Identifying School Climate Variables Associated with Students' Financial Literacy Outcomes

A Cross-Country Comparison Using PISA 2018 Data

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敬致父母

To my parents

Study hard what interests you the most in the most undisciplined, irreverent and original manner possible.

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Acknowledgement

Thank-you goes to

Popular Abstract

Preparing youth for life-long success with numeracy, literacy and science capability has been the foundation for all schooling. The post-global financial crises and post-COVID era, in addition, imposes increasing demand on financial literacy for school leavers. Since 2012, OECD has been a pioneer in measuring 15-year-olds' financial literacy through its triennial Programme for International Student Assessment (PISA) cycles. Subsequent analyses using PISA data, however, reported paradoxical results that more classroom interventions tend to be associated with either no differences or even lower scores in students' financial literacy measures. It is in educators' interest therefore to carefully examine the relationship between school climate variables, including the academic, safety, community and institutional envrionment aspects of students' lives, and their financial literacy outcomes.

Abstract

Repeated economic crises in recent memory have exposed the harsh consequences of financial *illiteracy* shared by high proportions of the general population. Policy makers experienced little resistance when identifying youth as the most effective group for bringing about improvement in citizens' ability to engage with economic and financial matters, but opinions quickly diverge over the optimal approaches for achieving such targeted outcome. Existing literature frequently reports the importance of family environment in cultivating students' financial literacy through the process of "financial socialisation" – [definition goes here] (reference). Such practice, however, encounters interrogation by educators over equity concerns should families remain the main arena for financial literacy development. Schools play vital roles in alleviating inequality in accessing education and training in general but scarce research so far has been devoted into identifying the specific classroom factors that are most effective in advancing students' financial literacy outcomes. The current study therefore attempts to contribute to this enquiry by investigating the relationship between school climate variables and students' financial literacy achievement with an aim of stimulating policy debate about the levers and instruments available to education interventionists for the purpose of improving young people's financial literacy and preparedness as they step into an increasingly uncertain world. Using the 2018 PISA dataset, this paper employs a three-level hierarchical model to conduct cross-country comparisons to highlight school climate variables that are most strongly associated with high financial literacy outcomes.

Chapter 1 Introduction

Repeated economic crises in recent memory have exposed the harsh consequences of financial *illiteracy* shared by high proportions of the general population. Low levels of financial literacy are observed not only in less developed countries such as India and Indonesia (Cole et al., 2009) but also in advanced economies such as the USA (Huston, 2012), Germany (Bucher-Koenen et al., 2016) and OECD countries (Lusardi, 2015). Amongst the many redress schemes aimed at promoting citizens' financial capability, the return on investment is the highest when intervention is applied early in life. Lusardi and Mitchell (2014) have shown that providing financial knowledge to the least educated before they enter the labour market increases their well-being by approximately 82% of their initial wealth, while the rate of return is around 56% for college graduates—results that are significant both statistically and economically.

Research efforts aiming at advancing youth's financial literacy over the years evolved into two strands: on the design and evaluation of school financial education programs, and on the influence of home environment through the process of financial socialisation—the intentional or involuntary transmission of financial concepts which are required to functioning successfully in society (Bowen, 2002). A recent meta-analysis conducted by Kaiser and Menkhoff (2020) found that while school financial education programs had sizeable impacts on *financial knowledge* (+0.33 SD) similar to education interventions in other domains, their effect on students' *financial behaviour* is quite small (+0.07 SD). This conclusion added to a list of weak or non-findings regarding the long-term behavioural effect brought about by school financial education programs. Brown et al. (2016), for instance, reported mixed outcome in students' long-term financial well-being depending on the programs received; whereas Cole et al. (2016) observed that traditional personal finance courses lacked any explanatory power in accounting for graduates' financial outcome once the additional mathematics training in which finance topics were packaged has been controlled for. Despite careful controls and thoughtful study designs, correlating classroom interventions and young people's financial literacy outcomes has repeatedly yielded paradoxical results of non-significant or even negative relationship; any positive findings remain small in magnitudes and/or are sensitive to robust analyses.

Optimism, fortunately, runs higher at the financial socialisation camp. Building on the

acknowledgement that families serve as information filters from the outside world (Danes & Haberman, 2007) as well as the foundation for youth's continued financial concept formation, Gudmunson and Danes (2011) put forward a family financial socialisation theory to accommodate both the *process* and the *outcome* for variations in young people's financial capabilities. Using structural equation modelling, Jorgensen and Savla (2010) was able to show that perceived parental influence had a direct and moderately significant influence on financial attitude, did *not* have an effect on *financial knowledge*, and had an indirect and moderately significant influence on financial behaviour, mediated through financial attitude. This attitude(A)–behaviour(B) –cognition(C) conceptualisation of financial literacy (Potrich et al., 2015) continues to influence subsequent research effort. More recently, Moreno-Herrero et al. (2018) continued this line of enquiry by applying multilevel regression analyses to 2015 PISA data and reported that students' financial literacy was associated mainly with understanding the value of saving and discussing money matters with parents. In addition, exposure and use of financial products, in particular holding a bank account, improved students' financial knowledge as well.

One chief concern for every research project is the quality of its data source. Amongst competing inventories, PISA stands out as a comprehensive and reliable source of data for measuring 15-year-olds' financial literacy outcomes thanks to OECD's careful sampling procedure and attention to construct validity of measurement. Following statistical theory, PISA designers firstly recognise the hierarchical nature of education research data such that students are nested in schools, and schools are further nested in countries. In addition, one student weight is assigned to each observation in order to account for the fact that not all schools in a country are equally likely to be sampled by the PISA organiser; and given a particular school that has been chosen, not every student in this school is equally likely to be asked to participate in the test (Rust, 2014). A third complication arises from the "planned missingness" in students' responses because each participant is only given a small number of questions relative to the entire test bank in order to ensure their responses are not undermined by tiredness (von Davier, 2014), leading to the outcome variables being represented by ten plausible values. Lastly, PISA consulted and synthesised multiple schools of thoughts (OECD, 2019) before constructing financial literacy as

> the knowledge and understanding of financial concepts and risks, and the skills, motivation and confidence to apply such knowledge and understanding in order to make effective decisions across a range of financial contexts, to improve the

financial well-being of individuals and society, and to enable participation in economic life. (p. 128)

As a result, 2018 PISA data set (OECD, 2020) provides not only variables measuring *cognitive* outcomes but also *affective* factors such as familiarity with concepts of finance and confidence about financial matters, enabling a nuanced study design involving decomposing the total effect of financial literacy development into its "brain" (cognitive) and "heart" (affective) pathways.

The current study wishes to take advantage of the latest wave of 2018 PISA results and investigate the covariation financial literacy outcomes share with the following four aspects of young people's daily lives, inspired by school climate literature (Wang & Degol, 2016): (a) academic training, including any financial education programs received at schools; (b) safety perception about their schools; (c) financial socialisation experienced at home; and (d) their schools' resource endowment. More specifically, this project aims to answer these three research questions:

RQ1. Having controlled for demographic characteristics such as socio-economic status, sex and immmigration history, to what extent can the variation in students' financial literacy outcomes be accounted for by each of the school climate variables mentioned above.

RQ2. How does the total effect each school climate variable carries decompose into cognitive and affect pathways.

RQ3. How do these effects differ across countries.

Chapter 2 Conceptual Framework

2.1 In-depth definitions of "financial literacy"

2.1.1 Every term my readers need in order to understand my research question

2.1.2 Survey not only PISA but also alternative definitions, even critiques of such definitions

2.1.3 Any practices that are common in maths/literature but uncommon in financial literacy? Meaning? Implies?

2.2 Country-level Financial Knowledge Index

PISA 2018 financial literacy dataset (OECD, 2020) provides rich information about students and schools. For the purpose of cross-country comparison, however, the country-level financial literacy information must be addressed separately by the researchers. Earlier attempts such as Moreno-Herrero et al. (2018) approximated this information using a variable "quality of math and science education" to control for country-level differences since consensus is yet to emerge about the most appropriate measure for countries' financial knowledge. Inspired by the UN's approach to forming Human Development Indices, a recent publication by Oliver-Márquez et al. (2020) proposed a macroeconomic measure for countries' general financial knowledge levels by examining their economic capability, educational training, existing practices in the financial markets as well as incentives to interact with financial products. More specifically, the authors considered a country's economic capability, represented by its GDP per capita, to be a key dimension in bringing about its financial knowledge index (FKI). Secondly, literature converges on the importance of educational training for a country's financial knowledge capability (OECD, 2005). Thirdly, countries with regular engagement with sophisticated financial products and financial markets should possess higher FKI. Lastly, countries with higher aggregate consumption levels and with ageing populations are likely to possess higher FKI due to more frequent exposure and pressure in retirement provision, respetively. Macroeconomic data needed for these computations can be sourced from the World Bank (World Bank, 2020) and the United Nations' *Human Development Reports* (United Nations, 2020).

Combining individual and institutional data cources can be a productive approach in international large-scale assessment (ILSA) research. According to the framework for comparative education analyses (Bray & Thomas, 1995), this project extends education outcome measures to a country level, addresses the aspect of society and labour market, and relates countries' entire populations to ILSA research (Strietholt & Scherer, 2018). By combining education outcome data with countries' economic performance indicators, this project remains most comparable to Hanushek and Woessmann (2012)—while these authors looked into the relationship between countries' education achievement and their GDP growth, the current investigation highlights how countries' GDP, along with other macroeconomic practices, in turn systematically impacts on their youth's educational performance.

Table 2.1

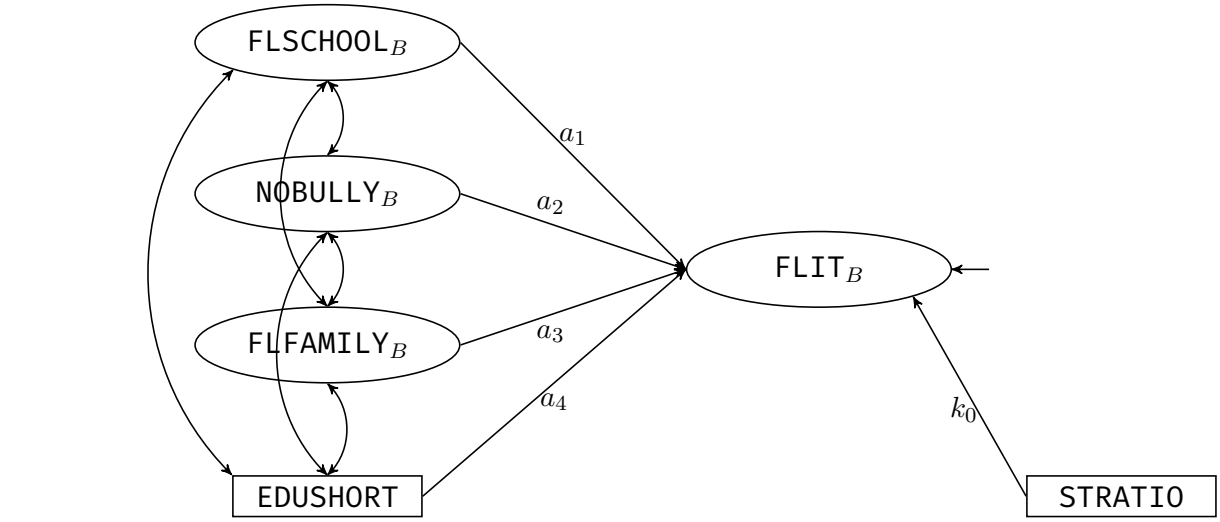
Percentages of Missing Values

CNT	MALE	IMMI1GEN	IMMI2GEN	ESCS	FCFMLRTY	FLCONFIN	PERFEED	TEACHINT	FLSCHOOL	DISCRIM†	BELONG	BULLY	FLFAMILY	CURSUPP†	PASCHPOL†	STRATIO	EDUSHT	STAFFSHT
BGR	0	6	6	3	12	27	10	10	21	28	19	31	22	100	100	8	3	3
BRA	0	5	5	2	12	34	9	8	21	36	23	40	24	17	19	12	6	7
CAN†	0	7	7	5	11	15	100	100	13	100	8	14	14	100	100	100	2	2
CHL	0	4	4	3	10	24	5	4	13	30	15	34	15	9	8	18	9	9
ESP	0	3	3	2	5	21	3	2	7	25	9	29	8	100	100	11	5	6
EST	0	3	3	3	4	8	3	3	6	9	5	11	6	100	100	0	0	0
FIN	0	2	2	2	4	10	3	3	6	100	6	11	7	100	100	2	7	7
GEO	0	5	5	2	9	26	9	9	17	100	15	22	21	4	5	1	2	2
IDN	0	3	3	1	3	6	3	2	5	3	2	5	5	100	100	23	14	14
ITA	0	4	4	3	7	17	4	4	10	23	10	27	12	16	17	9	3	3
LTU	0	3	3	3	4	12	3	3	5	17	8	20	7	100	100	0	0	0
LVA	0	2	2	2	5	9	3	3	6	14	6	15	7	100	100	6	3	4
NLD	0	3	3	2	3	5	3	2	4	100	4	8	4	100	100	11	5	5
PER	0	2	2	1	2	11	5	4	4	56	31	65	5	100	100	2	0	0
POL	0	1	1	1	3	7	2	1	5	9	3	11	5	100	100	0	0	0
PRT	0	6	6	5	8	11	6	6	10	15	8	17	10	10	10	11	1	1
RUS	0	3	3	2	8	13	5	4	11	13	8	15	11	100	100	3	3	3
SRB	0	3	3	1	10	25	8	7	18	25	15	27	19	100	100	8	1	1
SVK	0	2	2	1	4	12	4	3	7	14	6	17	8	100	100	6	6	7
USA	0	3	3	2	3	6	2	1	4	100	4	6	4	100	100	16	10	10

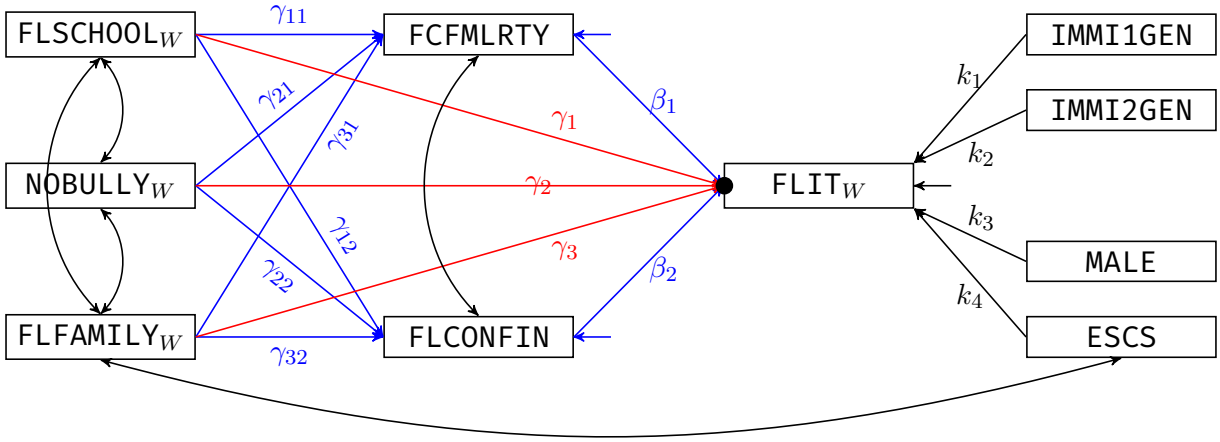
Note. Using shades of red in addition to numbers (measured in %), this table visualises the missing percentages by variable and by country. Variables DISCRIM, CURSUPP and PASCHPOL are no longer pursued in the model because too many countries chose not to respond to these questions. Canada (CAN) is not included due to 100 percent missings on multiple variables. † marks the country and variables that are excluded from subsequent analyses.

