UiO : CEMO – Centre for Educational Measurement
University of Oslo

Structural Equation Modeling of PIAAC Data

Data Analysis with M*plus*

Ronny Scherer

International PIAAC Research Conference
GESIS Mannheim
22.-23.03.2022

Welcome!

Some Housekeeping...

Agenda—Day 1

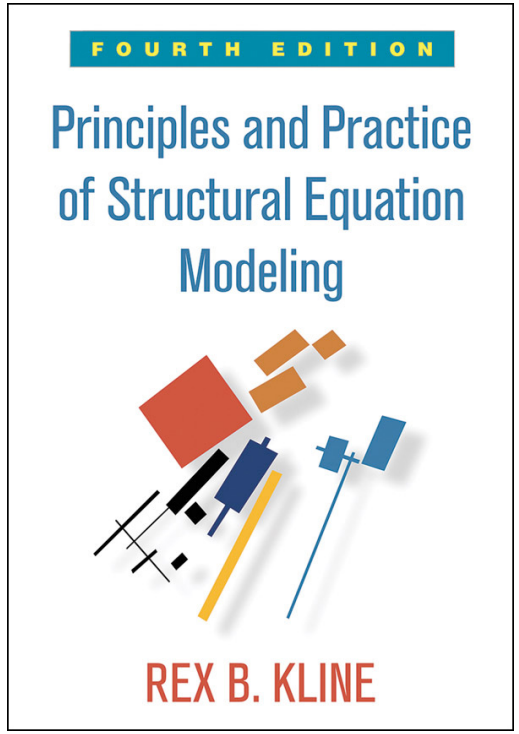
Time	Content
09:00-10:15	**Session 1:** Principles of structural equation modeling
10:15-10:30	*Second breakfast break*
10:30-11:45	**Session 2:** Complexities of the PIAAC data, regression and path models
11:45-12:30	*Lunch break*
12:30-14:00	**Session 3:** Confirmatory factor analysis, structural regression and indirect effects models

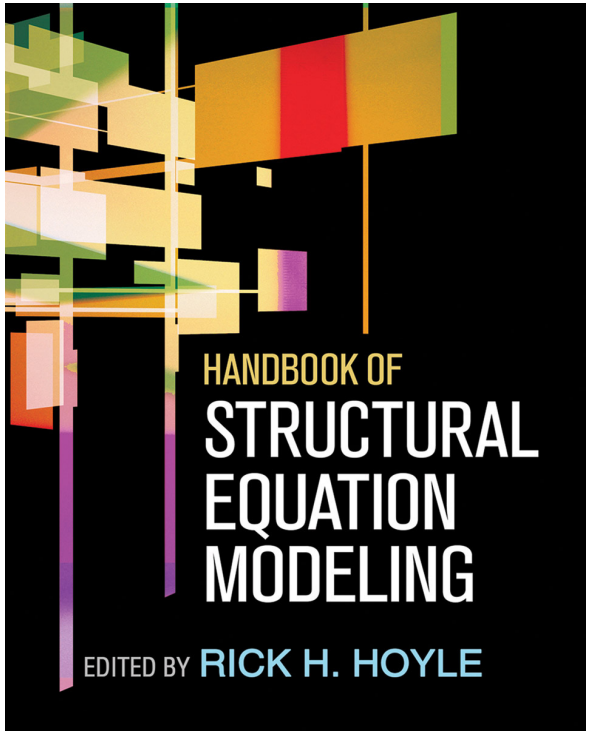
Agenda—Day 2

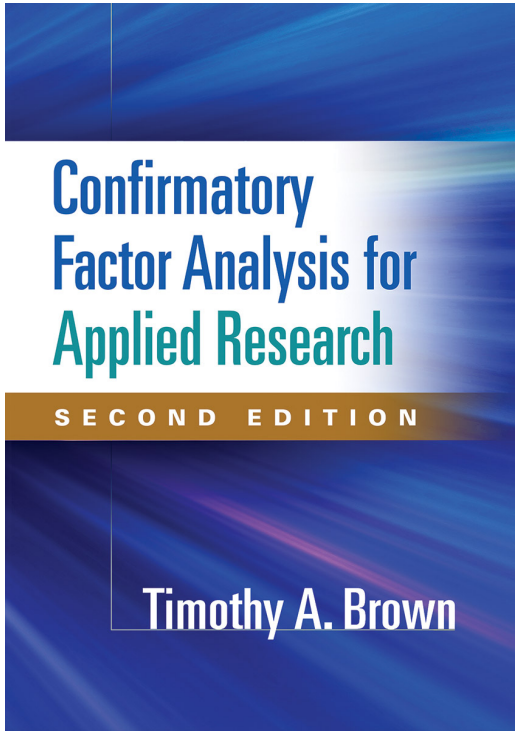
Time	Content
09:00-10:15	**Session 4:** Structural regression and indirect effects models (*cont'd*), multi-group structural equation modeling
10:15-10:30	*Second breakfast break*
10:30-11:45	**Session 5:** Multi-group structural equation modeling (*cont'd*), alignment optimization
11:45-12:30	*Lunch break*
12:30-14:00	**Session 6:** Multilevel structural equation modeling

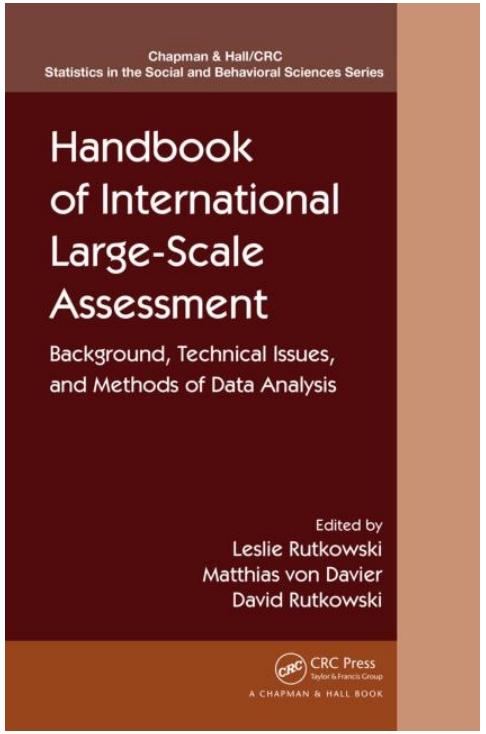
Resources

Some Recommended Literature

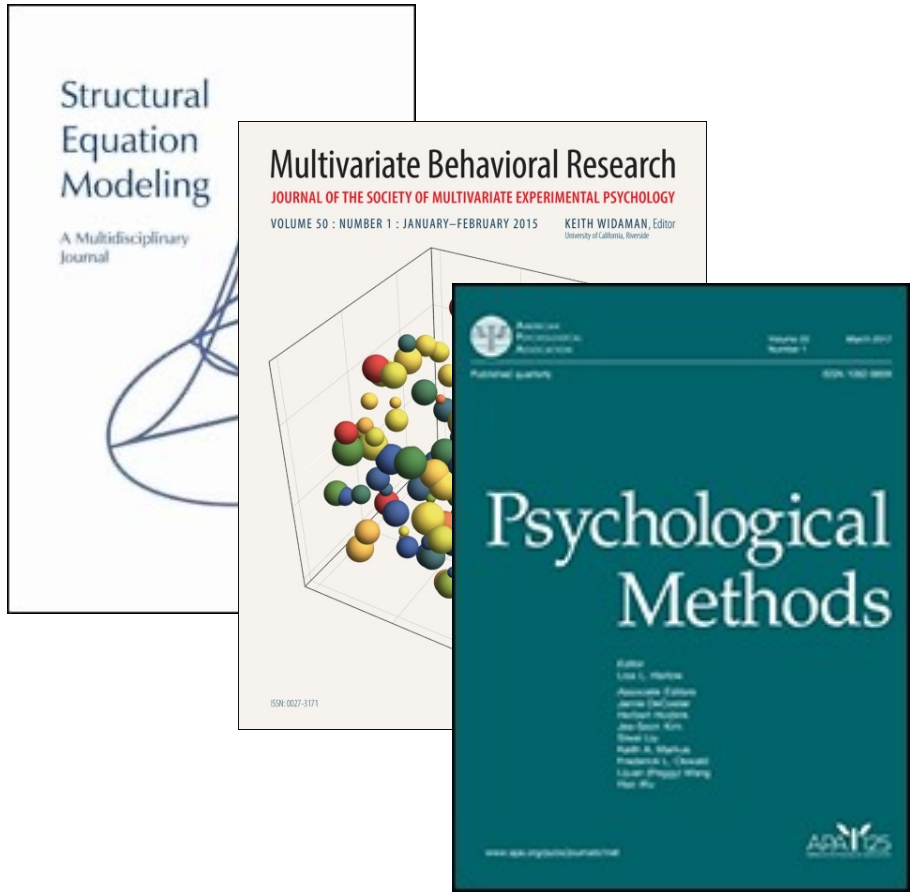


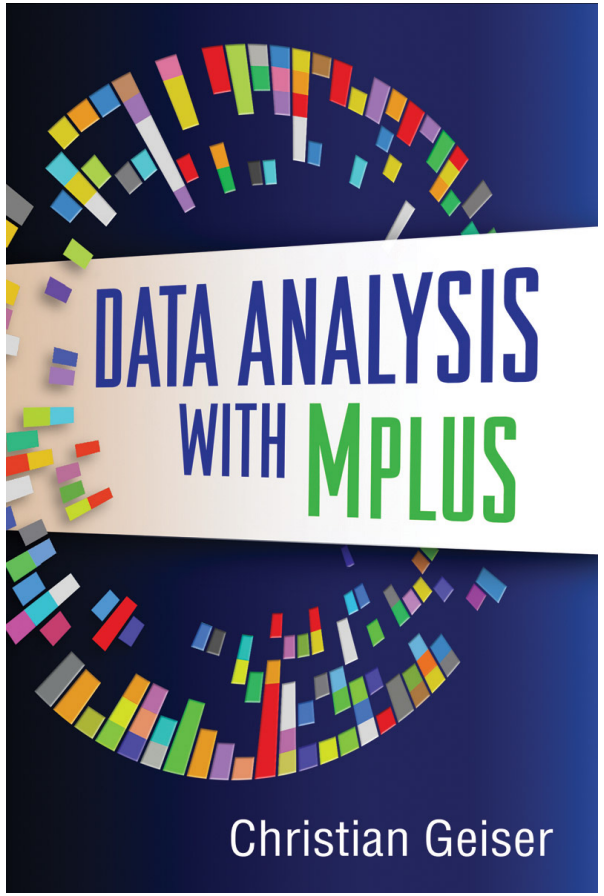


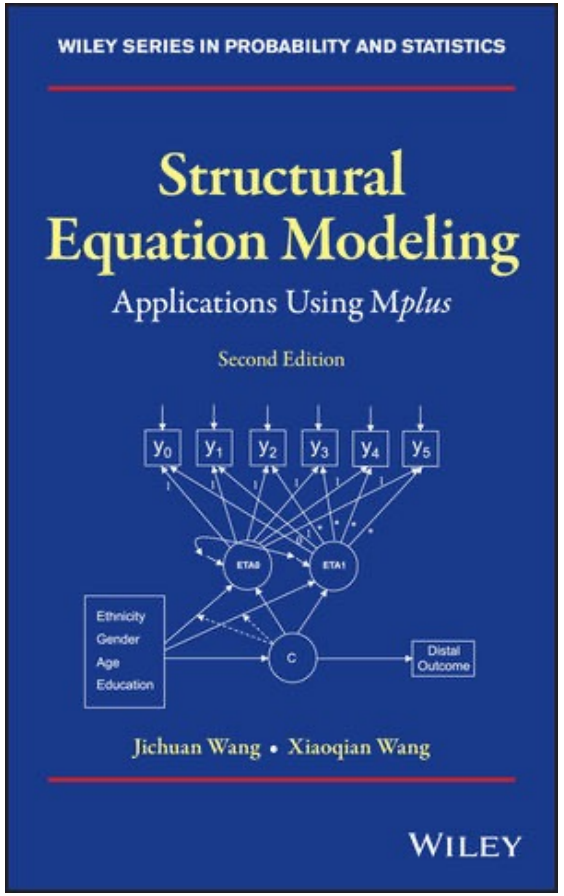




Some Recommended Literature



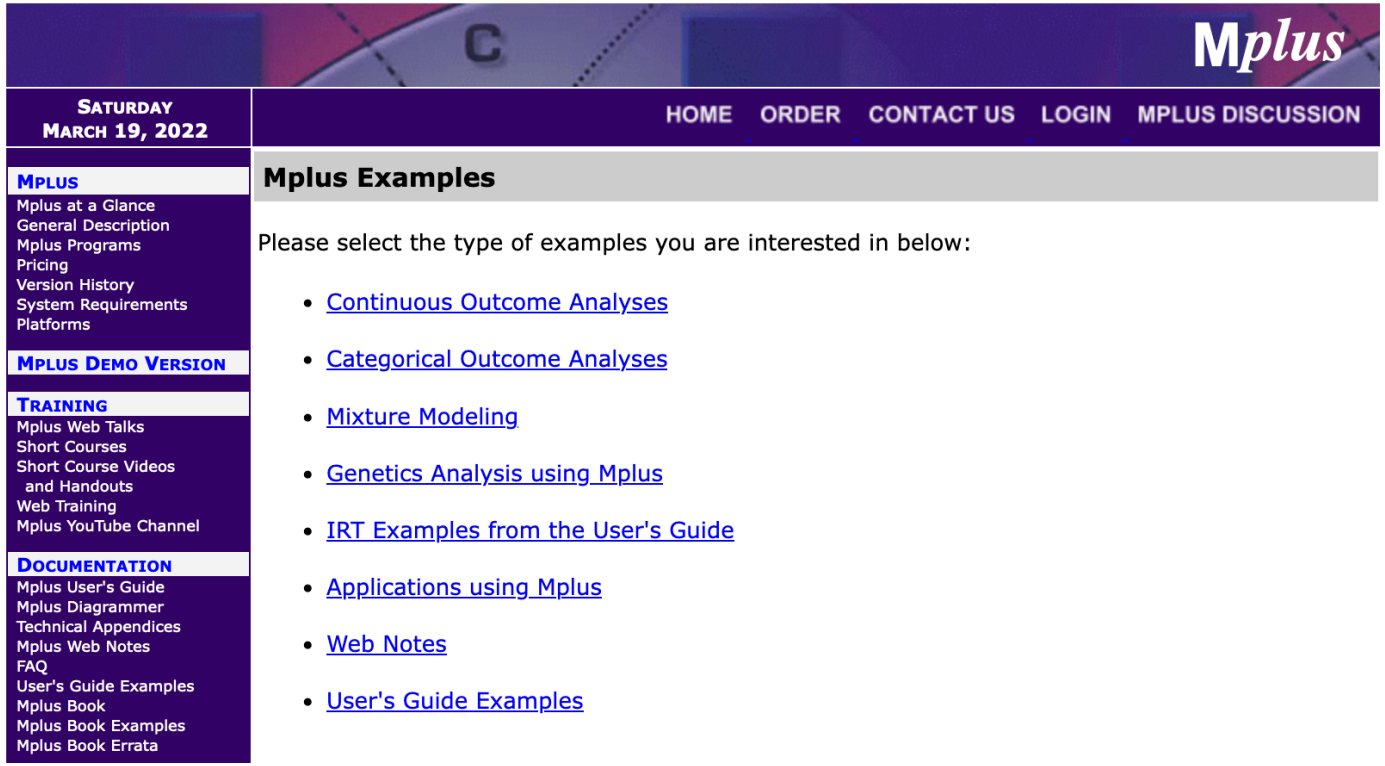




Software

Mplus

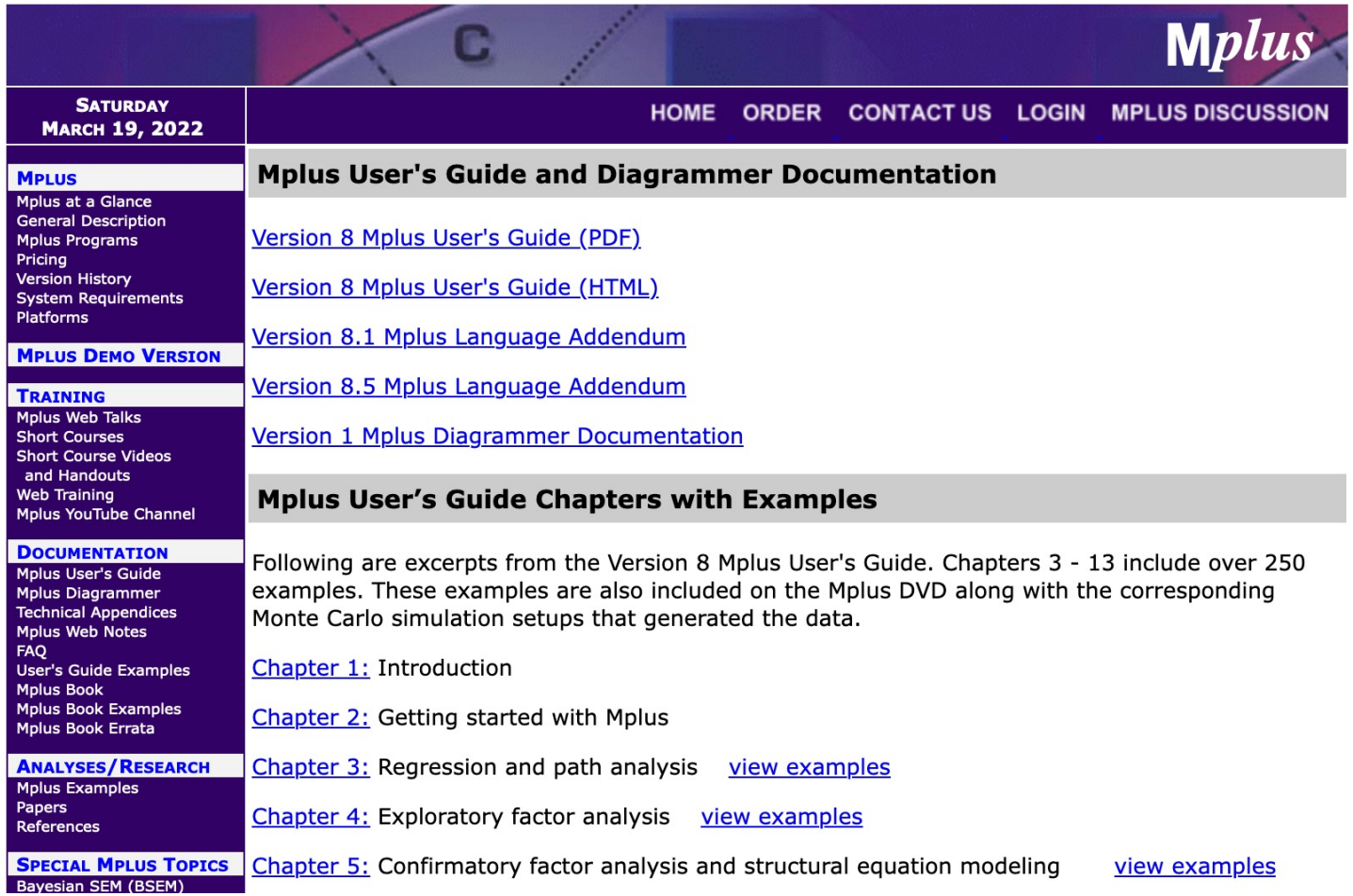
https://www.statmodel.com



Software

Mplus

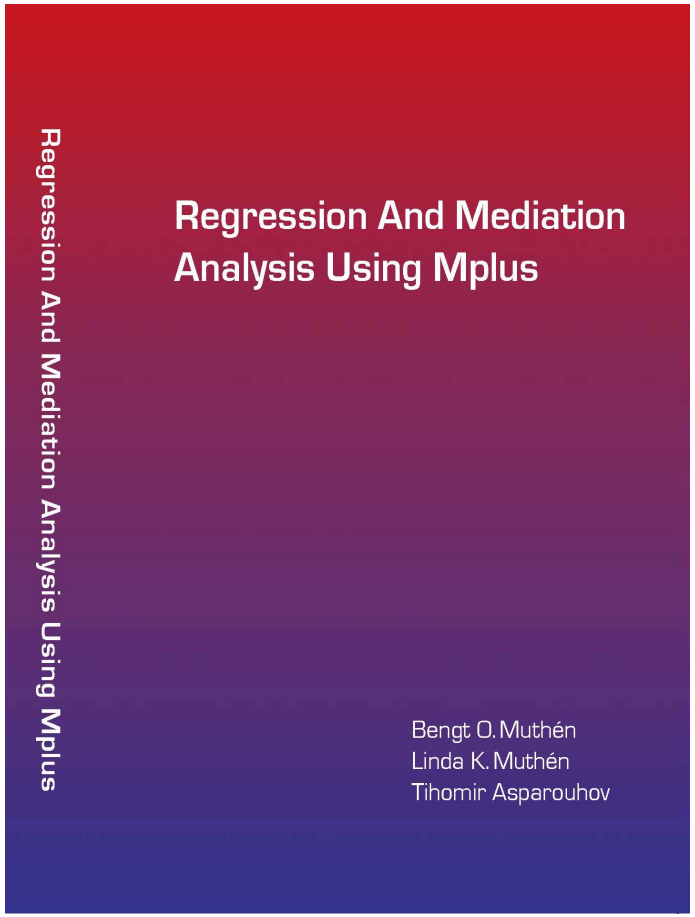
https://www.statmodel.com

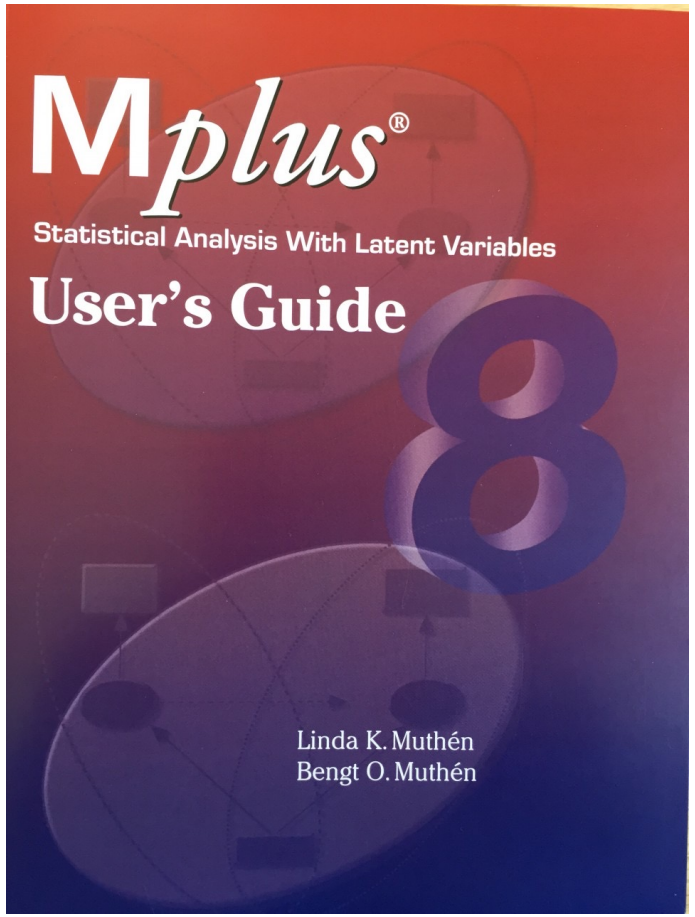


Software

Mplus

https://www.statmodel.com





Software

Mplus

https://www.statmodel.com

Diagrammer

Mplus Editor

MplusAutomation: An R Package for Facilitating Large-Scale Latent Variable Analyses in Mplus

Leverages the R language to automate latent variable model estimation and interpretation using 'Mplus', a powerful latent variable modeling program developed by Muthen and Muthen (<https://www.statmodel.com>). Specifically, this package provides routines for creating related groups of models, running batches of models, and extracting and tabulating model parameters and fit statistics.

Version:	1.0.0
Depends:	R (≥ 3.5.0), methods
Imports:	utils, boot, plyr, gsubfn, coda, xtable, lattice, texreg, pander, digest, parallel, ggplot2, data.table, rlang, fastDummies, checkmate
Suggests:	rhdf5, tcltk, relimp, knitr, testthat (≥ 3.0.0), rmarkdown
Published:	2021-07-01
Author:	Michael Hallquist [aut, cre], Joshua Wiley [aut], Caspar van Lissa [ctb], Daniel Morillo [ctb]
Maintainer:	Michael Hallquist <michael.hallquist at gmail.com>
License:	LGPL-3
URL:	https://github.com/michaelhallquist/MplusAutomation
NeedsCompilation:	no
Citation:	MplusAutomation citation info
Materials:	README NEWS
In views:	Psychometrics
CRAN checks:	MplusAutomation results

R

https://cran.r-project.org/web/packages/MplusAutomation/index.html

PIAAC Data and Workshop Material

OECD

https://www.oecd.org/skills/piaac/

OECD Skills Surveys

HOME | ABOUT | PIAAC DESIGN | EVENTS | DATA | PUBLICATIONS | ONLINE ASSESSMENT

Survey of Adult Skills (PIAAC)

About PIAAC

The **Programme for the International Assessment of Adult Competencies** (PIAAC) is a programme of assessment and analysis of adult skills. The major survey conducted as part of PIAAC is the **Survey of Adult Skills.** The Survey measures adults' proficiency in key information-processing skills - literacy, numeracy and problem solving - and gathers information and data on how adults use their skills at home, at work and in the wider community.

This international survey is conducted **in over 40 countries/economies** and workplace skills needed for individuals to participate in society and for econo

Learn more about how we measure and collect data.

