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# Financial Education and Conceptual Understanding: Learning from Best Practices in Mathematics

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Best practices in mathematics education require building conceptual understanding alongside procedural skill. We argue that financial education would benefit from adopting this strategy. Conceptual understanding is required to adapt to changing financial technology. In addition, research demonstrates that procedural skills are better remembered if they are coupled with conceptual understanding. Therefore, such understanding is required if the financial skills learned in school are to be retained and applied in adulthood. We show how current financial literacy curriculums do not emphasize conceptual understanding, and demonstrate how this can be rectified through a focus on the financial life cycle.





## 1. Introduction

There is a widely recognized need for improved levels of financial knowledge amongst adults in the US. One of the central parts of a strategy to bring this about is providing financial education. However, to date, such education programs have not been as successful as might be hoped. In order to improve the effectiveness of financial education efforts, a promising idea is to look for guidance from other areas of education. In particular, given the significant mathematical component to financial literacy, it makes particular sense to look at best practices in mathematics education, and explore any lessons for finance education.

This paper will focus on the precept in mathematics education that best practices require teaching students *conceptual understanding* rather than focusing on mere *procedural knowledge*. It is assumed in this field that possessing deep and interconnected knowledge of the subject matter is an essential component of effective education. We will argue that financial education would benefit from adopting this principle.

First, we will show the need for improved financial knowledge, and the shortcomings of current financial education programs. Next, we will review the evidence demonstrating the importance of conceptual understanding and explain how this applies to personal finance. In section 5 we will review some of the most influential financial education curriculums and show that they neglect conceptual understanding. Finally, we will offer a suggestion for *how* financial education programs can incorporate conceptual understanding effectively: by building the course around an understanding of Modigliani's Life Cycle Hypothesis.<sup>1</sup>

## 2. The Need for Improving Financial Education

The level of financial knowledge among adults in the US is much lower than it should be, given the complex financial choices people must navigate in modern Western society. A vast body of research demonstrates the breadth and depth of this issue. The successive 'Financial Capability Surveys' (FINRA, 2009, 2013, 2015) show that a large percentage of Americans

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<sup>1</sup>Ando and Modigliani (1963)



are unable to answer a number of questions on central financial concepts, and display a lack of knowledge in “fundamental economic principles” (Lusardi, 2011).

Further research backs these findings up: Hilgert, Hogarth and Beverly (2003) used an extensive test of financial knowledge and found similar failings; Lusardi, Mitchell and Curto (2014) found that older people in the US lack financial sophistication; while Lusardi and Mitchell (2011) showed that a lack of essential financial knowledge is widespread in countries around the world.

Of particular concern is the lack of financial knowledge in young people in the United States, given that the country is trending towards placing more individual responsibility on financial planning (Lusardi & Mitchell, 2014), and technological developments promise ever more complex financial instruments will be available in the future. It has been demonstrated that most high-school and undergraduate students fail basic financial literacy tests (Hastings, Madrian & Skimmyhorn 2012; Lusardi, Mitchell & Curto 2010; Mandell 2008; Markow & Bagnaschi, 2005; Shim, Barber, Card, Xiao & Serido, 2010; National Council on Economic Education [CEE] 2005).

Making this even more concerning is the fact that many people are making poor financial decisions, and experiencing negative financial outcomes. One third of Americans in their 50’s have failed to develop a retirement plan, leaving them with a precarious financial future (Lusardi, 2011). Hilgert et al. (2003) provide evidence that a significant proportion of adults self-report low scores in financial best practices. Many households fail to diversify their investment portfolios, or fail to refinance their mortgages at opportune moments, creating completely avoidable financial risk and interest expenses respectively (Campbell, 2006). In addition, young people are again particularly at risk, being “heavily reliant on debt” (Brown, Van der Klaauw, Wen, & Zafar, 2016). The flaws in financial decision making have been linked to widespread flaws and biases in our reasoning through research in behavioral economics, showing a theoretical underpinning to the trends observed (Kahneman, 2011; Sunstein & Thaler, 2008; Thaler 2015).

