

Principles of Measurement - Assignment 1

October 21, 2021

Instructions

- The assignment should be completed individually.
- Exercises 1 and 2 should be done by displaying the operations conducted to obtain the final answer.
- Exercise 3 uses the R workspace `MAE4011H21A1.RData`, found on Canvas.
- Write your answers in a word processor.
- Upload your assignment through the submission system in canvas.
- The deadline for handing in the assignment is November 5 18.00.

Exercise 1

The following covariance matrix was observed for a four-item screening tool of depression severity:

$$\hat{\Sigma}_{X_1, X_2, X_3, X_4} = \begin{pmatrix} 0.23 & 0.05 & 0.07 & 0.09 \\ 0.05 & 0.25 & 0.10 & 0.06 \\ 0.07 & 0.10 & 0.25 & 0.08 \\ 0.09 & 0.06 & 0.08 & 0.24 \end{pmatrix}.$$

The task:

- a Calculate the estimated variance of the sum score $X_1 + X_2 + X_3 + X_4$.
- b Calculate the estimated correlation matrix from the above covariance matrix.
- c Comment on the relationships between the variables X_1, X_2, X_3 and X_4 based on the observations.
- d What further information would you like to have in order better understand the relationships between the variables X_1, X_2, X_3 and X_4 ?

Exercise 2

Eleven dichotomous algebra items X_j (correct/incorrect - 0/1 scoring) were answered by a group of grade 7 students and the observed proportions agree are given in Table 1. The variance estimate for the sum score of the eleven dichotomous variables was 9.60.

Table 1: Proportion correctly answered for each item

Item	1	2	3	4	5	6	7	8	9	10	11
	0.88	0.75	0.62	0.67	0.52	0.71	0.40	0.65	0.56	0.53	0.60

The task:

Calculate the estimate of

$$\alpha_c = \frac{m}{m-1} \left[1 - \frac{\sum_{j=1}^m \text{Var}(X_j)}{\sigma_Y^2} \right],$$

where X_j denotes each item, m is the number of items and Y denotes the sum $X_1 + \dots + X_m$. Interpret the results in terms of what it says about the variable Y . Consider carefully the assumptions underlying the interpretations.

Exercise 3

We have collected data from a questionnaire and the data can be found in the R workspace `MAE4011H21A1.RData`. The object is named `QT1` and contains the answers to all the individual questions of the questionnaire in the order they were asked. The questionnaire itself can be found in the file `Principles of Measurement Questionnaire H21.pdf`. It is now your task to process and analyze the data.

The task:

- a The workspace contains data regarding which category was selected for each of the items for each respondent. Based on this information, score each of the items in each of the three parts according to a scoring rule you find appropriate. Motivate your choice of scoring rule.
- b Consider each of the three parts of the questionnaire and calculate summary statistics for one item in each of the questionnaire parts (i.e. consider in total three items). Motivate your choice of summary statistics and explain what they represent. Comment on the results.
- c Construct a composite score from the item scores in each of the three parts of the questionnaire. Motivate your choice of scoring rule.
- d Graphically represent the composite scores for the individuals of each part in some manner and motivate your choice of graphical display.