Multivariate Behavioral Research, 28 (1), 97-110

Invariant Standardized Estimated Parameter Change for Model Modification in Covariance Structure Analysis

Chih-Ping Chou
Department of Preventive Medicine
University of Southern California

P. M. Bentler
University of California, Los Angeles

An estimated parameter change (EPC) has recently been introduced as another criterion to be considered in the process of model modification in covariance structure analysis. Kaplan (1989) provided a standardized version of this statistic (SEPC-K). It has been found that SEPC-K is only partially standardized; specifically, it is not invariant under different scalings of latent and measured variables. In this article, a new SEPC that is invariant to the original metrics of the measured and latent variables is suggested for use in model modification. A multivariate estimated parameter change (MEPC) which estimates changes for a set of fixed parameters to be freed simultaneously is also introduced. A standardized MEPC (SMEPC) is, furthermore, provided. Because there are now three different types of standardized solutions in structural modeling programs, general discussion of standardized solution in covariance structure analysis is provided. The inappropriate use of standardization for scale-specific models is noted.

Model modification, or specification search, to improve a hypothesized model has been an inevitable process in the application of covariance structure analysis. Conventionally this process is carried out using the Likelihood Ratio (LR) test to compare nested models to determine a preferred model. More recently, the Lagrange Multiplier (LM) test for forward search and the Wald (W) test for backward search have been introduced (Bentler, 1989; Chou & Bentler, 1990; Satorra, 1989). The forward search improves models by adding free parameters, the backward search by reducing free parameters. The modification index (Jöreskog & Sörbom, 1988; Sörbom, 1989) can be considered as a univariate version of the LM statistic, and the square of the z ratio, or t ratio, as the univariate W statistic. The LR, LM, and W statistics are asymptotically equivalent under the null hypothesis that the restrictions differentiating the

This research was supported in part by USPHS grants DA03976, DA01070 and DA00017.
Address correspondence and reprint requests to C.-P. Chou, Department of Preventive Medicine, University of Southern California, 1000 S. Fremont Ave. Suite 641, Alhambra, CA 91803-1358.

more general model and the more restricted model are true. Satorra (1989) provides a thorough theoretical discussion of these three tests. Chou and Bentler (1990) also offer some empirical evidence for the equivalence of these tests in the context of covariance structure analysis.

For the forward search in which constraints are to be removed to increase free parameters in the model, another statistic has been proposed (Saris, Satorra, & Sörbom, 1987). This statistic yields an estimated value for a fixed parameter if it is to be freed for estimation in the new model, and is known as Estimated Parameter Change (EPC). Saris et al. suggest evaluating both LM and EPC statistics, and further propose that large EPC's provide strong evidence for misspecification even in absence of a large or significant LM statistic. Due to the arbitrary metrics of the variables involved in the EPC's, decisions based on the EPC's to free fixed parameters may be erroneous. For example, an EPC that is large under one scaling of the variables may not be large under another scaling. Kaplan (1989) proposed a Standardized version of EPC, here called SEPC-K, to provide a more appropriate basis for comparisons among a set of fixed parameters (see also Kaplan,1990).

It will be shown that Kaplan's SEPC (SEPC-K; 1989, 1990) can only be considered to be partly standardized, that is, it is not fully free from arbitrary modeling decisions. Alternative definitions of standardization are discussed in this article, and a full standardization is suggested such that comparisons of the new SEPC's can be more adequate. SEPC's are univariate one-parameter statistics. We also demonstrate how a Multivariate or Multiparameter Estimated Parameter Change (MEPC) can be defined and standardized. These statistics are then studied in some empirical applications.

Statistical Background

The statistical theories regarding forward search and standardization procedures required for the development of SEPC is summarized in this section. A factor analytic simultaneous equation model (e.g. Jöreskog & Sörbom, 1988) contains the following equations:

(1) $$y = \Lambda_y \eta + \varepsilon$$

(2) $$x = \Lambda_x \xi + \delta$$

(3) $$\eta = \beta\eta + \gamma\xi + \zeta$$

with parameters in matrices:

(4) $$\Lambda_y, \Lambda_x, \beta, \gamma, \Theta_\varepsilon, \Theta_\delta, \mathbf{\Phi}, \text{ and } \psi.$$

