*Multivariate Behavioral Research, 24* (3), 285-305


# Model Modification in Covariance Structure Analysis: Application of the Expected Parameter Change Statistic

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This paper examines the problem of model modification in covariance structure analysis. Two methods of model modification are studied: the Modification Index (MI) which suggests modifications based on the largest drop in the overall value of the test statistic, and the Expected Parameter Change Statistic (EPC) which suggests modifications based on the removal of large and interesting specifications errors. Following a detailed discussion of the theory behind the MI and EPC, these methods are studied and applied to two specifications of the Wisconsin status attainment model. Additionally, a standardized version of the EPC statistic (SEPC) is proposed and applied to one of these models. Results indicate that the MI tends to suggest freeing substantively implausible parameters. The EPC and SEPC, by contrast, suggest freeing substantively interesting parameters. Results are discussed in terms of the practice of covariance structure modeling.

In the routine practice of structural equation modeling, a researcher may find that his/her model is not in agreement with the data as evidenced by standard statistical tests. Lack of agreement may be due to the fact that distributional assumptions have been violated (Boomsma, 1983; Muthen & Kaplan, 1985; in press), problems of missing data (Muthen, Kaplan, & Hollis, 1987), and/or model misspecification. Also, it may be the case that the test statistic is overly sensitive to the size of the sample, such that trivial misspecifications are being detected. The problem of strong sensitivity to sample size is particularly relevant for models which are estimated on large samples.

The relationship between sample size and size of misspecification implies that the power of the test statistic needs to be considered. Power refers to the probability of rejection of the null hypothesis implied by the model when the null hypothesis is false. Assessing power allows us to determine the extent to which the test statistic is strongly sensitive to sample size. Recent theoretical developments by Satorra and Saris (1985) make it possible to calculate power

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The author would like to thank Bengt Muthen, Albert Satorra, Peter Bentler, and James Crouse for their valuable comments on an earlier version of this paper. Requests for reprints should be sent to the author, Department of Educational Studies, University of Delaware, Newark, DE 19716.

in the covariance structure modeling framework using standard software such as LISREL (see also Matsueda and Bielby, 1986). To calculate power, however, it is necessary to specify alternative models for which one wishes to detect an error. Specifying alternatives and calculating the power function may be tedious for large models. Also, assessing power may detract from the important issue of finding serious misspecifications and removing them. Removing potentially serious misspecifications involves modifying the model.

The most common method for modifying covariance structure models is the Modification Index (MI) (Jöreskog and Sörbom, 1986). Essentially, the MI gives the expected drop in the overall value of the test statistic when a restriction implied by the model is relaxed. Recent work by MacCallum (1986), utilizing MI's for specification searches, found that the MI was not particularly successful in recovering a misspecified parameter from a true model that was misspecified. This finding was also substantiated in a population study by Kaplan (1988).

In this paper, the MI is compared to the expected parameter change statistic (EPC) recently proposed by Saris, Satorra, and Sörbom (1987). In particular, Saris, Satorra, and Sörbom found that one can approximate the size of a misspecified parameter by examining the MI for each fixed parameter and conducting a sensitivity analysis. Such a sensitivity analysis could, however, become tedious. The expected change statistic developed by Saris et al. (1987) is based on the MI and a function of the first order derivative of the fitting function when evaluated at the fixed parameter. However, because the metric of the observed variables is often arbitrary, it is necessary to standardize the EPC in order to allow valid comparisions. This paper provides a standardized verison of the EPC and compares it to the unstandardized EPC and the MI for a selected set of empirical models. The MI and EPC were compared in a Monte Carlo study by Luijben, Boomsma, and Molenaar (1987), wherein, among other things, it was found that the EPC was more successful in pointing to the misspecified parameter than the MI.

The empirical models to be considered are taken from the status attainment literature for illustrative purposes only. Specifically, this paper utilizes the early work of Sewell, Haller, and Ohlendorf (1970), hereafter SHO, and the extension of their work by Hauser, Tsai, and Sewell (1983), hereafter HTS. These works embody what has traditionally been called the Wisconsin model of status attainment.

Because model modification is essentially a substantive problem, a brief summary of the nature of the Wisconsin model is needed. The model of SHO elaborates the seminal work of Blau and Duncan (1967) by considering social-psychological variables that mediate the effects of background variables such as socioeconomic status and mental ability on attainment outcomes. In particular, SHO estimated their model on samples from a variety of size-of-place communities. On the basis of inspecting residual correlation matrices, SHO concluded that there was support for the social-psychological interpretation of status attainment.

The model of HTS expanded on SHO by considering the problem of measurement error. Specifically, HTS obtained at least two indicators of each construct of interest, thereby allowing for the application of confirmatory factor analysis to purge response errors from the observed variables. Also of interest in the HTS paper is their measurement of socioeconomic status (SES), academic performance (AP), and significant others' influence (SOI). In all three cases, these constructs were formed as composites rather than as factors. Thus, instead of SES generating the responses on such variables as father's occupational status and mother's educational attainment, these variables are weighted in such a way as to form SES. The same is true for AP and SOI composites. In general HTS concluded that there is support for rejecting the factor specifications of SES, AP, and SOI in favor of the composite specification. Furthermore HTS found general support for the earlier SHO specification of the Wisconsin model.

The application of the MI and the EPC to the Wisconsin status attainment models is motivated by the fact that these models utilize structural equation methodology, and that in both cases the models do not fit the data as evidenced by standard statistical tests. In particular, the SHO status attainment model was reanalyzed via LISREL where it was found that the model was not consistent with the empirical data. These results are discussed in Section 4 below. Although SHO talk about model fit, it is in terms of the size of the elements in the residual correlation matrices. Lack of fit is a rather serious finding because (as shown in e.g. Kaplan, 1988) proper inferences based on the results depend on these models fitting the data. It is of interest then to determine if the MI or EPC (or both) provide useful information in improving the fit of these models.