Figure 2.1
Path Diagram

***L2*: School**



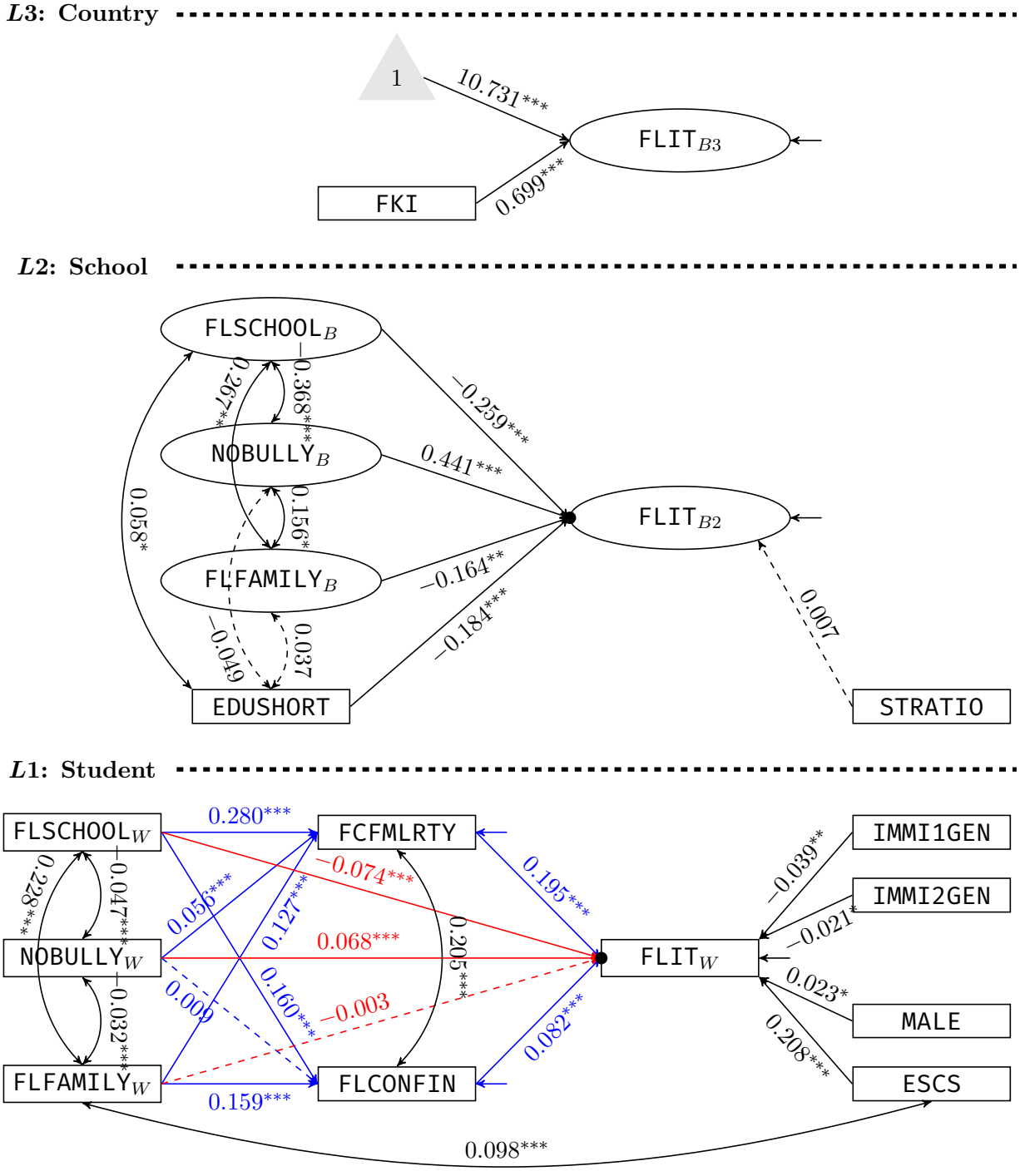
***L1*: Student**



Note. The direct and indirect paths of the structural component are represented in red and blue respectively.

Figure 2.2

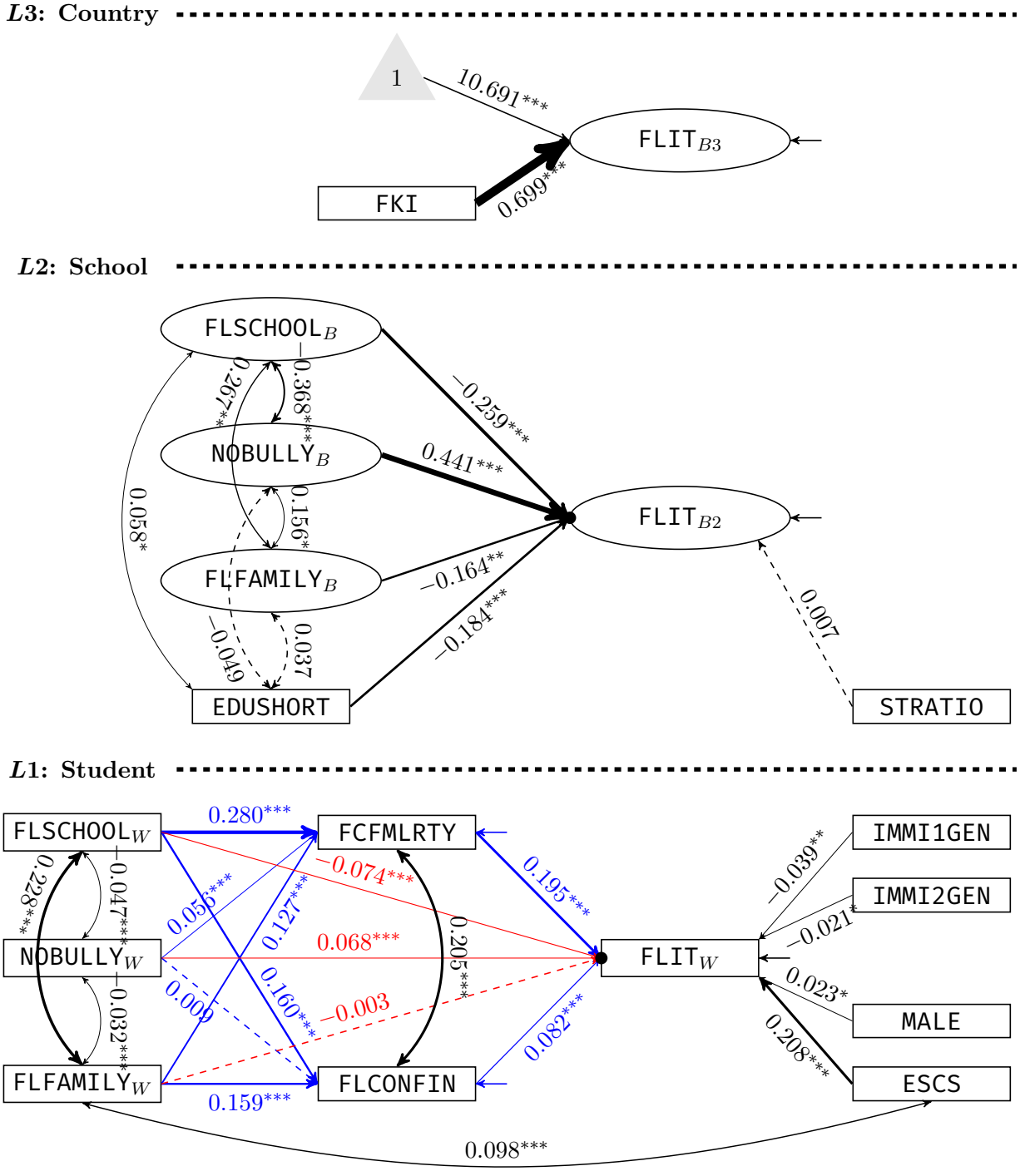
Three-level Structural Equation Model Predicting Youth's Financial Literacy Outcomes



Note. This multilevel structural equation model predicts youth's financial literacy outcomes using PISA 2018 data, with mediating effects of familiarity with, and confidence in, financial matters at student-level (direct and indirect pathways). Statistics are standardised regression coefficients. Dashed lines represent nonsignificant relations at $\alpha = 0.05$ level. FKI = financial knowledge indices, FLIT = financial literacy. Subscript $_W$ = within, $_B$ = between.
$^*p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

Figure 2.3

Three-level Structural Equation Model Predicting Youth's Financial Literacy Outcomes



Note. This multilevel structural equation model predicts youth's financial literacy outcomes using PISA 2018 data, with mediating effects of familiarity with, and confidence in, financial matters at student-level (direct and indirect pathways). Statistics are standardised regression coefficients. Dashed lines represent nonsignificant relations at $\alpha = 0.05$ level. FKI = financial knowledge indices, FLIT = financial literacy. Subscript $_W$ = within, $_B$ = between.
$^*p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

Chapter 3 Methods

3.1 Data / Sample / Participants

This study drew its primary data soruce from PISA 2018 database (OECD, 2020) containing 107,174 observations spanning 20 countries, in which students were asked about their demographic background, family lives and school experiences. For the financial literacy section, in particular, students responded to qustions about their familiarity with concepts of finance (`FCFMLRTY`), confidence about financial matters (`FLCONFIN`), their classroom (`FLSCHOOL`) as well as parental (`FLFAMILY`) involvement in matters of fianncial literacy. Ten plausible values were subsequently generated by PISA organisers as measures of students' financial literacy outcomes and were used as the dependent variable (`FLIT`).

Student-level independent variables are

School-level independent variables are

Country-level independent variables are

Missing data are handled using Mplus's multiple imputation procedure with ten imputations generated and pooled subsequently following Rubin's Rule (Rubin, 1976).

A three-level multigroup structural equation model was employed to account for the hiearchical structure of the PISA design, with private versus public school as the grouping variable.

3.2 Measurement of financial literacy

3.2.1 Background questions

3.2.2 Students' motivation of spending money

3.2.3 Four-point Likert scale

3.2.4 Averages

3.3 Model

The proposed model therefore is:

Student-level (L_1):

$$\begin{aligned}
\texttt{FCFMLRTY}_{ij} &= \gamma_{11}\texttt{FLSCHOOL}_{ij} + \gamma_{21}\texttt{NOBULLY}_{ij} + \gamma_{31}\texttt{FLFAMILY}_{ij} + r_{ij}^{M_1} \\
\texttt{FLCONFIN}_{ij} &= \gamma_{12}\texttt{FLSCHOOL}_{ij} + \gamma_{22}\texttt{NOBULLY}_{ij} + \gamma_{32}\texttt{FLFAMILY}_{ij} + r_{ij}^{M_2} \\
\texttt{FLIT}_{ij} &= \alpha_{0j} + \beta_1\texttt{FCFMLRTY}_{ij} + \beta_2\texttt{FLCONFIN}_{ij} \\
&\quad + \gamma_1\texttt{FLSCHOOL}_{ij} + \gamma_2\texttt{NOBULLY}_{ij} + \gamma_3\texttt{FLFAMILY}_{ij} \\
&\quad + k_1\texttt{IMMI1GEN}_{ij} + k_2\texttt{IMMI2GEN}_{ij} + k_3\texttt{MALE}_{ij} + k_4\texttt{ESCS}_{ij} + r_{ij}^{Y_W}
\end{aligned} \tag{3.1}$$

School-level (L_2):

$$\begin{aligned}
\alpha_{0j} &= \alpha_{00} + a_1\texttt{FLSCHOOL}_j + a_2\texttt{NOBULLY}_j + a_3\texttt{FLFAMILY}_j + a_4\texttt{EDUSHTG}_j \\
&\quad + k_0\texttt{STRATIO}_j + \varepsilon_j^{Y_B}
\end{aligned} \tag{3.2}$$

Using Kaplan's (2009) notation $\boldsymbol{y}_{ij} = \boldsymbol{\alpha}_j + \mathbf{B}_j\boldsymbol{y}_{ij} + \boldsymbol{\Gamma}_j\boldsymbol{x}_{ij} + \text{controls}^W + \boldsymbol{r}_{ij}$ for student-level (L_1) and random intercept $\alpha_{0j} = \alpha_{00} + \boldsymbol{a}\boldsymbol{w}_j + \text{control}^B + \varepsilon_j$ for school-level (L_2):

$$\begin{aligned}
\begin{bmatrix} \texttt{FCFMLRTY}_{ij} \\ \texttt{FLCONFIN}_{ij} \\ \texttt{FLIT}_{ij} \end{bmatrix} &= \begin{pmatrix} 0 \\ 0 \\ \alpha_{0j} \end{pmatrix} + \begin{pmatrix} 0 & 0 & \beta_1 \\ 0 & 0 & \beta_2 \\ 0 & 0 & 0 \end{pmatrix}^{\intercal} \begin{bmatrix} \texttt{FCFMLRTY}_{ij} \\ \texttt{FLCONFIN}_{ij} \\ \texttt{FLIT}_{ij} \end{bmatrix} \\
&\quad + \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_1 \\ \gamma_{21} & \gamma_{22} & \gamma_2 \\ \gamma_{31} & \gamma_{32} & \gamma_3 \end{pmatrix}^{\intercal} \begin{bmatrix} \texttt{FLSCHOOL}_{ij} \\ \texttt{NOBULLY}_{ij} \\ \texttt{FLFAMILY}_{ij} \end{bmatrix} + \begin{pmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \end{pmatrix}^{\intercal} \begin{bmatrix} \texttt{IMMI1GEN}_{ij} \\ \texttt{IMMI2GEN}_{ij} \\ \texttt{MALE}_{ij} \\ \texttt{ESCS}_{ij} \end{bmatrix} + \begin{pmatrix} r_{ij}^{M_1} \\ r_{ij}^{M_2} \\ r_{ij}^{Y_W} \end{pmatrix}
\end{aligned} \tag{3.3}$$

$$\alpha_{0j} = \alpha_{00} + \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ a_4 \end{pmatrix}^{\intercal} \begin{bmatrix} \texttt{FLSCHOOL}_j \\ \texttt{NOBULLY}_j \\ \texttt{FLFAMILY}_j \\ \texttt{EDUSHTG}_j \end{bmatrix} + k_0\texttt{STRATIO}_j + \varepsilon_j^{Y_B} \tag{3.4}$$

3.4 Country-level Financial Knowledge Index

This project closely follows Oliver-Márquez et al.'s (2020) procedure in developing country-level financial kowledge indices using four sub-indices: economic capability (`EC`), educational training (`ET`), existing practices in financial market (`Use`), and incentives (`Need`) to engage with financial products. The first sub-index `EC` is calculated using the logarithm of a country's GDP per capita in current international dollars (purchasing power parity adjusted). For the `ET` sub-index, a country's highly skilled workforce is represented by its postgraduate to total tertiary graduation ratio, and the mean years of schooling is used to measure its general education level. For the `Use` sub-index, gross portfolio equity assets (GPEA) and insurance company assets (ICA) are considered sophisticated financial products a country engages in. Additionally, in order to capture the central role of technology in amplifying the proliferation and use of financial assets, the proportion of a country's Internet users (IUS) enters the definition via

$$\texttt{Use} = (\text{GPEA} + \text{ICA})^{\text{IUS}}.$$

The final sub-index `Need` is compiled as

$$\texttt{Need} = (\text{PFA} + \text{AC})^{\text{AGEING}},$$

where PFA is the pension fund assets to GDP ratio. Aggregate consumption is defined as:

$$\text{AC} = \frac{2\% \times \text{household final consumption expenditure}}{\text{GDP}},$$

with the "2% rule" being drawn from Caliendo and Findley's (2013) derivation, and the proportion of ageing population is computed as

$$\text{AGEING} = \frac{\left[\frac{\text{population}(>65)}{\text{population}(20\sim64)}\right]_{2018} - \left[\frac{\text{population}(>65)}{\text{population}(20\sim64)}\right]_{2009}}{\left[\frac{\text{population}(>65)}{\text{population}(20\sim64)}\right]_{2009}}.$$

3.4.1 Data Collection and Missing Data Treatment

The data sources for FKI computation are documented in Table C.1 and its associated notes. Sub-indices `ET` and `Use` both contain missing observations for the year 2018. Majority of

such missing data appear to be the result of administrative delay, with historic observations available until 2017. It is therefore feasible to conduct time-series forecasts using prior year observations to best approximate 2018 values.

Table 3.1

Data Sources for FKI Computation

Database [a]	Country [b]	Series	Time
		Economic Capacity	
WB-dev	20	GDP per capita, PPP (current international $)	2018
		Educational Training	
WB-ed	20 \ Russia	Graduates from ISCED 7 programmes in tertiary education, both sexes (number)	2013–**2018**
		Graduates from ISCED 8 programmes in tertiary education, both sexes (number)	2013–**2018**
		Graduates from tertiary education, both sexes (number)	2013–**2018**
RS	Russia	PhD (Type 1) [c], PhD (Type 2) [d]	2018
RE	Russia	Master (Type 1) [e], Master (Type 2) [f], total tertiary *excluding* PhD [g]	2018
HDR	20	Dimension = Education; Education = Mean years of schooling (years)	2018
		Use	
WB-fin	20	Gross portfolio equity assets to GDP (%)	2011–**2018**
		Insurance company assets to GDP (%)	2011–**2018**
WB-dev	20	Individuals using the Internet (% of population)	2009–**2018**
		Need	
WB-fin	20 \ Georgia	Pension fund assets to GDP (%)	2008–**2018**
GP	Georgia	Minutes of the meeting of the investment board of the Pension Agency [h]	2019 [*]
GS	Georgia	GDP at current prices, billion GEL [i]	2018
WB-dev	20	Household and NPISHs final consumption expenditure, PPP (current international $)	2018
		GDP, PPP (current international $)	2018
		Population ages 0–14, male	2009, 2018
		Population ages 0–14, female	2009, 2018
		Population ages 15–64, male	2009, 2018
		Population ages 15–64, female	2009, 2018
		Population ages 65 and above, male	2009, 2018
		Population ages 65 and above, female	2009, 2018
		Population ages 15–19, male (% of male population)	2009, 2018
		Population ages 15–19, female (% of female population)	2009, 2018

Note. Sub-indices are shaded in gray. Bold font signifies this year contains missing data.