The basic model can be expressed equivalently in a *fully-standardized* form in which each of the variables in Equations 1 through 3 has unit variance. This is a natural convention typically adopted in traditional path analysis. Let y^*, x^*, η^*, ξ^*, ε^*, δ^*, ζ^* be the unit-variance standardized scores and $\mathbf{D}_y$, $\mathbf{D}_x$, $\mathbf{D}_\eta$, $\mathbf{D}_\xi$, $\mathbf{D}_\varepsilon$, $\mathbf{D}_\delta$, $\mathbf{D}_\zeta$ be the model-based diagonal matrices of standard deviations of y, x, η, ξ, ε, δ, ζ, respectively. Using a full standardization procedure, the equations for standardized scores can be expressed as:

(5) $$y^* = \mathbf{D}_y^{-1}y = (\mathbf{D}_y^{-1}\Lambda_y\mathbf{D}_\eta)(\mathbf{D}_\eta^{-1}\eta) + (\mathbf{D}_y^{-1}\mathbf{D}_\varepsilon)(\mathbf{D}_\varepsilon^{-1}\varepsilon) = \Lambda_y^*\eta^* + \mathbf{u}_\varepsilon^*\varepsilon^*$$

(6) $$x^* = \mathbf{D}_x^{-1}x = (\mathbf{D}_x^{-1}\Lambda_x\mathbf{D}_\xi)(\mathbf{D}_\xi^{-1}\xi) + (\mathbf{D}_x^{-1}\mathbf{D}_\delta)(\mathbf{D}_\delta^{-1}\delta) = \Lambda_x^*\xi^* + \mathbf{u}_\delta^*\delta^*$$

(7) $$\begin{aligned}\eta^* &= \mathbf{D}_\eta^{-1}\eta = (\mathbf{D}_\eta^{-1}\beta\mathbf{D}_\eta)(\mathbf{D}_\eta^{-1}\eta) + (\mathbf{D}_\eta^{-1}\gamma\mathbf{D}_\xi)(\mathbf{D}_\xi^{-1}\xi) + (\mathbf{D}_\eta^{-1}\mathbf{D}_\zeta)(\mathbf{D}_\zeta^{-1}\zeta)\\ &= \beta^*\eta^* + \gamma^*\xi^* + \mathbf{u}_\zeta^*\zeta^*,\end{aligned}$$

where Λ_y^*, Λ_x^*, β^*, and γ^* are matrices of standardized coefficients and $\mathbf{u}_\varepsilon^*$, $\mathbf{u}_\delta^*$, and $\mathbf{u}_\zeta^*$ are diagonal matrices for standardized coefficients for the residuals ε^*, δ^*, and ζ^* respectively. Associated with Equations 5 through 7 are the correlation matrices Θ_ε^*, Θ_δ^*, $\mathbf{\Phi}^*$, and ψ^* of the standardized variables ε^*, δ^*, ξ^*, and ζ^* respectively. These equations can be specialized for other models (e.g., Bentler, 1989). In fact, they define the standardized solution in EQS. This standardization is not available in LISREL; LISREL's two standardizations are given in the following.

Let θ be the vector of parameters selected from the matrices in Equation 4 for a specific model to be evaluated. Using maximum likelihood estimation as an example, the function to be minimized is:

(8) $$F(\theta) = \log|\Sigma(\theta)| + \mathrm{tr}[\mathbf{S}\Sigma(\theta)^{-1}] - \log|\mathbf{S}| - p,$$

where p is the number of variables, $\Sigma(\theta)$ is the model covariance matrix, and $\mathbf{S}$ is the sample covariance matrix. Assume that $\hat{\theta}$ is a vector of constrained estimators of θ that satisfies $\mathbf{h}(\theta) = \mathbf{0}$ while minimizing $F(\theta)$. There exists a vector λ of Lagrange Multipliers (LM) and matrices of derivatives $\Delta(\theta) = (\partial F/\partial\theta)$ and $\mathbf{L} = (\partial\mathbf{h}(\theta)/\partial\theta)$ evaluated at $\hat{\theta}$ such that

(9) $$\hat{\Delta}(\theta) + \hat{\mathbf{L}}'\hat{\lambda} = \mathbf{0} \text{ and } \mathbf{h}(\hat{\theta}) = \mathbf{0}.$$

It should be noted that $\mathbf{h}(\hat{\theta}) = \mathbf{0}$ can be any type of general constraint function imposed on θ, and that simple linear constraints are most typical, for example, $\hat{\theta}_1 - 2\hat{\theta}_2 = \mathbf{0}$. For simplicity and clarity, this article will only consider the case where parameters are constrained at $\mathbf{0}$, that is, the case of fixed parameters that are potentially able to be freed. We assume that the constraints are not linearly dependent, that is, not redundant.