+ PIAAC Data
+ Documentation
+ Tools for Analysis
+ Explore the Data

OSF HOME

PIAAC SEM Workshop 2022 | Files | Wiki | Analytics

PIAAC SEM Workshop 2022

Contributors: Ronny Scherer

Date created: 2022-03-05 05:54 PM | La

Identifier: DOI 10.17605/OSF.IO/5V2D9

Category: Project

https://osf.io/5v2d9/

Name

- PIAAC SEM Workshop 2022
- OSF Storage (Germany - Frankfurt)
  - Administration
  - Data
  - Examples
  - Slides

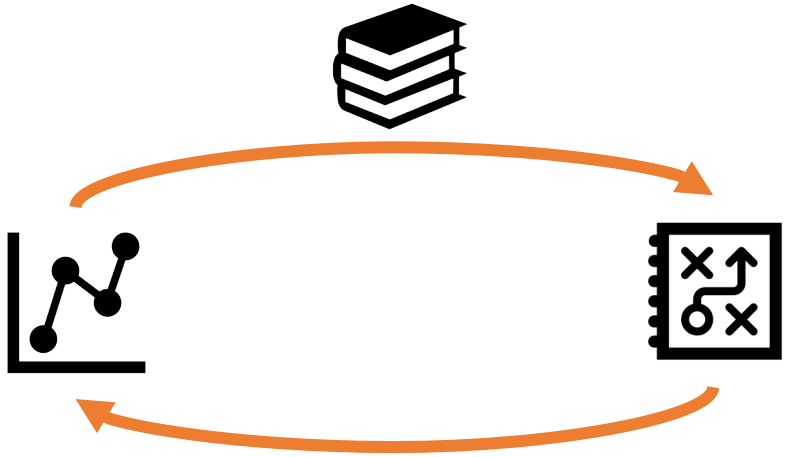
Principles of Structural Equation Modeling (SEM)

Structural Equation Modeling (SEM)—Why?

Definition

"Structural equation modeling can be defined as a class of methodologies that seeks to **represent hypotheses** about the *means, variances, and covariances* of observed data in terms of a smaller number of 'structural' parameters defined by a **hypothesized underlying conceptual or theoretical model.**"

(Kaplan, 2001, p. 15215)



SEM is a **multivariate** statistical modelling technique.
We postulate a model, which may or may not fit the data.

(Kline, 2016)

Structural Equation Modeling (SEM)—What?

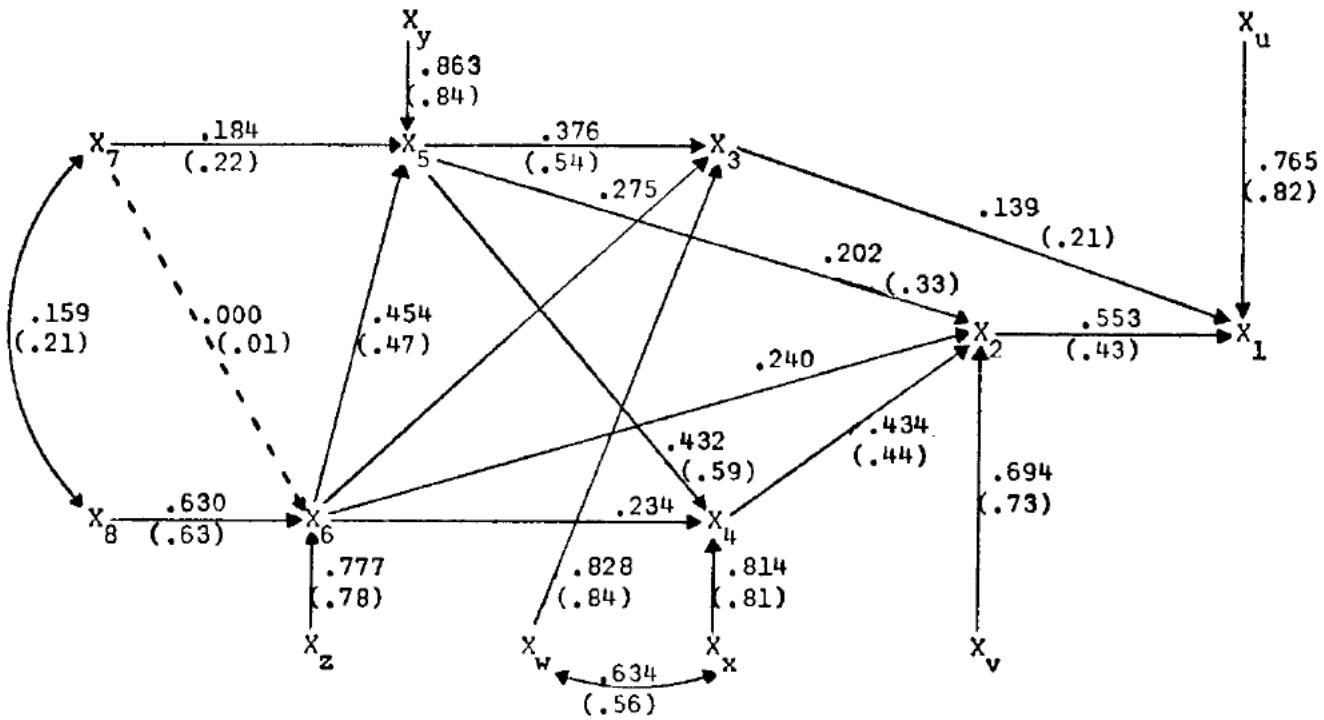
THE EDUCATIONAL AND EARLY OCCUPATIONAL STATUS ATTAINMENT PROCESS: REPLICATION AND REVISION [1]

WILLIAM H. SEWELL, ARCHIBALD O. HALLER AND GEORGE W. OHLENDORF
University of Wisconsin

American Sociological Review 1970, Vol. 35 (December):1014–1027

Relations between attainment, aspiration, socioeconomic status, mental ability, and academic performance

FIGURE 1. PATH COEFFICIENTS FOR ANTECEDENTS OF EDUCATIONAL AND OCCUPATIONAL ATTAINMENT WITH REVISED MODEL FOR FARM BOYS *



X_1 - Occupational Attainment
X_2 - Educational Attainment
X_3 - Level of Occupational Aspiration
X_4 - Level of Educational Aspiration
X_5 - Significant Others' Influence
X_6 - Academic Performance
X_7 - Socioeconomic Status
X_8 - Mental Ability

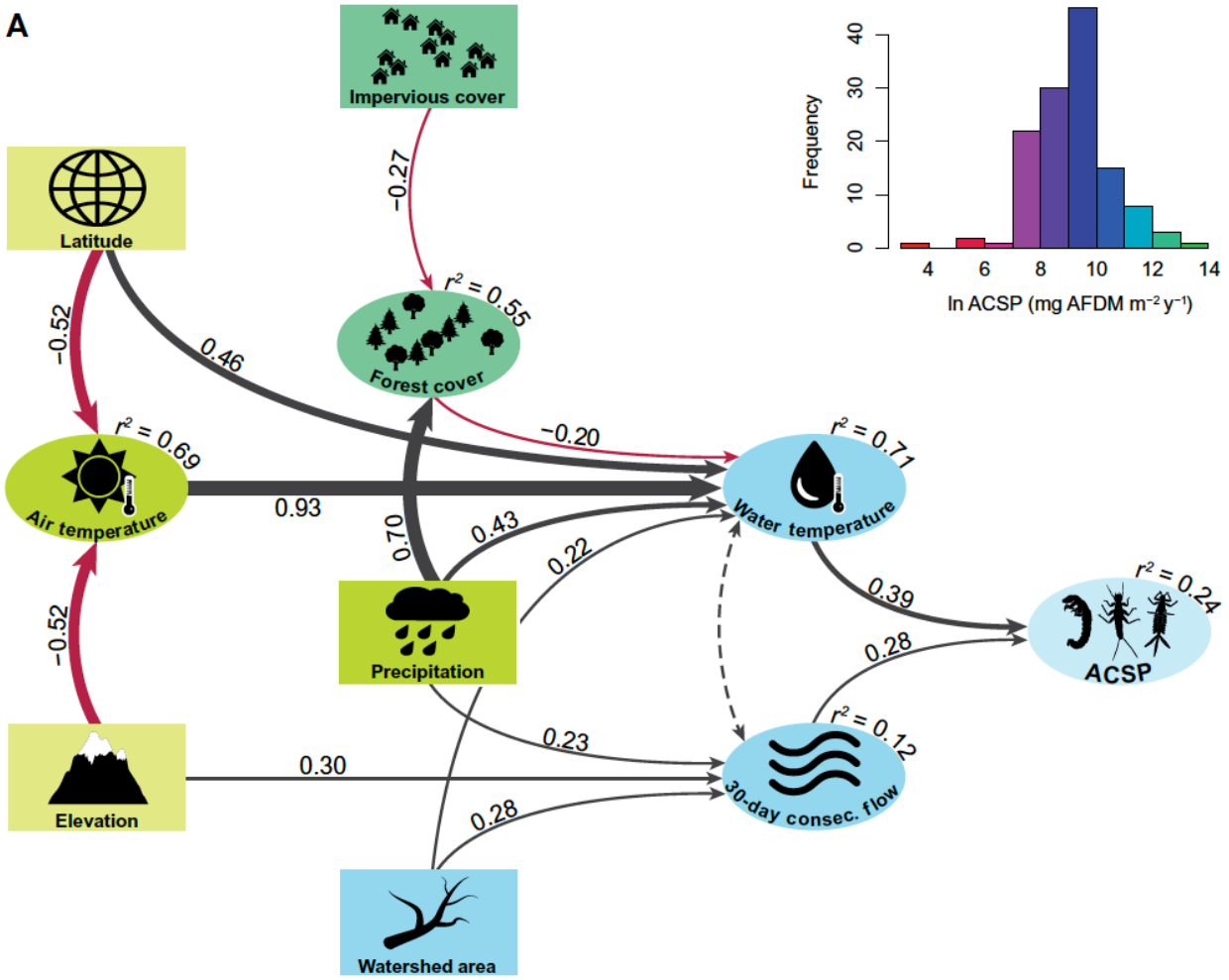
Structural Equation Modeling (SEM)—What?

SCIENCE ADVANCES | RESEARCH ARTICLE

ECOLOGY

Precipitation and temperature drive continental-scale patterns in stream invertebrate production

C. J. Patrick[1]*, D. J. McGarvey[2], J. H. Larson[3], W. F. Cross[4], D. C. Allen[5], A. C. Benke[6], T. Brey[7], A. D. Huryn[6], J. Jones[8], C. A. Murphy[9], C. Ruffing[10], P. Saffarinia[11], M. R. Whiles[12], J. B. Wallace[13], G. Woodward[14]



Identifying drivers of annual community secondary production (ACSP), defined as the sum of annual production of all invertebrate populations within a community (*17*), is particularly challenging ...

Structural Equation Modeling (SEM)—What?

Relations between creativity, intelligence, and executive functions

Intelligence 46 (2014) 73–83

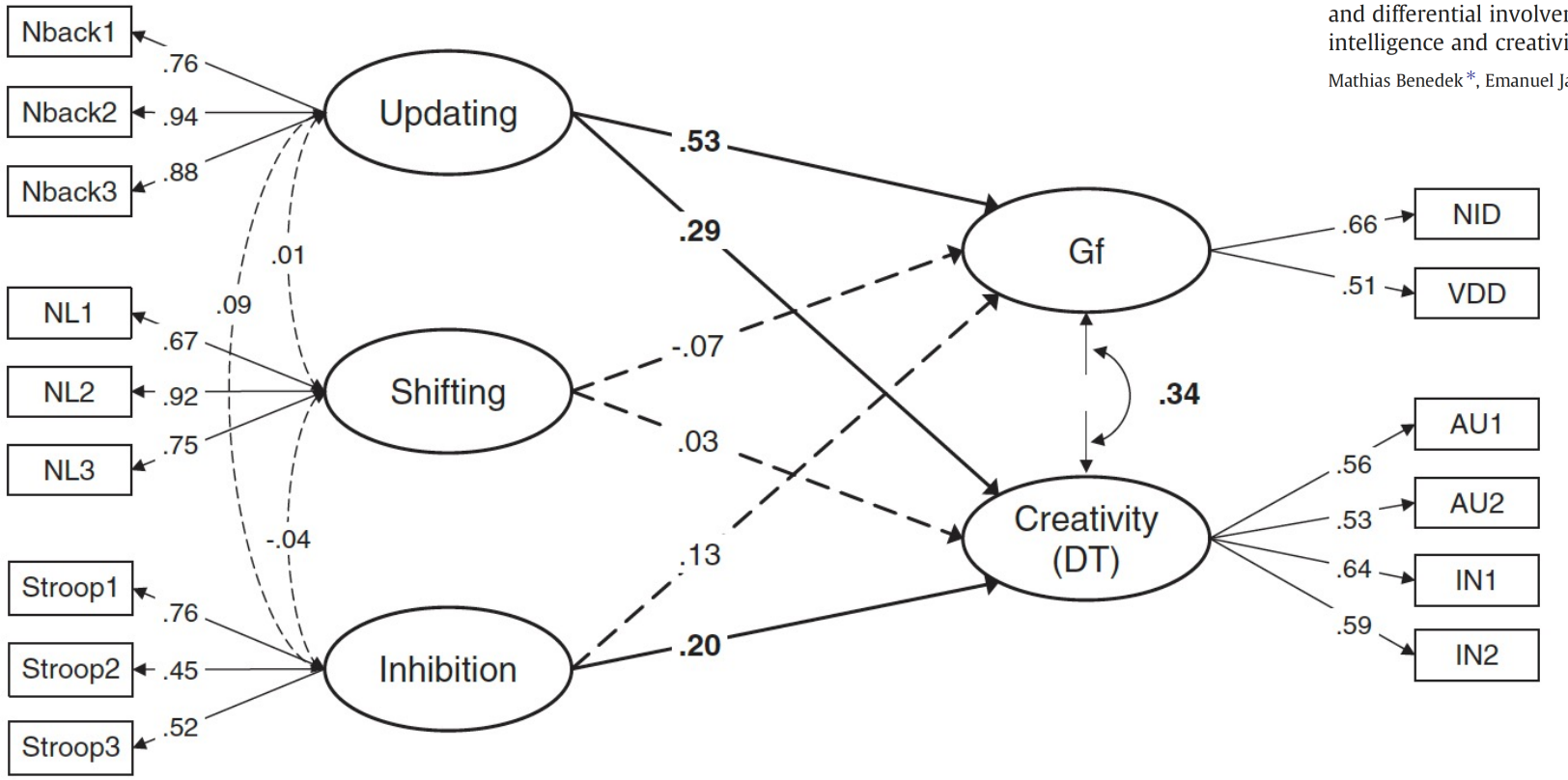
Contents lists available at ScienceDirect

Intelligence

ELSEVIER

Intelligence, creativity, and cognitive control: The common and differential involvement of executive functions in intelligence and creativity

Mathias Benedek*, Emanuel Jauk, Markus Sommer, Martin Arendasy, Aljoscha C. Neubauer



Structural Equation Modeling (SEM)—Why?

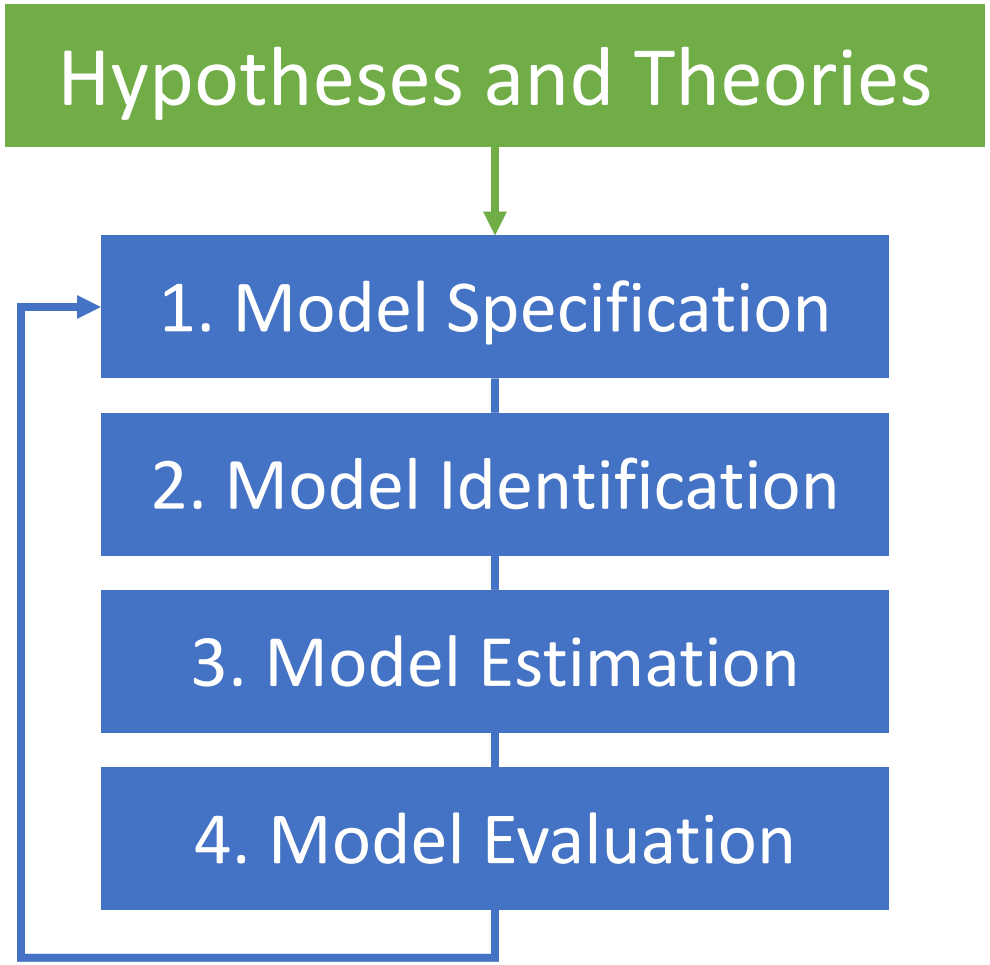
What is so special about SEM?

- SEM offers a broad range of models, including mediation models, multi-group models, multilevel models, growth curve models, …
- SEM can contain latent (unobserved) variables to represent constructs (i.e., measurement models) → Include random effects and control measurement error
- SEM can estimate derived effects, such as indirect or contextual effects.
- SEM can handle raw *and* summary data (i.e., covariance matrix and mean vector).
- SEM can be represented by path diagrams.

(Rosseel, 2020; Kline, 2016)

Structural Equation Modeling (SEM)—How?

(Hoyle, 2012)



Key steps to take in structural equation modeling

Structural Equation Modeling (SEM)—How?

(Hoyle, 2012)

Hypotheses and Theories

1. Model Specification
2. Model Identification
3. Model Estimation
4. Model Evaluation

The "Grammar" of a model:

Translate the hypotheses/theories into testable models.

- Specification equations
- Structural and measurement part
- Define regressions, (co-)variances, latent variables, means/intercepts, …

Some specification commands in M*plus*:

- Regression: `Y ON X;`
- Covariance: `Y WITH X;`
- Latent variable: `eta BY x1 x2 x3;`

Structural Equation Modeling (SEM)—How?

(Hoyle, 2012)

Hypotheses and Theories

1. Model Specification

2. Model Identification

3. Model Estimation

4. Model Evaluation

The "Feasibility Check" of a model:

Do we have enough information to estimate all model parameters?

- Observed information (from the data): variances, covariances, means
- Model parameters: variances, covariances, regression coefficients, intercepts, …

Rule:
Have at least the same amount of observed information than model parameters → degrees of freedom of the model ≥ 0

Structural Equation Modeling (SEM)—How?

(Hoyle, 2012)

Hypotheses and Theories

1. Model Specification

2. Model Identification

3. Model Estimation

4. Model Evaluation

Estimating model parameters:

Reduce the discrepancy between the observed covariance matrix (S) and the covariance matrix implied by the model (Σ).

- Maximum likelihood estimation (`ML`)
- Weighted least squares (`WLS`, `WLSMV`)
- Bayesian estimation (`BAYES`)
- Robust maximum likelihood (`MLR`)
- …

Structural Equation Modeling (SEM)—How?

(Hoyle, 2012)

Hypotheses and Theories

1. Model Specification
2. Model Identification
3. Model Estimation
4. Model Evaluation

Evaluate the model fit:

Well-fitting models show a small discrepancy between the observed covariance matrix (S) and the covariance matrix implied by the model (Σ).

Global fit evaluation:

- Chi-square statistic χ^2
- Comparative Fit Index (CFI)
- Root Mean Square Error of Approximation (RMSEA)
- Standardized Root Mean Square Residual (SRMR)
- …

Structural Equation Modeling (SEM)—How?

(Brown, 2015; Hu & Bentler, 1999; Marsh et al., 2005)

4. Model Evaluation

Common criteria for an acceptable model fit:

- Chi-square statistic χ^2 insignificant ($p > .05$)
- Comparative Fit Index (CFI) close to .95 or greater
- Root Mean Square Error of Approximation (RMSEA) close to .08 or smaller
- Standardized Root Mean Square Residual (SRMR) close to .08 or smaller

Local fit evaluation—Detecting where misfit may lie:

- **Model comparisons** (e.g., differences in fit indices)
- **Residuals (S-Σ)**—How well can (co-)variances be reproduced?
- **Modification indices**—Overall decrease in χ^2 when freeing a parameter (e.g., expected parameter change [EPC])

Structural Equation Modeling (SEM)—How?

4. Model Evaluation

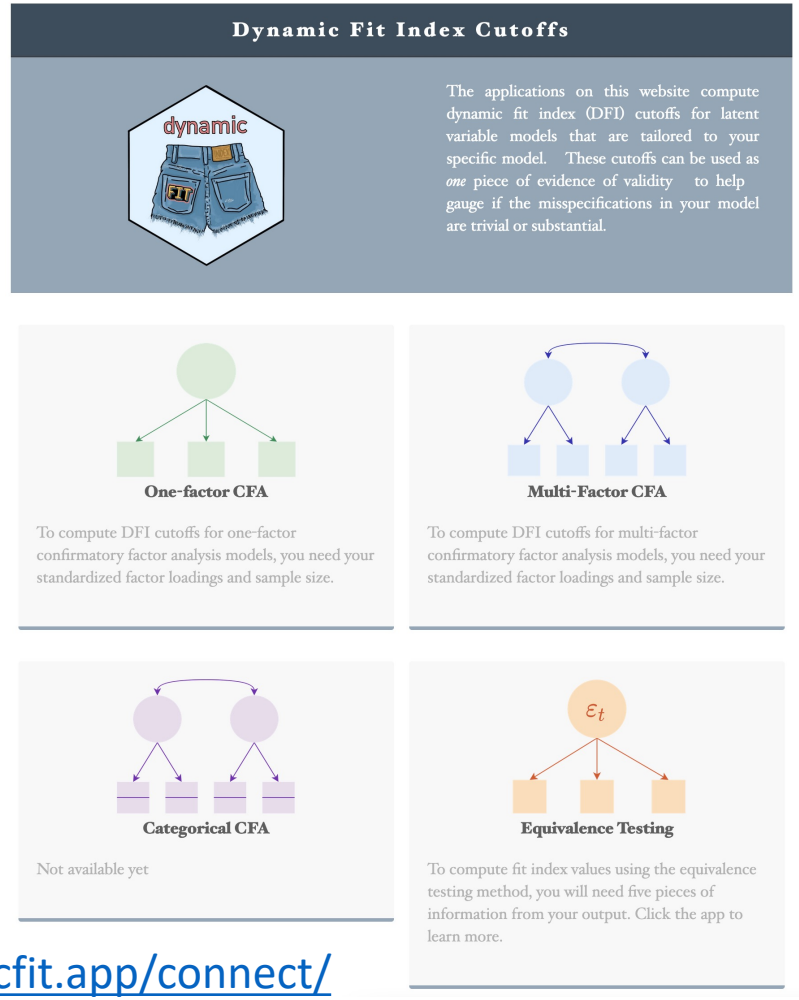
Problem:

Common guidelines were developed based on a set of limited factor models

Solution:

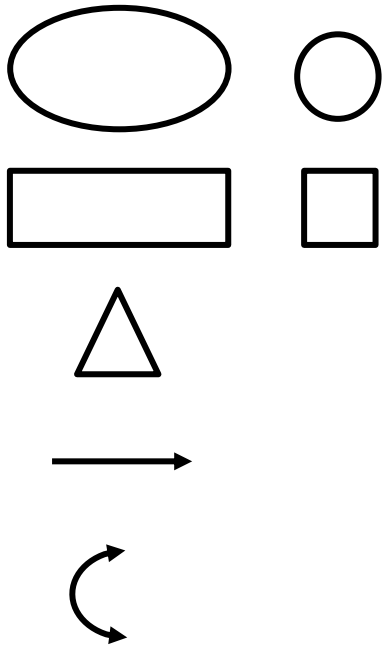
(McNeish & Wolf, 2021)

Dynamic Fit Index Cutoffs that are tailored to the specific structural equation model and its complexities (e.g., number of factors, sizes of factor loadings, residual covariances)



https://dynamicfit.app/connect/

Path Diagrams



Latent (unobserved) variables (e.g., residuals, constructs)

Manifest (observed) variables (e.g., items, indices)

Constants [1] (e.g., intercepts, factor means)

Directional relations (e.g., path coefficients, factor loadings)

Non-directional relations (e.g., variances, covariances)

Complexities of the PIAAC Data

PIAAC's Complex Sampling Design

PIAAC Technical Report

"Inferences will not be valid unless the corresponding variance estimators appropriately reflect all of the complex features of the PIAAC sample design (e.g., stratification and clustering). The replication approach is used for estimating variances for the international analyses of PIAAC data."

(OECD, 2019, chap. 15, p. 33)

Core elements:

Weighting, clustering, stratification, and plausible values

(e.g., Rust, 2014; Rutkowski & Zhou, 2014)

Weighting

Sampling weights

- Adjust for differences in random selection probabilities
- Adjust for non-responses
- Different scaling procedures possible

M*plus* options:

- `WEIGHT=SPFWT0`
- `WTSCALE=CLUSTER`
- `WTSCALE=SAMPLE`

Replicate weights

Under the replication approach, subsamples (also known as replicates) from the full sample are formed and statistics of the subsamples are used to estimate the variance of the full sample statistic.