[a] WB-dev = World Bank – World development indicators
WB-ed = World Bank – Education statistics – All indicators
WB-fin = World Bank – Global financial development
HDR = Human Development Reports – Data
RS = Russian Federal State Statistic Service
RE = Russian Ministry of Education and Science
GP = Pension Agency of Georgia
GS = National Statistics Office of Georgia

[b] "20" = the 20 participating countries in 2018 PISA financial literacy test: Brazil, Bulgaria, Canada, Chile, Estonia, Finland, Georgia, Indonesia, Italy, Latvia, Lithuania, the Netherlands, Peru, Poland, Portugal, Russian Federation, Serbia, Slovak Republic, Spain, and the USA. "\" = exluding or except

[c] https://rosstat.gov.ru/storage/mediabank/asp-2(1).xls, Sheet "по направлениям подготовки", Cell C7 = number of PhD graduates (Type 1)

[d] https://rosstat.gov.ru/storage/mediabank/asp-3.xls, Sheet "по научным специальностям", Cell B7 = number of PhD graduates (Type 2)

[e–g] https://minobrnauki.gov.ru/common/upload/download/VPO_1_2018.rar contains a spreadsheet СВОД_ВПО1_ВСЕГО.xls, Sheet "P2_1_3(1)", Cell E198 = number of master graduates (Type 1)[e], Cell E410 = number of master graduates (Type 2)[f], Cell E592 = total tertiary graduates *excluding* PhD[g]

[h] Minutes of the meeting of the investment board of the Pension Agency, p. 4, no. 3

[i] Gross domestic product (GDP), row = GDP at current prices, billion GEL, column = 2018

[*] Georgia started a new pension system on 1 January 2019. Since 2018 was a transitional period with scarce data, 2019 is used as the best approximation for Georgia's pension system for 2018.

Sub-index **ET**

The 2018 archive for the number of master (ISCED 7), PhD (ISCED 8), and total tertiary graduates are incomplete for all participating countries except Georgia, Indonesia and Serbia. Figure C.1 presents a time series plot of

$$\text{SKILLED} = \frac{\text{number of masters} + \text{number of PhDs}}{\text{total number of tertiary graduates}}$$

Figure 3.1
Proportion of Postgraduates to Total Tertiary Graduations

Note. "Postgraduate" is defined as master (ISCED 7) and PhD (ISCED 8) graduates. Countries not shown: GEO, IDN and SRB (2018 data available) and RUS (consult other sources)

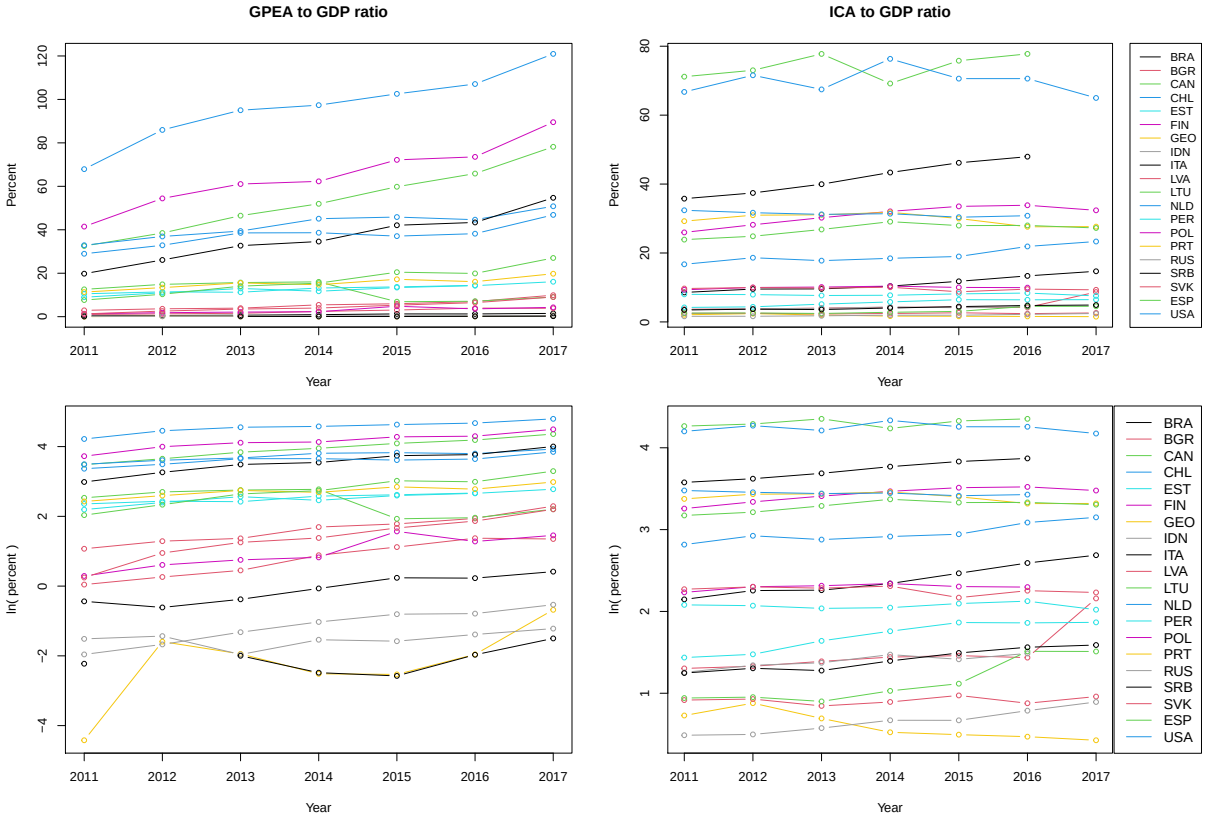
and suggests that this ratio is likely to be stable over time, especially between adjacent years. A "naive forecast", where the nearest available year's data are to be duplicated for 2018, is applied for SKILLED.

Sub-index **Use**

All series involved in calculating this sub-index, GPEA, ICA and IUS, contain missing data. When time series data contain only exponential growth but no underlying trend, a simple exponential smoothing would suffice (Garder, 1985); if trend is present, Holt-Winters method is superior (Chatfield, 1978). Figure C.2 facilitates this decision making by plotting both the original and log-transformed versions of GPEA and ICA series. Since curves after log-transformations have slopes, it is prudent to apply the Holt-Winters forecasting method in order to account for possible trends contained in the original series.

Figure 3.2

Time Series Trend Test



Note. The time series plots after natural logarithm transformations (bottom panels) are not flat, suggesting the original series (top panels) contain trends. Holt-Winters method therefore is preferred over simple exponential smoothing for 2018 forecasts.

The IUS series contains missing data for Canada, Chile and the United States. Similar Holt-Winters procedure is applied to recover 2018 IUS data.

Table 3.2
Data Utilised for Computing FKI

	Economic Capacity	Educational Training		Use			Need		
	GDP per capita	Skilled	Schooling	GPEA	ICA	IUS	PFA	AC	AGEING
BRA	9.612	6.484	7.8	1.683	16.259	70.434	11.827	1.210	0.288
BGR	10.026	45.294	11.8	4.114	7.044	64.782	13.577	1.091	0.234
CAN	10.821	15.832	13.3	84.010	77.728	93.588	96.205	1.068	0.271
CHL	10.117	16.371	10.4	51.755	25.591	89.531	73.225	1.073	0.214
EST	10.501	36.765	13.0	16.399	7.681	89.357	18.012	0.876	0.163
FIN	10.807	35.024	12.4	93.626	31.481	88.890	52.024	0.974	0.370
GEO	9.588	24.039	12.8	0.784	1.469	62.718	0.834	1.227	0.042
IDN	9.362	7.771	8.0	0.636	4.612	39.905	1.826	1.059	0.145
ITA	10.665	44.771	10.2	57.434	51.260	74.387	10.589	1.075	0.155
LVA	10.330	29.554	12.8	8.598	2.538	83.577	14.732	1.027	0.142
LTU	10.487	28.749	13.0	9.008	5.500	79.723	7.457	1.107	0.149
NLD	10.961	32.590	12.2	124.171	64.956	94.712	207.938	0.805	0.326
PER	9.479	13.577	9.2	16.027	6.505	52.540	22.530	1.187	0.227
POL	10.368	36.725	12.3	4.853	9.535	77.542	9.838	1.085	0.355
PRT	10.444	34.454	9.2	19.353	25.579	74.661	8.761	1.133	0.237
RUS	10.267	30.349	12.0	0.302	2.614	80.865	4.415	0.941	0.155
SRB	9.774	26.946	11.2	0.306	5.111	73.361	0.845	1.171	0.280
SVK	10.391	54.417	12.6	10.644	8.873	80.660	12.497	0.962	0.300
ESP	10.609	33.929	9.8	27.681	28.230	86.107	10.235	1.044	0.186
USA	11.048	24.825	13.4	55.505	30.183	84.881	150.040	1.364	0.252

Note. Full variable names: Skilled = Postgraduate to total tertiary ratio; Schooling = Mean year of schooling; GPEA = Gross portfolio to GDP ratio; ICA = Insurance company assets to GDP ratio; IUS = Number of Internet users per 100 population; PFA = Pension fund assets to GDP ratio; AC = 2% of household final consumption expenditure to GDP ratio; AGEING = Aged-to-productive-population ratio (% change between 2009 and 2018)

Other Items with Data Concerns

Russia reported 67.96% and 61.01% of its total university degree receipients to be postgraduates for the year 2013 and 2015 respectively (2014 missing). This figure rapidly declines to 41.6% in 2016 and further down to 25.69% in 2017. Such volatility goes against the stable patterns shared by most countries in Figure C.1, casting doubt on data reliability. Separate investigation is therefore conducted using Russian government archive (Notes c to g in Table C.1).

Georgia underwent pension reform in 2018 with fund balance gradually transitioning to State Pension Agency for its official resumption of duty on 1 January 2019. Resultantly, 2018 pension balance for this country is unavailable but to be best appoximated using 2019 official data (Notes h, i and * of Table C.1).

Table C.2 documents the results of the abovementioned data recovery process.

3.4.2 Standardisation, Weights and FKI

Following Oliver-Márquez et al. (2020)'s procedure, all series in Table C.2 undergo min-max normalisation such that the smallest entry receives a new score of 0.01 and the biggest number is re-coded to 0.99. This slight deviation from the original paper (where the min-max normalisation yields 0 to 1) is to avoid multiplying a series by zero or raising a base to the power of zero.

Variable weights are calculated following Oliver-Márquez et al. (2020)'s recipe to be the inverses of each series' standard deviations. Whereas a sub-index combines more than one series, each weight is further divided by the sum of the constituent weights so that total weights add to one.

FKI is finally computed by taking the geometric mean of all four sub-indices, subject to sub-index-weights similar to variable weights above, as presented in Table C.3.

Table 3.3
FKI and Sub-indices

	FKI	EC	ET	Use	Need
NLD	0.940	0.939	0.640	1.805	1.000
USA	0.937	0.990	0.589	0.856	1.406
CAN	0.784	0.858	0.409	1.637	0.953
ITA	0.762	0.767	0.602	1.069	0.807
FIN	0.724	0.850	0.685	1.127	0.562
ESP	0.627	0.735	0.464	0.635	0.726
LTU	0.613	0.664	0.632	0.243	0.836
PRT	0.591	0.639	0.401	0.630	0.762
BGR	0.583	0.396	0.760	0.384	0.729
EST	0.577	0.672	0.746	0.266	0.575
SVK	0.559	0.608	0.924	0.301	0.441
POL	0.555	0.595	0.699	0.286	0.572
LVA	0.550	0.573	0.633	0.161	0.795
CHL	0.544	0.449	0.302	0.761	0.908
RUS	0.450	0.536	0.597	0.083	0.639
GEO	0.424	0.141	0.547	0.210	0.997
SRB	0.423	0.249	0.500	0.193	0.742
PER	0.309	0.078	0.194	0.691	0.877
BRA	0.141	0.155	0.010	0.432	0.833
IDN	0.122	0.010	0.040	0.973	0.787

Note. Table sorted in descending order by countries' FKI. FKI = financial knowledge index, EC = Economic Capability, ET = Educational Training.

3.5 What exactly I was using to address my research question

3.5.1 Sum score? Averages? One particular question?

3.5.2 Factor loading? Latent variables?

3.5.3 Motivation for choosing these measures

3.6 Software and version

3.7 My models

3.7.1 Motivation for choosing this particular model

3.7.2 Refer to my research question

3.8 Estimators I obtained

3.8.1 Motivation why these estimators rather than others

3.9 Weights? Plausible values?

3.10 Missing data and how I treated missing data

3.11 Model comparison

3.12 Guidelines and indices

Chapter 4 Results

4.1 Descriptive statistics

4.2 Correlation matrices

4.2.1 Across countries

4.2.2 Across levels: Country | School | Students

4.3 Examination of measurement models

Table 4.1

Model Parameters and Fit Indices for Multilevel Regressions

Effect	Parameter [a]	Null model		1-level model		2-level saturated		2-level structured		3-level model	
		Coef	*SE*	Coef	*SE*	Coef	*SE*	Coef	*SE*	Coef	*SE*
FIXED EFFECTS											
Intercept	α_{00}	500.005	11.682***	451.806	1.781***	445.315	2.632***	489.884	4.568***	396.052	14.534***
Student-level Predictors											
FLSCHOOL (total)				−0.082	0.009***	−0.035	0.011**	−0.031	0.012*	−0.010	0.018
direct	γ_1			−0.141	0.009***	−0.100	0.011***	−0.096	0.012***	−0.082	0.015***
total indirect				0.059	0.003***	0.065	0.004***	0.065	0.004***	0.072	0.005***
via FCFMLRTY	$\gamma_{11}\beta_1$			0.055	0.003***	0.059	0.003***	0.057	0.004***	0.059	0.005***
via FLCONFIN	$\gamma_{12}\beta_2$			0.004	0.002*	0.006	0.002**	0.008	0.002***	0.014	0.002***
NOBULLY (total)				0.099	0.008***	0.070	0.010***	0.064	0.011***	0.086	0.009***
direct	γ_2			0.081	0.008***	0.060	0.010***	0.055	0.011***	0.074	0.009***
total indirect				0.018	0.002***	0.009	0.002***	0.008	0.002**	0.012	0.002***
via FCFMLRTY	$\gamma_{21}\beta_1$			0.018	0.002***	0.010	0.002***	0.009	0.002***	0.011	0.001***
via FLCONFIN	$\gamma_{22}\beta_2$			0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
FLFAMILY (total)				0.015	0.008†	0.010	0.010	0.005	0.011	0.033	0.015*
direct	γ_3			−0.019	0.008*	−0.020	0.010*	−0.024	0.010*	−0.004	0.014
total indirect				0.034	0.003***	0.030	0.003***	0.028	0.003***	0.037	0.004***
via FCFMLRTY	$\gamma_{31}\beta_1$			0.030	0.002***	0.025	0.002***	0.021	0.002***	0.024	0.003***
via FLCONFIN	$\gamma_{32}\beta_2$			0.004	0.002*	0.006	0.002**	0.007	0.002***	0.013	0.002***
IMMI1GEN	k_1			0.008	0.008	−0.040	0.012**	−0.042	0.012**	−0.039	0.015*
IMMI2GEN	k_2			0.082	0.009***	0.015	0.012	0.018	0.013	−0.023	0.010*
MALE	k_3			−0.010	0.008	0.014	0.010	0.009	0.011	0.025	0.012*
SES	k_4			0.462	0.008***	0.267	0.015***	0.261	0.015***	0.209	0.020***
School-level Predictors											
FLSCHOOL	a_1							−0.319	0.057***	−0.330	0.058***
NOBULLY	a_2							0.288	0.064***	0.378	0.055***
FLFAMILY	a_3							−0.189	0.055**	0.011	0.056
EDUSHORT	a_4							−0.323	0.039***	−0.182	0.044***
STRATIO [b]	k_0							−0.137	0.029***	0.009	0.079
Country-level Predictors											
FKI										0.699	0.090***