As a response to this problem, there has been a growth in a range of financial education programs across the county: from state-mandated high school courses to on-the-job training



sessions (CEE, 2016). Despite this, the evidence in favor of financial education has been mixed. Fernandes, Lynch, and Netemeyer (2013) showed that measured financial literacy can make significant predictions regarding later financial behavior; however, they found that financial education interventions intended to improve later financial behavior were largely ineffective, with a statistically significant but minuscule effect. Cole, Paulson, and Shastry (2014) found similar results across a range of states; Mandell (2008) found no evidence of improvement in financial knowledge due to financial education. On the basis of this, Willis (2011) argues that we should abandon the project of trying to provide such education altogether.

Other research has found that financial education can improve knowledge under the appropriate circumstances. Urban, Schmeiser, Collins, and Brown (2015) found that more rigorous state mandates for education in financial literacy had a greater effect on subsequent financial well-being than less demanding mandates. There were improved credit scores and reduced delinquency rates for young adults in states with rigorous state mandates, relative to those states that had less rigorous mandates, or none at all. This implies that financial literacy education can create change. While there are conflicting findings regarding financial education in general, rigorous in-depth financial literacy courses have been shown to be effective in improving financial well-being (Brown et al., 2016; Walstad, Rebeck, & MacDonald, 2010).

Given these mixed results, it's worth looking at what research in other areas of education can tell us about creating an effective course in personal finance.

### **3. Conceptual Understanding in Mathematics Education**

Consensus opinion on best practices in mathematics education emphasizes the importance of developing students' *conceptual understanding*, not just their *procedural knowledge*. This is summarized in the highly influential manifesto *Principles to Actions*, written by the National Council of Teachers of Mathematics (NCTM, 2014). They argue that the current state of mathematics education has the follow deficiencies:



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Too much focus is on learning procedures without any connection to meaning, understanding, or the applications that require these procedures... Too much weight is placed on results from assessments—particularly large-scale, high-stakes assessments—that emphasize skills and fact recall and fail to give sufficient attention to problem solving and reasoning. (p. 3)

NCTM (2014) offers a set of principles for effective mathematics teaching, designed in large part to combat this deficiency. In particular one of the standards states that teachers should: “Build procedural fluency from conceptual understanding. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.” (NCTM, 2014, p. 10)

This focus is not unique to NCTM, either. Proposals from the National Research Council (2001) and the National Governors Association Center for Best Practices and Council of Chief State School Officers (NGA Center and CCSSO, 2010) (these are the familiar common core standards for mathematics) both place an emphasis on the importance of conceptual understanding in mathematics education.

All of this naturally brings up the question: what is conceptual understanding? And relatedly, what is procedural knowledge? NCTM (2014) offers the following explanation: “Conceptual understanding (i.e., the comprehension and connection of concepts, operations, and relations) establishes the foundation, and is necessary, for developing procedural fluency (i.e., the meaningful and flexible use of procedures to solve problems).” (p. 7) Similarly, Rittle-Johnson, Siegler & Alibali (2001) give the following definition:

We define procedural knowledge as the ability to execute action sequences to solve problems. This type of knowledge is tied to specific problem types and therefore is not widely generalizable. To assess procedural knowledge researchers typically use routine tasks... In contrast to procedural knowledge, we define conceptual knowledge as implicit or explicit understanding of the principles that govern a domain and of the interrelations between units of knowledge in a domain. This knowledge is flexible and not tied to specific problem types and is therefore generalizable... To assess conceptual knowledge,



researchers often use novel tasks, such as counting in nonstandard ways or evaluating unfamiliar procedures. (p. 347)

Similar ideas are expressed in a large body of research on the topic (See Brown, Roediger, & McDaniel., 2014; Bullmaster-Day, 2006; Ellis and Worthington, 1994; Hiebert and Lefevre 1986).

Putting this all together, we have two key points:

1. Conceptual understanding in a given area requires that one knows a number of facts in that area, knows of the connections between these items of knowledge, and knows how knowledge in this area connects to other subject matters. In the terminology of Brown et al. (2014), one's knowledge forms a complex 'mental model'.<sup>2</sup>
2. Conceptual understanding can be flexibly deployed across a range of subject matters, where the principles one has learned are applied to problems one has not seen before.