The remainder of this paper is organized as follows: Section 2 provides a detailed background of the models and statistics considered. Section 3 gives the design of the study. Section 4 presents the results and Section 5 concludes with a summary and discussion of how the results bear on modeling practice.

## *Statistical Background*

### *General Model and Special Cases*

The models to be considered can be written in a general form as a system of linear simultaneous equations among a set of unknown or latent variables:

$$\eta = \mathbf{B}\eta + \Gamma\xi + \zeta, \tag{1}$$

where $\eta$ is a (m x 1) vector of latent endogenous factors, $\xi$ is a (n x 1) vector of latent exogenous factors, **B** and $\Gamma$ are coefficient matrices of order (m x m) and (m x n) respectively, and $\zeta$ is a (m x 1) vector of disturbances where Var($\zeta$) = $\Psi$ (m x m). The latent variables $\eta$ and $\xi$ are related to observed variables via the factor analytic equations

[2] $$\mathbf{y} = \Lambda_y \eta + \varepsilon,$$

and

[3] $$\mathbf{x} = \Lambda_x \xi + \delta,$$

where **y** and **x** are (p x 1) and (q x 1) vectors of observed variables, $\Lambda_y$ and $\Lambda_x$ are factor loading matrices of order (p x m) and (q x n) respectively, and $\varepsilon$ and $\delta$ are vectors of measurement errors where Var($\varepsilon$) = $\Theta_\varepsilon$ and Var($\delta$) = $\Theta_\delta$.

Under standard assumptions the model in Equations 1, 2, and 3 give rise to a covariance structure $\Sigma = \Sigma(\theta)$, where $\Sigma(\theta)$ is a (p + q) x (p + q) symmetric matrix valued function of an (s x 1) parameter vector $\theta$. It is assumed throughout this paper that the parameters of the model are identified; that is $\Sigma(\theta_1) = \Sigma(\theta_2)$ implies $\theta_1 = \theta_2$. This model has been discussed in Jöreskog (1977).

We will also be interested in the special case where B is lower triangular, $\Lambda_y = \mathbf{I}$ and $\Lambda_x = \mathbf{I}$, and $\Theta_\delta = \mathbf{0}$ and $\Theta_\varepsilon = \mathbf{0}$, where **I** is the identity matrix and **0** is the null matrix, each of appropriate order. With $\Psi$ lower triangular, the general model for latent variables becomes the block-recursive simultaneous equation model among observed variables of the type considered by SHO and Sewell, Haller, and Portes (1969).

The most common method for estimating parameters of the general model is maximum likelihood (ML). The ML fitting function is

[4] $$F(\mathbf{S}, \Sigma(\theta) = \log|\Sigma(\theta)| + \mathrm{tr}\{\mathbf{S}(\Sigma(\theta))^{-1}\} - \log|\mathbf{S}| - (p + q),$$

where S is the unbiased sample covariance matrix. ML estimates are derived from a solution to the first order conditions $fF/f\theta = \mathbf{0}$.

The likelihood ratio test (LR) used to test such models is defined as

[5] $$LR = n\{F(\mathbf{S}, \Sigma(\theta_\omega)) - F(\mathbf{S}, \Sigma(\theta_\Omega))\},$$

where $\theta_\omega$ and $\theta_\Omega$ are vectors minimizing $F(\mathbf{S}, \Sigma(\theta))$ under the null ($H_0$) and alternative ($H_1$) hypotheses respectively, and n= N-1. Rejection of $H_0$ occurs when LR exceeds a critical point $K_\alpha$.

*Model Modification via the Modification Index*

As stated in the introduction, the MI is commonly used in modifying misfitting models. The MI is a function of the Lagrange multiplier (LM) diagnostic and is calculated as the derivative of the fitting function evaluated at the fixed parameter, scaled to a chi-square metric (Jöreskog and Sörbom, 1986). The most recent version of the MI is identical in form to the Score statistic (Rao, 1973). The Score statistic and the LM test are equivalent and the Score form has historical precedence over the LM form (see e.g. Breusch and Pagan, 1980; Buse, 1982; Rao, 1973). Furthermore, the score statistic is asymptotically equivalent to the LR test. It follows then, that because the MI is the same as the Score statistic it is also asymptotically equivalent to the LR test statistic and hence shares the same asymptotic properties.

*Power and the Expected Parameter Change Statistic*

Power in the LR context is defined as

[6] $$\Pr\{ LR > K_{\alpha} | \theta_{A} \}$$

where the probability depends on the alternative parameter $\theta_A$ which is assumed to lie near $\omega$ (see e.g. Satorra and Saris, 1985).

A method for asessing the power of the likelihood rato test in the covariance structure modeling framework was developed by Satorra and Saris (1985) and can be easily implemented in standard software such as LISREL and EQS (Bentler, 1985). The theoretical background for deriving the power of the LR test rests on the observation that the noncentrality parameter of the noncentral chi-square distribution (corresponding to the distribution of the test statistic when $H_0$ is false) can be approximated by the value

[7] $$LR_A = n_0 F(\Sigma_A, \Sigma(\theta^*_A)),$$

where $n_0 = N_0 - 1$, $\Sigma_A = \Sigma(\theta_A)$, and $\theta^*_A$ is the vector minimizing $F(\Sigma_A, \Sigma(\theta))$ in $\omega$. The method for assessing power has been outlined in Saris and Stronkhorst (1984) and will not be discussed here.

The procedure for calculating power is based on asymptotic theory and gives an asymptotic approximation to the true power. The finite sample properties of this procedure are also of interest and these have been studied by Satorra and Saris (1982; 1985).

