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The role of participatory identity in learners' hybridization of activity across contexts

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ABSTRACT

Background: We explore how school-based mathematical experiences shape out-of-school mathematical experiences, developing the idea that learners hybridize norms and practices around authority and evaluation across these two contexts. To situate our study, we build on constructs of participatory identity and framing.

Methods: Drawing from a large corpus of video records capturing children's point-of-view, we present a case study of hybridization with two purposively sampled 12-year-old friends—Aimee and Dia—interacting in an out-of-school mathematics playspace. We use interaction analysis to articulate grounded theories of hybridization.

Findings: We present a thick description of how *children hybridize their activity in out-of-school spaces and how such hybridization is consequential for engagement*. Dia's case illustrates how traditional norms and practices around authority and evaluation can lead to uncertainty and dissatisfaction, while Aimee's illustrates how playful norms and practices can lead to exploration and pleasure in making. We argue that their school-based mathematics experiences and identities influenced these differences.

Contribution: This report strengthens theoretical and methodological tools for understanding how activity and identity development in one context become relevant and shape activity in another by connecting analytic constructs of identity, framing, and hybridizing.

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Introduction

Over the last 20 years, efforts to foster meaningful engagement in STEM have often focused on creating opportunities to connect knowledge and practices from informal spaces into school (Banks et al., 2007; González et al., 2001). Such efforts may create an opportunity for children to see school mathematics as relevant to their lives (Abreu & Cline, 2003) and to see out-of-school experiences as relevant to school (Gutiérrez et al., 1999). The focus of this scholarship—whether and how out of school experiences might make school experiences better—underscores a tacit belief: that the ultimate place where mathematical thinking matters is in schools.

In this paper, we ask a different question: how do in-school experiences influence learners' out-of-school mathematical activity? In particular, when offered the opportunity to explore mathematical ideas through unstructured playful activities, how might school experiences influence what learners make of these informal spaces and ultimately what they do? This issue is important because informal, enjoyable mathematical experiences may transform learners' relationship with mathematics in enduring ways (Banks et al., 2007; Petrich et al., 2013; Quinn & Bell, 2013). In fact, a survey of STEM professionals found that 94% of those surveyed reported that out-of-school experiences were important in their choices of STEM careers (Jones et al., 2011). However, the causal direction of the benefit of out-of-school experiences is unclear: did those who had positive experiences in out-of-school activities also enjoy school mathematics, or did enjoyment of school math prompt them to seek out-of-school experiences? Better understanding the relationship between in- and out-of-school mathematics activities has implications for our understanding of how to invite and sustain participation in mathematics, particularly when some students' experiences with school math leave them feeling disenfranchised and alienated (Anderson, 1997; Boaler, 2000). This paper contributes by examining learners' activity in an out-of-school context, offering a deeper understanding of how school mathematics might shape engagement even in playful out-of-school contexts.

To do so we build on the idea of *hybridizing* (Bakhtin, 1981), the process in which people integrate two or more cultural activities in a way that is more than the sum of their parts, resulting in *new* activities (Gutiérrez et al., 1999). Specifically, this paper looks closely at two learners' interactions in an out-of-school mathematical playspace, documenting the mathematical identities that are on display and asking *how do children hybridize their activity in out-of-school spaces and how is this hybridization consequential for engagement?*

Identities within and across contexts

We begin with an assumption about participation, which is simply that people's behavior can be understood to be an interaction among what there are opportunities to do, which of those opportunities they notice, and which they choose to act on (Greeno, 1991). This claim centers attention on how environments invite different forms of participation, the result of interactions among the design of a space, individuals' histories, and intentions.

For this paper, we draw on a model of identity proposed by Gresalfi and Hand (2019), which conceptualizes identity as an act of connecting the resources of *broad cultural narratives* and in-the-moment *interactions* by drawing on *frames* (see Figure 1). Specifically, identity is defined as *participation in and across social activities and the sense one makes of oneself in relation to these activities* (Hand & Gresalfi, 2015). Below, we focus specifically on what we know about the first two levels of the identity model, as those are most relevant for the current analysis.

Norms and practices that position people through interactions

A fundamental resource for identity comes through interactions among people and tools in particular contexts that position people in particular

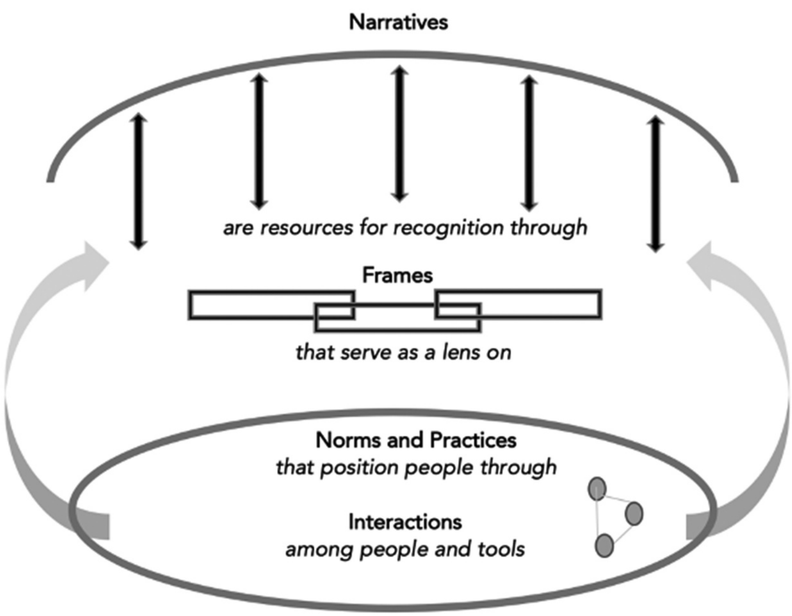

Figure 1. Gresalfi and Hand (2019) model of resources for identity construction as they interact in mathematics classrooms, reprinted with permission from Gresalfi and Hand (2019), FIZ Karlsruhe.

ways. Decades of research in mathematics education has documented that learners' ideas about mathematics learning—what counts as mathematics, how they are expected to engage with mathematics, and ultimately what they think about themselves in relation to the mathematics—develop through their experiences with the norms and practices of different settings (Boaler & Greeno, 2000; Gresalfi et al., 2009; Schoenfeld, 1988). Here, norms refer to the general obligations and expectations for behavior that are taken-as-shared among participants in a setting (Yackel & Cobb, 1996). Practices refer to the specific kinds activities that are routinely undertaken in that setting (Cobb et al., 2001).

These two terms are often used together because they are related: the norms of a setting influence the practices that are likely to dominate. For example, if a norm is "maintain 6 feet of distance" a practice in a grocery store might be to stop and wait in an aisle while someone looks for a product on a shelf. Before that norm was in place, a far more common practice would have been for the person to simply go around the shopper who had stopped to search.

