Time-Varying Leverage Demand and Predictability of Betting-Against-Beta*

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Abstract

The leverage aversion theory implies that returns to the betting-against-beta (BAB) strategy are predictable by past market returns: An outward shift in investors' aggregate demand function simultaneously increases market prices and increases the expected future BAB return. I confirm the prediction empirically and find that the BAB strategy performs better in times when and in countries where past market returns have been high. I construct timing-strategies that are long BAB half the time and short BAB half the time, based on past market returns, and show that these timing strategies have realized strong historical performance.

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1 Introduction

In the Capital Asset Pricing Model (CAPM) of Sharpe (1964) and Lintner (1965), the risk of an asset is measured by its market beta and the expected return on an asset is proportional to its beta such that the relation between beta and expected returns, the security market line (SML), has a slope equal to the equity risk premium and an intercept equal to the risk-free rate. Empirically, Black et al. (1972) finds that the SML is too flat, and Black (1972) and Frazzini and Pedersen (2014) (FP hereafter) suggest that the flat SML is due to leverage constrained investors. Instead of applying leverage, constrained investors obtain an expected return higher than the expected return on the market by over-weighting high-beta stocks and under-weighting low-beta stocks in their portfolios, thereby lowering future risk-adjusted returns on high-beta stocks and increasing future risk-adjusted returns on low-beta stocks. FP construct a betting-against-beta (BAB) portfolio that goes long low-beta assets, leveraged to a beta of one, and short high-beta assets, also leveraged to a beta of one. The leverage aversion theory in FP predicts that the SML should be too flat on average and that the BAB factor should have a positive average return, which FP also find empirically.

I show that according to the leverage aversion theory, a shift in investors' aggregate demand function moves the economy to a new equilibrium in which both prices and expected returns on the BAB factor have changed. Therefore, past market returns forecast future BAB returns. For instance, consider an outward shift in investors' demand functions. For markets to clear, prices must increase and we observe a positive return on the market. In the new equilibrium, constrained investors have increased the over-weight of high-beta stocks in their portfolio, relative to the market portfolio, such that high-beta stocks have become more overvalued relative to the CAPM and the expected return on the BAB factor has increased. Therefore, high past returns on the market forecast high future returns on the BAB factor. Further, since expected returns move opposite to prices, high market returns lead to contemporaneously low returns on the BAB factor.

More subtly, the effect of an outward shift in investors' demand function on the expected BAB return is theoretically ambiguous, since there are two opposing effects: First, holding expected returns constant, constrained investors become more constrained and increase the over-weight to high-beta stocks, such that the expected return on the BAB factor increases. Second, since prices increase and expected returns decline for markets to clear, the improvement in utility of over-weighting high-beta stocks decline, such that investors become less constrained and the expected BAB return declines. Thus, if investors become more constrained following an outward shift in their demand functions, the shift leads to an increase in the expected BAB return, but if investors become less constrained it leads to a decrease in the expected BAB return.

It seems natural that investors become more constrained following an outward shift in their demand functions, and I show that empirically high past market returns forecast high future BAB returns and that daily market returns are negatively correlated with contemporaneous BAB returns. I demonstrate this for both U.S. BAB returns and international BAB returns, as well as for BAB factors formed from country indices.

I show that past market returns predict future BAB returns using both regressions, sorts, and by constructing a trading strategy. First, regressing BAB returns on contemporaneous and lagged market returns shows that daily BAB returns have a strong negative correlation with daily market returns and that past market returns positively predict future BAB returns. Second, sorting BAB returns on past market returns shows that realized BAB returns are higher following high past market returns. The difference in BAB returns following high and low market returns is not explained by different exposures to any of the Fama and French (1993) or Carhart (1997) factors, nor is it explained by variation in the realized volatility of the BAB strategy. I demonstrate this effect for both the United States as well as for a sample of 23 other countries. Third, I construct BAB-timing strategies that take a position in the BAB factor in a given country proportional to the past 12 month market return of that country. Across the 24 international countries, the BAB timing strategies have generated positive alpha to a two-factor model with the market and BAB in all but 2 countries, and the alpha is significant in 12 of the 24 countries.

Further, the variation in expected BAB returns coming from past returns is not only a time series effect, it is also a cross sectional effect. I show that past market returns help explain cross-country differences in future BAB returns. Sorting countries based on their past market return shows that future BAB returns are higher in countries for which the past market return was higher than the cross-country average market return, and future BAB returns are lower in countries for which the past market return was below the cross-country average.

These results show a strong pattern in the time-series and cross-country variation of expected returns. Low-beta stocks outperform high-beta stocks on average, on a risk-adjusted basis, but the outperformance is stronger when and where past market returns are high.

I also test the predictability of BAB using country indices. Here, the BAB strategy goes long country indices with a low beta to a global index and short indices with a high beta to a global index. FP show that the BAB strategy generates positive excess returns in a region of 13 developed countries, and extending their results I show that the BAB strategy also generates positive returns in a region of 25 emerging countries. Further, I show that in each region the returns on the BAB strategy are predictable by past returns on a cap-weighted index of the countries in the region.

My results help understand time-series variation in the cross-section of expected returns. The standard CAPM cannot explain the cross-section of unconditional stock returns (Fama and French (1993)) or conditional stock returns (Lewellen and Nagel (2006)). However, I find that the deviation of expected returns from the CAPM, when sorting stocks on their betas, is stronger when past market returns are high. Further, my results add to our understanding of low-risk anomalies by showing that the low-beta anomaly is strongest when past market returns are high and that the anomaly disappears when past market returns are low.

Other papers document evidence consistent with the leverage aversion theory. Boguth and Simutin (2018) use the average beta of actively managed mutual funds to proxy for their leverage demand and show that this variable predicts BAB returns in the U.S. I include their measure of mutual fund beta in my regressions and show that both past market returns as well as the aggregate mutual fund beta remain significant predictors of future BAB returns.[1] Adrian et al. (2014) find that BAB returns are linked to the leverage of financial intermediaries, and Malkhozov et al. (2017) find that BAB returns are linked to international illiquidity. Barosso and Maio (2016) show that the BAB factor performs better when scaled to constant volatility.[2] The leverage aversion theory predicts this: when market volatility is low, all else equal, investors increase their demand for leverage and future expected BAB returns are high. Since market volatility and the volatility of the BAB factor are positively correlated, scaling BAB factors to constant volatility improves the performance.

2 The Leverage Aversion Theory and Predictability of BAB

FP consider an overlapping generations model with leverage-constrained investors. Investor i's problem is

$$\max_x x' \left(E_t(P_{t+1} + \delta_{t+1}) - (1 + r^f)P_t\right) - \frac{\gamma_t^i}{2} x'\Omega_t x, \tag{1}$$

where P_t is the vector of prices at time t, δ_{t+1} are dividends, and Ω_t is the covariance matrix of $P_{t+1} + \delta_{t+1}$. γ_t^i is a discount rate parameter that determines the discount rate in equilibrium.

[1]See also Huang et al. (2015) who measure 'beta-arbitrage activity' using the correlation of low-beta stocks and show that this variable predicts future BAB returns in the U.S. Hong and Sraer (2016) show that measures of disagreement forecast the slope of the SML.

[2]See also Moreira and Muir (2017) who analyze volatility-managed portfolios.

Agent i is subject to a leverage constraint

$$m_t^i \sum_s x^s P_t^s \leq W_t^i \tag{2}$$

where m_i determines the amount of leverage investor i can apply and W_i is investor i's wealth. Let ψ_t^i be the Lagrange multiplier on the investor's budget constraint and define the aggregate discount rate parameter γ_t by $1/\gamma_t = \sum_i 1/\gamma_t^i$. Then, defining ψ_t as the weighted average Lagrange multiplier $\psi_t = \sum_i \frac{\gamma_t}{\gamma_t^i}\psi_t^i$, FP derive equilibrium expected returns and show that the expected return on the BAB factor is

$$E_t(r_{t+1}^{BAB}) = \frac{\beta_t^H - \beta_t^L}{\beta_t^L \beta_t^H}\psi_t \geq 0 \tag{3}$$

where β_t^H is the beta of the short leg consisting of assets with β higher than 1 and β_t^L is the beta of the long leg consisting of assets with beta less than 1.

Note that the expected return is increasing in aggregate funding tightness ψ_t. In other words, the leverage aversion theory predicts that the expected return on the BAB factor is higher when constrained investors become more constrained and increase their demand for high-beta assets, or previously unconstrained investors become constrained and as a result increase their demand for high-beta assets.

