

**Identifying Inter-subject Difficulties in Norwegian  
GPA Data Using Item Response Theory**

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Continuous Draft

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## Abstract

### Research Topic

The Grade Point Average (GPA, *skolepoeng* in Norwegian) plays a determining role in Norway's tertiary admission process. The academic track in Norwegian upper secondary education offers students a set of compulsory joint core subjects as well as a wide range of elective subjects for different specialisations. Since different elective subjects are treated *equally* in its calculation, GPA implicitly assumes that grades across different specialised subjects are *equivalent* indicators of students' preparedness for higher education—an assumption that remains untested and questioned by descriptive statistics (Utdanningsdirektoratet, 2022). This paper focuses on the comparability of difficulty levels across subjects to provide a test of the hidden assumption in the current procedure for producing the GPA.

### Theoretical Framework

Fairness is both an essential and an elusive integral of educational assessment. Following Gipps and Stobart's (2009) social-cultural framing of assessment fairness and Tierny's (2017) democratic–measurement–pedagogical construction, the current study models GPA as a selection device (Kane, 2013) for accessing privileged social resources (Bourdieu, 1973). It addresses the construct validity of GPAs by examining any construct-irrelevant variance (Messick, 1989) related to students' subject choices.

### Methodology

Item response theory is particularly suitable for extracting item difficulty information in order to study assessment's selection fairness. This study considers each GPA subject as an item and each candidate as a person. Using marginal maximum likelihood (MML) estimation, the analyses will ascertain difficulty parameters for all major subjects in Norwegian upper secondary schools. Registry data containing Norwegian students' GPA performance in 2019 are first regularised by removing subjects with fewer than 1,000 candidates and by only including candidates who have received valid GPAs through upper secondary school completions. Next, subject difficulty parameters will be extracted using generalised partial credit models (GPCM, Muraki, 1992). Lastly, group invariance tests are applied to assess the extent to which selection bias had impacted on subject difficulty parameter estimates.

## **Expected Results**

The registry data set will be available for analysis in short time and the described analyses will be presented and discussed at the conference. We expect Norway's GPA subjects to differ in difficulties (He et al., 2018) and to exhibit significant selection effects (Korobko et al., 2008).

## **Relevance to Nordic Educational Research**

Given that university entries in Europe is largely based on the final grades from secondary schooling, the presented analysis is likely to be relevant to other countries using grades as the selection criteria into tertiary education. The issue of potential unequal treatment of students with different specialisation in upper secondary school applies beyond the Norwegian context. By testing the assumption that grades from different specialities support GPA's selection purpose equally well, this study lends statistical support to evidence-based policy formation process commonly practised in the Nordic community and serves to strengthen the fairness of our merit-based university admission decisions.

## Methods

### Sample

Year 10 students' GPA (*grunnskolepoeng*) and teacher-assigned grades (*standpunktkarakter*) were extracted from the Norwegian register for the administrative year ending in June 2019 (*avgangdato* = 201906). Attainment records were subsequently re-formatted with each row representing one candidate and each column being one subject. This process yielded 64,918 students and 200 subjects.

Under the Norwegian education system, students shall complete 13 compulsory subjects as well as electives. This study focuses on these compulsory subjects but excludes one course "Norwegian as a Second Language" due to large number of missings and its sensitivity to factors such as candidates' native languages. We apply equal treatment to courses instructed in Norwegian and in Sami language by merging these records. Twelve subjects are retained for our analysis: Written Norwegian (NORW), Oral Norwegian (NORO), Written English (ENGW), Oral English (ENGO), Mathematics (MATH), Natural Sciences (NATS), Social Sciences (SOCS), Religion (RELI), Music (MUSI), Arts and Handcraft (HAND), Food and Health (FOOD), and Physical Education (PHED). After dropping students without valid GPA records (data loss  $n^- = 4,300$  cases, loss rate  $r^- = 6.62\%$ ), we impose the selection criteria a) four or more records among NORW, NORO, ENGW, ENGO, MATH, NATS, SOCS ( $n^- = 1,101$ ,  $r^- = 1.82\%$ ), and b) three or more among RELI, MUSI, HAND, FOOD and PHED ( $n^- = 1,787$ ,  $r^- = 3\%$ ) in order to retain only cases with more observed information than missings. The final data set contains  $n = 57,730$  students and  $i = 12$  subjects. At this stage, the existence of missing data no longer poses any problems for our analyses thanks to sufficient overlap across subjects in the score matrix. The ability to deal with incomplete data is one major advantage of using the Rasch model for studying inter-subject comparability (He et al., 2018).