With respect to mathematics, learners' experiences are usually predominantly located in classrooms, which for many American students are characterized by "traditional" school mathematics norms and practices (Jacobs et al., 2006; Litke, 2015). In such classrooms, the teacher and textbook are sources of *authority* (Herbel-Eisenmann, 2007; Mehan, 1979)—an interactional feature of schooling in which teachers and textbooks are often expected to offer claims about what is right and how mathematical thinking should unfold (c.f. Engle & Conant, 2002). Likewise, classrooms can sometimes include discourse patterns that emphasize compliance over sensemaking (Cazden, 2001), communicating to students that *how* they solve mathematical problems is the most valued measure of mathematical thinking. This version of authority contrasts starkly with authority as it is typically exercised among mathematicians, which involves exploration, problem-posing, justification to peers, and constructing aesthetically-appealing solutions (Sinclair et al., 2006).

A second, related feature of schooling is *evaluation*, which leads to the inevitable classification of students as either failing or succeeding. This includes informal labels (e.g., fast, slow, lazy, "bubble" kids) that become resources for students' identity development (Anderson, 2009; Gargroetzi et al., in press; Horn, 2007, 2008) as well as more formal, institutionalized labels (e.g., low-tracked, high risk, gifted, ADHD, learning disabled) that more directly position students as particular kinds of people (McDermott, 2001). For many students—including high achieving students—traditional evaluation practices can lead to disaffiliation with mathematics (e.g., the common phrase, "I'm not a math person," Boaler & Greeno, 2000; Pope, 2001).

These experiences in school can be very influential. Because schools, as institutions, have the power to certify and sanction certain forms of knowledge, people can demonstrate knowing mathematics outside of school but be evaluated as not knowing within its walls (Nasir, 2002; Noss et al., 2002; Saxe, 1988). What is more, ideas and expectations from school can carry over into other contexts. For example, Griffin (1994, 2004) demonstrated that when children invoke school-based expectations of learning in museums, they often do not see their activity as meaningful for learning unless given a worksheet to fill out while there. This suggests that some of the characteristic practices of schooling are easily invoked in out-of-school spaces, becoming a benchmark to make sense of content, engagement, and learning.

Research on out-of-school STEM spaces—commonly referred to as *free-choice learning environments* (e.g., Falk & Dierking, 2002)—has demonstrated that often a very different set of practices and expectations abound. For example, in relation to *authority*, free-choice environments invite visitors to determine their own goals and how to pursue them. This supports multiple trajectories through a space and myriad possible experiences that visitors might have. Although at times designers have attempted to deepen inquiry by semi-structuring visitors' goals, often this can lead to "killing the playfulness" that is fundamental to learning in these spaces (Falk & Dierking, 2013). For example, Gutwill and Allen (2012) designed a "juicy question" participation structure that succeeded in supporting learners to pose more questions, interpret their results, and produce collaborative explanations. Yet, some children and chaperones noted that the formal structure of the intervention killed the interest-driven nature typical of their prior museum experiences.

In relation to *evaluation* in free-choice environments, visitors self-evaluate based on whether they are satisfied with their activity and whether they meet their own goals; "failure" to meet one's goals is rarely automatically attributed to the visitor themselves. Visitors are unlikely to say "I failed at that science museum today" but rather "the new exhibit isn't that great." Thus, free-choice environments not only allow visitors to determine for themselves if they have met their goals, but they also offer a different set of resources to make sense of a disappointing experience.

Frames

Math On-A-Stick (MOAS) offers at least two possible broader narratives, *school mathematics* and *play*. Which narratives people recognize and make relevant in their interactions is a matter of *framing* (Goffman, 1974), with frames being constituted by the sum of cultural activities evoked for an individual in a given situation. *What* is evoked—what constitutes the particular cultural activity for a particular individual—has to do with an individual's history of participation across contexts. Operationally, framing is the phenomenon of interpreting a situation to determine (often unconsciously) "what is going on here," and relatedly, how it is "desirable, appropriate, or at least socially acceptable" to behave, move, and speak (Engle, 2006, p. 455). Framing usually happens unconsciously, as people seamlessly enter new spaces without problem. While frames are almost never stated explicitly in interaction, they are most analytically visible when they are called into question (Hammer et al., 2005) or when participants' expectations are broken.

In relation to mathematics, novel activities that disrupt both typical processes of engaging with mathematics and broader narratives around what counts as mathematics invite *hybridizing* (Ma, 2016) because they elicit uncertainty. Hybridizing is the act of integrating two or more activities, resulting in a new activity that is different from either (Bakhtin, 1981; Calabrese Barton & Tan, 2009; Gutiérrez et al., 1999; Ma, 2016). In considering how uncertainty functions in the invocation of framing, Goffman (1974) stated that:

> It is perfectly possible for individuals, especially one at a time and briefly, to be in doubt about what it is that is going on [. . .] And insofar as the individual is moved to engage in action of some kind—a very usual possibility—the ambiguity will be translated into felt uncertainty and hesitancy. Note, ambiguity as here defined is itself of two kinds: one, where there is question as to *what could possibly be going on here*; the other as to *which one of two or more clearly possible things is going on* (pp. 302, 303, *emphasis* added).

In this way, novel activities that have less established—and thus, more ambiguous—norms and practices may invite hybridizing since participants must recruit their resources from familiar cultural activities to resolve their uncertainty in how to proceed (c.f. Lavie et al., 2019). We refer to such activities as *hybridization-inviting*, and we consider this recruiting of resources to be a process of *framing*.

Research design

Research questions

As mentioned, this paper asks two related questions:

(1) How do children hybridize their activity in out-of-school spaces?
(2) How is this hybridization consequential for engagement?

We examine these questions by looking closely at two learners, Aimee and Dia, who traveled the exhibit together but demonstrated very different patterns of engagement and had very different reflections on their activity. Understanding these differences motivated our in-depth look at these cases, as described in greater detail below.

Setting

Data for this study come from a mathematics playspace called MOAS, located at the Minnesota State Fair. In the United States, state fairs happen once a year in each state for 1–2 weeks. They are public places of leisure and entertainment, with thrilling roller coaster rides, games with prizes, and indulgent foods that are often served on a wooden skewer (hence the space's playful name, MOAS). MOAS was tucked away in a quiet corner of the fairgrounds near educational booths and buildings.

