

Identifying School Climate Variables Associated with Students' Financial Literacy Outcomes

*A Cross-Country Comparison
Using PISA 2018 Data*

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敬致父母

To my parents

*Study hard what interests you the most in the
most undisciplined, irreverent and original manner
possible.*

Richard P. Feynman

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Thank-you goes to

Popular Abstract

This is a press release style abstract.

Abstract

Repeated economic crises in recent memory have exposed the harsh consequences of financial *illiteracy* shared by high proportions of the general population. Policy makers experienced little resistance when identifying youth as the most effective group for bringing about improvement in citizens' ability to engage with economic and financial matters, but opinions quickly diverge over the optimal approaches for achieving such targeted outcome. Existing literature frequently reports the importance of family environment in cultivating students' financial literacy through the process of "financial socialisation" – [definition goes here] (reference). Such practice, however, encounters interrogation by educators over equity concerns should families remain the main arena for financial literacy development. Schools play vital roles in alleviating inequality in accessing education and training in general but scarce research so far has been devoted into identifying the specific classroom factors that are most effective in advancing students' financial literacy outcomes. The current study therefore attempts to contribute to this enquiry by investigating the relationship between school climate variables and students' financial literacy achievement with an aim of stimulating policy debate about the levers and instruments available to education interventionists for the purpose of improving young people's financial literacy and preparedness as they step into an increasingly uncertain world. Using the 2018 PISA dataset, this paper employs a three-level hierarchical model to conduct cross-country comparisons to highlight school climate variables that are most strongly associated with high financial literacy outcomes.

Chapter 1 Introduction

Repeated economic crises in recent memory have exposed the harsh consequences of financial *illiteracy* shared by high proportions of the general population. Low levels of financial literacy are observed not only in less developed countries such as India and Indonesia (Cole et al., 2009) but also in advanced economies such as the USA (Huston, 2012), Germany (Bucher-Koenen et al., 2016) and OECD countries (Lusardi, 2015). Amongst the many redress schemes aimed at promoting citizens' financial capability, the return on investment is the highest when intervention is applied early in life. Lusardi and Mitchell (2014) have shown that providing financial knowledge to the least educated before they enter the labour market increases their well-being by approximately 82% of their initial wealth, while the rate of return is around 56% for college graduates—results that are significant both statistically and economically.

Research efforts aiming at advancing youth's financial literacy over the years evolved into two strands: on the design and evaluation of school financial education programs, and on the influence of home environment through the process of financial socialisation—the intentional or involuntary transmission of financial concepts which are required to functioning successfully in society (Bowen, 2002). A recent meta-analysis conducted by Kaiser and Menkhoff (2020) found that while school financial education programs had sizeable impacts on *financial knowledge* (+0.33 *SD*) similar to education interventions in other domains, their effect on students' *financial behaviour* is quite small (+0.07 *SD*). This conclusion added to a list of weak or non-findings regarding the long-term behavioural effect brought about by school financial education programs. Brown et al. (2016), for instance, reported mixed outcome in students' long-term financial well-being depending on the programs received; whereas Cole et al. (2016) observed that traditional personal finance courses lacked any explanatory power in accounting for graduates' financial outcome once the additional mathematics training in which finance topics were packaged has been controlled for. Despite careful controls and thoughtful study designs, correlating classroom interventions and young people's financial literacy outcomes has repeatedly yielded paradoxical results of non-significant or even negative relationship; any positive findings remain small in magnitudes and/or are sensitive to robust analyses.

Optimism, fortunately, runs higher at the financial socialisation camp. Building on the

acknowledgement that families serve as information filters from the outside world (Danes & Haberman, 2007) as well as the foundation for youth’s continued financial concept formation, Gudmunson and Danes (2011) put forward a family financial socialisation theory to accommodate both the *process* and the *outcome* for variations in young people’s financial capabilities. Using structural equation modelling, Jorgensen and Savla (2010) was able to show that perceived parental influence had a direct and moderately significant influence on financial attitude, did *not* have an effect on *financial knowledge*, and had an indirect and moderately significant influence on financial behaviour, mediated through financial attitude. This attitude(A)–behaviour(B)–cognition(C) conceptualisation of financial literacy (Potrich et al., 2015) continues to influence subsequent research effort. More recently, Moreno-Herrero et al. (2018) continued this line of enquiry by applying multilevel regression analyses to 2015 PISA data and reported that students’ financial literacy was associated mainly with understanding the value of saving and discussing money matters with parents. In addition, exposure and use of financial products, in particular holding a bank account, improved students’ financial knowledge as well.

One chief concern for every research project is the quality of its data source. Amongst competing inventories, PISA stands out as a comprehensive and reliable source of data for measuring 15-year-olds’ financial literacy outcomes thanks to OECD’s careful sampling procedure and attention to construct validity of measurement. Following statistical theory, PISA designers firstly recognise the hierarchical nature of education research data such that students are nested in schools, and schools are further nested in countries. In addition, one student weight is assigned to each observation in order to account for the fact that not all schools in a country are equally likely to be sampled by the PISA organiser; and given a particular school that has been chosen, not every student in this school is equally likely to be asked to participate in the test (Rust, 2014). A third complication arises from the “planned missingness” in students’ responses because each participant is only given a small number of questions relative to the entire test bank in order to ensure their responses are not undermined by tiredness (von Davier, 2014), leading to the outcome variables being represented by ten plausible values. Lastly, PISA consulted and synthesised multiple schools of thoughts (OECD, 2019) before constructing financial literacy as

the knowledge and understanding of financial concepts and risks, and the skills, motivation and confidence to apply such knowledge and understanding in order to make effective decisions across a range of financial contexts, to improve the

financial well-being of individuals and society, and to enable participation in economic life. (p. 128)

As a result, 2018 PISA data set (OECD, [2020](#)) provides not only variables measuring *cognitive* outcomes but also *affective* factors such as familiarity with concepts of finance and confidence about financial matters, enabling a nuanced study design involving decomposing the total effect of financial literacy development into its “brain” (cognitive) and “heart” (affective) pathways.

The current study wishes to take advantage of the latest wave of 2018 PISA results and investigate the covariation financial literacy outcomes share with the following four aspects of young people’s daily lives, inspired by school climate literature (Wang & Degol, [2016](#)):

(a) academic training, including any financial education programs received at schools; (b) safety perception about their schools; (c) financial socialisation experienced at home; and (d) their schools’ resource endowment. More specifically, this project aims to answer these three research questions:

- RQ1. Having controlled for demographic characteristics such as socio-economic status, sex and immigration history, to what extent can the variation in students’ financial literacy outcomes be accounted for by each of the school climate variables mentioned above.
- RQ2. How does the total effect each school climate variable carries decompose into cognitive and affect pathways.
- RQ3. How do these effects differ across countries.

Chapter 2 Conceptual Framework

2.1 In-depth definitions of “financial literacy”

2.1.1 Every term my readers need in order to understand my research question

2.1.2 Survey not only PISA but also alternative definitions, even critiques of such definitions

2.1.3 Any practices that are common in maths/literature but uncommon in financial literacy? Meaning? Implies?

2.2 Country-level Financial Knowledge Index

PISA 2018 financial literacy dataset (OECD, 2020) provides rich information about students and schools. For the purpose of cross-country comparison, however, the country-level financial literacy information must be addressed separately by the researchers. Earlier attempts such as Moreno-Herrero et al. (2018) approximated this information using a variable “quality of math and science education” to control for country-level differences since consensus is yet to emerge about the most appropriate measure for countries’ financial knowledge. Inspired by the UN’s approach to forming Human Development Indices, a recent publication by Oliver-Márquez et al. (2020) proposed a macroeconomic measure for countries’ general financial knowledge levels by examining their economic capability, educational training, existing practices in the financial markets as well as incentives to interact with financial products. More specifically, the authors considered a country’s economic capability, represented by its GDP per capita, to be a key dimension in bringing about its financial knowledge index (FKI). Secondly, literature converges on the importance of educational training for a country’s financial knowledge capability (OECD, 2005). Thirdly, countries with regular engagement with sophisticated financial products and financial markets should possess higher FKI. Lastly, countries with higher aggregate consumption levels and with ageing populations are likely to possess higher FKI due to more frequent exposure and pressure in retirement provision, respectively. Macroeconomic data needed for these computations can be sourced from the World Bank (World Bank, 2020) and the United Nations’ *Human Development Reports* (United Nations, 2020).

Combining individual and institutional data sources can be a productive approach in international large-scale assessment (ILSA) research. According to the framework for comparative education analyses (Bray & Thomas, 1995), this project extends education outcome measures to a country level, addresses the aspect of society and labour market, and relates countries’ entire populations to ILSA research (Strietholt & Scherer, 2018). By combining education outcome data with countries’ economic performance indicators, this project remains most comparable to Hanushek and Woessmann (2012)—while these authors looked into the relationship between countries’ education achievement and their GDP growth, the current investigation highlights how countries’ GDP, along with other macroeconomic practices, in turn systematically impacts on their youth’s educational performance.

Table 2.1
Percentages of Missing Values

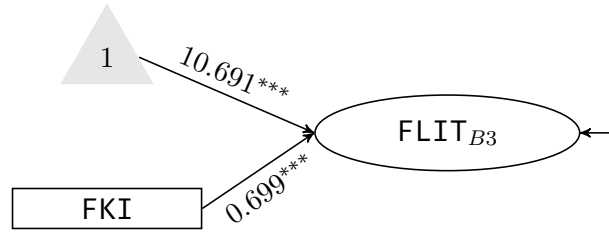
CNT	MALE	IMMI1GEN	IMMI2GEN	ESCS	FCFMLRTY	FLCONFIN	PERFEED	TEACHINT	FLSCHOOL	DISCRIM [†]	BELONG	BULLY	FLFAMILY	CURSUPP [†]	PASCHPOL [†]	STRATIO	EDUSHT	STAFFSHT
BGR	0	6	6	3	12	27	10	10	21	28	19	31	22	100	100	8	3	3
BRA	0	5	5	2	12	34	9	8	21	36	23	40	24	17	19	12	6	7
CAN [†]	0	7	7	5	11	15	100	100	13	100	8	14	14	100	100	100	2	2
CHL	0	4	4	3	10	24	5	4	13	30	15	34	15	9	8	18	9	9
ESP	0	3	3	2	5	21	3	2	7	25	9	29	8	100	100	11	5	6
EST	0	3	3	3	4	8	3	3	6	9	5	11	6	100	100	0	0	0
FIN	0	2	2	2	4	10	3	3	6	100	6	11	7	100	100	2	7	7
GEO	0	5	5	2	9	26	9	9	17	100	15	22	21	4	5	1	2	2
IDN	0	3	3	1	3	6	3	2	5	3	2	5	5	100	100	23	14	14
ITA	0	4	4	3	7	17	4	4	10	23	10	27	12	16	17	9	3	3
LTU	0	3	3	3	4	12	3	3	5	17	8	20	7	100	100	0	0	0
LVA	0	2	2	2	5	9	3	3	6	14	6	15	7	100	100	6	3	4
NLD	0	3	3	2	3	5	3	2	4	100	4	8	4	100	100	11	5	5
PER	0	2	2	1	2	11	5	4	4	56	31	65	5	100	100	2	0	0
POL	0	1	1	1	3	7	2	1	5	9	3	11	5	100	100	0	0	0
PRT	0	6	6	5	8	11	6	6	10	15	8	17	10	10	10	11	1	1
RUS	0	3	3	2	8	13	5	4	11	13	8	15	11	100	100	3	3	3
SRB	0	3	3	1	10	25	8	7	18	25	15	27	19	100	100	8	1	1
SVK	0	2	2	1	4	12	4	3	7	14	6	17	8	100	100	6	6	7
USA	0	3	3	2	3	6	2	1	4	100	4	6	4	100	100	16	10	10

Note. Using shades of red in addition to numbers (measured in %), this table visualises the missing percentages by variable and by country. Variables DISCRIM, CURSUPP and PASCHPOL are no longer pursued in the model because too many countries chose not to respond to these questions. Canada (CAN) is not included due to 100 percent missings on multiple variables. [†] marks the country and variables that are excluded from subsequent analyses.

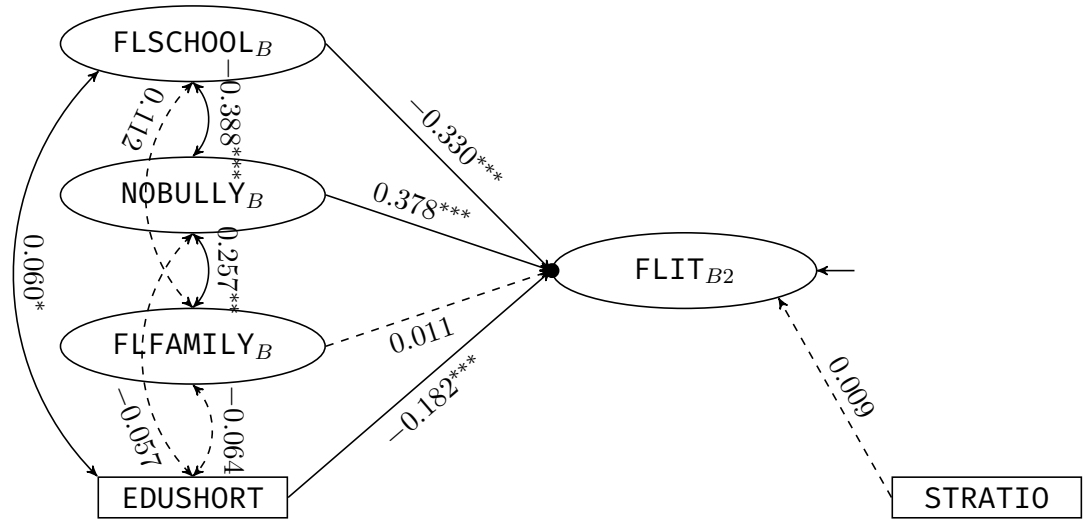
Figure 2.1

Three-level Structural Equation Model Predicting Youth's Financial Literacy Outcomes