(OECD, 2019, chap. 15, p. 33)

M*plus* options:

- `REPWEIGHTS=SPFWT1-SPFWT80`
- `REPSE=JACKKNIFE`
- `REPSE=JACKKNIFE2`
- `REPSE=BOOTSTRAP`

Clustering and Stratification

Clustering and stratification

- Multi-stage sampling design
- Several sampling strata (e.g., age, gender, SES, regions)

Table 14.8a: Main study sample units for countries with two stages of sampling – Round 1

Country		Stage 1	Stage 2
Cyprus [15]		Households	Persons
France		Area PSUs	Persons
Germany		Communities	Persons
Japan		Cho/Chome/Aza administrative districts	Persons
Poland	Urban	Towns/Cities	Persons

(OECD, 2019, chap. 14, p. 17)

Table 14.15a: Main Study stratification/sorting variables and methods for countries with one stage of selection – Round 1

Country	Description
Austria	Sort by province, urban/rural, age, gender and citizenship
Denmark	Strata: age categories, immigration status
Estonia	Strata: gender and age categories Within strata: sort by region and age
Finland	Strata: native language (Finnish and other languages than Swedish, and Swedish) Within strata: sort by region, age, educational attainment, and gender
Flanders (Belgium)	Strata: province Within strata: sort by postal code, gender and age
Netherlands	Strata: municipality
Norway	Strata: level of education and age group
Sweden	Strata: gender, age, country of birth, level of education

(OECD, 2019, chap. 14, p. 17)

M*plus* options:

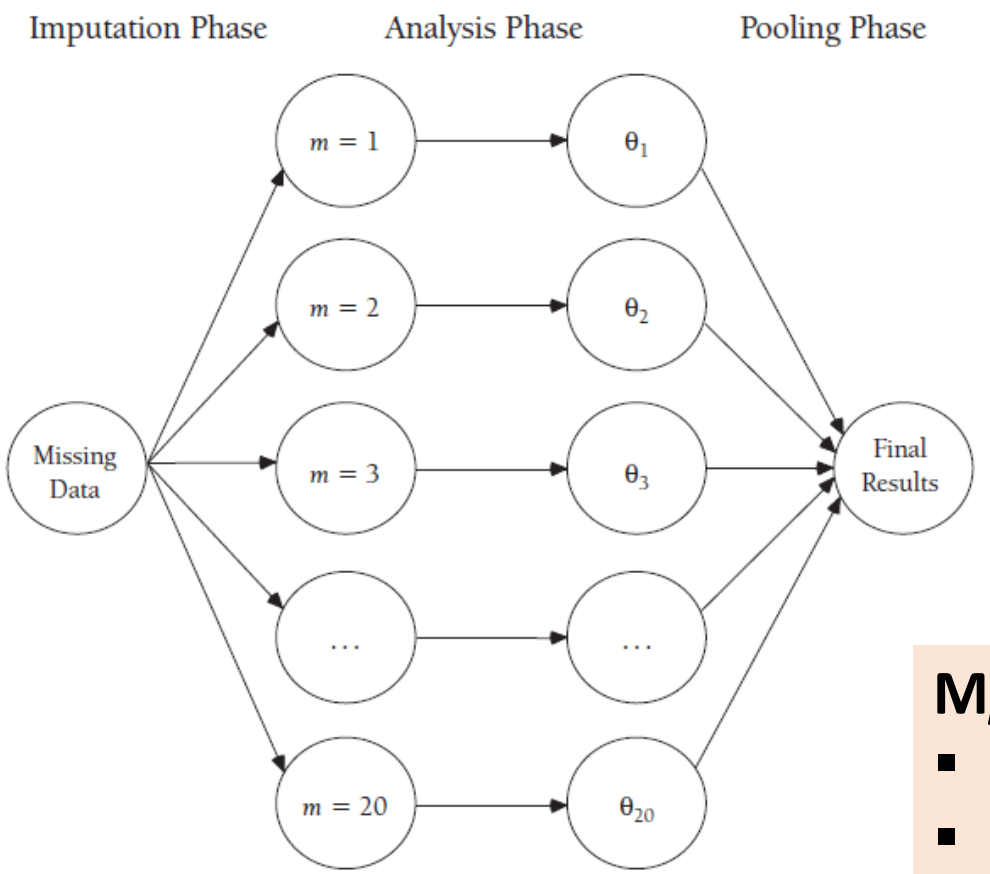
- `CLUSTER=CNTRYID`
- `CLUSTER=VARUNIT`
- `TYPE=COMPLEX`
- `TYPE=TWOLEVEL`
- `TYPE=TWOLEVEL RANDOM`
- `ESTIMATOR=MLR`
- `STRATIFICATION=VARSTRAT`

Plausible Values (PVs)

Population achievement estimation

- Rotated matrix design with missing data
- PVs as a representation of a range of abilities a participant may have
- Random draws from the distribution of abilities

(e.g., von Davier et al., 2009; Wu, 2005)



(Enders, 2010, p. 188)

Pooled parameter estimate

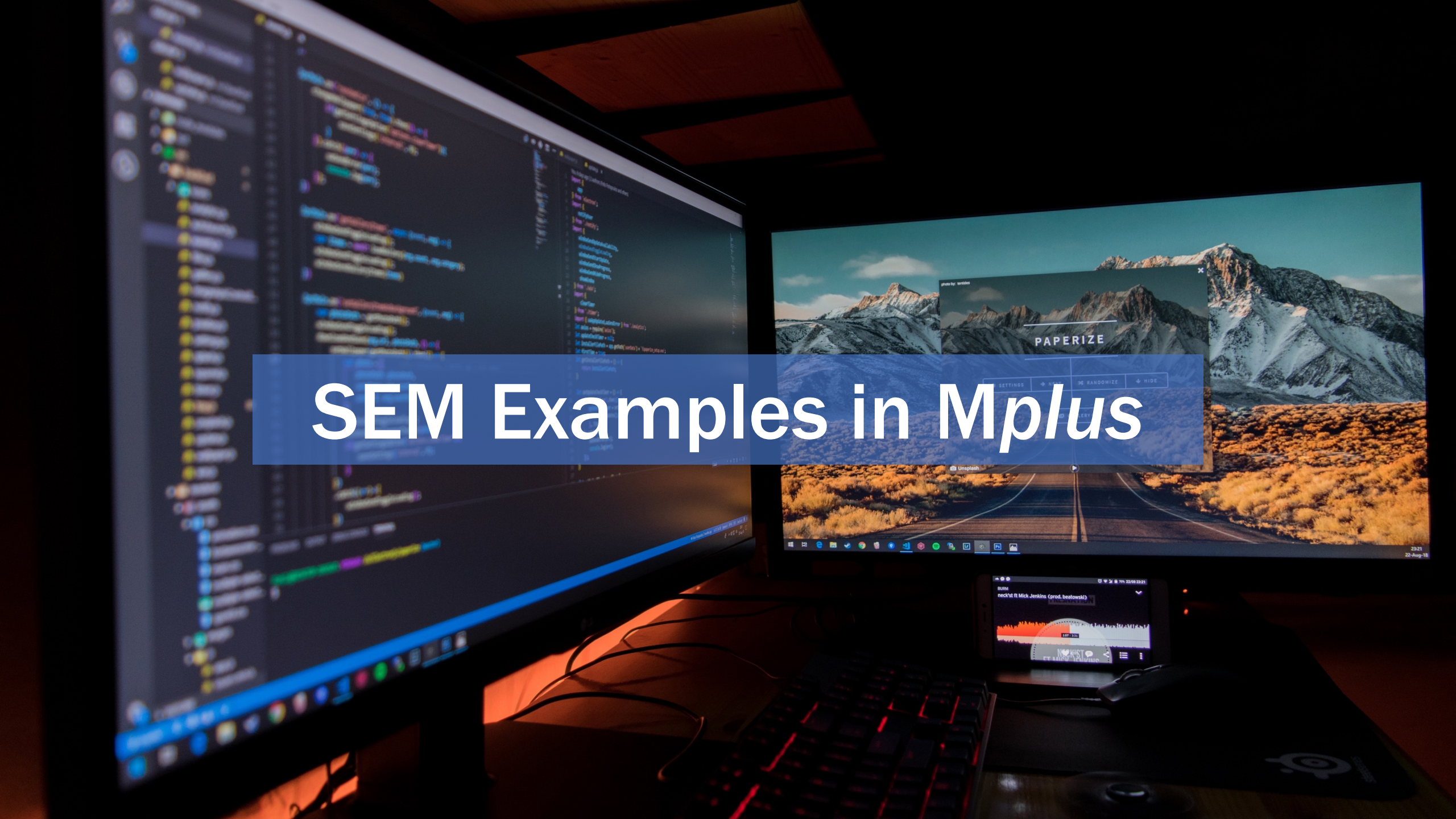
$$\theta = \frac{1}{m}\sum_{i=1}^{m}\theta_i$$

Total variance estimate

$$V_T = V_W + V_B + \frac{V_B}{m}$$

M*plus* options:

- `TYPE=IMPUTATION`
- Multiple data sets and one control file



SEM Examples in M*plus*

Main Construct in the Examples: Curiosity

No.	Items from PIAAC (original variable name)
1	When I hear or read about new ideas, I try to relate them to real life situations to which they might apply. (I_Q04b)
2	I like learning new things. (I_Q04d)
3	When I come across something new, I try to relate it to what I already know. (I_Q04h)
4	I like to get to the bottom of difficult things. (I_Q04j)
5	I like to figure out how different ideas fit together. (I_Q04l)
6	If I don't understand something, I look for additional information to make it clearer. (I_Q04m)

"I would now like to ask you some questions about how you deal with problems and tasks you encounter. To what extent do the following statements apply to you?"

Response options:
1 (not at all), 2 (very little), 3 (to some extent), 4 (to a high extent), 5 (to a very high extent)

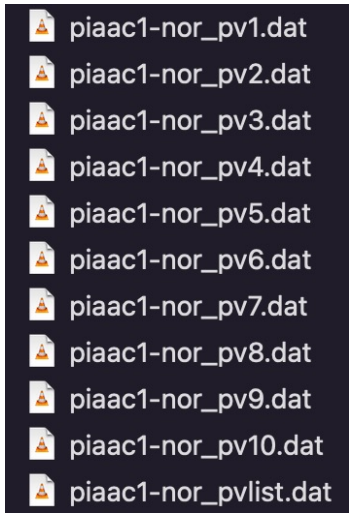
(Gorges et al., 2016, p. 5)

M*plus* and Data Files

piaac1-nor_pv1.dat × Data file for PV1 (.dat)

Users > ronnysc > Documents > CEMO > TEACHING > PI

1	19	1	5	2	2	3	1	4	4	4
2	32	0	3	2	3	4	4	1	4	3
3	19	1	2	3	4	2	3	3	3	4
4	27	0	8	3	3	3	1	4	4	4
5	58	1	6	2	2	2	2	4	1	3
6	64	0	12	-99	-99	-99	-99	-99	-99	-99
7	40	1	8	3	2	4	1	2	0	1
8	55	0	8	3	4	3	2	4	4	4



Data files (.dat)

```
piaac1-nor_pv1.dat
piaac1-nor_pv2.dat
piaac1-nor_pv3.dat
piaac1-nor_pv4.dat
piaac1-nor_pv5.dat
piaac1-nor_pv6.dat
piaac1-nor_pv7.dat
piaac1-nor_pv8.dat
piaac1-nor_pv9.dat
piaac1-nor_pv10.dat
```

Control list file (.dat)

Mplus - ALIGN1a.inp

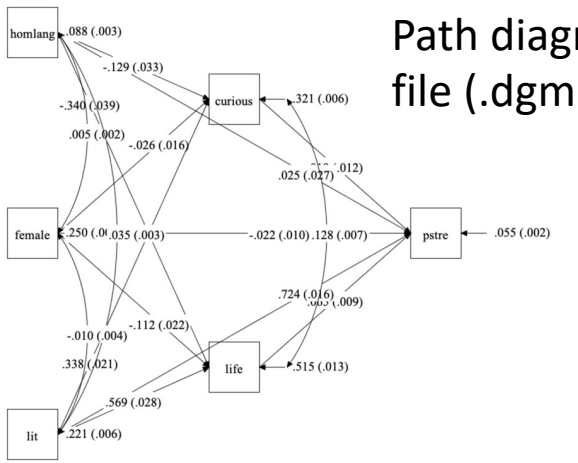
File Edit Mplus View Diagram Window Help

ALIGN1a.inp

```
TITLE: Structural equation modeling with PIAAC data
       Multi-group CFA
       Single-factor model of curiosity and continuous treatment--modified
       Complex design: Total weights
       Grouping by: country (CNTRYID)
       FREE ALIGNMENT METHOD

DATA:  FILE IS piaac1-all-curiosity.dat;
       ! TYPE = IMPUTATION;
       ! Not needed here as the model does not contain any PV

VARIABLE:
       NAMES ARE
              CNTRYID
              I_Q04b I_Q04d I_Q04h I_Q04j I_Q04l I_Q04m
              SPFWT0
              HDI2012 UAI;
```

Line: 1 Column: 1

Input file (.inp)



Path diagram file (.dgm)

ALIGN1a.out

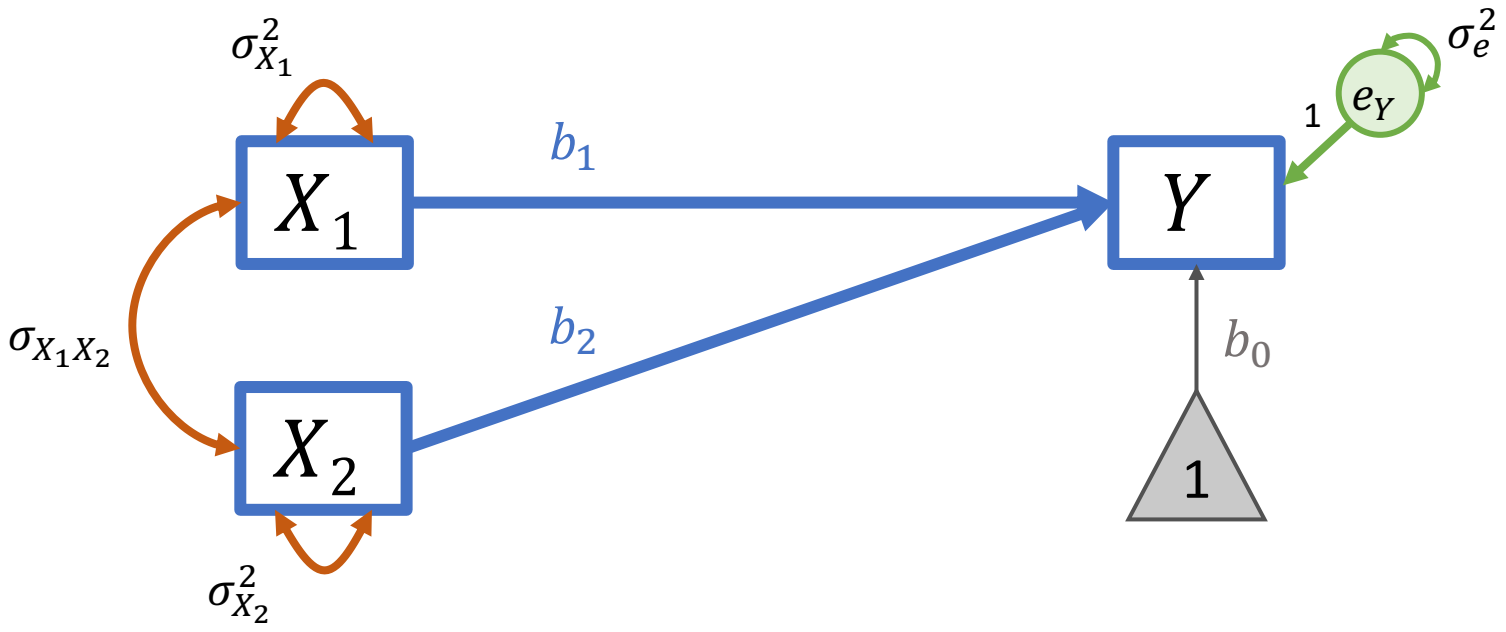
```
Mplus VERSION 8.2 (Mac)
MUTHEN & MUTHEN
03/19/2022   6:55 PM

INPUT INSTRUCTIONS

  TITLE:     Structural equation modeling with PIAAC data
       Multi-group CFA
       Single-factor model of curiosity and continuous treatment--mod:
       Complex design: Total weights
       Grouping by: country (CNTRYID)
       FREE ALIGNMENT METHOD

  DATA:      FILE IS piaac1-all-curiosity.dat;
       ! TYPE = IMPUTATION;
       ! Not needed here as the model does not contain any PV
```

Output file (.out)

Regression and Path Models

Regression Model with Multiple Predictors

Simple linear regression model with two predictors



$$Y_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + e_{Yi}$$

$$e_Y \sim N(0, \sigma_e^2)$$

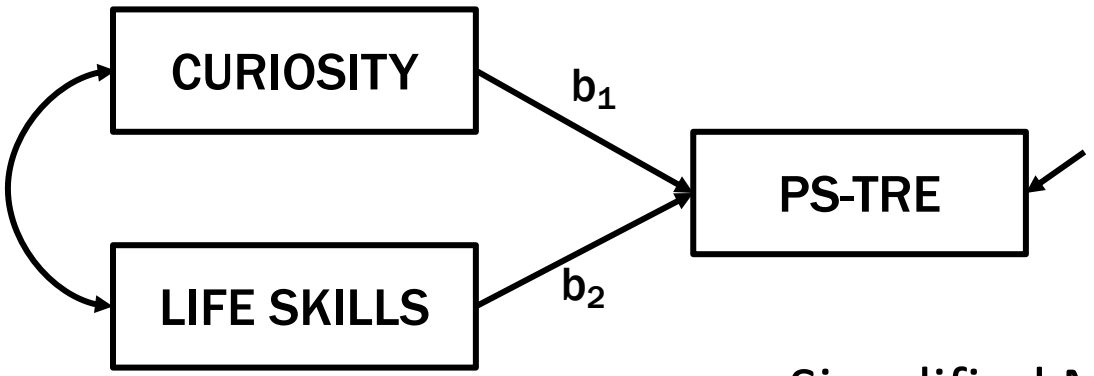
Data

(y_i, x_{1i}, x_{2i}) for persons $i = 1, \ldots, N$

Y	Problem-solving in TRE (PV1-PV10)
X_1	Curiosity (average scale score)
X_2	Everyday life skills (average scale score)

Regression Model with Multiple Predictors

Simple linear regression model with two predictors



Simplified M*plus* path diagram

$$Y_i = b_0 + b_1 X_{1i} + b_2 X_{2i} + e_{Yi}$$

$$e_Y \sim N(0, \sigma_e^2)$$

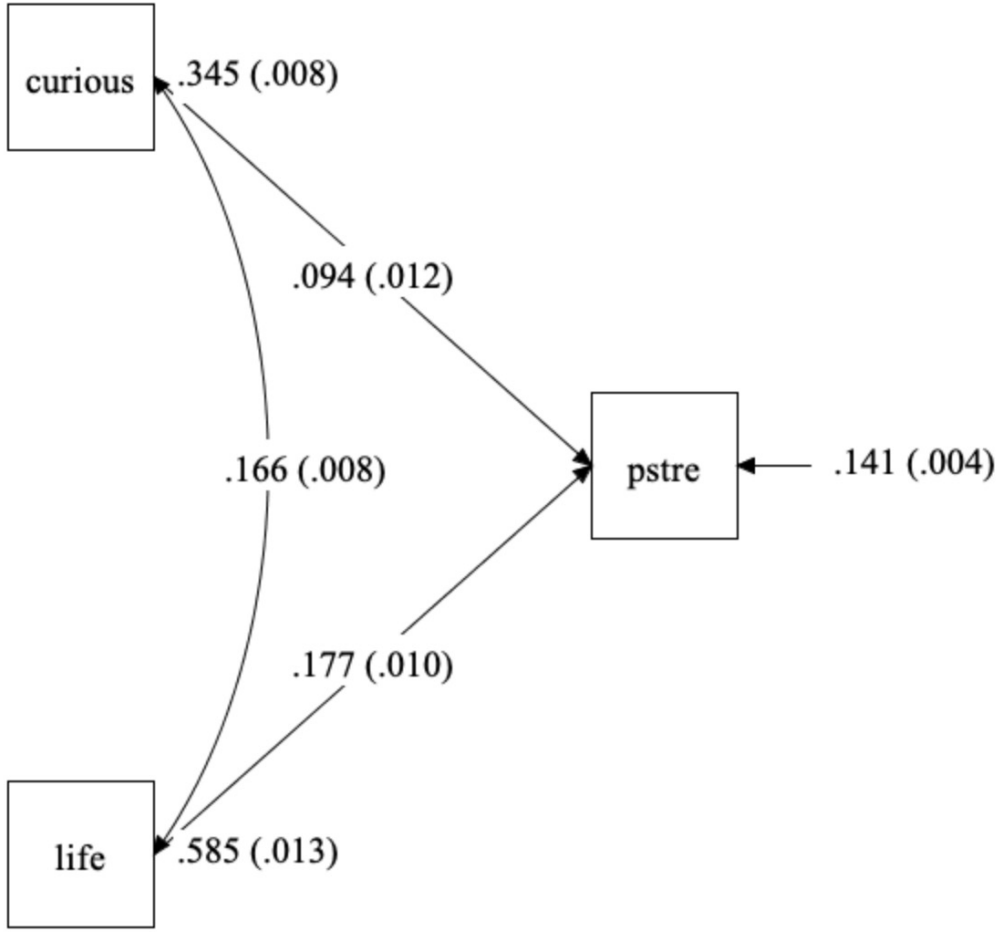
Data
(y_i, x_{1i}, x_{2i}) for persons
$i = 1, \ldots, N$

Y	Problem-solving in TRE (PV1-PV10)
X_1	Curiosity (average scale score)
X_2	Everyday life skills (average scale score)

Regression Model with Multiple Predictors

Model specification

```
MODEL:
! Regression coefficients
PSTRE ON CURIOUS
         LIFE;

! Variances of predictors
CURIOUS;
LIFE;

! Covariance between predictors
CURIOUS WITH LIFE;
```

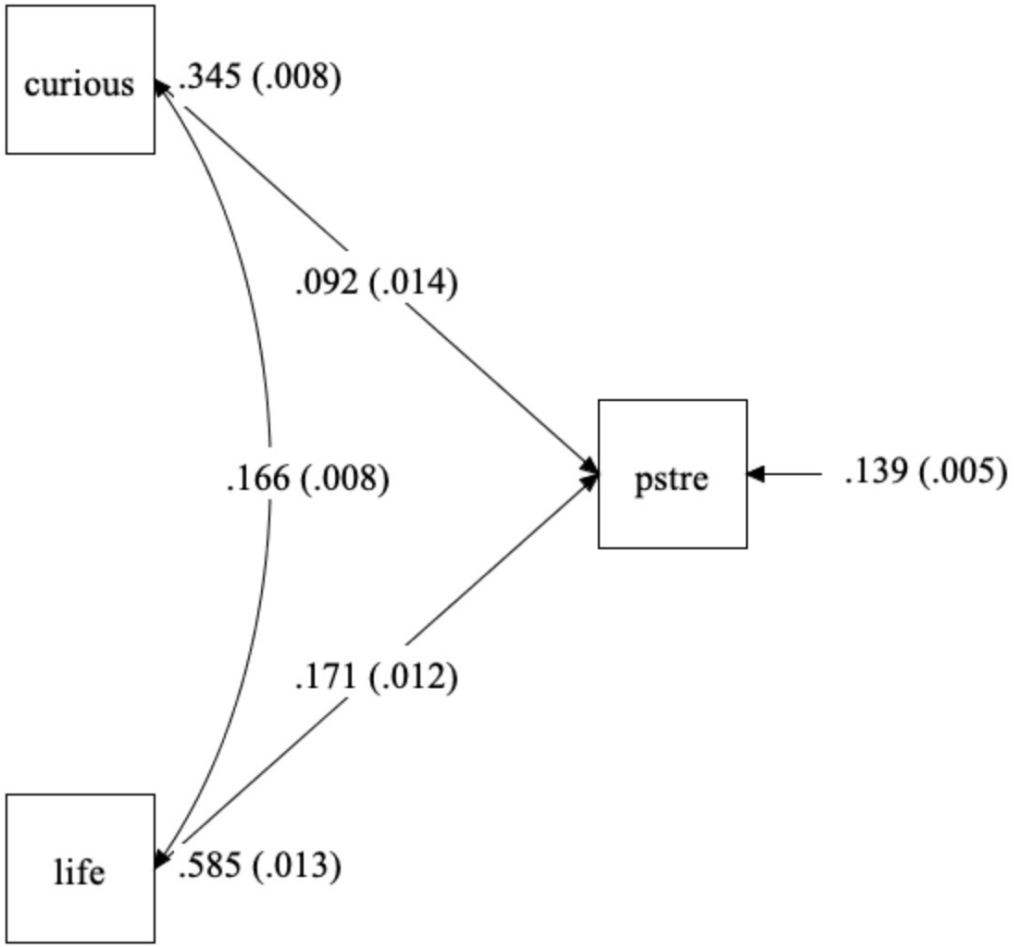


Model REG1a

Regression Model with Multiple Predictors

Model specification with PV1-PV10

```
DATA:
FILE IS piaac1-nor_pvlist.dat;
TYPE = IMPUTATION;

MODEL:
PSTRE ON CURIOUS LIFE;
CURIOUS;
LIFE;
CURIOUS WITH LIFE;
```



Model REG1b

Regression Model with Multiple Predictors

Model specifications with elements of the complex survey design

Model	Model specification elements
REG1b	PV1-PV10, total weights
REG1c	PV1-PV10, total weights, clustering, stratification
REG1d	PV1-PV10, total weights, replication weights