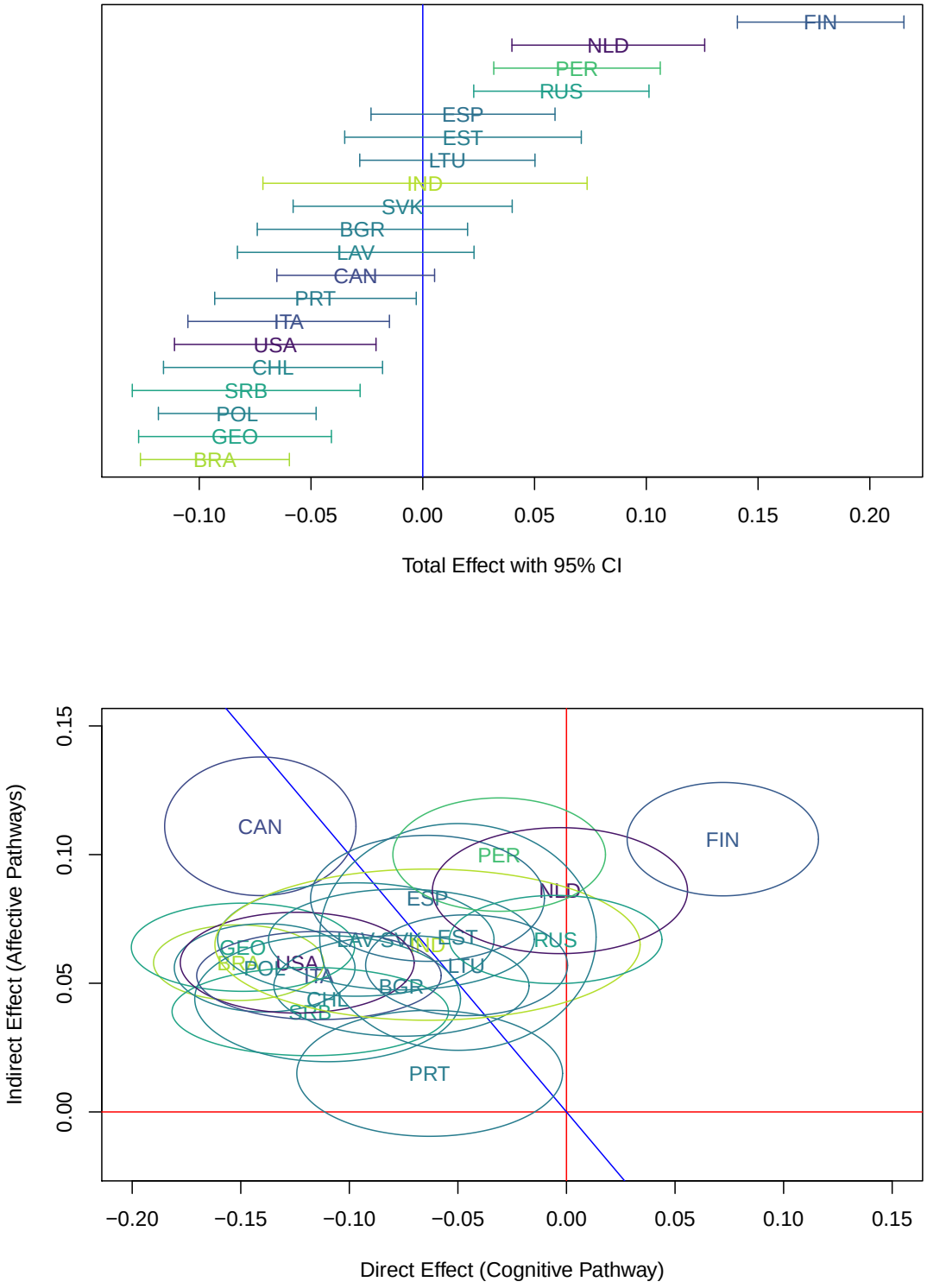
Continued

Variable	Parameter [a]	Null model		1-level model		2-level saturated		2-level structured		3-level model	
		Coef	*SE*	Coef	*SE*	Coef	*SE*	Coef	*SE*	Coef	*SE*
RANDOM EFFECTS (residual variances of `FLIT`)											
Student-level	var $\left(r_{ij}^{Y_W}\right)$	6095.860	300.449	7436.535	110.908	5639.376	129.367	5628.797	143.549	5561.267	228.973
School-level	var $\left(\varepsilon_{j}^{Y_B}\right)$	2675.574	338.740			3084.154	192.395	1455.695	122.133	1017.969	146.516
Country-level		5232.035	0.209							701.990	212.127
MODEL FIT INDICES											
		Est	*SD*	Est	*SD*	Est	*SD*	Est	*SD*	Est	*SD*
AIC		1340755	357	2747149	841	2781389	1152	2324572	1108	2542030	409
BIC		1340793	357	2747443	841	2781902	1152	2325020	1108	2542530	409
χ^2 Test of Model Fit				1708.802	0.817	1182.265	1.763	1120.977	1.899	1467.739	1.760
RMSEA				0.029	0.000	0.024	0.000	0.023	0.000	0.023	0.000
CFI				0.828	0.001	0.815	0.001	0.797	0.002	0.742	0.003
TLI				0.794	0.002	0.500	0.004	0.543	0.004	0.500	0.006
SRMR L1				0.056	0.000	0.052	0.000	0.052	0.001	0.048	0.000
SRMR L2						0.063	0.001	0.082	0.001	0.065	0.000
SRMR L3										0.287	0.001

Note. All p values in this table are two-tailed.
[a] Notations for the 2-level structured model. [b] Omitted for Canada due to whole-country missing and Estonia due to convergence issue.
$^{\dagger}p < 0.10$, $^{*}p < 0.05$, $^{**}p < 0.01$, $^{***}p < 0.001$

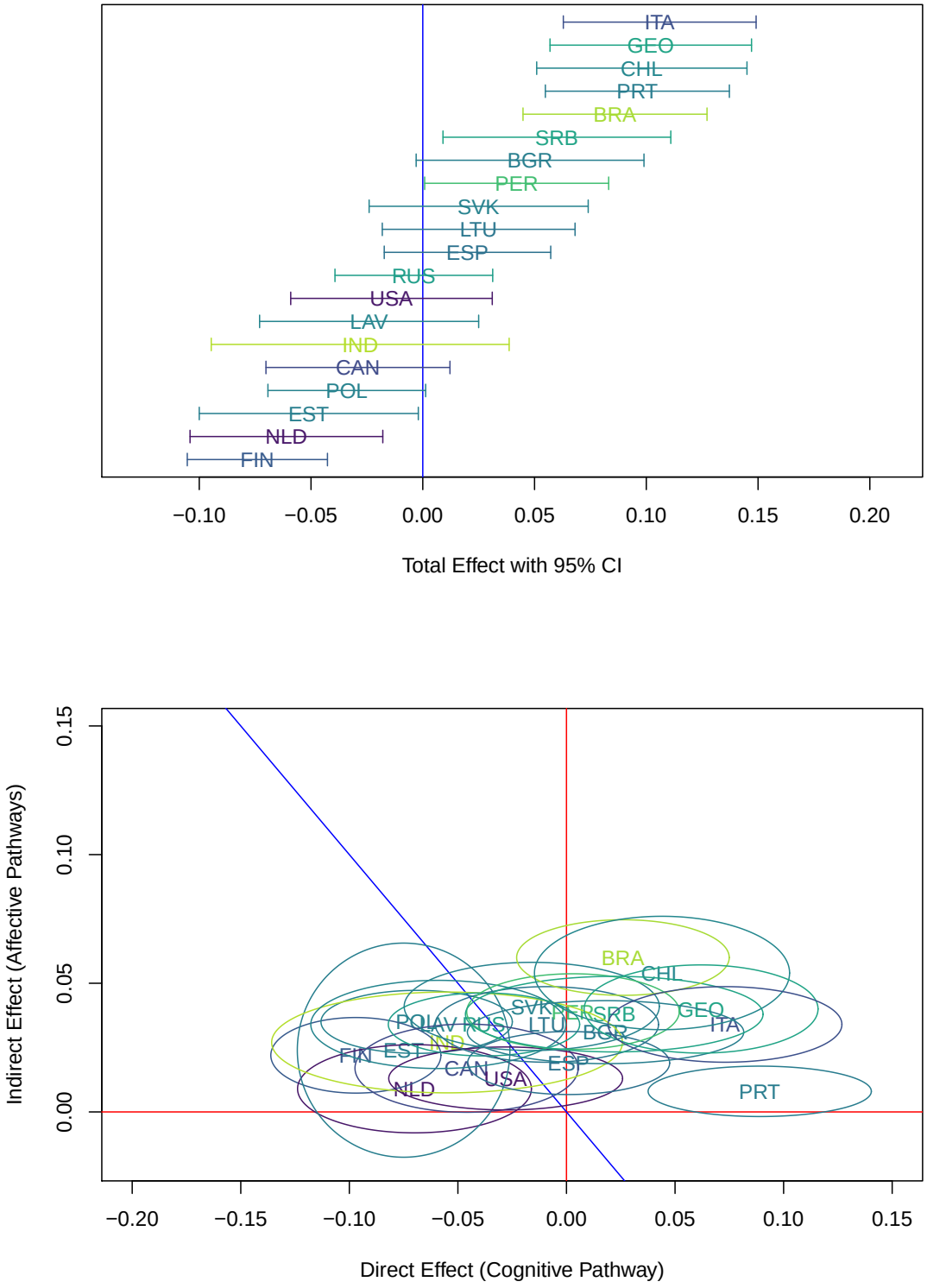
4.4 Address the research question

Figure 4.1
*Total, Direct and Indirect Effects of School Intervention (*FLSCHOOL*)*



Note. Countries with high (low) financial knowlege indices are represented in darker (lighter) colours. Top panel: Estimates with 95% confidence intervals crossing the verticle blue line are not statistically significant. Lower panel: Estimates whose 95% confidence ellipses cross the verticle, horizontal and $-45°$ lines are not significant for direct, indirect and total effect, respectively.

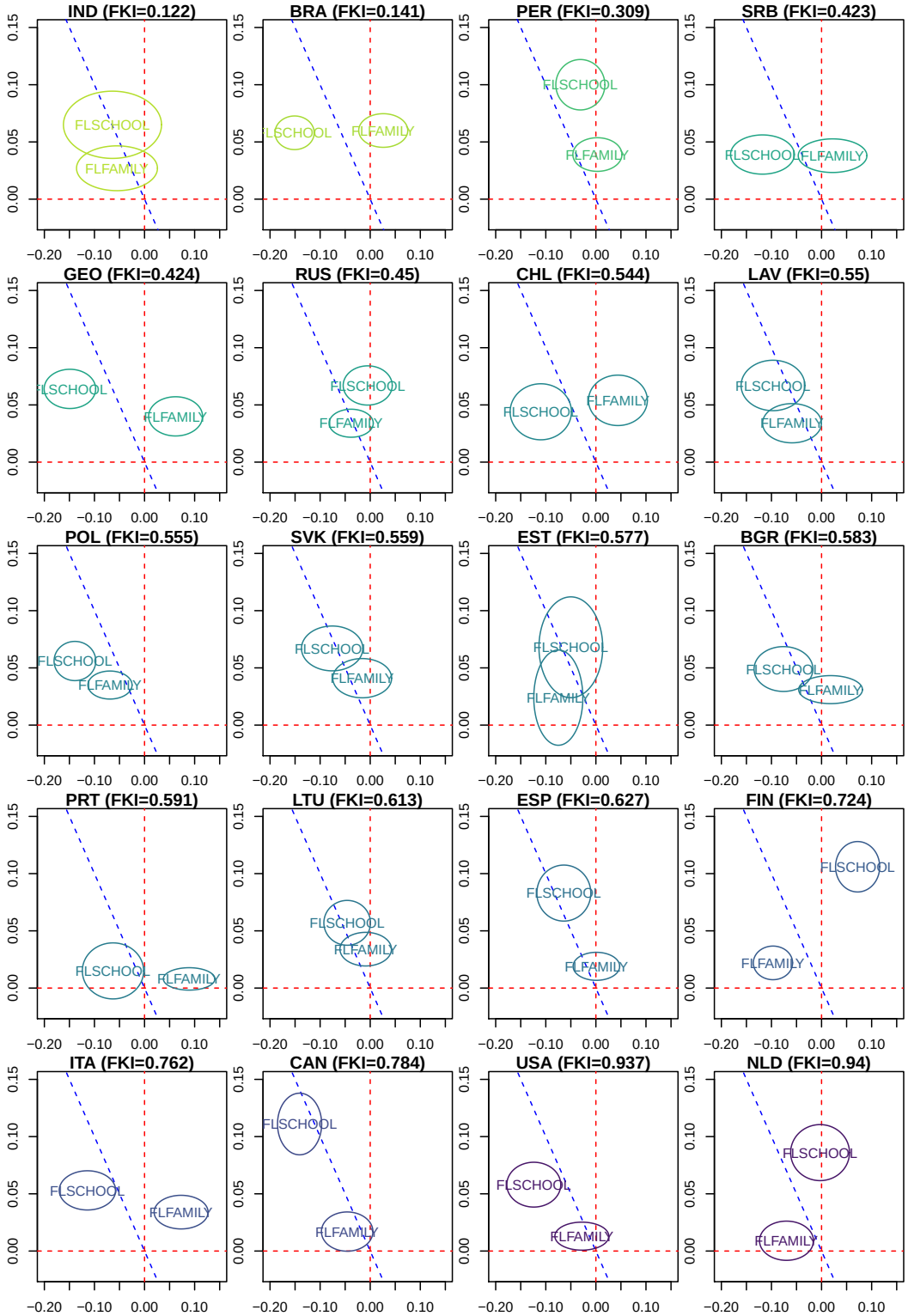
Figure 4.2
Total, Direct and Indirect Effects of Financial Socialisation (`FLFAMILY`*)*



Note. Countries with high (low) financial knowlege indices are represented in darker (lighter) colours. Top panel: Estimates with 95% confidence intervals crossing the verticle blue line are not statistically significant. Lower panel: Estimates whose 95% confidence ellipses cross the verticle, horizontal and −45° lines are not significant for direct, indirect and total effect, respectively.

Figure 4.3

School-Family Effect Decomposition by Country



Note. Cognitive and affective effects are represented on hozitonal and vertical axes respectively.

Chapter 5 Discussion

5.1 Brief summary

5.1.1 Remind readers what my research questions are

5.2 The implication of this study

5.3 Limitation and future directions

5.3.1 Word in positive form

5.4 Bird-eye view

5.4.1 What conclusion I can draw from this paper/study

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Appendices

Appendix A GDPR Documentation and Ethical Approval

This research project discharges its duty imposed by the EEA's general data protection regulation (GDPR) by following Norwegian Centre for Research Data (NSD)'s notification test on Friday, 11 September 2020. Both PISA 2018 Database and the World Bank Open Data contain only aggregated and de-personalised datasets with no possibility of back-tracing to any particular participant. Resultantly, no identifiable personal data were collected or used at any stage of this research. The NSD's assessment letter outlines the agency's decision of not subjecting this project to the GDPR notification. The NSD decision letter also satisfies University of Oslo's ethical approval requirement and concludes the approval process.

Will you be processing personal data?

Are you unsure whether your project is subject to notification? Feel free to try our informal Notification test. Note that the test is intended as a guidance and is not a formal assessment.

Will you be collecting/processing directly identifiable personal data? ○ Yes ◉ No

A person will be directly identifiable through name, social security number, or other uniquely personal characteristics.

Read more about personal data (/personvernombud/en/help/vocabulary.html?id=8) and notification (/personvernombud/en/notify/index.html).

NB! Even though information is to be anonymized in the final thesis/report, check the box if identifying personal data is to be collected/processed in connection with the project.

Will directly identifiable personal information be linked to the data (e.g. through a reference number which refers to a separate list of names/scrambling key)? ○ Yes ◉ No

Note that the project will be subject to notification even if you cannot access the scrambling key (/personvernombud/en/help/vocabulary.html?id=11), as the procedure often is when using a data processor (/personvernombud/en/help/vocabulary.html?id=3), or in register-based studies (/personvernombud/en/help/research_methods/register_studies.html).

Will you be collecting/processing background information that may identify individuals (indirectly identifiable personal data)? ○ Yes ◉ No

A person will be indirectly identifiable if it is possible to identify him/her through a combination of background information (such as place of residence or workplace/school, combined with information such as age, gender, occupation, etc.).

Will there be registered personal data (directly/indirectly/via IP or email address, etc.) using online surveys? ○ Yes ◉ No

Please note that the project will be subject to notification even if you as a student/researcher cannot access the link to the IP or email address, as the procedure often is when using a data processor.

Read more about online surveys (/personvernombud/en/help/research_methods/online_surveys.html).

Will there be registered personal data using digital photo or video files? ○ Yes ◉ No

Photo/video recordings of faces will be regarded as identifiable personal data. In order for a voice to be considered as identifiable, it must be registered in combination with other background information, in such a way that people can be recognized.