### Generalised Partial Credit Model (GPCM)

A unidimensional generalised partial credit model (Muraki, 1992) with the probability that Candidate  $n$ 's score in Subject  $i$  ( $x_{ni}$ ) being Grade  $j$  ( $j = 0, \dots, m$ ) is given by

$$p(x_{ni} = j | d_{ni} = 1; \theta_n) = \frac{e^{j\alpha_i\theta_n - \sum_{h=1}^j \beta_{ih}}}{1 + \sum_{h=1}^m e^{h\alpha_i\theta_n - \sum_{k=1}^h \beta_{ik}}}, \quad (1)$$

where  $\theta_n$  is the unidimensional proficiency parameter that represents the overall proficiency of Candidate  $n$ .

### ***Log-likelihood***

In MML, a likelihood function ( $\ell$ ) is maximised where the candidates' proficiency parameters ( $\theta$ ) are integrated out of the likelihood. The marginal log-likelihood for a unidimensional GPCM is given by

$$\ell_{\text{unidimensional}} = \sum_p \sum_{n|p} \log \int \prod_i p(x_{ni}|d_{ni}; \theta) g(\theta; \mu_p, \sigma^2) d\theta, \quad (2)$$

where  $x_{ni}$  is the observed grade,  $p(\cdot)$  is equal to [Equation \(1\)](#) evaluated at  $x_{ni}$  if  $d_{ni} = 1$ , and  $p(\cdot) = 1$  if  $d_{ni} = 0$ . In addition,  $g(\theta; \mu_p, \sigma^2)$  is the normal pdf with mean  $\mu_p$  and variance  $\sigma^2$ . The model can be identified by choosing a standard normal  $\mathcal{N}(0, 1)$  (Korobko et al., [2008](#)).

### ***Multidimensionality***

There exists strong believes among educational scientists that learners' proficiency is multidimensional, such as one proficiency factor for STEM subjects, for example, and another one for languages. If  $F$  proficiency dimensions are required to model the grades, the proficiency can be represented by a vector of proficiency parameters  $\boldsymbol{\theta}_n = (\theta_{n1}, \dots, \theta_{nF})^\top$  with the corresponding GPCM:

$$p(x_{ni} = j | d_{ni} = 1; \boldsymbol{\theta}_n) = \frac{e^{j \left( \sum_{f=1}^F \alpha_{if} \theta_{nf} \right) - \sum_{h=1}^j \beta_{ih}}}{1 + \sum_{h=1}^m e^{h \left( \sum_{f=1}^F \alpha_{if} \theta_{nf} \right) - \sum_{k=1}^h \beta_{ik}}}. \quad (3)$$

with  $\boldsymbol{\theta}_n$  following a multivariate normal distribution with mean  $\boldsymbol{\mu}_p$  and variance-covariance matrix  $\boldsymbol{\Sigma}$ . Similar to the unidimensional case, [Equation \(3\)](#) is identified by setting  $\boldsymbol{\mu}_p = \mathbf{0}$  and  $\boldsymbol{\Sigma} = \mathbf{I}$  the identity matrix. The log-likelihood of a multidimensional GPCM then becomes:

$$\ell_{\text{multidimensional}} = \sum_p \sum_{n|p} \log \int \cdots \int \prod_i p(x_{ni}|d_{ni}; \boldsymbol{\theta}) g(\boldsymbol{\theta}; \boldsymbol{\mu}_p, \boldsymbol{\Sigma}) d\boldsymbol{\theta}, \quad (4)$$

with each component sharing similar interpretations to the unidimensional counterpart in [Equation \(2\)](#).

### Interaction between Subject Choice and Proficiency

Under the advisory of Korobko et al. (2008), a latent variable  $\theta^+$  is introduced to reflect student's propensity of choosing a particular subject. Augmenting  $\theta^+$  to  $\boldsymbol{\theta} = (\theta_1, \dots, \theta_F)^\top$  yields  $\boldsymbol{\theta}^+ = (\theta_1, \dots, \theta_F, \theta^+)^\top$ , with a corresponding marginal likelihood:

$$\ell_{\text{interaction}} = \sum_p \sum_{n|p} \log \int \cdots \int \prod_i \left[ p(x_{ni}|d_{ni}; \boldsymbol{\theta}) p(d_{ni}; \theta^+) \right] g(\boldsymbol{\theta}^+; \boldsymbol{\mu}_p, \boldsymbol{\Sigma}) d\boldsymbol{\theta}^+. \quad (5)$$