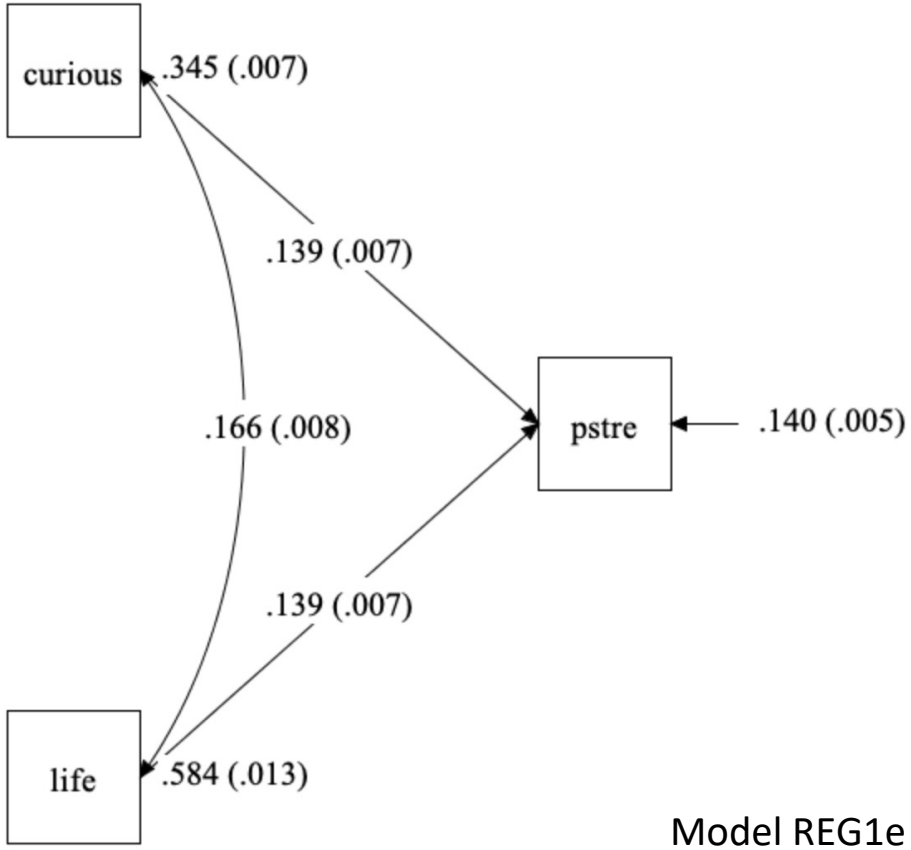
VARIABLE:

- `WEIGHT = SPFWT0;`
- `CLUSTER = VARUNIT;`
- `STRATIFICATION = VARSTRAT;`
- `REPWEIGHTS = SPFWT1- SPFWT80;`

Regression Model with Multiple Predictors

Model with equal regression coefficients—**Option 1**

```
MODEL:
! Regression coefficients
PSTRE ON CURIOUS(a)
         LIFE(b);
[…]

MODEL CONSTRAINT:

! Equality of coefficients
0 = a-b;
```

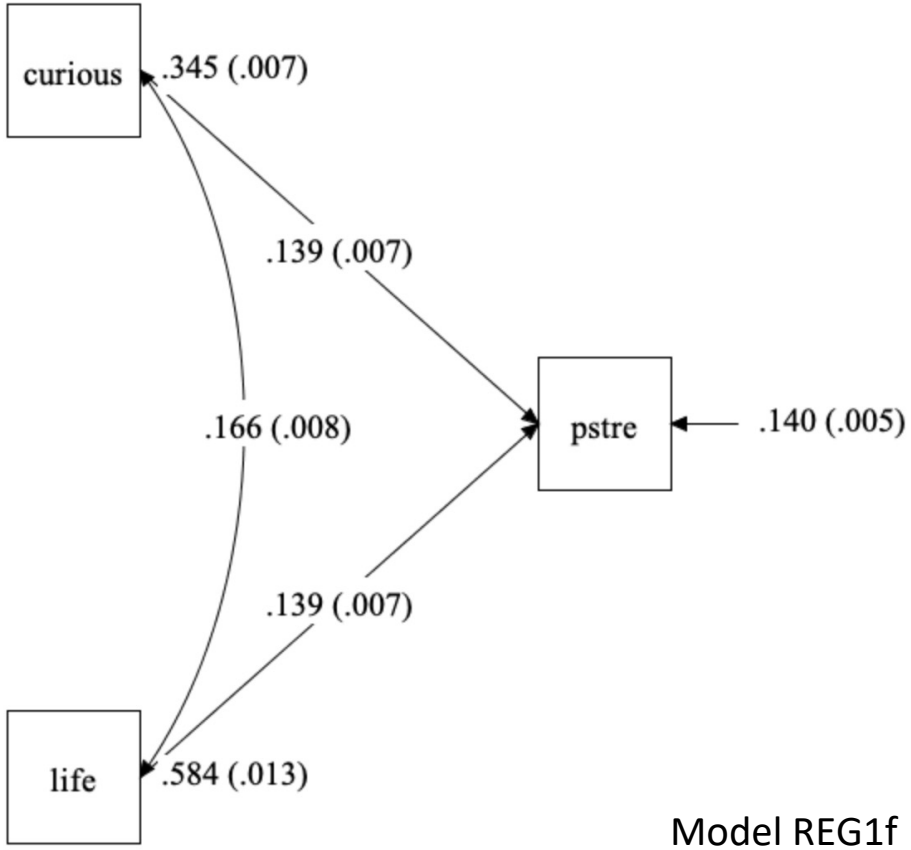


Model REG1e

Regression Model with Multiple Predictors

Model with equal regression coefficients—**Option 2**

```
MODEL:
! Regression coefficients
PSTRE ON CURIOUS(a)
         LIFE(a);

! Variances of predictors
CURIOUS;
LIFE;

! Covariance between predictors
CURIOUS WITH LIFE;
```



Model REG1f

Model Comparison

Likelihood-Ratio Test (LRT)

(Enders, 2010; Satorra & Bentler, 2010)

- Test competing models based on the likelihood value
- Test statistic: Ratio of the (log-)likelihoods for the two models against a χ^2 distribution with degrees of freedom ΔNpar

$$D = \Delta(-2 \log L) = -2 \cdot (\log L_{H0} - \log L_{H1})$$

- For complex data sets: Satorra-Bentler correction

Satorra-Bentler using the scaled Likelihood-Ratio Test

	Model 1	Model 2
LogL	-11211,3	-11223,5
Npar	9	8
SCF	1	1

cd	1
TRd	24,4
p-value	0,000

SB-LRT.xlsx

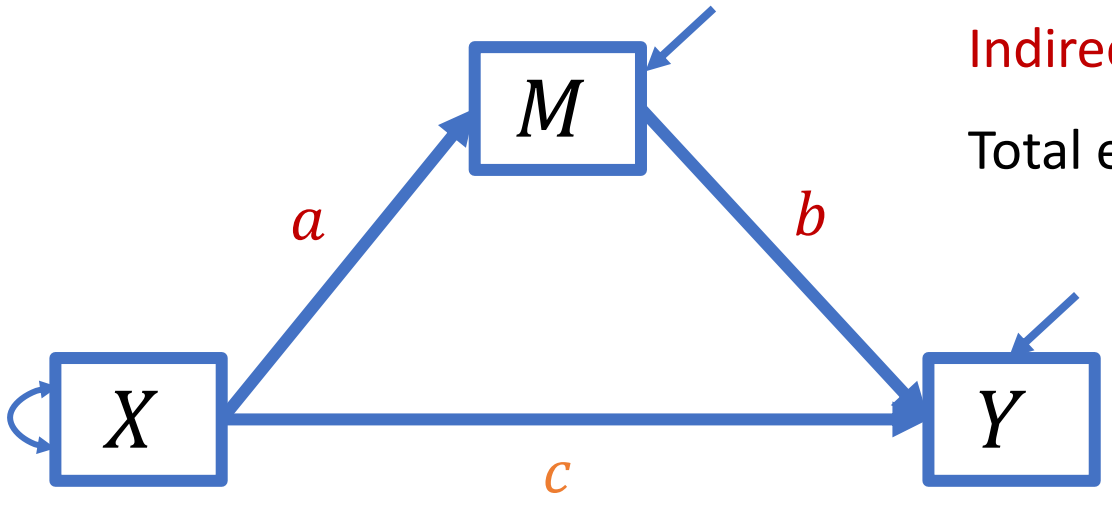
Caution:
For multiply imputed data sets (with all PVs), pooled LRTs should be performed.
(Meng & Rubin, 1992; Grund et al., 2021, https://doi.org/10.31234/osf.io/d459g)

Mediation Model $X \rightarrow M \rightarrow Y$

Mediation model with the direct effect (c-path)

$$M_i = i_M + aX_i + e_{Mi}$$
$$Y_i = i_Y + bM_i + cX_i + e_{Yi}$$

$$e_M \sim N(0, \sigma^2_{e_M}), e_Y \sim N(0, \sigma^2_{e_Y})$$



Direct effect c

Indirect effect ab

Total effect $ab + c$

Data

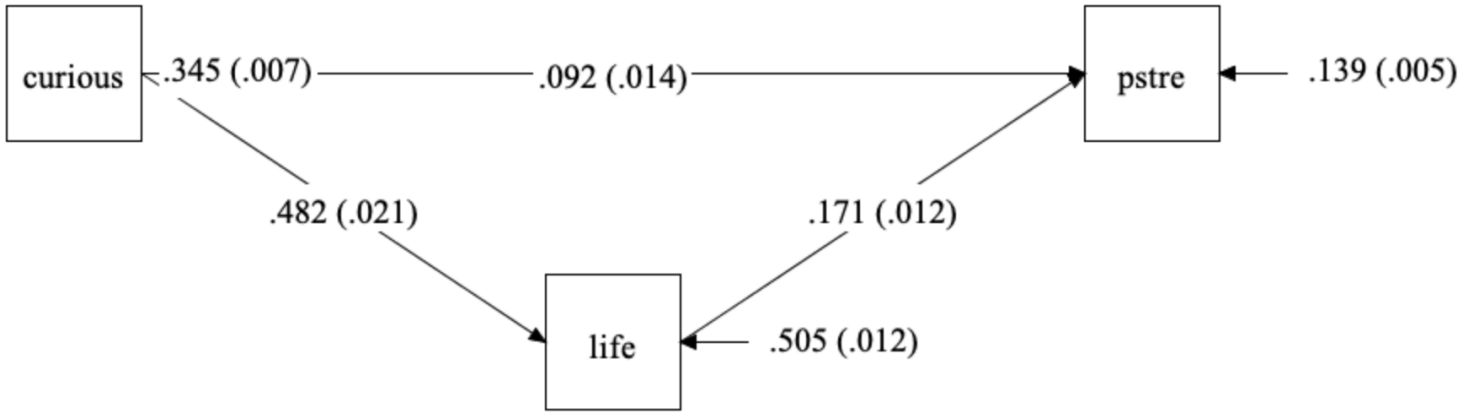
(y_i, m_i, x_i) for persons $i = 1, \ldots, N$

Y	Problem-solving in TRE (PV1-PV10)
X	Curiosity (average scale score)
M	Everyday life skills (average scale score)

Mediation Model $X \rightarrow M \rightarrow Y$

Model specification with the c-path

```
MODEL:
PSTRE ON CURIOUS(c)
          LIFE(b);
LIFE ON CURIOUS(a);
CURIOUS;

MODEL INDIRECT:
PSTRE IND CURIOUS;

MODEL CONSTRAINT:
new(ind);
ind = a*b;
```

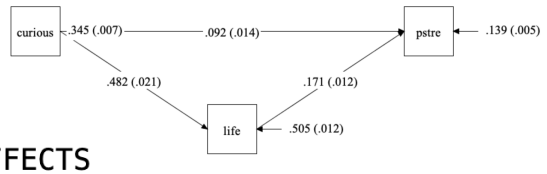



```
New/Additional Parameters
    IND                0.083      0.007     11.998      0.000
    TOT                0.175      0.013     13.095      0.000
```

Model PATH2a

Mediation Model $X \rightarrow M \rightarrow Y$

Model specification with the c-path




```
MODEL:
PSTRE ON CURIOUS(c)
         LIFE(b);
LIFE ON CURIOUS(a);
CURIOUS;

MODEL INDIRECT:
PSTRE IND CURIOUS;

MODEL CONSTRAINT:
new(ind);
ind = a*b;
```

```
TOTAL, TOTAL INDIRECT, SPECIFIC INDIRECT, AND DIRECT EFFECTS

                                                    Two-Tailed
                    Estimate       S.E.  Est./S.E.    P-Value

Effects from CURIOUS to PSTRE

  Total                0.175      0.013     13.095      0.000
  Total indirect       0.083      0.007     11.998      0.000

  Specific indirect

    PSTRE
    LIFE
    CURIOUS            0.083      0.007     11.998      0.000

  Direct
    PSTRE
    CURIOUS            0.092      0.014      6.558      0.000
```

Model PATH2a

Multi-Group Mediation Model $X \rightarrow M \rightarrow Y$

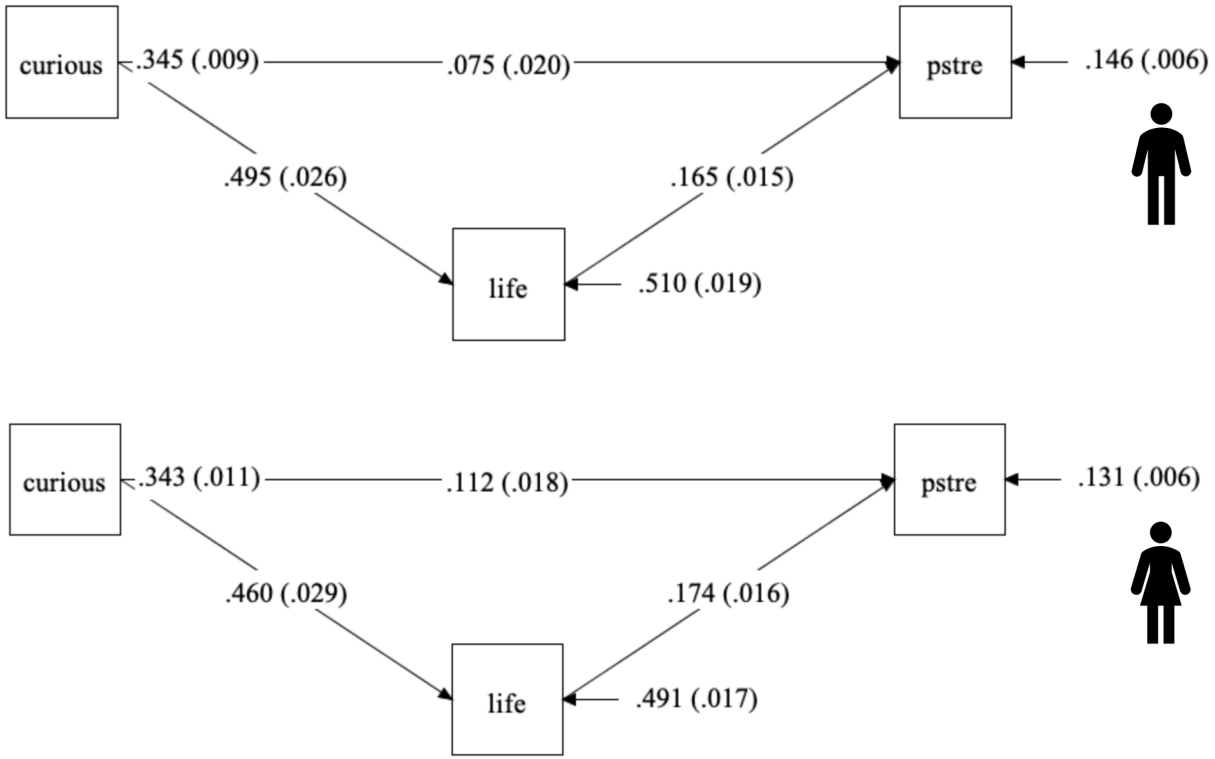
Model specification across gender

```
VARIABLE: […]
GROUPING IS
      FEMALE(0=MEN 1=WOMEN);
```

Define groups

```
MODEL:
PSTRE ON CURIOUS
           LIFE(b);
LIFE ON CURIOUS;
CURIOUS;
```

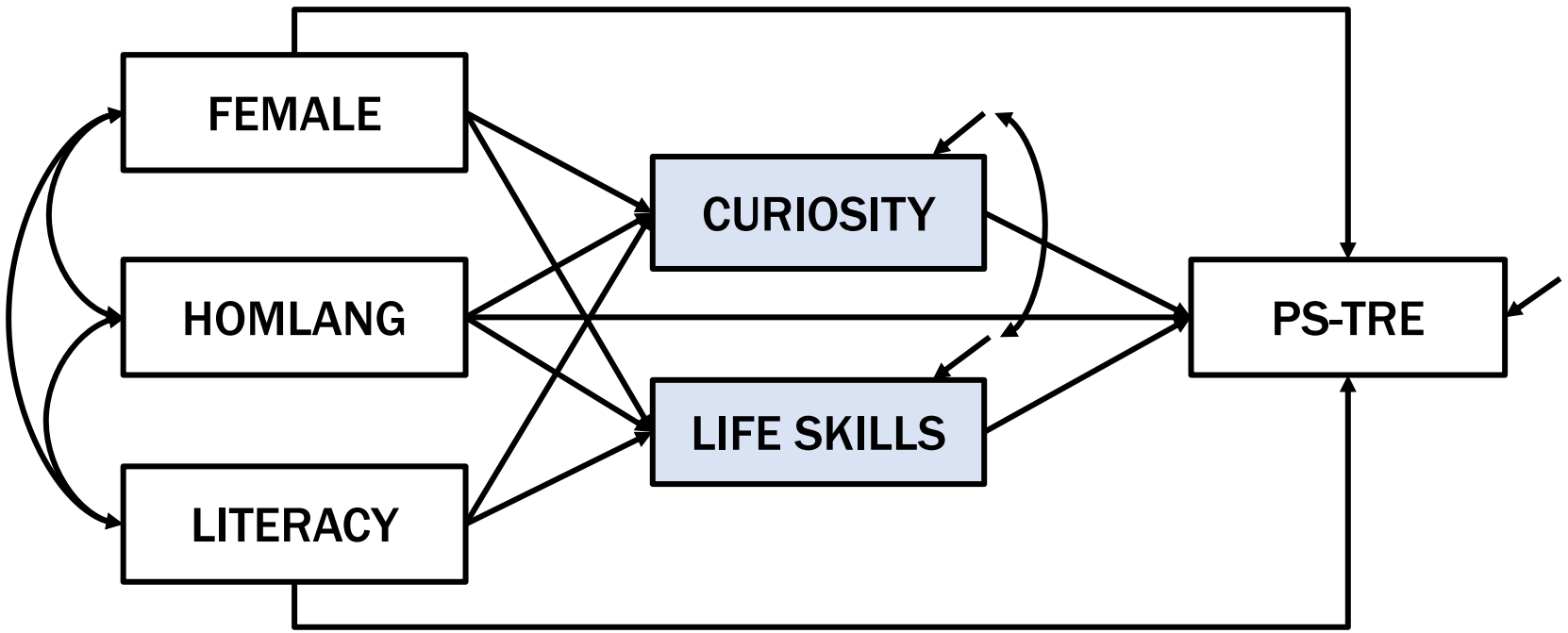
Test the equality of path coefficients



Models PATH2c-e

Complex Path Model

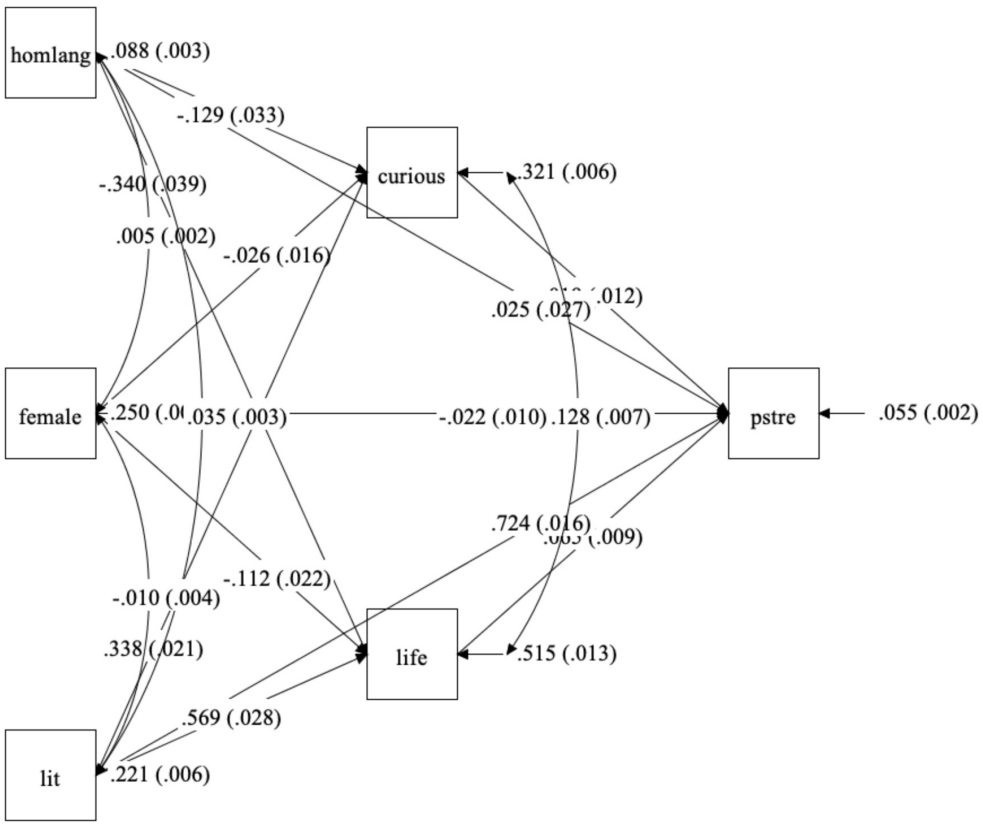
Path model with multiple mediators and predictors

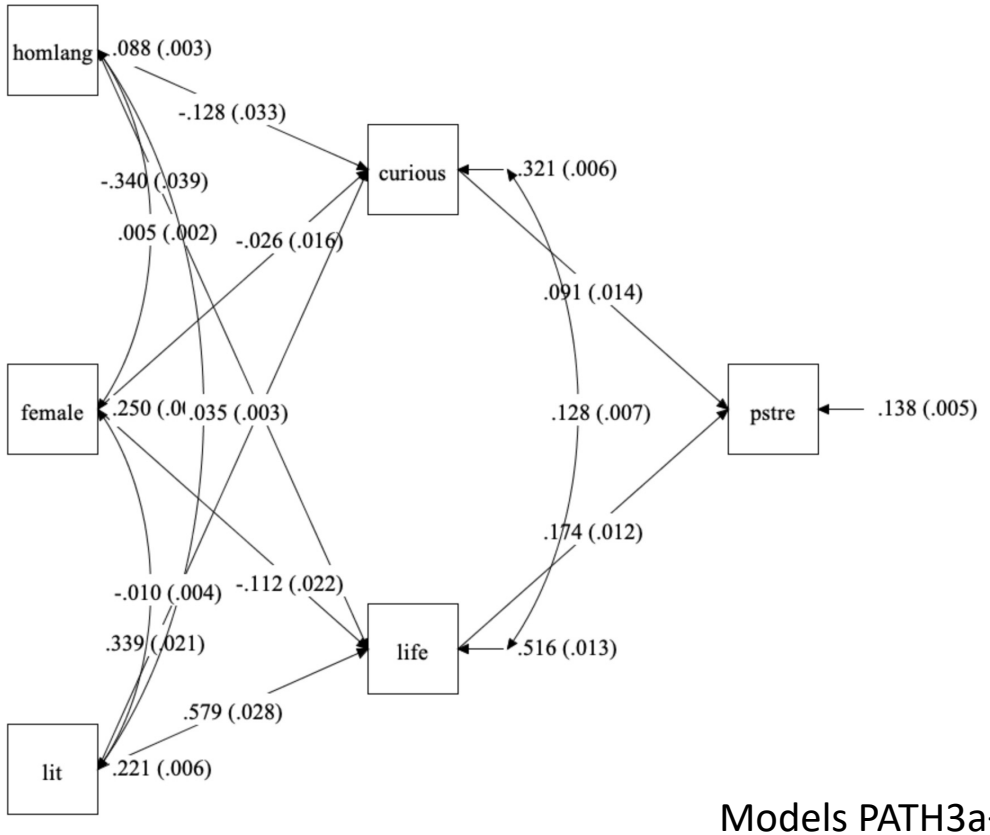


- Observed variables
- Multiple indirect paths
- Multiple direct paths

Complex Path Model

Parameters of the models with or without the direct *c*-paths



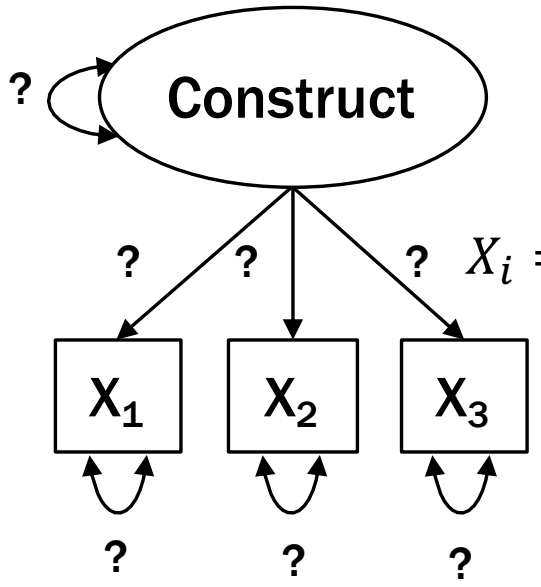


Models PATH3a+b

Confirmatory Factor Analysis (CFA)