Show results

Notify project

Do I have to notify my project? (/personvernombud/en/notify/index.html)

Notification Form (/personvernombud/en/notify/meldeskjema_link)

Notifying changes (/personvernombud/en/notify/notifying_changes.html)

Get help notifying your project

Processing the notification (/personvernombud/en/help/index.html)

Frequently asked questions (/personvernombud/en/help/faq.html)

Vocabulary (/personvernombud/en/help/vocabulary.html)

Research topics (/personvernombud/en/help/research_topics/)

Research methods (/personvernombud/en/help/research_methods/)

Information and consent (/personvernombud/en/help/information_consent/)

Other approvals (/personvernombud/en/help/other_approvals/)

© NSD - Norsk senter for forskningsdata • Kontakt NSD (/om/kontakt.html) • Personvern og informasjonskapsler (cookies) (/om/personvern.html)

Result of Notification Test: Not Subject to Notification

You have indicated that neither directly or indirectly identifiable personal data will be registered in the project.

If no personal data is to be registered, the project will not be subject to notification, and you will not have to submit a notification form.

Please note that this is a guidance based on information that you have given in the notification test and not a formal confirmation.

For your information: *In order for a project not to be subject to notification, we presuppose that all information processed using electronic equipment in the project remains anonymous.*

Anonymous information is defined as information that cannot identify individuals in the data set in any of the following ways:

- *directly, through uniquely identifiable characteristic (such as name, social security number, email address, etc.)*
- *indirectly, through a combination of background variables (such as residence/institution, gender, age, etc.)*
- *through a list of names referring to an encryption formula or code, or*
- *through recognizable faces on photographs or video recordings.*

Furthermore, we presuppose that names/consent forms are not linked to sensitive personal data.

Kind regards,
NSD Data Protection

Appendix B Analysis Code

B.1 Chapter 1

There is no analysis code in Chapter 1.

B.2 Chapter 2

B.2.1 Data Import

```r
# Housekeeping
library(Orcs)
setwdOS(lin = "~/", win = Sys.getenv("USERPROFILE"))

# Import SPSS file into R
library(intsvy)
finlit ← pisa.select.merge(
    student.file = "CY07_MSU_FLT_QQQ.SAV", # file ext in capital
    school.file = "CY07_MSU_SCH_QQQ.sav", # file ext in lower case
    student = c(
        # Control variables
        "ST004D01T", # Student (Standardized) Gender
        "IMMIG", # Index Immigration status
        "ESCS", # Index of economic, social and cultural status
        # Mediators
        "FCFMLRTY", # Familiarity with concepts of finance (Sum)
        "FLCONFIN", # Confidence about financial matters (WLE)
        # Academic
        "PERFEED", # Perceived feedback (WLE)
        "TEACHINT", # Perceived teacher's interest (WLE)
        "FLSCHOOL", # Financial education in school lessons (WLE)
        # Safety
        "DISCRIM", # Discriminating school climate (WLE)
        "BELONG", # Sense of belonging to school (WLE)
        "BEINGBULLIED", # Student's experience of being bullied (WLE)
        # Community
        "FLFAMILY", # Parental involvement in matters of Financial Literacy (WLE)
        "CURSUPP", # Current parental support for learning at home (WLE)
        "PASCHPOL" # School policies for parental involvement (WLE)
    ),
    school = c(
        "STRATIO", # Student-teacher ratio
        "EDUSHORT", # Shortage of educational material (WLE)
        "STAFFSHORT" # Shortage of educational staff (WLE)
    ),
    countries = c(
        "BGR", "BRA", "CAN", "CHL", "ESP",
        "EST", "FIN", "GEO", "IDN", "ITA",
        "LTU", "LVA", "NLD", "PER", "POL",
        "PRT", "QMR", "QRT", "RUS", "SRB",
        "SVK", "USA"
    )
)

# Inspect table header
names(finlit)
```

```r

# Remove columns that I do not need
finlit ← finlit[, -c(5, 7:86)] # 5 = BOOKID; 7:86 = resampling weights

# Some var need recording
library(car)

# Re-code Russian territories to RUS
finlit$CNT ← recode(finlit$CNT, "
    'QMR' = 'RUS';
    'QRT' = 'RUS'
")

# Recode ST004D01T from Sex to Male
MALE ← finlit$ST004D01T - 1

# Recode IMMIG to 1st and 2nd generation
IMMI1GEN ← recode(finlit$IMMIG, "
    1 = 0;
    2 = 0;
    3 = 1
")

IMMI2GEN ← recode(finlit$IMMIG, "
    1 = 0;
    2 = 1;
    3 = 0
")

# Revert coding direction: bigger number ⇒ safer school
NOBULLY ← finlit$BEINGBULLIED * (-1)

# Stitch spreadsheets together
finlit ← cbind(
    finlit[, c(1:35)],
    MALE, IMMI1GEN, IMMI2GEN,
    finlit[, c(38:45)],
    NOBULLY,
    finlit[, c(47:53)]
)

# Use data.table for better RAM management
library(data.table); setDTthreads(0) # 0 means all the available cores
# Export data into a CSV file for faster import next time
fwrite(finlit,
    file = "finlit.csv",
    na = "NA", row.names = F, col.names = T
)
```

B.2.2 Missing Pattern Inspection

```r
library(Orcs)
setwdOS(lin = "~/", win = Sys.getenv("USERPROFILE"))

library(data.table); setDTthreads(0)
finlit ← fread("finlit.csv", nThread = getDTthreads())

library(dplyr)
# Record how many missings each country has for each var
missings ← finlit %>%
    select(everything()) %>%
```

```
    group_by(CNT) %>%
    summarise_all(funs(sum(is.na(.))))
# Give me the headcount for each country
headcount ← finlit %>%
    group_by(CNT) %>%
    summarize(n())
# Stitch these two tables together
missing_table ← tibble(headcount, missings[, -1])
# Save this file
fwrite(missing_table, "missing_table.csv", row.names = F, col.names = T)

# Inspect the missing table using Excel
# Throw away the following countries
#   CAN: 100% missing on too many var
#   BRA, FIN, LVA, NLD, RUS, SRB: private/public info missing
# Throw away these var
#   DISCRIM, CURSUPP, PASCHPOL: Too many countries have 100% missing
```

B.2.3 Financial Knowledge Index

```
# Section 0: Housekeeping
library(Orcs) # Set working directory depending on operating system
setwdOS(
    lin = "~/uio/", win = "M:/",
    ext = "pc/Dokumenter/MSc/Thesis/Data/L3/"
)

# Set up a "bookshelf" to hold variables nessary to compute FKI
fki_raw ← matrix(NA,
    nrow = 20, ncol = 10, dimnames = list(
        c( # row names
            "BRA", "BGR", "CAN", "CHL", "EST",
            "FIN", "GEO", "IDN", "ITA", "LVA",
            "LTU", "NLD", "PER", "POL", "PRT",
            "RUS", "SRB", "SVK", "ESP", "USA"
        ),
        c( # column names
            "gdp_per_capita", # economic capability (sub_ind_ec)
            "highly_skilled", "mean_year_of_schooling", # (sub_ind_et)
            "gpea", "ica", "ius", # use (sub_ind_use)
            "pfa", "ac", "gdp", "ageing" # need (sub_ind_need)
        )
    ) # End list()
) # End matrix()


# Section 1: Economic Capacity (EC)

gdp_per_capita ← read.csv("gdp_per_capita.csv", header = T, sep = "\t")

fki_raw[, 1] ← log(gdp_per_capita[, 2])

rm(gdp_per_capita)


# Section 2: Educational Training (ET)
    # Subsection 2.1: Highly skilled
        # Masters
isced_7 ← read.csv("isced_7.csv", header = T, sep = "\t")
        # PhDs
isced_8 ← read.csv("isced_8.csv", header = T, sep = "\t")
```

```r
        # Total tertiary
total_tertiary ← read.csv("total_tertiary.csv", header = T, sep = "\t")

# Compute highly skilled (master + PhD) to total tertiary ratio
highly_skilled ← ts(
    (isced_7 + isced_8) / total_tertiary,
    start = 2013, end = 2018, frequency = 1
)

# # Visualise highly_skilled. Turn off GEO (#7), IDN (#8), SRB (#17) and RUS (#16)
# pdf("../../Figures/skilled.pdf")
# ts.plot(100 * highly_skilled[, -c(7, 8, 16, 17)],
#     type = "b", col = 1:15,
#     xlab = "Year", ylab = "Percent"
# )
# legend("topright", colnames(highly_skilled[, -c(6, 7, 15, 16)]),
#     col = 1:15, lty = 1, cex = 0.65
# )
# dev.off()

# Decision: naive forcasts, i.e., copy-paste nearest available year
library(forecast)
# Create a placeholder matrix
placeholder ← matrix(NA, nrow = 20, ncol = 1)

# Run a loop to foreecast all 20 countries, using naive method
for (.i in 1:20) {
    m_naive_i ← naive(highly_skilled[, .i], h = 1)
    placeholder[.i] ← data.frame(unlist(m_naive_i[5])[6])[1, 1]
}
# [5] = fitted values; [6] = 2018; [1,1] = only the numeric value

# GEO and IDN have 2018 data, plug actual numbers back
placeholder[c(7, 8)] ← highly_skilled[6, c(7, 8)]

# RUS needs separate calculation
# ISCED 7 = 101766 (Type 1) + 170437 (Type 2) = 272203 (total masters)
# ISCED 8 = 15465 (Type 1) + 330 (Type 2) = 15795 (total PhDs)
# Total tertiary WITHOUT PhD = 933153
# ⇒ Total tertiary = 933153 + 15795 = 948948
# highly_skilled (RUS) = (272203 + 15795) / 948948 = 0.30349187
placeholder[16] ← 0.30349187

# Save results to "bookshelf"
fki_raw[, 2] ← placeholder * 100

rm(
    isced_7, isced_8, total_tertiary,
    highly_skilled, placeholder, m_naive_i
)

# Sub-section 2.2: Mean year of schooling
mean_year_of_schooling ← read.csv("mean_year_of_schooling.csv",
    header = F, sep = "\t"
)
fki_raw[, 3] ← mean_year_of_schooling[, 2]

rm(mean_year_of_schooling)


# Section 3: Use
```

```r

gpea <- read.csv("gpea.csv", header = T, sep = "\t")
gpea <- ts(gpea, start = 2011, end = 2017, frequency = 1)

# # Visualise data in both original and ln forms. Contain trend?
# pdf("../../Figures/use.pdf", width = 12.94, height = 9.15)

# # Re-set canvas layout to 2x2
# par(mfcol = c(2, 2))

# # Add extra space to the right of plot area
# par(mar = c(5.1, 4.1, 4.1, 2.1), xpd = TRUE)

# # Plot GPEA in original form
# ts.plot(gpea,
#     type = "b", col = 1:20,
#     xlab = "Year", ylab = "Percent", main = "GPEA to GDP ratio"
# )

# # Remove extra gap between the two graphs
# par(mar = c(5.1, 4.1, 0, 2.1), xpd = TRUE)

# # Repeat GPEA, but for the ln() version
# ts.plot(log(gpea),
#     type = "b", col = 1:20,
#     xlab = "Year", ylab = "ln( percent )"
# )

# # Plot ICA in original form
# par(mar = c(5.1, 4.1, 4.1, 6.1), xpd = TRUE)
# ts.plot(ica,
#     type = "b", col = 1:20,
#     xlab = "Year", ylab = "Percent", main = "ICA to GDP ratio"
# )
# # Add the legend
# legend("topright",
#     inset = c(-0.2, 0), colnames(ica),
#     col = 1:20, lty = 1, cex = 0.875
# )

# # Remove extra gap between the two graphs
# par(mar = c(5.1, 4.1, 0, 6.1), xpd = TRUE)

# # Repeat, but for the ln()
# ts.plot(log(ica),
#     type = "b", col = 1:20,
#     xlab = "Year", ylab = "ln( percent )"
# )
# # Add the legend
# legend("topright",
#     inset = c(-0.2, 0), colnames(ica),
#     col = 1:20, lty = 1, cex = 1.07
# )
# dev.off()

# Decision: since the ln() version is not flat, original time series
# contain trend. Use Holt method rather than simple exponential smoothing.

# Run a time series forecast using Holt method

```

```r
# Create a placeholder matrix
placeholder <- matrix(NA, nrow = 20, ncol = 1)

# Run a loop to forecast all 13 countries, using Holt method
for (.i in 1:20) {
    m_holt_i <- holt(gpea[, .i], h = 1)
    placeholder[.i] <- m_holt_i[2]
} # Ignore warnings

# Only keep the 2018 forecasts
placeholder <- unlist(placeholder)

# Run PER (#13) separately because it misses both 2017 and 2018 data
m_holt_PER <- holt(gpea[, 13], h = 2); summary(m_holt_PER)
placeholder[13] <- 16.02698

# Push placeholder to fki_raw
fki_raw[, 4] <- placeholder

rm(gpea, placeholder, m_holt_i, m_holt_PER)

# Sub-section 3.2: Insurance company assets (ica)

ica <- read.csv("ica.csv", header = T, sep = "\t")
ica <- ts(ica, start = 2011, end = 2017, frequency = 1)

placeholder <- matrix(NA, nrow = 20, ncol = 1)

for (.i in 1:20) {
    m_holt_i <- holt(ica[, .i], h = 1)
    placeholder[.i] <- m_holt_i[2]
} # Ignore warnings

placeholder <- unlist(placeholder)

m_holt_CAN <- holt(ica[, 3], h = 2); summary(m_holt_CAN)
m_holt_IND <- holt(ica[, 8], h = 2); summary(m_holt_IND)
m_holt_ITA <- holt(ica[, 9], h = 2); summary(m_holt_ITA)
m_holt_POL <- holt(ica[, 14], h = 2); summary(m_holt_POL)
m_holt_USA <- holt(ica[, 20], h = 2); summary(m_holt_USA)

placeholder[c(3, 8, 9, 14, 20)] <- c(
    77.72768, 4.611597, 51.2596, 9.534750, 30.18295
)

fki_raw[, 5] <- placeholder

rm(ica, placeholder, list = ls(pattern = "^m.holt"))

# Sub-section 3.3: Individuals using the Internet (ius)

ius <- read.csv("ius.csv", header = T, sep = "\t")
ius <- ts(ius, start = 2009, end = 2018, frequency = 1)

m_holt_CAN <- holt(ius[1:9, 3], h = 1); summary(m_holt_CAN)
m_holt_CHL <- holt(ius[1:9, 4], h = 1); summary(m_holt_CHL)
m_holt_USA <- holt(ius[1:9, 20], h = 1); summary(m_holt_USA)

ius_2018 <- ius[10, ] # Only want 2018 data
ius_2018[3] <- 93.58751 # CAN
```

```r
ius_2018[4] ← 89.5309 # CHL
ius_2018[20] ← 84.88108 # USA

fki_raw[, 6] ← ius_2018

rm(list = ls(pattern = "^ius"))
rm(list = ls(pattern = "^m_holt_"))


# Section 4: Need

# Subsection 4.1: Pension fund assets (pfa)
pfa ← read.csv("pfa.csv", header = T, sep = "\t")
# Delete GEO (#7) due to all missing. Will come back to it later.
pfa ← ts(pfa[, -7], start = 2008, end = 2017, frequency = 1)

placeholder ← matrix(NA, nrow = 19, ncol = 1)

for (.i in 1:19) {
    m_holt_i ← holt(pfa[, .i], h = 1)
    placeholder[.i] ← m_holt_i[2]
}

placeholder ← unlist(placeholder)

# Calculate GEO
# From Georgia Pension Agency:
#   2019 mesub_ind_eting minute: 372,113,933 GEL
# From GeoStat website:
#   2018 gdp = 44.6 billion GEL

fki_raw[, 7] ← c(
    placeholder[1:6],
    372113934 / 44600000000 * 100, # Insert GEO figure
    placeholder[7:19]
)

rm(pfa, placeholder, m_holt_i)

# Subsection 4.2: Aggregate consumption (ac)

ac ← read.csv("ac.csv", header = F, row.names = 1, sep = "\t")
gdp ← read.csv("gdp.csv", header = F, row.names = 1, sep = "\t")

fki_raw[, 8] ← unlist(ac * 0.02 / gdp * 100)

fki_raw[, 9] ← unlist(gdp)

rm(ac, gdp)

# Subsection 4.3: Ageing

ageing ← read.csv("ageing.csv", header = T, sep = "\t")
attach(ageing)
names(ageing)

# Calculate total population
poptotal_f ← pop0to14_f + pop15to64_f + pop65plus_f
poptotal_m ← pop0to14_m + pop15to64_m + pop65plus_m
# Calculate population between 15 and 19
# Need to divide by 100 to get decimals
```

```r
pop15to19_f ← poptotal_f * per15to19_f / 100
pop15to19_m ← poptotal_m * per15to19_m / 100
# Calculate population between 0 and 19
pop0to19_f ← pop0to14_f + pop15to19_f
pop0to19_m ← pop0to14_m + pop15to19_m
# Calculate population between 20 and 64
pop20to64_f ← poptotal_f - pop0to19_f - pop65plus_f
pop20to64_m ← poptotal_m - pop0to19_m - pop65plus_m
# Calculate 64+ / 20-to-64 ratio    'GEO' = 0.419;
ageing_ratio ← I(
    (pop65plus_f + pop65plus_m) / (pop20to64_f + pop20to64_m)
)
# Split data into 2018 [ , 1] and 2009 [ , 2] portions
ageing ← cbind(ageing_ratio[1:20], ageing_ratio[21:40])
fki_raw[, 10] ← (ageing[, 1] - ageing[, 2]) / ageing[, 2]

rm(ageing, ageing_ratio, list = ls(pattern = "^pop"))


# Section 5: FKI

fki_raw ← fki_raw[, -9] # Throw away gdp (already in ac)
round(fki_raw, digits = 3) # Inspect data

# Save data to an external file
library(data.table); setDTthreads(0)
fwrite(round(fki_raw, digits = 3), file = "fki_raw.csv", row.names = T)

# Subection 5.0: Standardise each variable to [0.01,0.99] range
fki_stand ← matrix(NA, nrow = dim(fki_raw)[1], ncol = dim(fki_raw)[2])
dimnames(fki_stand) ← dimnames(fki_raw)

library(scales) # I wish this function could have "by.col = T". Oh well.
for (.j in 1:dim(fki_raw)[2]) {
    fki_stand[, .j] ← rescale(fki_raw[, .j], to = c(0.01, 0.99))
}

rm(fki_raw)

fki_stand ← data.frame(fki_stand)
attach(fki_stand)

# Subsection 5.1: Economic capacity (sub_ind_ec)

sub_ind_ec ← gdp_per_capita

# Subsection 5.2: Education and training (sub_ind_et)

wt_highly_skilled ← 1 / sd(highly_skilled)
wt_mean_year_of_schooling ← 1 / sd(mean_year_of_schooling)

sub_ind_et ← (highly_skilled^wt_highly_skilled *
    mean_year_of_schooling^wt_mean_year_of_schooling)^
    (1 / (wt_highly_skilled + wt_mean_year_of_schooling))

rm(list = ls(pattern = "^wt"))

# Subsection 5.3: Use (sub_ind_use)

sub_ind_u ← (gpea + ica)^ius
```

```r

# Subsection 5.4: Need (sub_ind_need)

sub_ind_n ← (pfa + ac)^ageing

## Subsection 5.5: FKI

wt_ec ← 1 / sd(sub_ind_ec)
wt_et ← 1 / sd(sub_ind_et)
wt_u ← 1 / sd(sub_ind_u)
wt_n ← 1 / sd(sub_ind_n)

fki ← (
    sub_ind_ec^wt_ec *
        sub_ind_et^wt_et *
        sub_ind_u^wt_u *
        sub_ind_n^wt_n
) ^ (
    1 / (wt_ec + wt_et + wt_u + wt_n)
)

rm(list = ls(pattern = "^wt"))

l3 ← data.frame(
    round(
        cbind(fki, sub_ind_ec, sub_ind_et, sub_ind_u, sub_ind_n),
        digits = 3
    )
)
rownames(l3) ← rownames(fki_stand)
attach(l3)

rm(fki_stand, fki, sub_ind_ec, sub_ind_et, sub_ind_u, sub_ind_n)

# Display country-level FKI, default by country code
l3

# Sort FKI by country (highest to lowest)
l3_ordered ← l3[order(-fki), ]
l3_ordered
fwrite(l3_ordered, file = "fki.csv", row.names = T)

pdf("../../Figures/FKI.pdf")
    barplot(l3_ordered$fki,
        names.arg = rownames(l3_ordered),
        xlab = "Country", las = 2, ylab = "Financial Knowledge Index (FKI)",
        ylim = c(0, 1), main = "FKI of 20 participating countries"
    )
dev.off()
```