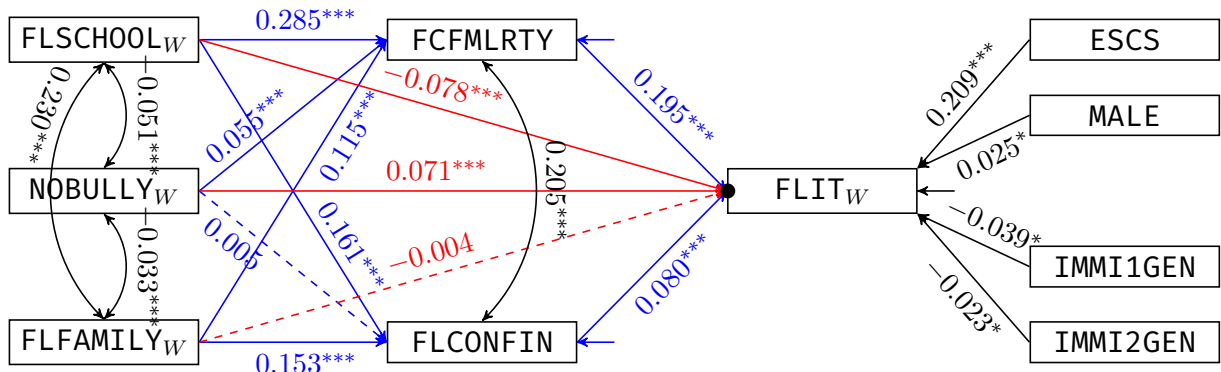
L3: Country



L2: School



L1: Student



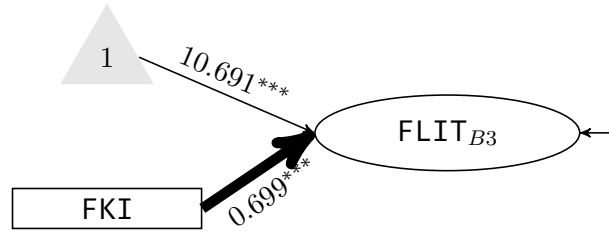
Note. This multilevel structural equation model predicts youth's financial literacy outcomes using PISA 2018 data, with mediating effects of familiarity with, and confidence in, financial matters at student-level (direct and indirect pathways). Statistics are standardised regression coefficients. Dashed lines represent nonsignificant relations at $\alpha = 0.05$ level. FK1 = financial knowledge indices, FLIT = financial literacy. Subscript W = within, B = between.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

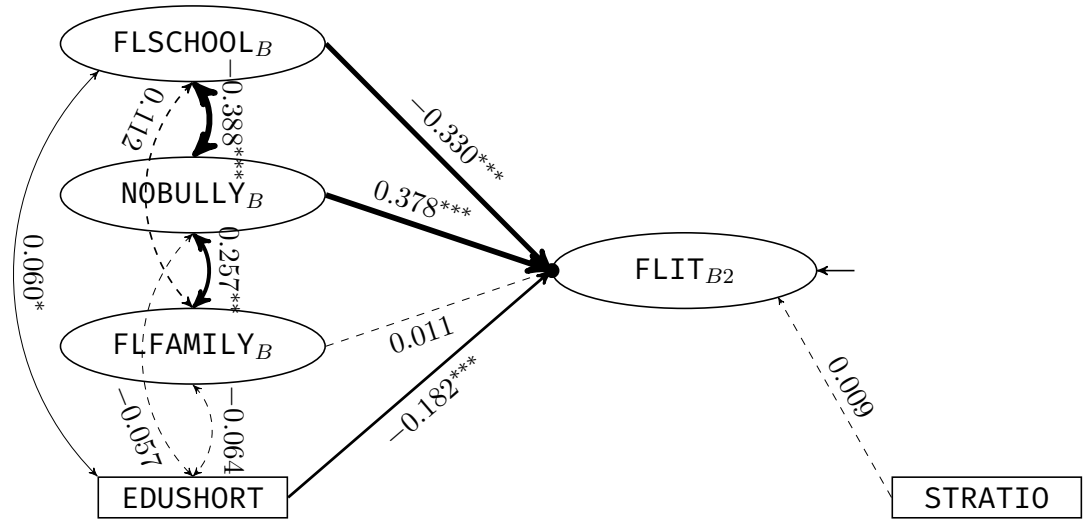
Figure 2.2

Three-level Structural Equation Model Predicting Youth's Financial Literacy Outcomes

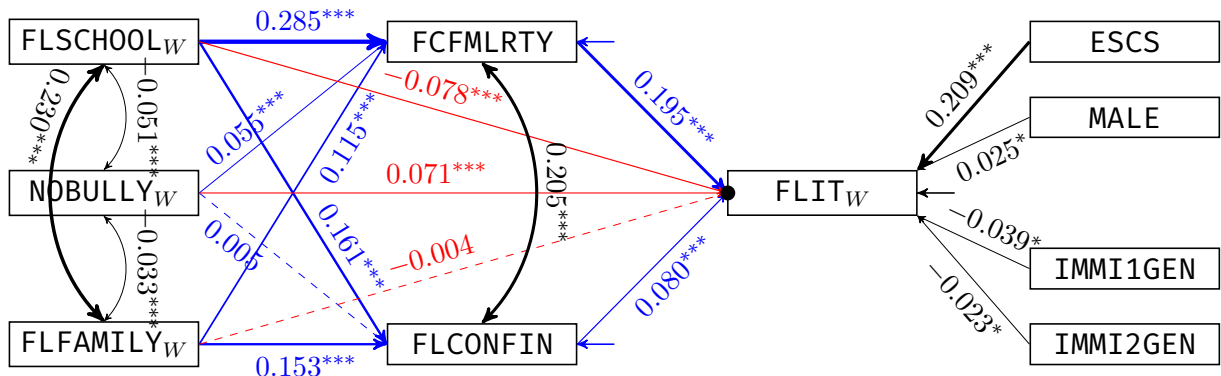
L3: Country



L2: School



L1: Student



Note. This multilevel structural equation model predicts youth's financial literacy outcomes using PISA 2018 data, with mediating effects of familiarity with, and confidence in, financial matters at student-level (direct and indirect pathways). Statistics are standardised regression coefficients. Dashed lines represent nonsignificant relations at $\alpha = 0.05$ level. FKI = financial knowledge indices, FLIT = financial literacy. Subscript W = within, B = between.

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Chapter 3 Methods

3.1 Data / Sample / Participants

This study drew its primary data source from PISA 2018 database (OECD, [2020](#)) containing 107,174 observations spanning 20 countries, in which students were asked about their demographic background, family lives and school experiences. For the financial literacy section, in particular, students responded to questions about their confidence about financial matters, familiarity with concepts of finance, their parental involvement in matters of financial literacy. Ten plausible values were subsequently generated by PISA organisers as measures of students' financial literacy outcomes and were used as the dependent variable.

Student-level independent variables are

School-level independent variables are

Country-level independent variables are

Missing data are handled using Mplus's multiple imputation procedure with ten imputations generated and pooled subsequently following Rubin's Rule (Rubin, 1976).

A three-level multigroup structural equation model was employed to account for the hierarchical structure of the PISA design, with private versus public school as the grouping variable.

3.2 Measurement of financial literacy

3.2.1 Background questions

3.2.2 Students' motivation of spending money

3.2.3 Four-point Likert scale

3.2.4 Averages

3.3 Country-level Financial Knowledge Index

This project closely follows Oliver-Márquez et al.’s (2020) procedure in developing country-level financial knowledge indices using four sub-indices: economic capability (**EC**), educational training (**ET**), existing practices in financial market (**Use**), and incentives (**Need**) to engage with financial products. The first sub-index **EC** is calculated using the logarithm of a country’s GDP per capita in current international dollars (purchasing power parity adjusted). For the **ET** sub-index, a country’s highly skilled workforce is represented by its postgraduate to total tertiary graduation ratio, and the mean years of schooling is used to measure its general education level. For the **Use** sub-index, gross portfolio equity assets (GPEA) and insurance company assets (ICA) are considered sophisticated financial products a country engages in. Additionally, in order to capture the central role of technology in amplifying the proliferation and use of financial assets, the proportion of a country’s Internet users (IUS) enters the definition via

$$\text{Use} = (\text{GPEA} + \text{ICA})^{\text{IUS}}.$$

The final sub-index **Need** is compiled as

$$\text{Need} = (\text{PFA} + \text{AC})^{\text{AGEING}},$$

where PFA is the pension fund assets to GDP ratio. Aggregate consumption is defined as:

$$\text{AC} = \frac{2\% \times \text{household final consumption expenditure}}{\text{GDP}},$$

with the “2% rule” being drawn from Caliendo and Findley’s (2013) derivation, and the proportion of ageing population is computed as

$$\text{AGEING} = \frac{\left[\frac{\text{population}(> 65)}{\text{population}(20 \sim 64)} \right]_{2018} - \left[\frac{\text{population}(> 65)}{\text{population}(20 \sim 64)} \right]_{2009}}{\left[\frac{\text{population}(> 65)}{\text{population}(20 \sim 64)} \right]_{2009}}.$$

3.3.1 Data Collection and Missing Data Treatment

The data sources for FKI computation are documented in [Table C.1](#) and its associated notes.

Sub-indices **ET** and **Use** both contain missing observations for the year 2018. Majority of

such missing data appear to be the result of administrative delay, with historic observations available until 2017. It is therefore feasible to conduct time-series forecasts using prior year observations to best approximate 2018 values.

Table 3.1*Data Sources for FKI Computation*

Database ^a	Country ^b	Series	Time
Economic Capacity			
WB-dev	20	GDP per capita, PPP (current international \$)	2018
Educational Training			
WB-ed	20 \ Russia	Graduates from ISCED 7 programmes in tertiary education, both sexes (number)	2013– 2018
		Graduates from ISCED 8 programmes in tertiary education, both sexes (number)	2013– 2018
		Graduates from tertiary education, both sexes (number)	2013– 2018
RS	Russia	PhD (Type 1) ^c , PhD (Type 2) ^d	2018
RE	Russia	Master (Type 1) ^e , Master (Type 2) ^f , total tertiary <i>excluding</i> PhD ^g	2018
HDR	20	Dimension = Education; Education = Mean years of schooling (years)	2018
Use			
WB-fin	20	Gross portfolio equity assets to GDP (%)	2011– 2018
		Insurance company assets to GDP (%)	2011– 2018
WB-dev	20	Individuals using the Internet (% of population)	2009– 2018
Need			
WB-fin	20 \ Georgia	Pension fund assets to GDP (%)	2008– 2018
GP	Georgia	Minutes of the meeting of the investment board of the Pension Agency ^h	2019 [*]
GS	Georgia	GDP at current prices, billion GEL ⁱ	2018
WB-dev	20	Household and NPISHs final consumption expenditure, PPP (current international \$)	2018
		GDP, PPP (current international \$)	2018
		Population ages 0–14, male	2009, 2018
		Population ages 0–14, female	2009, 2018
		Population ages 15–64, male	2009, 2018
		Population ages 15–64, female	2009, 2018
		Population ages 65 and above, male	2009, 2018
		Population ages 65 and above, female	2009, 2018
		Population ages 15–19, male (% of male population)	2009, 2018
		Population ages 15–19, female (% of female population)	2009, 2018

Note. Sub-indices are shaded in gray. Bold font signifies this year contains missing data.

- ^a WB-dev = [World Bank – World development indicators](#)
 WB-ed = [World Bank – Education statistics – All indicators](#)
 WB-fin = [World Bank – Global financial development](#)
 HDR = [Human Development Reports – Data](#)
 RS = [Russian Federal State Statistic Service](#)
 RE = [Russian Ministry of Education and Science](#)
 GP = [Pension Agency of Georgia](#)
 GS = [National Statistics Office of Georgia](#)
- ^b “20” = the 20 participating countries in 2018 PISA financial literacy test: Brazil, Bulgaria, Canada, Chile, Estonia, Finland, Georgia, Indonesia, Italy, Latvia, Lithuania, the Netherlands, Peru, Poland, Portugal, Russian Federation, Serbia, Slovak Republic, Spain, and the USA. “\” = excluding or except
- ^c [https://rosstat.gov.ru/storage/mediabank/asp-2\(1\).xls](https://rosstat.gov.ru/storage/mediabank/asp-2(1).xls), Sheet “по направлениям подготовки”, Cell C7 = number of PhD graduates (Type 1)
- ^d <https://rosstat.gov.ru/storage/mediabank/asp-3.xls>, Sheet “по научным специальностям”, Cell B7 = number of PhD graduates (Type 2)
- ^{e-g} https://minobrnauki.gov.ru/common/upload/download/VPO_1_2018.rar contains a spreadsheet [СВОД_БПО1_БСЕГО.xls](#), Sheet “P2_1_3(1)”, Cell E198 = number of master graduates (Type 1)^e, Cell E410 = number of master graduates (Type 2)^f, Cell E592 = total tertiary graduates *excluding* PhD^g
- ^h [Minutes of the meeting of the investment board of the Pension Agency](#), p. 4, no. 3
- ⁱ [Gross domestic product \(GDP\)](#), row = GDP at current prices, billion GEL, column = 2018
- ^{*} Georgia started a [new pension system](#) on 1 January 2019. Since 2018 was a transitional period with scarce data, 2019 is used as the best approximation for Georgia’s pension system for 2018.