Latent Variables as Construct Representations

Construct specification (before estimation)

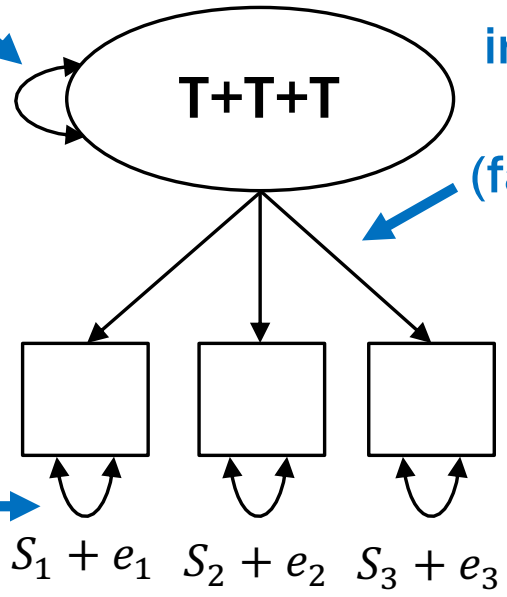


$$X_i = T_i + (S_i + e_i)$$

Key estimates for a construct

Factor variance (amount of common information)

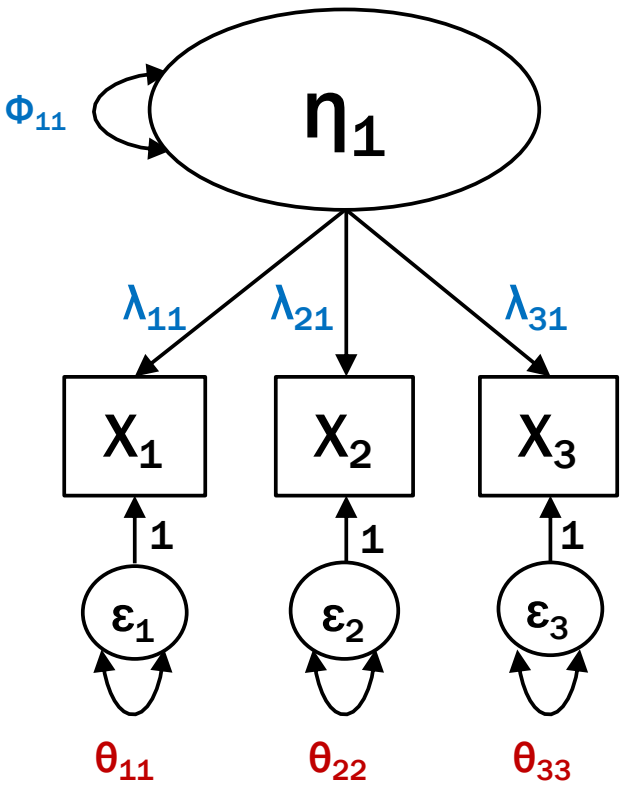
Amount of contribution of each indicator to the construct (factor loadings)



Amount of information unique to each indicator

(Little, 2013)

Model Specification



Model-implied covariance matrix:

	X_1	X_2	X_3
X_1	$\lambda_{11}\phi_{11}\lambda_{11} + \theta_{11}$		
X_2	$\lambda_{11}\phi_{11}\lambda_{21}$	$\lambda_{21}\phi_{11}\lambda_{21} + \theta_{22}$	
X_3	$\lambda_{11}\phi_{11}\lambda_{31}$	$\lambda_{21}\phi_{11}\lambda_{31}$	$\lambda_{31}\phi_{11}\lambda_{31} + \theta_{33}$

Fundamental SEM Equation: $\boldsymbol{\Sigma} = \boldsymbol{\Lambda\Phi\Lambda}' + \boldsymbol{\Theta}$

Σ: Model-implied covariance matrix
Λ: Matrix of factor loadings
Φ: Matrix of construct variances and covariances
Θ: Matrix of residual variances and covariances

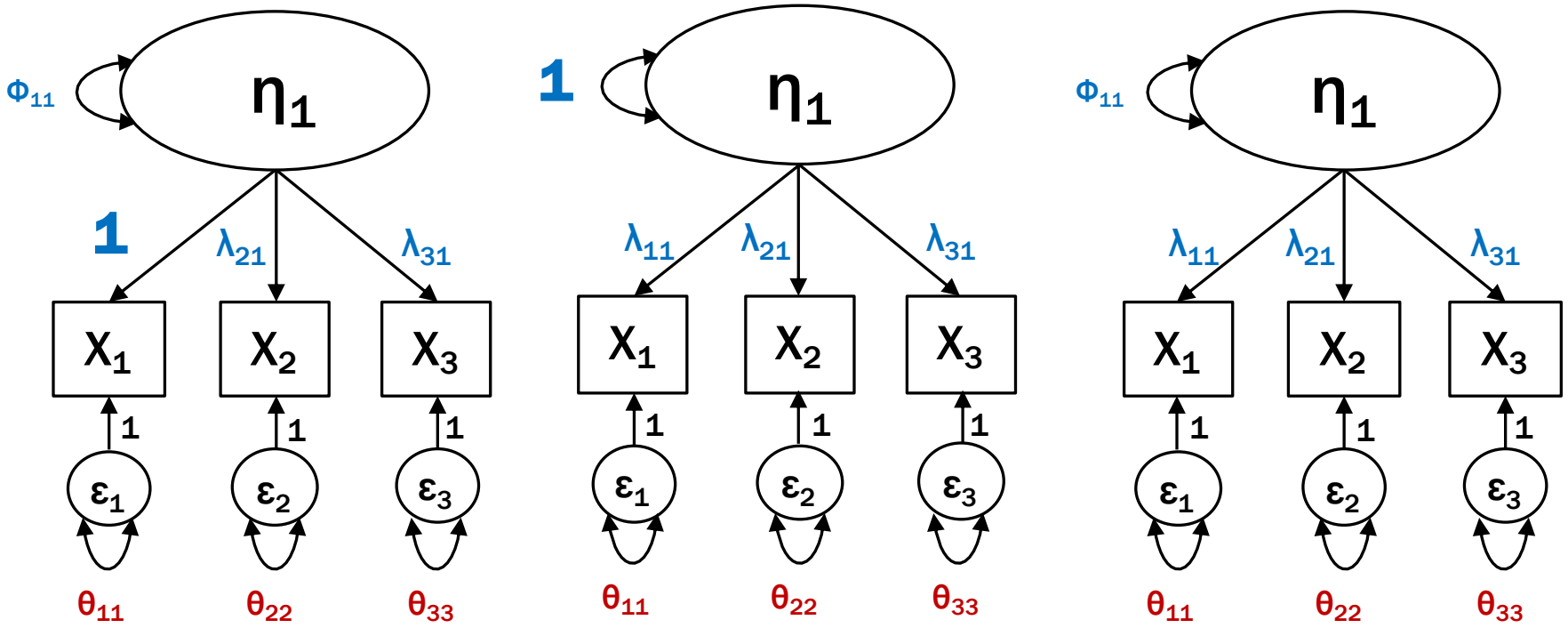
(Brown, 2015; Jöreskog et al., 2016)

Scale Setting

Setting the scale of the latent variable in CFA—Three options:

(Brown, 2015)

$$\lambda_{11} = 3 - \lambda_{21} - \lambda_{31}$$



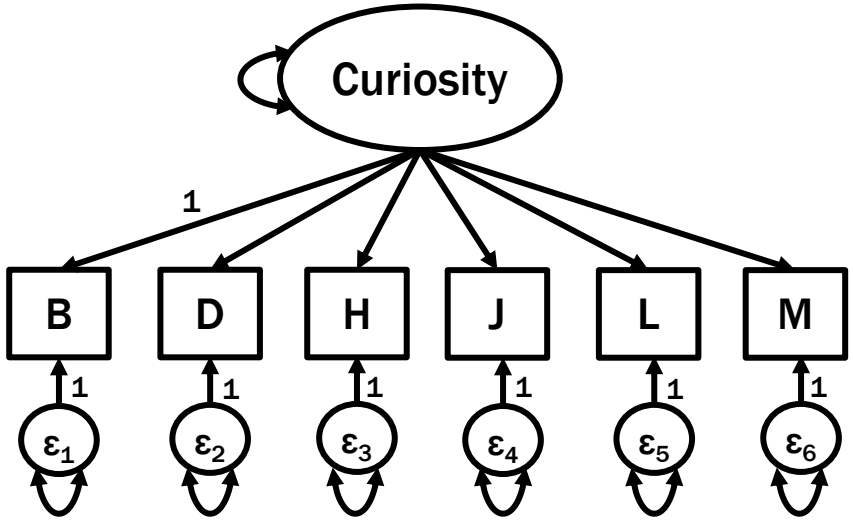
Reference variable **Factor scaling** **Effects coding**

Single-Factor CFA

Hypothesized Measurement Model of **Curiosity**:

CFA model specification

- One factor
- Six manifest indicators
- Continuously or categorically treated item responses



Single-Factor CFA

Model specification

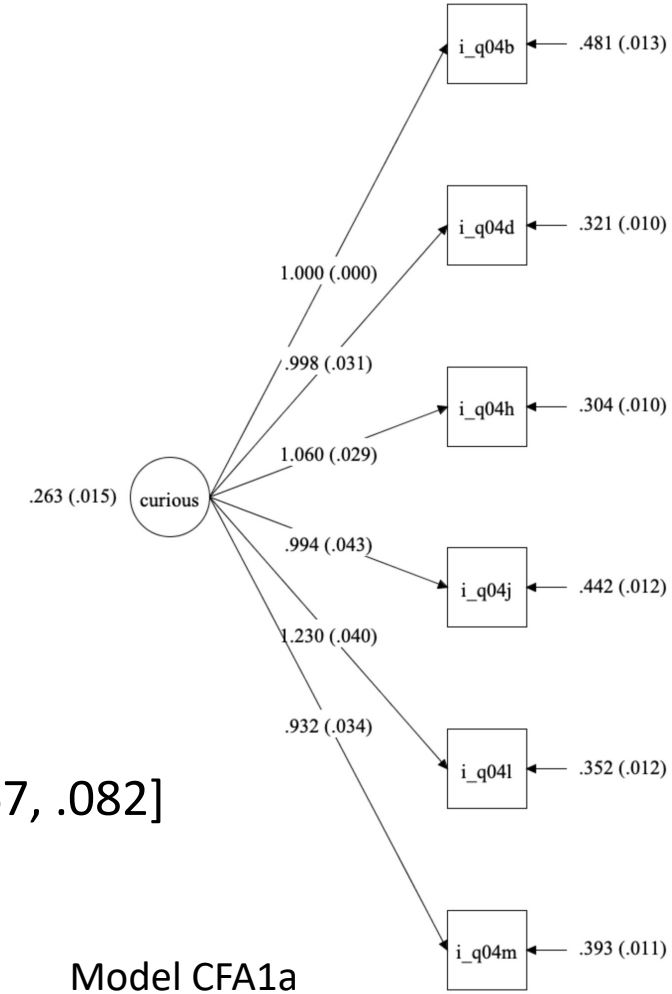
```
MODEL:
! Measurement model
CURIOUS BY
      I_Q04b@1
      I_Q04d
      I_Q04h
      I_Q04j
      I_Q04l
      I_Q04m;

CURIOUS*;
```

Model fit indices:

$\chi^2(9) = 254.0, p < .001$

RMSEA=.074, 90% CI [.067, .082]

CFI=.955, SRMR=.032



Model CFA1a

CFA: Dynamic Fit Index Cutoffs

Single-factor CFA

Model specification using the `lavaan` language:

CURIOUS =~ 0.595*I_Q04B+0.671*I_Q04D+0.702*I_Q04H+
0.609*I_Q04J+0.729*I_Q04L+0.607*I_Q04M

Instructions | Results | Plots | References

These are the dynamic fit index cutoff values for your model:

	SRMR	RMSEA	CFI
Level 1: 95/5	.029	.074	.972
Level 1: 90/10	--	--	--
Level 2: 95/5	.043	.113	.942
Level 2: 90/10	--	--	--
Level 3: 95/5	.055	.149	.908
Level 3: 90/10	--	--	--

Single-Factor CFA

Model specification with reliability estimation

```
MODEL:
CURIOUS BY
      I_Q04b*(L1)
      I_Q04d(L2)
      I_Q04h(L3)
      I_Q04j(L4)
      I_Q04l(L5)
      I_Q04m(L6);

CURIOUS@1;
! Residual variances
I_Q04b-I_Q04m(R1-R6);
```

McDonald's Omega

$$\omega = \frac{\left(\sum_{i=1}^{k} \lambda_i\right)^2}{\left(\sum_{i=1}^{k} \lambda_i\right)^2 + \sum_{i=1}^{k} \theta_{ii}}$$

```
MODEL CONSTRAINT:
new(rel);

rel=((L1+L2+L3+L4+L5+L6)^2)/
((L1+L2+L3+L4+L5+L6)^2 +
R1+R2+R3+R4+R5+R6);
```

```
New/Additional Parameters
    REL                    0.816      0.005
```

Model CFA1a-Rel

Single-Factor CFA with Modifications

Model specification

```
MODEL:
! Measurement model
CURIOUS BY
      I_Q04b-I_Q04m;

! Modifications
! Residual covariances
I_Q04B WITH I_Q04H;
I_Q04B WITH I_Q04J;
```

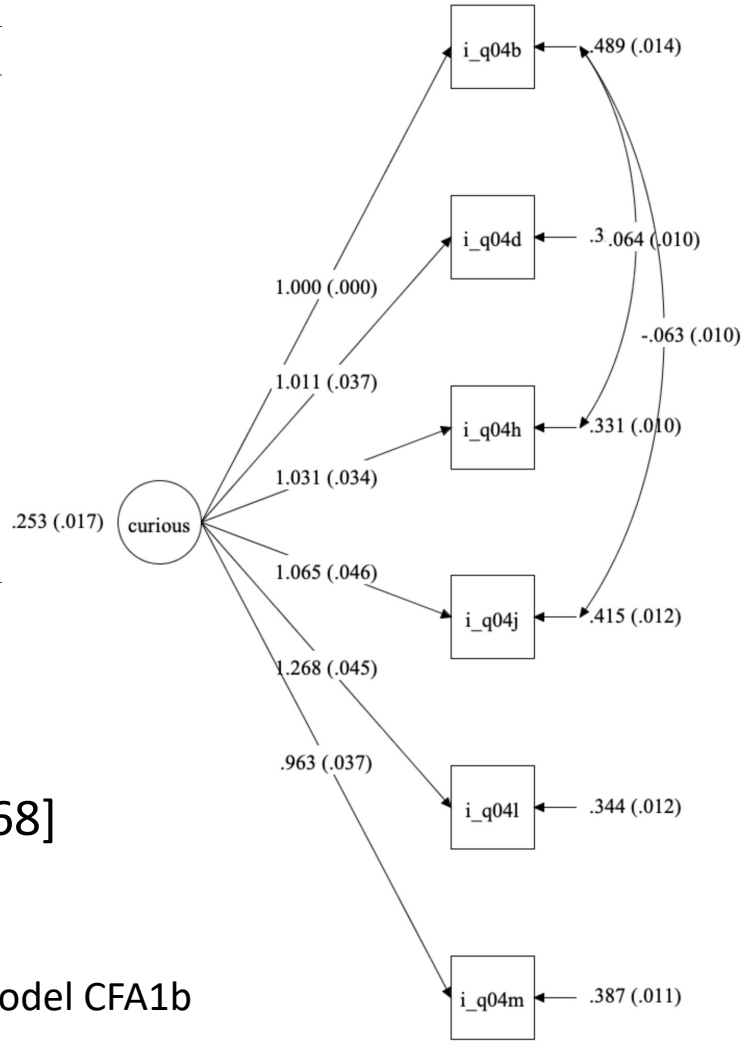
No.	Items from PIAAC (original variable name)
1	When I hear or read about new ideas, I try to relate them to real life situations to which they might apply. (I_Q04b)
2	I like learning new things. (I_Q04d)
3	When I come across something new, I try to relate it to what I already know. (I_Q04h)
4	I like to get to the bottom of difficult things. (I_Q04j)
5	I like to figure out how different ideas fit together. (I_Q04l)
6	If I don't understand something, I look for additional information to make it clearer. (I_Q04m)

Model fit indices:

$\chi^2(7) = 126.2, p < .001$

RMSEA=.059, 90% CI [.050, .068]

CFI=.978, SRMR=.022



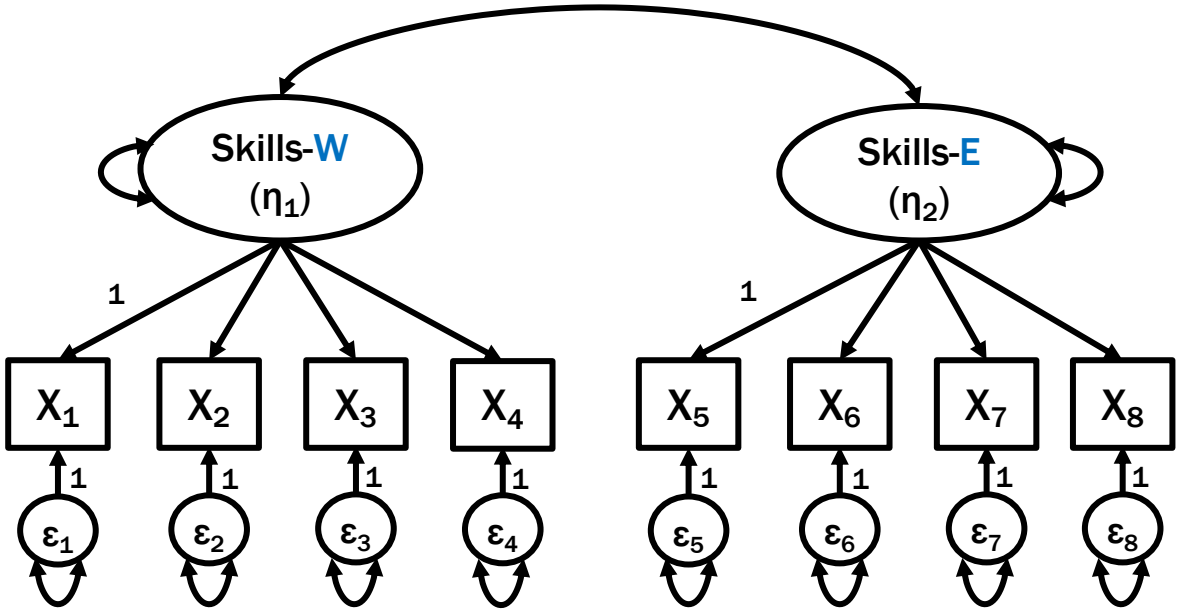
Model CFA1b

Multiple-Factors CFA

Hypothesized Measurement Model of **Skills:**

ICT skills at Work (W) or in Everyday Life (E)

- Frequency of using ICT for specified purposes
- *Emailing, using the Internet, Excel, and Word*
- *Items Q05a,c,e,f*



Multiple-Factors CFA

Model specification

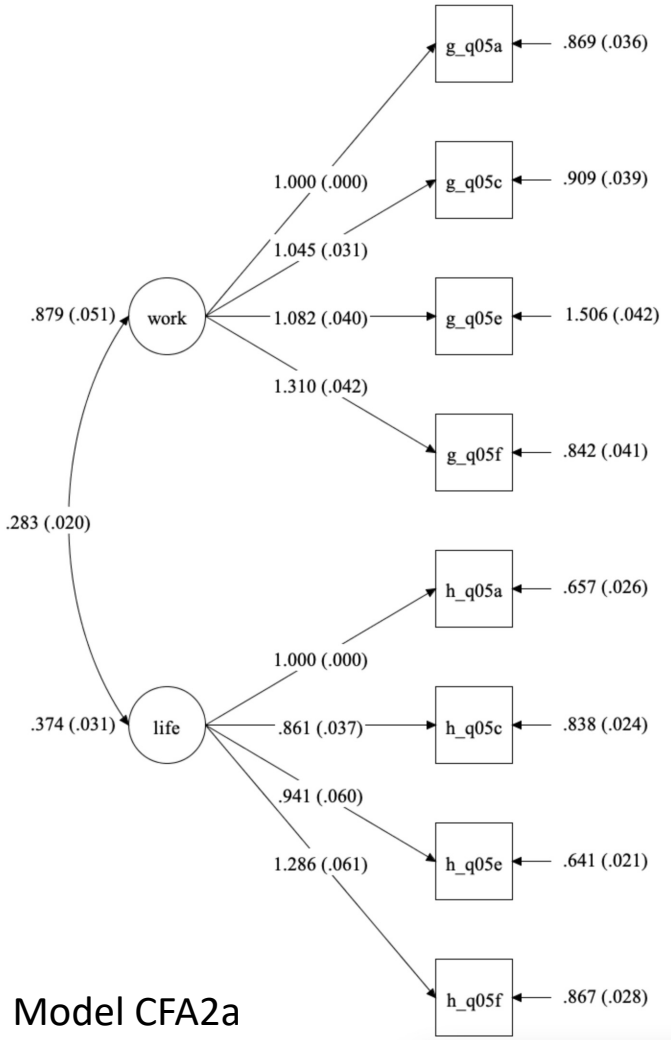
```
MODEL:
! Measurement model
WORK BY G_Q05a G_Q05c G_Q05e G_Q05f;
LIFE BY H_Q05a H_Q05c H_Q05e H_Q05f;

! Factor covariance
WORK WITH LIFE;
```

Model fit indices:

$\chi^2(19) = 1047.6, p < .001$

RMSEA=.107, 90% CI [.101, .112]

CFI=.841, SRMR=.064



Model CFA2a

Multiple-Factors CFA with Modifications

Model specification

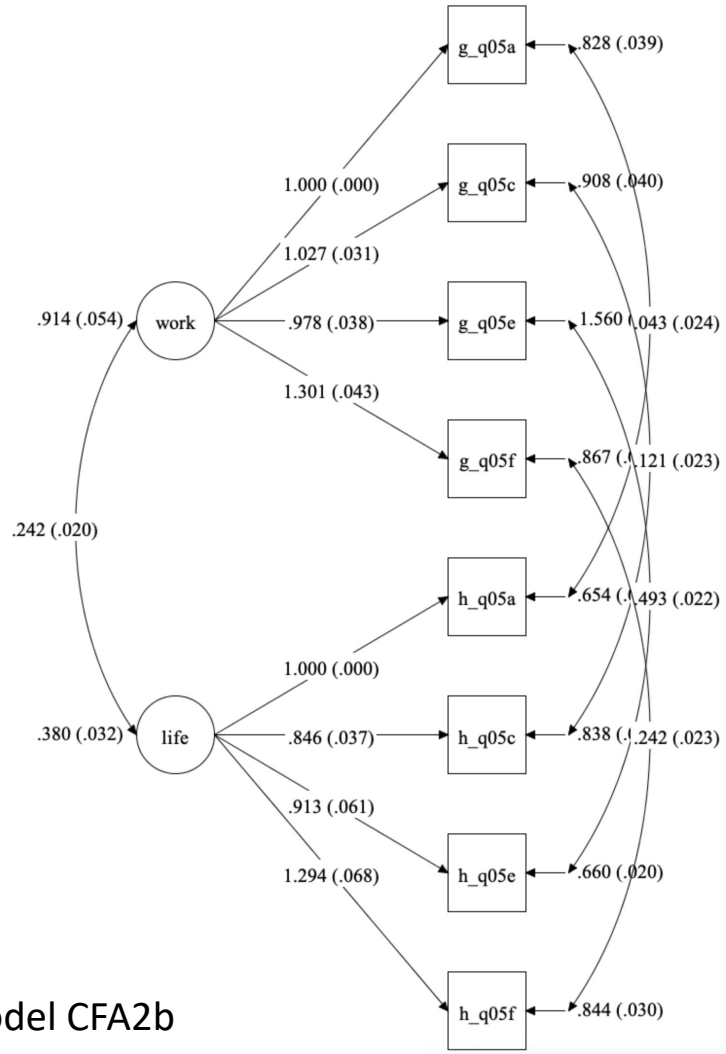
```
MODEL:
! Measurement model
WORK BY G_Q05a G_Q05c G_Q05e G_Q05f;
LIFE BY H_Q05a H_Q05c H_Q05e H_Q05f;
WORK WITH LIFE;

! Model modifications
G_Q05A WITH H_Q05A;
G_Q05C WITH H_Q05C;
G_Q05E WITH H_Q05E;
G_Q05F WITH H_Q05F;
```

Model fit indices:

$\chi^2(15) = 365.5, p < .001$

RMSEA=.070, 90% CI [.064, .076]

CFI=.946, SRMR=.042



Model CFA2b

Structural Equation Models (SEM)

SEM Elements

Structural Model

Measurement Models

Structural Equation Models (SEM)