Despite its mathematical label, the materials at MOAS looked different than most school-based mathematics materials. The materials were designed to be aesthetically pleasing—appealing to the eye, the hand, and even theear—to invite encounters with mathematical concepts. For example, the *Tiles and Patterns* and *Pentagons* exhibits included shapes that tiled the plane without necessarily requiring or producing symmetry, while the *Pattern Machine* and *Eggs and Crates* exhibits included grid-like materials with affordances for encountering different mathematical concepts based on whether the arrays were square or rectangle, with even or odd numbers of grid spaces.

MOAS offered visitors little structure in terms of instructions. Typically, visitors spent their entire time at MOAS playing as they wished; mostly, volunteers helped if asked or offered suggestions if pressed. Most visitors to MOAS engaged in solo or parallel play with friends or family rather than in collaborative play, perhaps due to the design of the materials.

Though MOAS was designed to encourage play, not all fairgoers found it equally inviting. Our fieldnotes capture this poignantly. Sitting on the sidewalk of MOAS watching passersby, we heard some rejecting MOAS with statements like, "Mathematics at the fair! No thank you!" or "Math? Ugh, keep walking." Despite this clear indication of self-selection based on (dis)affiliation with mathematics, our study includes participants who claimed that mathematics was their least favorite subject.

Data collection

Because we are interested in how children hybridize their activity and the consequences of hybridization on their engagement, our primary data comes from line-of-sight video that captures the locus of children's visual attention. Participants wore GoPro™ cameras on baseball caps aimed downwards and slightly forward to capture their talk, gestures, and object manipulation (Figure 2). The final data set included a total of approximately 179 hours of video from 337 participant video records, with the average visit lasting approximately 32 minutes (*sd* = ~15 min).

Participants were given an intake survey (see Appendix A) to capture demographic information, an exit survey (see Appendix B) to capture

Figure 2. A participant wearing the GoPro™ camera.

experiences with school mathematics, and a brief semi-structured exit interview (Appendix C) to capture immediate reflections on activity at MOAS.

Analysis

Phase 1: Coding the corpus

Our first task in our larger study was to understand learners' activity in general at MOAS. Thus, we first coded all 337 videos (using Studiocode™ software) for when participants were at each exhibit. Of the 179 hours of video collected, 57 hours contained records of participants engaged in transitional activities, such as checking in with their parents or eating snacks (~10.5 minutes of transitional time, on average). The remaining 122 hours contained 101 hours of participants in our focal age band (7–12 years old, n = 277, see Figure 3) and 21 hours of participants outside of our age band (n = 58) or with no age recorded (n = 13).[1] This initial viewing offered a sense of typical activity at each exhibit.

[1]The number of participants accounted for here and in Figure 7 (348) differs from the number of video files analyzed (337). This is due to (a) some video files failing to save and (b) some entrance surveys (the source of age data) being filled out more than once due to perceived issues with the Google form and internet at the fair. We consented 345 participants, so this number is not off by much.

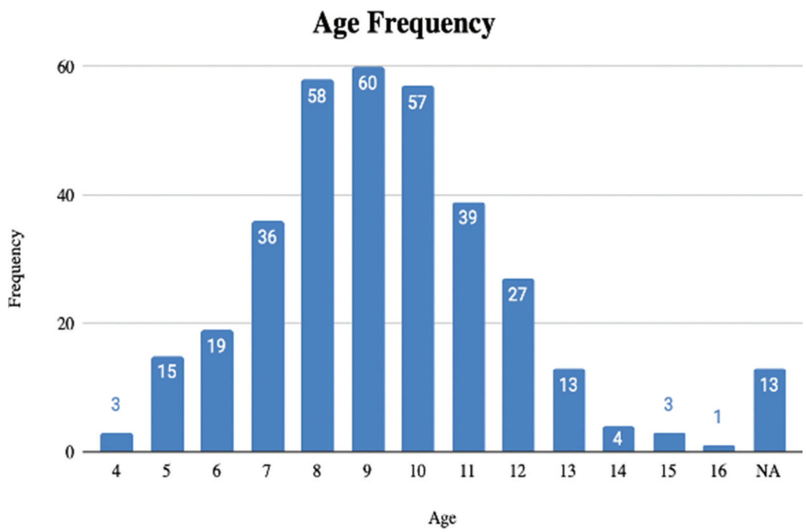

Figure 3. Distribution of participants' age in years.

Phase 2: Data reduction

During Phase 1, we noticed that participants with longer durations at particular exhibits seemed to offer a richer understanding of the potential for *mathematical* engagement, an interest of the larger project. This was relevant to the current analysis as it allowed for a focus on how school norms and practices might affect learners' framing of their activity in a playful math space.

Following Gutwill and Allen (2012) in their study of inquiry in museums, engagement was first conceptualized as duration at each exhibit. We used the median duration for the corpus as an indication of typical engagement, then compared each child's duration at each exhibit to the median and selected participants with longer stay times.

Phase 3: Analyzing patterns in engagement

Because seeing framing requires finding meaningful chunks of activity in which an individual might orient to the question "What is happening here," we segmented participants' activity into episodes.

Centering children's perspectives in video records lends itself to interaction analysis since this approach seeks to understand participants' meaning in activity (Hall & Stevens, 2016; Jordan & Henderson, 1995). We began by determining appropriate analytic chunks of the video, which we term *Episodes of Making* (EOMs). An EOM is an instance of activity that is oriented toward a particular goal *or* end-state; the making of "something." To emically define EOMs, we identified beginnings and endings of activity from participants' perspectives. According to Jordan and Henderson (1995), these "starting up" and "winding down" segments of interaction tend to be the temporal location of significant events. Delineating the boundaries of EOMs can be difficult due to goal shifting in play, so ultimately, we specified the endpoint of each EOM by attending to when participants "erased" their prior work (i.e., making themselves a "blank slate" by destroying what they built) or made major revisions that resulted in a new object. We then rewound the video to determine when that activity had begun (often right after the end of the previous EOM). If the child made a blank slate and re-attempted their previous EOM, we rewound the video and coded the entire instance as one. Together, Phases 1–3 took over a year, with the first author and multiple research assistants parsing and re-parsing the data.

Phase 4: Selecting focal participants for contrasting engagement

Because our research question asks *how* children hybridize and *what* the consequences are for their engagement, we selected a case likely to provide deep understanding with maximum variation (Flyvbjerg, 2006).

While coding for EOMs, we identified a pair of participants, Aimee and Dia[2] (pseudonyms), who were friends moving through MOAS together, yet also engaged the space very differently. They were the only two participants who stayed at all four selected exhibits for longer than was typical, making them our two "most engaged" participants (following Gutwill & Allen, 2012). However, by other measures their engagement was fairly representative. For example, their stay times at each exhibit were longer than the median stay time for all 7-to-12-year-olds, yet their stay times were usually closer to the median of all 7- to 12-year-olds than to the median of 7-to-12-year-olds who stayed longer than typical (Table 1). As a note, 62 participants in our age band stayed longer than typical at more than one exhibit, with 13 of them staying longer than typical at three exhibits, 45 staying longer than typical at 2 exhibits, and two staying longer than typical at only one exhibit.