Investors may become more constrained either due to increases in the cost of leverage (which can be interpreted as an increase in the margin constraint m_t^i) or due to an increase in their demand for leverage. FP focus on time-variation in margin requirements and use the TED spread as a proxy of funding conditions, whereas I focus on time-variation in the demand for leverage.

The leverage aversion theory predicts that changes in investors' aggregate demand function simultaneously lead to changes in prices and changes in the expected return on the BAB factor. Therefore, past market returns predict future BAB returns. This can be seen by considering changes in the parameters of the model. Three parameters, exogenous in the model, cause shifts in investors' demand functions: the discount rate parameters γ_i, volatility Ω_t, and expected payoffs at $t+1$, $E_t(P_{t+1} + \delta_{t+1})$. I emphasize that the focus is on the shift in investors' demand functions, and the parameters of the leverage aversion theory provide a mechanism for analyzing the effect of these shifts.

First, consider the effect of a change in investors' discount rate parameters. Specifically, consider a decrease in the discount rate parameter for all agents such that investors' demand functions shift out. The effect of a decrease in discount rates on the equilibrium expected return on the BAB factor is ambiguous, depending on whether constrained investors become more constrained or not. To see this, suppose first that investors' discount rate parameters are so high that no investor wants to apply leverage in equilibrium and all investors are effectively unconstrained. In this case, the leverage aversion theory collapses to the CAPM, all investors hold the market portfolio, and the expected return on the BAB factor is zero.

Next, suppose that investors' discount rate parameters decrease. Their demand functions shift out and in equilibrium prices must increase for markets to clear. Thus, we observe a positive return on the market. In the new equilibrium, investors' expenditures $x_i' P_t$ are higher and *a)* some investors are constrained (an investor is constrained when the Lagrange multiplier on his budget constraint is positive), *b)* constrained investors overweight high-beta stocks relative to the market portfolio (and unconstrained investors underweight high-beta stocks and apply leverage), and *c*) high-beta stocks are overpriced relative to the CAPM and low-beta stocks are underpriced relative to the CAPM. In this case, a decrease in discount rates results in a positive market return and an increase in the expected return on the BAB factor.

Now, consider what happens as we continue to decrease investors' discount rate parameters. Since prices continue to increase to clear markets, expected returns decline. Intuitively, the Lagrange multiplier measures the utility for a constrained investor of having one extra dollar of wealth. In equilibrium, there are two opposing effects of an outward shift in an investor's demand function on the utility of an extra dollar. One the one hand, holding prices and expected returns fixed, the higher demand increases the utility of an extra dollar of wealth and thus increases the Lagrange multiplier. On the other hand, since prices adjust in equilibrium, the lower expected returns lower the utility of an extra dollar of wealth and thus decrease the Lagrange multiplier. When discount rates are high, the first effect dominates, but when discount rates are low enough, the second effect dominates.

Thus, as we continue to decrease investors' discount rate parameters we arrive at a new equilibrium, where *a)* investors are less constrained, *b)* constrained investors continue to increase the overweight of high-beta stocks relative to the market portfolio (and unconstrained investors increase the underweight of high-beta stocks and apply more leverage), and *c*) high-beta stocks are less overpriced relative to the CAPM and low-beta stocks are less underpriced relative to the CAPM. Because the aggregate Lagrange multiplier declines, the expected return on the BAB factor declines. In this case, a decrease in discount rates results in a positive market return and a decrease in the expected return on the BAB factor.

In the limit where discount rates are zero and all investors are risk-neutral, the expected return on all assets is zero, there is no point in over-weighting high-beta stocks, all investors become unconstrained and the expected return on the BAB factor is zero.

A change in volatility impacts the equilibrium of the model in the same way as a change to all agents' discount rate parameters, since only the product of an investor's discount rate parameter and variance enters into the investor's utility function. Specifically, a decrease in volatility will cause all investors' demand functions to shift out and will always increase market prices, but the effect on the expected return on the BAB factor is ambiguous depending on whether investors become more constrained or not. If investors become more constrained, the economy will move to an equilibrium in which the expected return on the BAB factor is higher, whereas if investors become less constrained the economy will move to an equilibrium in which the expected return on the BAB factor is lower.

Finally, an increase in expected payoffs at time $t+1$, $E_t(P_{t+1} + \delta_{t+1})$, will again shift out investors' demand functions and increase market prices. Higher expected payoffs will always make investors more constrained and increase the expected return on the BAB factor.

In Appendix A I illustrate these effects in a simple example with two investors, one constrained and one unconstrained. I solve numerically for the equilibrium in the example since both prices and betas change in equilibrium when the model parameters change.

Note that the leverage aversion theory predicts that investors will trade when moving from one equilibrium to another; since there are two types of agents, constrained and unconstrained, and their allocations to low-beta versus high-beta stocks change, the two groups of investors will trade with each other to arrive at the new equilibrium.

In summary, an outward shift in investors' demand functions always results in a positive market return but can either increase or decrease the expected return on the BAB factor. If investors become more constrained following an outward shift in the demand function, the expected return on the BAB factor increases, otherwise it decreases. While it is an empirical question which type of equilibrium we are in, it seems natural that investors become more constrained when their demand functions shift out and discount rates decrease. Indeed, I find empirically that past market returns positively predict future BAB returns over a wide range of look-back periods, from 1 day up to 1 year, and that daily positive market returns are associated with negative contemporaneous BAB returns.[3]

To predict expected returns on the BAB factor, we do not need to observe changes in the underlying parameters of the model since changes in market prices are a sufficient statistic. For example, volatility, while not directly observable, can be estimated from past market returns. Assuming that investors become more constrained when their demand functions

[3]Koijen and Yogo (2017) examine the demand for different types of stocks from a set of heterogeneous investors and show that the demand for high-beta stocks is pro-cyclical for all groups of investors. Their results are consistent with my finding that market returns predict returns on the BAB strategy.

shift out, the leverage aversion theory predicts that volatility should negatively predict future BAB returns. Specifically, holding everything else constant, a decrease in volatility should cause higher market prices and higher expected returns on the BAB factor. In reality, of course, there is no mechanical relation between market volatility and returns since other variables may change simultaneously with volatility, and if a decrease in volatility does not lead to an outward shift in demand functions and higher prices there should be no effect on the expected BAB return. The empirical prediction is thus that volatility, in a univariate model, should negatively predict future BAB returns, but controlling for market returns should drive out volatility as a predictor.

3 Data and Methodology

3.1 Data

Equities Data for U.S. and international equities are from the union of the Center for Research in Security Prices (CRSP) tape and the Xpressfeed Global database. The U.S. data include all available common stocks on CRSP between January 1926 and January 2018. The international data include all available common stocks on the Xpressfeed Global daily security file. Table 1 shows summary statistics for the equity data. BAB returns start in 1931 for the U.S. market and in 1988 or later for the remaining 23 countries in the data set.

I compute excess returns on the BAB factor and each market index in both USD and local currency. Excess returns in USD are in excess of the U.S. Treasury bill rate. Excess returns in local currencies are in excess of the 90 interbank offer rate from Bloomberg, for each country. The main analyses below are based on returns in USD but I perform robustness checks using local returns.

Daily and monthly excess returns in USD for the BAB-portfolios, the Fama and French

(1993) SML and HML factors, and the Carhart (1997) UMD factor for U.S. equities as well as 23 international countries are also available directly from the web site accompanying FP.[4]

[Table 1 about here.]

Equity Indices Daily returns on international equity index futures are from Bloomberg. Returns are in USD and in excess of the U.S. Treasury bill rate. Table 2 shows summary statistics for the equity index futures. I split the countries into a developed region with 13 countries and an emerging region with 25 countries. For each region, the beta of a country is computed with respect to a market-cap weighted portfolio of the indices in the region. To construct the BAB factors for the developed and emerging regions I follow the methodology in FP which I briefly outline in Appendix B. The 13 developed countries correspond to the equity indices considered in FP, while the 25 emerging countries provide new evidence on the robustness of the BAB-factor as well as the predictability of the factor.

[Table 2 about here.]

Controls FP use the TED spread as a proxy for the tightness of leverage constraints, and I obtain the TED spread from the Federal Reserve Bank of St. Louis. FP interpret the TED spread as a cost of leverage and therefore expect that future BAB returns are positively correlated with the TED spread, however, they empirically find a negative relation between the current TED spread and future BAB returns.