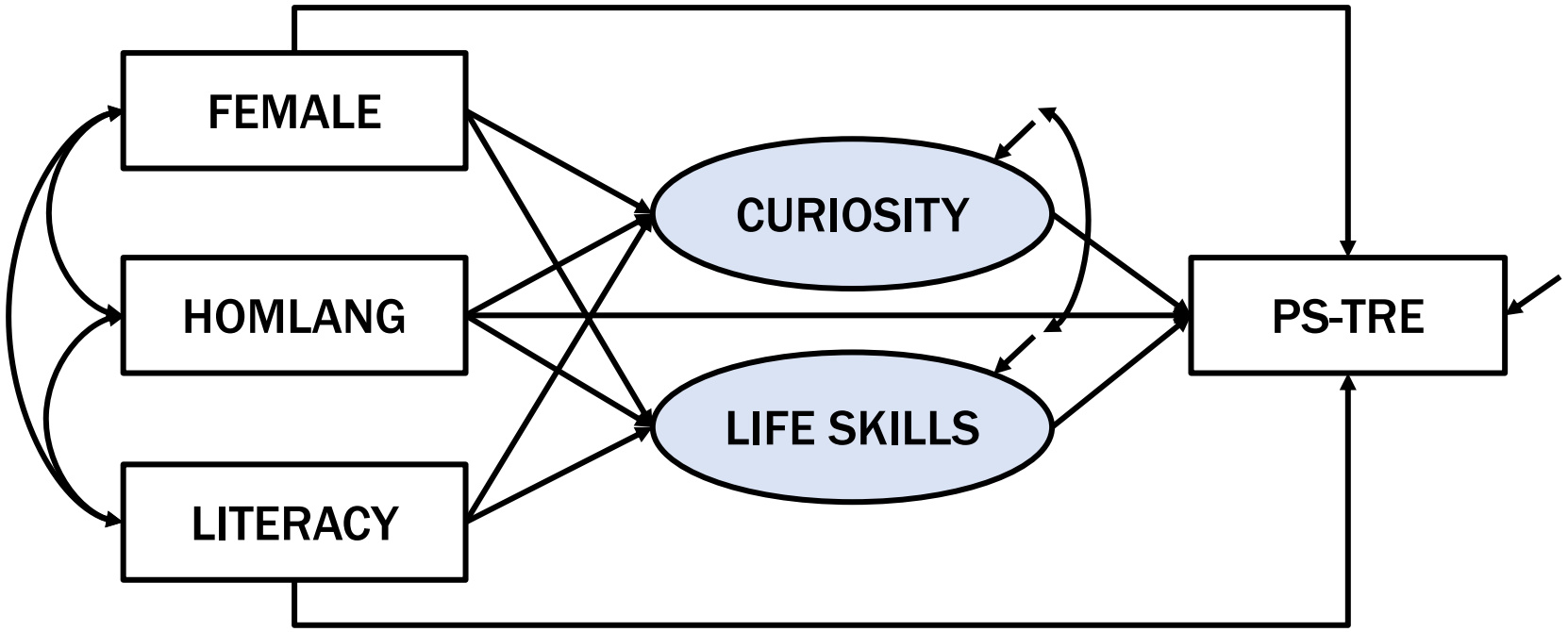
Define the relationships among variables at the level of constructs

Define latent variables and construct representations

(Kline, 2016)

Complex Structural Equation Model

SEM with multiple mediators and predictors

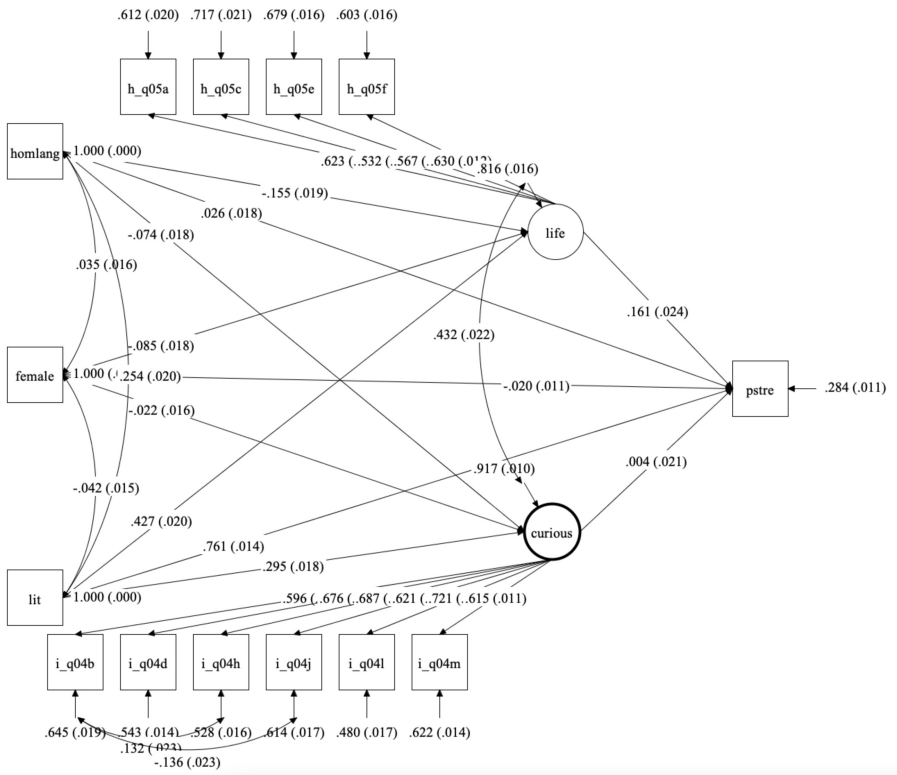


- Observed and latent variables
- Multiple indirect paths
- Multiple direct paths

Complex Structural Equation Model

Model specification

```
MODEL:
! Measurement models
CURIOUS BY I_Q04b-I_Q04m; […]
LIFE BY H_Q05a H_Q05c H_Q05e H_Q05f;

! Structural model
PSTRE ON CURIOUS LIFE FEMALE HOMLANG LIT;
CURIOUS ON FEMALE HOMLANG;
LIFE ON FEMALE HOMLANG;
CURIOUS WITH LIFE;
[…]
```



Model SEM1a

Multi-Group Structural Equation Models (MGSEM)

Multi-Group CFA and Measurement Invariance

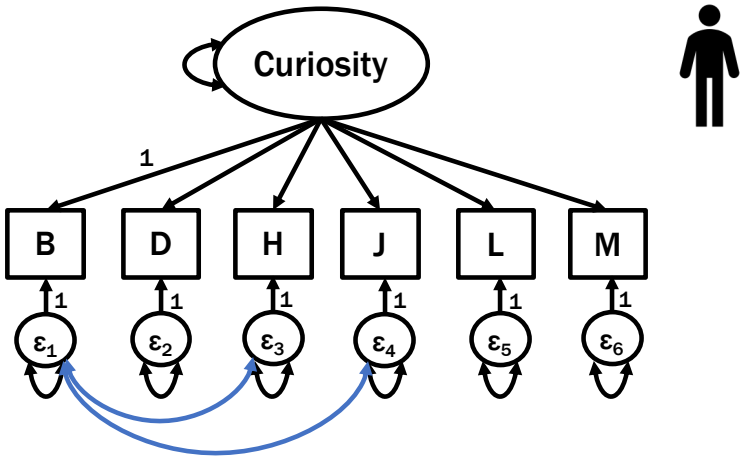
Invariance testing via MG-CFA:

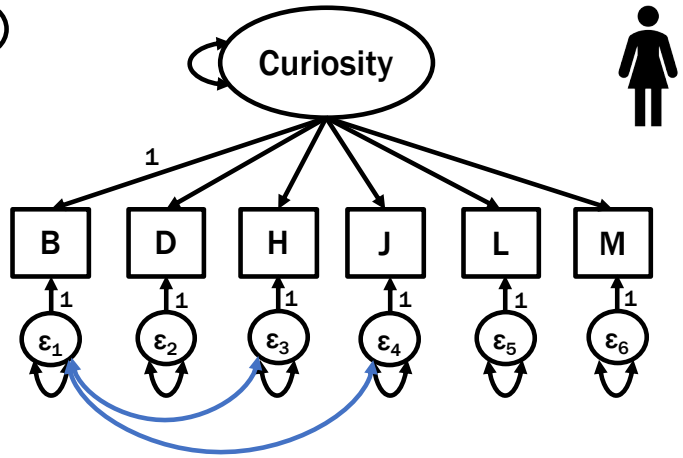
How comparable is the measurement model across groups? To what extent are the model parameters invariant?

Series of nested models with **equality constraints on model parameters**

- Configural invariance
- Metric (weak) invariance
- Scalar (strong) invariance
- Strict (residual) invariance

(Brown, 2015; Millsap, 2011)





Multi-Group CFA and Measurement Invariance

Invariance testing via MG-CFA: Model comparisons

Accept the model with more equality constraints based on the folloiwng evidence:

- Non-significant χ^2 difference test result
- Small differences in fit indices: ΔCFI, ΔRMSEA, ΔSRMR

Comparisons of Cutoff Criteria for ΔGOF Indices in the Bifactor Model to Prior Studies

	This Study	*Chen (2007)*	*Cheung and Rensvold (2002)*	*Meade, Johnson, and Braddy (2008)*
ΔGamma	−.016 to −.023	−.005 to −.008	−.001	N/A
ΔMcDonald's	−.077 to −.101[a]	−.025 to −.030	−.02	−.008
ΔCFI	−.003 to −.004	−.005 to −.010	−.01	−.002
ΔSRMR	.021 to .030[a]	.025 to .030	N/A	N/A
ΔRMSEA	.030 to .034[a]	.010 to .015	N/A	N/A

(Khojasteh & Lo, 2015, p. 540)

Challenge:
Sensitivity of cutoffs on the number of factors, categorical vs. continuous treatment of indicators, complex samples, type of invariance etc.

(e.g., Brown, 2015; Rutkowski & Svetina, 2013)

Multi-Group CFA and Measurement Invariance

Model specification: MG-CFA across **gender**

```
VARIABLE: […]
GROUPING IS FEMALE(0=MEN 1=WOMEN);

ANALYSIS:
! Invariance level(s)
MODEL = CONFIGURAL METRIC SCALAR;

MODEL:
! Measurement model
CURIOUS BY I_Q04b-I_Q04m;
I_Q04B WITH I_Q04H;
I_Q04B WITH I_Q04J;
```

Convenience function to specify multiple levels of invariance

Additional invariance constraints on the residual (co-)variances and other parameters possible

```
MODEL:
[…]
! Residual variances
I_Q04b-I_Q04m(R1-R6);
```

Models MGCFA1a-e

Multi-Group CFA and Measurement Invariance

Model specification: MG-CFA across **gender**

Overview of model fit

Model	χ^2 (*df*)	CFI	RMSEA	SRMR	$\Delta\chi^2$
Configural	138.3 (14)*	.978	.060	.023	-
Metric	147.7 (19)*	.978	.052	.024	3.3, *p*=.65
Scalar	170.6 (24)*	.975	.050	.027	21.0, *p*=.02
Strict	174.7 (32)*	.975	.042	.025	28.1, *p*=.06

*p<.01

Comparisons to the configural model

Result: We can assume strict invariance across gender.

Models MGCFA1b-e

Multi-Group CFA and Measurement Invariance

Model specification: MG-CFA across **countries**

```
VARIABLE: […]
GROUPING IS CNTRYID (56=BEL 124=CAN
      152=CHL 203=CZE 208=DNK 233=EST
      246=FIN 250=FRA 276=DEU 300=GRC
      372=IRL 376=ISR 380=ITA 392=JPN
      410=KOR 440=LTU 528=NLD 554=NZL
      578=NOR 616=POL 702=SGP 703=SVK
      705=SVN 724=ESP 752=SWE 826=GBR
      840=USA);
```

Model comparisons:
Configural, metric, and scalar invariance

MODEL FIT INFORMATION

Invariance Testing

Model	Number of Parameters	Chi-Square	Degrees of Freedom	P-Value
Configural	540	7472.147	189	0.0000
Metric	410	9506.127	319	0.0000
Scalar	280	23723.617	449	0.0000

Models Compared	Chi-Square	Degrees of Freedom	P-Value
Metric against Configural	1581.761	130	0.0000
Scalar against Configural	17033.379	260	0.0000
Scalar against Metric	16218.288	130	0.0000

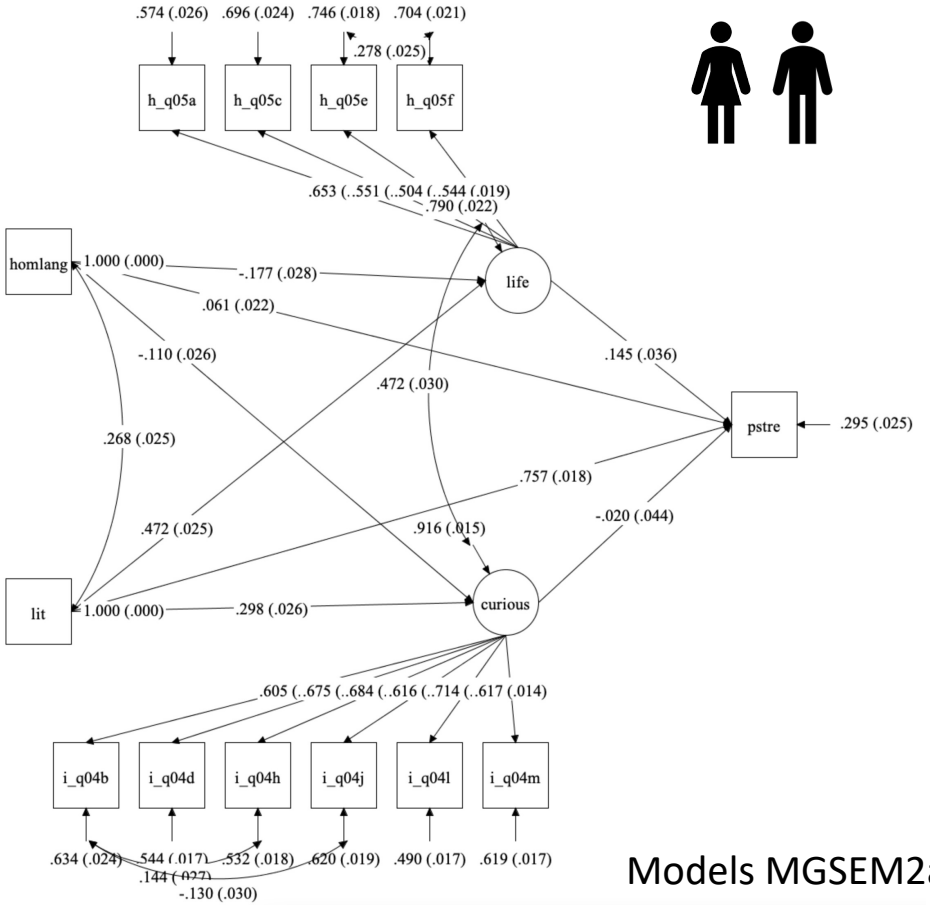
Problem:
Many groups make the invariance of intercepts unlikely.

Models MGCFA3a-d

Multi-Group SEM and Regression Invariance

Extending the invariance testing to the structural model: MG-SEM

```
MODEL:
! Measurement models […]
! Structural model […]

MODEL WOMEN:
! Measurement models […]
! Structural model […]

MODEL MEN:
! Measurement models […]
! Structural model […]
```



Models MGSEM2a-c

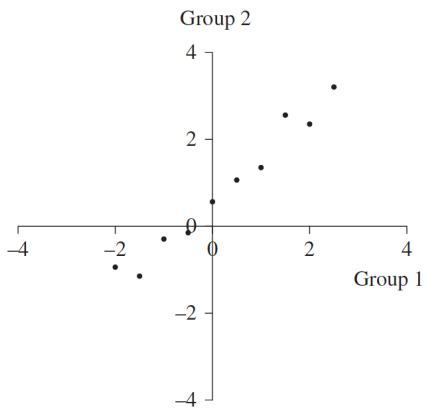
Alignment Optimization

Free or Fixed Alignment Optimization

Alignment Optimization

Estimate factor means and variances **without equality constraints** on item intercepts and factor loadings, **accounting for differences** in item intercepts and factor loadings

(Asparouhov & Muthén, 2014; Muthén & Aspraouhov, 2018)



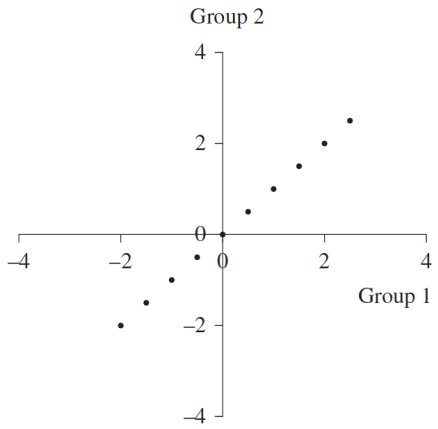
Unaligned: Configural model (mean=0, variance=1 in both groups)

Step 1—Configural invariance model

Estimation of a configural invariance model with freely estimated intercepts and loadings, factor variances@1, and factor means@0

Step 2—Alignment model

Freely estimated factor means and variances with a minimal number of non-invariance intercepts and loadings (i.e., minimizing a simplicity function)



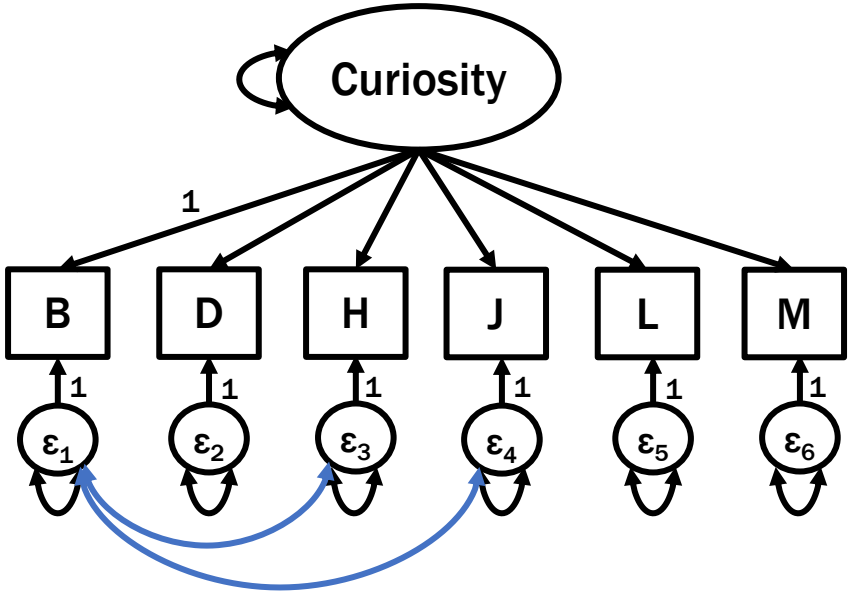
Aligned: Taking into account the group differences in means and variances

(Asparouhov & Muthén, 2014, p. 4)

Alignment Optimization

```
VARIABLE:
CLASSES=c(27);
KNOWNCLASS=c(CNTRYID=56 124 152 203
      208 233 246 250 276 300 372
      376 380 392 410 440 528 554
      578 616 702 703 705 724 752
      826 840);

ANALYSIS:                          OUTPUT:
TYPE=MIXTURE;                      ALIGN;
ALIGNMENT=FREE;                    SVALUES;

MODEL: %OVERALL%
CURIOUS BY I_Q04b-I_Q04m;
I_Q04B WITH I_Q04H; I_Q04B WITH I_Q04J;
```

Alignment method with the FREE option (all factor means freely estimated)

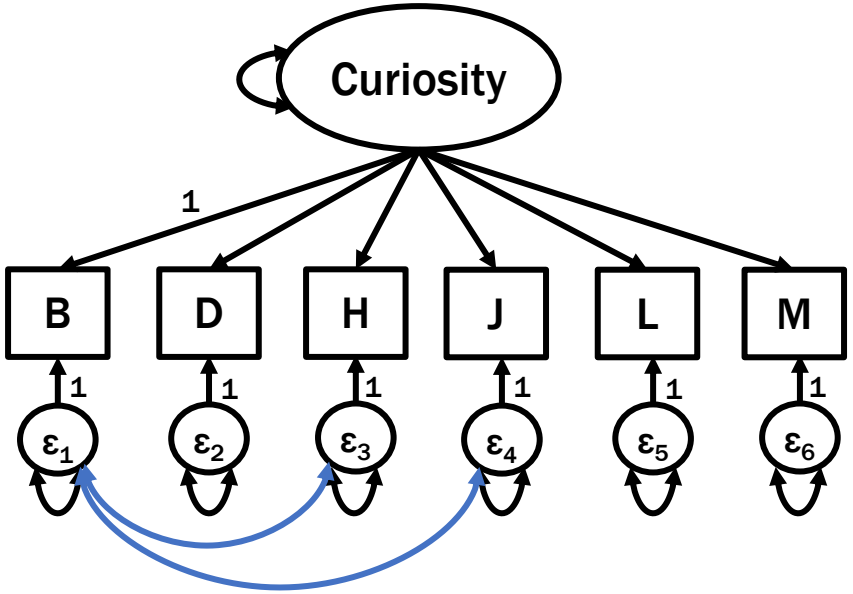


Model ALIGN1a

Alignment Optimization

```
APPROXIMATE MEASUREMENT INVARIANCE (NONINVARIANCE) FOR GROUPS

 Intercepts/Thresholds
   I_Q04B      (56) 124 (152) 203 (208) (233) (246) 250 276 (300) (372) 376 (380) (392)
               (410) (440) (528) (554) (578) 616 (702) (703) (705) 724 (752) (826) 840
   I_Q04D      (56) (124) (152) (203) (208) (233) 246 250 (276) 300 (372) (376) (380)
               392 (410) (440) 528 (554) 578 (616) (702) (703) (705) 724 (752) (826) 840
   I_Q04H      (56) (124) 152 (203) (208) (233) (246) (250) 276 300 372 (376) (380) 392
               (410) (440) (528) 554 (578) 616 702 (703) (705) 724 (752) 826 840
   I_Q04J      (56) 124 152 (203) 208 (233) (246) (250) (276) (300) (372) (376) (380)
               (392) (410) (440) (528) 554 (578) (616) (702) 703 705 (724) (752) (826) 840
   I_Q04L      (56) 124 152 203 208 233 (246) 250 (276) (300) (372) 376 (380) (392) 410
               (440) (528) 554 (578) (616) 702 (703) 705 (724) (752) (826) 840
   I_Q04M      56 124 (152) (203) (208) 233 (246) 250 (276) 300 372 376 (380) (392) 410
               (440) 528 (554) (578) (616) (702) (703) (705) (724) (752) 826 840


 Loadings for CURIOUS
   I_Q04B      56 124 152 203 208 233 246 250 (276) 300 372 376 380 392 410 (440) 528
               554 578 (616) (702) (703) 705 724 752 826 840
   I_Q04D      56 124 152 203 208 (233) 246 250 (276) 300 372 376 380 (392) 410 (440)
               528 554 578 (616) (702) 703 (705) 724 (752) 826 840
   I_Q04H      56 124 152 203 208 233 (246) (250) (276) 300 372 376 380 392 410 440 528
               554 578 616 702 703 705 (724) (752) 826 840
   I_Q04J      (56) 124 152 203 208 (233) 246 (250) 276 300 372 (376) 380 392 410 440
               528 554 (578) 616 (702) 703 705 724 (752) 826 840
   I_Q04L      (56) 124 (152) 203 208 233 246 250 276 (300) 372 376 (380) (392) (410)
               440 528 554 578 (616) (702) (703) (705) (724) (752) 826 840
   I_Q04M      56 124 152 (203) 208 (233) 246 (250) 276 (300) 372 376 380 392 (410) 440
               528 554 578 (616) 702 (703) 705 724 752 826 840
```

Alignment method with the FREE option (all factor means freely estimated)

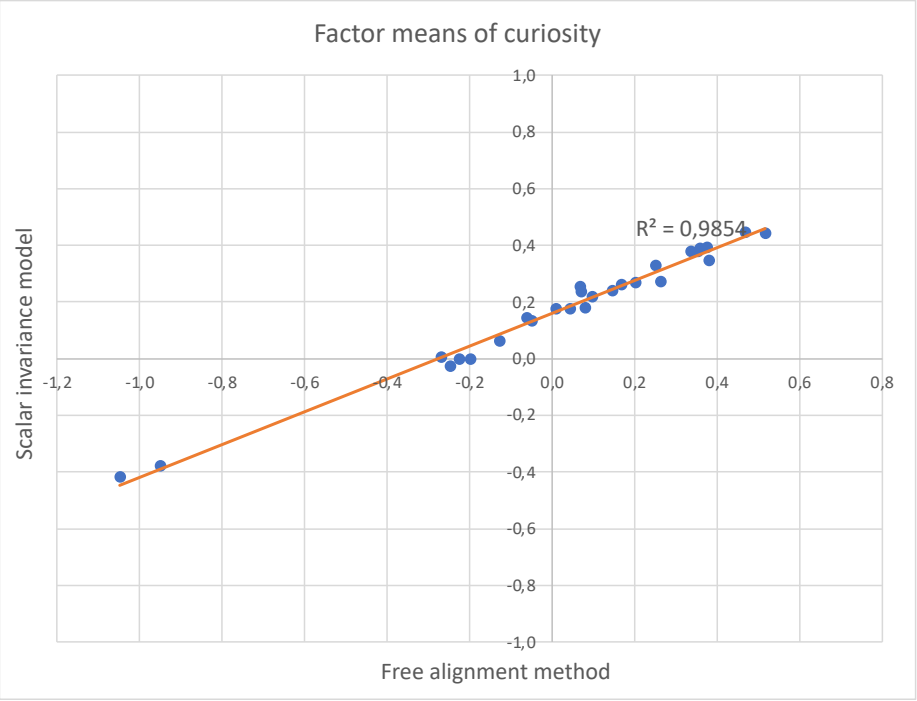


Model ALIGN1a

Alignment Optimization

Alignment method with the FREE option (all factor means freely estimated)