Sub-index ET

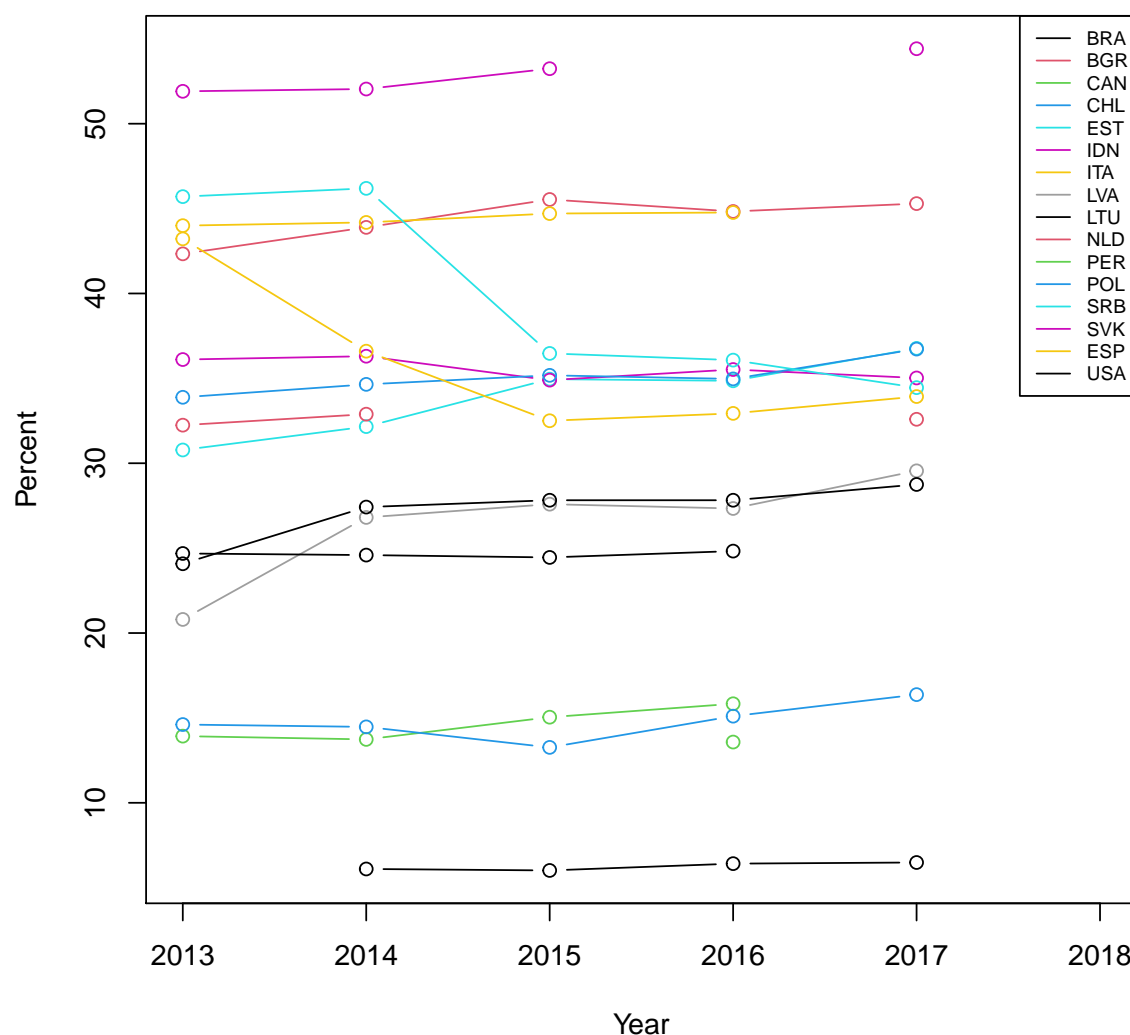
The 2018 archive for the number of master (ISCED 7), PhD (ISCED 8), and total tertiary graduates are incomplete for all participating countries except Georgia, Indonesia and Serbia.

Figure C.1 presents a time series plot of

$$\text{SKILLED} = \frac{\text{number of masters} + \text{number of PhDs}}{\text{total number of tertiary graduates}}$$

Figure 3.1

Proportion of Postgraduates to Total Tertiary Graduations



Note. “Postgraduate” is defined as master (ISCED 7) and PhD (ISCED 8) graduates. Countries not shown: GEO, IDN and SRB (2018 data available) and RUS (consult other sources)

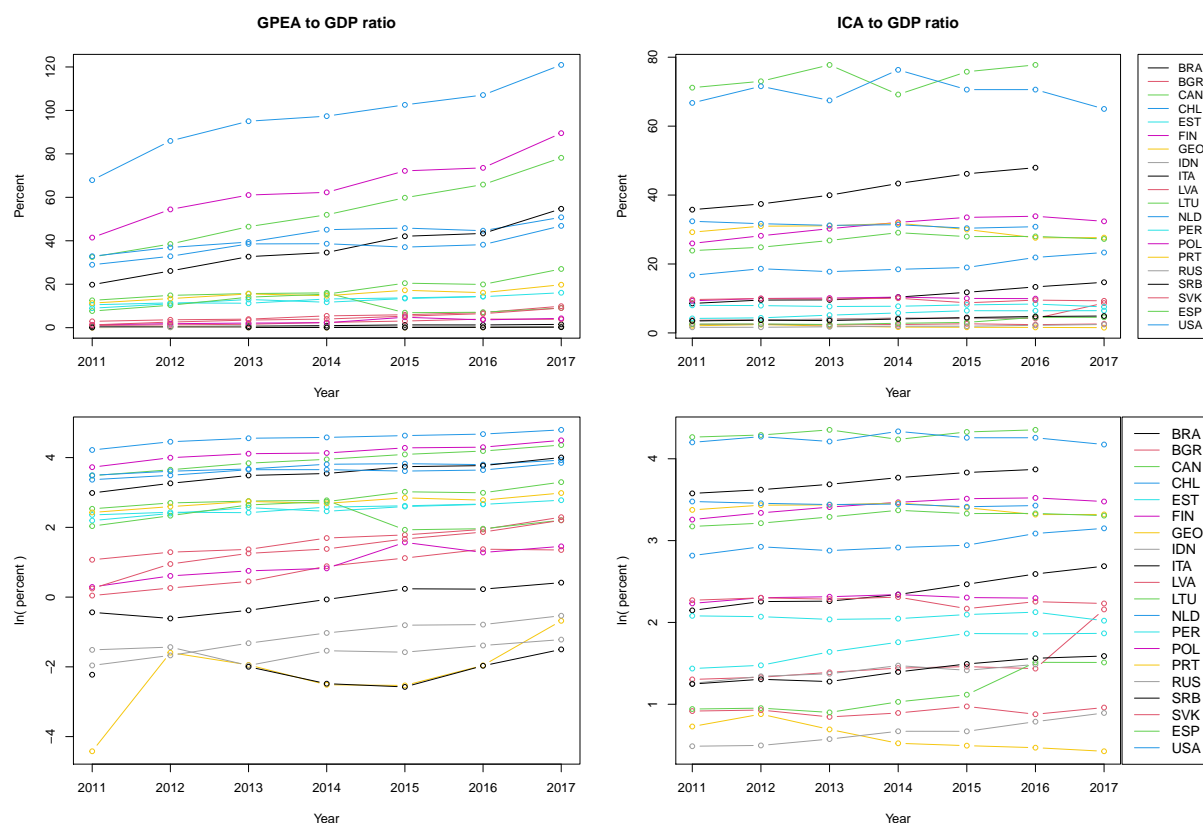
and suggests that this ratio is likely to be stable over time, especially between adjacent years. A “naive forecast”, where the nearest available year’s data are to be duplicated for 2018, is applied for SKILLED.

Sub-index Use

All series involved in calculating this sub-index, GPEA, ICA and IUS, contain missing data. When time series data contain only exponential growth but no underlying trend, a simple exponential smoothing would suffice (Gardner, 1985); if trend is present, Holt-Winters method is superior (Chatfield, 1978). Figure C.2 facilitates this decision making by plotting both the original and log-transformed versions of GPEA and ICA series. Since curves after log-transformations have slopes, it is prudent to apply the Holt-Winters forecasting method in order to account for possible trends contained in the original series.

Figure 3.2

Time Series Trend Test



Note. The time series plots after natural logarithm transformations (bottom panels) are not flat, suggesting the original series (top panels) contain trends. Holt-Winters method therefore is preferred over simple exponential smoothing for 2018 forecasts.

The IUS series contains missing data for Canada, Chile and the United States. Similar Holt-Winters procedure is applied to recover 2018 IUS data.

Table 3.2*Data Utilised for Computing FKI*

	Economic Capacity	Educational Training		Use			Need		
	GDP per capita	Skilled	Schooling	GPEA	ICA	IUS	PFA	AC	AGEING
BRA	9.612	6.484	7.8	1.683	16.259	70.434	11.827	1.210	0.288
BGR	10.026	45.294	11.8	4.114	7.044	64.782	13.577	1.091	0.234
CAN	10.821	15.832	13.3	84.010	77.728	93.588	96.205	1.068	0.271
CHL	10.117	16.371	10.4	51.755	25.591	89.531	73.225	1.073	0.214
EST	10.501	36.765	13.0	16.399	7.681	89.357	18.012	0.876	0.163
FIN	10.807	35.024	12.4	93.626	31.481	88.890	52.024	0.974	0.370
GEO	9.588	24.039	12.8	0.784	1.469	62.718	0.834	1.227	0.042
IDN	9.362	7.771	8.0	0.636	4.612	39.905	1.826	1.059	0.145
ITA	10.665	44.771	10.2	57.434	51.260	74.387	10.589	1.075	0.155
LVA	10.330	29.554	12.8	8.598	2.538	83.577	14.732	1.027	0.142
LTU	10.487	28.749	13.0	9.008	5.500	79.723	7.457	1.107	0.149
NLD	10.961	32.590	12.2	124.171	64.956	94.712	207.938	0.805	0.326
PER	9.479	13.577	9.2	16.027	6.505	52.540	22.530	1.187	0.227
POL	10.368	36.725	12.3	4.853	9.535	77.542	9.838	1.085	0.355
PRT	10.444	34.454	9.2	19.353	25.579	74.661	8.761	1.133	0.237
RUS	10.267	30.349	12.0	0.302	2.614	80.865	4.415	0.941	0.155
SRB	9.774	26.946	11.2	0.306	5.111	73.361	0.845	1.171	0.280
SVK	10.391	54.417	12.6	10.644	8.873	80.660	12.497	0.962	0.300
ESP	10.609	33.929	9.8	27.681	28.230	86.107	10.235	1.044	0.186
USA	11.048	24.825	13.4	55.505	30.183	84.881	150.040	1.364	0.252

Note. Full variable names: Skilled = Postgraduate to total tertiary ratio; Schooling = Mean year of schooling; GPEA = Gross portfolio to GDP ratio; ICA = Insurance company assets to GDP ratio; IUS = Number of Internet users per 100 population; PFA = Pension fund assets to GDP ratio; AC = 2% of household final consumption expenditure to GDP ratio; AGEING = Aged-to-productive-population ratio (% change between 2009 and 2018)

Other Items with Data Concerns

Russia reported 67.96% and 61.01% of its total university degree recipients to be postgraduates for the year 2013 and 2015 respectively (2014 missing). This figure rapidly declines to 41.6% in 2016 and further down to 25.69% in 2017. Such volatility goes against the stable patterns shared by most countries in [Figure C.1](#), casting doubt on data reliability. Separate investigation is therefore conducted using Russian government archive (Notes c to g in [Table C.1](#)).

Georgia underwent pension reform in 2018 with fund balance gradually transitioning to State Pension Agency for its official resumption of duty on 1 January 2019. Resultantly, 2018 pension balance for this country is unavailable but to be best approximated using 2019 official data (Notes h, i and * of [Table C.1](#)).

[Table C.2](#) documents the results of the abovementioned data recovery process.

3.3.2 Standardisation, Weights and FKI

Following Oliver-Márquez et al. ([2020](#))’s procedure, all series in [Table C.2](#) undergo min-max normalisation such that the smallest entry receives a new score of 0.01 and the biggest number is re-coded to 0.99. This slight deviation from the original paper (where the min-max normalisation yields 0 to 1) is to avoid multiplying a series by zero or raising a base to the power of zero.

Variable weights are calculated following Oliver-Márquez et al. ([2020](#))’s recipe to be the inverses of each series’ standard deviations. Whereas a sub-index combines more than one series, each weight is further divided by the sum of the constituent weights so that total weights add to one.

FKI is finally computed by taking the geometric mean of all four sub-indices, subject to sub-index-weights similar to variable weights above, as presented in [Table C.3](#).

Table 3.3*FKI and Sub-indices*

	FKI	EC	ET	Use	Need
NLD	0.940	0.939	0.640	1.805	1.000
USA	0.937	0.990	0.589	0.856	1.406
CAN	0.784	0.858	0.409	1.637	0.953
ITA	0.762	0.767	0.602	1.069	0.807
FIN	0.724	0.850	0.685	1.127	0.562
ESP	0.627	0.735	0.464	0.635	0.726
LTU	0.613	0.664	0.632	0.243	0.836
PRT	0.591	0.639	0.401	0.630	0.762
BGR	0.583	0.396	0.760	0.384	0.729
EST	0.577	0.672	0.746	0.266	0.575
SVK	0.559	0.608	0.924	0.301	0.441
POL	0.555	0.595	0.699	0.286	0.572
LVA	0.550	0.573	0.633	0.161	0.795
CHL	0.544	0.449	0.302	0.761	0.908
RUS	0.450	0.536	0.597	0.083	0.639
GEO	0.424	0.141	0.547	0.210	0.997
SRB	0.423	0.249	0.500	0.193	0.742
PER	0.309	0.078	0.194	0.691	0.877
BRA	0.141	0.155	0.010	0.432	0.833
IDN	0.122	0.010	0.040	0.973	0.787

Note. Table sorted in descending order by countries'
 FKI. FKI = financial knowledge index, EC =
 Economic Capability, ET = Educational Training.

3.4 What exactly I was using to address my research question

3.4.1 Sum score? Averages? One particular question?

3.4.2 Factor loading? Latent variables?

3.4.3 Motivation for choosing these measures

3.5 Software and version

3.6 My models

3.6.1 Motivation for choosing this particular model

3.6.2 Refer to my research question

3.7 Estimators I obtained

3.7.1 Motivation why these estimators rather than others

3.8 Weights? Plausible values?

3.9 Missing data and how I treated missing data

3.10 Model comparison

3.11 Guidelines and indices

Chapter 4 Results

4.1 Descriptive statistics

4.2 Correlation matrices

4.2.1 Across countries

4.2.2 Across levels: Country | School | Students

4.3 Examination of measurement models

4.4 Address the research question

Table 4.1
Parameter Estimate

	Model 0		Model 1		Model 2	
	Par.	SE	Par.	SE	Par.	SE
FIXED EFFECTS						
Intercept	481.17	(10.50)	376.22	(13.47)	27.80	(4.61)
Student-level Predictors						
Academic						
Safety						
Financial Socialisation						
Familiarity						
Confidence						
SES					0.30 [†]	(0.53)
Male					7.30	(1.40)
1st Generation Migrant					0.05 [†]	(3.65)
2nd Generation Migrant					−1.25 [†]	(2.17)
Numeracy					0.57	(0.01)
Literacy					0.38	(0.02)
School-level Predictors						
Academic						
Safety						
Resource shortage						
Student-teacher ratio						
Country-level Predictors						
Financial Knowledge Index			151.91	(21.42)	0.55	(0.05)
	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>	<i>M</i>	<i>SD</i>
RANDOM EFFECTS						
Student-level	6034.80	(317.12)	6004.72	(313.32)	1645.19	(52.95)
School-level	2775.10	(328.71)	2882.72	(356.54)	56.89	(12.27)
Country-level	5163.06		5158.35		110.97	(36.57)
MODEL FIT INDICES						
Log-likelihood	−0.63	(189.96)	−0.63	(190.95)	−0.53	(373.04)
AIC	1.25	(379.91)	1.25	(381.89)	1.06	(746.08)
BIC	1.25	(379.91)	1.25	(381.89)	1.06	(746.08)
SABIC	1.25	(379.91)	1.25	(381.89)	1.06	(746.08)
χ^2 Test of Model Fit			25.00	(3.52)	0.55	(0.14)
RMSEA						
CFI			0.40	(0.35)		
TLI						
SRMR L1			0.00	(0.00)		
SRMR L2			0.06	(0.01)		
SRMR L3			0.20	(0.01)	0.08	(0.01)

Note. Par. = parameter estimate, S.E. = standard error, [†] = estimate that failed to reach 0.05 significance level, *M* = mean estimate over ten plausible values, pooled using Rubin's Rule, *SD* = standard deviation of estimates over ten plausible values. Mean estimates for log-likelihood, AIC, BIC and SABIC are recorded in $\times 10^6$ (i.e., millions), whose standard deviations are in original scale.