Boguth and Simutin (2018) compute the aggregate beta of actively managed mutual funds and show that it positively predicts future BAB returns in the U.S. They interpret the aggregate beta as a proxy for leverage constraint tightness, and denote their measure LCT. The authors first compute the market beta of individual stocks from daily returns

[4] http://people.stern.nyu.edu/afrazzin/data_library.htm contains updated BAB-returns for U.S. and international equities.

within a month with Dimson (1979) lags and then compute the aggregate mutual fund beta as a weighted average of individual stock betas, where the weights are inferred from the aggregate holdings of domestically managed active mutual funds. The average value of the LCT measure is 1.08 and the 10th and 90t percentiles are 0.96 and 1.21, and the LCT measure has a low and insignificant correlation with the TED spread. The preferred measure of leverage constraint tightness in Boguth and Simutin (2018) is a 12 month moving average of LCT. I obtain the LCT factor from Boguth's web site.[5]

Asness et al. (2018) construct a leverage measure based on the amount of margin debt held by customers at NYSE member organizations. The data are available starting in 1959 and are published on the NYSE website.[6] At the end of each month, Asness et al. (2018) calculate their margin debt measure as the ratio of margin debt to the market capitalization of NYSE stocks:

$$MD_t = \frac{\text{Margin debt}_t}{\text{Market capitalization of NYSE firms}_t}. \tag{4}$$

When margin debt is low, Asness et al. (2018) interpret this as leverage constraints being tight. Consistent with this interpretation they find a negative correlation between MD and the TED spread.

3.2 Construction of BAB Factors

I follow the methodology in FP, which I briefly outline in Appendix B, to construct BAB factors. The BAB factor constructed in FP is scaled such that the long and short legs both have a beta of 1 at each rebalancing. The BAB factor has therefore neither constant dollar exposure nor is it constant volatility. Moreira and Muir (2017) and Barosso and Maio (2016) analyze volatility managed portfolios and Barosso and Maio (2016) find that

[5] http://www.public.asu.edu/ oboguth/research.html

[6] The data can be found at http://www.nyxdata.com/nysedata/asp/factbook/viewer_edition.asp?mode=table&key=3153&category=8

the BAB factor has a higher Sharpe ratio when scaled to constant volatility. The leverage aversion theory predicts this result: When market volatility is low, investors become more constrained and expected BAB returns increase. Since I focus on time-variation in expected returns driven by past market returns, I first scale the monthly BAB factors to constant volatility. Specifically, I use an exponentially weighted moving average of squared daily returns, where the exponentially weighted moving average has a half-life of 22 days, to scale all BAB factors to an *ex-ante* volatility of 10% (results are robust to using a slower volatility model). My results are then driven by time-varying risk exposure to the BAB factor, and not by implicitly re-scaling it to constant volatility. In the remainder of the paper, all BAB factors are scaled to constant 10% *ex-ante* volatility.

4 Predictability of BAB

4.1 Regressions on Past Market Returns

I start by examining the time-series predictability of BAB returns by market returns across different horizons. In univariate regressions, I regress the excess BAB return on contemporaneous and lagged excess market returns for various daily and monthly lags h:[7]

$$r_t^{BAB} = \alpha + \beta_h r_{t-h}^{MKT} + \varepsilon_t \tag{5}$$

where r_t^{BAB} is a (constant vol) return series for the BAB strategy and r_{t-h}^{MKT} is the market return for period $t - h$.

Figure 1 shows the estimates of β_h as bars, as well as the associated t-statistics as a solid line. The top left plot shows the results for daily returns, using U.S. equities over the period February 1931 to February 2018. The leverage aversion theory predicts that when the

[7] Moskowitz et al. (2012) estimate a similar regression to analyze time series momentum.

market return is positive, investors contemporaneously re-balance into high-beta stocks to get more market exposure, driving up the prices of these stocks and causing BAB to realize contemporaneous negative returns. We would therefore expect the contemporaneous market beta of BAB to be negative. This intuition is confirmed in the top left plot in Figure 1, which shows that BAB returns have a daily beta of -0.3 to the market with a t-statistic of -40. Thus, on days where the market increases, BAB realizes lower than average returns. Following a positive market return, high-beta stocks have become more overvalued relative to the CAPM and have lower expected returns, and/or low-beta stocks have become more undervalued relative to the CAPM and have higher expected returns. Therefore, a positive market return should predict positive future BAB returns. We see that this is the case empirically, with lagged daily market returns consistently predicting future BAB returns out to one month.

The top right plot in Figure 1 shows the results using monthly returns for U.S. equities. The monthly beta of BAB to the market is close to zero. The monthly beta of BAB is determined by the sum of the negative daily beta and the positive cross-correlations between daily BAB returns and lagged daily market returns up to a one month lag and is close to zero. Intuitively, a positive market returns is associated with a negative contemporaneous BAB return, but predicts positive future BAB returns such that the monthly beta of BAB to the market is close to zero.[8] Past monthly market returns predict future BAB returns out to over 1 year. The magnitude is economically important, with a 1% (abnormal) monthly market return predicting a 12 bps abnormal monthly return for BAB over the following month. Given that the historical monthly standard deviation of the U.S. stock market is around 5.2%, a one standard deviation monthly market return predicts a 64 bps abnormal

[8]FP shrink the raw beta estimates for individual stocks towards 1 when forming the BAB portfolio. The amount of shrinkage does affect the beta of BAB to the market since it affects how the long and short legs are levered. However, the fact that the monthly beta of BAB to the market is higher than the daily beta of BAB to the market is not explained by the amount of shrinkage, but by the predictability of BAB returns.

BAB return over the following month for a BAB strategy with 10% annualized volatility. This translates into a 7.7% abnormal annualized return, i.e., a 0.77 change in the Sharpe ratio of the BAB strategy.

The bottom left and right plots in Figure 1 repeat the analysis using a pooled regression of international equities, excluding the U.S. The sample for the international equities is January 1989 to February 2018. The patterns are the same as for the U.S. with lagged daily market returns positively predicting BAB returns for a month, and lagged monthly market returns positively predicting BAB returns for up to 10 months. The t-statistics in the pooled regression are clustered by time and are above 2 for the first 4 month lags. The magnitude of the predictability of BAB in international equity returns is lower than in the U.S. with a 1% (abnormal) monthly market return predicting a 6.9 bps abnormal BAB return over the next month. Still, a 6% monthly return on the market, roughly the standard deviation of monthly returns in the international sample, predicts a 5% abnormal annualized return on a 10% volatility BAB strategy, i.e., a change in the Sharpe ratio of 0.5.

[Figure 1 about here.]

4.2 Sorts

Next, I consider the performance of the U.S. BAB factor sorted on past market returns over different horizons. Table 3 shows performance statistics of the BAB factor, sorted by past 1, 6, and 12 months market returns. Average returns and alphas are reported in monthly values, while volatilities and Sharpe ratios are annualized. I sort each month in the sample into 1 of 5 groups, $A_1, \ldots, A_5$, determined by the past market return at the beginning of the month. The breakpoints are determined using the full sample, such that they do not reflect an implementable timing-strategy, and each of the five groups contains the same number of months. I then estimate the regression $r_t^{BAB} = \sum_{i=1}^{5} \mu_i 1_{(t \in A_i)} + \varepsilon_t$, i.e., the estimate of μ_i is the average return in each group, and test $\mu_1 = \mu_5$ using a Wald test. Standard errors are corrected for heteroscedasticity.

In Panel A of Table 3 I sort on the past 1 month market return. Following the worst market returns, where the market on average has dropped 6.4% over the previous month, monthly BAB returns are on average −43 bps over the next month (significant at the 10% level). Following the largest stock market rallies, where the market on average has realized a 6.5% return over the previous month, BAB has realized an average of 164 bps over the next month, which is significant at the 1 percent level. The difference of 207 bps is significant at the 1 percent level.[9]

The difference in BAB returns following positive and negative market returns is not explained by differences in market beta. To show this, I estimate the regression $r_t^{BAB} = \sum_{i=1}^{5} \mu_i 1_{(t\in A_i)} + \sum_{i=1}^{5} \beta_i r_t^{MKT} 1_{(t\in A_i)} + \varepsilon_t$, such that the market beta is allowed to vary across groups, and test $\mu_1 = \mu_5$ using a Wald test. Panel A in Table 3 shows that the CAPM alpha increases from −40 bps per month following the worst 1 month market declines to 171 bps following the highest 1 month market returns, and that the difference is significant at the 1 percent level. Similarly, I estimate a regression with the four Fama-French-Carhart factors, allowing the beta to each factor to vary across groups. The 4-factor alpha increases from −55 bps per month to 163 bps per month across groups. Finally, the difference in BAB returns across groups is not due to differences in realized volatility of the *ex-ante* constant volatility returns. The realized Sharpe ratio of the BAB factor increases from −0.43 to 1.64 across the groups.

[Table 3 about here.]

Panel B and C show the results for sorting on 6 and 12 months past market returns, respectively. Again, excess returns, CAPM alphas, 4-factor alphas, and Sharpe ratios increase

[9] The column H-L does not show the return earned by a trading strategy, since the returns in the High and Low columns occur at different times in the sample.

across the groups, showing that U.S. BAB returns have been higher following high market returns and lower following low market returns.