Table 1. Aimee's and Dia's stay times at selected exhibits.

	Minutes/station for all 7-to-12-year-old participants		Stay times (in minutes) at selected exhibits for 7–12-year-olds who stayed longer than typical		Aimee and Dia's stay times	
Exhibit	*n*	*Mdn*	*n**	*Mdn**	*Aimee*	*Dia*
Tiles and Patterns	180	2.15	69	4.91	6.54	6.45
Pentagons	156	4.07	70	7.71	5.34	5.86
Pattern Machine	237	4.32	104	7.63	6.42	6.51
Eggs and Crates	156	2.76	68	4.58	5.03	3.53
Lizards and Turtles	183	4.95	––	––	––	––
Spiral Machine	90	3.26	––	––	––	––

[2]Although not central to our analysis, Aimee's and Dia self-reported their race/ethnicity. Aimee reported *Chinese American* and Dia reported *American*. Both girls struggled to identify their race/ethnicity, consulting a guardian before reporting their identification.

The girls' affinities for mathematics are the primary dimension along which they varied. As Dia recounted in her exit interview, the girls visited MOAS because they spotted it on their way to the fair's science center, which they were heading to because science was her favorite subject. As Aimee recounted in her exit interview, they came to MOAS "because I love math." Although both girls expressed affinities for elements of STEM disciplines (science, technology, engineering, and mathematics) as bringing them to MOAS, Aimee and Dia described contrasting experiences of mathematics in their entrance surveys, with Aimee claiming mathematics as her favorite subject and Dia claiming mathematics as her least favorite.

Although their affinities for mathematics differed, Dia and Aimee had many similar opinions about mathematics as they experienced it in school, as indicated by a short Likert scale exit survey.[3] They both thought that mathematics is moderately challenging (*sometimes*), that it consists of steps (*usually*) and facts to remember (*sometimes*), and that they needed their teacher to tell them whether their answers are correct or not (*sometimes*). They both also responded that they had the agency to figure out whether or not their answers were correct on their own. Interestingly, Dia—whose least favorite subject was mathematics—provided a higher agency response (*strongly agree*) than Aimee (*agree*).

Their survey responses differed in relation to whether they found mathematics interesting or boring, with Dia finding mathematics very boring (*strongly agree*) and Aimee finding it very interesting (*strongly agree*). In this way, we interpret Dia's dislike for mathematics not to be related to her feelings of competence or agency but rather to her sense of aesthetic—her feelings about whether or not mathematics is interesting. Yet, as we will show in the findings, Dia positioned herself as failing to produce interesting EOMs at MOAS—that is, as failing to find interesting problems—and thus also evaluated herself as not fully competent in this out-of-school context.

Phase 5: Frame analysis of hybridity between school mathematics and play

We drew on frame theory as a means of orienting our analysis into whether and how Aimee and Dia hybridized their activity in the mathematics playspace of MOAS with school-mathematics norms and practices around authority and evaluation. We looked for indicators of the girls' frames by attending to the content of their talk, the majority of which involved making comments about their work. In order to make sense of their co-navigation of and parallel engagement in MOAS as a joint endeavor, we viewed their

[3]Notably, our video records cut off before the girls began their exit surveys so it is possible that they copied one another's answers, but we find this unlikely as other questions on the survey have non-matching responses, as well as because our presence collecting the data leads us to believe that such copying of responses was uncommon.

Figure 4. A screenshot of Aimee's and Dia's videos stitched together such that their activity was synced in time. Aimee's arms can be seen reaching across Dia's body to grab eggs in both Aimee's video (right) and Dia's video (left).

videos side by side (Figure 4). Such a view gave us the ability to analyze points of renegotiation and confusion during framing. We then transcribed the entirety of their stay at MOAS using a joint transcript that attended to talk and manipulation of exhibit materials.

The next step in the analysis was to investigate their framing. To do this, we focused on practices that are distinctly different in school math and play, involving authority and evaluation.

Authority in task determination. We looked for *authority* by attending to whether or not the girls oriented to the exhibits in self-determined ways. For example, we coded statements such as "So I just play the games?", "What are you supposed to do?", and "So now we just make more designs?" as evidence for an external locus of authority. Attending to who *posed* orienting questions and who *answered* them helped to clarify how participants positioned themselves and each other. We also carefully attended to the time lapse between when participants first began manipulating materials at each exhibit and when they began making their first EOMs at each exhibit. We consider this span of time to be *exploratory* activity preceding play (Hutt, 1966; Pellegrini, 2009), with exploratory activity as an indication that the girls expected to exercise personal authority to make sense of materials.

Evaluation of correctness. We looked for *evaluation* by attending to how the girls judged their EOMs. Following Allen's (2003) study of learning through talk in museums, we coded our video data for participants' evaluations (or requests for evaluations) of their own performance, actions, or abilities. We also coded affective talk as evaluations because comments such as "beautiful," "cool," "I like that," "ugly," "whoa!," and "ooooh" can be considered evaluations of the things produced by participants at MOAS. Because MOAS is a mathematical playspace—it is composed of exhibits with aesthetically pleasing objects designed to afford mathematical encounters—we take aesthetic evaluations at MOAS (e.g., "No that's ugly") as particular instances of evaluations for correctness. This is consistent with mathematicians' evaluations for correctness: an answer may be known but a solution is not reached until the presentation of the solution satisfies the mathematician's sense of aesthetic (Sinclair, 2004; Sinclair et al., 2006).

We then classified the girls' evaluations as positive (expressing satisfaction), negative (expressing dissatisfaction), or neutral (expressing neither satisfaction nor dissatisfaction). We coded statements such as "I like it!" and "There!" as *positive evaluations*; statements such as "I can't make it," "No that's ugly," as *negative evaluations*; and statements such as "I don't know what I'm making" and "It's hard to figure out" as *neutral evaluations*. Subsequent viewing of the video led us to add gestures to our account of evaluations. The gestures were all coded as positive evaluations, for example, tracing designs or rubbing hands together with pleasure after completing designs. In the interest of not over-interpreting the data, we left uncoded any gestures that may have been neutral and negative as these were more difficult to interpret.

We coded the entire joint transcript (including gestures) and found evidence for authority and evaluation throughout both girls' EOMs.