Chapter 5 Discussion

5.1 Brief summary

5.1.1 Remind readers what my research questions are

5.2 The implication of this study

5.3 Limitation and future directions

5.3.1 Word in positive form

5.4 Bird-eye view

5.4.1 What conclusion I can draw from this paper/study

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Appendices

Appendix A GDPR Documentation and Ethical Approval

This research project discharges its duty imposed by the EEA's general data protection regulation (GDPR) by following Norwegian Centre for Research Data (NSD)'s [notification test](#) on Friday, 11 September 2020. Both [PISA 2018 Database](#) and the [World Bank Open Data](#) contain only aggregated and de-personalised datasets with no possibility of back-tracing to any particular participant. Resultantly, no identifiable personal data were collected or used at any stage of this research. The NSD's assessment letter outlines the agency's decision of not subjecting this project to the GDPR notification. The NSD decision letter also satisfies University of Oslo's [ethical approval requirement](#) and concludes the approval process.

About us (/personvernombud/en/about_us.html)

Norwegian (/personvernombud/meld_prosjekt/meldeplikttest.html)

NSD (</>) > Personvern tjenester (</personvernombud/>) > Data Protection Services (</personvernombud/en/>) > Notify project (</personvernombud/en/notify/>) > Notification Test

Denne siden på norsk (/personvernombud/meld_prosjekt/meldeplikttest.html)

Will you be processing personal data?

Are you unsure whether your project is subject to notification? Feel free to try our informal Notification test. Note that the test is intended as a guidance and is not a formal assessment.

Will you be collecting/processing directly identifiable personal data?

☐ Yes

☒ No

A person will be directly identifiable through name, social security number, or other uniquely personal characteristics.

Read more about personal data (</personvernombud/en/help/vocabulary.html?id=8>) and notification (</personvernombud/en/notify/index.html>).

NB! Even though information is to be anonymized in the final thesis/report, check the box if identifying personal data is to be collected/processed in connection with the project.

Will directly identifiable personal information be linked to the data (e.g. through a reference number which refers to a separate list of names/scrambling key)?

☐ Yes

☒ No

Note that the project will be subject to notification even if you cannot access the scrambling key (</personvernombud/en/help/vocabulary.html?id=11>), as the procedure often is when using a data processor (</personvernombud/en/help/vocabulary.html?id=3>), or in register-based studies (/personvernombud/en/help/research_methods/register_studies.html).

Will you be collecting/processing background information that may identify individuals (indirectly identifiable personal data)?

☐ Yes

☒ No

A person will be indirectly identifiable if it is possible to identify him/her through a combination of background information (such as place of residence or workplace/school, combined with information such as age, gender, occupation, etc.).

Will there be registered personal data (directly/indirectly/via IP or email address, etc.) using online surveys?

☐ Yes

☒ No

Please note that the project will be subject to notification even if you as a student/researcher cannot access the link to the IP or email address, as the procedure often is when using a data processor.

Read more about online surveys (/personvernombud/en/help/research_methods/online_surveys.html).

Will there be registered personal data using digital photo or video files?

☐ Yes☒ No

Photo/video recordings of faces will be regarded as identifiable personal data. In order for a voice to be considered as identifiable, it must be registered in combination with other background information, in such a way that people can be recognized.

Show results

Notify project

Do I have to notify my project? (</personvernombud/en/notify/index.html>)

Notification Form (/personvernombud/en/notify/meldeskjema_link)

Notifying changes (/personvernombud/en/notify/notifying_changes.html)

Get help notifying your project

Processing the notification (</personvernombud/en/help/index.html>)

Frequently asked questions (</personvernombud/en/help/faq.html>)

Vocabulary (</personvernombud/en/help/vocabulary.html>)

Research topics (/personvernombud/en/help/research_topics/)

Research methods (/personvernombud/en/help/research_methods/)

Information and consent (/personvernombud/en/help/information_consent/)

Other approvals (/personvernombud/en/help/other_approvals/)

Result of Notification Test: Not Subject to Notification

You have indicated that neither directly or indirectly identifiable personal data will be registered in the project.

If no personal data is to be registered, the project will not be subject to notification, and you will not have to submit a notification form.

Please note that this is a guidance based on information that you have given in the notification test and not a formal confirmation.

For your information: *In order for a project not to be subject to notification, we presuppose that all information processed using electronic equipment in the project remains anonymous.*

Anonymous information is defined as information that cannot identify individuals in the data set in any of the following ways:

- directly, through uniquely identifiable characteristic (such as name, social security number, email address, etc.)*
- indirectly, through a combination of background variables (such as residence/institution, gender, age, etc.)*
- through a list of names referring to an encryption formula or code, or*
- through recognizable faces on photographs or video recordings.*

Furthermore, we presuppose that names/consent forms are not linked to sensitive personal data.

Kind regards,
NSD Data Protection

Appendix B Analysis Code

B.1 Chapter 1

There is no analysis code in [Chapter 1](#).

B.2 Chapter 2

B.2.1 Data Import

```
1 | # Housekeeping
2 | library(Orcs)
3 | setwdOS(lin = "~/", win = Sys.getenv("USERPROFILE"))
4 |
5 | # Import SPSS file into R
6 | library(intsvy)
7 | finlit <- pisa.select.merge(
8 |   student.file = "CY07_MSU_FLT_QQQ.SAV", # file ext in capital
9 |   school.file = "CY07_MSU_SCH_QQQ.sav", # file ext in lower case
10 |   student = c(
11 |     # Control variables
12 |     "ST004D01T", # Student (Standardized) Gender
13 |     "IMMIG", # Index Immigration status
14 |     "ESCS", # Index of economic, social and cultural status
15 |     # Mediators
16 |     "FCFMLRTY", # Familiarity with concepts of finance (Sum)
17 |     "FLCONFIN", # Confidence about financial matters (WLE)
18 |     # Academic
19 |     "PERFEED", # Perceived feedback (WLE)
20 |     "TEACHINT", # Perceived teacher's interest (WLE)
21 |     "FLSCHOOL", # Financial education in school lessons (WLE)
22 |     # Safety
23 |     "DISCRIM", # Discriminating school climate (WLE)
24 |     "BELONG", # Sense of belonging to school (WLE)
25 |     "BEINGBULLIED", # Student's experience of being bullied (WLE)
26 |     # Community
27 |     "FLFAMILY", # Parental involvement in matters of Financial Literacy (WLE)
28 |     "CURSUPP", # Current parental support for learning at home (WLE)
29 |     "PASCHPOL" # School policies for parental involvement (WLE)
30 |   ),
31 |   school = c(
32 |     "STRATIO", # Student-teacher ratio
33 |     "EDUSHORT", # Shortage of educational material (WLE)
34 |     "STAFFSHORT" # Shortage of educational staff (WLE)
35 |   ),
36 |   countries = c(
37 |     "BGR", "BRA", "CAN", "CHL", "ESP",
38 |     "EST", "FIN", "GEO", "IDN", "ITA",
39 |     "LTU", "LVA", "NLD", "PER", "POL",
40 |     "PRT", "QMR", "QRT", "RUS", "SRB",
41 |     "SVK", "USA"
42 |   )
43 | )
44 |
45 | # Inspect table header
46 | names(finlit)
```

```

47|
48| # Remove columns that I do not need
49| finlit <- finlit[, -c(5, 7:86)] # 5 = BOOKID; 7:86 = resampling weights
50|
51| # Some var need recording
52| library(car)
53|
54| # Re-code Russian territories to RUS
55| finlit$CNT <- recode(finlit$CNT, "
56|   'QMR' = 'RUS';
57|   'QRT' = 'RUS'
58| ")
59|
60| # Recode ST004D01T from Sex to Male
61| MALE <- finlit$ST004D01T - 1
62|
63| # Recode IMMIG to 1st and 2nd generation
64| IMMI1GEN <- recode(finlit$IMMIG, "
65|   1 = 0;
66|   2 = 0;
67|   3 = 1
68| ")
69|
70| IMMI2GEN <- recode(finlit$IMMIG, "
71|   1 = 0;
72|   2 = 1;
73|   3 = 0
74| ")
75|
76| # Revert coding direction: bigger number => safer school
77| NOBULLY <- finlit$BEINGBULLIED * (-1)
78|
79| # Stitch spreadsheets together
80| finlit <- cbind(
81|   finlit[, c(1:35)],
82|   MALE, IMMI1GEN, IMMI2GEN,
83|   finlit[, c(38:45)],
84|   NOBULLY,
85|   finlit[, c(47:53)]
86| )
87|
88| # Use data.table for better RAM management
89| library(data.table); setDTthreads(0) # 0 means all the available cores
90| # Export data into a CSV file for faster import next time
91| fwrite(finlit,
92|   file = "finlit.csv",
93|   na = "NA", row.names = F, col.names = T
94| )

```

B.2.2 Missing Pattern Inspection

```

1| library(Orcs)
2| setwdOS(lin = "~/", win = Sys.getenv("USERPROFILE"))
3|
4| library(data.table); setDTthreads(0)
5| finlit <- fread("finlit.csv", nThread = getDTthreads())
6|
7| library(dplyr)
8| # Record how many missings each country has for each var
9| missings <- finlit %>%
10|   select(everything()) %>%

```



```

11|     group_by(CNT) %>%
12|     summarise_all(funs(sum(is.na(.))))
13| # Give me the headcount for each country
14| headcount <- finlit %>%
15|     group_by(CNT) %>%
16|     summarize(n())
17| # Stitch these two tables together
18| missing_table <- tibble(headcount, missings[, -1])
19| # Save this file
20| fwrite(missing_table, "missing_table.csv", row.names = F, col.names = T)
21|
22| # Inspect the missing table using Excel
23| # Throw away the following countries
24| #   CAN: 100% missing on too many var
25| #   BRA, FIN, LVA, NLD, RUS, SRB: private/public info missing
26| # Throw away these var
27| #   DISCRIM, CURSUPP, PASCHPOL: Too many countries have 100% missing

```

B.2.3 Financial Knowledge Index

```

1| # Section 0: Housekeeping
2| library(Orcs) # Set working directory depending on operating system
3| setwdOS(
4|   lin = "~/uio/", win = "M:/",
5|   ext = "pc/Dokumenter/MSc/Thesis/Data/L3/"
6| )
7|
8| # Set up a "bookshelf" to hold variables nessary to compute FKI
9| fki_raw <- matrix(NA,
10|   nrow = 20, ncol = 10, dimnames = list(
11|     c( # row names
12|       "BRA", "BGR", "CAN", "CHL", "EST",
13|       "FIN", "GEO", "IDN", "ITA", "LVA",
14|       "LTU", "NLD", "PER", "POL", "PRT",
15|       "RUS", "SRB", "SVK", "ESP", "USA"
16|     ),
17|     c( # column names
18|       "gdp_per_capita", # economic capability (sub_ind_ec)
19|       "highly_skilled", "mean_year_of_schooling", # (sub_ind_et)
20|       "gpea", "ica", "ius", # use (sub_ind_use)
21|       "pfa", "ac", "gdp", "ageing" # need (sub_ind_need)
22|     )
23|   ) # End list()
24| ) # End matrix()
25|
26|
27| # Section 1: Economic Capacity (EC)
28|
29| gdp_per_capita <- read.csv("gdp_per_capita.csv", header = T, sep = "\t")
30|
31| fki_raw[, 1] <- log(gdp_per_capita[, 2])
32|
33| rm(gdp_per_capita)
34|
35|
36| # Section 2: Educational Training (ET)
37|   # Subsection 2.1: Highly skilled
38|     # Masters
39| isced_7 <- read.csv("isced_7.csv", header = T, sep = "\t")
40|     # PhDs
41| isced_8 <- read.csv("isced_8.csv", header = T, sep = "\t")

```

```

42|         # Total tertiary
43| total_tertiary ← read.csv("total_tertiary.csv", header = T, sep = "\t")
44|
45| # Compute highly skilled (master + PhD) to total tertiary ratio
46| highly_skilled ← ts(
47|     (isced_7 + isced_8) / total_tertiary,
48|     start = 2013, end = 2018, frequency = 1
49| )
50|
51| # Visualise highly_skilled. Turn off GEO (#7), IDN (#8), SRB (#17) and RUS (#16)
52| pdf("../Figures/skilled.pdf")
53| ts.plot(100 * highly_skilled[, -c(7, 8, 16, 17)],
54|     type = "b", col = 1:15,
55|     xlab = "Year", ylab = "Percent"
56| )
57| legend("topright", colnames(highly_skilled[, -c(6, 7, 15, 16)]),
58|     col = 1:15, lty = 1, cex = 0.65
59| )
60| dev.off()
61|
62| # Decision: naive forecasts, i.e., copy-paste nearest available year
63| library(forecast)
64| # Create a placeholder matrix
65| placeholder ← matrix(NA, nrow = 20, ncol = 1)
66|
67| # Run a loop to forecast all 20 countries, using naive method
68| for (.i in 1:20) {
69|     m_naive_i ← naive(highly_skilled[, .i], h = 1)
70|     placeholder[.i] ← data.frame(unlist(m_naive_i[5])[6])[1, 1]
71| }
72| # [5] = fitted values; [6] = 2018; [1,1] = only the numeric value
73|
74| # GEO and IDN have 2018 data, plug actual numbers back
75| placeholder[c(7, 8)] ← highly_skilled[6, c(7, 8)]
76|
77| # RUS needs separate calculation
78| # ISCED 7 = 101766 (Type 1) + 170437 (Type 2) = 272203 (total masters)
79| # ISCED 8 = 15465 (Type 1) + 330 (Type 2) = 15795 (total PhDs)
80| # Total tertiary WITHOUT PhD = 933153
81| # ⇒ Total tertiary = 933153 + 15795 = 948948
82| # highly_skilled (RUS) = (272203 + 15795) / 948948 = 0.30349187
83| placeholder[16] ← 0.30349187
84|
85| # Save results to "bookshelf"
86| fki_raw[, 2] ← placeholder * 100
87|
88| rm(
89|     isced_7, isced_8, total_tertiary,
90|     highly_skilled, placeholder, m_naive_i
91| )
92|
93| # Sub-section 2.2: Mean year of schooling
94| mean_year_of_schooling ← read.csv("mean_year_of_schooling.csv",
95|     header = F, sep = "\t"
96| )
97| fki_raw[, 3] ← mean_year_of_schooling[, 2]
98|
99| rm(mean_year_of_schooling)
100|
101|
102| # Section 3: Use

