Consider the original regression

$$y = \hat{\beta}_0 + \hat{\beta}_1 x_1 + \hat{\beta}_2 x_2 + e, \tag{1}$$

where e is the residual from this regression.

Johan wanted two regressions

$$x_1 = \hat{\gamma}_0 + \hat{\gamma}_1 x_2 + g, \tag{2}$$

$$y = \hat{\delta}_0 + \hat{\delta}_1 g + h, \tag{3}$$

and wanted you to show $\hat{\delta}_1 = \hat{\beta}_1$.

From Equation (2), we have

$$g = x_1 - \hat{\gamma}_0 - \hat{\gamma}_1 x_2. \tag{4}$$

Substitute Equation (4) into Equation (3):

$$\begin{aligned} y &= \hat{\delta}_0 + \hat{\delta}_1 (x_1 - \hat{\gamma}_0 - \hat{\gamma}_1 x_2) + h \\ &= \hat{\delta}_0 + \hat{\delta}_1 x_1 - \hat{\delta}_1 \hat{\gamma}_0 - \hat{\delta}_1 \hat{\gamma}_1 x_2 + h \\ &= \left(\hat{\delta}_0 - \hat{\delta}_1 \hat{\gamma}_0\right) + \hat{\delta}_1 x_1 + \left(-\hat{\delta}_1 \hat{\gamma}_1\right) x_2 + h. \end{aligned} \tag{5}$$

Compare Equation (5) with Equation (1), we see that $\hat{\delta}_1 = \hat{\beta}_1$, which is what Johan wanted.