Saris and Satorra (1987) have studied how various model characteristics affect the power approximation procedure. Of importance to this study is their finding that the LR test has unequal power for the same size misspecfication in different places within a model. This means that only some misspecifications will be detected. Saris, Satorra, and Sörbom (1987) replicated this finding and argued that because the overall model test examines multiple hypotheses, only a very elaborate power study of many possible misspecifications could give enough information to draw any conclusions. A power study of that scope could become tedious for large models.

Recently, Satorra (1989) found that the MI could be used to approximate the noncentrality parameter for some alternative covariance matrix $\Sigma_A$. This finding was based on the known asymptotic equivalence between the LM and LR tests (see e.g. Buse, 1982) and the fact that, as stated above, the MI is equivalent to the LM test. Because there is an MI associated with each fixed parameter, one can obtain power for each restriction in the model. Satorra has shown that approximating power using the MI as the noncentrality parameter and approximating power using the Saris-Satorra procedure are asymptotically equal.

In a related investigation, Saris, Satorra, and Sörbom (1987) found that one can approximate the size of a misspecified parameter by examining the MI for the fixed parameter and conducting a sensitivity analysis. In particular, they found that by examining power for a variety of possible values of the misspecified parameter and comparing the obtained noncentrality parameter associated with each value with the MI for that parameter, the size of the misspecified parameter could be approximated. They found that an examination of the MI alone might lead to freeing a parameter that was small in absolute value. In combination with a sensitivity analysis, however, they were able to determine which parameter would yield a large value if freed. Thus they argued for the importance of assessing the size of a misspecified parameter.

To estimate the size of a misspecified parameter, Saris et al. (1987) developed an index of the expected change in the value of a parameter if that parameter was freed. Following their discussion, let $\theta_i$ be a parameter that takes on the value $\theta_0$ under $H_0$, and let $d\theta_i = f\ln L(\theta)/f\theta_i$ evaluated at $\hat{\theta}_i$. Saris et al. show that the expected change or shift in the parameter can be derived as

$$[8] \qquad \theta_i - \theta_0 = MI/d_{\theta i}.$$

The proof of Equation 8 is given in Saris et al. Computationally, the index in Equation 8 can be calculated in LISREL as -(N - 1) times the "FIRST ORDER DERIVATIVES", where N is the sample size. In many applications $\theta_i$ is fixed to zero indicating the absence of an effect. Asymptotic theory for the EPC is given in Satorra (1989).

Saris et al. (1987) discuss four possible outcomes that might occur using the EPC statistic. These outcomes are of importance to this study. First, a large MI might be associated with a large EPC. Here one would be justified in freeing the parameter, especially if there is theoretical justification for doing so. Second, a large MI might be associated with a small EPC. Here, Saris et al. argue that it does not make sense to free this parameter despite the large drop in chi-square, because the obtained parameter estimate is likely to be trivial. In the third situation, a small MI might be associated with a large EPC. Here the situation is ambiguous and might be due to sampling variability or the fact that the test statistic is not sensitive to this parameter. A more detailed power analysis might be necessary. Finally, the fourth outcome might be a small MI associated with a small EPC. Clearly, there would be little interest in freeing this parameter.

*Standardized Expected Parameter Change Statistic*

The EPC statistic represents a shift of focus away from improving model fit in terms of chi-square and toward removing large and perhaps theoretically important misspecifications. For a given model there exists an EPC for each fixed parameter. However, because the metric of the observed variables is often arbitrary, it is necessary to standardize the EPC in order to allow valid comparisons.

In the context of ML estimation of path analysis models using correlation matrices, the EPC is in a standardized metric. In structural models among latent variables, however, the metric problem is not removed by simply using correlation matrices.

To standardize the EPC, it is necessary to recognize that the same logic applies here as when standardizing any free parameter. For a general structural model such as that given in Equation 1, standardization of the EPC requires the standard deviations of the endogenous and exogenous constructs associated with the parameter of interest. For example, the SEPC associated with a fixed element of $\Gamma$, say $\theta_{\gamma}^{sepc}$ would be calculated as $[\hat{V}(\xi)/\hat{V}(\eta)]^{1/2} * \theta_{\gamma}^{epc}$, where $\hat{V}(\xi)$ is an appropriate diagonal element of $\hat{\Phi}$ and $\hat{V}(\eta)$ is expressed in terms of other model parameters. It should be noted that even though the variances of the endogenous constructs are themselves functions of other model parameters, the information is directly obtainable from the output of LISREL. Thus, SEPC's for the regression of endogenous constructs on other endogenous constructs (contained in **B**) can be easily obtained. Having obtained the standardized expected parameter change statistic (hereafter referred to as SEPC) it is possible to compare values for all fixed parameters of the model.

Before turning to the design of the study two points need to be made. First, EPC methodology involves modifying the proposed model in the direction of adding parameters. Although adding parameters will reduce the value of the test statistic and perhaps yield information about large specification errors, there will be an increase in degrees-of-freedom and some loss of parsimony. With the addition of more parameters it might be useful to "trim" the model by fixing nonsignificant or otherwise trivial parameters. An assessment of the effects of deleting parameters on the test statistic can be obtained by squaring the t-value associated with the parameter of interest. This is equivalent to the Wald test, and because the Wald test is asymptotically equivalent to the LR test (Buse, 1982), it gives the expected increase in the overall value of the test statistic if that parameter is fixed.

Second, it is important to emphasize that EPC and SEPC methodology is only capable of establishing if there are large specification errors given the specific set of variables included. The importance of these errors is a matter of theoretical concern. Thus the "internal" specification of the model is examined for omitted paths that may be large. The problem of omitted variables and their relationships to the variables included in the model is not addressed by these methods.