Table 4 shows the results for a pooled analysis of international data, excluding the United States. First, for each country n, I divide the sample into five subsamples $A_{n,1},\dots,A_{n,5}$ based on the market return for country n over the past 1, 6, or 12 months. I then estimate the pooled regression

$$r_{n,t}^{BAB}=\sum_{i=1}^{5}\mu_i 1_{(t\in A_{n,i})}+\varepsilon_{n,t},\quad n=1,\dots,23$$

and test $\mu_1=\mu_5$ using a Wald test. Standard errors are clustered by time (monthly). I also calculate CAPM alphas by allowing the beta of the BAB factors to the market to vary by group (allowing them to vary by group and country do not materially alter the results):

$$r_{n,t}^{BAB}=\sum_{i=1}^{5}\mu_i 1_{(t\in A_{n,i})}+\sum_{i=1}^{5}\beta_i r_{n,t}^{MKT} 1_{(t\in A_{n,i})}+\varepsilon_{n,t},\quad n=1,\dots,23$$

4-Factor alphas are calculated analogously. For all three look-back periods, the excess returns, CAPM alphas, 4-factor alphas, and Sharpe ratios of BAB increase monotonically across the groups, showing that BAB returns are higher following high market returns and lower following low market returns. The differences between group 1 and 5 are always significant at the 1% level. The differences in returns are economically large. In Panel C of Table 4, in the 20% of the sample where the past 1 year market return is in its lowest quintile with an average return of −26.5%, future monthly BAB returns are insignificantly different from zero, and the realized Sharpe ratio of BAB is close to zero as well. In contrast, in the 20% of the sample where the past 1 year market return is in its highest quintile with an average return of 43.3%, future monthly BAB returns are around 100 bps per month and the BAB factor has realized a Sharpe ratio of 1.18.

[Table 4 about here.]

4.3 Multivariate Regressions

I now estimate the regression

$$r_{t+1}^{BAB} = \alpha + \beta X_t + \varepsilon_t \tag{6}$$

where X is a vector of explanatory variables. To test if past market returns predict future BAB returns, I include the average 1, 6, and 12 month return on the market. In addition, I include the average 1 and 12 month lagged BAB return to control for any reversal or momentum effect in the BAB factor. Further, I include the current level of volatility as well as the change in volatility over the previous 6 months. The leverage aversion theory predicts that volatility, in a univariate regression, should negatively predict future BAB returns, but that volatility should be driven out by past returns on the market. As controls, I include the TED spread, the margin debt measure from Asness et al. (2018), and the leverage constraint tightness measure LCT from Boguth and Simutin (2018). As in Boguth and Simutin (2018), I use a 12 month average of LCT, denoted LCT^{12M}.

Table 5 shows the results for various specifications of equation (6), using returns for the U.S. Standard errors are corrected for heteroscedasticity. The row 'Past Market Wald' reports the F-statistic from the test of the null that the three coefficients on lagged market returns are all zero. In specification (1), we see that past market returns predict future BAB returns, and that the past 1 month and 6 month returns drive out the past 12 month return. In column (2), both past 1 month and 12 month returns on the BAB factor predict future returns on the factor. The economic magnitude is large, with a coefficient of 0.41 on the past 12 month average BAB return. Thus, a 10 percent return over the past 12 months predict an abnormal 4 percent annualized return on the BAB factor over the next month. Thus, there is no sign of reversal in the BAB factor, which supports the model prediction that the variations in expected returns on the factor are due to equilibrium effects and not short-term price pressure.

As predicted by the leverage aversion theory, volatility as well as changes in volatility negatively predict future BAB returns in columns (3) and (4).

The TED spread negatively predicts future BAB returns as seen in column (5), which is consistent with the results in FP. FP suggest that this is the 'wrong sign', since they hypothesize that the TED spread proxies for the cost of leverage and that aggregate funding tightness therefore should be high when the TED spread is high. However, it seems plausible that the demand for leverage is low when the TED spread is high, and that future BAB returns are therefore low when the TED spread is high. This interpretation is consistent with the negative sign on the TED spread.

The margin debt measure shows no predictability of BAB returns in column (6). This is consistent with Asness et al. (2018) who find weak predictability of the margin debt measure, but also find that contemporaneous changes in the margin debt measure are positively correlated with BAB returns; when margin debt goes up, it is because leverage constraints become less tight and investors contemporaneously shift into low beta assets, thus generating a positive return on the BAB factor.

The leverage constraint tightness measure positively predicts future BAB returns, consistent with the findings in Boguth and Simutin (2018). If the average 12 month aggregate mutual fund beta is higher by 0.1, future BAB returns are on average higher by 1.4 percent over the following month.

Finally, columns (8) and (9) show the results including all controls, with column (8) including levels of volatility and column (9) including changes in volatility (since levels and changes in volatility are highly correlated, I do not include both in the same regression). Past market returns remain significant predictors of future BAB returns, and the row 'Past Market Wald' shows that the hypothesis that the coefficients on past market returns are all zero is rejected at the 1% level. Past BAB returns remain positive predictors of future BAB returns, even when including controls. Past BAB returns could proxy for unobserved characteristics that influence equilibrium expected returns on the BAB factor, and if these characteristics are persistent, past BAB returns should forecast future BAB returns. Both levels and changes in volatility are driven out in the multivariate regression, as predicted by the leverage aversion theory. The TED spread remains a significant predictor of BAB returns, and the margin debt measure remains insignificant. The LCT measure of Boguth and Simutin (2018) remains significant at the 1% level.

[Table 5 about here.]

Table 6 shows the corresponding results for the panel regression using the international sample, excluding the U.S. Standard errors are clustered by date, and all specifications include country fixed effects. The results are similar to the results for the U.S. sample: In the univariate regressions, both past market returns and past BAB returns predict future BAB returns. Both levels and changes in volatility negatively predict future BAB returns. Interestingly, the TED spread remains a significant predictor of BAB returns in international data: when the TED spread is higher by 1 percentage point, future monthly BAB returns are on average lower by 1 percentage point across countries. The LCT measure of Boguth and Simutin (2018) is not significant in the international sample, which is not surprising since the LCT measure reflects the aggregate mutual fund beta of domestic mutual funds.

In specification (8) and (9) which include past market returns as well as the controls, past market returns and past BAB returns remain positive predictors of future BAB returns. The Wald test that the coefficients on past market returns are zero is rejected at the 1% level. Moreover, the TED spread remains a significant negative predictor of future BAB returns. In conclusion, past market returns remain significant predictors of future BAB returns even when including various controls.

[Table 6 about here.]

4.4 BAB-Timing Strategies

I now form trading strategies with time-varying BAB exposure based on past market returns. To construct a strategy that has time-varying BAB exposure and is unconditionally uncorrelated to BAB itself, I consider a strategy that takes a position in BAB proportional to the past market return. I first demean the market return using an expanding mean, which ensures that the timing strategy is approximately long BAB half the time and short BAB half the time. Without demeaning the market return, the timing strategy would be long BAB more than half of the time, mechanically resulting in positive returns. The timing strategies are then

$$\text{BAB-Timing} = \text{BAB} \times (\text{Past } N\text{-month demeaned market return}) \tag{7}$$

where I vary the look-back horizon N.

Table 7 reports the two-factor alphas of the timing portfolios to the market and BAB for each country. The sample is 1931 to 2018 for U.S. equities, and 1988 or later to 2018 for the remaining countries. The timing strategies are scaled to 10% annualized volatility *ex-post* to make the alphas comparable across countries. I include BAB as a factor on the right-hand-side to ensure that the results are not driven by the timing strategies being long BAB more or less than half of the time. Using the past 12 month market return to time BAB, the timing strategies have positive alpha for all but two countries and the alphas are significant at the 5% level or better for 12 out of the 24 countries. The cap weighted global timing strategy generates a 2-factor alpha of 91 bps per month with a t-statistic of 5.0. The results are slightly stronger for the two shorter look-back periods. With a 6 month look-back period, the cap weighted global timing strategy generates a 2-factor alpha of 130 bps per month with a t-statistic of 7.1, and with a 1 month look-back the cap weighted timing strategy generates a 2-factor alpha of 109 bps per month with a t-statistic of 5.2.

[Table 7 about here.]