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## Analysis Code, Additional Tables and Figures

### Register Data Re-format

```

1 # Only keep 2019 data
  # STP (Teacher assigned marks)
  teacher_mk <- gpa[which(gpa$AVGDATE == 201906), c(1:4, 7)]
  # Save the total number of students
5 n_student <- dim(teacher_mk)[1] # Should be 1,073,204 obs
  if (interactive()) {n_student}

  # Inspect unusual marks in the "STP" column
  if (interactive()) {
10     table(unlist(teacher_mk$STP))
  }
  # These marks are not usable:
  # '' empty [n = 20,042],
  # 7 [n = 33],
  # D [n = 58,182],
15 # F [n = 37,273],
  # GK [n = 55],
  # IM [n = 2],
  # IV [n = 12,576].

20 # Recode un-usable STP into NA
  teacher_mk$STP <- car::recode(teacher_mk$STP, "
    c('', '7', 'D', 'F', 'GK', 'IM', 'IV') = NA
  ")
25

  # Part 1: Re-shape teacher-assigned marks columns: one subject per column

30 # How many subjects there are? (Answer: 200 different subjects in total)
  # How many times each subject name appeared (with or without valid score)?
  subject_frequency <- sort(table(unlist(teacher_mk$FAGKODE)), decreasing = T)
  if (interactive()) {subject_frequency}
  # Save subject list
35 subject_list <- as.character(data.frame(subject_frequency)[, 1])
  # Save total number of subjects
  n_subject <- length(subject_list)
  if (interactive()) {n_subject} # Should be 200 subjects in total

40 # Create a placeholder spreadsheet
  stp_spreadsheet <- data.frame(matrix(NA, nrow = n_student, ncol = n_subject))
  colnames(stp_spreadsheet) <- subject_list

  # Stitch STP and this empty placeholder spreadsheet together
45 teacher_reshape <- cbind(teacher_mk, stp_spreadsheet)
  if (interactive()) {names(teacher_reshape)}

  # Set up a progress bar
  n_iter <- dim(teacher_reshape)[2] # Set the progress bar's end point
50 pb <- progress::progress_bar$new( # Refresh progress bar's internal definition
    format = "(:spin) [:bar] :percent [Elapsed time: :elapsedfull || Estimated time remaining: :
      ↪ eta]",
    total = n_iter,
    complete = "=",
    incomplete = "-",
55    current = ">",
    clear = F,
    width = 100
  )

60 for (j in 6:n_iter) { # 200 cycles
  # Insert progress bar here
  progress::pb$tick() # Update progress bar

  # Create a placeholder list
65 temp <- rep(names(teacher_reshape)[j], n_student)
  # Test whether subject names match

```



```

equal_test <- temp == teacher_reshape[, 4]
# Turn FALSE/TRUE to 0/1
equal_test <- equal_test + 0
70
# If subject name matches, copy-paste teacher-assign marks
# into the temp_subject column
temp_subject <- equal_test * teacher_reshape[, 5]
# Turn off list property (in order to recode)
75 temp_subject <- as.numeric(unlist(temp_subject))
# Recode 0 to NA
teacher_reshape[, j] <- car::recode(temp_subject, "0 = NA")
}
cat("\n") # Start a new line once progress bar is full
80
# Remove subject name and STP columns
teacher_reshaped <- teacher_reshape[, -c(4, 5)]
# Inspect the newly shaped data set
if (interactive()) {head(teacher_reshaped, 20)}
85
# Save to external file.
if (Sys.info()["sysname"] == "Windows") {
  data.table::fwrite(teacher_reshaped,
    "M:/p1708-tctan/Documents/teacher0.csv",
90   row.names = F
  )
} else {
  data.table::fwrite(teacher_reshaped,
    "/tsd/p1708/home/p1708-tctan/Documents/teacher0.csv",
95   row.names = F
  )
}
# Should be 239,329 KB in size
100
# Part 2: Re-shape rows: one student per row
# How many (unique) students there are? (Answer: 64,918 unique students)
105 # How many times each student ID appeared (with or without valid score)?
student_frequency <- data.frame(sort(
  table(unlist(teacher_reshaped$w21_4952_lopenr_person)),
  decreasing = T
))
110 # Display the top 20 students who took the most number of subjects
head(student_frequency, 20)
# Display the bottom 20 students who took the least number of subjects
tail(student_frequency, 20)
# Save student list
115 student_list <- as.character(student_frequency[, 1])
# Save total number of unique students
(n_unique_student <- length(student_list)) # 64,918 unique students
# Set up a placeholder spreadsheet
120 teacher_reshaped_final <- matrix(
  nrow = n_unique_student, ncol = dim(teacher_reshaped)[2]
)
colnames(teacher_reshaped_final) <- names(teacher_reshaped)
teacher_reshaped_final <- data.frame(teacher_reshaped_final)
125
# Prepare multi-core processing
if (Sys.info()["sysname"] == "Windows") { # Windows can only use single core
  n_cores <- 1
} else { # Both Linux and Mac can implement multicore
130   n_cores <- parallel::detectCores() # Count the total number of CPU cores
  n_cores <- n_cores - 1 # Reserve one core for system admin
}
# Set up a progress bar
135 n_iter <- n_unique_student # Set the progress bar's end point
pb <- progress::progress_bar$new( # Refresh progress bar's internal definition
  format = "(:spin) [:bar] :percent [Elapsed time: :elapsedfull || Estimated time remaining: :
    ↪ eta]",
  total = n_iter,