## *Design*

The assessment of the specification status for the SHO model will be carried out as follows: Of the six size-of-place communities, three will be chosen to keep tables at a minimum: Large City (N=686), Farm (N=857), and Total (N=4388). For each community (ordered from smallest sample size to largest), the SEPC will be calculated for all relevant fixed parameters. Note that because the SHO model is a path analysis model and the data are standardized to have unit variances, the EPC and SEPC are identical. The fixed parameter with the largest EPC and largest MI will be freed first, subject to the condition that the choice is based on reasonable sociological theory. These parameters will be underlined in the tables for ease of reading. For the purposes of this study, we consider an EPC > .10 substantively large. After freeing the parameter, the EPC statistics will be recomputed and the next largest parameter will be freed. This will continue until no substantively meaningful parameters remain.

It should be pointed out that the results presented for the SHO analysis are based on secondary analyses of published correlation matrices. Characteristics of the data such as normality and missing data are not known though for the latter problem, Sewell and Hauser (1975) suggest that the sample nonresponse in the Wisconsin data set was very low. This implies that if the model still does not fit after this process, it may be due to (a) normality issues, (b) sensitivity of the test statistic to the sample size, such that trivial errors are being detected, or (c) additional misspecifications such as omitted exogenous variables. Of course, all three problems may be occurring simultaneously.

The specification status of the HTS model will be evaluated via the MI and SEPC statistic in the same manner as the SHO, model. Here the EPC and SEPC are not the same. Focus of attention will be on the "modified SHO" specification (Hauser, et al., 1983, p. 29) and will be assessed under the assumption of multivariate normality. An inspection of the distributions of the raw data for HTS revealed that the assumption of marginal univariate normality roughly holds. Therefore, it is argued that not much improvement in the overall value of the test statistic will be gained when utilizing an asymptotic distribution-free estimator such as ADF (Browne, 1984), or the categorical variable methodology estimator of Muthen (1984; see also Muthen & Kaplan, 1985; in press).

## *Results*

### *SHO LISREL Reanalysis*

Figure 1 displays the SHO model along with definitions of the acronyms for the variables in the model. Table 1 presents the LISREL reanalysis of the three SHO communities considered. It can be seen that in all cases the model does not fit the data. For the Farm sample, model fit could be argued on the basis of the low $\chi^2$/df ratio. However, Kaplan (1988) has shown that this ratio could lead investigators to entertain models possessing unacceptable parameter estimate bias. In addition to lack of statistical fit, inspection of the maximum modification index (MMI) reveals that in all cases the parameter to be freed cannot logically be justified. Specifically, the MMI points to the parameter $\beta_{52}$ which refers to the regression of SOI on Educational Attainment (EA), where EA was measured later in time than SOI. Although temporal difference is not the crucial factor, it should be noted that SOI was measured as a retrospective self report, whereas EA was measured as the number of years of actual schooling. Hence, it does not seem logically justifiable to include this affect in the model. Thus, it can be seen that for this model the MMI is an unreliable index for model inprovement.

In addition to the MMI, Table 1 gives the EPC associated with the MMI. It can be seen that in all cases the EPC is negative. This is implausible given the interpretation of $\beta_{52}$. Furthermore, for the Large City community and Farm Community, the EPC's are greater than 1.0 which is inadmissible considering that the variables are in a standardized metric.

Table 1
Chi-Square Goodness of Fit and MMI for Revised SHO Model

| Community | N | $\chi^2(13)$ | $\chi^2/df$ | MMI | EPC |
|---|---|---|---|---|---|
| Large City | 686 | 132.04 | 10.157 | $\beta_{52}$ | -1.373 |
| Village | 816 | 50.09 | 3.853 | $\beta_{54}$ | -0.577 |
| Farm | 857 | 93.47 | 7.190 | $\beta_{52}$ | -1.043 |
| Medium City | 935 | 144.85 | 11.142 | $\beta_{52}$ | -0.763 |
| Small City | 1084 | 160.22 | 12.325 | $\beta_{52}$ | -0.869 |
| Total | 4388 | 669.70 | 51.515 | $\beta_{52}$ | -0.950 |

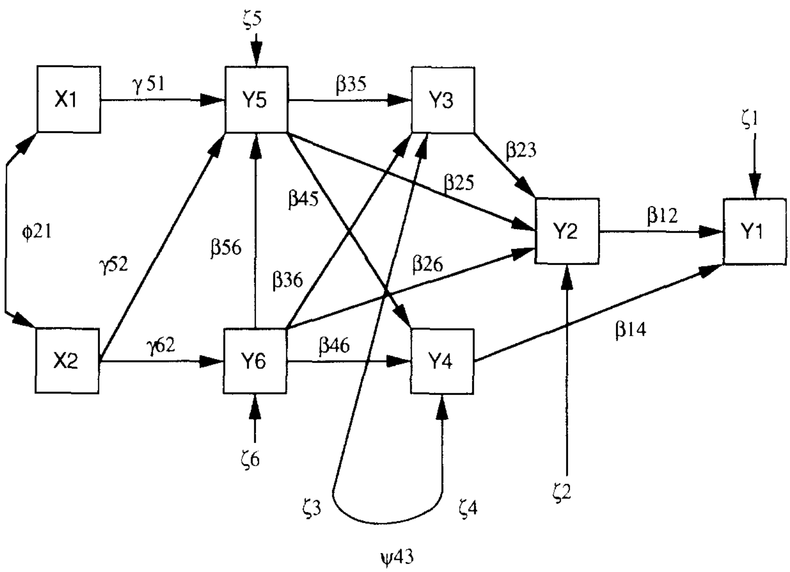


**Figure 1.**
Sewell, Haller, and Ohlendorf (1970) Wisconsin Status Attainment Model. Note: X1=Socioeconomic Status (SES), X2=Mental Ability (MA), Y1=Occupational Attainment (OA), Y2=Educational Attainment (EA), Y3=Level of Educational Aspiration (LEA), Y4=Level of Occupational Aspiration (LOA), Y5=Significant Others' Influence (SOI), Y6=Academic Performance (AP).