```

```

103|
104| gpea ← read.csv("gpea.csv", header = T, sep = "\t")
105| gpea ← ts(gpea, start = 2011, end = 2017, frequency = 1)
106|
107| # # Visualise data in both original and ln forms. Contain trend?
108| # pdf("../Figures/use.pdf", width = 12.94, height = 9.15)
109|
110| # # Re-set canvas layout to 2x2
111| # par(mfcol = c(2, 2))
112|
113| # # Add extra space to the right of plot area
114| # par(mar = c(5.1, 4.1, 4.1, 2.1), xpd = TRUE)
115|
116| # # Plot GPEA in original form
117| # ts.plot(gpea,
118| #   type = "b", col = 1:20,
119| #   xlab = "Year", ylab = "Percent", main = "GPEA to GDP ratio"
120| # )
121|
122| # # Remove extra gap between the two graphs
123| # par(mar = c(5.1, 4.1, 0, 2.1), xpd = TRUE)
124|
125| # # Repeat GPEA, but for the ln() version
126| # ts.plot(log(gpea),
127| #   type = "b", col = 1:20,
128| #   xlab = "Year", ylab = "ln( percent )"
129| # )
130|
131| # # Plot ICA in original form
132| # par(mar = c(5.1, 4.1, 4.1, 6.1), xpd = TRUE)
133| # ts.plot(ica,
134| #   type = "b", col = 1:20,
135| #   xlab = "Year", ylab = "Percent", main = "ICA to GDP ratio"
136| # )
137| # # Add the legend
138| # legend("topright",
139| #   inset = c(-0.2, 0), colnames(ica),
140| #   col = 1:20, lty = 1, cex = 0.875
141| # )
142|
143| # # Remove extra gap between the two graphs
144| # par(mar = c(5.1, 4.1, 0, 6.1), xpd = TRUE)
145|
146| # # Repeat, but for the ln()
147| # ts.plot(log(ica),
148| #   type = "b", col = 1:20,
149| #   xlab = "Year", ylab = "ln( percent )"
150| # )
151| # # Add the legend
152| # legend("topright",
153| #   inset = c(-0.2, 0), colnames(ica),
154| #   col = 1:20, lty = 1, cex = 1.07
155| # )
156| # dev.off()
157|
158| # Decision: since the ln() version is not flat, original time series
159| # contain trend. Use Holt method rather than simple exponential smoothing.
160|
161| # Run a time series forecast using Holt method
162|

```

```

163 | # Create a placeholder matrix
164 | placeholder ← matrix(NA, nrow = 20, ncol = 1)
165 |
166 | # Run a loop to forecast all 13 countries, using Holt method
167 | for (.i in 1:20) {
168 |   m_holt_i ← holt(gpea[, .i], h = 1)
169 |   placeholder[.i] ← m_holt_i[2]
170 | } # Ignore warnings
171 |
172 | # Only keep the 2018 forecasts
173 | placeholder ← unlist(placeholder)
174 |
175 | # Run PER (#13) separately because it misses both 2017 and 2018 data
176 | m_holt_PER ← holt(gpea[, 13], h = 2); summary(m_holt_PER)
177 | placeholder[13] ← 16.02698
178 |
179 | # Push placeholder to fki_raw
180 | fki_raw[, 4] ← placeholder
181 |
182 | rm(gpea, placeholder, m_holt_i, m_holt_PER)
183 |
184 | # Sub-section 3.2: Insurance company assets (ica)
185 |
186 | ica ← read.csv("ica.csv", header = T, sep = "\t")
187 | ica ← ts(ica, start = 2011, end = 2017, frequency = 1)
188 |
189 | placeholder ← matrix(NA, nrow = 20, ncol = 1)
190 |
191 | for (.i in 1:20) {
192 |   m_holt_i ← holt(ica[, .i], h = 1)
193 |   placeholder[.i] ← m_holt_i[2]
194 | } # Ignore warnings
195 |
196 | placeholder ← unlist(placeholder)
197 |
198 | m_holt_CAN ← holt(ica[, 3], h = 2); summary(m_holt_CAN)
199 | m_holt_IND ← holt(ica[, 8], h = 2); summary(m_holt_IND)
200 | m_holt_ITA ← holt(ica[, 9], h = 2); summary(m_holt_ITA)
201 | m_holt_POL ← holt(ica[, 14], h = 2); summary(m_holt_POL)
202 | m_holt_USA ← holt(ica[, 20], h = 2); summary(m_holt_USA)
203 |
204 | placeholder[c(3, 8, 9, 14, 20)] ← c(
205 |   77.72768, 4.611597, 51.2596, 9.534750, 30.18295
206 | )
207 |
208 | fki_raw[, 5] ← placeholder
209 |
210 | rm(ica, placeholder, list = ls(pattern = "^m.holt"))
211 |
212 | # Sub-section 3.3: Individuals using the Internet (ius)
213 |
214 | ius ← read.csv("ius.csv", header = T, sep = "\t")
215 | ius ← ts(ius, start = 2009, end = 2018, frequency = 1)
216 |
217 | m_holt_CAN ← holt(ius[1:9, 3], h = 1); summary(m_holt_CAN)
218 | m_holt_CHL ← holt(ius[1:9, 4], h = 1); summary(m_holt_CHL)
219 | m_holt_USA ← holt(ius[1:9, 20], h = 1); summary(m_holt_USA)
220 |
221 | ius_2018 ← ius[10, ] # Only want 2018 data
222 | ius_2018[3] ← 93.58751 # CAN

```

```

223 ius_2018[4] ← 89.5309 # CHL
224 ius_2018[20] ← 84.88108 # USA
225
226 fki_raw[, 6] ← ius_2018
227
228 rm(list = ls(pattern = "^ius"))
229 rm(list = ls(pattern = "^m_holt_"))
230
231
232 # Section 4: Need
233
234 # Subsection 4.1: Pension fund assets (pfa)
235 pfa ← read.csv("pfa.csv", header = T, sep = "\t")
236 # Delete GEO (#7) due to all missing. Will come back to it later.
237 pfa ← ts(pfa[, -7], start = 2008, end = 2017, frequency = 1)
238
239 placeholder ← matrix(NA, nrow = 19, ncol = 1)
240
241 for (.i in 1:19) {
242   m_holt_i ← holt(pfa[, .i], h = 1)
243   placeholder[.i] ← m_holt_i[2]
244 }
245
246 placeholder ← unlist(placeholder)
247
248 # Calculate GEO
249 # From Georgia Pension Agency:
250 #   2019 mesub_ind_eting minute: 372,113,933 GEL
251 # From GeoStat website:
252 #   2018 gdp = 44.6 billion GEL
253
254 fki_raw[, 7] ← c(
255   placeholder[1:6],
256   372113934 / 446000000000 * 100, # Insert GEO figure
257   placeholder[7:19]
258 )
259
260 rm(pfa, placeholder, m_holt_i)
261
262 # Subsection 4.2: Aggregate consumption (ac)
263
264 ac ← read.csv("ac.csv", header = F, row.names = 1, sep = "\t")
265 gdp ← read.csv("gdp.csv", header = F, row.names = 1, sep = "\t")
266
267 fki_raw[, 8] ← unlist(ac * 0.02 / gdp * 100)
268
269 fki_raw[, 9] ← unlist(gdp)
270
271 rm(ac, gdp)
272
273 # Subsection 4.3: Ageing
274
275 ageing ← read.csv("ageing.csv", header = T, sep = "\t")
276 attach(ageing)
277 names(ageing)
278
279 # Calculate total population
280 poptotal_f ← pop0to14_f + pop15to64_f + pop65plus_f
281 poptotal_m ← pop0to14_m + pop15to64_m + pop65plus_m
282 # Calculate population between 15 and 19
283 # Need to divide by 100 to get decimals

```

```

284 pop15to19_f ← poptotal_f * per15to19_f / 100
285 pop15to19_m ← poptotal_m * per15to19_m / 100
286 # Calculate population between 0 and 19
287 pop0to19_f ← pop0to14_f + pop15to19_f
288 pop0to19_m ← pop0to14_m + pop15to19_m
289 # Calculate population between 20 and 64
290 pop20to64_f ← poptotal_f - pop0to19_f - pop65plus_f
291 pop20to64_m ← poptotal_m - pop0to19_m - pop65plus_m
292 # Calculate 64+ / 20-to-64 ratio      'GEO' = 0.419;
293 ageing_ratio ← I(
294   (pop65plus_f + pop65plus_m) / (pop20to64_f + pop20to64_m)
295 )
296 # Split data into 2018 [ , 1] and 2009 [ , 2] portions
297 ageing ← cbind(ageing_ratio[1:20], ageing_ratio[21:40])
298 fki_raw[, 10] ← (ageing[, 1] - ageing[, 2]) / ageing[, 2]
299
300 rm(ageing, ageing_ratio, list = ls(pattern = "^pop"))
301
302
303 # Section 5: FKI
304
305 fki_raw ← fki_raw[, -9] # Throw away gdp (already in ac)
306 round(fki_raw, digits = 3) # Inspect data
307
308 # Save data to an external file
309 library(data.table); setDTthreads(0)
310 fwrite(round(fki_raw, digits = 3), file = "fki_raw.csv", row.names = T)
311
312 # Subection 5.0: Standardise each variable to [0.01,0.99] range
313 fki_stand ← matrix(NA, nrow = dim(fki_raw)[1], ncol = dim(fki_raw)[2])
314 dimnames(fki_stand) ← dimnames(fki_raw)
315
316 library(scales) # I wish this function could have "by.col = T". Oh well.
317 for (.j in 1:dim(fki_raw)[2]) {
318   fki_stand[, .j] ← rescale(fki_raw[, .j], to = c(0.01, 0.99))
319 }
320
321 rm(fki_raw)
322
323 fki_stand ← data.frame(fki_stand)
324 attach(fki_stand)
325
326 # Subsection 5.1: Economic capacity (sub_ind_ec)
327
328 sub_ind_ec ← gdp_per_capita
329
330 # Subsection 5.2: Education and training (sub_ind_et)
331
332 wt_highly_skilled ← 1 / sd(highly_skilled)
333 wt_mean_year_of_schooling ← 1 / sd(mean_year_of_schooling)
334
335 sub_ind_et ← (highly_skilled^wt_highly_skilled *
336   mean_year_of_schooling^wt_mean_year_of_schooling)^
337   (1 / (wt_highly_skilled + wt_mean_year_of_schooling))
338
339 rm(list = ls(pattern = "^wt"))
340
341 # Subsection 5.3: Use (sub_ind_use)
342
343 sub_ind_u ← (gpea + ica)^ius

```

```

344|
345| # Subsection 5.4: Need (sub_ind_need)
346|
347| sub_ind_n ← (pfa + ac)^ageing
348|
349| ## Subsection 5.5: FKI
350|
351| wt_ec ← 1 / sd(sub_ind_ec)
352| wt_et ← 1 / sd(sub_ind_et)
353| wt_u ← 1 / sd(sub_ind_u)
354| wt_n ← 1 / sd(sub_ind_n)
355|
356| fki ← (
357|   sub_ind_ec^wt_ec *
358|   sub_ind_et^wt_et *
359|   sub_ind_u^wt_u *
360|   sub_ind_n^wt_n
361| ) ^ (
362|   1 / (wt_ec + wt_et + wt_u + wt_n)
363| )
364|
365| rm(list = ls(pattern = "^wt"))
366|
367| l3 ← data.frame(
368|   round(
369|     cbind(fki, sub_ind_ec, sub_ind_et, sub_ind_u, sub_ind_n),
370|     digits = 3
371|   )
372| )
373| rownames(l3) ← rownames(fki_stand)
374| attach(l3)
375|
376| rm(fki_stand, fki, sub_ind_ec, sub_ind_et, sub_ind_u, sub_ind_n)
377|
378| # Display country-level FKI, default by country code
379| l3
380|
381| # Sort FKI by country (highest to lowest)
382| l3_ordered ← l3[order(-fki), ]
383| l3_ordered
384| fwrite(l3_ordered, file = "fki.csv", row.names = T)
385|
386| pdf("../..//Figures/FKI.pdf")
387| barplot(l3_ordered$fki,
388|   names.arg = rownames(l3_ordered),
389|   xlab = "Country", las = 2, ylab = "Financial Knowledge Index (FKI)",
390|   ylim = c(0, 1), main = "FKI of 20 participating countries"
391| )
392| dev.off()

```

B.2.4 Data Reimport

```

1| # Housekeeping
2| library(Orcs)
3| setwdOS(lin = "~/", win = Sys.getenv("USERPROFILE"))
4|
5| # Import SPSS file into R
6| library(intsvy)
7| finlit ← pisa.select.merge(
8|   student.file = "CY07_MSU_FLT_QQQ.SAV", # file ext in capital