Figure 2 shows the cumulative log returns for the U.S. and global BAB-timing portfolios. In the United States, the constant volatility BAB strategy has generated a Sharpe ratio of 0.99 over the period February 1931 to February 2018. The BAB-timing portfolios based on 1, 6 and 12 month past market returns have generated Sharpe ratios of 0.67, 0.71, and 0.55, respectively. The global cap weighted BAB strategy (including the United States), in which the BAB strategies for each country are scaled to an *ex-ante* volatility of 10% and then combined using market cap weights, has generated a Sharpe ratio of 1.52 over the period January 1989 to February 2018, and the three timing strategies have generated Sharpe ratios of 1.24, 1.35, and 0.97 respectively.

[Figure 2 about here.]

As a robustness check, consider a timing strategy that goes long or short one unit of BAB depending on the sign of the past market return:

$$\text{BAB-Timing-sign} = \text{BAB} \times \text{sign}(\text{Past } N\text{-month demeaned market return}) \tag{8}$$

This timing strategy is always scaled to *ex-ante* 10% volatility since the BAB returns are *ex-ante* 10% volatility. Table 8 shows the 2-factor alphas of the constant volatility BAB-timing strategies for each country in the sample. As expected, the average alphas are slightly lower than for the timing strategies in Table 7, but the differences are small. The global cap weighted timing-strategies now generate 2-factor alphas of 99 bps, 106 bps, and 90 bps for 1, 6, and 12 month past market returns, compared to 109 bps, 130 bps, and 91 bps in Table 7. This shows that the magnitude of the past market return does contain information about the magnitude of future BAB returns as we also see in Table 3 and 4, but that basing the timing strategy solely on the sign of the past market return still generates large and significant alphas. Further, the timing strategies based on the magnitude of past market returns have a small but significant alpha to the timing strategies based on the sign of past market returns (I omit the results for brevity).

[Table 8 about here.]

4.5 Local Returns

As a robustness check, I re-run all analyses above using returns in local currencies. The results are virtually unchanged, and I therefore do not report them. This shows that BAB returns are predictable from the point of view of local investors in the different countries.

4.6 Long/Short Cross-Country BAB Strategy

To provide further support for the leverage aversion theory and the predictive power of market returns, I now test if cross-country differences in past market returns forecast cross-country differences in realized BAB returns.[10] If all investors have a global perspective, there should be one global Lagrange multiplier that applies to BAB strategies in all countries. For instance, suppose that there are positive news about future dividends in a particular country, and that investors bid up prices to restore equilibrium expected returns. A global investor who is leverage constrained would want to *i)* overweight high-beta stock even more in the country with positive news, increasing expected future BAB returns for that country, as well as *ii)* reallocate capital from other countries to the country with positive news. In doing this, he effectively becomes more constrained in other countries as well and as a result he overweights high-beta stocks more everywhere, increasing the expected return on BAB portfolios in all countries. On the other hand, if investors have a preference for local stocks, there would be a separate Lagrance multiplier associated with the expected BAB return for

[10] I am indebted to Lars Nielsen for suggesting this analysis.

each country. Cross-country differences in past market returns should predict cross-country differences in the Lagrange multipliers and thus differences in expected BAB returns.

To test this, for each of the 24 countries in Table 1, I calculate the past 1, 6, and 12 month return on the market. I then sort the countries into 5 quintiles based on the past market return, and within each quintile I form an equal-weighted portfolio of the BAB strategies for the countries in the quintile.

Table 9 shows summary statistics for the returns of the quintile portfolios. In Panel A I sort countries into quintiles based on the past 1 month return on the market index. The BAB strategies perform better in countries where the past market return has been higher than the cross-country average, and BAB strategies perform worse in countries where the past market return has been lower than average. The average excess return of the BAB-strategies increases from 14 bps per month in countries where the past market return is low to an average of 100 bps per month in countries where the past market return is high. The difference of 86 bps per month is significant at the 1% level.

The row 2-Factor Alpha shows the alpha wrt. a 2-factor model with the global cap-weighted market and global cap-weighted BAB portfolio, where the global BAB portfolio is constructed by cap-weighting the constant volatility BAB strategies for each country (the global BAB portfolio is not constant volatility). Using an equal-weighted BAB portfolio does not materially change the results. The 2-factor alphas increase almost monotonically from -10 bps per month in quintile 1 to 74 bps per month in quintile 5. That is, in countries where the past market return is lower than the cross-country average the BAB portfolios on average have negative alpha to the global market and global BAB portfolio, and in countries where the past market return is higher than the cross-country average the BAB portfolios on average have positive alpha to the global market and global BAB portfolio. Similarly, the 5-factor alphas to the global MKT, SMB, HML, UMD, and BAB portfolios increase almost monotonically across groups. These differences are not explained by differences in volatility, and the Sharpe ratio of BAB increases from 0.30 in the first group to 2.10 in the group of countries where the past market return is highest. I also form a portfolio that goes long BAB in the quintile of countries where the past market return has been high and short BAB in the quintile of countries where the past Sharpe ratio has been low. This H-L BAB strategy has realized an excess return of 86 bps per month, a global 2-factor alpha of 84 bps per month, a 5-factor alpha of 85 bps per month, and a Sharpe ratio of 1.60. The excess return, 2-, and 5-factor factor alpha are significant at the 1 percent level.

Panel B and C in Table 9 repeat the analysis using past 6 and 12 month market returns. Again, excess returns, 2-, and 5-factor alphas, and Sharpe ratios increase monotonically across groups. These results are consistent with the leverage aversion theory and indicate that investors have become more constrained in countries where the past market return has been higher than the cross-country average. In these countries, investors have increased their demand for equities the most and are now more constrained than investors in other countries, and future BAB returns are therefore higher.

[Table 9 about here.]

Finally, I construct a cross-country BAB-timing factor which goes long BAB in half the countries and short BAB in half the countries. The weights are determined by ranking the past market return of the countries, demeaning the ranks, and taking a position in BAB for a country proportional to the demeaned rank of the country. Table 10 shows the results. Using the past 12 month market returns, the cross-country long/short BAB-timing strategy generates an excess return of 54 bps per month at an annualized volatility of 5.2%. This corresponds to a Sharpe ratio of 1.26. Table 10 also reports 2-factor alphas wrt. the global market and BAB, where the global BAB portfolio is again a market-cap weighted average of the constant volatility BAB strategies for each country (result are similar when using an equal-weighted BAB index). Using the past 12 month market return to determine the country weights in the cross-country BAB strategy, the 2-factor alpha wrt. the global market and the global BAB portfolio is 58 bps per month, and the 5-factor alpha wrt. the global market, SML, HML, momentum, and BAB strategies is 61 bps per month with a t-statistic of 6.9. The results are similar using 1- and 6 month market returns to determine the country weights.

This implies that the global cap-weighted BAB strategy can be improved by over-weighting BAB in some countries relative to other countries, based on the past return of the countries' market indices. This suggests that at least some investors have a local bias when investing in stocks.

[Table 10 about here.]

4.7 Equity Indices

I now consider BAB- and BAB-timing portfolios for international equity indices. For the 13 developed countries in Table 2 I construct a market-cap weighted index and compute the beta of each country with respect to the cap-weighted index. The BAB strategy goes long low-beta indices and short high-beta indices, levered to be beta-neutral (see Appendix B for details). I scale the BAB strategy to an *ex-ante* annualized volatility of 10%. The BAB portfolio for the 25 countries in the emerging region is constructed similarly.

The BAB-timing portfolios are based on the return of the cap-weighted index for the respective regions over the previous 1, 6, and 12 months, and I scale the BAB-timing portfolios to an *ex-post* annualized volatility of 10% to make the alphas comparable. Table 11 shows performance statistics of both the BAB and BAB-timing portfolios.

Data for the developed region span 1981 to 2015, and the emerging region uses data for 1991 to 2015. The BAB strategies have realized positive and significant alpha in both regions, with 37 bps per month in the developed region and 66 bps per month in the emerging region. The BAB-timing strategies have realized positive alphas to the market and BAB, for both regions and for all look-back periods, but the results are stronger for shorter look-back horizons. Using the 1 month past market return to time BAB, the BAB-timing strategies have realized roughly the same returns as the BAB strategy itself while being long BAB half the time and short BAB half the time.

A constrained investor with a global perspective who wants to increase his exposure to global equities, either due to positive news about future cashflows or due to changes in discount rates, will overweight high-beta countries more and thereby increase the expected return on the BAB strategy formed on country indices. Thus, the finding that expected returns on a BAB strategy formed on country indices is predictable is not contradictory to the finding that cross-country differences in past market returns predict cross-country differences in future BAB returns, but together the results suggest that some investors have a local bias whereas others have a global perspective.

[Table 11 about here.]