*SHO Large City Sample*

Table 2 displays the MI and EPC for the Large City sample. If we focus on only those parameters that make substantive sense, it can be seen that the parameter with the largest MI and largest EPC is $\gamma_{22}$, the direct effect of Mental Ability (MA) on EA. Freeing this parameter yields a significant improvement in the overall fit of the model ($\Delta\chi^2_{(1)} = 40.74$).

With $\gamma_{22}$ included, the EPC was recomputed and the results are also displayed in Table 2. It can be seen that the next parameter to be freed on the basis on the EPC should be $\beta_{65}$, the regression of AP on SOI. It was decided not to free this parameter because the MI was low and it would change the recursive nature of the initial model. The next parameter with the largest EPC associated with a large MI is $\gamma_{21}$, the direct effect of SES on EA. Freeing this parameter further improves the model fit ($\Delta\chi^2_{(1)} = 23.24$).

An inspection of Table 2 also shows that the EPC's and MI's after $\gamma_{22}$ and $\gamma_{21}$ have been included. It can be seen that there are no remaining parameters (other than $\beta_{65}$) that have an EPC that equals or exceeds .10. Thus the final model for the Large City Sample using the EPC contains two substantively plausable paths from SES to EA and from MA to EA. These results suggest that in a large city with a presumably heterogenous population, educational attainment might be dependent on background factors over and above the social-psychological

Table 2
MI and EPC for SHO Large City Sample

| | No Error | | $\gamma_{22}$ Included | | $\gamma_{21}$,$\gamma_{22}$ Included | |
|---|---|---|---|---|---|---|
| Param.[a] | MI | EPC | MI | EPC | MI | EPC |
| $\beta_{65}$ | 0.661 | 0.193 | 0.661 | 0.193 | 0.661 | 0.193 |
| $\gamma_{11}$ | 3.888 | 0.061 | 3.928 | 0.062 | 4.191 | 0.066 |
| $\gamma_{12}$ | 0.581 | 0.025 | 0.655 | 0.028 | 0.661 | 0.028 |
| $\gamma_{21}$ | 30.137 | 0.152 | <u>21.915</u> | <u>0.129</u> | ----- | ----- |
| $\gamma_{22}$ | <u>38.655</u> | <u>0.199</u> | ----- | ----- | ----- | ----- |
| $\gamma_{31}$ | 2.845 | 0.044 | 2.845 | 0.044 | 2.845 | 0.044 |
| $\gamma_{32}$ | 10.511 | 0.097 | 10.511 | 0.097 | 10.511 | 0.097 |
| $\gamma_{41}$ | 13.635 | 0.095 | 13.635 | 0.095 | 13.635 | 0.095 |
| $\gamma_{42}$ | 1.702 | 0.039 | 1.702 | 0.039 | 1.701 | 0.039 |
| $\gamma_{61}$ | 0.661 | 0.028 | 0.661 | 0.028 | 0.661 | 0.028 |

[a] Only structural parameters that yield plausable results are shown.

factors postulated by SHO. It should be pointed out, however, that though there has been a significant reduction in the overall value of the goodness-of-fit statistic, the model is still not in agreement with the data.

*SHO Farm Sample*

The MI's and EPC's for the Farm sample are displayed in Table 3. The parameter associated with the largest MI and largest EPC is $\gamma_{21}$, the effect of SES on EA. With $\gamma_{21}$ included, the fit of the model significantly improves ($\Delta\chi^2_{(1)}$ = 19.45) but not enough to conclude that the model fits the data. Recomputing the MI's and EPC's after inclusion of $\gamma_{21}$ shows no other parameter could be freed that would significantly improve the fit of the model.

Table 3
MI and SEPC for SHO Farm Sample

| | No Error | | $\gamma_{21}$ Included | |
|---|---|---|---|---|
| Param. | MI | EPC | MI | EPC |
| $\gamma_{11}$ | 0.066 | 0.007 | 0.068 | 0.007 |
| $\gamma_{12}$ | 0.519 | 0.020 | 0.522 | 0.020 |
| $\gamma_{21}$ | 19.020 | 0.106 | ----- | ----- |
| $\gamma_{22}$ | 9.065 | 0.093 | 7.055 | 0.081 |
| $\gamma_{31}$ | 0.266 | -0.011 | 0.266 | -0.012 |
| $\gamma_{32}$ | 10.255 | 0.091 | 10.255 | 0.091 |
| $\gamma_{41}$ | 7.141 | 0.059 | 7.141 | 0.059 |
| $\gamma_{42}$ | 0.412 | 0.018 | 0.412 | 0.018 |
| $\gamma_{61}$ | 5.388 | 0.059 | 5.388 | 0.059 |

*SHO Total Sample*

For the Total sample, the test statistic is quite large as can be seen from Table 1. The EPC's and MI's are displayed in Table 4. As with other communities, the most substantial improvement both in terms of model fit and parameter change would occur by freeing $\gamma_{21}$. Freeing this parameter substantially improves the fit of the model ($\Delta\chi^2_{(1)}$ = 180.71), nevertheless the model still does not fit the data.

An inspection of Table 4 also shows the EPC's and MI's after inclusion of $\gamma_{21}$. Here it can be seen that many parameters are associated with large MI's, but the associated EPC's are small by our criterion. This corresponds to the second situation that might occur using the EPC — namely that if the goal is to correct substantively large and interesting misspecifications, then in this case there are no more parmeters to be freed (see Saris et al., 1987). Freeing $\gamma_{32}$, for example, would likely improve the fit of the model but would yield a rather trivial estimate for that parameter. We might conclude that the large MI's remaining are due mostly to the substantially larger sample, when compared to the separate communities. Thus for the Total sample, the power is too high probably owing to the very large sample size.