B.2.4 Data Reimport

```r
# Housekeeping
library(Orcs)
setwdOS(lin = "~/", win = Sys.getenv("USERPROFILE"))

# Import SPSS file into R
library(intsvy)
finlit ← pisa.select.merge(
    student.file = "CY07_MSU_FLT_QQQ.SAV", # file ext in capital
```

```
    school.file = "CY07_MSU_SCH_QQQ.sav", # file ext in lower case
    student = c(
    # Control variables
        "ST004D01T", # Student (Standardized) Gender
        "IMMIG", # Index Immigration status
        "ESCS", # Index of economic, social and cultural status
    # Mediators
        "FCFMLRTY", # Familiarity with concepts of finance (Sum)
        "FLCONFIN", # Confidence about financial matters (WLE)
    # Academic
        "FLSCHOOL", # Financial education in school lessons (WLE)
    # Safety
        "BEINGBULLIED", # Student's experience of being bullied (WLE)
    # Community
        "FLFAMILY" # Parental involvement in matters of Financial Literacy (WLE)
    ),
    school = c(
        "STRATIO", # Student-teacher ratio
        "EDUSHORT" # Shortage of educational material (WLE)
    ),
    countries = c(
        "BRA", "BGR", "CAN", "CHL", "EST",
        "FIN", "GEO", "IDN", "ITA", "LVA",
        "LTU", "NLD", "PER", "POL", "PRT",
        "RUS", "QMR", "QRT", # Russian Federation and other regions
        "SRB", "SVK", "ESP", "USA"
    )
)

names(finlit)
# Throw away columns that I do not need
finlit ← finlit[, -c(5, 7:86)] # 5 = BOOKID; 7:86 = resampling weights
names(finlit)

# Some var need recording
library(car)

# Re-code Russian territories to RUS
finlit$CNT ← recode(finlit$CNT, "
    'QMR' = 'RUS';
    'QRT' = 'RUS'
")

finlit$CNTRYID ← recode(finlit$CNTRYID, "
    982 = 643;
    983 = 643
")

# Input country-level FKI
FKI ← recode(finlit$CNT, "
    'NLD' = 0.940;
    'USA' = 0.937;
    'CAN' = 0.784;
    'ITA' = 0.762;
    'FIN' = 0.724;
    'ESP' = 0.627;
    'LTU' = 0.613;
    'PRT' = 0.591;
    'BGR' = 0.583;
    'EST' = 0.577;
    'SVK' = 0.559;
```

```
    'POL' = 0.555;
    'LVA' = 0.550;
    'CHL' = 0.544;
    'RUS' = 0.450;
    'GEO' = 0.424;
    'SRB' = 0.423;
    'PER' = 0.309;
    'BRA' = 0.141;
    'IDN' = 0.122
")

# Recode ST004D01T from Sex to Male
MALE ← finlit$ST004D01T - 1

# Recode IMMIG to 1st and 2nd generation
IMMI1GEN ← recode(finlit$IMMIG, "
    1 = 0;
    2 = 0;
    3 = 1
")

IMMI2GEN ← recode(finlit$IMMIG, "
    1 = 0;
    2 = 1;
    3 = 0
")

# Revert coding direction: bigger number ⇒ safer school
NOBULLY ← finlit$BEINGBULLIED * (-1)

# Stitch spreadsheet together
names(finlit)
finlit ← cbind(FKI, finlit[, c(2:35)], MALE, IMMI1GEN, IMMI2GEN, finlit[, c(38:41)], NO
head(finlit)
names(finlit)

# Remove cases whose school weights (col #45) are NA
obs0 ← dim(finlit)[1]
finlit ← finlit[complete.cases(finlit[, 45]), ]
obs1 ← dim(finlit)[1]
obs0 - obs1 # 12 cases contained missing school weights and have been dropped
rm(obs0, obs1)

# Use data.table for better RAM management
library(data.table); setDTthreads(0) # 0 means all the available cores
# Export data into a CSV file for faster import next time
fwrite(finlit, file = "finlit.csv", na = "NA", row.names = F, col.names = T)
```

Appendix C Derivation of Country-level Financial Knowledge Indices

C.1 Theoretical Foundation

PISA 2018 financial literacy dataset (OECD, 2020) provides rich information about students and schools. For the purpose of cross-country comparison, however, the country-level data must be addressed separately by the researchers. Moreno-Herrero et al. (2018), for instance, introduced a variable "quality of math and science education" to control for country-level differences since consensus is yet to emerge about the most appropriate measure for "countries' financial knowledge". Inspired by the UN's approach to forming Human Development Indices, a recent publication Oliver-Márquez et al. (2020) highlighted four aspects of countries' macroeconomic practices in their attempt to develop country-level financial knowledge indices (FKI).

Oliver-Márquez and colleagues consider a country's economic capability, represented by its GDP per capita, to be a key dimension in bringing about its FKI. Secondly, literature converges on the importance of education training for a country's financial knowledge capability (OECD, 2005). Thirdly, countries with regular engagement with sophisticated financial products and financial markets should possess higher FKI. Lastly, countries with higher aggregate consumption levels and with ageing populations are likely to possess higher FKI due to more frequent exposure and pressure in retirement provision, respetively.

More specifically, Oliver-Márquez et al. (2020) suggests using the logarithm of GDP per capital in current international dollars (purchasing power parity adjusted) as a measure for the `Economic Capability` sub-index. For the `Education Training` sub-index, the authors consider postgraduate-to-total-tertiary-graduation ratios as a reflection of "highly skilled" workforce and the mean years of schooling as a measure of countries' general education levels. For the `Use` sub-index, gross portfolio equity assets (GPEA) and insurance company assets (ICA) are considered sophisticated financial products countries engage themselves in. Additionally, in order to capture the central role of technology in amplifying the proliferation

and use of financial assets, the proportion of Internet users (IUS) enters the definition via

$$\texttt{Use} = (\text{GPEA} + \text{ICA})^{\text{IUS}}.$$

For the final sub-index `Need`, the authors define

$$\texttt{Need} = (\text{PFA} + \text{AC})^{\text{AGEING}},$$

where PFA is the pension-fund-assets-to-GDP ratio. Aggregate consumption is defined as:

$$\text{AC} = \frac{2\% \times \text{household final consumption expenditure}}{\text{GDP}},$$

where the "2% rule" is drawn from Caliendo and Findley (2013) and the proportion of ageing population is computed as

$$\text{AGEING} = \frac{\left[\frac{\text{population}(>65)}{\text{population}(20\sim64)}\right]_{2018} - \left[\frac{\text{population}(>65)}{\text{population}(20\sim64)}\right]_{2009}}{\left[\frac{\text{population}(>65)}{\text{population}(20\sim64)}\right]_{2009}}.$$

C.2 Data Collection and Missing Data Treatment

The data sources for FKI computation are documented in Table C.1 and its associated notes. The sub-indices `Educational Training` and `Use` both contain missing observations for the year 2018. Majority of such missing data appear to be the result of administrative delay, with historic observations available until 2017. It is therefore feasible to conduct time-series forecasts using prior year observations to best approximate 2018 values.

Table C.1

Data Sources for FKI Computation

Database [a]	Country [b]	Series	Time
		Economic Capacity	
WB-dev	20	GDP per capita, PPP (current international $)	2018
		Educational Training	
WB-ed	20 \ Russia	Graduates from ISCED 7 programmes in tertiary education, both sexes (number)	2013–**2018**
		Graduates from ISCED 8 programmes in tertiary education, both sexes (number)	2013–**2018**
		Graduates from tertiary education, both sexes (number)	2013–**2018**
RS	Russia	PhD (Type 1) [c], PhD (Type 2) [d]	2018
RE	Russia	Master (Type 1) [e], Master (Type 2) [f], total tertiary *excluding* PhD [g]	2018
HDR	20	Dimension = Education; Education = Mean years of schooling (years)	2018
		Use	
WB-fin	20	Gross portfolio equity assets to GDP (%)	2011–**2018**
		Insurance company assets to GDP (%)	2011–**2018**
WB-dev	20	Individuals using the Internet (% of population)	2009–**2018**
		Need	
WB-fin	20 \ Georgia	Pension fund assets to GDP (%)	2008–**2018**
GP	Georgia	Minutes of the meeting of the investment board of the Pension Agency [h]	2019 [*]
GS	Georgia	GDP at current prices, billion GEL [i]	2018
WB-dev	20	Household and NPISHs final consumption expenditure, PPP (current international $)	2018
		GDP, PPP (current international $)	2018
		Population ages 0–14, male	2009, 2018
		Population ages 0–14, female	2009, 2018
		Population ages 15–64, male	2009, 2018
		Population ages 15–64, female	2009, 2018
		Population ages 65 and above, male	2009, 2018
		Population ages 65 and above, female	2009, 2018
		Population ages 15–19, male (% of male population)	2009, 2018
		Population ages 15–19, female (% of female population)	2009, 2018

Note. Sub-indices are shaded in gray. Bold font signifies this year contains missing data.

[a] WB-dev = World Bank – World development indicators
WB-ed = World Bank – Education statistics – All indicators
WB-fin = World Bank – Global financial development
HDR = Human Development Reports – Data
RS = Russian Federal State Statistic Service
RE = Russian Ministry of Education and Science
GP = Pension Agency of Georgia
GS = National Statistics Office of Georgia

[b] "20" = the 20 participating countries in 2018 PISA financial literacy test: Brazil, Bulgaria, Canada, Chile, Estonia, Finland, Georgia, Indonesia, Italy, Latvia, Lithuania, the Netherlands, Peru, Poland, Portugal, Russian Federation, Serbia, Slovak Republic, Spain, and the USA. "\" = exluding or except

[c] https://rosstat.gov.ru/storage/mediabank/asp-2(1).xls, Sheet "по направлениям подготовки", Cell C7 = number of PhD graduates (Type 1)

[d] https://rosstat.gov.ru/storage/mediabank/asp-3.xls, Sheet "по научным специальностям", Cell B7 = number of PhD graduates (Type 2)

[e–g] https://minobrnauki.gov.ru/common/upload/download/VPO_1_2018.rar contains a spreadsheet СВОД_ВПО1_ВСЕГО.xls, Sheet "P2_1_3(1)", Cell E198 = number of master graduates (Type 1)[e], Cell E410 = number of master graduates (Type 2)[f], Cell E592 = total tertiary graduates *excluding* PhD[g]

[h] Minutes of the meeting of the investment board of the Pension Agency, p. 4, no. 3

[i] Gross domestic product (GDP), row = GDP at current prices, billion GEL, column = 2018

[*] Georgia started a new pension system on 1 January 2019. Since 2018 was a transitional period with scarce data, 2019 is used as the best approximation for Georgia's pension system for 2018.

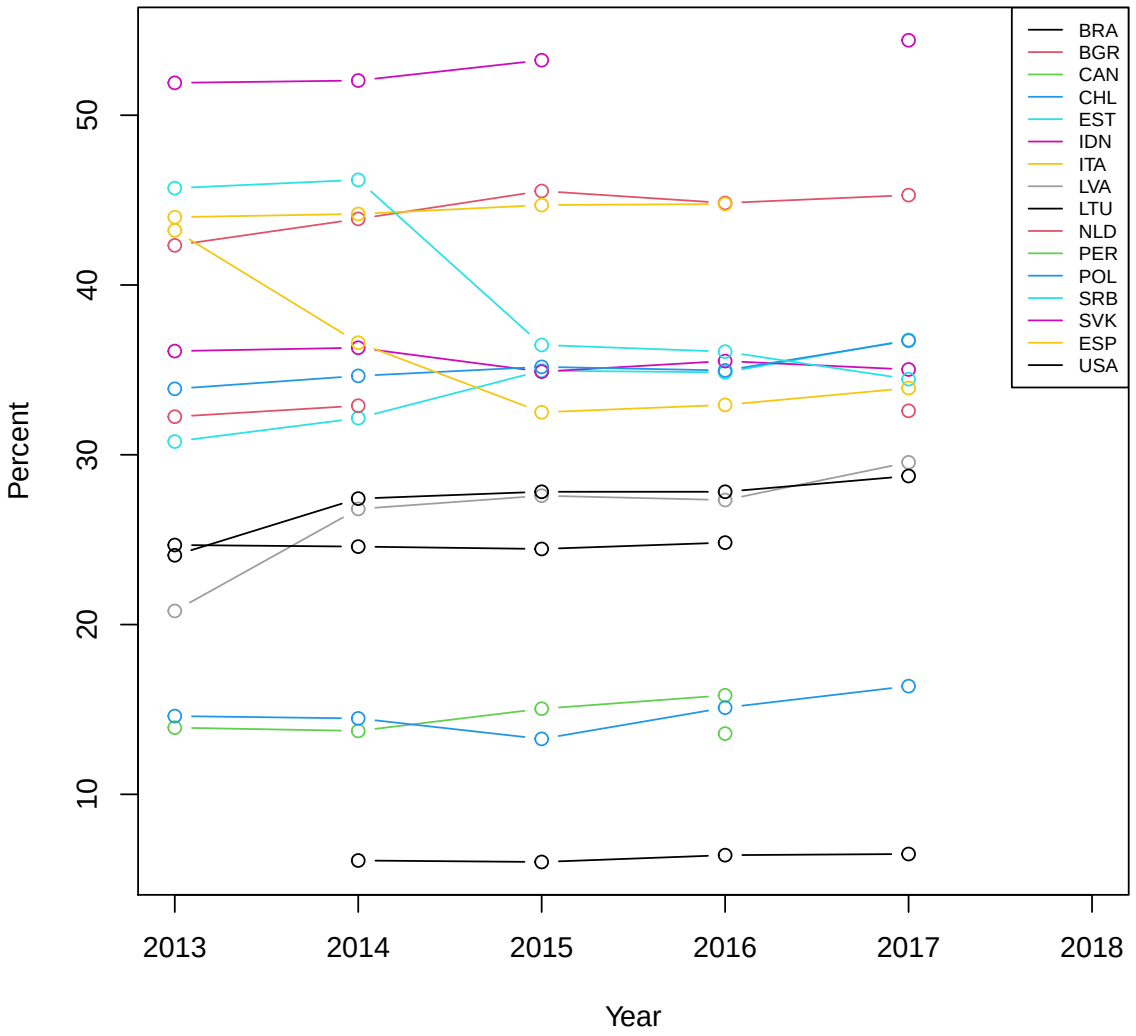
C.2.1 Sub-index **`Educational Training`**

The 2018 archive for the number of master (ISCED 7), PhD (ISCED 8), and total tertiary graduates are incomplete for all participating countries except Georgia, Indonesia and Serbia. Figure C.1 presents a time series plot of

$$\texttt{highly skilled} = \frac{\text{number of masters} + \text{number of PhDs}}{\text{total number of tertiary graduates}}$$

and suggests that this ratio is likely to be stable over time, especially between adjacent years. A "naive forecast", where the nearest available year's data are to be duplicated for 2018, is applied for `highly skilled`.