```

```

140   complete = "=",
      incomplete = "-",
      current = ">",
      clear = F,
      width = 100
145 )
for(i in 1:n_iter) {
  # Insert progress bar here
  progress::pb$tick()

150   # Pull out lines that share the same Student ID
      student_temp <- teacher_reshaped[which(
          teacher_reshaped[, 1] == student_list[i]
      ), ]
      # Collapse multiple lines into one line
155   student_temp_teacher <- parallel::mclapply(student_temp[, -c(1:3)],
      function(x) max(x, na.rm = T), mc.cores = n_cores)
      # In cases where, same person, same subject, but multiple marks,
      # take the maximum, because I do not know which score was given first.
      # When I asked R to compute max from a column containing NA only,
160   # R produced -Inf and a warning.
      # Safe to ignore these warnings and turn -Inf to NA.
      # Recode 0 and -Inf to NA
      student_temp_teacher <- car::recode(student_temp_teacher, "
165   c('0', '-Inf') = NA
      ")
      # Stitch admin variables to student_temp_teacher (need transpose)
      teacher_reshaped_final[i, ] <- data.frame(cbind(
          student_temp[1, c(1:3)], t(student_temp_teacher)
      ))
170 }
cat("\n") # Start a new line once progress bar is full

# Save the standard Student ID list for subsequent work
if (Sys.info()["sysname"] == "Windows") {
175   write.table(teacher_reshaped_final[, 1],
      "M:/p1708-tctan/Documents/student_id.csv",
      row.names = F, col.names = c("student_id")
    )
} else {
180   write.table(teacher_reshaped_final[, 1],
      "/tsd/p1708/home/p1708-tctan/Documents/student_id.csv",
      row.names = F, col.names = "student_id"
    )
}
185 # Should be 888 KB in size

# Save teacher-assigned marks
if (Sys.info()["sysname"] == "Windows") {
190   data.table::fwrite(teacher_reshaped_final,
      "M:/p1708-tctan/Documents/teacher1.csv",
      row.names = F
    )
} else {
195   data.table::fwrite(teacher_reshaped_final,
      "/tsd/p1708/home/p1708-tctan/Documents/teacher1.csv",
      row.names = F
    )
}
# Should be 15,345 KB in size

```

## Retain Students with Valid GPA and Sufficient Teacher-assigned Grades

```

1 # Full data set: N = 64,918

# Drop students without valid GPAs
teacher_gpa <- teacher[!is.na(teacher$GRUNNSKOLEPOENG), ]
5 if (interactive()) {dim(teacher_gpa)} # 60,618 students remain

# Compute data loss rate (n = 4,300, % = 6.62)