In contrast:

1. Procedural knowledge is limited in scope and disconnected from other areas of knowledge.
2. Procedural knowledge allows one to perform a specific task in a specific context, and is not easily adapted to unfamiliar circumstances.

This can be illustrated by example. Consider knowledge of multiplication. Here a form of procedural knowledge would be the ability to reliably use the algorithm for multiplication typically taught in the United States. Conceptual understanding would require knowing about our decimal number system, place value, and the connections between multiplication and the other arithmetic operations – for example, that multiplication can be understood as 'repeated addition'. Conceptual understanding could also involve a geometric as well as a symbolic conception of the operation.

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<sup>2</sup> NCTM (2014) promotes this aspect of conceptual understanding in their discussion of how best practices require the use of "*purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships*" The most sophisticated level of questioning is describes as: "Encouraging reflection and justification: Students reveal deeper understanding of their reasoning and actions, including making an argument for the validity of their work." (p. 36-37) Brown et al. (2014) are explicit that engaging in 'reflection and justification' is how one creates a complex mental model. See also McDaniel and Donnelly (1996).



As a result, conceptual understanding of multiplication enables students to tackle a range of problems, whereas procedural knowledge only helps with questions of the form ‘What is  $a \cdot b$ ?’ For example, if one had to calculate the number of squares inside a rectangle drawn on graph paper, conceptual understanding allows one to multiply the numbers of squares up with the number of squares across – even if one has not encountered such a problem before. By contrast, procedural knowledge of the multiplication algorithm is no help in this context.

#### **4. The Need for Conceptual Understanding in Finance Education**

Conceptual understanding is a central component of mathematics education, but why does this mean that it should be a part of *finance* education? The central goals of finance education are mostly *practical* in nature: we want people to acquire the skills necessary to effectively manage their finances. This might suggest that while flexible problem solving is an essential part of mathematical proficiency, it’s the concrete procedural knowledge that is required for financial literacy. It would certainly be nice if students could appreciate the beauty and interconnectedness of financial theory, one might argue; however, it’s a luxury we can’t afford to dedicate scarce educational resources to, given the pressing need for more mundane financial skills.

This line of argument is mistaken for two reasons:

1. Our rapidly changing financial world requires the flexible problem solving that comes from conceptual understanding.
2. Conceptual understanding is required to effectively learn and retain procedural knowledge.

The first point should be self-evident. The kind of concrete procedures that are taught today might well be completely obsolete in 10 or 20 years’ time. For example, even now, students are taught to balance a checkbook, despite the fact that many of them will never have to write a check in their adult lives.<sup>3</sup> Similarly, it is of limited use to teach students the details of the current tax system, given that it will likely be overhauled multiple times over their working lives. In our rapidly changing financial environment, procedural knowledge that allows one to

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<sup>3</sup> See, e.g., Gerver and Sgroi (2018, p. 73).





perform a specific task in a specific context becomes quickly outdated. A deeper and more flexible understanding of finance is required so that students are able to apply their knowledge to the new financial technologies that will undoubtedly emerge over their lifetimes.

The second point is the result of a significant body of research in education and psychology. The fundamental idea is that even if all you care about is giving your students procedural knowledge, the most effective way to do this requires developing students' conceptual understanding as well. NCTM summarizes the point:

Student learning is greatest in classrooms where the tasks consistently encourage high-level student thinking and reasoning and least in classrooms where the tasks are routinely procedural in nature. (NCTM 2014 p17)

They further note that when there is conceptual understanding “students have better retention of the procedures and are more able to apply them in new situations” (p. 42) Similarly, Resnick, Bill, Lesgold, and Leer (1991) argue that in a course that emphasizes conceptual understanding “children can acquire the traditional basic skills in the process of reasoning and solving problems.” (p. 137.)

This is not a result idiosyncratic to mathematics education, but broadly confirmed by research into memory. As Brown et al. (2014) argue, we remember things better when they are embedded within complex ‘mental models’.<sup>4</sup> This phenomenon also naturally connects with how certain specialists use ‘memory palaces’ to remember vast amounts of disconnected pieces of information – such as the order of a shuffled deck of cards. In their imagination, they place each item to be remembered in a location in a physical environment (such as their house), then they ‘walk through’ the environment in their imagination to recall the information. In doing so, they find an ingenious way to push isolated items of knowledge into an interconnected mental representation – the kind of thing we are better able to remember.<sup>5</sup>

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<sup>4</sup> See also McDaniel and Donnelly (1996).