Let $\mathbf{H} = \mathbf{H}(\theta)$ be the information matrix of θ. The asymptotic covariance matrix for θ with constraints imposed can be obtained by the following augmented matrix:

$$(10) \quad \begin{bmatrix} \mathbf{H} & \mathbf{L}' \\ \mathbf{L} & \mathbf{0} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{H}^{-1} - \mathbf{H}^{-1}\mathbf{L}'(\mathbf{L}\mathbf{H}^{-1}\mathbf{L}')^{-1}\mathbf{L}\mathbf{H}^{-1} & \mathbf{H}^{-1}\mathbf{L}'(\mathbf{L}\mathbf{H}^{-1}\mathbf{L}')^{-1} \\ (\mathbf{L}\mathbf{H}^{-1}\mathbf{L}')^{-1}\mathbf{L}\mathbf{H}^{-1} & -(\mathbf{L}\mathbf{H}^{-1}\mathbf{L}')^{-1} \end{bmatrix} = \begin{bmatrix} \mathbf{M} & \mathbf{T}' \\ \mathbf{T} & -\mathbf{R} \end{bmatrix},$$

where $\mathbf{L}$ is defined earlier and $\mathbf{0}$ is a null matrix. Under the null hypothesis, the joint distribution of random variables $\sqrt{n}(\hat{\theta} - \theta)$ and $\sqrt{n}\hat{\lambda}$ is multivariate normal with mean zero and covariance matrix

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} \end{bmatrix}.$$

See Lee and Bentler (1980) for proof and details. The multivariate LM-statistic is based on the estimate $\hat{\lambda}$ and a consistent estimator $\hat{\mathbf{R}}$, and is distributed asymptotically as a χ^2 variate

$$(11) \quad \mathrm{LM} = n\hat{\lambda}'\hat{\mathbf{R}}^{-1}\hat{\lambda} \sim \chi_r^2,$$

where r is the number of nondependent constraints in $\mathbf{h}(\theta) = \mathbf{0}$. The vector of Multivariate EPC (MEPC) is:

$$(12) \quad \mathrm{MEPC} = \hat{\lambda}'\hat{\mathbf{R}}^{-1}.$$

The univariate LM and EPC for a specific fixed parameter, say θ, are

$$(13) \quad \mathrm{LM}_i = n\hat{\lambda}_i^2/\hat{\mathbf{R}}_{ii} \sim \chi_1^2, \text{ and}$$

$$(14) \quad \mathrm{EPC}_i = \hat{\lambda}_i/\hat{\mathbf{R}}_{ii},$$

where $\hat{\mathbf{R}}_{ii}$ is the i^{th} diagonal of the $\hat{\mathbf{R}}$ matrix. Computational versions of Equations 11 through 14 are given in Bentler (1989, Chapter 10; see also Satorra, 1989).

The univariate EPC can be conceptualized as the estimate for a fixed parameter if it is freed, while the multivariate MEPC refers to the estimates for a set of fixed parameters if freed simultaneously in the new model. However, EPC and MEPC are not invariant with respect to rescaling of the variables in a model. This can be seen as follows. Each θ_i is an element of one of the matrices in Equation 4. Let $\mathbf{A}$ be one of these matrices. Then $\theta_i = [\mathbf{A}_{kl}]$, for example. But since $\mathbf{A}$ can be rescaled within most models (see *Discussion*) to the form $\mathbf{A}^* = \mathbf{D}_1\mathbf{A}\mathbf{D}_2$, where $\mathbf{D}_1$ and $\mathbf{D}_2$ are diagonal matrices with elements d_{1k} and d_{2l}, θ_i becomes $\theta_i^* = d_{1k}d_{2l}[\mathbf{A}_{kl}] = d_{1k}d_{2l}\theta_i$. EPC and MEPC yield estimates of the value that θ_i would take if estimated freely rather than fixed at zero, as it is in the constrained solution. In the parameterization $\mathbf{A}$, $\hat{\theta}_i$ is an appropriate estimate. But in the parameterization $\mathbf{A}^*$, $\hat{\theta}_i^*$ is the appropriate estimate. Thus the largest EPC in $\mathbf{A}$, which is typically sought in model modification, may not be the largest in $\mathbf{A}^*$. This is the logical problem with the Saris et. al (1987) procedure. Fully standardized versions of EPC and MEPC assure that $\mathbf{A}$ cannot be rescaled to $\mathbf{A}^*$. This is done by defining SEPC and SMEPC as the value of EPC and MEPC in the standardized solution given by Equations 5 through 7. That is, Equations 12 and 14 are computed for the standardized parameter estimates.

