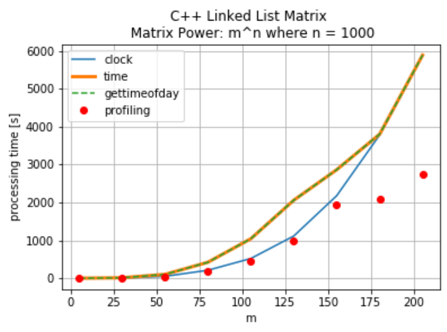
(a) Double Pointer fixed m



(b) Linked List fixed m



(c) Double Pointer fixed n



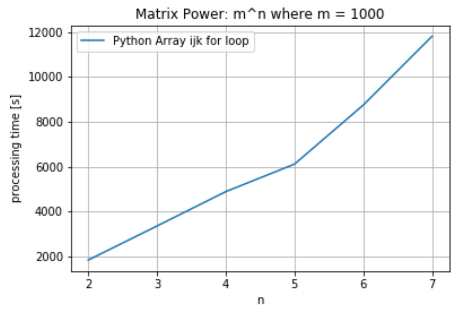
(d) Linked List fixed n

Figure 2: C++ Running Time Using Profiler and Clock

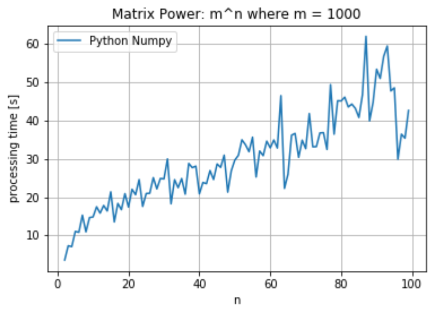
The figures 2 (a) and (b) show the double pointer solution and the linked list solution respectively, both have processed a matrix power operation with fixed m, we can observe that these figures follow a linear trend, this behavior is supported by the equation 2 which shows a linear equation when we make the m constant. In contrast, the figures 2 (c) and (d) show the result of matrix power operation with fixed n, the graph seems to follow a polynomial behavior, this can also be demonstrated by the equation 2 when we make the n constant, the equation becomes polynomial function.

Additionally, the figures show that the double pointer matrix is much faster than the linked list, this is because that the double pointer matrix memory is contiguous in each row and the memory access can be faster. In contrast, for the linked list it needs to access no contiguous memory location each time, also it requires memory allocation for each node in different time, and the memory deletion must be done separately for each node, and for the object based programming it need to copy each node when we use assignment operator (=) and copy constructor. All these factors make the linked list slower.

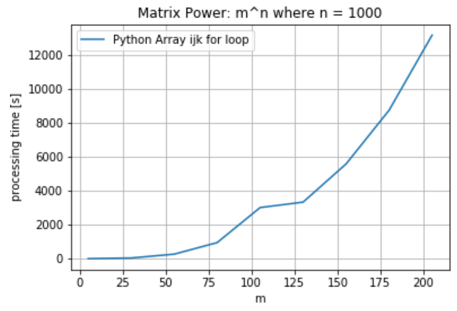
The figure 3 follows the same pattern as described above. One observation is that the python for loop is tremendously slow because python is a scripting language. However, we can also observe from the graph that the numpy matrix_power() function is extremely fast, even it is much faster than c++ solution, this is because that the numpy library is written in a very optimized c code [3].



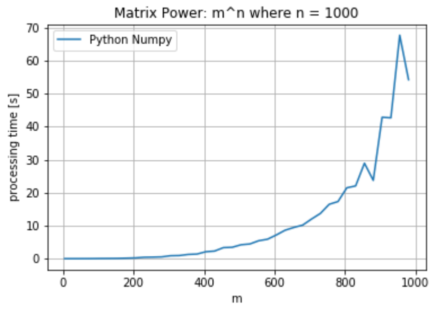
(a) for loop fixed m



(b) numpy matrix_power() fixed m



(c) for loop fixed n



(d) numpy matrix_power() fixed n

Figure 3: Python Running Time Using time library

4.2 Profiler and Optimization

For this project, we have used profiler, clock(), time() and gettimeofday() functions to calculate the process running time, despite the overflow problem, the profiler method gave us a more detailed result, and this allows us to identify which function of the code is consuming more time. For example, the listing 4 shows us the profiling result of matrix power using linked list where $m = 80$ and $n = 1000$.