Trustworthiness of the analytic methods

Our analytic approach sought to ensure the truth value, consistency, and neutrality of our findings (Lincoln & Guba, 1985). We safeguarded for credibility through prolonged engagement and persistent observation, both in the field when collecting data and in repeated engagement with the video data of all 277 7–12-year-old participants. The first author was present for all 10 days of the fair, for most of the time MOAS was open each day. We took multiple roles during our time at the fair, from exhibit facilitator to participant observer playing at the tables to taking fieldnotes as an observer. Our analysis included multiple iterations of viewing and coding of the data, spending almost a year with multiple analysts talking through what we saw and refining our codes, before beginning this particular analysis. These systematic data analysis procedures help to ensure the dependability of our analysis, as our procedures arose as we pursued emergent patterns in the data. Finally, our attempts to maintain neutrality include triangulating our interpretations of the girls' interactions with their exit interviews and surveys, peer debriefing in interaction analysis labs with scholars from multiple fields of study not associated with the project, and offering raw data to allow the reader to form their own judgments. These neutrality safeguards establish the confirmability of our analysis.

Findings

Our analysis of Aimee and Dia indicates that they hybridized their activity differently, framing their activity in ways that evoked dissimilar norms and practices, leading to engagement with dissimilar characteristics. As primary evidence of this claim, we present how the girls engaged at each exhibit by appealing to external authority or to themselves and how they evaluated their engagement. We focus first on the most readily observable aspect of identity as described by Gresalfi and Hand (2019): interactions among people and tools, Level 1 of the identity model. We then turn to Level 2 of the identity model, considering whether and how Aimee and Dia's observed interactions might indicate different framing of their activity. In the discussion we consider Level 3 of the identity model—the role that broader narratives and experiences might have played in the ways that the girls hybridized their activity.

Interaction among people and tools

At a large grainsize, Aimee's and Dia's joint interaction at MOAS can be described as a form of co-navigation structured by both their friendship (they stayed together most of the time) and their individual preferences, with the greatest difference in time spent at each exhibit being just over half a minute (Table 2). However, the way they led each other through MOAS was in some ways quite different. When Aimee left exhibits first, it was almost always because their adult chaperone suggested that they move on to another exhibit. An exception is when the girls were at the Visiting Mathematician exhibit. Unlike other exhibits which allowed children to determine their tasks, the Visiting Mathematician exhibit was structured by a facilitating adult mathematician. Aimee left this exhibit of her own accord and went to an exhibit with more open engagement. In contrast, Dia left multiple exhibits before Aimee but stayed longer than Aimee at the one exhibit that was structured by an adult—the Visiting Mathematician. We interpret this as Dia having a preference for exhibits that are more structured by adults while Aimee preferred exhibits that were more open.

Our analysis of each girls' exhibit-level activity focused on how they approached each exhibit, how they engaged in their respective EOMs, and how they appeared to feel about the things they produced.

Dia's activity: Uncertainty and dissatisfaction

In the exploratory phase, Dia frequently explicitly questioned what she was *supposed* to be doing. Within the first 13 minutes of her 35-minute visit, Dia offered nine utterances that focused on the purpose of her participation, saying things like, "So I just play the games?" or, "We have to do all of them,

Table 2. Description of Aimee and Dia's exhibit exits.

Aimee and Dia's Exhibit Exits

Explanation	Difference (Dia-Aimee)	Dia Duration	Aimee Duration	Exhibit	Exhibit Order
Dia started late because of consent process; Aimee *agreed to leave* first because of adult suggestion but Dia walked away first	**−1.52**	**3.58**	**5.09**	Eggs & Crate	1
Aimee left first because of adult suggestion	0.22	6.52	6.30	Pattern Machine	2
Dia left first after completing multiple EOMs and Aimee followed, also having completed multiple EOMs	−0.14	6.39	6.52	Tiles & Patterns	3
Aimee pulled Dia away to go to the Stepping Stones	0.00	0.56	0.56	Visiting Mathematician	4
Aimee did more repetitions at Stepping Stones and Dia went back to watch Visiting Mathematician (exhibits were right next to each other and both were standing rather than sitting exhibits)	**−0.59**	**0.32**	**0.90**	Stepping Stones	5
Both left Visiting Mathematician at the same time because of adult suggestion	**0.51**	**0.76**	**0.26**	Visiting Mathematician	6
Aimee stayed longer at the playful mathematical books exhibit and Dia went to Pentagons	**−0.46**	**3.80**	**4.26**	WWDB Book	7
Aimee left Pentagons first because of adult suggestion	**0.53**	**5.86**	**5.33**	Pentagons	8
The adult guardian suggested it was time to leave MOAS and go find lunch, but Dia said she could not leave until she finished her current design. Dia decided she was done when she had a rectangle; Aimee's design was not in a "clean" shape when she decided to leave the exhibit after Dia did.	−0.18	4.72	4.90	Lizards & Turtles	9
Before heading over to return the Go-Pros and do their exit surveys and interviews, the girls pose for a picture taken by their adult guardian. Before the picture was taken, Dia moved her sticker with her participant number onto the front of her t-shirt where it is was highly visible to the camera (she said she wanted her number in the picture).					

I think." Thus, Dia appeared to be searching for directions, indicating that, for her, the openness of the activity was an ambiguity that needed to be resolved, and relatedly, that she was not the person who was accountable for establishing the rules at the exhibit.

Dia's physical interaction with the materials at MOAS further indicated an expectation of predetermined well-structured tasks rather than using the materials to explore the parameters of what was possible and interesting at each exhibit. Dia rarely explored the materials of the exhibits before using them in an EOM; her explorations generally lasted only 2–16 seconds.

Instead, she almost always integrated the first materials she touched into her first EOM.

This suggests that Dia did not exercise her personal agency to make sense of the materials on her own. It seemed that, in the absence of finding defined rules or instructions, Dia used materials to comply with the expectation that she immediately "make something."

Despite the apparent uncertainty with which Dia approached the exhibits, Dia evaluated her activity quite frequently (Table 3): while working on her EOMs, she produced 27 self-evaluations over 14 EOMS. Her frequent evaluations suggest Dia felt that some EOMs were more valuable or correct than others.

In her self-evaluations, Dia appeared to demonstrate concern for doing things correctly based on an external standard. As Table 3 illustrates, Dia made negative self-evaluations in six of 14 EOMs and positive evaluations in two EOMs. She also made neutral statements like "I don't know what I'm making" in seven out of 14 EOMs, and these statements functioned discursively to devalue her work. For example, at *Tiles and Patterns*, she repeatedly made neutral comments about not knowing ("I don't know what I'm making;" "What is this, nobody knows") and negative comments about being unsuccessful ("Dang it I can't do it;" "Great" in a sarcastic tone while destroying her design; "No that's ugly" while trying to create an "m" on the *Pattern Machine*). Despite these evaluations, Dia's EOMs resulted in emergent patterns (Figure 5).