```

```

if (interactive()) {
  dim(teacher)[1] - dim(teacher_gpa)[1]
10   round((dim(teacher)[1] - dim(teacher_gpa)[1]) / dim(teacher)[1] * 100, 2)
}

# Sort columns by number of valid entries
teacher_sorted <- teacher_gpa[, order(colSums(is.na(teacher_gpa)))]
15

# Keep the order of admin variables
teacher_export <- cbind(teacher_gpa[, c(1:8)], teacher_sorted[, -c(1:8)])

# Save the subject list
20 n_valid <- dim(teacher_export)[1] - colSums(is.na(teacher_export[, -c(1:8)]))
r_valid <- round(n_valid / dim(teacher_export)[1] * 100, 2)
teacher_valid <- cbind(n_valid, r_valid)

# Preserve subject list to an external file
25 data.table::fwrite(data.frame(teacher_valid), "subject_list.csv", row.names = T)

# Save data set containing anyone with valid GPA
data.table::fwrite(teacher_export, "./Rolf/stp_valid_gpa.csv", row.names = F)

30 # Save "the 12 subjects" including both Norwegian and Sami as instruction lang
teacher_export_subj_12 <- teacher_export[, c(
  # English x 2
  # ENGW:
  'ENG0012', # 1 English (written)
35  # ENGO:
  'ENG0013', # 2 English (oral)
  # HAND: Handcraft
  'KHV0010', # 3 Handcraft
  'KHV0020', # 4 Duoji (Sami handcraft)
40  # PHED: Physical Education
  'KRO0020', # 5 P.E.
  # MATH: Mathematics
  'MAT0010', # 6 Mathematics
  # FOOD: Food and Health
45  'MHE0010', # 7 Food and Health
  'MHE0020', # 8 Food and Health (instructed in Sami)
  # MUSI: Music
  'MUS0010', # 9 Music
  'MUS0020', # 10 Music (instructed in Sami)
50  # NATS: Natural Sciences
  'NAT0010', # 11 Natural Sciences
  'NAT0020', # 12 Natural Sciences (instructed in Sami)
  # Norwegian x 2
  # NORW:
55  'NOR0214', # 13 Norwegian (written)
  'NOR0041', # 14 Norwegian (written, native language Sami)
  # NORO:
  'NOR0216', # 15 Norwegian (oral)
  'NOR0042', # 16 Norwegian (oral, native language Sami)
60  # RELI: Religion
  'RLE0030', # 17 Religion
  'RLE0040', # 18 Religion (instructed in Sami)
  # SOCS: Social Sciences
  'SAF0010', # 19 Social Sciences
65  'SAF0020', # 20 Social Sciences (instructed in Sami)
)]

# Add admin variables to "the 12 subjects" list
teacher_export_12 <- cbind(teacher_export[, c(1:8)], teacher_export_subj_12)
# Save "the 12 subjects"
70 data.table::fwrite(teacher_export_12, "./Rolf/stp_12.csv", row.names = F)

# Merge Norwegian- and Sami-instructed marks
75

# Create a placeholder matrix
subj_12 <- data.frame(matrix(NA, nrow= dim(teacher_export_12)[1], ncol = 12))
names(subj_12) <- c(
  "ENGW", "ENGO", # 1, 2
80  "HAND", # 3

```

```

    "PHED",      # 4
    "MATH",      # 5
    "FOOD",      # 6
    "MUSI",      # 7
    "NATS",      # 8
    "NORW", "NORO", # 9, 10
    "RELI",      # 11
    "SOCS"       # 12
  )
)
# Copy-paste subjects that do not need merges
subj_12[, 1] <- teacher_export_subj_12[, 1] # ENGW: English (written)
subj_12[, 2] <- teacher_export_subj_12[, 2] # ENGO: English (oral)
subj_12[, 4] <- teacher_export_subj_12[, 5] # PHED: Physical education
subj_12[, 5] <- teacher_export_subj_12[, 6] # MATH: Mathematics

# Set up a progress bar
library(progress)

n_iter <- dim(teacher_export_12)[1] # Set the progress bar's end point
pb <- progress_bar$new( # Refresh progress bar's internal definition
  format = "(:spin) [:bar] :percent [Elapsed time: :elapsedfull || Estimated time remaining: :
    ↪ eta]",
  total = n_iter,
  complete = "=",
  incomplete = "-",
  current = ">",
  clear = F,
  width = 100
)

# Merge HAND
for (i in 1:n_iter) {
  # Insert progress bar here
  pb$tick() # Update progress bar

  subj_12[i, 3] <- max(
    teacher_export_subj_12[i, 3],
    teacher_export_subj_12[i, 4],
    na.rm = T
  )
}
cat("\n") # Start a new line once progress bar is full

# Reset progress bar
n_iter <- dim(teacher_export_12)[1] # Set the progress bar's end point
pb <- progress_bar$new( # Refresh progress bar's internal definition
  format = "(:spin) [:bar] :percent [Elapsed time: :elapsedfull || Estimated time remaining: :
    ↪ eta]",
  total = n_iter,
  complete = "=",
  incomplete = "-",
  current = ">",
  clear = F,
  width = 100
)

# Merge FOOD
for (i in 1:n_iter) {
  # Insert progress bar here
  pb$tick() # Update progress bar

  subj_12[i, 6] <- max(
    teacher_export_subj_12[i, 7],
    teacher_export_subj_12[i, 8],
    na.rm = T
  )
}
cat("\n") # Start a new line once progress bar is full

# Reset progress bar
n_iter <- dim(teacher_export_12)[1] # Set the progress bar's end point