Listing 4: matrix_double_pointer class

```
Flat  profile:

Each sample counts as 0.01 seconds.
%   cumulative   self              self     total
time   seconds   seconds    calls   s/call   s/call   name
64.37    518.02   518.02 1022976000   0.00     0.00   matrix_linked_list::get_value(int, int)
34.78    797.90   279.88 511488000   0.00     0.00   matrix_linked_list::add_value(long double, int, int)
0.77     804.07     6.17        1     6.17   804.58   matrix_linked_list::operator^(int)
0.07     804.60     0.53                              matrix_linked_list:: print_values_ij ()
0.04     804.92     0.32     2004     0.00     0.00   matrix_linked_list::delete_all_node()
```

```
0.02    805.11    0.19    1000    0.00    0.00   matrix_linked_list ::operator=(matrix_linked_list const&)
0.00    805.11    0.00       5    0.00    0.00   matrix_linked_list ::~ matrix_linked_list ()
0.00    805.11    0.00       2    0.00    0.00   matrix_linked_list :: matrix_linked_list ( matrix_linked_list const&)
0.00    805.11    0.00       2    0.00    0.00   matrix_linked_list :: matrix_linked_list ()
0.00    805.11    0.00       1    0.00    0.00  _GLOBAL__sub_I__ZN21matrix_double_pointerC2Ei
0.00    805.11    0.00       1    0.00    0.00    __static_initialization_and_destruction_0 (int , int )
0.00    805.11    0.00       1    0.00    0.00   matrix_linked_list ::load(long double**, int)
0.00    805.11    0.00       1    0.00    0.00   matrix_linked_list :: matrix_linked_list (int )
0.00    805.11    0.00       1    0.00    0.00  matrix_double_pointer::get_matrix()
0.00    805.11    0.00       1    0.00    0.00  matrix_double_pointer:: init_rand ()
0.00    805.11    0.00       1    0.00    0.00  matrix_double_pointer::matrix_double_pointer(int)
0.00    805.11    0.00       1    0.00    0.00  matrix_double_pointer::~matrix_double_pointer()
```

As displayed in listing 4, the total processing time of the programa was 805.11 seconds, the operator $\wedge$ consumed 804.58 seconds per call, inside the operator $\wedge$ we have get_value() function that consumed 518.02 seconds and add_value() function that consumed 279.88 seconds. Consequently, with this information we have performed some changes to improve the processing time in the linked list code, the performance of these changes is displayed in figure 4 :

a) Without any change, so the address of head_node and node will reset in each loop iteration.

b) We keep the head_node address of the previous loop in the get_value() function.

c) We keep the head_node and the node addresses of the previous loop in the get_value() function.

d) We keep the head_node and the node addresses of the previous loop in get_value() and add_value() function.

we can see clearly how these changed made the processing time of the program faster.

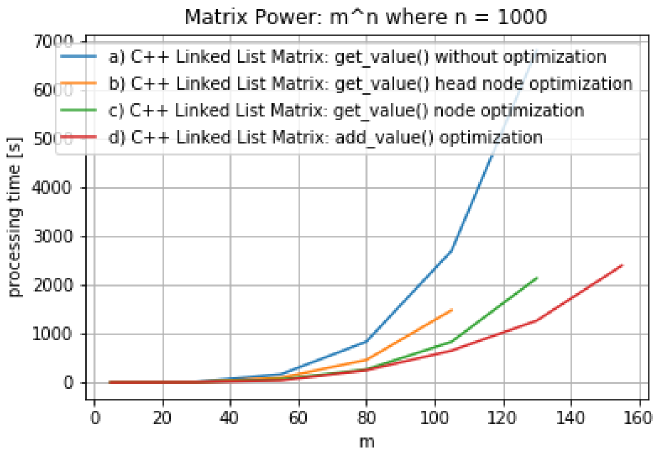


Figure 4: C++ linked list running time (clock) after applying changes

5 Conclusion

In conclusion, we have implemented four versions of matrix multiplication solution, two in c++ and two in python, we found that in c++ the double pointer matrix was faster than the linked list one, because in the linked list version there is a lot of memory access and allocation work, and we also noticed that the python ijk loop was much slower than c++ solutions, this is because python is a scripting language. However, the version with matrix_power() from numpy library performed much faster than the c++ solutions since the numpy library is coded in high optimized c code.

Moreover, we have compared the different meaning of timing functions, then we found that clock() time and profiling time was better since it calculates the CPU processing time, and we detected a potential bug in g++ profiling related with overflow.

In addition, we have demonstrated with figures that the behavior of the processing time follow the equation trend of the equation: $[2m^3 - m^2] * (n-1)$.

Finally, we consider that the profiling method is very useful for coding optimization, and we were able to improve the running time linked list with the help of profiling.

References

[1] Measure time in python – time.time() vs time.clock(). `https://www.pythoncentral.io/measure-time-in-python-time-time-vs-time-clock/`. Accessed: 2019-04-10.

[2] Classes with pointer data members. `http://pages.cs.wisc.edu/~hasti/cs368/CppTutorial/NOTES/CLASSES-PTRS.html`. Accessed: 2019-04-08.

[3] Numpy. `https://introcs.cs.princeton.edu/python/appendix_numpy/`. Accessed: 2019-04-10.