Table 4
MI and SEPC for SHO Total Sample Sample

| | No Error | | $\gamma_{21}$ Included | |
|---|---|---|---|---|
| Param. | MI | EPC | MI | EPC |
| $\gamma_{11}$ | 25.571 | 0.060 | 27.442 | 0.065 |
| $\gamma_{12}$ | 15.251 | 0.050 | 15.511 | 0.051 |
| $\gamma_{21}$ | 167.601 | 0.136 | ----- | ----- |
| $\gamma_{22}$ | 75.091 | 0.109 | 49.483 | 0.088 |
| $\gamma_{31}$ | 41.695 | 0.064 | 41.695 | 0.064 |
| $\gamma_{32}$ | 55.454 | 0.088 | 55.454 | 0.088 |
| $\gamma_{41}$ | 50.506 | 0.069 | 50.506 | 0.069 |
| $\gamma_{42}$ | 1.071 | 0.012 | 1.071 | 0.012 |
| $\gamma_{61}$ | 4.350 | 0.027 | 4.350 | 0.027 |

*HTS "Modified SHO" Model*

The HTS "Modified SHO" model is displayed in Figure 2 along with a description of each construct. HTS refer to this model as Model 4. In specifying this model, HTS were required to fix certain relevant parameters to zero because in earlier analyses, these parameters gave implausible results. In particular, the parameter $\beta_{17,15}$, the direct effect of educational attainment on mid-life occupational status, was negative. From Kaplan (1988), we note that the occurrance of an implausible effect such as this may be indicative of specification error elsewhere in the model. Nevertheless, even after fixing this parameter, Model 4 is not consistent with the data as evidenced by the large chi-square value ($\chi^2_{176}$ = 443.91, $p < .05$).

It should be noted that the MMI associated with Model 4 points to freeing $\psi_{16,13}$. The SEPC associated with this parameter is -0.089. Although this is a small effect, the substantive interpretation suggests that there is some variation between educational aspiration and early occupational status not accounted for by the current structural form of the model.

In examining the usefulness of the SEPC, focus of attention will be on structural regression coefficients. Table 5 presents the MI's, EPC's and SEPC's for a selected set of structural parameters of Model 4. The first fifteen of these parameters were fixed in the original specification (Model 1 of Table 2 in HTS) so as to define the "Modified SHO" model. An inspection of the MI's and SEPC's for the fixed parameters of the model suggest that $\beta_{16,13}$ should be freed

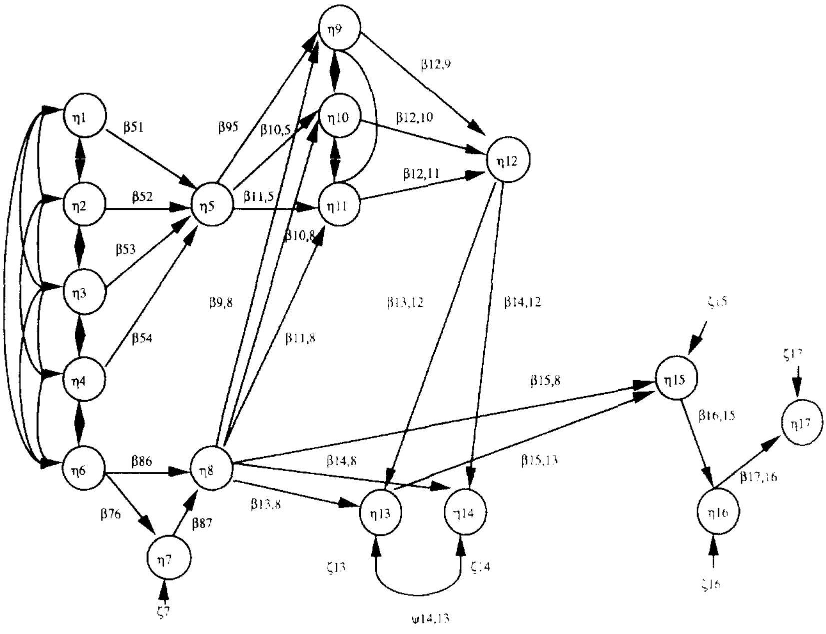


**Figure 2.**

Hauser, Tsai, and Sewell (1983) "Modified SHO" Wisconsin Status Attainment Model.[a] Note: η1=father's educational attainment, η2=mother's educational attainment, η3=father's occupational status, η4=parents' income, η5=socioeconomic status*, η6=mental ability, η7=rank in high school, η8=academic performance*, η9=parents' encouragement to attend college, η10=teachers' encouragement to attend college, η11=friends' encouragement to attend college, η12=significant others' influence on college attendance*, η13=educational aspiration, η14=occupational status aspirations, η15=educational attainment, η16=early occupational status, η17=mid-life occupational status. [a] Shown are latent variables. * Intervening latent variable without direct measurements.

Table 5
MI, EPC and SEPC for HTS "Modified SHO" model

| Parameter | MI | EPC | SEPC |
|---|---|---|---|
| $\beta_{7,5}$ | 2.779 | -0.010 | -0.033 |
| $\beta_{13,5}$ | 6.893 | -0.015 | -0.059 |
| $\beta_{14,5}$ | 4.943 | 0.013 | 0.051 |
| $\beta_{15,5}$ | 6.968 | 0.017 | 0.059 |
| $\beta_{16,5}$ | 13.251 | 0.017 | 0.068 |
| $\beta_{17,5}$ | 3.655 | 0.010 | 0.037 |
| $\beta_{16,8}$ | 0.792 | -0.005 | -0.018 |
| $\beta_{17,8}$ | 1.345 | -0.007 | -0.023 |
| $\beta_{16,12}$ | 0.862 | -0.028 | -0.045 |
| $\beta_{17,12}$ | 5.466 | -0.062 | -0.092 |
| $\beta_{16,13}$ | 61.917 | -0.349 | -0.351 |
| $\beta_{17,13}$ | 25.291 | -0.210 | -0.197 |
| $\beta_{15,14}$ | 0.895 | 0.049 | 0.043 |
| $\beta_{17,14}$ | 0.505 | -0.031 | -0.029 |
| $\beta_{17,15}$ | <u>48.507</u> | <u>-0.322</u> | <u>-0.341</u> |
| $\beta_{16,3}$ | 19.740 | 0.079 | 0.083 |
| $\beta_{14,6}$ | 10.352 | 0.065 | 0.079 |
| $\beta_{17,3}$ | 5.645 | 0.050 | 0.049 |
| $\beta_{8,2}$ | 0.976 | 0.240 | 0.063 |
| $\beta_{8,3}$ | <u>6.896</u> | <u>0.846</u> | <u>0.247</u> |
| $\beta_{8,1}$ | 0.374 | 0.184 | 0.051 |