In practice, it is simpler to compute EPC and MEPC using Equations 12 and 14 for the unstandardized estimate. The computed parameter changes are then adjusted to yield the effect of standardization. Specifically, if EPC or MEPC is computed for the (k, l) element of the estimates in matrices in Equation 4, as shown below on the right hand side, the corresponding SEPC and SMEPC estimates are given by the terms on the left:

$$\hat{\Lambda}_y^*(k, l) = \hat{\mathbf{D}}_y^{-1}(k, k) \quad \hat{\Lambda}_y(k, l) \quad \hat{\mathbf{D}}_\eta(l, l) \tag{15}$$

$$\hat{\Lambda}_x^*(k, l) = \hat{\mathbf{D}}_x^{-1}(k, k) \quad \hat{\Lambda}_x(k, l) \quad \hat{\mathbf{D}}_\xi(l, l) \tag{16}$$

$$\hat{\beta}^*(k, l) = \hat{\mathbf{D}}_\eta^{-1}(k, k) \quad \hat{\beta}(k, l) \quad \hat{\mathbf{D}}_\eta(l, l) \tag{17}$$

$$\hat{\gamma}^*(k, l) = \hat{\mathbf{D}}_\eta^{-1}(k, k) \quad \hat{\gamma}(k, l) \quad \hat{\mathbf{D}}_\xi(l, l) \tag{18}$$

$$\hat{\Phi}^*(k, l) = \hat{\mathbf{D}}_\xi^{-1}(k, k) \quad \hat{\Phi}(k, l) \quad \hat{\mathbf{D}}_\xi^{-1}(l, l) \tag{19}$$

$$\hat{\psi}^*(k, l) = \hat{\mathbf{D}}_\zeta^{-1}(k, k) \quad \hat{\psi}(k, l) \quad \hat{\mathbf{D}}_\zeta^{-1}(l, l) \tag{20}$$

(21) $$\hat{\Theta}_\varepsilon^*(k, l) = \hat{\mathbf{D}}_\varepsilon^{-1}(k, k) \quad \hat{\Theta}_\varepsilon(k, l) \quad \hat{\mathbf{D}}_\varepsilon^{-1}(l, l)$$

(22) $$\hat{\Theta}_\delta^*(k, l) = \hat{\mathbf{D}}_\delta^{-1}(k, k) \quad \hat{\Theta}_\delta(k, l) \quad \hat{\mathbf{D}}_\delta^{-1}(l, l)$$

This type of full standardization has been implemented in EQS since its inception.

The second edition of LISREL 7 (Jöreskog & Sörbom, 1989) uses a different standardization. In their "completely standardized" solution, the variables y, x, η, and ξ are standardized to unit variances, but the residual variables ζ, ε, and δ are not standardized to unit variance. As a result, LISREL's "completely standardized" solution is equivalent to Equations 15 through 19. However, Equations 20 through 22 are not computed in LISREL, but rather Equation 20 is replaed by

$$\hat{\psi}^*(k, l) = \hat{\mathbf{D}}_\eta^{-1}(k, k) \quad \hat{\psi}(k, l) \quad \hat{\mathbf{D}}_\eta^{-1}(l, l),$$

Equation 21 is replaced by

$$\hat{\Theta}_\varepsilon^*(k, l) = \hat{\mathbf{D}}_y^{-1}(k, k) \quad \hat{\Theta}_\varepsilon^*(k, l) \quad \hat{\mathbf{D}}_y^{-1}(l, l),$$

and Equation 22 is replaced by

$$\hat{\Theta}_\delta^*(k, l) = \hat{\mathbf{D}}_x^{-1}(k, k) \quad \hat{\Theta}_\delta^*(k, l) \quad \hat{\mathbf{D}}_x^{-1}(l, l).$$

The matrices are rescaled covariance matrices; they are not correlation matrices. It does not seem accurate to describe this second standardization as a "complete standardized" solution.

The most recent version of LISREL also provides a "standardized solution" in which neither observed variables y and x nor residual variables ζ, ε, and δ are standardized. This is the standardization that is familiar to most LISREL users since it has been available in LISREL for over a decade (e.g., Jöreskog & Sörbom, 1988), and is discussed in secondary sources (e.g., Hayduck, 1987). The SEPC-K (Kaplan, 1989) parameter estimates are computed based on this third standardization, which clearly yields a partially standardized solution. The partly standardized solution is given by Equations 23 through 30 (cf. Equations 1.31 through 1.38, Jöreskog & Sörbom, 1988).