Illustration of uncertainty and dissatisfaction. In the *Pattern Machine* transcript excerpt in Table 4, Dia repeatedly questioned what she was supposed to be doing (see timestamps 06:37, 07:04, 07:23) and negatively evaluated her work. Although Dia showed interest when suggesting the *Pattern Machine* as their next exhibit ("Let's do this, this looks interesting"), she immediately followed her suggestion with a request for guidance in understanding the task. Aimee answered, telling her that the point of the exhibit was to make patterns. While noting that they had been doing a lot of pattern-making at MOAS and that she liked pattern-making, Dia also denigrated her pattern

Table 3. Summary of Dia's and Aimee's evaluations during each EOM*.

	# of EOMs		Positive		Neutral		Negative		Total	
Exhibit	Dia	Aimee	Dia	Aimee	Dia	Aimee	Dia	Aimee	Dia	Aimee
Eggs (1)	2	3	5	2	2	0	1	0	**8**	**2**
Pattern Machine (2)	4	6	0	2	2	1	2	1	**4**	**4**
Tiles and Patterns (3)	4	4	0	2	5	0	2	0	**7**	**2**
Pentagons (8)	3	3	0	0	0	0	3	0	**3**	**0**
Lizards and Turtles (9)	1	1	3	0	2	1	0	0	**5**	**1**
Total			**8**	**6**	**11**	**2**	**8**	**1**	**27**	**9**

*Exhibits 4–7 are not included in this table because they did not involve EOMs.

Figure 5. The product of Dia's four EOMs at *tiles and patterns* (tiles are multi-colored).

Table 4. An excerpt of transcript from the pattern machine*.

Time	Actor	Talk	Action
06:34	Dia	Let's do this, this looks interesting	
06:37		What are you supposed to do?	*((beginning to touch Pattern Machine))*
06:41	Aimee	You make a pattern	*((beginning to touch Pattern Machine))*
06:43	Dia	There's a lot of just making patterns	
06:44		This is the kind of math I like	
06:57		I'm tired of standing	*((grabs Pattern Machine and moves to opposite side of table, such that she and Aimee are face-to-face across the table from each other))*
07:04		I don't know what this is supposed to be	*((sits down))*
07:23		Ohhh are we supposed to count?	*((beginning to push down design))*
07:26	Aimee	No	*((beginning to make heart from the bottom))*
07:37			*((finishes up the last bit of a heart, and after clicking down the last buttons moves her hands away quickly as if to draw attention to the fact that she has finished))*
07:56	Dia	Tah-dah	*((dull tone, picks up her Pattern Machine to show what she has made))*
07:59	Aimee	Can I see!?	
08:00	Dia		*((turns her Pattern Machine so Aimee can see it))*
08:13	Aimee	Oh. I want to pop up-	*((begins popping all of the buttons up quickly, meanwhile Dia is pushing her design down))*

*This transcript has been modified to include talk that centers Dia, and so talk between Aimee and their chaperone has been omitted for the sake of clarity and page limits.

twice, once while creating it, saying, "I don't know what this is supposed to be" and once at the end of making her pattern, saying, "Tah-dah," with a downward intonation that communicated the opposite of excitement or pride in work (i.e., sarcasm). Thus, this analysis of Dia's activity suggests framing around authority and evaluation that led to uncertainty in and dissatisfaction with her activity.

Aimee's activity: Exploration and pleasure in making

Although Aimee and Dia traveled through the exhibit together, Aimee's experience appeared very different. In general, Aimee determined her own goals for her activity, actively displayed pleasure in the process of making, and did not negatively self-evaluate.

In contrast to Dia, Aimee appeared to treat the open, unstructured exhibits as an invitation to explore and did not seem to struggle to consider whether her activity was aligned with any externally-defined goals. Indeed, occasionally when Dia posed questions about their purpose, Aimee's responses suggest her ownership of her own activity. For example, when Dia and Aimee approached the *Pattern Machine*, Dia asked "What are you supposed to do, just press things?" Aimee answered by offering an alternative to Dia's proposal, saying, "Or you make a pattern." This is not to say that Aimee immediately knew what to do when she moved to a new exhibit. Whereas Dia approached the first three exhibits asking what she was supposed to do, Aimee approached the same exhibits by picking up materials and exploring their properties, before leveraging the structure of the materials toward making deliberate designs. For example, Aimee took almost twice as long as Dia to get started with her first EOMs at all exhibits (with generous coding on what counted as Dia's exploration). While the first materials Dia touched often remained part of her first EOM, Aimee's rarely did, with Aimee taking 20–34 seconds to explore and Dia taking between 2–16 seconds to explore.

This suggests more intentionality in Aimee's work than in Dia's. For example, at *Tiles and Patterns*, Dia used the first two tiles she put together in her first EOM in exactly the way she first put them together. Aimee, on the other hand, explored for approximately 33 seconds, which was enough time for her to select only green colored tiles and to notice that there were two kinds of green colored tiles. It took Dia until her third EOM at this exhibit to notice this design feature (as indicated by her surprise, selection and rejection of tiles, and talk). The products of their EOMs at this exhibit strongly reflect this difference (Figures 5 and 6), with Aimee's designs having a clear repeating structure. At *Pentagons*, Dia explored for 14 seconds and then attempted a pinwheel with red and dark brown pentagons but soon gave up, saying "Dang it I can't do it." She went on to make a pinwheel without red pentagons and evaluated it as a "boring circle." Aimee, on the other hand, explored for 22 seconds, much of which was spent examining the red pentagons (which required special attention to orientation due to being red on one side and light brown on the other), and then successfully completed a red and dark brown pinwheel. We interpret Aimee's extended exploration as an indication of her sense of personal authority to make sense of the exhibit materials and to use them for her own patterning interests.

Aimee did not appear to engage in frequent self-evaluation; she produced only one third the number of evaluations as Dia, with 15 evaluations total—less than one evaluation per EOM (Table 3). These evaluations took place at the end of her activity, when she stepped back to comment on what she had made. Most (six out of nine) of these evaluations were positive. However, Aimee did have one negative evaluation. At the *Pattern Machine*, she attempted to make an "A" for Aimee but was unable to make it the way she wanted. Dia showed Aimee how to make an A, but Aimee discounted Dia's suggestion by saying that she wanted to make her A fill-up the board and have a point at the top (i.e., rather than an A with a flat top), which is impossible in the nine-by-nine grid of the *Pattern Machine*. In this way, Aimee's negative evaluation was quite different from Dia's evaluations, as Aimee's was related to the constraints of the tool in relation to her goals. In

Figure 6. The product of Aimee's four EOMs at *tiles and patterns* (tiles are colored only green and black).

a

b

c

Figure 7. Aimee's TWO evaluations in her third EOM at *tiles and patterns* (tiles are colored only green and black).

contrast, Dia's negative evaluations were based on her affective orientation toward her own design work. This pattern led to an observable difference in the girls' affective experiences at MOAS.