5 Conclusion

Black (1972) and FP show that low-beta stocks on average have higher risk-adjusted returns than high-beta stocks, and FP constructs a BAB factor that exploits this by going long low-beta stocks and short high-beta stocks. I find that returns to the BAB strategy are time-varying and predictable by past market returns. These findings are consistent with the leverage aversion theory in which the BAB premium depends on the tightness of funding constraint of constrained investors. An outward shift in investors' demand functions simultaneously causes an increase in prices and makes investors more constrained, such that future BAB returns are higher. Thus, time-variation in investors' demand for leverage generates time-variation in the expected returns of the BAB strategy.

The variation in investors' demand for leverage generates both time-series and cross-sectional predictability of the BAB factor. BAB returns are higher in periods following high past market returns, and BAB returns are higher in countries with high past market returns. The time-series predictability of BAB returns is robust across a sample of 24 countries as well as for a BAB factor formed from equity indices, and the differences in returns across different periods are not explained by exposure to any standard risk factors. The cross-sectional results show that BAB returns are higher in countries with high past market returns, suggesting that investors are more constrained in those countries.

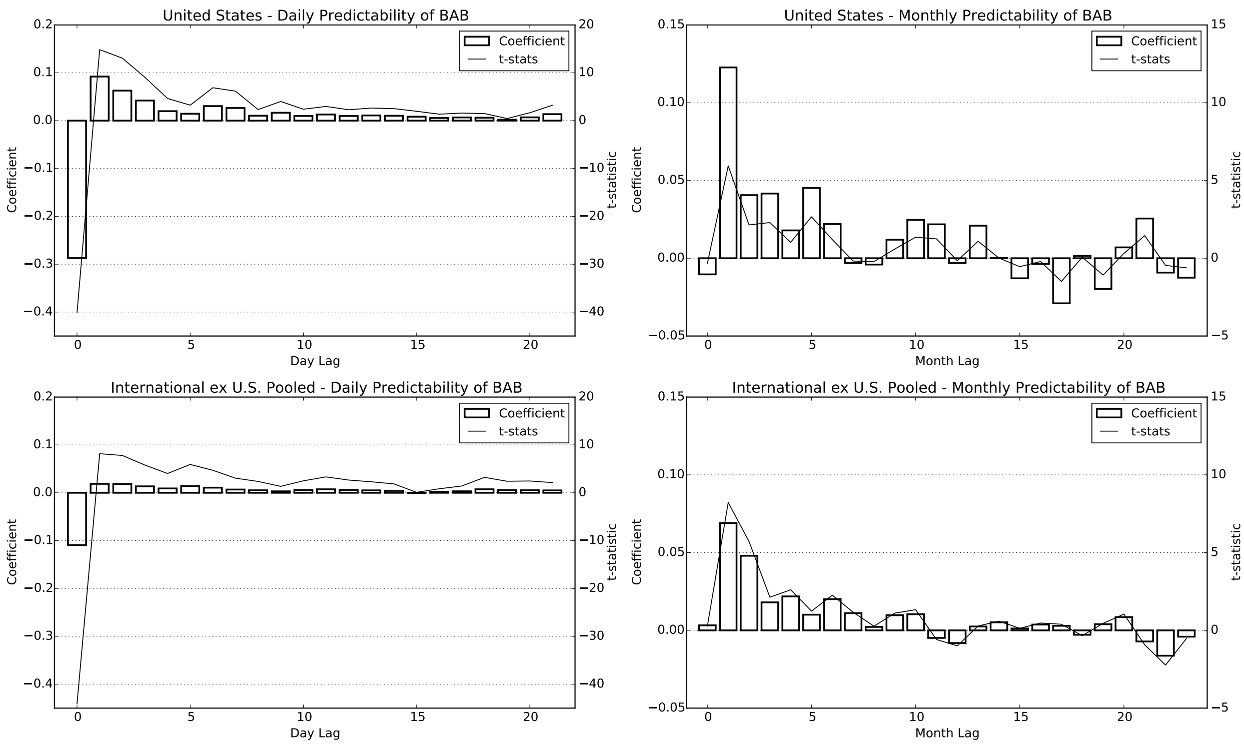

Figure 1: Time-Series Predictability of BAB by Past Market Returns

I estimate univariate regressions of constant *ex-ante* volatility BAB returns on past market returns,

$$r_t^{BAB} = \alpha + \beta_h r_{t-h}^M + \varepsilon_t$$

for various lags h. The top left/right plots show the results for daily/monthly U.S. returns. The bottom left/right plots show the results for a pooled regression of daily/monthly international equity returns, excluding the U.S. The t-statistics are corrected for heteroscedasticity for the U.S. regression and are clustered by time for the pooled international regression. The sample is February 1931 to February 2018 for the U.S. and January 1989 to February 2018 for international equities.

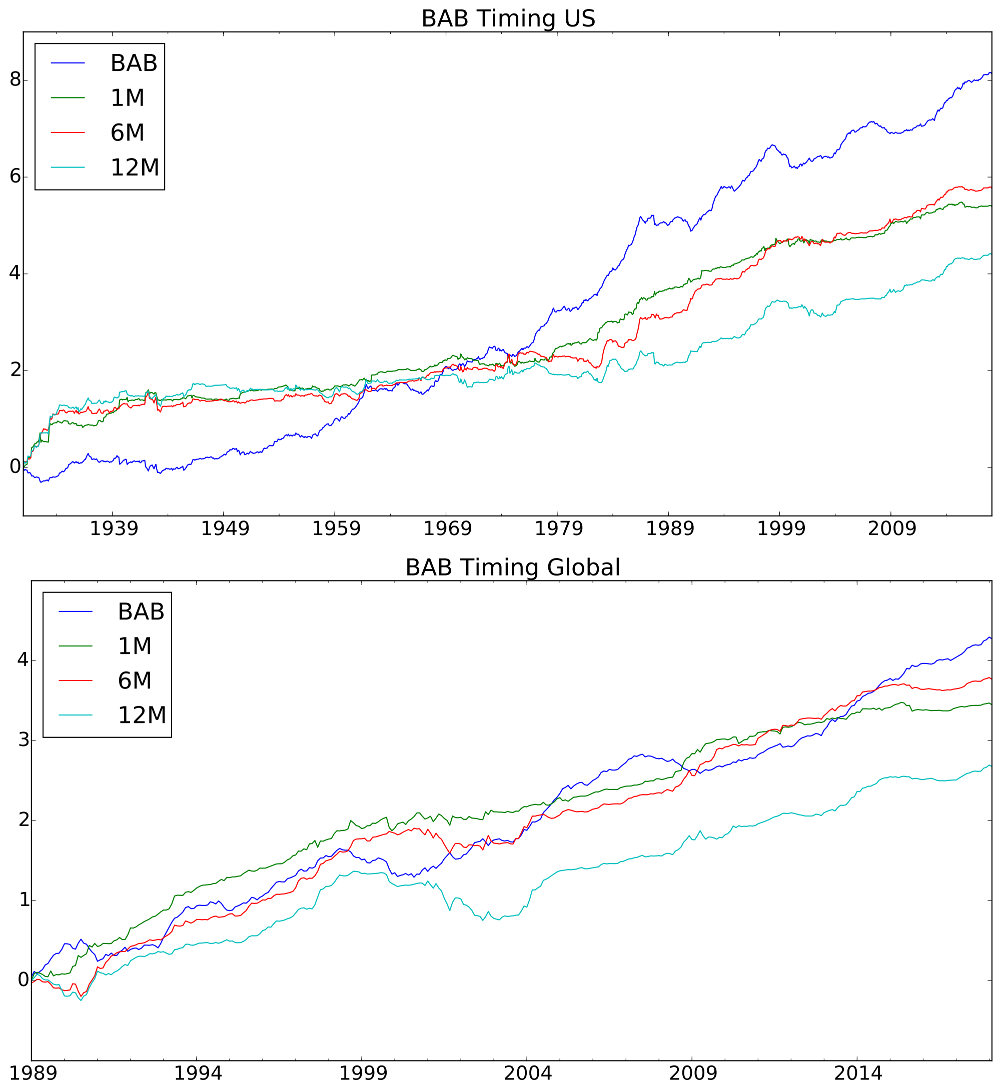

Figure 2: Cumulative Returns for BAB and BAB-Timing Portfolios

The top plot shows the cumulative log returns to the constant *ex-ante* volatility BAB factor for the U.S., as well as to three BAB-timing strategies using 1, 6, and 12 month past market returns to determine the BAB exposure. The BAB-timing portfolios are constructed as

$$\text{BAB-Timing} = \text{BAB} \times (\text{Past } N\text{-month demeaned market return}), N = 1, 6, 12$$

where the BAB strategy has been scaled to *ex-ante* 10% vol and the market returns has been demeaned using an expanding mean. The bottom plot shows the cumulative returns for global BAB- and BAB-timing strategies. The global BAB strategy is a cap-weighted average of individual country BAB strategies, where the BAB strategy for each country is scaled to constant *ex-ante* volatility. The BAB-timing strategies are cap-weighted averages of the individual country BAB-timing strategies, where the BAB-timing strategies have been scaled to 10% annualized volatility *ex-post* in each country. The sample is February 1931 to January 2016 for the U.S. and January 1989 to January 2016 for international equities.