```

```

9| school.file = "CY07_MSU_SCH_QQQ.sav", # file ext in lower case
10| student = c(
11|   # Control variables
12|     "ST004D01T", # Student (Standardized) Gender
13|     "IMMIG", # Index Immigration status
14|     "ESCS", # Index of economic, social and cultural status
15|   # Mediators
16|     "FCFMLRTY", # Familiarity with concepts of finance (Sum)
17|     "FLCONFIN", # Confidence about financial matters (WLE)
18|   # Academic
19|     "FLSCHOOL", # Financial education in school lessons (WLE)
20|   # Safety
21|     "BEINGBULLIED", # Student's experience of being bullied (WLE)
22|   # Community
23|     "FLFAMILY" # Parental involvement in matters of Financial Literacy (WLE)
24| ),
25| school = c(
26|   "STRATIO", # Student-teacher ratio
27|   "EDUSHORT" # Shortage of educational material (WLE)
28| ),
29| countries = c(
30|   "BRA", "BGR", "CAN", "CHL", "EST",
31|   "FIN", "GEO", "IDN", "ITA", "LVA",
32|   "LTU", "NLD", "PER", "POL", "PRT",
33|   "RUS", "QMR", "QRT", # Russian Federation and other regions
34|   "SRB", "SVK", "ESP", "USA"
35| )
36| )
37|
38| names(finlit)
39| # Throw away columns that I do not need
40| finlit <- finlit[, -c(5, 7:86)] # 5 = BOOKID; 7:86 = resampling weights
41| names(finlit)
42|
43| # Some var need recording
44| library(car)
45|
46| # Re-code Russian territories to RUS
47| finlit$CNT <- recode(finlit$CNT, "
48|   'QMR' = 'RUS';
49|   'QRT' = 'RUS'
50| ")
51|
52| finlit$CNTRYID <- recode(finlit$CNTRYID, "
53|   982 = 643;
54|   983 = 643
55| ")
56|
57| # Input country-level FKI
58| FKI <- recode(finlit$CNT, "
59|   'NLD' = 0.940;
60|   'USA' = 0.937;
61|   'CAN' = 0.784;
62|   'ITA' = 0.762;
63|   'FIN' = 0.724;
64|   'ESP' = 0.627;
65|   'LTU' = 0.613;
66|   'PRT' = 0.591;
67|   'BGR' = 0.583;
68|   'EST' = 0.577;
69|   'SVK' = 0.559;

```



```

70 | 'POL' = 0.555;
71 | 'LVA' = 0.550;
72 | 'CHL' = 0.544;
73 | 'RUS' = 0.450;
74 | 'GEO' = 0.424;
75 | 'SRB' = 0.423;
76 | 'PER' = 0.309;
77 | 'BRA' = 0.141;
78 | 'IDN' = 0.122
79 | ")
80 |
81 | # Recode ST004D01T from Sex to Male
82 | MALE ← finlit$ST004D01T - 1
83 |
84 | # Recode IMMIG to 1st and 2nd generation
85 | IMMI1GEN ← recode(finlit$IMMIG, "
86 |   1 = 0;
87 |   2 = 0;
88 |   3 = 1
89 | ")
90 |
91 | IMMI2GEN ← recode(finlit$IMMIG, "
92 |   1 = 0;
93 |   2 = 1;
94 |   3 = 0
95 | ")
96 |
97 | # Revert coding direction: bigger number ⇒ safer school
98 | NOBULLY ← finlit$BEINGBULLIED * (-1)
99 |
100 | # Stitch spreadsheet together
101 | names(finlit)
102 | finlit ← cbind(FKI, finlit[, c(2:35)], MALE, IMMI1GEN, IMMI2GEN, finlit[, c(38:41)], NOBULLY)
103 | head(finlit)
104 | names(finlit)
105 |
106 | # Remove cases whose school weights (col #45) are NA
107 | obs0 ← dim(finlit)[1]
108 | finlit ← finlit[complete.cases(finlit[, 45]), ]
109 | obs1 ← dim(finlit)[1]
110 | obs0 - obs1 # 12 cases contained missing school weights and have been dropped
111 | rm(obs0, obs1)
112 |
113 | # Use data.table for better RAM management
114 | library(data.table); setDTthreads(0) # 0 means all the available cores
115 | # Export data into a CSV file for faster import next time
116 | fwrite(finlit, file = "finlit.csv", na = "NA", row.names = F, col.names = T)

```


Appendix C Derivation of Country-level Financial Knowledge Indices

C.1 Theoretical Foundation

PISA 2018 financial literacy dataset (OECD, 2020) provides rich information about students and schools. For the purpose of cross-country comparison, however, the country-level data must be addressed separately by the researchers. Moreno-Herrero et al. (2018), for instance, introduced a variable “quality of math and science education” to control for country-level differences since consensus is yet to emerge about the most appropriate measure for “countries’ financial knowledge”. Inspired by the UN’s approach to forming Human Development Indices, a recent publication Oliver-Márquez et al. (2020) highlighted four aspects of countries’ macroeconomic practices in their attempt to develop country-level financial knowledge indices (FKI).

Oliver-Márquez and colleagues consider a country’s economic capability, represented by its GDP per capita, to be a key dimension in bringing about its FKI. Secondly, literature converges on the importance of education training for a country’s financial knowledge capability (OECD, 2005). Thirdly, countries with regular engagement with sophisticated financial products and financial markets should possess higher FKI. Lastly, countries with higher aggregate consumption levels and with ageing populations are likely to possess higher FKI due to more frequent exposure and pressure in retirement provision, respectively.

More specifically, Oliver-Márquez et al. (2020) suggests using the logarithm of GDP per capital in current international dollars (purchasing power parity adjusted) as a measure for the **Economic Capability** sub-index. For the **Education Training** sub-index, the authors consider postgraduate-to-total-tertiary-graduation ratios as a reflection of “highly skilled” workforce and the mean years of schooling as a measure of countries’ general education levels. For the **Use** sub-index, gross portfolio equity assets (GPEA) and insurance company assets (ICA) are considered sophisticated financial products countries engage themselves in. Additionally, in order to capture the central role of technology in amplifying the proliferation and use of financial assets, the proportion of Internet users (IUS) enters the definition via

$$\text{Use} = (\text{GPEA} + \text{ICA})^{\text{IUS}}.$$

For the final sub-index **Need**, the authors define

$$\text{Need} = (\text{PFA} + \text{AC})^{\text{AGEING}},$$

where PFA is the pension-fund-assets-to-GDP ratio. Aggregate consumption is defined as:

$$\text{AC} = \frac{2\% \times \text{household final consumption expenditure}}{\text{GDP}},$$

where the “2% rule” is drawn from Caliendo and Findley (2013) and the proportion of ageing population is computed as

$$\text{AGEING} = \frac{\left[\frac{\text{population}(>65)}{\text{population}(20\sim64)} \right]_{2018} - \left[\frac{\text{population}(>65)}{\text{population}(20\sim64)} \right]_{2009}}{\left[\frac{\text{population}(>65)}{\text{population}(20\sim64)} \right]_{2009}}.$$

C.2 Data Collection and Missing Data Treatment

The data sources for FKI computation are documented in [Table C.1](#) and its associated notes. The sub-indices **Educational Training** and **Use** both contain missing observations for the year 2018. Majority of such missing data appear to be the result of administrative delay, with historic observations available until 2017. It is therefore feasible to conduct time-series forecasts using prior year observations to best approximate 2018 values.

Table C.1*Data Sources for FKI Computation*

Database ^a	Country ^b	Series	Time
Economic Capacity			
WB-dev	20	GDP per capita, PPP (current international \$)	2018
Educational Training			
WB-ed	20 \ Russia	Graduates from ISCED 7 programmes in tertiary education, both sexes (number)	2013– 2018
		Graduates from ISCED 8 programmes in tertiary education, both sexes (number)	2013– 2018
		Graduates from tertiary education, both sexes (number)	2013– 2018
RS	Russia	PhD (Type 1) ^c , PhD (Type 2) ^d	2018
RE	Russia	Master (Type 1) ^e , Master (Type 2) ^f , total tertiary <i>excluding</i> PhD ^g	2018
HDR	20	Dimension = Education; Education = Mean years of schooling (years)	2018
Use			
WB-fin	20	Gross portfolio equity assets to GDP (%)	2011– 2018
		Insurance company assets to GDP (%)	2011– 2018
WB-dev	20	Individuals using the Internet (% of population)	2009– 2018
Need			
WB-fin	20 \ Georgia	Pension fund assets to GDP (%)	2008– 2018
GP	Georgia	Minutes of the meeting of the investment board of the Pension Agency ^h	2019 [*]
GS	Georgia	GDP at current prices, billion GEL ⁱ	2018
WB-dev	20	Household and NPISHs final consumption expenditure, PPP (current international \$)	2018
		GDP, PPP (current international \$)	2018
		Population ages 0–14, male	2009, 2018
		Population ages 0–14, female	2009, 2018
		Population ages 15–64, male	2009, 2018
		Population ages 15–64, female	2009, 2018
		Population ages 65 and above, male	2009, 2018
		Population ages 65 and above, female	2009, 2018
		Population ages 15–19, male (% of male population)	2009, 2018
		Population ages 15–19, female (% of female population)	2009, 2018

Note. Sub-indices are shaded in gray. Bold font signifies this year contains missing data.

- ^a WB-dev = [World Bank – World development indicators](#)
 WB-ed = [World Bank – Education statistics – All indicators](#)
 WB-fin = [World Bank – Global financial development](#)
 HDR = [Human Development Reports – Data](#)
 RS = [Russian Federal State Statistic Service](#)
 RE = [Russian Ministry of Education and Science](#)
 GP = [Pension Agency of Georgia](#)
 GS = [National Statistics Office of Georgia](#)
- ^b “20” = the 20 participating countries in 2018 PISA financial literacy test: Brazil, Bulgaria, Canada, Chile, Estonia, Finland, Georgia, Indonesia, Italy, Latvia, Lithuania, the Netherlands, Peru, Poland, Portugal, Russian Federation, Serbia, Slovak Republic, Spain, and the USA. “\” = excluding or except
- ^c [https://rosstat.gov.ru/storage/mediabank/asp-2\(1\).xls](https://rosstat.gov.ru/storage/mediabank/asp-2(1).xls), Sheet “по направлениям подготовки”, Cell C7 = number of PhD graduates (Type 1)
- ^d <https://rosstat.gov.ru/storage/mediabank/asp-3.xls>, Sheet “по научным специальностям”, Cell B7 = number of PhD graduates (Type 2)
- ^{e-g} https://minobrnauki.gov.ru/common/upload/download/VPO_1_2018.rar contains a spreadsheet [СВОД_БПО1_БСЕГО.xls](#), Sheet “P2_1_3(1)”, Cell E198 = number of master graduates (Type 1)^e, Cell E410 = number of master graduates (Type 2)^f, Cell E592 = total tertiary graduates *excluding* PhD^g
- ^h [Minutes of the meeting of the investment board of the Pension Agency](#), p. 4, no. 3
- ⁱ [Gross domestic product \(GDP\)](#), row = GDP at current prices, billion GEL, column = 2018
- ^{*} Georgia started a [new pension system](#) on 1 January 2019. Since 2018 was a transitional period with scarce data, 2019 is used as the best approximation for Georgia’s pension system for 2018.

C.2.1 Sub-index Educational Training

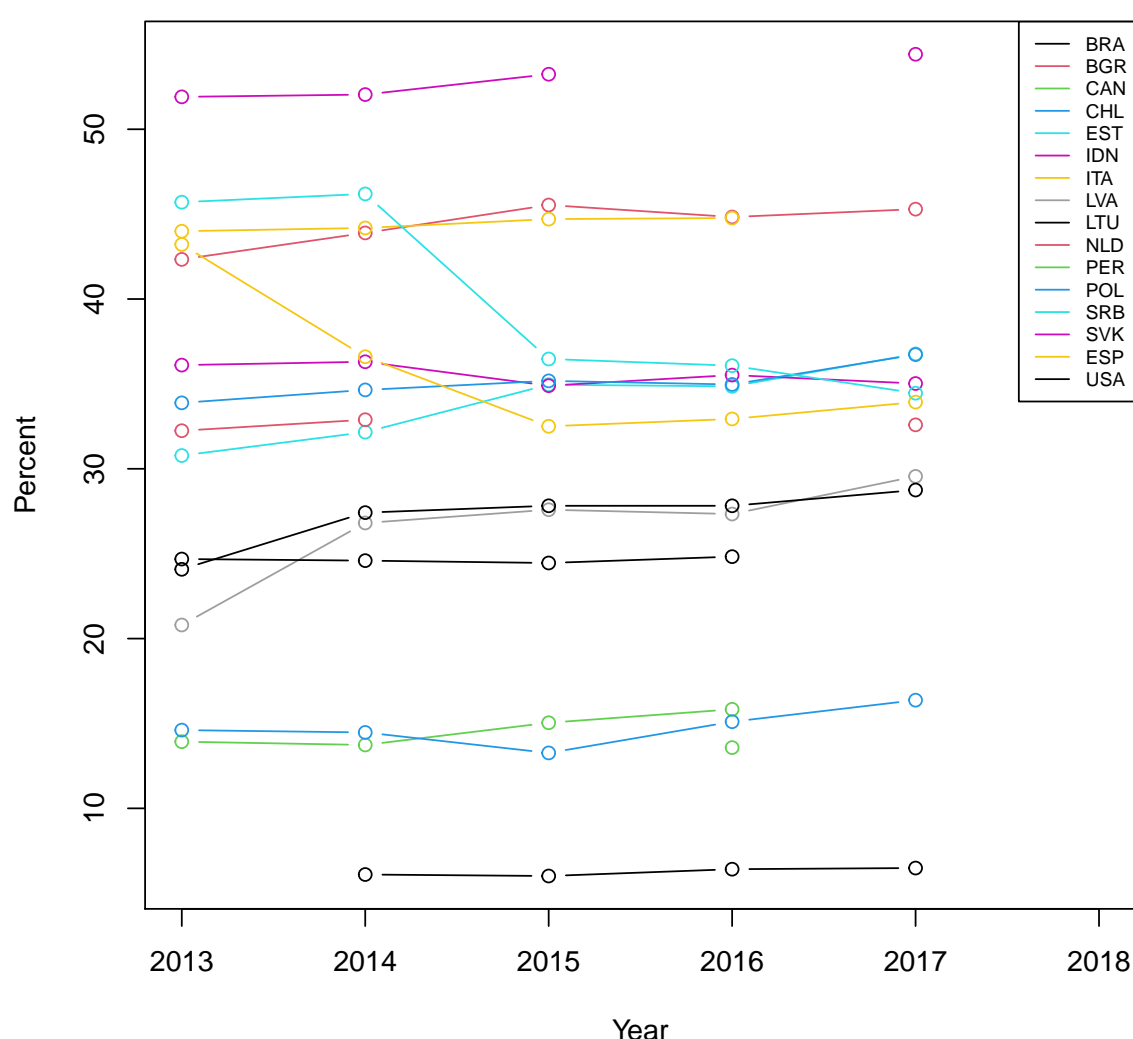
The 2018 archive for the number of master (ISCED 7), PhD (ISCED 8), and total tertiary graduates are incomplete for all participating countries except Georgia, Indonesia and Serbia. **Figure C.1** presents a time series plot of

$$\text{highly skilled} = \frac{\text{number of masters} + \text{number of PhDs}}{\text{total number of tertiary graduates}}$$

and suggests that this ratio is likely to be stable over time, especially between adjacent years. A “naive forecast”, where the nearest available year’s data are to be duplicated for 2018, is applied for **highly skilled**.