Figure C.1
Proportion of Postgraduates to Total Tertiary Graduations



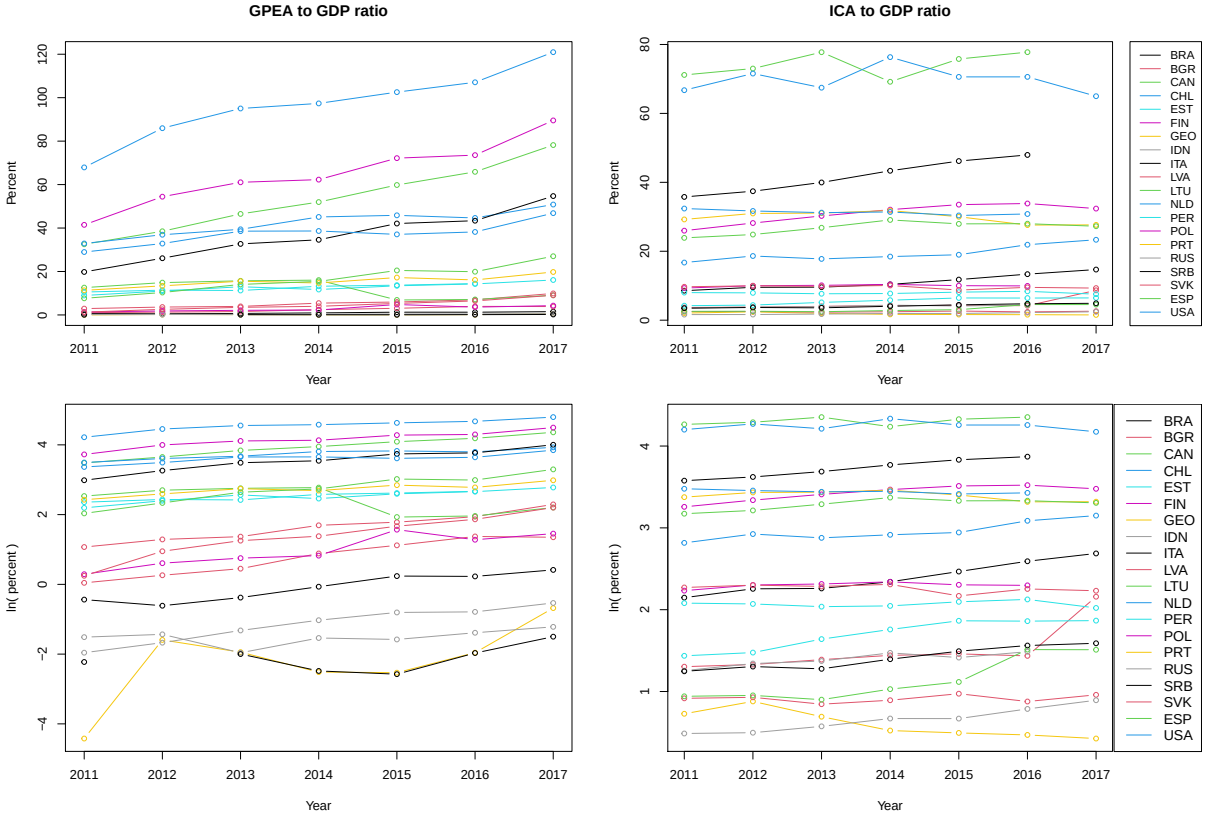
Note. "Postgraduate" is defined as master (ISCED 7) and PhD (ISCED 8) graduates. Countries not shown: GEO, IDN and SRB (2018 data available) and RUS (consult other sources)

C.2.2 Sub-index **Use**

All series involved in calculating this sub-index, GPEA, ICA and IUS, contain missing data. When time series data contain only exponential growth but no underlying trend, a simple exponential smoothing would suffice (Garder, 1985); if trend is present, Holt-Winters method is superior (Chatfield, 1978). Figure C.2 facilitates this decision making by plotting both the original and log-transformed versions of GPEA and ICA series. Since curves after log-transformations have slopes, it is prudent to apply the Holt-Winters forecasting method in order to account for possible trends contained in the original series.

Figure C.2

Time Series Trend Test



Note. The time series plots after natural logarithm transformations (bottom panels) are not flat, suggesting the original series (top panels) contain trends. Holt-Winters method therefore is preferred over simple exponential smoothing for 2018 forecasts.

The IUS series contains missing data for Canada, Chile and the United States. Similar Holt-Winters procedure is applied to recover 2018 IUS data.

Table C.2
Data Utilised for Computing FKI

	Economic Capacity	Educational Training		Use			Need		
	GDP per capita	Skilled	Schooling	GPEA	ICA	IUS	PFA	AC	AGEING
BRA	9.612	6.484	7.8	1.683	16.259	70.434	11.827	1.210	0.288
BGR	10.026	45.294	11.8	4.114	7.044	64.782	13.577	1.091	0.234
CAN	10.821	15.832	13.3	84.010	77.728	93.588	96.205	1.068	0.271
CHL	10.117	16.371	10.4	51.755	25.591	89.531	73.225	1.073	0.214
EST	10.501	36.765	13.0	16.399	7.681	89.357	18.012	0.876	0.163
FIN	10.807	35.024	12.4	93.626	31.481	88.890	52.024	0.974	0.370
GEO	9.588	24.039	12.8	0.784	1.469	62.718	0.834	1.227	0.042
IDN	9.362	7.771	8.0	0.636	4.612	39.905	1.826	1.059	0.145
ITA	10.665	44.771	10.2	57.434	51.260	74.387	10.589	1.075	0.155
LVA	10.330	29.554	12.8	8.598	2.538	83.577	14.732	1.027	0.142
LTU	10.487	28.749	13.0	9.008	5.500	79.723	7.457	1.107	0.149
NLD	10.961	32.590	12.2	124.171	64.956	94.712	207.938	0.805	0.326
PER	9.479	13.577	9.2	16.027	6.505	52.540	22.530	1.187	0.227
POL	10.368	36.725	12.3	4.853	9.535	77.542	9.838	1.085	0.355
PRT	10.444	34.454	9.2	19.353	25.579	74.661	8.761	1.133	0.237
RUS	10.267	30.349	12.0	0.302	2.614	80.865	4.415	0.941	0.155
SRB	9.774	26.946	11.2	0.306	5.111	73.361	0.845	1.171	0.280
SVK	10.391	54.417	12.6	10.644	8.873	80.660	12.497	0.962	0.300
ESP	10.609	33.929	9.8	27.681	28.230	86.107	10.235	1.044	0.186
USA	11.048	24.825	13.4	55.505	30.183	84.881	150.040	1.364	0.252

Note. Full variable names: Skilled = Postgraduate to total tertiary ratio; Schooling = Mean year of schooling; GPEA = Gross portfolio to GDP ratio; ICA = Insurance company assets to GDP ratio; IUS = Number of Internet users per 100 population; PFA = Pension fund assets to GDP ratio; AC = 2% of household final consumption expenditure to GDP ratio; AGEING = Aged-to-productive-population ratio (% change between 2009 and 2018)

C.2.3 Other Items with Data Concerns

Russia reported 67.96% and 61.01% of its total university degree receipients to be postgraduates for the year 2013 and 2015 respectively (2014 missing). This figure rapidly declines to 41.6% in 2016 and further down to 25.69% in 2017. Such volatility goes against the stable patterns shared by most countries in Figure C.1, casting doubt on data reliability. Separate investigation is therefore conducted using Russian government archive (Notes c to g in Table C.1).

Georgia underwent pension reform in 2018 with fund balance gradually transitioning to State Pension Agency for its official resumption of duty on 1 January 2019. Resultantly, 2018 pension balance for this country is unavailable but to be best appoximated using 2019 official data (Notes h, i and * of Table C.1).

Table C.2 documents the results of the abovementioned data recovery process.

C.3 Standardisation, Weights and FKI

Following Oliver-Márquez et al. (2020)'s procedure, all series in Table C.2 undergo min-max normalisation such that the smallest entry receives a new score of 0.01 and the biggest number is re-coded to 0.99. This slight deviation from the original paper (where the min-max normalisation yields 0 to 1) is to avoid multiplying a series by zero or raising a base to the power of zero.

Variable weights are calculated following Oliver-Márquez et al. (2020)'s recipe to be the inverses of each series' standard deviations. Whereas a sub-index combines more than one series, each weight is further divided by the sum of the constituent weights so that total weights add to one.

FKI is finally computed by taking the geometric mean of all four sub-indices, subject to sub-index-weights similar to variable weights above, as presented in Table C.3.

Table C.3
FKI and Sub-indices

	FKI	EC	ET	Use	Need
NLD	0.940	0.939	0.640	1.805	1.000
USA	0.937	0.990	0.589	0.856	1.406
CAN	0.784	0.858	0.409	1.637	0.953
ITA	0.762	0.767	0.602	1.069	0.807
FIN	0.724	0.850	0.685	1.127	0.562
ESP	0.627	0.735	0.464	0.635	0.726
LTU	0.613	0.664	0.632	0.243	0.836
PRT	0.591	0.639	0.401	0.630	0.762
BGR	0.583	0.396	0.760	0.384	0.729
EST	0.577	0.672	0.746	0.266	0.575
SVK	0.559	0.608	0.924	0.301	0.441
POL	0.555	0.595	0.699	0.286	0.572
LVA	0.550	0.573	0.633	0.161	0.795
CHL	0.544	0.449	0.302	0.761	0.908
RUS	0.450	0.536	0.597	0.083	0.639
GEO	0.424	0.141	0.547	0.210	0.997
SRB	0.423	0.249	0.500	0.193	0.742
PER	0.309	0.078	0.194	0.691	0.877
BRA	0.141	0.155	0.010	0.432	0.833
IDN	0.122	0.010	0.040	0.973	0.787

Note. Table sorted in descending order by countries' FKI. FKI = financial knowledge index, EC = Economic Capability, ET = Educational Training.

Appendix D Review of Matrix Calculus

D.1 Notations

Let us first establish the notation. This is important because bad notation is a serious obstacle to elegant mathematics and coherent exposition and it can be misleading.

Unless specified otherwise, φ denotes a scalar function; $\boldsymbol{f}$ a vector function and $\boldsymbol{F}$ a matrix function. Also, x denotes a scalar argument, $\boldsymbol{x}$ a vector argument and $\boldsymbol{X}$ a matrix argument. For example, we write

$$\begin{array}{lll} \varphi(x) = x^2 & \varphi(\boldsymbol{x}) = \boldsymbol{a}^\mathsf{T}\boldsymbol{x} & \varphi(\boldsymbol{X}) = \operatorname{tr}\left(\boldsymbol{X}^\mathsf{T}\boldsymbol{X}\right) \\ \boldsymbol{f}(x) = \begin{pmatrix} x \\ x^2 \end{pmatrix} & \boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{A}\boldsymbol{x} & \boldsymbol{f}(\boldsymbol{X}) = \boldsymbol{X}\boldsymbol{a} \\ \boldsymbol{F}(x) = x^2\boldsymbol{I}_m & \boldsymbol{F}(\boldsymbol{x}) = \boldsymbol{x}\boldsymbol{x}^\mathsf{T} & \boldsymbol{F}(\boldsymbol{X}) = \boldsymbol{X}^\mathsf{T} \end{array}$$

Since the prime notation $'$ may easily cause confusion between derivatives and transposes, preference is given to the Leibniz notation $\frac{\mathrm{d}}{\mathrm{d}x}$ for derivatives and $^\mathsf{T}$ for transposes—unless this system becomes too cumbersome, in which case $\boldsymbol{f}'(\boldsymbol{x})$ will denote derivatives and $\boldsymbol{f}(\boldsymbol{x})'$ for transposes.

D.2 Derivatives and differentials

D.2.1 Derivative

Definition D.2.1 (Derivatives)**.** If $\boldsymbol{f}$ is an $m \times 1$ vector function of an $n \times 1$ vector $\boldsymbol{x}$, then the *derivative* (or *Jacobian matrix*) of $\boldsymbol{f}$ is the $m \times n$ matrix

$$\mathrm{D}\boldsymbol{f}(\boldsymbol{x}) := \frac{\partial \boldsymbol{f}(\boldsymbol{x})}{\partial \boldsymbol{x}^\mathsf{T}}, \tag{D.1}$$

whose elements are the partial derivatives

$$\frac{\partial f_i(\boldsymbol{x})}{\partial x_j}, \text{ for } \begin{array}{l} i = 1, \cdots, m, \\ j = 1, \cdots, n. \end{array}$$

D.2.2 Differential

In the one dimensional case, the equation

$$\lim_{u \to 0} \frac{\varphi(x+u) - \varphi(x)}{u} = \varphi'(x) \tag{D.2}$$

defines the derivative of φ at x. Rewriting Equation (D.2) gives

$$\varphi(x+u) = \varphi(x) + \varphi'(x)u + O(u), \tag{D.3}$$

where the remainder term $O(u)$ quickly vanishes as u approaches 0.

Definition D.2.2 (Differential)**.** We define the (first) *differential* of φ at x (with increment u) as

$$\mathrm{d}\varphi(x; u) = \varphi'(x)u. \tag{D.4}$$

For example, for $\varphi(x) = x^2$, we have $\mathrm{d}\varphi(x; u) = 2xu$. In practice, we write $\mathrm{d}x$ instead of u, so that $\mathrm{d}\varphi(x) = \varphi'(x)\mathrm{d}x = 2x\mathrm{d}x$.

In the vector case, similar to Equation (D.3), we have

$$\boldsymbol{f}(\boldsymbol{x}+\boldsymbol{u}) = \boldsymbol{f}(\boldsymbol{x}) + [\mathrm{D}\boldsymbol{f}(x)]\boldsymbol{u} + O(\boldsymbol{u}), \tag{D.5}$$

and the (first) differential is defined as

$$\mathrm{d}\boldsymbol{f}(\boldsymbol{x}; \boldsymbol{u}) = [\mathrm{D}\boldsymbol{f}(x)]\boldsymbol{u}. \tag{D.6}$$

Although rarely used in econometrics, for completeness, the matrix case can be obtained from the vector case by writing $\boldsymbol{f} := \mathrm{vec}\,(\boldsymbol{F})$ and $\boldsymbol{x} := \mathrm{vec}\,(\boldsymbol{X})$.

D.2.3 Which to use?

For practical rather than theoretical reasons, the treatment of matrix calculus is based on differentials ($\mathrm{d}\boldsymbol{f}$) rather than derivatives ($\mathrm{D}\boldsymbol{f}$) because the former yields a result with the same dimension as $\boldsymbol{f}$. For example, consider $\underset{m\times 1}{\boldsymbol{f}}(\underset{n\times 1}{\boldsymbol{x}})$ (reading "$\boldsymbol{f}$ being an $m \times 1$ vector function of an $n \times 1$ vector $\boldsymbol{x}$"), $\mathrm{D}\boldsymbol{f}(\boldsymbol{x})$ is an $m \times n$ matrix (due to Definition D.2.1) whereas $\mathrm{d}\boldsymbol{f}(\boldsymbol{x})$ remains an $m \times 1$ vector (same as $\boldsymbol{f}$). The advantage is even larger for matrices: for

$\underset{m\times p}{\boldsymbol{F}}(\underset{n\times q}{\boldsymbol{X}})$, $\mathrm{d}\boldsymbol{F}(\boldsymbol{X})$ has the same dimension as $\boldsymbol{F}$ irrespective of the dimension of $\boldsymbol{X}$, but $\mathrm{D}\boldsymbol{F}(\boldsymbol{X})$ is going to be a horrendous $mp \times nq$ matrix.