(23) $$\hat{\Lambda}_y^{**}(k, l) = \hat{\Lambda}_y(k, l) \quad \hat{\mathbf{D}}_\eta(l, l)$$

(24) $$\hat{\Lambda}_x^{**}(k, l) = \hat{\Lambda}_x(k, l) \quad \hat{\mathbf{D}}_\xi(l, l)$$

(25) $\hat{\beta}^*(k, l) = \hat{\mathbf{D}}_\eta^{-1}(k, k) \quad \hat{\beta}(k, l) \quad \hat{\mathbf{D}}_\eta(l, l)$

(26) $\hat{\gamma}^*(k, l) = \hat{\mathbf{D}}_\eta^{-1}(k, k) \quad \hat{\gamma}(k, l) \quad \hat{\mathbf{D}}_\xi(l, l)$

(27) $\hat{\Phi}^*(k, l) = \hat{\mathbf{D}}_\xi^{-1}(k, k) \quad \hat{\Phi}(k, l) \quad \hat{\mathbf{D}}_\xi^{-1}(l, l)$

(28) $\hat{\psi}^{**}(k, l) = \hat{\mathbf{D}}_\eta^{-1}(k, k) \quad \hat{\psi}(k, l) \quad \hat{\mathbf{D}}_\eta^{-1}(l, l)$

(29) $\hat{\Theta}_\varepsilon^{**}(k, l) = \hat{\Theta}_\varepsilon(k, l)$

(30) $\hat{\Theta}_\delta^{**}(k, l) = \hat{\Theta}_\delta(k, l)$

The matrices with double asterisks (**) are not the same under the first standardization (see Equations 15 through 22) and the third standardization, while the matrices with a single asterisk (*) are the same. It is clear from the above definitions that only the standardized solutions for parameter estimates in β, γ, and Φ matrices are the same in EQS and all versions of LISREL. Jöreskog and Sörbom specifically pointed out that their partial standardization does not change the original metric of the observed variables. They suggested that the correlation matrix, instead of the covariance matrix, needs to be analyzed to obtain standardized solutions for the observed variables. However, analysis of the sample correlation matrix may or may not yield model matrices that are fully standardized. If the empirical sample standard deviation of the y and x variables happens to be equal to the model-based estimates $\hat{\mathbf{D}}_y$ and $\hat{\mathbf{D}}_x$, then analysis of the correlation matrix will standardize the model solution. Such an equality will not occur with all models and estimates: As Cudeck (1989) notes, analyzing the correlation matrix in covariance structure analysis is typically not a good idea. This point is also recognized in the most recent LISREL manual (Jöreskog & Sörbom, 1988, p.38).

The SEPC proposed in this research can be summarized from Equations 15 through 22. The i^{th} fixed parameter can be defined as

(31) $$\text{SEPC}_i = \text{EPC}_i \hat{t}_i,$$

where $\hat{t}_i$ is the scaling weight transforming EPC_i to SEPC_i. The corresponding SMEPC can similarly be expressed as

(32) $$\text{SMEPC} = \text{MEPC} \times \hat{\mathbf{T}} = \hat{\lambda}'\hat{\mathbf{R}}^{-1}\hat{\mathbf{T}}$$

where $\hat{\mathbf{T}}$ is a diagonal matrix with the scaling weights $\hat{t}_i$ on the diagonal. The new SEPC's for the fixed parameters in the Θ_ε^*, Θ_δ^*, Φ^*, and ψ^* matrices can be interpreted as correlations among unique parts of y, unique parts of x, independent factors, ξ, and disturbances, ζ, respectively, while the remaining SEPC's can be interpreted as standardized regression coefficients.