```

```

pb <- progress_bar$new( # Refresh progress bar's internal definition
  format = "(:spin) [:bar] :percent [Elapsed time: :elapsedfull || Estimated time remaining: :
    ↪ eta]",
  total = n_iter,
155 complete = "=",
  incomplete = "-",
  current = ">",
  clear = F,
  width = 100
160 )

# Merge MUSI
for (i in 1:n_iter) {
  # Insert progress bar here
165 pb$tick() # Update progress bar

  subj_12[i, 7] <- max(
    teacher_export_subj_12[i, 9],
    teacher_export_subj_12[i, 10],
170 na.rm = T
  )
}
cat("\n") # Start a new line once progress bar is full

175 # Reset progress bar
n_iter <- dim(teacher_export_12)[1] # Set the progress bar's end point
pb <- progress_bar$new( # Refresh progress bar's internal definition
  format = "(:spin) [:bar] :percent [Elapsed time: :elapsedfull || Estimated time remaining: :
    ↪ eta]",
  total = n_iter,
180 complete = "=",
  incomplete = "-",
  current = ">",
  clear = F,
  width = 100
185 )

# Merge NATS
for (i in 1:n_iter) {
  # Insert progress bar here
190 pb$tick() # Update progress bar

  subj_12[i, 8] <- max(
    teacher_export_subj_12[i, 11],
    teacher_export_subj_12[i, 12],
195 na.rm = T
  )
}
cat("\n") # Start a new line once progress bar is full

200 # Reset progress bar
n_iter <- dim(teacher_export_12)[1] # Set the progress bar's end point
pb <- progress_bar$new( # Refresh progress bar's internal definition
  format = "(:spin) [:bar] :percent [Elapsed time: :elapsedfull || Estimated time remaining: :
    ↪ eta]",
  total = n_iter,
205 complete = "=",
  incomplete = "-",
  current = ">",
  clear = F,
  width = 100
210 )

# Merge NORW
for (i in 1:n_iter) {
  # Insert progress bar here
215 pb$tick() # Update progress bar

  subj_12[i, 9] <- max(
    teacher_export_subj_12[i, 13],
    teacher_export_subj_12[i, 14],
220 na.rm = T
  )
}

```

```

}
cat("\n") # Start a new line once progress bar is full

225 # Reset progress bar
n_iter <- dim(teacher_export_12)[1] # Set the progress bar's end point
pb <- progress_bar$new( # Refresh progress bar's internal definition
  format = "(:spin) [:bar] :percent [Elapsed time: :elapsedfull || Estimated time remaining: :
    ↪ eta]",
  total = n_iter,
230 complete = "=",
  incomplete = "-",
  current = ">",
  clear = F,
  width = 100
235 )

# Merge NORO
for (i in 1:n_iter) {
  # Insert progress bar here
240 pb$tick() # Update progress bar

  subj_12[i, 10] <- max(
    teacher_export_subj_12[i, 15],
    teacher_export_subj_12[i, 16],
245 na.rm = T
  )
}
cat("\n") # Start a new line once progress bar is full

250 # Reset progress bar
n_iter <- dim(teacher_export_12)[1] # Set the progress bar's end point
pb <- progress_bar$new( # Refresh progress bar's internal definition
  format = "(:spin) [:bar] :percent [Elapsed time: :elapsedfull || Estimated time remaining: :
    ↪ eta]",
  total = n_iter,
255 complete = "=",
  incomplete = "-",
  current = ">",
  clear = F,
  width = 100
260 )

# Merge RELI
for (i in 1:n_iter) {
  # Insert progress bar here
265 pb$tick() # Update progress bar

  subj_12[i, 11] <- max(
    teacher_export_subj_12[i, 17],
    teacher_export_subj_12[i, 18],
270 na.rm = T
  )
}
cat("\n") # Start a new line once progress bar is full

275 # Reset progress bar
n_iter <- dim(teacher_export_12)[1] # Set the progress bar's end point
pb <- progress_bar$new( # Refresh progress bar's internal definition
  format = "(:spin) [:bar] :percent [Elapsed time: :elapsedfull || Estimated time remaining: :
    ↪ eta]",
  total = n_iter,
280 complete = "=",
  incomplete = "-",
  current = ">",
  clear = F,
  width = 100
285 )

# Merge SOCS
for (i in 1:n_iter) {
  # Insert progress bar here
290 pb$tick() # Update progress bar