<sup>5</sup> On memory palaces, see, for example, Foer (2011).



Note that this does not require the strong thesis that conceptual knowledge is in some sense *prior* to procedural knowledge – that students must first develop conceptual understanding before it’s possible to acquire procedural understanding. Instead, we just need the weaker, and more plausible thesis that conceptual understanding and procedural knowledge are *mutually reinforcing*. This idea is articulated, and backed up by research in a paper by Rittle-Johnson et al. (2001):

Conceptual and procedural knowledge develop iteratively, with increases in one type of knowledge leading to increases in the other type of knowledge, which trigger new increases in the first... Increases in one type of knowledge lead to gains in the other type of knowledge, which in turn lead to further increases in the first. Knowledge of a particular type is often incomplete, and a variety of experiences, such as problem solving, observation of other people's activities, direct verbal instruction, and reflection, may initiate knowledge change. Past research is consistent with this gradual, bidirectional model of conceptual and procedural knowledge development... Conceptual and procedural knowledge may develop in a hand-over-hand process, rather than one type strictly preceding the other. (p. 347)

Bullmaster-Day (2006) makes a similar point, noting that “Conceptual understanding and procedural fluency are not “either/or” elements of mathematical knowledge – they grow together” (p. 1; *see also* Star, 2005 and Yu, 1999).

It’s worth elaborating on a fact alluded to above: conceptual understanding helps not just with acquiring procedural knowledge, but with *retaining* it. The importance of retention for financial knowledge is perhaps even more important than with other subject matters. When learning traditional subjects, such as mathematics, many of the goals are relatively short term: from graduating high-school, to getting accepted at a college, to acquiring the abilities to get started on a demanding major at college – it’s not necessarily going to be important for the specific skills learned in high school math to be applied in adulthood. For financial education, on the other hand, the central goal is for students to carry what they learn into adulthood, and apply it then.



A great deal of research combining results from psychology and education has looked at what forms of learning promote long term retention of knowledge.<sup>6</sup> A course that emphasizes conceptual understanding fits well with this: it's not just that, as we have discussed, acquiring conceptual understanding in itself improves memory, but that it also facilitates other strategies for better retention.

The central finding in memory research is that learning leads to better retention when the process is *effortful* (Bjork, 1994). One remembers better when one takes notes than when one listens passively to a lecture. One learns even better when one's notetaking involves putting the key ideas in one's own words, rather than writing down exactly what the lecturer dictates (Dunlosky, Rawson, Marsh, Nathan, & Willingham, 2013). Similarly, when going over material a second time, it's more effective to take mini-tests, then look up the answers where one made mistakes, rather than rereading notes or course materials (Roediger, Putnam, & Smith, 2011).

Further, it's better to test oneself intermittently across all the topics one has been learning (a process known as 'interleaving'), rather than to practice topics one at a time as one learns them (a process known as 'chunking'). Answering questions on a topic is more effortful when there has been a delay since one was introduced to the topic, as partial forgetting has set in so that retrieving the information is more difficult. In addition, taking a test in which the topics vary unpredictably requires more effort as one must pause and think about what procedures a particular question requires, rather than applying the same one mechanically (McDaniel, 2012).

A course that emphasizes conceptual understanding naturally leads to interleaving in the questions students must answer. That's because it requires exploring the *connections* between the different topics in the course, so there will be questions which integrate ideas from previous topics into current topics. In addition, conceptual understanding is a prerequisite for the kind of high-effort note taking we discussed, such as rewriting key points in one's own words.

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<sup>6</sup> Brown et al (2014) provides a comprehensive overview of the field.



A final technique for improving retention that's relevant for our purposes is *generative learning*. (Bjork & Bjork, 2011). This occurs when one attempts to solve a problem in a new area *before* being given the information necessary to solve it. That is, if one learns through a process of discovery, one learns better than if one is told how to solve a given type of problem, then asked to apply that method to a series of examples, in a rote manner.