Table 1: Summary Statistics for Equities

The table shows summary statistics for the equity data. All data are obtained from the web site accompanying Frazzini and Pedersen (2014). The average market cap is in billions of U.S. dollars.

Country	Index	Avg Mkt Cap	Start Year	End Year
Australia	MSCI Australia	538	1988	2018
Austria	MSCI Austria	58	1988	2018
Belgium	MSCI Belgium	166	1988	2018
Canada	MSCI Canada	819	1985	2018
Denmark	MSCI Denmark	100	1988	2018
Finland	MSCI Finland	119	1988	2018
France	MSCI France	1,093	1988	2018
Germany	MSCI Germany	905	1988	2018
Greece	MSCI Greece	68	1991	2018
Hong Kong	MSCI Hong Kong	803	1988	2018
Ireland	MSCI Ireland	58	1988	2018
Israel	MSCI Israel	96	1997	2018
Italy	MSCI Italy	421	1988	2018
Japan	MSCI Japan	3,443	1988	2018
Netherlands	MSCI Netherlands	385	1988	2018
New Zealand	MSCI New Zealand	23	1988	2018
Norway	MSCI Norway	119	1988	2018
Portugal	MSCI Portugal	51	1991	2018
Singapore	MSCI Singapore	220	1988	2018
Spain	MSCI Spain	380	1988	2018
Sweden	MSCI Sweden	246	1988	2018
Switzerland	MSCI Switzerland	527	1988	2018
United Kingdom	MSCI United Kingdom	2,055	1988	2018
United States	CRSP value-weighted index	11,316	1931	2018

Table 2: Summary Statistics for Equity Index Futures

The table shows summary statistics for the equity index data.

Asset Class	Instrument	Avg Mkt Cap	Start Year	End Year
Equity Indices, Developed	Australia	311	1978	2015
	Germany	466	1978	2015
	Canada	439	1978	2015
	Spain	193	1980	2015
	France	522	1978	2015
	Hong Kong	146	1980	2015
	Italy	191	1978	2015
	Japan	1,611	1978	2015
	Netherlands	210	1978	2015
	Sweden	153	1980	2015
	Switzerland	404	1978	2015
	United Kingdom	1,235	1978	2015
	United States	6,213	1978	2015
Index	Cap-weighted		1978	2015
Equity Indices, Emerging	Brazil	237	1994	2015
	Colombia	13	1993	2015
	China	293	1996	2015
	Chile	35	1990	2015
	Czech Republic	11	1995	2015
	Egypt	11	2001	2015
	Greece	29	1988	2015
	Hungary	12	1995	2015
	Indonesia	40	1992	2015
	India	125	1993	2015
	Israel	38	1988	2015
	South Korea	226	1988	2015
	Mexico	96	1988	2015
	Malaysia	78	1990	2015
	Peru	12	1998	2015
	Philippines	14	1988	2015
	Poland	26	1993	2015
	Qatar	17	2007	2015
	Russia	122	1996	2015
	South Africa	136	1988	2015
	Singapore	92	1988	2015
	Taiwan	205	1988	2015
	Thailand	39	1988	2015
	Turkey	36	1996	2015
	United Arab Emirates	18	2006	2015
Index	Cap-weighted		1988	2015

Table 3: BAB Returns Sorted on Past Market Returns U.S. Equities

The table shows performance statistics for constant volatility BAB returns, sorted on past market returns. Average returns and alphas are reported in percent per month while volatilities and Sharpe ratios are annualized. For excess returns and CAPM alphas I estimate the models

$$r_t^{BAB} = \sum_{i=1}^{5} \mu_i 1_{(t \in A_i)} + \varepsilon_t \quad \text{and} \quad r_t^{BAB} = \sum_{i=1}^{5} \mu_i 1_{(t \in A_i)} + \sum_{i=1}^{5} \beta_i r_t^{MKT} 1_{(t \in A_i)} + \varepsilon_t$$

where A_1 to A_5 are five non-contiguous sub-samples based on the past market return. I estimate the equivalent regression for 4-factor alphas. t-statistics are reported in brackets and are based on White's heteroscedasticity consistent covariance matrix. In the column H-L I test the hypothesis $\mu_1 = \mu_5$ using a Wald test and report the F-value in brackets. The sample is 1931-02 to 2016-01.

Panel A: BAB Returns Sorted on Past 1 Month Market Return

	Low				High	H-L
Sorting Variable	−6.41	−1.53	0.81	2.85	6.53	
Excess Return	−0.43*	0.69***	1.54***	1.73***	1.64***	2.07***
t-Stat/Wald F	[−1.79]	[2.73]	[6.77]	[7.53]	[6.78]	[36.99]
CAPM Alpha	−0.40*	0.66***	1.53***	1.89***	1.71***	2.11***
t-Stat/Wald F	[−1.72]	[2.70]	[6.67]	[8.45]	[7.28]	[40.72]
Four-Factor Alpha	−0.55**	0.47*	1.52***	1.84***	1.63***	2.19***
t-Stat/Wald F	[−2.28]	[1.92]	[7.06]	[7.61]	[7.03]	[42.35]
Volatility	11.86	12.53	11.26	11.36	12.01	
Sharpe Ratio	−0.43	0.66	1.64	1.82	1.64	

Panel B: BAB Returns Sorted on Past 6 Months Market Return

	Low				High	H-L
Sorting Variable	−14.69	−1.81	4.42	9.90	21.07	
Excess Return	−0.19	0.84***	1.30***	1.61***	1.61***	1.81***
t-Stat/Wald F	[−0.84]	[3.47]	[5.28]	[6.36]	[6.95]	[30.63]
CAPM Alpha	−0.18	0.92***	1.23***	1.46***	1.63***	1.82***
t-Stat/Wald F	[−0.81]	[3.85]	[4.66]	[4.89]	[7.24]	[32.28]
Four-Factor Alpha	−0.21	0.90***	1.15***	1.37***	1.66***	1.87***
t-Stat/Wald F	[−1.01]	[3.65]	[4.21]	[4.57]	[7.55]	[38.24]
Volatility	11.37	11.95	12.22	12.53	11.51	
Sharpe Ratio	−0.20	0.84	1.28	1.54	1.68	

Panel C: BAB Returns Sorted on Past 12 Months Market Return

	Low				High	H-L
Sorting Variable	−20.19	−0.66	8.83	16.95	32.92	
Excess Return	0.02	0.88***	1.37***	1.46***	1.43***	1.41***
t-Stat/Wald F	[0.09]	[3.85]	[6.22]	[5.63]	[5.39]	[15.99]
CAPM Alpha	0.03	0.90***	1.38***	1.44***	1.31***	1.29***
t-Stat/Wald F	[0.12]	[3.77]	[6.11]	[5.25]	[4.34]	[11.46]
Four-Factor Alpha	0.09	0.64***	1.16***	1.61***	1.49***	1.40***
t-Stat/Wald F	[0.42]	[2.68]	[5.00]	[5.85]	[5.63]	[17.66]
Volatility	11.54	11.39	10.90	12.84	13.17	
Sharpe Ratio	0.02	0.93	1.51	1.36	1.31	

***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 4: BAB Returns Sorted on Past Market Returns for International Equities

The table shows performance statistics for the constant volatility BAB factor, sorted on past market returns. Average returns and alphas are reported in percent per month, while volatilities and Sharpe ratios are annualized. CAPM- and 4-factor alphas are computed for each country with respect to country-specific factor portfolios. For excess returns and alphas, I first sort the observations in each country n into 5 groups $A_{n,1}, \dots, A_{n,5}$. I then estimate the constrained panel regressions

$$r^{BAB}_{n,t} = \sum_{i=1}^{5} \mu_i 1_{(t \in A_{n,i})} + \varepsilon_{n,t} \quad \text{and} \quad r^{BAB}_{n,t} = \sum_{i=1}^{5} \mu_i 1_{(t \in A_{n,i})} + \sum_{i=1}^{5} \beta_i r^{MKT}_{n,t} 1_{(t \in A_{n,i})} + \varepsilon_{n,t}$$

where $n = 1, \dots, 23$ denotes the country, and $A_{n,i}, i = 1, \dots, 5$ are the five non-contiguous subsamples for country n, based on the past market return for country n. t-statistics are reported in brackets and are clustered by time. In the rightmost column I test the hypothesis $\mu_1 = \mu_5$ using a Wald test and report the F-value in brackets. The sample is 1988 to 2016 and excludes the United States.

