Exercise 2

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L5 Task 1

Two alternate-form tests were given to the same population, yielding estimated standard deviation of $s_1 = 7.5$, $s_2 = 8$ and an estimated covariance of $s_{12} = 12$. Calculate the estimated standard error of measurement for the test

L5 Task 1: Solution

We first estimate the reliability coefficient

$$\text{Population: } \rho_{YY'} = \frac{\mathrm{Cov}(Y, Y')}{\sigma_Y \sigma_{Y'}} = \frac{\sigma_T^2}{\sqrt{\sigma_Y^2}\sqrt{\sigma_{Y'}^2}} = \frac{\sigma_T^2}{\sigma_Y^2}$$

$$\text{Sample: } \widehat{\rho}_{YY'} = \frac{S_{12}}{s_1 \cdot s_2} = \frac{12}{7.5 \times 8} = 0.2.$$

We then obtain the standard error of measurement as

$$\widehat{\text{SEM}}(Y) = \sqrt{s_Y^2(1 - \widehat{\rho}_{YY'})} = \sqrt{7.5 \times 8 \times (1 - 0.2)} \approx 6.93.$$

L5 Task 2

- We have autumn (A) and spring (S) test-takers of the SweSAT with $A \sim \mathcal{N}(60, 12)$ and $S \sim \mathcal{N}(70, 16)$ true score distributions respectively.
- The test takers are given the same test which is defined by the classical true score model with an error score $E \sim \mathcal{N}(0, 4)$.

Calculate the reliability of the test for population A and S.

L5 Task 2: Solution

Recall that the reliability coefficient is defined as

$$\rho_{YY'} = \frac{\sigma_T^2}{\sigma_Y^2} = \frac{\sigma_T^2}{\sigma_T^2 + \sigma_E^2}.$$

For population A, we have

$$\widehat{\rho}_{YY'}^{A} = \frac{12}{12+4} = 0.75.$$

Similarly for population S

$$\widehat{\rho}_{YY'}^{S} = \frac{16}{16+4} = 0.80.$$

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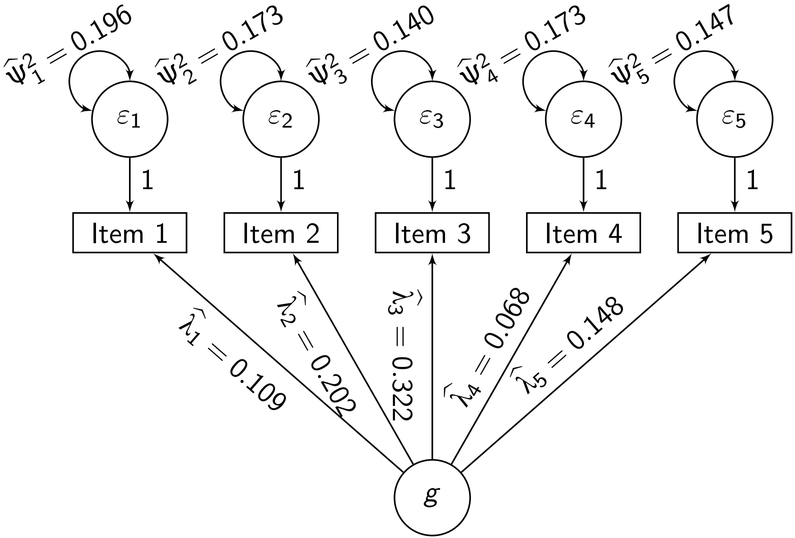
L6 Task 1

The following estimated factor loadings and error variances were obtained from a single factor model for five mathematics items

	Item 1	Item 2	Item 3	Item 4	Item 5
$\widehat{\lambda}_j$	0.109	0.202	0.322	0.068	0.148
$\widehat{\Psi}_j^2$	0.196	0.173	0.140	0.173	0.147

Calculate the estimated coefficient omega.

L6 Task 1: Diagram



L6 Task 1: Solution

Coefficient omega is equal to the reliability of the sum score when a single factor models holds. We obtain the following estimate of the reliability with the estimated factor model parameters:

$$\widehat{\omega} = \frac{\left(\sum_{j=1}^{5} \widehat{\lambda}_j\right)^2}{\left(\sum_{j=1}^{5} \widehat{\lambda}_j\right)^2 + \sum_{j=1}^{5} \widehat{\Psi}_j^2} \approx \frac{0.849^2}{0.849^2 + 0.829} \approx 0.465.$$

If the factor model is the true model, $\widehat{\omega}$ is an unbiased estimator of the reliability of the sum score.

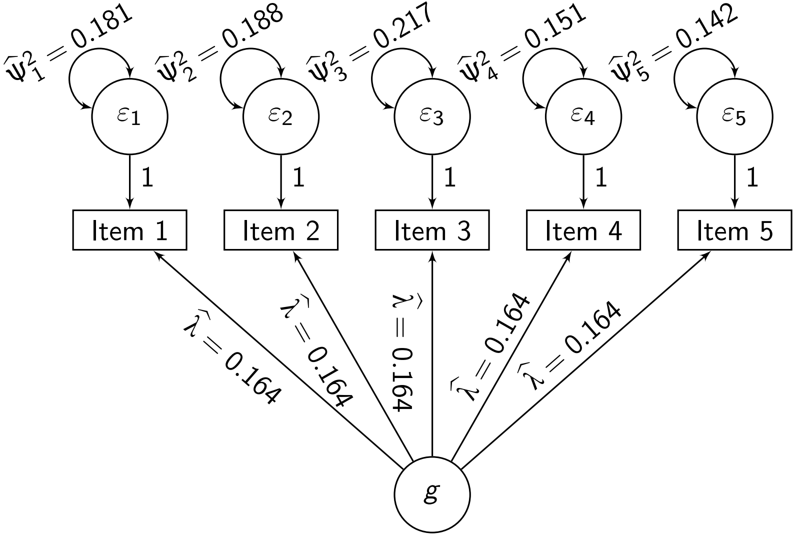
L6 Task 2

A restricted model with the factor loadings set equal was also estimated. The estimated factor loading was $\widehat{\lambda} = 0.164$ and the error variances are given in the table below:

	Item 1	Item 2	Item 3	Item 4	Item 5
$\widehat{\Psi}_j^2$	0.181	0.188	0.217	0.151	0.142

Calculate the estimated coefficient alpha/omega.

L6 Task 2: Diagram



L6 Task 2: Solution

$$\widehat{\alpha} = \widehat{\omega} = \frac{5^2 \times \widehat{\lambda}^2}{5^2 \times \widehat{\lambda}^2 + \sum_{j=1}^{5} \widehat{\Psi}_j^2} = \frac{5^2 \times 0.164^2}{5^2 \times 0.164^2 + 0.879} \approx 0.433$$

Comparing Task 2 with Task 1: Cronbach's alpha is only valid under tau-equivalence and it underestimates the reliability of the scale.