Since SEPC-K is computed based on the "standardized solution" of LISREL given by Equations 23 through 30, which provides only a partially standardized solution, it also will be affected by change in the metrics of variables. The new SEPC will, in contrast, remain invariant when the metrics of variables are changed since the fully standardized solution is always the same. Another important feature of the new SEPC is due to the correlation approach to standardizing parameters in the Φ, ψ, Θ_ε, and Θ_δ matrices (see Equations 19 through 22). This approach assigns equal weights to the parameters in these four matrices as those in the Λ_y, Λ_x, β, and γ matrices. This would seem to provide a more appropriate comparison in considering the fixed parameters to be freed. This property can be illustrated by reevaluating the HTS model (Hauser, Tsai, & Sewell, 1983). The EPC for $\psi(17,16)$ is estimated at .079. Using the variances of η_{16} and η_{17} estimated at .820 and .880, respectively, the SEPC-K yielded a covariance estimate of 0.109 (Kaplan, 1989, p. 299). As a covariance, this estimate is difficult to interpret without also considering the variances. Kaplan selected this parameter for freeing in the new model due to substantive considerations and its relative numerical importance. In contrast, the new SEPC standardizes using the variances of ζ_{16} and ζ_{17}, which are .179 and .240, respectively, and results in a correlation estimate of 0.380. This larger value of an interpretable quantity, a correlation rather than covariance, offers a more convincing numerical reason to free the substantively important parameter $\psi(17,16)$. Allowing $\psi(17,16)$ free, the reestimated model yields a final estimate of 0.080 for $\psi(17,16)$, and giving "standardized" estimate of .106 (a covariance) using Equation 28 and .429 (a correlation) using Equation 20.

Illustration

Two examples are provided in this section to illustrate the application of MEPC, SEPC, and SMEPC in the process of model modification. The first example analyzes a population matrix of 8 variables with 2 factors (4 corresponding indicators for each factor) to illustrate the application of MEPC. Each variable is assumed to have variance of 1 and factor loading of 0.8 on its corresponding factor. The correlation between the two factors is 0.5. Correlated errors also were assumed between the uniqueness of corresponding indicators across factors. The covariances for these four pairs of correlated unique variables are 0.05, 0.10, 0.15., and 0.20, respectively. The population covariance matrix is presented in Table 1. A model is then evaluated without the four correlated residuals, or with them as fixed parameters. Results of EPC and MEPC are summarized in Table 2. The EPC only provides the estimated change of each of the four unique covariances one at a time, while the MEPC shows the simultaneous change of all four fixed parameters. Because the population covariance matrix is analyzed and only correlated errors are misspecified in the model, it is not surprising that the MEPC's, shown in the second column, actually reflect the true values while the EPC's provide biased results.

To illustrate the invariance of SEPC under different scalings of the latent variables, a reparameterization technique using equivalent models suggested by Luijben (1989, 1991) was adopted. Models are considered equivalent if they have the same χ^2 goodness-of-fit test statistics with same degrees of freedom when analyzing the same data set. It should be noted that Luijben (1989) concluded that SEPC-K's are not invariant under his reparameterization procedure. A true model containing six variables and two correlated factors with three variables for each factor was constructed. A sample, S1 (see Table

Table 1
A Population Covariance Matrix

	X1	X2	X3	X4	X5	X6	X7	X8
X1	1.000							
X2	.640	1.000						
X3	.640	.640	1.000					
X4	.640	.640	.640	1.000				
X5	.306	.256	.256	.256	1.000			
X6	.256	.356	.256	.256	.640	1.000		
X7	.256	.256	.406	.256	.640	.640	1.000	
X8	.256	.256	.256	.256	.640	.640	.640	1.000

Table 2
Comparisons of EPC and MEPC

Parameters	EPC	MEPC
δ_{51}	.083	.050
δ_{62}	.129	.100
δ_{73}	.176	.150
δ_{84}	.222	.200

3), was randomly drawn by simulation from the population covariance matrix. Two misspecified but equivalent models were evaluated to fit the data. Model 1, M1, contains 6 unknown factor loadings, 6 error variances as free parameters, and 2 factor variances fixed at 1. Model 2, M2, has factor loadings of variables 1 and 4 fixed at 1 and the other four unknown loadings, the variances of factors, and variances of errors freed. The latent variables are standardized in Model 1, but not in Model 2. Both models are misspecified since the factor correlation (true population value = .5) is not included. A rescaled covariance matrix, S2, was also constructed and analyzed by both M1 and M2 models. The S2 matrix basically increased the metrics of variables 1 and 4 by a scale of 2.

Table 3
A Simulated Sample Covariance Matrix

	X1	X2	X3	X4	X5	X6
X1	.943					
X2	.344	1.075				
X3	.194	.175	1.023			
X4	.105	.159	.052	.971		
X5	.186	.166	.081	.346	.974	
X6	.194	.154	.108	.275	.336	1.070

The results of EPC, LM, and SEPC statistics are summarized in Table 4. The EPCs for the fixed Λ parameters, factor loadings, are affected by reparameterization and different scaling of observed variables. The uniqueness and correlated errors, δs, varied only under different scaling. More specifically, the magnitudes of expected change of correlated errors, δs, increase with the scaling of observed variables. Both LM statistics and SEPCs, however, remain the same under all those different conditions. It is obvious from Table 4 that although the EPCs are not invariant under different parameterization and scaling, the new SEPCs are invariant as are the LM statistics. The lack of invariance in EPCs shows how incorrect conclusions in model modification could be based on EPCs. In Table 4, ϕ_{21} has the largest value for EPC under M_1, but λ_{12} has the largest value under the scaling M_2 when analyzing S1. But λ_{12} is truly zero in the population. In contrast, the largest value of SEPC is the same under both M_1 and M_2, and it corresponds to the misspecified element ϕ_{21}.