The idea of discovery as the ideal form of learning goes back to at least the Ancient Greeks: in a famous passage in Plato's *Meno*, a slave boy is able to discover the Pythagorean Theorem by answering a series of leading questions (Cooper & Hutchinson, 1997). Plato's hunch has now been verified by empirical research.<sup>7</sup> Significantly, research shows that retention is better when discovery is *attempted*, whether the attempt is successful or not. This is important in a realistic classroom setting, since students are not always going to succeed in solving a novel problem by themselves. As long as they put effort into attempting a solution, when they are told how to do it, they remember better than if they just been told straightaway. This means that unsuccessful discovery attempts are not a waste of time, and the teacher can in good conscience step in with help before the process becomes counter-productive. NCTM (2014) emphasizes the importance of attempting discovery in learning in its promotion of *productive struggle*.

Teaching for conceptual understanding is essential for generative learning. One needs background knowledge that one can flexibly deploy in order to have any chance at solving a new kind of problem. A well-conceived discovery task must be set up in such a way that the students' existing knowledge gives them the tools to *reason* their way to a solution. This by definition requires conceptual understanding.

## 5. Current Curriculums

We have established that an effective financial literacy course *should* develop conceptual understanding. A survey of the most prominent financial literacy curriculums and standards in use in the US, suggest that this is not currently being achieved. We examined: the

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<sup>7</sup> Though the research has not yet confirmed Plato's corollary that our ability to discover mathematical theorems entails that we have lived past lives, and the discovery is really recollection of what we knew in a past life.



Department of Treasury's Financial Education Core Competencies; the Jump\$tart standards for financial literacy; the Money as You Grow 20 Mile Stones; The PISA standards; and the Council of Economic Education (CEE) National Standards in Financial Literacy.<sup>8</sup>

Some of the standards are simply a list of procedural skills students should learn, specifically: Department of the Treasury (2010); Money as You Grow (2012); PISA (2012). The Jump\$tart and CEE standards are more thorough and more structured, but still do not explicitly emphasize conceptual understanding. Neither specify 'Essential Questions' or 'Big Ideas' for the curriculum, or units they contain, in line with best practices for curriculum design (McTighe & Wiggins 2012).<sup>9</sup> The Jump\$tart standards do provide guiding principles for each unit, but these are in the form of 'Overall Competencies' which emphasize procedural skill over conceptual understanding. For example, the Overall Competency for the 'Spending and Saving' unit is: "Apply strategies to monitor income and expenses, plan for spending and save for future goals."

Additionally, neither of these curriculums explore the inter-connections between different financial topics: for example Jump\$tart puts discussion of investing and discussion of borrowing into separate units, and does not emphasize the parallels between these two processes.<sup>10</sup>

In some ways, the CEE standards come closest to what is required for a focus on conceptual understanding: they frame their standards around a foundational understanding of *economics*. Though such an understanding is valuable for many reasons, it is not what is required for a conceptual understanding of financial matters. The issues of how supply and demand shape price equilibriums are too far removed from personal finance to provide the

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<sup>8</sup> See Department of the Treasury (2010); Jump\$tart Coalition for Personal Financial Literacy (2015); Money as You Grow (2012); PISA (2012); CEE (2013).

<sup>9</sup> Charles (2005) writes "A Big Idea is a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole." Zimmerman (2001) suggests that presenting students with the Big Ideas for a course and improve their academic achievement.

<sup>10</sup> NCTM (2014) write: "An excellent mathematics program includes a curriculum that develops important mathematics along coherent learning progressions and develops connections among areas of mathematical study and between mathematics and the real world." (p. 7). This is related to the importance of 'learning trajectories' in a curriculum that promotes understanding (Clements & Sarama 2004; Sztajn, Confrey, Holt Wilson, & Edgington, 2012).



appropriate understanding. This is like trying to develop an understanding of human biology and starting with sub-atomic physics.

To be clear, none of this is to say that the standards discussed are ‘wrong’ or must be scrapped entirely. It is perfectly possible to create a curriculum that covers the topics described by these standards in a way that creates understanding. One just needs to first appreciate the central concepts and then build the topics around them. The best way to illustrate this is to show what a course focused on a conceptual understanding of personal finance would look like. This will be the topic of the final section.