Aimee's apparent enjoyment of her work carried through all her EOMs. For example, at the *Pattern Machine*, Aimee described the materials as "strangely fascinating" and often traced over her final designs. At *Tiles and Patterns*, she gleefully exclaimed "I made something!" and then rubbed her hands together with apparent pleasure and continued to extend her design. Upon finishing this design, she again rubbed her hands together and then traced units of her pattern (black and green diamonds) before carefully disassembling it (Figure 7).

Illustration of exploration and pleasure in making. In the *Eggs and Crates* transcript excerpt in Table 5, Aimee set a goal of making a heart. Before beginning, she attended to the midline of the crate, rotating the crate so that the side with five slots was parallel to her body and created a center row that would allow her to make a heart with one egg at the upper and lower points. When she finished making her heart, she rescued it from Dia's eager hands attempting to steal the blue eggs that provided a background for her heart. When Dia retreated and asked Aimee what she had made, Aimee raised the pitch of her voice so that it sounded quite sweet and said, "It's a heart," while tracing the mirrored sides of the heart using both hands. We take this preservation, tone change, and tracing to indicate that Aimee found enjoyment and pride in her activity.

Table 5. Aimee's third EOM at eggs and crates.*

Time	Actor	Talk	Action
03:25	Aimee	Ohh wait, I know what I wanna do	*((dumps crate, grabs 2 pink eggs, rotates crate so side with 5 (not 6) slots is parallel to her body, places egg in middle slot —second row from top, continues to make pink heart))*
04:54	Aimee	Can I have more blues?	*((filling up background of heart in crate))*
04:55	Dia	Yeah	
04:55	Aimee		*((reaches over Dia's egg crate to grab more blues))*
05:09	Dia	Wait no I want the bluuuue	*((grabs to take blues out of Aimee's crate, which is now completed with its blue background))*
05:10	Aimee	NO NO! Wait wait, Wa-wait wait wait	*((covers Dia's hands, preventing removal of blue eggs))*
05:12	Dia	What is it?	*((removes hands from Aimee's crate))*
05:14	Aimee	It's a heart	*((traces heart with fingers using both hands in a coordinated motion))*
05:15	Dia	Oh! Well can I-	*((reaches to grab blue eggs out of Aimee's crate again))*
05:15	Aimee		*((dumps out all eggs from her crate before Dia grabs any out, many of them landing on Dia's crate))*
05:17	Dia	Not on mine!	*((removes eggs that landed on her crate from Aimee dumping eggs))*
05:26	Aimee		*((starts putting blue eggs back into her crate, then takes them out and replaces them with green eggs))*

Summary of interactions: Authority and evaluation

Authority. Our analyses suggest that the girls exercised authority differently at MOAS. For Aimee, the openness of the activities appeared to invite exploration and then unproblematic movement into plans and designs of her own, with an eye toward her own interest and enjoyment. In contrast, Dia approached the exhibits with an eye toward the external rules with which she was expected to comply.

Evaluation. The quantity and nature of the girls' evaluations suggest that they saw their activity differently. At MOAS, evaluations from "authorities" were rarely available, which may have contributed to Dia's uncertainty about her activity. Overall, Dia engaged in significantly more evaluations of her work than Aimee, debating about whether what she was doing was "right" or "made sense." These comments and their frequency suggest that Dia was aware of or concerned about the audience for her work. In contrast, Aimee was comfortable with the exploratory and unstructured nature of the environment, making changes to her design without comment, and then indicating her satisfaction with her work upon its completion.

In chronicling and attempting to make sense of these differences between Aimee and Dia, it seems to us that although they worked side-by-side and commented on each other's work, the girls were ultimately engaged in different activities. For that reason, we turn to the concept of *framing* to help us to make sense of why these differences might have occurred.

Inferences on the role of frames in hybridizing in activity

The novelty of MOAS made important norms and practices open for interpretation and re-construction. Everyone who entered the exhibit had to answer the question, "What is going on here?" Likewise, Dia and Aimee had to draw on their personal histories and interactions to decide what kinds of activities—and their associated norms and practices—were relevant.

How might Aimee and Dia's prior experiences with school mathematics have shaped the way they each differently hybridized mathematics and play in the interactions we observed? While we have no direct observations of Aimee's or Dia's prior mathematical experiences, we make conjectures based on available evidence. As a starting point, in the exit interview we asked the girls if MOAS resembled their experience in school math: Dia said no and Aimee said sometimes.

School-math framing can be defined as the phenomenon of interpreting a situation as one in which the best discourse to use is that of school mathematics, with its associated norms and practices. Dia's talk contrasted activity at MOAS with school mathematics and found MOAS to be likable but school math not ("This is the kind of math I like," Table 4). This verbal contrasting of the two activities suggests that Dia tried to make sense of MOAS by foregrounding school math, repeatedly finding misalignments between the norms and practices of the math she disliked at school and the playful math she did like at MOAS. Evidence of math being foregrounded were suggested when Dia engaged in activities that mimicked expectations of school, namely expectations of fulfilling tasks predetermined by higher authorities (e.g., "What are we supposed to do here?") and an orientation toward doing so correctly (e.g., "I don't know what I'm making").[4] In traditional school math, tasks are fairly well-defined and typically everyone "makes" the same thing—the one acceptable correct answer. Yet, even as she posed these questions, Dia played with and explored the materials; her unique and bold designs show traces of play that align with her vision of herself, as seen in her response to the question "What is it?" with "Nobody knows, it's my creation so that makes sense." In her exit interview, Dia said that MOAS was unlike mathematics class and we argue that her activity was shaped by this misalignment. Specifically, in drawing on both school math and play as a frame, Dia hybridized her activity in a way that ultimately resulted in uncertainty and dissatisfaction, as she demonstrated expectations for a kind of certainty and external evaluation that was absent, while she engaged in the design activities of the exhibit. Although she did appear to explore the materials and make her own things, the joy that is typically associated with play was generally absent from Dia's activity.

In contrast, Aimee said that MOAS *is* like what she gets to do at a math play table in her math class after she finishes her work. Thus, in asking "What is going on here," school math as a frame was not such a noticeable mismatch with the affordances of the activity. Her resulting engagement—hybridizing school math and play—offered a more expansive and enjoyable experience. Indeed, it is difficult to say where school math began and play ended, at least with respect to the way Aimee appeared to frame her activity. In contrast to foregrounding evaluative expectations of school math, Aimee's school mathematics frame made space for play.