```

```

    subj_12[i, 12] <- max(
      teacher_export_subj_12[i, 19],
      teacher_export_subj_12[i, 20],
      na.rm = T
    )
  }
  cat("\n") # Start a new line once progress bar is full
# Turn -Inf to NA column-by-column
# Create a placeholder matrix
subj_12_clean <- matrix(NA, nrow = dim(subj_12)[1], ncol = dim(subj_12)[2])
n_iter <- dim(subj_12_clean)[2] # Set the progress bar's end point
pb <- progress_bar$new( # Refresh progress bar's internal definition
  format = "(:spin) [:bar] :percent [Elapsed time: :elapsedfull || Estimated time remaining: :
    ↪ eta]",
  total = n_iter,
  complete = "=",
  incomplete = "-",
  current = ">",
  clear = F,
  width = 100
)
for (j in 1:n_iter) { # 12 cycles
  # Insert progress bar here
  pb$tick() # Update progress bar

  subj_12_clean[, j] <- car::recode(subj_12[, j], "
    '-Inf' = NA
  ")
}
subj_12_clean <- data.frame(subj_12_clean)
names(subj_12_clean) <- names(subj_12)

# Re-order subjects
subj_12_clean <- subj_12_clean[, c(9,10,1,2,5,8,12,4,7,6,3,11)]
# New order:
# NORW: Norwegian (written)
# NORO: Norwegian (oral)
# ENGW: English (wrItten)
# ENGO: English (oral)
# MATH: Mathematics
# NATS: Natural Sciences
# SOCS: Social Sciences
# PHED: Physical Education
# MUSI: Music
# FOOD: Food and Health
# HAND: Arts and Handcraft
# RELI: Religion

# Count the number of missings for each student
missing_12 <- rowSums(is.na(subj_12_clean)) # Total number of missings
missing_7 <- rowSums(is.na(subj_12_clean[, c(1:7)])) # 7 major subjects
missing_5 <- rowSums(is.na(subj_12_clean[, c(8:12)])) # 5 minor subjects

# Stitch admin, missing counts and marks together
teacher_final <- cbind(teacher_export_12[, c(1:8)], # Admin variables
  missing_12, missing_7, missing_5, # Missing counts
  subj_12_clean # Teacher-assigned marks
)

# Save teacher_final
data.table::fwrite(teacher_final, "./Rolf/60618.csv", row.names = F)

# Keep students with 4 or more of the 7-major subjects
major_4_plus <- teacher_final[which(teacher_final$missing_7 < 4), ]
if (interactive()) {dim(major_4_plus)} # 59,517 students remain

# Compute data loss rate (n = 1,101, % = 1.82)
if (interactive()) {
  dim(teacher_final)[1] - dim(major_4_plus)[1]
  round((dim(teacher_final)[1] - dim(major_4_plus)[1]) / dim(teacher_final)[1] * 100, 2)
}

```

```

365 # Save major_4_plus to an external file
data.table::fwrite(major_4_plus[, -c(9:11)], "./Rolf/59517.csv", row.names = F)

# Keep students with 3 or more of the 5-minor subjects

370 minor_3_plus <- major_4_plus[which(major_4_plus$missing_5 < 3), ]
dim(minor_3_plus) # 57,730 students remain

# Compute data loss rate (n = 1,787, % = 3)
if (interactive()) {
375   dim(major_4_plus)[1] - dim(minor_3_plus)[1]
   round((dim(major_4_plus)[1] - dim(minor_3_plus)[1]) / dim(major_4_plus)[1] * 100, 2)
}

# Save minor_3_plus to an external file
380 data.table::fwrite(minor_3_plus[, -c(9:11)], "./Rolf/57730.csv", row.names = F)