Table 4
The LM, EPC, and SEPC for Various Fixed Parameters under Two Models with Two Covariance Matrices

	EPC[b]				LM[c]	SEPC[c]
	S1		S2			
Parameter[a]	M1	M2	M1	M2		
ϕ_{21}	.468	.155	.468	.618	14.236	.468
λ_{12}	.180	.338	.361	.338	4.777	.186
λ_{22}	.166	.311	.166	.155	3.515	.160
λ_{32}	.059	.110	.059	.055	.426	.058
λ_{41}	.070	.114	.141	.114	.654	.072
λ_{51}	.164	.265	.164	.132	3.651	.166
λ_{61}	.184	.297	.184	.149	3.931	.177
δ_{41}	-.009	-.009	-.035	-.035	.023	-.014
δ_{42}	.077	.077	.154	.154	1.529	.106
δ_{43}	-.002	-.002	-.004	-.004	.001	-.003
δ_{51}	.079	.079	.158	.158	1.931	.142
δ_{52}	.042	.042	.042	.042	.462	.064
δ_{53}	.013	.013	.013	.013	.044	.018
δ_{61}	.089	.089	.178	.178	2.101	.133
δ_{62}	.028	.028	.028	.028	.185	.036
δ_{63}	.047	.047	.047	.047	.487	.054

[a] The standardized true value for ϕ_{21} is .5. The rest of the parameters are fixed at 0. [b] S1 and S2 are two covariance matrices with different metrics on variables 1 and 4. M1 and M2 are two equivalent models. [c] Since all four corresponding entries for LM or SEPC are the same, only one entry is reported.

The behavior of the MEPC and SMEPC is studied next. These statistics are computed for the factor covariance parameter and the four fixed factor loadings having the largest univariate LM statistics. Due to the linear dependency among the estimates of these fixed parameters, not all 6 factor loadings can be included simultaneously in the computation. Results obtained from analyzing S1 are summarized in Table 5. As expected, the SMEPC possesses the invariance property that was exhibited by SEPC across the two equivalent models. In addition, when comparing Tables 4 and 5, it is apparent that the estimates of MEPC and SMEPC for all four factor loadings are closer to the true value of zero than are the corresponding estimates of SEPC. For example, for λ_{12}, SEPC predicts a value of .186 while SMEPC yields the value of .014, which is closer to the true value of zero. Thus the SMEPC provides a more accurate picture of the estimate to be expected when a truly nonzero value is considered for freeing. On the other hand, SMEPC is more misleading in the case of ϕ_{21} than is SEPC, since the SEPC for ϕ_{21} at .468 is closer to the true value of .5 than is the SMEPC estimate of .359. However, this apparently paradoxical finding is to be expected since only ϕ_{21} should be freed theoretically, that is, a multivariate change is not appropriate. In fact, the empirical multivariate forward-stepping LM test also clearly yields this conclusion. That is, the computation of MEPC and SMEPC should in fact be terminated right after ϕ_{21} is included (i.e., when it equals SEPC), since the multivariate LM test procedure yields the result that none of the factor loadings makes a significant contribution to improve the model, once ϕ_{21} has been considered.

Discussion

The estimated parameter change has been recommended as a criterion to supplement the modification index, and Lagrange multiplier statistic, in the forward search of a model modification process. To overcome limitations of scale arbitrariness, Kaplan (1989) furthermore developed a standardized version of this statistic to yield conclusions uninfluenced by arbitrary model features. This article completes Kaplan's work on standardization to yield an SEPC that provides an even more adequate basis for model comparisons. In addition, the new problem of standardizing the multivariate estimated parameter change also was addressed.