Figure C.1

Proportion of Postgraduates to Total Tertiary Graduations



Note. “Postgraduate” is defined as master (ISCED 7) and PhD (ISCED 8) graduates. Countries not shown: GEO, IDN and SRB (2018 data available) and RUS (consult other sources)

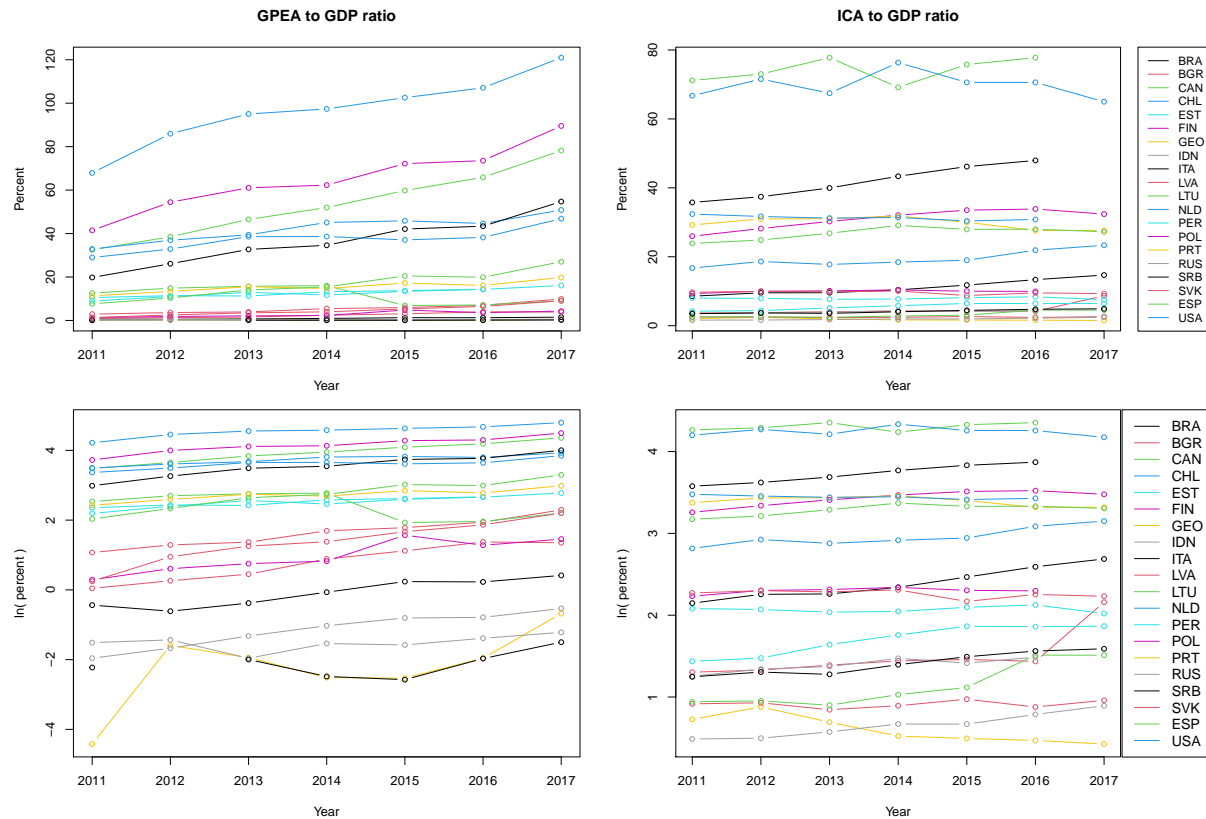
C.2.2 Sub-index Use

All series involved in calculating this sub-index, GPEA, ICA and IUS, contain missing data. When time series data contain only exponential growth but no underlying trend, a simple

exponential smoothing would suffice (Gardner, 1985); if trend is present, Holt-Winters method is superior (Chatfield, 1978). **Figure C.2** facilitates this decision making by plotting both the original and log-transformed versions of GPEA and ICA series. Since curves after log-transformations have slopes, it is prudent to apply the Holt-Winters forecasting method in order to account for possible trends contained in the original series.

Figure C.2

Time Series Trend Test



Note. The time series plots after natural logarithm transformations (bottom panels) are not flat, suggesting the original series (top panels) contain trends. Holt-Winters method therefore is preferred over simple exponential smoothing for 2018 forecasts.

The IUS series contains missing data for Canada, Chile and the United States. Similar Holt-Winters procedure is applied to recover 2018 IUS data.

Table C.2*Data Utilised for Computing FKI*

	Economic Capacity	Educational Training		Use			Need		
	GDP per capita	Skilled	Schooling	GPEA	ICA	IUS	PFA	AC	AGEING
BRA	9.612	6.484	7.8	1.683	16.259	70.434	11.827	1.210	0.288
BGR	10.026	45.294	11.8	4.114	7.044	64.782	13.577	1.091	0.234
CAN	10.821	15.832	13.3	84.010	77.728	93.588	96.205	1.068	0.271
CHL	10.117	16.371	10.4	51.755	25.591	89.531	73.225	1.073	0.214
EST	10.501	36.765	13.0	16.399	7.681	89.357	18.012	0.876	0.163
FIN	10.807	35.024	12.4	93.626	31.481	88.890	52.024	0.974	0.370
GEO	9.588	24.039	12.8	0.784	1.469	62.718	0.834	1.227	0.042
IDN	9.362	7.771	8.0	0.636	4.612	39.905	1.826	1.059	0.145
ITA	10.665	44.771	10.2	57.434	51.260	74.387	10.589	1.075	0.155
LVA	10.330	29.554	12.8	8.598	2.538	83.577	14.732	1.027	0.142
LTU	10.487	28.749	13.0	9.008	5.500	79.723	7.457	1.107	0.149
NLD	10.961	32.590	12.2	124.171	64.956	94.712	207.938	0.805	0.326
PER	9.479	13.577	9.2	16.027	6.505	52.540	22.530	1.187	0.227
POL	10.368	36.725	12.3	4.853	9.535	77.542	9.838	1.085	0.355
PRT	10.444	34.454	9.2	19.353	25.579	74.661	8.761	1.133	0.237
RUS	10.267	30.349	12.0	0.302	2.614	80.865	4.415	0.941	0.155
SRB	9.774	26.946	11.2	0.306	5.111	73.361	0.845	1.171	0.280
SVK	10.391	54.417	12.6	10.644	8.873	80.660	12.497	0.962	0.300
ESP	10.609	33.929	9.8	27.681	28.230	86.107	10.235	1.044	0.186
USA	11.048	24.825	13.4	55.505	30.183	84.881	150.040	1.364	0.252

Note. Full variable names: Skilled = Postgraduate to total tertiary ratio; Schooling = Mean year of schooling; GPEA = Gross portfolio to GDP ratio; ICA = Insurance company assets to GDP ratio; IUS = Number of Internet users per 100 population; PFA = Pension fund assets to GDP ratio; AC = 2% of household final consumption expenditure to GDP ratio; AGEING = Aged-to-productive-population ratio (% change between 2009 and 2018)

C.2.3 Other Items with Data Concerns

Russia reported 67.96% and 61.01% of its total university degree recipients to be postgraduates for the year 2013 and 2015 respectively (2014 missing). This figure rapidly declines to 41.6% in 2016 and further down to 25.69% in 2017. Such volatility goes against the stable patterns shared by most countries in [Figure C.1](#), casting doubt on data reliability. Separate investigation is therefore conducted using Russian government archive (Notes c to g in [Table C.1](#)).

Georgia underwent pension reform in 2018 with fund balance gradually transitioning to State Pension Agency for its official resumption of duty on 1 January 2019. Resultantly, 2018 pension balance for this country is unavailable but to be best approximated using 2019 official data (Notes h, i and * of [Table C.1](#)).

[Table C.2](#) documents the results of the abovementioned data recovery process.

C.3 Standardisation, Weights and FKI

Following Oliver-Márquez et al. ([2020](#))’s procedure, all series in [Table C.2](#) undergo min-max normalisation such that the smallest entry receives a new score of 0.01 and the biggest number is re-coded to 0.99. This slight deviation from the original paper (where the min-max normalisation yields 0 to 1) is to avoid multiplying a series by zero or raising a base to the power of zero.

Variable weights are calculated following Oliver-Márquez et al. ([2020](#))’s recipe to be the inverses of each series’ standard deviations. Whereas a sub-index combines more than one series, each weight is further divided by the sum of the constituent weights so that total weights add to one.

FKI is finally computed by taking the geometric mean of all four sub-indices, subject to sub-index-weights similar to variable weights above, as presented in [Table C.3](#).

Table C.3
FKI and Sub-indices

	FKI	EC	ET	Use	Need
NLD	0.940	0.939	0.640	1.805	1.000
USA	0.937	0.990	0.589	0.856	1.406
CAN	0.784	0.858	0.409	1.637	0.953
ITA	0.762	0.767	0.602	1.069	0.807
FIN	0.724	0.850	0.685	1.127	0.562
ESP	0.627	0.735	0.464	0.635	0.726
LTU	0.613	0.664	0.632	0.243	0.836
PRT	0.591	0.639	0.401	0.630	0.762
BGR	0.583	0.396	0.760	0.384	0.729
EST	0.577	0.672	0.746	0.266	0.575
SVK	0.559	0.608	0.924	0.301	0.441
POL	0.555	0.595	0.699	0.286	0.572
LVA	0.550	0.573	0.633	0.161	0.795
CHL	0.544	0.449	0.302	0.761	0.908
RUS	0.450	0.536	0.597	0.083	0.639
GEO	0.424	0.141	0.547	0.210	0.997
SRB	0.423	0.249	0.500	0.193	0.742
PER	0.309	0.078	0.194	0.691	0.877
BRA	0.141	0.155	0.010	0.432	0.833
IDN	0.122	0.010	0.040	0.973	0.787

Note. Table sorted in descending order by countries' FKI. FKI = financial knowledge index, EC = Economic Capability, ET = Educational Training.

Appendix D Review of Matrix Calculus

D.1 Notations

Let us first establish the notation. This is important because bad notation is a serious obstacle to elegant mathematics and coherent exposition and it can be misleading.

Unless specified otherwise, φ denotes a scalar function; \mathbf{f} a vector function and \mathbf{F} a matrix function. Also, x denotes a scalar argument, \mathbf{x} a vector argument and \mathbf{X} a matrix argument. For example, we write

$$\begin{aligned}\varphi(x) &= x^2 & \varphi(\mathbf{x}) &= \mathbf{a}^\top \mathbf{x} & \varphi(\mathbf{X}) &= \text{tr}(\mathbf{X}^\top \mathbf{X}) \\ \mathbf{f}(x) &= \begin{pmatrix} x \\ x^2 \end{pmatrix} & \mathbf{f}(\mathbf{x}) &= \mathbf{A}\mathbf{x} & \mathbf{f}(\mathbf{X}) &= \mathbf{X}\mathbf{a} \\ \mathbf{F}(x) &= x^2 \mathbf{I}_m & \mathbf{F}(\mathbf{x}) &= \mathbf{x}\mathbf{x}^\top & \mathbf{F}(\mathbf{X}) &= \mathbf{X}^\top\end{aligned}$$

Since the prime notation $'$ may easily cause confusion between derivatives and transposes, preference is given to the Leibniz notation $\frac{d}{dx}$ for derivatives and $^\top$ for transposes—unless this system becomes too cumbersome, in which case $\mathbf{f}'(\mathbf{x})$ will denote derivatives and $\mathbf{f}(\mathbf{x})'$ for transposes.

D.2 Derivatives and differentials

D.2.1 Derivative

Definition D.2.1 (Derivatives). If \mathbf{f} is an $m \times 1$ vector function of an $n \times 1$ vector \mathbf{x} , then the *derivative* (or *Jacobian matrix*) of \mathbf{f} is the $m \times n$ matrix

$$\mathbf{D}\mathbf{f}(\mathbf{x}) := \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}^\top}, \quad (\text{D.1})$$

whose elements are the partial derivatives

$$\frac{\partial f_i(\mathbf{x})}{\partial x_j}, \text{ for } \begin{matrix} i = 1, \dots, m, \\ j = 1, \dots, n. \end{matrix}$$

D.2.2 Differential

In the one dimensional case, the equation

$$\lim_{u \rightarrow 0} \frac{\varphi(x+u) - \varphi(x)}{u} = \varphi'(x) \quad (\text{D.2})$$

defines the derivative of φ at x . Rewriting [Equation \(D.2\)](#) gives

$$\varphi(x+u) = \varphi(x) + \varphi'(x)u + O(u), \quad (\text{D.3})$$

where the remainder term $O(u)$ quickly vanishes as u approaches 0.

Definition D.2.2 (Differential). We define the (first) *differential* of φ at x (with increment u) as

$$d\varphi(x; u) = \varphi'(x)u. \quad (\text{D.4})$$

For example, for $\varphi(x) = x^2$, we have $d\varphi(x; u) = 2xu$. In practice, we write dx instead of u , so that $d\varphi(x) = \varphi'(x)dx = 2xdx$.

In the vector case, similar to Equation (D.3), we have

$$\mathbf{f}(\mathbf{x} + \mathbf{u}) = \mathbf{f}(\mathbf{x}) + [\mathbf{D}\mathbf{f}(\mathbf{x})]\mathbf{u} + O(\mathbf{u}), \quad (\text{D.5})$$

and the (first) differential is defined as

$$d\mathbf{f}(\mathbf{x}; \mathbf{u}) = [\mathbf{D}\mathbf{f}(\mathbf{x})]\mathbf{u}. \quad (\text{D.6})$$

Although rarely used in econometrics, for completeness, the matrix case can be obtained from the vector case by writing $\mathbf{f} := \text{vec}(\mathbf{F})$ and $\mathbf{x} := \text{vec}(\mathbf{X})$.

D.2.3 Which to use?