## **6. Conceptual Understanding and the Financial Life Cycle**

The various topics involved in personal finance are tied together by the notion of the *financial life cycle* – as explained in Modigliani’s Life Cycle Hypothesis.<sup>11</sup> This makes it a natural foundation for building conceptual understanding of this subject matter. As Lusardi and Mitchell (2014) note, when discussing financial illiteracy in the US:

[A] fully rational and well-informed individual will consume less than his income in times of high earnings, and he will save to support consumption when income falls (e.g. after retirement). In this context... the consumer is posited to arrange his optimal saving and decumulation patterns to smooth marginal utility over his lifetime. Many studies have shown how such a life cycle optimization process can be shaped by consumer preferences... [However] this is far from true in the real world: very few people possess the extensive financial knowledge conducive to making and executing complex plans. (p. 3)

The central idea here is that a typical individual has different financial needs and different levels of income over the course of their lifetime. To successfully manage over this life cycle, an individual must ensure that they allocate their lifetime earnings in such a way that they are able to meet their financial needs at all points in time: they must have additional resources available for periods when income is low; and they must effectively reallocate surplus income in periods when income is high (roughly speaking). This is what Lusardi and Mitchell

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<sup>11</sup> See Ando and Modigliani (1963)



mean when they say that “the consumer is posited to arrange his optimal saving and decumulation patterns to smooth marginal utility over his lifetime.”

As the authors note, this is a description of what is required to *successfully* manage financial matters – it describes the actions of “a fully rational and well-informed individual”. The majority of people are not perfectly rational and well-informed, and consequently they fail to allocate their life time income appropriately. The aim of financial education programs is to rectify this situation.

It is clear that the general problem of navigating the financial life cycle is a giant task that is made up of a huge array of sub-problems. Solving these sub-problems requires, as Lusardi and Mitchell write, “extensive financial knowledge conducive to making and executing complex plans.” What is not made explicit here, and is omitted from most personal finance curriculums, is that this extensive financial knowledge is unified by conceptual understanding of the financial life cycle.

When one understands the idea of the lifecycle, background knowledge about how people live at different stages of their life naturally leads one to see the need for the various sub-steps alluded to above. With the conceptual understanding in place, many of the key topics in personal finance can be learned through *discovery*. When it is ignored, retaining knowledge of the myriad procedural sub-tasks becomes much more difficult. We’ll discuss three central examples to illustrate this.

1. *Wealth*: Understanding the life cycle hypothesis, requires a sophisticated understanding of consumption and its relationship to wealth. Once one learns to distinguish income, expense, asset and liability, many other topics fall into place. For example, one sees why borrowing money to purchase an *asset*, such as a house or car, does not reduce one’s wealth, and so does not jeopardize future consumption. By contrast, borrowing to pay for expenses (in contrast to investment) does reduce wealth. Such borrowing reduces lifetime consumption and should therefore be employed judiciously.
2. *Transferring Wealth over Time*: The life cycle hypothesis tells us we must use transfer our wealth backwards and forwards through time so that we are able to meet our





consumption needs at all life stages. From this, one can see the rationality of borrowing for education earlier in life and investing money during one's career for use in retirement. Further, one can see the connections and symmetry between borrowing and investing, creating a more complex mental model.

3. *Managing Risk*: Another crucial factor in the financial life cycle is one's willingness to take on risk. Where a person is facing more risk than they can handle, financial tools allow them to reduce this risk. Similarly, when they are willing to take on additional risk, they can receive compensation for doing so. This understanding connects to financial lessons on what kinds of investments are appropriate at what points in life: for example, high risk investments are more appropriate early in one's career, while low risk investments are more appropriate when approaching retirement.

From this, we can see that there are strong theoretical grounds for building a financial literacy course around conceptual understanding of the life cycle hypothesis. Doing so allows students to create the kind of complex mental model that gives them the best chance of retaining the skills necessary to manage their financial lives in adulthood.

## 7. Conclusion

Given the importance of improving the financial knowledge of young people, it's crucial to draw upon all available tools in implementing financial education programs. As we have shown, there is a wealth of research in mathematics education that is particularly well-suited for this purpose. A central starting point is making sure personal finance courses emphasize conceptual understanding. To do this, we must focus on the financial life cycle.

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