and that this parameter will give the largest SEPC. However, the obtained value would be negative, and this is not reasonable given our discussion above. In addition, $\beta_{17,15}$, the direct effect of educational attainment on mid-life occcupational status, is also associated with a large MI but a negative though large SEPC. Recalling that this parameter was fixed by HTS in the original specification, it would not make sense to free this parameter. The remainder of the first fifteen structural parameters reveals either small MI's associated with small SEPC's or large MI's associated with small or implausible SEPC's.

It was decided to first see if the signs on the parameters $\beta_{16,13}$ and $\beta_{17,15}$ would switch if certain residual correlations were freed. The parameter $\psi_{17,16}$ represents the residual correlation between the equations for early occupational status and mid-life occupational status. The MI associated with this parameter is 35.566. The EPC and SEPC for this parameter are 0.079 and 0.109 respectively. Because the SEPC is reasonably large and the MI suggests some improvement could be made, the parameter was freed.

With $\psi_{17,16}$ free, the EPC's and SEPC's were recomputed for the remaining parameters. These results are displayed in Table 6. An inspection of the SEPC's reveal that the largest value is associated with the parameter $\beta_{8,3}$, the direct effect of father's occupational status on academic performance. This is an interesting effect because it suggests that the variables defining SES in the composite equation also have direct effects on later variables. A further inspection of Table 6 shows that the SEPC for $\beta_{17,14}$ is also large and associated with a large MI. However, the SEPC for $\beta_{8,3}$ is larger and it was decided to free this one next.

Table 6

MI, EPC and SEPC for HTS "Modified SHO" model with $\psi_{17,16}$ Included

| Parameter | MI | EPC | SEPC |
|---|---|---|---|
| $\beta_{7,5}$ | 2.763 | -0.010 | -0.032 |
| $\beta_{13,5}$ | 6.859 | -0.015 | -0.059 |
| $\beta_{14,5}$ | 4.862 | 0.013 | 0.051 |
| $\beta_{15,5}$ | 6.502 | 0.016 | 0.055 |
| $\beta_{16,5}$ | 10.961 | 0.016 | 0.063 |
| $\beta_{17,5}$ | 8.541 | 0.016 | 0.057 |
| $\beta_{16,8}$ | 0.774 | -0.005 | -0.018 |
| $\beta_{17,8}$ | 0.496 | -0.004 | -0.013 |
| $\beta_{16,12}$ | 0.904 | -0.030 | -0.048 |
| $\beta_{17,12}$ | 5.068 | 0.080 | 0.116 |
| $\beta_{16,13}$ | 60.399 | -0.357 | -0.357 |
| $\beta_{17,13}$ | 1.407 | -0.063 | -0.057 |
| $\beta_{15,14}$ | 0.877 | 0.048 | 0.042 |
| $\beta_{17,14}$ | <u>14.167</u> | <u>0.211</u> | <u>0.158</u> |
| $\beta_{17,15}$ | 14.168 | -0.302 | -0.312 |
| $\beta_{16,3}$ | 16.852 | 0.075 | 0.078 |
| $\beta_{14,6}$ | 10.591 | 0.070 | 0.067 |
| $\beta_{17,3}$ | 10.184 | 0.064 | 0.077 |
| $\beta_{8,2}$ | 0.983 | 0.241 | 0.070 |
| $\beta_{8,3}$ | <u>6.536</u> | <u>0.802</u> | 0.233 |
| $\beta_{8,1}$ | 0.335 | 0.164 | 0.045 |

Table 7 gives the EPC's and SEPC's for Model 4 with $\psi_{17,16}$ and $\beta_{8,3}$ included. It can be seen that the next parameter to free is $\beta_{17,14}$, the direct effect of occupational status aspiration on mid-life occupational status. The results of the model with $\beta_{17,14}$ are given in Table 8. It is interesting to note that when $\beta_{17,14}$ is included in the model $\beta_{17,15}$ is no longer identified. It was decided to stop improving the model at this point. Nevertheless, there is some suggestion that $\beta_{8,2}$, the direct effect of mother's educational attainment on academic performance, would yield a moderately large parameter estimate.

Table 7
MI, EPC and SEPC for HTS "Modified SHO" model with $\psi_{17,16}$ and $\beta_{f8,3}$ Included