Popular computer programs such as LISREL and EQS provide three different types of standardization that are not equivalent. LISREL's two types of standardized solutions (the "standardized" and "completely standardized"

Table 5
The MEPC and SMEPC for Selected Fixed Parameters

	MEPC		SMEPC	
Parameter	M1	M2	M1	M2
ϕ_{21}	.359	.118	.359	.359
λ_{12}	.013	.025	.014	.014
λ_{22}	.035	.066	.034	.034
λ_{51}	.035	.085	.040	.041
λ_{61}	.102	.165	.098	.098

solutions) do not rescale all variables to have unit variance. In contrast, the "standardized solution" in EQS rescales all variables to unit variance; we call this a *full* standardization. The new SEPCs proposed in this article are based on the full standardization of all the variables and factors involved in a covariance structure model. For convenience, equations were provided for the LISREL-type model, but the results specialize immediately to other models such as that used in EQS. An important property of our new statistic is that it is invariant under the different scalings of the observed variables and the latent variables as illustrated by the reparameterization procedure suggested by Luijben (1989). A disadvantage of the full standardization is that it may not be able to be implemented at times even though it is theoretically appropriate. The theoretical standardization given in Equations 5 through 7 requires model-based diagonal metrics of standard deviations of y, x, η, ξ, ε, δ, and ζ. Estimates of these matrices, which are required in Equations 15 through 22, may not be able to be computed; for example, estimates of variance parameters are negative.

If one wants to consider LM tests and estimated parameter changes simultaneously in model modification, as suggested by Saris et al. (1987), the new SEPCs developed in this article seem to be more appropriate in most circumstances than the EPCs considered by Saris et al. The SEPCs have the same invariance property as the LM statistics, which means that conclusions will not be arbitrary. Furthermore, the SMEPCs are also invariant, and can be anticipated to more adequately reflect the estimated change that might occur in a set of parameters that is modified simultaneously. After all, few researchers remove only a single restriction when modifying a model.

There are some situations, however, in which standardization of the estimated parameter changes may not make sense. If a model contains matrices containing fixed nonzero parameters beyond those needed for identification, the parameter matrix cannot be arbitrarily rescaled. Similarly, if a model matrix contains fixed equality constraints, rescaling the matrix would destroy the equalities. As an example, consider the restricted factor model $\Sigma = \Lambda\Lambda' + \psi$ where $\Lambda = \alpha\mathbf{1}$ with α a scalar and $\mathbf{1}$ a unit vector. Then rescaling Σ would destroy the structure of the model. Standardization would make sense only when the structure of the model is maintained.

References

Bentler, P. M. (1989). *EQS structural equations program manual.* Los Angeles: BMDP Statistical Software.

Chou, C. -P. & Bentler, P. M. (1990). Model modification in covariance structure modeling: A comparison among likelihood ratio, Lagrange multiplier, and Wald tests. *Multivariate Behavioral Research*, *25*, 115-136.

Cudeck, R. (1989). Analysis of correlation matrices using covariance structure models. *Psychological Bulletin, 105*, 317-327.

Hauser, R. M., Tsai, S-L., & Sewell, W. H. (1983). A model of stratification with response error in social and psychological variables. *Sociology of Education, 56*, 20-46.

Hayduck, L. A. (1987). *Structural equation modeling with LISREL.* Baltimore: Johns Hopkins.

Jöreskog, K. G. & Sörbom, D. (1988). *LISREL 7: A guide to the program and applications.* Chicago: SPSS.

Jöreskog, K. G. & Sörbom, D. (1989). *LISREL 7: A guide to the program and applications* (2nd edition). Chicago: SPSS.

Kaplan, D. (1989). Model modification in covariance structure analysis: Application of the expected parameter change statistic. *Multivariate Behavioral Research, 24*, 285-305.

Kaplan, D. (1990). Evaluating and modifying covariance structure models: A review and recommendation. *Multivariate Behavioral Research, 25*, 137-155.

Lee, S. -Y., & Bentler, P. M. (1980). Some asymptotic properties of constrained generalized least squares estimation in covariance structure models. *South African Statistical Journal, 14*, 121-136.

Luijben, T. C. W. (1989). *Statistical guidance for model modification in covariance structure analysis.* Amsterdam: Sociometric Research Foundation.

Luijben, T. C. W. (1991). Equivalent models in covariance structure analysis. *Psychometrika, 56*, 653-665.

Saris, W. E., Satorra, A., & Sörbom, D. (1987). The detection and correction of specification errors in structural equation models. In C. C. Clogg (Ed.), *Sociological Methodology* (pp. 105 - 129). San Francisco: Jossey-Bass.

Satorra, A. (1989). Alternative test criteria in covariance structure analysis: A unified approach. *Psychometrika, 50*, 83-90.

Sörbom, D. (1989). Model modification. *Psychometrika, 54*, 371-384.

Accepted July, 1992.