The girls' different reported histories make it possible that the environment may have cued Aimee and Dia differently such that they engaged in contrasting ways at MOAS, with Aimee making her playful experiences in math class relevant (i.e., the cue was play with mathematical objects, likely pattern making, after completing classwork) and Dia making her experiences in school relevant by searching and seemingly failing to find the well-structured tasks so common in mathematics classrooms.

[4]Interestingly, when we compare the products of Aimee's EOMs to the product of Dia's EOMs, we notice that Aimee's EOMs look much more like something we would find in school than Dia's because Aimee's make more visible use of the structure and symmetry of the materials.

Discussion: Connecting with identities

In this study, we asked: *How do children hybridize their activity in out-of-school spaces and how is such hybridization consequential for engagement*? To answer this question, we conducted a close interaction analysis of two 12-year-old girls who co-navigated MOAS as friends and engaged in parallel play at exhibits. Their interactions suggest that the two girls experienced the space quite differently, with different school-math frames evoked. Here, we describe how those frames might have connected with their prior experiences and broader narratives about school mathematics and play.

Although we cannot know with certainty *why* the two girls framed their activity differently, it seems likely that their conceptions of mathematics—and themselves in relation to mathematics—contributed to the ways they invoked different frames. In particular, we conjecture that something about the design of the space, the name of the space as *Math On-A-Stick*, and/or participation in the *Playful Mathematics Learning* research project elicited their mathematics histories. We argue that Dia made her not-a-mathematics-person claim relevant in ways that repeatedly engaged in negative and neutral evaluations of her own play, with comments like, "I can't do it," "[I made a] boring circle" (as opposed to the not-boring circle she originally tried to make), and "I don't know what I'm making." We conjecture the opposite is true for Aimee: her self-identification as a mathematics person also played out in her EOMs, as she repeatedly engaged in positive evaluations of her own activity, making comments like "I made something," clapping and rubbing her hands together in pleasure, and tracing the elements of her designs before carefully disassembling them.

Secondary data and existing literature provide more substance to this conjecture. As mentioned, when asked why they came to MOAS in their exit interviews, Aimee claimed they came because "I love math" whereas Dia claimed they saw it as they were on their way to the nearby EcoExperience building (a science center, Dia's favorite subject). It seems likely that the girls' persistent experiences in and ideas about math—their mathematical identities—played a role in whether and how particular frames were cued. Indeed, the disciplinary identities of individual learners in addition to local practices have been found to influence the actual learning opportunities that are available (Esmonde, 2009). Research on free-choice environments documents how visitors' motivations for engaging are a larger determinant of what they learn than the design itself (Falk, 2006).

We conjecture that MOAS may have provided Aimee with an opportunity to engage in playful, out-of-school mathematics experiences in ways that buttressed her self-identification and interests as a mathematics person. It seems unlikely that Dia benefited from her time at MOAS in the same way Aimee might have. While out-of-school experiences are known to correlate with interest development and the pursuit of STEM careers (Jones et al., 2011), our analysis suggests that free-choice environments may help "the rich get richer" in terms of benefits for learning and identity development.

Alternate interpretations

Of course, there are multiple possibilities unrelated to mathematics that may explain why Aimee and Dia framed their activity differently. For example, discomfort with ambiguity, tendencies to be mastery- or performance-oriented, or different senses of aesthetic could explain the contrasts in the girls' engagement. We do not see these alternative interpretations as in conflict with school mathematics frames, although such traits may certainly have influenced the girls' activity.

An additional alternative interpretation is that Dia may have been engaging in face-saving because of her positioning with Aimee, where Aimee was the leader and Dia was the follower. In this case, Dia's actions (e.g., negative evaluations and questions for authoritative others) could be interpreted as having nothing to do with school mathematics' norms and practices. We believe that this is unlikely because the girls quite equally led each other from one exhibit to the other (Table 2), and both girls were comfortable to be at exhibits alone for short periods of time (e.g., Dia at the Visiting Mathematician and Aimee at the Stepping Stones, Table 2). Even more, Dia's voice was confident and assertive when she initiated leaving exhibits. For example, when leaving the *Tiles and Patterns* exhibit, Dia firmly said, "I'm going somewhere else" and then got up to leave the exhibit. Aimee responded, "I'll come with" and then proceeded to put her tiles back in a neat pile while Dia left her tiles strewn randomly. We think this indicates that the girls' relationship had a fair distribution of power and agency and thus find it unlikely that Dia was engaging in face-saving in front of Aimee.

Regardless of prior experiences with mathematics, we find it significant that *open activity structures can create opportunities for learners to devalue their activity, as participants bring in potentially undesired frames*. We posit that this happened with Dia because of the ambiguity of the norms of MOAS and the strong norms and practices that could be invoked by the name of the space—*Math On-A-Stick*—and by the name of our project, *Playful Mathematics Learning*. In the future, we hope to better understand how to invite a counter-narrative for what counts as mathematics such that participants understand that correctness is not sought and exercising personal authority is valued. Such future research would add nuance to studies that attend to how participant motivations for engaging impact what is learned (Falk, 2006) by helping us better understand how creating norms around playfulness can (re)shape motivations and engagement.

Limitations

With respect to limitations, we note that we did not conduct a negative case analysis to systematically look for disconfirming evidence in our entire corpus of data; the commonality of the hybridity we describe and its typical influences on activity is an open and interesting question. Second, we note that there are alternative interpretations of our focal participants' activity, and we have given voice to some of these alternative interpretations after presenting our findings. Third, we note our thin data on our participants' prior histories with mathematics, relying primarily on a short, self-report, Likert-scale questionnaire. Future studies should include denser child-level data, ideally with re-occurring qualitative observations of classroom and out-of-school experiences, as well as iterative in-depth interviews to achieve a nuanced understanding of salient experiences inside and outside of school mathematics.

Directions for future research

This study prompts many questions for future research. How can we design for open engagement in domain-specific free-choice learning environments such that children do not hybridize in-school experiences in ways that limit their ability to learn from the open design? While we do not claim that hybridity between school mathematics and play would emerge identically in other contexts, this analysis suggests that assumptions that school math and play are distinct are too simplistic, and instead that hybrid spaces like mathematical playgrounds, museums, and maker spaces must think critically about the ways they invite and engage broader narratives about what it means to do mathematics. Might opening up opportunities for children to engage in creative and self-driven activity in school mathematics—such as finding problems rather than only solving predetermined problems—help us begin to understand the complexities of producing positive mathematics experiences? Such studies are needed in-and-out of school. Future research will benefit from across-context studies that can trace participants' practices and engagement across school and free-choice learning environments.

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