D.3 Layout convention

Under the *numerator layout*, when we differentiate a scalar function φ with respect to a column vector $\underset{n\times 1}{\boldsymbol{x}}$, we get a *row* vector of dimension $1 \times n$. If we want our result to be in the column form, we must differentiate φ with respect to a row vector to start with. This is why the denominator in Equation (D.1) contains a transpose.

D.4 Application in OLS

D.4.1 Background

Imagine we are interested in learning the return on education. We might propose a rather simple model

$$\mathtt{inc} = \beta_0 + \beta_1 \mathtt{edu} + \beta_2 \mathtt{exp} + \varepsilon \tag{D.7}$$

where `inc` is one's income, `edu` and `exp` denote years of formal education and years spent in the labour market, respectively.

We managed to collect survey data from n respondents and organised this information in the following system of equations:

$$\begin{cases} \mathtt{inc}_1 = \beta_0 + \beta_1 \mathtt{edu}_1 + \beta_2 \mathtt{exp}_1 + \varepsilon_1 \\ \mathtt{inc}_2 = \beta_0 + \beta_1 \mathtt{edu}_2 + \beta_2 \mathtt{exp}_2 + \varepsilon_2 \\ \quad \dots \\ \mathtt{inc}_n = \beta_0 + \beta_1 \mathtt{edu}_n + \beta_2 \mathtt{exp}_n + \varepsilon_n \end{cases} \tag{D.8}$$

This system of linear equations can be represented in the matrix notation using

$$\underset{n\times 1}{\boldsymbol{y}} = \begin{pmatrix} \mathtt{inc}_1 \\ \mathtt{inc}_2 \\ \dots \\ \mathtt{inc}_n \end{pmatrix}, \ \underset{n\times 3}{\boldsymbol{X}} = \begin{pmatrix} 1 & \mathtt{edu}_1 & \mathtt{exp}_1 \\ 1 & \mathtt{edu}_2 & \mathtt{exp}_2 \\ \dots & & \\ 1 & \mathtt{edu}_n & \mathtt{exp}_n \end{pmatrix}, \ \underset{3\times 1}{\boldsymbol{\beta}} = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \end{pmatrix}, \text{ and } \underset{n\times 1}{\boldsymbol{\varepsilon}} = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \dots \\ \varepsilon_n \end{pmatrix} \tag{D.9}$$

as

$$\boldsymbol{y} = \boldsymbol{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}. \tag{D.10}$$

D.4.2 Ordinary least squares

The objective of OLS is to minimise the *sum of squared* error terms. A handy way of representing sum of squared ε is

$$\text{SSE} = \sum_{i=1}^{n} \varepsilon_i^2 = \varepsilon_1^2 + \varepsilon_2^2 + \cdots + \varepsilon_n^2 = \begin{pmatrix} \varepsilon_1 & \varepsilon_2 & \cdots & \varepsilon_n \end{pmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \cdots \\ \varepsilon_n \end{pmatrix} = \boldsymbol{\varepsilon}^\intercal \boldsymbol{\varepsilon}. \tag{D.11}$$

In fact, $\boldsymbol{x}^\intercal \boldsymbol{x}$ is the mathematical translation of "sum of squared" of $\boldsymbol{x}$.

Now we are ready to continue. We want to carefully choose a combination of β_0, β_1 and β_2 in order to make SSE as small as possible, ie

$$\min_{\boldsymbol{\beta}} \{\boldsymbol{\varepsilon}^\intercal \boldsymbol{\varepsilon}\} = \min_{\boldsymbol{\beta}} \{(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})^\intercal (\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})\} \tag{D.12}$$

(the equal sign is due to Equation (D.10)).

Two observations can be made from the minimisation problem in Equation (D.12):

1. both $\boldsymbol{y}$ and $\boldsymbol{X}$ are collected data therefore can no longer be changed by the researcher; but we are free to adjust $\boldsymbol{\beta}$ in whatever way we want, meaning $\boldsymbol{\beta}$ is the "independent variable" and SSE is a function of $\boldsymbol{\beta}$, and

2. $\boldsymbol{\varepsilon}^\intercal \boldsymbol{\varepsilon}$ is a scalar function (please verify).

Then,

$$\begin{aligned} \varphi(\boldsymbol{\beta}) = \boldsymbol{\varepsilon}^\intercal \boldsymbol{\varepsilon} &= (\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta})^\intercal (\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}) \\ &= \left(\boldsymbol{y}^\intercal - \boldsymbol{\beta}^\intercal \boldsymbol{X}^\intercal\right) (\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}) \\ &= \boldsymbol{y}^\intercal \boldsymbol{y} - \boldsymbol{y}^\intercal \boldsymbol{X}\boldsymbol{\beta} - \boldsymbol{\beta}^\intercal \boldsymbol{X}^\intercal \boldsymbol{y} + \boldsymbol{\beta}^\intercal \boldsymbol{X}^\intercal \boldsymbol{X}\boldsymbol{\beta} \end{aligned} \tag{D.13}$$

We now differentiate $\varphi(\boldsymbol{\beta})$ with respect to $\boldsymbol{\beta}$:

$$\begin{aligned}
\frac{\mathrm{d}\varphi(\boldsymbol{\beta})}{\mathrm{d}\boldsymbol{\beta}} &= -\boldsymbol{y}^\mathsf{T}\boldsymbol{X} - \frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\beta}}\left[\left(\boldsymbol{\beta}^\mathsf{T}\boldsymbol{X}^\mathsf{T}\boldsymbol{y}\right)^\mathsf{T}\right] + \boldsymbol{\beta}^\mathsf{T}\boldsymbol{X}^\mathsf{T}\boldsymbol{X} + \frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\beta}}\left[\left(\boldsymbol{\beta}^\mathsf{T}\boldsymbol{X}^\mathsf{T}\boldsymbol{X}\boldsymbol{\beta}\right)^\mathsf{T}\right] \\
&= -\boldsymbol{y}^\mathsf{T}\boldsymbol{X} - \frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\beta}}\left[\boldsymbol{y}^\mathsf{T}\boldsymbol{X}\boldsymbol{\beta}\right] + \boldsymbol{\beta}^\mathsf{T}\boldsymbol{X}^\mathsf{T}\boldsymbol{X} + \frac{\mathrm{d}}{\mathrm{d}\boldsymbol{\beta}}\left[\boldsymbol{\beta}^\mathsf{T}\boldsymbol{X}^\mathsf{T}\boldsymbol{X}\boldsymbol{\beta}\right] \\
&= -\boldsymbol{y}^\mathsf{T}\boldsymbol{X} - \boldsymbol{y}^\mathsf{T}\boldsymbol{X} + \boldsymbol{\beta}^\mathsf{T}\boldsymbol{X}^\mathsf{T}\boldsymbol{X} + \boldsymbol{\beta}^\mathsf{T}\boldsymbol{X}^\mathsf{T}\boldsymbol{X} \\
&= -2\boldsymbol{y}^\mathsf{T}\boldsymbol{X} + 2\boldsymbol{\beta}^\mathsf{T}\boldsymbol{X}^\mathsf{T}\boldsymbol{X}
\end{aligned} \tag{D.14}$$

(We were able to liberally apply transpose to terms containing $\boldsymbol{\beta}^\mathsf{T}$ and not to others because φ is a scalar function and each term in it must also be 1×1 in dimension, whose transpose must be equal to itself.)

Apply first order condition to Equation (D.14). An optimal $\widehat{\boldsymbol{\beta}}$ must satisfy

$$\begin{aligned}
-2\boldsymbol{y}^\mathsf{T}\boldsymbol{X} + 2\widehat{\boldsymbol{\beta}}^\mathsf{T}\boldsymbol{X}^\mathsf{T}\boldsymbol{X} &= \boldsymbol{O} \\
2\widehat{\boldsymbol{\beta}}^\mathsf{T}\boldsymbol{X}^\mathsf{T}\boldsymbol{X} &= 2\boldsymbol{y}^\mathsf{T}\boldsymbol{X} \\
\widehat{\boldsymbol{\beta}}^\mathsf{T}\boldsymbol{X}^\mathsf{T}\boldsymbol{X} &= \boldsymbol{y}^\mathsf{T}\boldsymbol{X} \\
\left(\widehat{\boldsymbol{\beta}}^\mathsf{T}\boldsymbol{X}^\mathsf{T}\boldsymbol{X}\right)^\mathsf{T} &= \left(\boldsymbol{y}^\mathsf{T}\boldsymbol{X}\right)^\mathsf{T} \\
\boldsymbol{X}^\mathsf{T}\boldsymbol{X}\widehat{\boldsymbol{\beta}} &= \boldsymbol{X}^\mathsf{T}\boldsymbol{y} \\
\widehat{\boldsymbol{\beta}} &= \left(\boldsymbol{X}^\mathsf{T}\boldsymbol{X}\right)^{-1}\boldsymbol{X}^\mathsf{T}\boldsymbol{y}
\end{aligned} \tag{D.15}$$

Notice that another transpose was applied to Line 4 of Equation (D.15) in order to correct $\widehat{\boldsymbol{\beta}}^\mathsf{T}$ (due to Section D.3) back to its column form $\widehat{\boldsymbol{\beta}}$. In fact, it would be better to do $\frac{\mathrm{d}\varphi(\boldsymbol{\beta})}{\mathrm{d}\boldsymbol{\beta}^\mathsf{T}}$ in Equation (D.14) to avoid this later flipping. But the downside of this approach is a pedagogical one: most students would find differentiating with respect to $\boldsymbol{\beta}^\mathsf{T}$ out of blue while with respect to $\boldsymbol{\beta}$ is much more natural. In further derivations, $\frac{\mathrm{d}\varphi(\boldsymbol{\beta})}{\mathrm{d}\boldsymbol{\beta}^\mathsf{T}}$ will be used.

Derivative of quadratic forms

The derivative of a quadratic form $q(\boldsymbol{x}) = \boldsymbol{x}^\mathsf{T}\boldsymbol{A}\boldsymbol{x}$ is

$$\frac{\mathrm{d}q}{\mathrm{d}\boldsymbol{x}} = \boldsymbol{x}^\mathsf{T}\left(\boldsymbol{A} + \boldsymbol{A}^\mathsf{T}\right),$$

which can be further simplified to $\mathrm{d}q/\mathrm{d}\boldsymbol{x} = 2\boldsymbol{x}^\mathsf{T}\boldsymbol{A}$, if $\boldsymbol{A}$ is symmetric.

Name the expression in the facebook post φ, which is a function of $\boldsymbol{\beta}$:

$$\varphi(\boldsymbol{\beta}) = \frac{1}{\sigma^2}\left(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\right)^\mathsf{T}\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\mathsf{T}\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\mathsf{T}\boldsymbol{\Omega}^{-1}\left(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\right).$$

Typo

There is a typo in the original post: all $\boldsymbol{\Omega}$ should be in the inverse form $\boldsymbol{\Omega}^{-1}$, including the one sandwiched between $\boldsymbol{Z}^\mathsf{T}$ and $\boldsymbol{Z}$.

Scalar function

Note that φ is a scalar function:

$$\underset{1\times 1}{\varphi}\left(\underset{k\times 1}{\boldsymbol{\beta}}\right) = \frac{1}{\sigma^2}\left(\underset{n\times 1}{\boldsymbol{y}} - \underset{n\times k}{\boldsymbol{X}}\underset{k\times 1}{\boldsymbol{\beta}}\right)^\mathsf{T}\underset{n\times n}{\boldsymbol{\Omega}^{-1}}\underset{n\times k}{\boldsymbol{Z}}\left(\underset{k\times n}{\boldsymbol{Z}^\mathsf{T}}\underset{n\times n}{\boldsymbol{\Omega}^{-1}}\underset{n\times k}{\boldsymbol{Z}}\right)^{-1}\underset{k\times n}{\boldsymbol{Z}^\mathsf{T}}\underset{n\times n}{\boldsymbol{\Omega}^{-1}}\left(\underset{n\times 1}{\boldsymbol{y}} - \underset{n\times k}{\boldsymbol{X}}\underset{k\times 1}{\boldsymbol{\beta}}\right).$$

When differentiating a scalar function φ with respect to a column vector $\boldsymbol{\beta}$, the result is a *row* vector. If this is undesirable, differentiate the scalar function φ with respect to the *transpose* of that vector $\boldsymbol{\beta}^\mathsf{T}$.

I want to know

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\boldsymbol{\beta}} = \frac{\mathrm{d}\varphi}{\mathrm{d}\left(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\right)}\frac{\mathrm{d}\left(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\right)}{\mathrm{d}\boldsymbol{\beta}} = \frac{\mathrm{d}\varphi}{\mathrm{d}\left(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\right)}\left(-\boldsymbol{X}\right),$$

so I first calculate (using the result from quadratic form derivatives stated at the beginning)

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\left(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\right)} = \frac{2}{\sigma^2}\left(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\right)^\mathsf{T}\left[\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\mathsf{T}\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\mathsf{T}\boldsymbol{\Omega}^{-1}\right].$$

Therefore,

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\boldsymbol{\beta}} = -\frac{2}{\sigma^2}\left(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\right)^\mathsf{T}\left[\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\mathsf{T}\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\mathsf{T}\boldsymbol{\Omega}^{-1}\right]\boldsymbol{X}$$

$$\frac{\mathrm{d}\varphi}{\mathrm{d}\boldsymbol{\beta}^\mathsf{T}} = -\frac{2}{\sigma^2}\boldsymbol{X}^\mathsf{T}\left[\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\mathsf{T}\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\mathsf{T}\boldsymbol{\Omega}^{-1}\right]\left(\boldsymbol{y} - \boldsymbol{X}\boldsymbol{\beta}\right)$$

(The second line is to avoid working with row vectors.)

Second derivative

The second derivative of φ is

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}\boldsymbol{\beta}^\intercal \mathrm{d}\boldsymbol{\beta}} = \frac{2}{\sigma^2}\boldsymbol{X}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{X},$$

which is a positive definite $k \times k$ matrix (another quadratic form). This implies that the result from the first order condition below is a minimum.

Impose the first order condition:

$$\begin{aligned}\boldsymbol{X}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\left(\boldsymbol{y} - \boldsymbol{X}\widehat{\boldsymbol{\beta}}_{\text{GLS-IV}}\right) &= \boldsymbol{0} \\ \boldsymbol{X}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{y} &= \boldsymbol{X}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{X}\widehat{\boldsymbol{\beta}}_{\text{GLS-IV}}\end{aligned}$$

Therefore:

$$\begin{aligned}\widehat{\boldsymbol{\beta}}_{\text{GLS-IV}} &= \left(\boldsymbol{X}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{X}\right)^{-1}\boldsymbol{X}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{y} \\ &= \left(\boldsymbol{X}^\intercal\boldsymbol{\Omega}^{-1}\widehat{\boldsymbol{X}}\right)^{-1}\boldsymbol{X}^\intercal\boldsymbol{\Omega}^{-1}\widehat{\boldsymbol{y}} \\ &\neq \left(\widehat{\boldsymbol{X}}^\intercal\boldsymbol{\Omega}^{-1}\widehat{\boldsymbol{X}}\right)^{-1}\widehat{\boldsymbol{X}}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{y}, \text{ satisfying the claim in the facebook post.}\end{aligned}$$

However, since $\widehat{\boldsymbol{X}}$ is the GLS-IV-estimator of $\boldsymbol{X}$ onto the $\boldsymbol{Z}$-space:

$$\begin{aligned}\widehat{\boldsymbol{X}} &= \boldsymbol{Z}\left(\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{X}, \text{ and} \\ \widehat{\boldsymbol{X}}^\intercal &= \boldsymbol{X}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\intercal.\end{aligned}$$

The last expression in the facebook post then becomes:

$$\begin{aligned}&\left(\widehat{\boldsymbol{X}}^\intercal\boldsymbol{\Omega}^{-1}\widehat{\boldsymbol{X}}\right)^{-1}\widehat{\boldsymbol{X}}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{y} \\ =&\left(\boldsymbol{X}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{X}\right)^{-1}\boldsymbol{X}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{y} \\ =&\left(\boldsymbol{X}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{X}\right)^{-1}\boldsymbol{X}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\left(\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{Z}\right)^{-1}\boldsymbol{Z}^\intercal\boldsymbol{\Omega}^{-1}\boldsymbol{y} \\ =&\widehat{\boldsymbol{\beta}}_{\text{GLS-IV}}\end{aligned}$$

After all, “one might have guessed” correctly!