For practical rather than theoretical reasons, the treatment of matrix calculus is based on differentials ($d\mathbf{f}$) rather than derivatives ($\mathbf{D}\mathbf{f}$) because the former yields a result with the same dimension as \mathbf{f} . For example, consider $\mathbf{f} \left(\begin{smallmatrix} \mathbf{x} \\ m \times 1 \quad n \times 1 \end{smallmatrix} \right)$ (reading “ \mathbf{f} being an $m \times 1$ vector function of an $n \times 1$ vector \mathbf{x} ”), $\mathbf{D}\mathbf{f}(\mathbf{x})$ is an $m \times n$ matrix (due to Definition D.2.1) whereas $d\mathbf{f}(\mathbf{x})$ remains an $m \times 1$ vector (same as \mathbf{f}). The advantage is even larger for matrices: for $\mathbf{F} \left(\begin{smallmatrix} \mathbf{X} \\ m \times p \quad n \times q \end{smallmatrix} \right)$, $d\mathbf{F}(\mathbf{X})$ has the same dimension as \mathbf{F} irrespective of the dimension of \mathbf{X} , but $\mathbf{D}\mathbf{F}(\mathbf{X})$ is going to be a horrendous $mp \times nq$ matrix.

D.3 Layout convention

Under the *numerator layout*, when we differentiate a scalar function φ with respect to a column vector $\mathbf{x} \left(\begin{smallmatrix} n \times 1 \end{smallmatrix} \right)$, we get a row vector of dimension $1 \times n$. If we want our result to be in the column form, we must differentiate φ with respect to a row vector to start with. This is why the denominator in Equation (D.1) contains a transpose.

D.4 Application in OLS

D.4.1 Background

Imagine we are interested in learning the return on education. We might propose a rather simple model

$$\text{inc} = \beta_0 + \beta_1 \text{edu} + \beta_2 \text{exp} + \varepsilon \quad (\text{D.7})$$

where inc is one’s income, edu and exp denote years of formal education and years spent in the labour market, respectively.

We managed to collect survey data from n respondents and organised this information in the following system of equations:

$$\begin{cases} \text{inc}_1 = \beta_0 + \beta_1 \text{edu}_1 + \beta_2 \text{exp}_1 + \varepsilon_1 \\ \text{inc}_2 = \beta_0 + \beta_1 \text{edu}_2 + \beta_2 \text{exp}_2 + \varepsilon_2 \\ \dots \\ \text{inc}_n = \beta_0 + \beta_1 \text{edu}_n + \beta_2 \text{exp}_n + \varepsilon_n \end{cases} \quad (\text{D.8})$$

This system of linear equations can be represented in the matrix notation using

$$\underset{n \times 1}{\mathbf{y}} = \begin{pmatrix} \text{inc}_1 \\ \text{inc}_2 \\ \dots \\ \text{inc}_n \end{pmatrix}, \underset{n \times 3}{\mathbf{X}} = \begin{pmatrix} 1 & \text{edu}_1 & \text{exp}_1 \\ 1 & \text{edu}_2 & \text{exp}_2 \\ \dots & \dots & \dots \\ 1 & \text{edu}_n & \text{exp}_n \end{pmatrix}, \underset{3 \times 1}{\boldsymbol{\beta}} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}, \text{ and } \underset{n \times 1}{\boldsymbol{\varepsilon}} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dots \\ \varepsilon_n \end{pmatrix} \quad (\text{D.9})$$

as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}. \quad (\text{D.10})$$

D.4.2 Ordinary least squares

The objective of OLS is to minimise the *sum of squared* error terms. A handy way of representing sum of squared ε is

$$\text{SSE} = \sum_{i=1}^n \varepsilon_i^2 = \varepsilon_1^2 + \varepsilon_2^2 + \dots + \varepsilon_n^2 = (\varepsilon_1 \quad \varepsilon_2 \quad \dots \quad \varepsilon_n) \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dots \\ \varepsilon_n \end{pmatrix} = \boldsymbol{\varepsilon}^\top \boldsymbol{\varepsilon}. \quad (\text{D.11})$$

In fact, $\mathbf{x}^\top \mathbf{x}$ is the mathematical translation of “sum of squared” of \mathbf{x} .

Now we are ready to continue. We want to carefully choose a combination of β_0 , β_1 and β_2 in order to make SSE as small as possible, ie

$$\min_{\boldsymbol{\beta}} \{\boldsymbol{\varepsilon}^\top \boldsymbol{\varepsilon}\} = \min_{\boldsymbol{\beta}} \{(\mathbf{y} - \mathbf{X}\boldsymbol{\beta})^\top (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})\} \quad (\text{D.12})$$

(the equal sign is due to [Equation \(D.10\)](#)).

Two observations can be made from the minimisation problem in [Equation \(D.12\)](#):

1. both \mathbf{y} and \mathbf{X} are collected data therefore can no longer be changed by the researcher; but we are free to adjust $\boldsymbol{\beta}$ in whatever way we want, meaning $\boldsymbol{\beta}$ is the “independent variable” and SSE is a function of $\boldsymbol{\beta}$, and
2. $\boldsymbol{\varepsilon}^\top \boldsymbol{\varepsilon}$ is a scalar function (please verify).

Then,

$$\begin{aligned} \varphi(\boldsymbol{\beta}) &= \boldsymbol{\varepsilon}^\top \boldsymbol{\varepsilon} = (\mathbf{y} - \mathbf{X}\boldsymbol{\beta})^\top (\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) \\ &= (\mathbf{y}^\top - \boldsymbol{\beta}^\top \mathbf{X}^\top) (\mathbf{y} - \mathbf{X}\boldsymbol{\beta}) \\ &= \mathbf{y}^\top \mathbf{y} - \mathbf{y}^\top \mathbf{X}\boldsymbol{\beta} - \boldsymbol{\beta}^\top \mathbf{X}^\top \mathbf{y} + \boldsymbol{\beta}^\top \mathbf{X}^\top \mathbf{X}\boldsymbol{\beta} \end{aligned} \quad (\text{D.13})$$

We now differentiate $\varphi(\boldsymbol{\beta})$ with respect to $\boldsymbol{\beta}$:

$$\begin{aligned} \frac{d\varphi(\boldsymbol{\beta})}{d\boldsymbol{\beta}} &= -\mathbf{y}^\top \mathbf{X} - \frac{d}{d\boldsymbol{\beta}} \left[(\boldsymbol{\beta}^\top \mathbf{X}^\top \mathbf{y})^\top \right] + \boldsymbol{\beta}^\top \mathbf{X}^\top \mathbf{X} + \frac{d}{d\boldsymbol{\beta}} \left[(\boldsymbol{\beta}^\top \mathbf{X}^\top \mathbf{X}\boldsymbol{\beta})^\top \right] \\ &= -\mathbf{y}^\top \mathbf{X} - \frac{d}{d\boldsymbol{\beta}} [\mathbf{y}^\top \mathbf{X}\boldsymbol{\beta}] + \boldsymbol{\beta}^\top \mathbf{X}^\top \mathbf{X} + \frac{d}{d\boldsymbol{\beta}} [\boldsymbol{\beta}^\top \mathbf{X}^\top \mathbf{X}\boldsymbol{\beta}] \\ &= -\mathbf{y}^\top \mathbf{X} - \mathbf{y}^\top \mathbf{X} + \boldsymbol{\beta}^\top \mathbf{X}^\top \mathbf{X} + \boldsymbol{\beta}^\top \mathbf{X}^\top \mathbf{X} \\ &= -2\mathbf{y}^\top \mathbf{X} + 2\boldsymbol{\beta}^\top \mathbf{X}^\top \mathbf{X} \end{aligned} \quad (\text{D.14})$$

(We were able to liberally apply transpose to terms containing $\boldsymbol{\beta}^\top$ and not to others because φ is a scalar function and each term in it must also be 1×1 in dimension, whose transpose must be equal to itself.)

Apply first order condition to Equation (D.14). An optimal $\hat{\beta}$ must satisfy

$$\begin{aligned}
-2\mathbf{y}^\top \mathbf{X} + 2\hat{\beta}^\top \mathbf{X}^\top \mathbf{X} &= \mathbf{O} \\
2\hat{\beta}^\top \mathbf{X}^\top \mathbf{X} &= 2\mathbf{y}^\top \mathbf{X} \\
\hat{\beta}^\top \mathbf{X}^\top \mathbf{X} &= \mathbf{y}^\top \mathbf{X} \\
\left(\hat{\beta}^\top \mathbf{X}^\top \mathbf{X}\right)^\top &= \left(\mathbf{y}^\top \mathbf{X}\right)^\top \\
\mathbf{X}^\top \mathbf{X} \hat{\beta} &= \mathbf{X}^\top \mathbf{y} \\
\hat{\beta} &= \left(\mathbf{X}^\top \mathbf{X}\right)^{-1} \mathbf{X}^\top \mathbf{y}
\end{aligned} \tag{D.15}$$

Notice that another transpose was applied to Line 4 of Equation (D.15) in order to correct $\hat{\beta}^\top$ (due to Section D.3) back to its column form $\hat{\beta}$. In fact, it would be better to do $\frac{d\varphi(\beta)}{d\beta^\top}$ in Equation (D.14) to avoid this later flipping. But the downside of this approach is a pedagogical one: most students would find differentiating with respect to β^\top out of blue while with respect to β is much more natural. In further derivations, $\frac{d\varphi(\beta)}{d\beta^\top}$ will be used.

Derivative of quadratic forms

The derivative of a quadratic form $q(\mathbf{x}) = \mathbf{x}^\top \mathbf{A} \mathbf{x}$ is

$$\frac{dq}{d\mathbf{x}} = \mathbf{x}^\top (\mathbf{A} + \mathbf{A}^\top),$$

which can be further simplified to $dq/d\mathbf{x} = 2\mathbf{x}^\top \mathbf{A}$, if \mathbf{A} is symmetric.

Name the expression in the facebook post φ , which is a function of β :

$$\varphi(\beta) = \frac{1}{\sigma^2} (\mathbf{y} - \mathbf{X}\beta)^\top \Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} (\mathbf{y} - \mathbf{X}\beta).$$

Typo

There is a typo in the original post: all Ω should be in the inverse form Ω^{-1} , including the one sandwiched between \mathbf{Z}^\top and \mathbf{Z} .

Scalar function

Note that φ is a scalar function:

$$\varphi \left(\begin{matrix} \beta \\ 1 \times 1 \end{matrix} \right) = \frac{1}{\sigma^2} \left(\begin{matrix} \mathbf{y} & - & \mathbf{X} & \beta \\ n \times 1 & & n \times k & k \times 1 \end{matrix} \right)^\top \begin{matrix} \Omega^{-1} \\ n \times n \end{matrix} \mathbf{Z} \begin{matrix} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \\ k \times n \quad n \times n \quad n \times k \end{matrix} \mathbf{Z}^\top \Omega^{-1} \begin{matrix} \left(\mathbf{y} - \mathbf{X} \beta \right) \\ n \times 1 \quad n \times k \quad k \times 1 \end{matrix}.$$

When differentiating a scalar function φ with respect to a column vector β , the result is a *row* vector. If this is undesirable, differentiate the scalar function φ with respect to the *transpose* of that vector β^\top .

I want to know

$$\frac{d\varphi}{d\beta} = \frac{d\varphi}{d(\mathbf{y} - \mathbf{X}\beta)} \frac{d(\mathbf{y} - \mathbf{X}\beta)}{d\beta} = \frac{d\varphi}{d(\mathbf{y} - \mathbf{X}\beta)} (-\mathbf{X}),$$

so I first calculate (using the result from quadratic form derivatives stated at the beginning)

$$\frac{d\varphi}{d(\mathbf{y} - \mathbf{X}\beta)} = \frac{2}{\sigma^2} (\mathbf{y} - \mathbf{X}\beta)^\top \left[\Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \right].$$

Therefore,

$$\begin{aligned} \frac{d\varphi}{d\beta} &= -\frac{2}{\sigma^2} (\mathbf{y} - \mathbf{X}\beta)^\top \left[\Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \right] \mathbf{X} \\ \frac{d\varphi}{d\beta^\top} &= -\frac{2}{\sigma^2} \mathbf{X}^\top \left[\Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \right] (\mathbf{y} - \mathbf{X}\beta) \end{aligned}$$

(The second line is to avoid working with row vectors.)

Second derivative

The second derivative of φ is

$$\frac{d^2\varphi}{d\beta^\top d\beta} = \frac{2}{\sigma^2} \mathbf{X}^\top \Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \mathbf{X},$$

which is a positive definite $k \times k$ matrix (another quadratic form). This implies that the result from the first order condition below is a minimum.

Impose the first order condition:

$$\begin{aligned} \mathbf{X}^\top \Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \left(\mathbf{y} - \mathbf{X} \hat{\boldsymbol{\beta}}_{\text{GLS-IV}} \right) &= \mathbf{0} \\ \mathbf{X}^\top \Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \mathbf{y} &= \mathbf{X}^\top \Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \mathbf{X} \hat{\boldsymbol{\beta}}_{\text{GLS-IV}} \end{aligned}$$

Therefore:

$$\begin{aligned} \hat{\boldsymbol{\beta}}_{\text{GLS-IV}} &= \left(\mathbf{X}^\top \Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \mathbf{X} \right)^{-1} \mathbf{X}^\top \Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \mathbf{y} \\ &= \left(\mathbf{X}^\top \Omega^{-1} \hat{\mathbf{X}} \right)^{-1} \mathbf{X}^\top \Omega^{-1} \hat{\mathbf{y}} \\ &\neq \left(\hat{\mathbf{X}}^\top \Omega^{-1} \hat{\mathbf{X}} \right)^{-1} \hat{\mathbf{X}}^\top \Omega^{-1} \mathbf{y}, \text{ satisfying the claim in the facebook post.} \end{aligned}$$

However, since $\hat{\mathbf{X}}$ is the GLS-IV-estimator of \mathbf{X} onto the \mathbf{Z} -space:

$$\begin{aligned} \hat{\mathbf{X}} &= \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \mathbf{X}, \text{ and} \\ \hat{\mathbf{X}}^\top &= \mathbf{X}^\top \Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top. \end{aligned}$$

The last expression in the facebook post then becomes:

$$\begin{aligned} &\left(\hat{\mathbf{X}}^\top \Omega^{-1} \hat{\mathbf{X}} \right)^{-1} \hat{\mathbf{X}}^\top \Omega^{-1} \mathbf{y} \\ &= \left(\mathbf{X}^\top \Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \mathbf{X} \right)^{-1} \mathbf{X}^\top \Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \mathbf{y} \\ &= \left(\mathbf{X}^\top \Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \mathbf{X} \right)^{-1} \mathbf{X}^\top \Omega^{-1} \mathbf{Z} \left(\mathbf{Z}^\top \Omega^{-1} \mathbf{Z} \right)^{-1} \mathbf{Z}^\top \Omega^{-1} \mathbf{y} \\ &= \hat{\boldsymbol{\beta}}_{\text{GLS-IV}} \end{aligned}$$

After all, “one might have guessed” correctly!