Ranking	Latent Class	Group Value	Factor Mean	Groups With Significantly Smaller Factor Mean
1	7	246	0.517	840 752 152 208 124 554 724 578 376 250 380 703 300 372 826 705 616 203 276 233 528 56 440 702 392 410
2	27	840	0.468	752 152 208 124 554 724 578 376 250 380 703 300 372 826 705 616 203 276 233 528 56 440 702 392 410
3	25	752	0.382	724 578 376 250 380 703 300 372 826 705 616 203 276 233 528 56 440 702 392 410
4	3	152	0.377	724 578 376 250 380 703 300 372 826 705 616 203 276 233 528 56 440 702 392 410
5	5	208	0.360	724 578 376 250 380 703 300 372 826 705 616 203 276 233 528 56 440 702 392 410
6	2	124	0.354	724 578 376 250 380 703 300 372 826 705 616 203 276 233 528 56 440 702 392 410
7	18	554	0.337	724 578 376 250 380 703 300 372 826 705 616 203 276 233 528 56 440 702 392 410
8	24	724	0.264	376 250 380 703 300 372 826 705 616 203 276 233 528 56 440 702 392 410
9	19	578	0.252	376 250 380 703 300 372 826 705 616 203 276 233 528 56 440 702 392 410
10	12	376	0.203	380 703 300 372 826 705 616 203 276 233 528 56 440 702 392 410
11	8	250	0.169	703 300 372 826 705 616 203 276 233 528 56 440 702 392 410
12	13	380	0.146	300 372 826 705 616 203 276 233 528 56 440 702 392 410
13	22	703	0.097	705 616 203 276 233 528 56 440 702 392 410
14	10	300	0.081	616 203 276 233 528 56 440 702 392 410
15	11	372	0.070	616 203 276 233 528 56 440 702 392 410
16	26	826	0.069	616 203 276 233 528 56 440 702 392 410
17	23	705	0.044	203 276 233 528 56 440 702 392 410
18	20	616	0.009	276 233 528 56 440 702 392 410
19	4	203	-0.048	233 528 56 440 702 392 410
20	9	276	-0.061	233 528 56 440 702 392 410
21	6	233	-0.126	528 56 440 702 392 410
22	17	528	-0.197	702 392 410
23	1	56	-0.224	392 410
24	16	440	-0.246	392 410
25	21	702	-0.268	392 410
26	14	392	-0.950	410
27	15	410	-1.048	



FACTOR MEAN COMPARISON AT THE 5% SIGNIFICANCE LEVEL IN DESCENDING ORDER

Results for Factor CURIOUS

Model ALIGN1a

Alignment Optimization

ALIGNMENT OUTPUT

INVARIANCE ANALYSIS

Intercepts/Thresholds
Intercept for I_Q04B

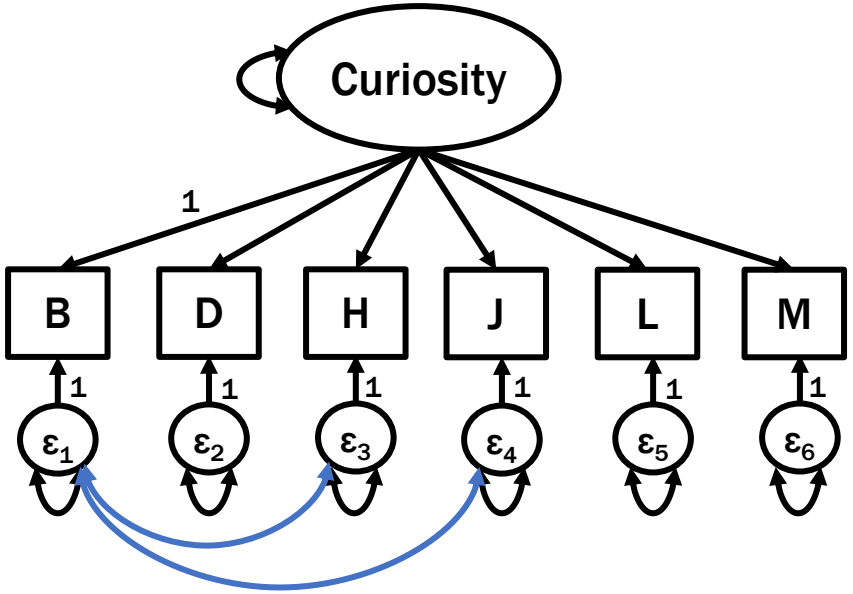
Group	Group	Value	Value	Difference	SE	P-value
124	56	2.179	2.386	-0.208	0.014	0.000
152	56	2.069	2.386	-0.318	0.024	0.000
152	124	2.069	2.179	-0.110	0.021	0.000
203	56	2.143	2.386	-0.243	0.019	0.000
203	124	2.143	2.179	-0.035	0.016	0.027
203	152	2.143	2.069	0.075	0.025	0.003
208	56	2.107	2.386	-0.279	0.017	0.000
208	124	2.107	2.179	-0.071	0.013	0.000
208	152	2.107	2.069	0.039	0.022	0.077

Approximate Measurement Invariance Holds For Groups:
124 203 250 276 376 616 724 840
Weighted Average Value Across Invariant Groups: 2.173
R-square/Explained variance/Invariance index: 0.213

Invariant Group Values, Difference to Average and Significance

Group	Value	Difference	SE	P-value
124	2.179	0.006	0.009	0.553
203	2.143	-0.030	0.015	0.045
250	2.207	0.034	0.012	0.004
276	2.149	-0.024	0.010	0.018
376	2.160	-0.013	0.012	0.259
616	2.177	0.004	0.011	0.715
724	2.201	0.028	0.010	0.007
840	2.168	-0.005	0.006	0.407

Alignment method with the FREE option (all factor means freely estimated)

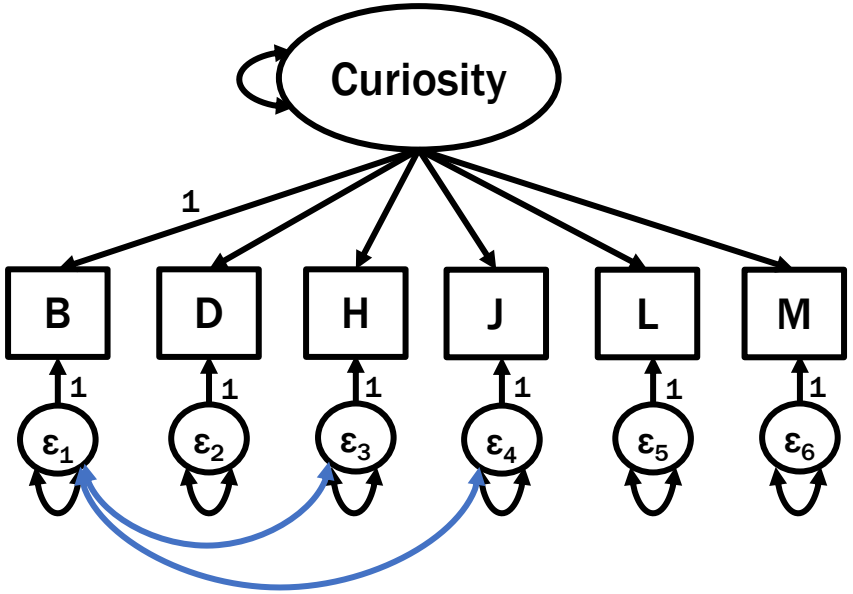


Model ALIGN1a

Alignment Optimization

```
VARIABLE:
CLASSES=c(27);
KNOWNCLASS=c(CNTRYID=56 124 152 203
      208 233 246 250 276 300 372
      376 380 392 410 440 528 554
      578 616 702 703 705 724 752
      826 840);

ANALYSIS:                          OUTPUT:
TYPE=MIXTURE;                      ALIGN;
ALIGNMENT=FIXED(616);              SVALUES;

MODEL: %OVERALL%
CURIOUS BY I_Q04b-I_Q04m;
I_Q04B WITH I_Q04H; I_Q04B WITH I_Q04J;
```

Alignment method with the FIXED option (one factor mean fixed to zero)



Model ALIGN1b

Alignment-Within-CFA (AwC)

Alignment-within-CFA

Extending the alignment optimization into the multi-group structural equation modeling framework

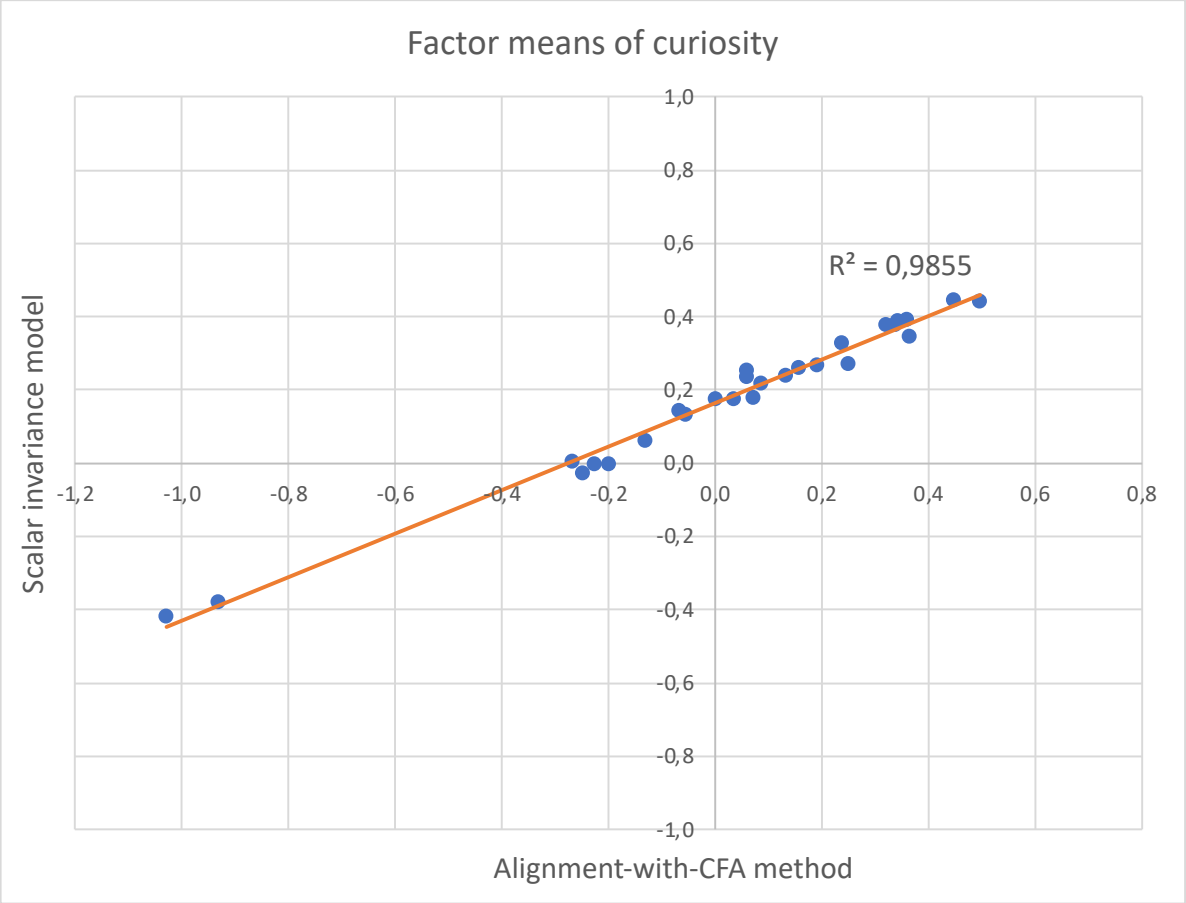
(Marsh et al., 2018)

Step 1—Alignment optimization

Free or fixed alignment optimization

Step 2—MG-CFA

Multi-group CFA with the model parameters from the alignment optimization as starting values



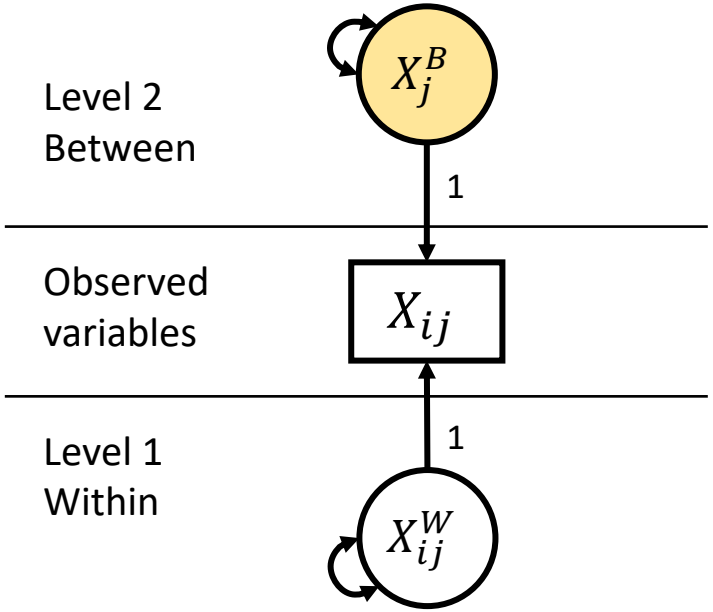
Model ALIGN2

Multilevel Structural Equation Models (MSEM)

Main Ideas behind MSEM

Latent Decomposition:

- Decompose all **observed variables into their latent within and between parts**
- Parsing the variance of the manifest variables in a model
- Dependent and independent variables



Additional features:

- Measurement at multiple levels
- Invariance across levels
- Invariance across many groups
- Inclusion of random slopes

$$\boldsymbol{\Sigma}_T = \boldsymbol{\Sigma}_W + \boldsymbol{\Sigma}_B$$

$$X_{ij} = X_{ij}^W + X_j^B$$

Level-specific fit indices can be computed from the chi-square values of a baseline and the hypothesized model

(Stapleton, 2013; Ryu & West, 2009; Kim et al., 2016)

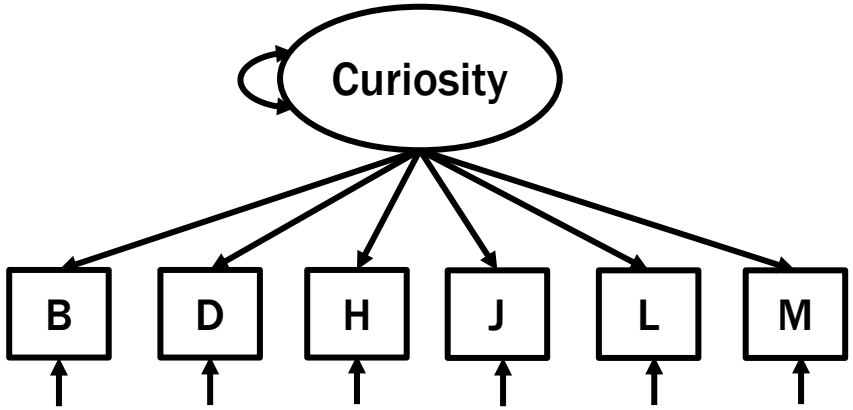
Multilevel Confirmatory Factor Analysis

Multilevel CFA

```
VARIABLE:
CLUSTER=CNTRYID;
```

```
ANALYSIS:
TYPE = TWOLEVEL; ESTIMATOR=MLR;
```

Participants (level 1, within level)




```
MODEL:
%WITHIN%
CURIOUS BY I_Q04b-I_Q04m;
```

Countries (level 2, between level)

Different models can be specified here.

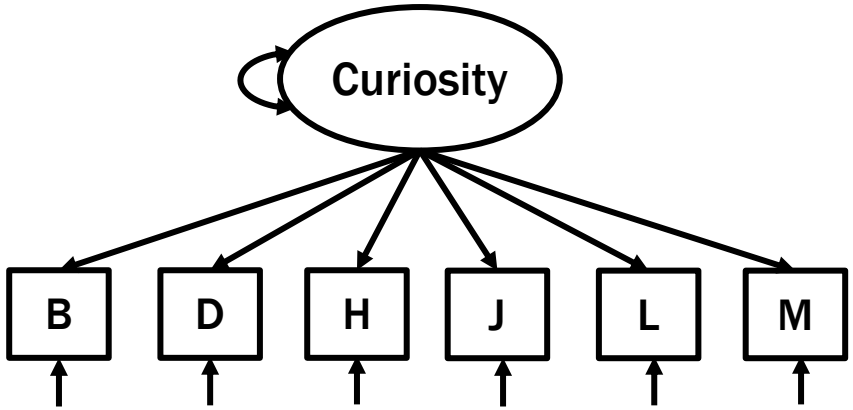
Models MCFA1a-j

```
%BETWEEN%
```

Multilevel Confirmatory Factor Analysis

Multilevel CFA

Participants (level 1, within level)

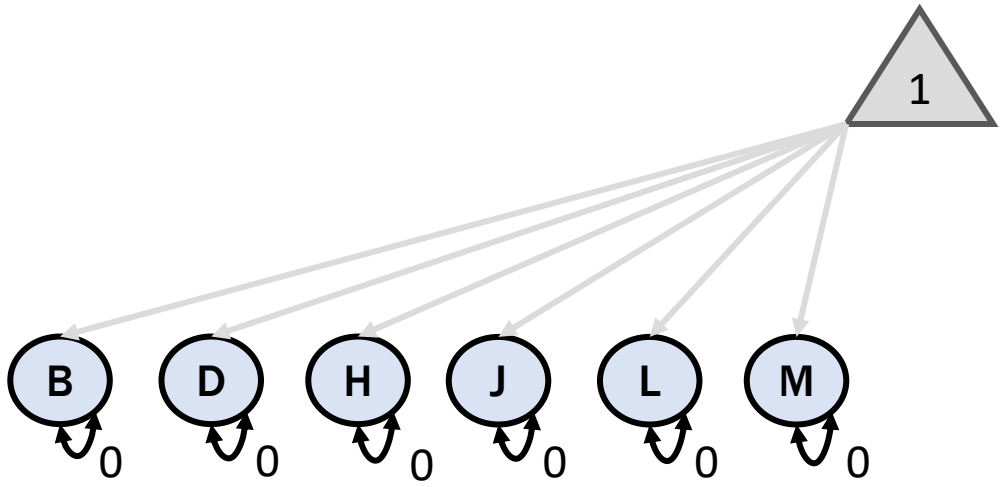



```
MODEL:
%WITHIN%
CURIOUS BY I_Q04b-I_Q04m;
```

Null model

Model MCFA1a

Countries (level 2, between level)

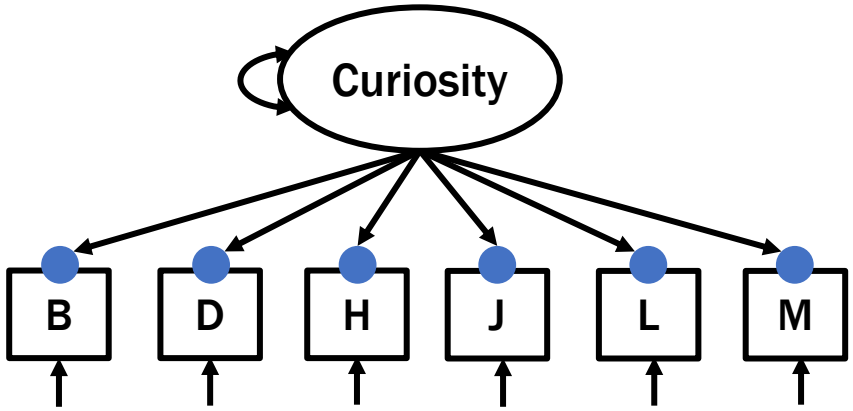



```
%BETWEEN%
[I_Q04b-I_Q04m]; ! Intercepts at L2
I_Q04b-I_Q04m@0;
I_Q04b-I_Q04m WITH I_Q04b-I_Q04m@0;
```

Multilevel Confirmatory Factor Analysis

Multilevel CFA

Participants (level 1, within level)

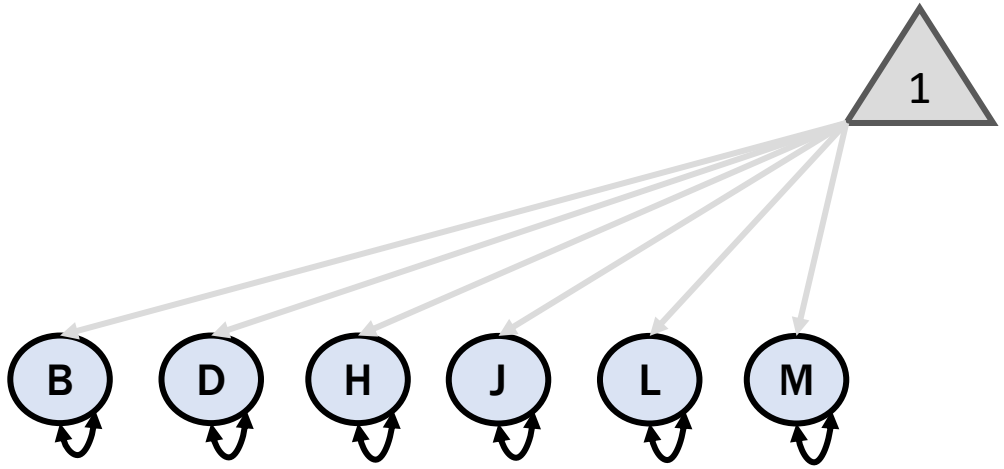



```
MODEL:
%WITHIN%
CURIOUS BY I_Q04b-I_Q04m;
```

Independence model

Model MCFA1b

Countries (level 2, between level)

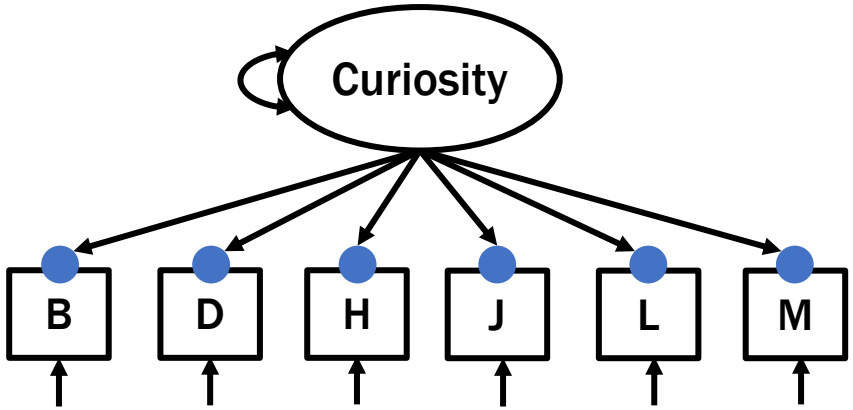



```
%BETWEEN%
[I_Q04b-I_Q04m];
I_Q04b-I_Q04m;
I_Q04b-I_Q04m WITH I_Q04b-I_Q04m@0;
```

Multilevel Confirmatory Factor Analysis

Multilevel CFA

Participants (level 1, within level)

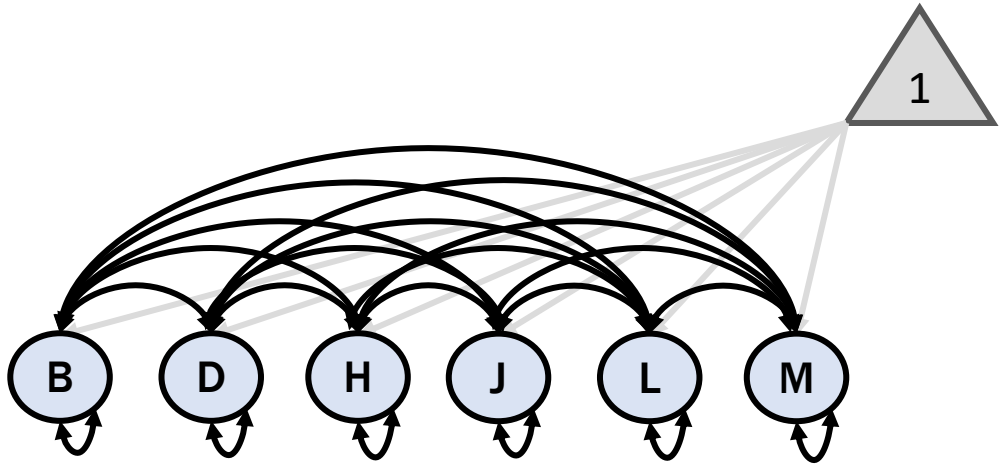



```
MODEL:
%WITHIN%
CURIOUS BY I_Q04b-I_Q04m;
```

Saturated model

Model MCFA1c

Countries (level 2, between level)

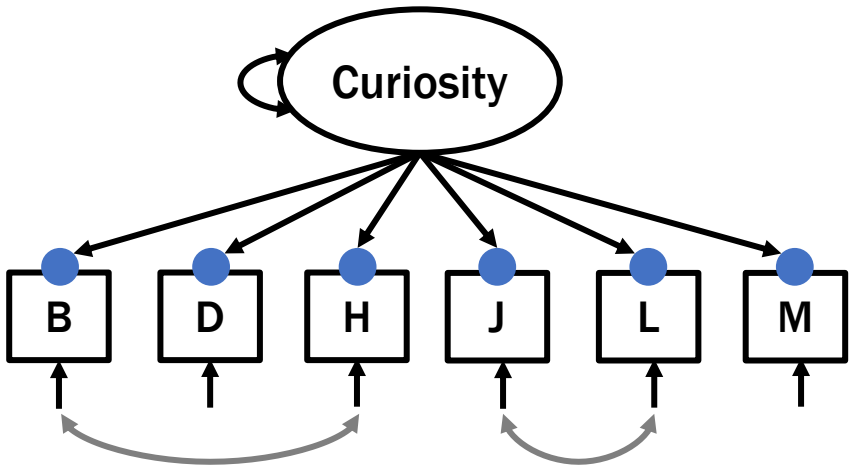



```
%BETWEEN%
[I_Q04b-I_Q04m];
I_Q04b-I_Q04m;
I_Q04b-I_Q04m WITH I_Q04b-I_Q04m;
```

Multilevel Confirmatory Factor Analysis

Multilevel CFA

Participants (level 1, within level)

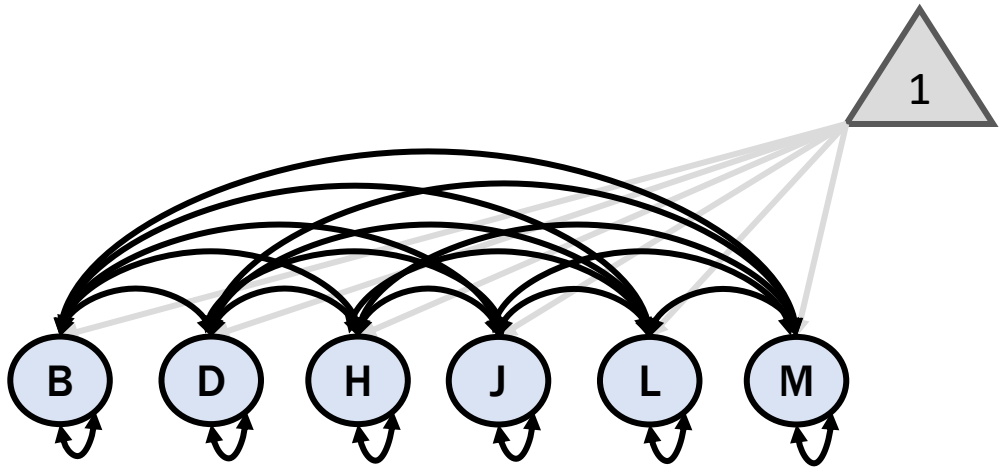



```
MODEL:
%WITHIN%
CURIOUS BY I_Q04b-I_Q04m;
I_Q04b WITH I_Q04h; I_Q04l WITH I_Q04j;
```