```

## Subject Difficulty Analysis using GPCM

```

1 # Load R package 'mirt'
  suppressWarnings(suppressMessages(library(mirt)))

# Generalised partial credit model
5 gpcm <- mirt(difficulty[,c(9:20)], itemtype = "gpcm", SE = T)
  coef(gpcm, printSE = T, IRTpars = T)
  data.table::fwrite(coef(gpcm, printSE = T, IRTpars = T, as.data.frame = T),
    "./Rolf/parameter.csv",
    now.names = T
10 )

# Save subjects' codes and names
  subj_code <- names(difficulty)[-c(1:8)]
  subj_name <- c(
15   "Written Norwegian",
    "Oral Norwegian",
    "Written English",
    "Oral English",
    "Mathematics",
20   "Natural Sciences",
    "Social Sciences",
    "Physical Education",
    "Music",
    "Food and Health",
25   "Arts and Handcraft",
    "Religion"
  )

# Item characteristic curves
30 # Auto-print is off in loops, causing corrupted PDFs. Insert print().
  for (i in 1:12) {
    pdf(file = paste0("./Rolf/trace/trace_", subj_code[i], ".pdf"))
    print(directlabels::direct.label(
      itemplot(gpcm, item = i, type = 'trace',
35       xlim = c(-6.5, 6.5),
       main = paste0(
         "Trace Plot for ", subj_code[i], " (", subj_name[i], ")"
       )
      ), 'top.points'
40   ))
    dev.off()
  }

# Expected scores
45 for (i in 1:12) {
  pdf(file = paste0("./Rolf/score/score_", subj_code[i], ".pdf"))
  print(itemplot(gpcm, item = i, type = 'score', CE = T,
    xlim = c(-6.5, 6.5),
    main = paste0(
50     "Expected Score for ", subj_code[i], " (", subj_name[i], ")"
    )
  )

```



```

    ))
    dev.off()
}
55
# Information and standard errors
for (i in 1:12) {
  pdf(file = paste0("./Rolf/info/infoSE_", subj_code[i], ".pdf"))
  print(itemplot(gpcm, item = i, type = 'infoSE', CE = T,
60      xlim = c(-6.5,6.5),
      main = paste0(
        "Information and SE for ", subj_code[i], " (" , subj_name[i], ")")
      )
  )
  ))
65
  dev.off()
}

```

# IRT Analysis Output

**Table 1**

*Generalised Partial Credit Model (GPCM) Parameter Estimates*

Subject Code	Subject Name	$a$	$b_1$	$b_2$	$b_3$	$b_4$	$b_5$
NORW	Written Norwegian	3.021 (0.025)	-2.882 (0.026)	-1.535 (0.010)	-0.403 (0.006)	0.627 (0.007)	1.789 (0.011)
NORO	Oral Norwegian	3.346 (0.028)	-3.024 (0.031)	-1.845 (0.011)	-0.817 (0.007)	0.154 (0.006)	1.300 (0.008)
ENGW	Written English	1.790 (0.015)	-2.875 (0.029)	-1.638 (0.013)	-0.559 (0.009)	0.534 (0.008)	1.741 (0.013)
ENGO	Oral English	1.689 (0.014)	-3.185 (0.041)	-2.024 (0.016)	-1.001 (0.010)	0.175 (0.008)	1.473 (0.011)
MATH	Mathematics	1.715 (0.014)	-2.773 (0.024)	-1.027 (0.010)	-0.113 (0.009)	0.619 (0.009)	1.694 (0.013)
NATS	Natural Sciences	2.656 (0.022)	-2.877 (0.026)	-1.602 (0.010)	-0.640 (0.007)	0.206 (0.007)	1.266 (0.009)
SOCS	Social Sciences	3.397 (0.028)	-2.920 (0.027)	-1.766 (0.010)	-0.826 (0.007)	0.074 (0.006)	1.170 (0.008)
RELI	Religion and Ethics	3.154 (0.026)	-2.850 (0.025)	-1.715 (0.011)	-0.800 (0.007)	0.108 (0.006)	1.181 (0.008)
MUSI	Music	1.331 (0.011)	-3.832 (0.077)	-2.558 (0.026)	-1.493 (0.014)	-0.140 (0.009)	1.559 (0.013)
HAND	Arts and Handcraft	1.138 (0.010)	-4.129 (0.101)	-2.924 (0.032)	-1.616 (0.016)	-0.124 (0.010)	1.751 (0.016)
FOOD	Food and Health	1.429 (0.012)	-4.565 (0.173)	-3.263 (0.037)	-1.644 (0.014)	-0.208 (0.009)	1.479 (0.012)
PHED	Physical Education	0.804 (0.008)	-4.483 (0.138)	-3.264 (0.050)	-2.295 (0.026)	-0.658 (0.015)	1.750 (0.019)

*Note.* A generalised partial credit model (GPCM) computes the discrimination ( $a$ ) and difficulty ( $b$ ) parameters for each subject. Standard errors are enclosed in parenthesis below point estimates. All estimates are significant at .001 level.