| Parameter | MI | EPC | SEPC |
|---|---|---|---|
| $\beta_{7,5}$ | 2.597 | -0.013 | -0.033 |
| $\beta_{13,5}$ | 7.861 | -0.019 | -0.058 |
| $\beta_{14,5}$ | 2.377 | 0.011 | 0.033 |
| $\beta_{15,5}$ | 2.529 | 0.013 | 0.035 |
| $\beta_{16,5}$ | 9.130 | 0.018 | 0.055 |
| $\beta_{17,5}$ | 7.588 | 0.019 | 0.053 |
| $\beta_{16,8}$ | 0.001 | -0.000 | -0.000 |
| $\beta_{17,8}$ | 2.027 | -0.009 | -0.031 |
| $\beta_{16,12}$ | 1.834 | -0.041 | -0.068 |
| $\beta_{17,12}$ | 3.978 | 0.067 | 0.102 |
| $\beta_{16,13}$ | 61.275 | -0.358 | -0.358 |
| $\beta_{17,13}$ | 1.472 | -0.066 | -0.060 |
| $\beta_{15,14}$ | 1.503 | 0.061 | 0.054 |
| $\beta_{17,14}$ | <u>14.129</u> | <u>0.204</u> | <u>0.186</u> |
| $\beta_{17,15}$ | 14.128 | -0.301 | -0.312 |
| $\beta_{16,3}$ | 16.168 | 0.074 | 0.077 |
| $\beta_{14,6}$ | 10.322 | 0.070 | 0.067 |
| $\beta_{17,3}$ | 7.603 | 0.056 | 0.068 |
| $\beta_{8,2}$ | 4.169 | 0.409 | 0.101 |
| $\beta_{8,1}$ | 0.153 | 0.075 | 0.019 |

Table 8

MI, EPC and SEPC for HTS "Modified SHO" model with $\psi_{17,16}$, $\beta_{8,3}$, and $\beta_{17,14}$ Included

| Parameter | MI | EPC | SEPC |
|---|---|---|---|
| $\beta_{7,5}$ | 2.602 | -0.013 | -0.033 |
| $\beta_{13,5}$ | 8.049 | -0.019 | -0.058 |
| $\beta_{14,5}$ | 2.720 | 0.012 | 0.036 |
| $\beta_{15,5}$ | 2.191 | 0.013 | 0.040 |
| $\beta_{16,5}$ | 6.742 | 0.016 | 0.049 |
| $\beta_{17,5}$ | 5.013 | 0.015 | 0.042 |
| $\beta_{16,8}$ | 0.095 | -0.002 | -0.008 |
| $\beta_{17,8}$ | 0.780 | 0.006 | 0.021 |
| $\beta_{16,12}$ | 2.456 | -0.048 | -0.080 |
| $\beta_{17,12}$ | 0.239 | 0.017 | 0.026 |
| $\beta_{16,13}$ | 57.590 | -0.349 | -0.349 |
| $\beta_{17,13}$ | 3.924 | -0.107 | -0.098 |
| $\beta_{15,14}$ | 1.259 | 0.056 | 0.050 |
| $\beta_{17,15}$ | NI[a] | NI | NI |
| $\beta_{16,3}$ | 12.483 | 0.066 | 0.069 |
| $\beta_{14,6}$ | 8.225 | 0.059 | 0.071 |
| $\beta_{17,3}$ | 7.464 | 0.060 | 0.057 |
| $\beta_{8,2}$ | <u>3.983</u> | <u>0.391</u> | <u>0.097</u> |
| $\beta_{8,1}$ | 0.149 | 0.073 | 0.019 |

[a] Not Identifiable

## *Summary and Discussion*

With regards to the SHO model, the final chi-square goodness-of-fit statistics after model modification indicates that some improvement had been made in the overall fit of the model using the EPC. The parameter which emerged as the most common for all models was $\gamma_{21}$, the direct effect of SES on EA. In addition, it appears that for the Large City sample, $\gamma_{22}$, the direct effect of MA on EA was also important.

Whereas other specification error searches are possible via the EPC and SEPC, it is interesting to note that $\gamma_{21}$ emerged as an important parameter regardless of the size of the sample or the community under consideration. Caution must be exercised when interpreting this result, however, because the community models are still not consistent with the observed data. Nevertheless, the findings suggest that the process of status attainment originally specified by SHO incorrectly ignored the large direct effect of SES as a predictor of educational attainment for individuals within and across communities. It appears that ascribed characteristics of individuals such as SES, exert an important direct influence on ultimate educational attainment.

With regards to the HTS model, overall, the inclusion of the three parameters suggested by the SEPC statistic gives rise to a statistically significant improvement in the fit of the model ($\Delta\chi^2_{(3)} = 58.20$). There are, of course, many other possible specification searches using the SEPC which might yield even more improvement. Nevertheless, this search revealed that $\beta_{17,14}$, which was fixed to define the "modified SHO model" needs to be considered an important parameter.

The HTS specification search using the SEPC also revealed a new parameter to be included which was not considered in any of the models tested by HTS — namely $\beta_{8,3}$, the effect of fathers' occupational status on academic performance. This parameter, in addition to $\beta_{8,2}$ suggests direct effects from the components of SES to academic performance. Moreover, these results may have bearing on earlier historical specifications of the Wisconsin model. In particular, the early work of Sewell, Haller, and Portes (1969) included a path from SES to AP, but this path was removed by SHO because it was found to be negligible. For the HTS specification, the components of this path were found to be moderately large and statistically significant. Again, caution needs to be exercised when interpreting these results.

Overall, EPC and SEPC were found useful in uncovering large and perhaps important specification errors. The results were somewhat equivocal however, especially in light of the fact that the EPC and SEPC did not lower the test statistics to the point that the models fit the data. Nevertheless, there remained only small errors after including the paths suggested by these statistics.

These results may have bearing on the practice of covariance structure modeling. Specifically, if small errors remain after a detailed specification error search is conducted via the EPC or SEPC, presentation of the estimates might perhaps be justified so long as other reasons for misfit are ruled out. However, as stated earlier, EPC methodology is only capable of detecting internal specification errors — not errors resulting from the exclusion of important exogenous variables. Hence presentation and interpretation of parameter estimates should be of a provisional nature only. It follows from this that the common practice of presenting parameter estimates from misfitting models should perhaps be avoided when such a search is not conducted. If other reasons for misfit are ruled out (e.g. non-normality or missing data), then the model should be rejected in favor of a new "causal" ordering of the variables at hand, and/or new variables altogether. With incorporation of the parameter change methodology in LISREL and EQS, it will be of interest to see if this methodology leads to interesting specification searches in other substantive studies.

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