Saturated model

Model MCFA1d

Countries (level 2, between level)



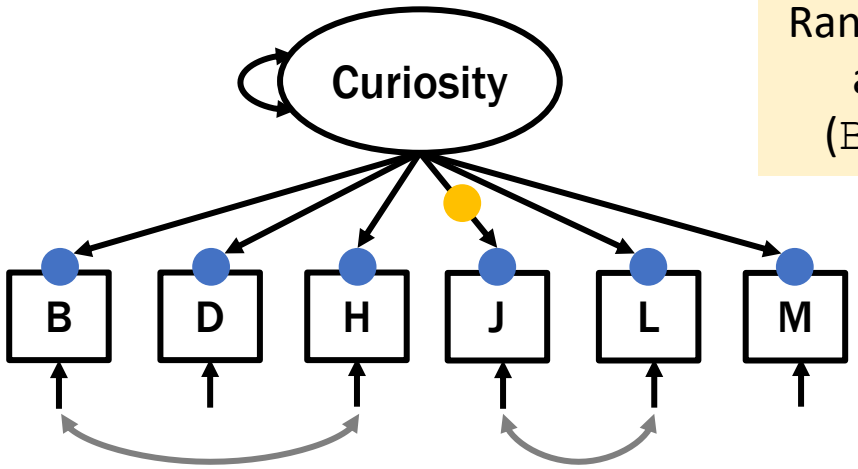

```
%BETWEEN%
[I_Q04b-I_Q04m];
I_Q04b-I_Q04m;
I_Q04b-I_Q04m WITH I_Q04b-I_Q04m;
```

Multilevel Confirmatory Factor Analysis

Multilevel CFA

(Muthén & Aspraouhov, 2018; Kim et al., 2017)

Participants (level 1, within level)



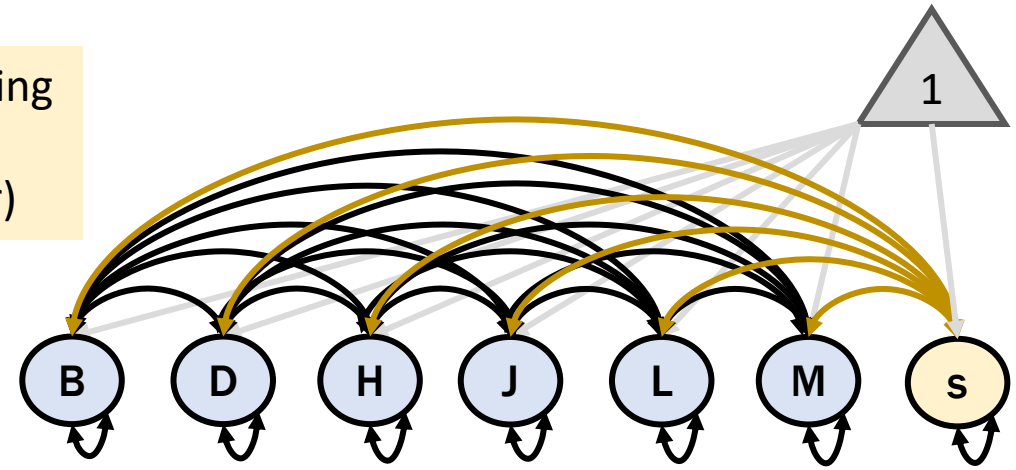

```
MODEL:
%WITHIN%
CURIOUSw BY I_Q04b I_Q04d I_Q04h I_Q04l I_Q04m;
s | CURIOUSw BY I_Q04j;        […]
```

Random factor loading across countries (BAYES estimator)

Saturated model

Model MCFA1j

Countries (level 2, between level)

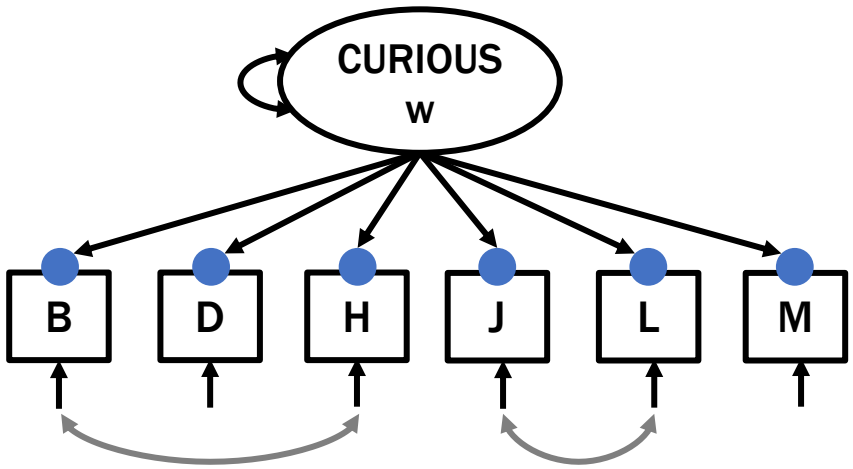



```
%BETWEEN%
I_Q04b-I_Q04m WITH I_Q04b-I_Q04m;
[s]; s;
s WITH I_Q04b-I_Q04m;
```

Multilevel Confirmatory Factor Analysis

Multilevel CFA

Participants (level 1, within level)

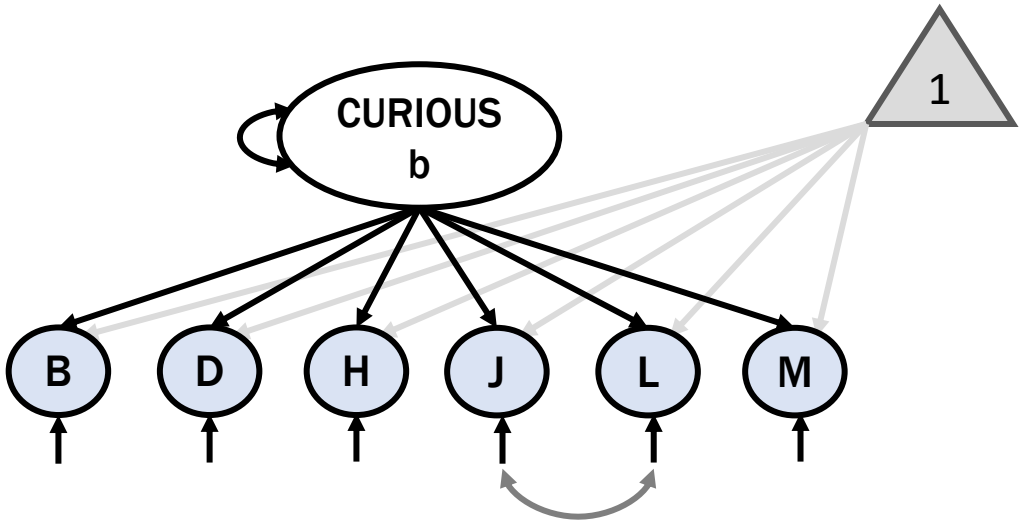



```
MODEL:
%WITHIN%
CURIOUSw BY I_Q04b-I_Q04m;
I_Q04b WITH I_Q04h; I_Q04l WITH I_Q04j;
```

Factor model

Model MCFA1g

Countries (level 2, between level)

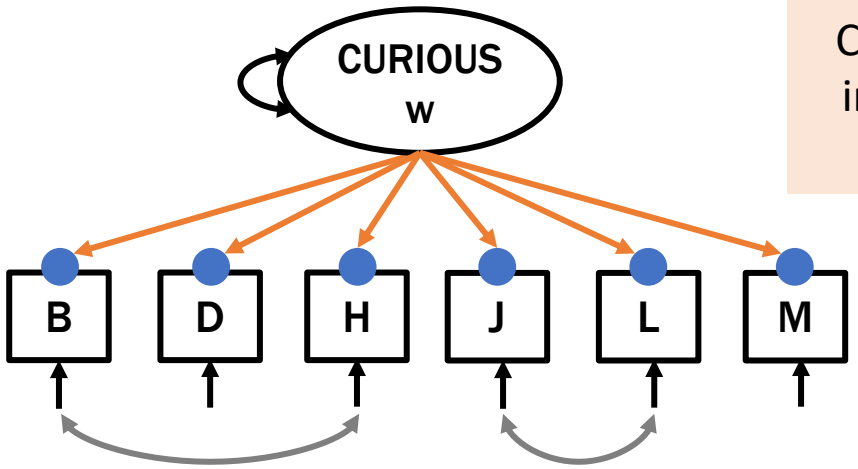



```
%BETWEEN%
CURIOUSb BY I_Q04b-I_Q04m;
I_Q04l WITH I_Q04j;
```

Multilevel Confirmatory Factor Analysis

Multilevel CFA

Participants (level 1, within level)



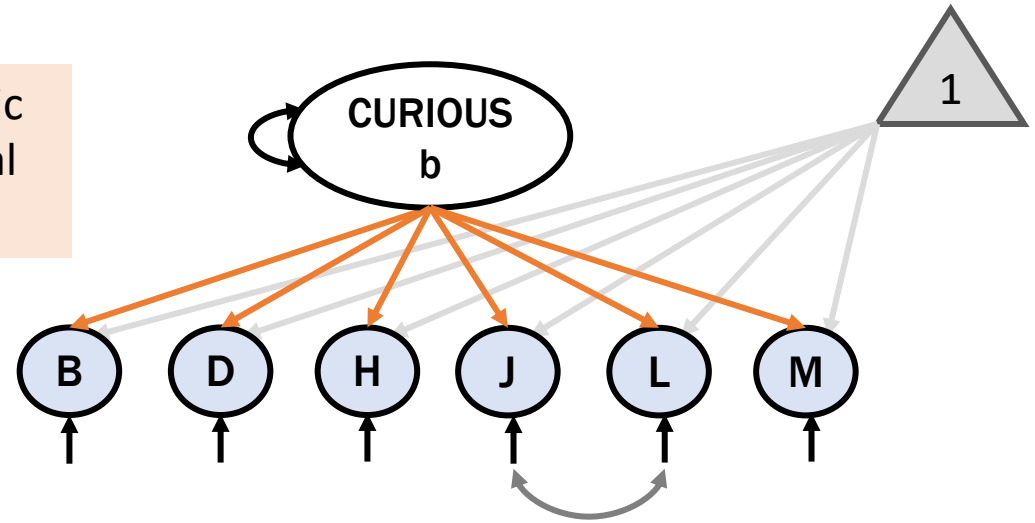

```
MODEL:
%WITHIN%
CURIOUSw BY I_Q04b I_Q04d-I_Q04m(L2-L6);
[…]
```

Cross-level metric invariance (equal factor loadings)

Factor model

Model MCFA1h

Countries (level 2, between level)

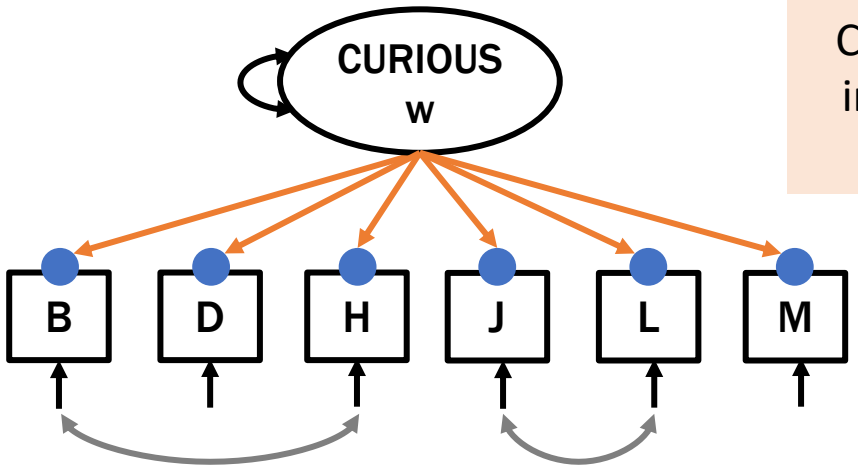



```
%BETWEEN%
CURIOUSb BY I_Q04b I_Q04d-I_Q04m(L2-L6);
[…]
```

Multilevel Confirmatory Factor Analysis

Multilevel CFA

Participants (level 1, within level)



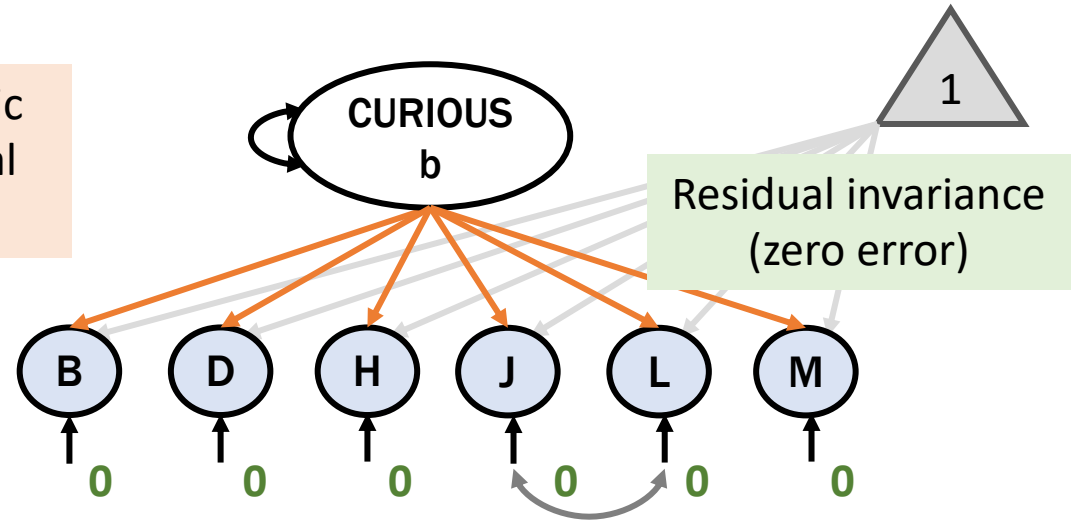

```
MODEL:
%WITHIN%
CURIOUSw BY I_Q04b I_Q04d-I_Q04m(L2-L6);
[…]
```

Cross-level metric invariance (equal factor loadings)

Factor model

Model MCFA1i

Countries (level 2, between level)

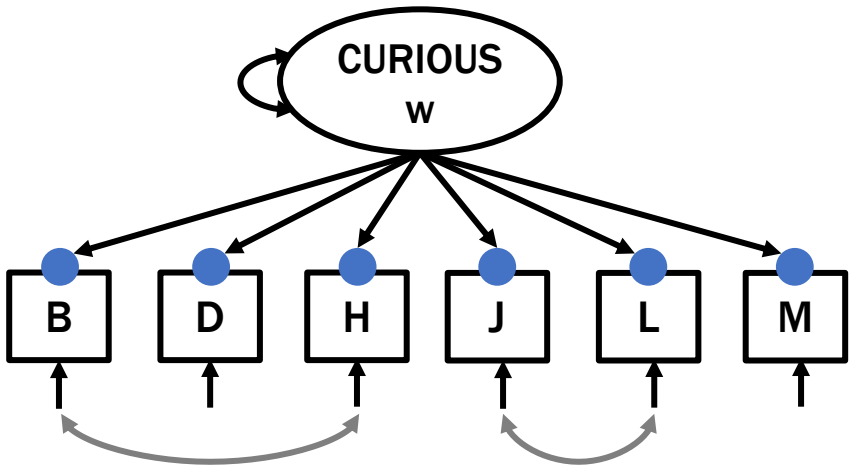



```
%BETWEEN%
CURIOUSb BY I_Q04b I_Q04d-I_Q04m(L2-L6);
I_Q04b-I_Q04m@0;     […]
```

Multilevel Structural Equation Model

Multilevel CFA

Participants (level 1, within level)

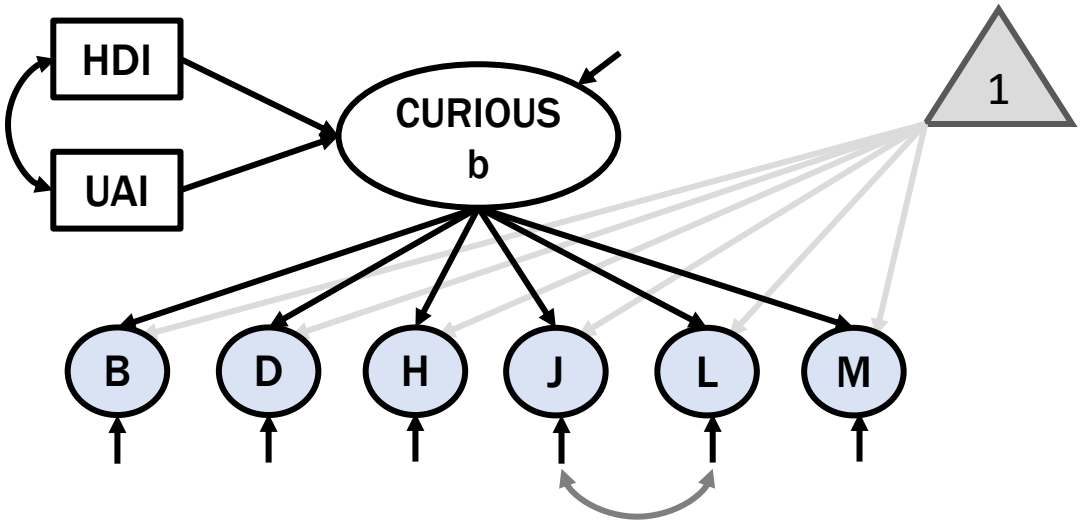



```
MODEL:
%WITHIN%
CURIOUSw BY I_Q04b-I_Q04m;
I_Q04b WITH I_Q04h; I_Q04l WITH I_Q04j;
```

Factor and structural model

Model MSEM2b

Countries (level 2, between level)




```
%BETWEEN%
CURIOUSb BY I_Q04b-I_Q04m; I_Q04l WITH I_Q04j;
CURIOUSb ON HDI2012 UAI;
HDI2012; UAI; HDI2012 WITH UAI;
```

Take-Home Messages

Some Recommendations

1. Implement the complexities of the PIAAC data (weighting, clustering, stratification)—if in doubt, compare multiple things.
2. When achievement domains are part of the SEM, always include all PVs and apply Rubin's combination rules.
3. When comparing construct scores across groups, make sure to test for measurement invariance and adapt your strategy if it is not fully met.
4. Make sure that your SEM is identified and correctly specified, so that the model parameter make sense.

Thank you!

References

Asparouhov, T., & Muthén, B. (2014). Multiple-Group Factor Analysis Alignment. *Structural Equation Modeling: A Multidisciplinary Journal*, *21*(4), 495-508. https://doi.org/10.1080/10705511.2014.919210

Brown, T. A. (2015). *Confirmatory Factor Analysis for Applied Research* (2nd ed.). The Guilford Press.

Enders, C. K. (2010). *Applied missing data analysis*. Guilford Press.

Gorges, J., Maehler, D. B., Koch, T., & Offerhaus, J. (2016). Who likes to learn new things: measuring adult motivation to learn with PIAAC data from 21 countries. *Large-scale Assessments in Education*, *4*(1), 9. https://doi.org/10.1186/s40536-016-0024-4

Hoyle, R. J. (2012). Introduction and overview. In R. H. Hoyle (Ed.), *Handbook of Structural Equation Modeling* (pp. 3-16). Guilford Press.

Hu, L. t., & Bentler, P. M. (1999). Cutoff criteria for fit indexes in covariance structure analysis: Conventional criteria versus new alternatives. *Structural Equation Modeling: A Multidisciplinary Journal*, *6*(1), 1-55. https://doi.org/10.1080/10705519909540118

Jöreskog, K. G., Olsson, U. H., & Wallentin, F. Y. (2016). *Multivariate Analysis with LISREL*. Springer. https://doi.org/10.1007/978-3-319-33153-9

Kaplan, D. (2001). Structural Equation Modeling. In N. J. Smelser & P. B. Baltes (Eds.), *International Encyclopedia of the Social & Behavioral Sciences* (pp. 15215-15222). Pergamon. https://doi.org/https://doi.org/10.1016/B0-08-043076-7/00776-2

Khojasteh, J., & Lo, W.-J. (2015). Investigating the Sensitivity of Goodness-of-Fit Indices to Detect Measurement Invariance in a Bifactor Model. *Structural Equation Modeling: A Multidisciplinary Journal*, *22*(4), 531-541. https://doi.org/10.1080/10705511.2014.937791

Kim, E. S., Cao, C., Wang, Y., & Nguyen, D. T. (2017). Measurement Invariance Testing with Many Groups: A Comparison of Five Approaches. *Structural Equation Modeling: A Multidisciplinary Journal*, *24*(4), 524-544. https://doi.org/10.1080/10705511.2017.1304822

Kim, E. S., Dedrick, R. F., Cao, C., & Ferron, J. M. (2016). Multilevel Factor Analysis: Reporting Guidelines and a Review of Reporting Practices. *Multivariate Behavioral Research*, *51*(6), 881-898. https://doi.org/10.1080/00273171.2016.1228042

Kline, R. B. (2016). *Principles and practice of structural equation modeling, 4th ed*. Guilford Press.

Little, T. D. (2013). *Longitudinal structural equation modeling*. The Guilford Press.

Marsh, H. W., Guo, J., Parker, P. D., Nagengast, B., Asparouhov, T., Muthén, B., & Dicke, T. (2018). What to do when scalar invariance fails: The extended alignment method for multi-group factor analysis comparison of latent means across many groups. *Psychol Methods*, *23*(3), 524-545. https://doi.org/10.1037/met0000113

Marsh, H. W., Hau, K.-T., & Grayson, D. (2005). Goodness of Fit in Structural Equation Models. In A. Maydeu-Olivares & J. J. McArdle (Eds.), *Contemporary psychometrics: A festschrift for Roderick P. McDonald* (pp. 275-340). Lawrence Erlbaum Associates Publishers.

References

McNeish, D., & Wolf, M. G. (2021). Dynamic fit index cutoffs for confirmatory factor analysis models. *Psychological Methods*. https://doi.org/10.1037/met0000425

Meng, X.-L., & Rubin, D. B. (1992). Performing Likelihood Ratio Tests with Multiply-Imputed Data Sets. *Biometrika*, *79*(1), 103-111. https://doi.org/10.2307/2337151

Millsap, R. E. (2011). *Statistical approaches to measurement invariance*. Routledge/Taylor & Francis Group.

Muthén, B. O., & Asparouhov, T. (2018). Recent Methods for the Study of Measurement Invariance With Many Groups: Alignment and Random Effects. *Sociological Methods & Research*, *47*(4), 637-664. https://doi.org/10.1177/0049124117701488

OECD. (2019). *Technical Report of the Survey of Adult Skills (PIAAC)* (3rd ed.). OECD Publishing. https://www.oecd.org/skills/piaac/publications/PIAAC_Technical_Report_2019.pdf

Rosseel, Y. (2020). *Structural equation modeling with lavaan* CISA, Geneva. https://users.ugent.be/~yrosseel/lavaan/geneve2020/lavaan_oneday_geneve2020.pdf

Rust, K. (2014). Sampling, Weighting, and Variance Estimation in International Large-Scale Assessments. In L. Rutkowski, M. von Davier, & D. Rutkowski (Eds.), *Handbook of international large-scale assessment: Background, technical issues, and methods of data analysis* (pp. 117-154). CRC Taylor & Francis. https://doi.org/10.1201/b16061

Rutkowski, L., & Svetina, D. (2013). Assessing the Hypothesis of Measurement Invariance in the Context of Large-Scale International Surveys. *Educational and Psychological Measurement*, *74*(1), 31-57. https://doi.org/10.1177/0013164413498257

Rutkowski, L., & Zhou, Y. (2014). Using Structural Equation Models to Analyze ILSA Data. In L. Rutkowski, M. von Davier, & D. Rutkowski (Eds.), *Handbook of international large-scale assessment: Background, technical issues, and methods of data analysis* (pp. 425-450). CRC Press. https://doi.org/https://doi.org/10.1201/b16061

Ryu, E., & West, S. G. (2009). Level-specific evaluation of model fit in multilevel structural equation modeling. *Structural Equation Modeling*, *16*(4), 583-601. https://doi.org/10.1080/10705510903203466

Satorra, A., & Bentler, P. M. (2010). Ensuring Positiveness of the Scaled Difference Chi-square Test Statistic. *Psychometrika*, *75*(2), 243-248. https://doi.org/10.1007/s11336-009-9135-y

Stapleton, L. M. (2013). Multilevel structural equation modeling with complex sample data. In G. R. Hancock & R. O. Mueller (Eds.), *Structural Equation Modeling: A Second Course* (2nd ed., pp. 521-562). Informationn Age Publishing, Inc.

von Davier, M., Gonzalez, E., & Mislevy, R. J. (2009). What are Plausible Values and Why are They Useful? *IERI Monograph Series: Issues and Methodologies in Large-Scale Assessments*, *2*, 9-36. http://www.ierinstitute.org/fileadmin/Documents/IERI_Monograph/IERI_Monograph_Volume_02_Chapter_01.pdf

Wu, M. (2005). The role of plausible values in large-scale surveys. *Studies in Educational Evaluation*, *31*(2), 114-128. https://doi.org/10.1016/j.stueduc.2005.05.005