	Low				High	H-L
Panel A: BAB Returns Sorted on Past 1 Month Market Return						
Past Mkt Return	−8.33	−2.18	0.73	3.56	9.02	
Excess Return	−0.11	0.35***	0.84***	1.10***	1.12***	1.23***
t-stat/Wald F	[−1.01]	[3.68]	[8.09]	[10.95]	[10.02]	[64.73]
CAPM Alpha	−0.11	0.35***	0.83***	1.11***	1.13***	1.24***
t-stat/Wald F	[−0.98]	[3.64]	[8.03]	[11.16]	[10.12]	[65.13]
4-Factor Alpha	−0.12	0.33***	0.83***	1.09***	1.11***	1.23***
t-stat/Wald F	[−1.10]	[3.42]	[7.98]	[10.88]	[9.96]	[64.34]
Volatility	8.94	9.46	9.79	9.90	9.78	
Sharpe ratio	−0.10	0.40	0.90	1.27	1.32	
Panel B: BAB Returns Sorted on Past 6 Month Market Return	Low				High	H-L
Past Mkt Return	−19.68	−4.11	4.13	11.95	27.43	
Excess Return	−0.14	0.47***	0.76***	1.01***	1.19***	1.33***
t-stat/Wald F	[−1.01]	[5.04]	[7.65]	[10.33]	[10.26]	[54.86]
CAPM Alpha	−0.14	0.48***	0.74***	1.01***	1.19***	1.33***
t-stat/Wald F	[−1.00]	[5.16]	[7.40]	[10.21]	[10.27]	[54.97]
4-Factor Alpha	−0.14	0.47***	0.73***	0.99***	1.19***	1.33***
t-stat/Wald F	[−1.05]	[5.05]	[7.27]	[10.04]	[10.26]	[56.83]
Volatility	9.42	9.29	9.72	9.38	10.11	
Sharpe ratio	−0.19	0.53	0.91	1.17	1.41	
Panel C: BAB Returns Sorted on Past 12 Month Market Return	Low				High	H-L
Past Mkt Return	−26.46	−5.59	8.17	21.22	43.26	
Excess Return	0.03	0.56***	0.77***	0.88***	1.04***	1.01***
t-stat/Wald F	[0.21]	[5.51]	[8.23]	[8.84]	[9.49]	[33.30]
CAPM Alpha	0.03	0.56***	0.77***	0.87***	1.02***	1.00***
t-stat/Wald F	[0.21]	[5.51]	[8.18]	[8.76]	[9.19]	[31.87]
4-Factor Alpha	0.04	0.53***	0.74***	0.88***	1.04***	1.00***
t-stat/Wald F	[0.29]	[5.13]	[7.86]	[8.85]	[9.43]	[33.51]
Volatility	9.38	9.31	9.97	9.65	9.86	
Sharpe ratio	0.05	0.76	0.80	1.04	1.18	

***,**, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 5: Multivariate Regressions, United States

The table shows the estimates from a regression of monthly BAB returns on explanatory variables for the United States. The explanatory variables are the market return over the past 1, 6 and 12 months, the BAB return over the past 1 and 12 months, volatility σ_t, the change in volatility over the past 6 months $\Delta\sigma_{t-6,t}$, the TED spread, the margin debt measure from Asness et al. (2018), and the leverage constraint tightness measure from Boguth and Simutin (2018). Returns are available starting in 1931 and the TED spread starts in 1986. The margin debt measure is calculated as the ratio of margin debt of NYSE member organizations to the market capitalization of NYSE stocks, and is available from 1959 to 2015. The leverage constraint measure is the aggregate beta of domestic mutual funds, averaged over 12 months, and is available 1980 to 2012. The row 'Past Market Wald' reports the F-statistic from the Wald test of the null that all three coefficients on past market returns equal zero. Standard errors are corrected for heteroscedasticity and the right-hand-side variables are demeaned to make the intercepts comparable across specifications.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Intercept	1.02***	1.02***	0.99***	1.00***	1.12***	1.29***	1.40***	1.12***	1.12***
	[9.65]	[9.82]	[9.32]	[9.33]	[6.42]	[9.38]	[7.97]	[6.52]	[6.51]
$r^{M}_{t-1,t}$	0.09***							0.14***	0.15***
	[4.00]							[2.85]	[2.70]
$r^{M}_{t-6,t}$	0.12**							0.35**	0.37**
	[1.96]							[2.34]	[2.34]
$r^{M}_{t-12,t}$	0.08							−0.09	−0.26
	[0.93]							[−0.47]	[−1.21]
$r^{BAB}_{t-1,t}$		0.13***						0.13*	0.13*
		[4.04]						[1.87]	[1.82]
$r^{BAB}_{t-12,t}$		0.41***						0.41***	0.34***
		[5.08]						[2.95]	[2.58]
σ_t			−0.06***					0.05	
			[−5.76]					[1.58]	
$\Delta\sigma_{t-6,t}$				−0.04***					0.04
				[−3.84]					[1.39]
TED_t					−2.27***			−1.40**	−1.36**
					[−4.06]			[−2.13]	[−1.99]
MD_t						−0.17		−0.07	0.10
						[−0.78]		[−0.20]	[0.31]
LCT^{12M}_t							14.14***	12.37***	10.89***
							[4.47]	[3.17]	[2.96]
Past Market Wald	49.05***							28.20***	23.20***
Adjusted R^2	0.04	0.07	0.02	0.01	0.08	−0.00	0.04	0.21	0.21
#obs	1035	1034	1046	1041	385	683	408	348	348

***,**, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 6: Multivariate Regressions, International Sample

The table shows the estimates from a panel regression of monthly BAB returns on explanatory variables for 23 countries excluding the U.S. The explanatory variables are the market return over the past 1, 6 and 12 months, the BAB return over the past 1 and 12 months, volatility σ_t, the change in volatility over the past 6 months $\Delta\sigma_{t-6,t}$, the TED spread, the margin debt measure from Asness et al. (2018), and the leverage constraint tightness measure from Boguth and Simutin (2018). Returns are available starting in 1931 for the U.S. and 1988 or later for the remaining countries, and the TED spread starts in 1986. The margin debt measure is calculated as the ratio of margin debt of NYSE member organizations to the market capitalization of NYSE stocks, and is available from 1959 to 2015. The leverage constraint measure is the aggregate beta of domestic mutual funds, averaged over 12 months, and is available 1980 to 2012. The TED spread, margin debt, and leverage constraint tightness is the same for all countries. The row 'Past Market Wald' reports the F-statistic from the Wald test of the null that all three coefficients on past market returns equal zero. Standard errors are clustered by date and all regressions include country fixed effects.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
$r^M_{t-1,t}$	0.05***							0.05***	0.05***
	[5.57]							[5.27]	[5.38]
$r^M_{t-6,t}$	0.09***							0.05**	0.05*
	[3.54]							[2.09]	[1.91]
$r^M_{t-12,t}$	0.03							−0.01	0.00
	[0.74]							[−0.23]	[0.12]
$r^{BAB}_{t-1,t}$		0.07***						0.06***	0.06***
		[4.09]						[3.18]	[3.20]
$r^{BAB}_{t-12,t}$		0.24***						0.20***	0.21***
		[4.48]						[3.62]	[3.74]
σ_t			−0.03***					−0.01	
			[−5.62]					[−1.09]	
$\Delta\sigma_{t-6,t}$				−0.03***					−0.00
				[−4.32]					[−0.52]
TED_t					−0.96***			−0.50***	−0.53***
					[−6.45]			[−2.70]	[−2.91]
MD_t						−0.03		0.01	−0.02
						[−0.32]		[0.09]	[−0.20]
LCT^{12M}_t							−0.96	0.03	0.21
							[−0.77]	[0.02]	[0.16]
Past Market Wald	83.54***							46.86***	49.41***
Adjusted R^2	0.04	0.02	0.02	0.02	0.02	0.01	0.01	0.06	0.06
#obs	7865	7589	7865	7865	7865	7267	6991	6715	6715

***,**, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 7: Alpha of BAB-Timing Portfolios wrt. the Market and BAB

The table shows the alpha of BAB-timing portfolios wrt. the market and BAB. Alphas are in percent per month. The BAB-timing portfolios are constructed as

$$\text{BAB-Timing} = \text{BAB} \times (N\text{-month past market return}), N = 1, 6, 12$$

where the BAB strategy has been scaled to *ex-ante* 10% vol and the market returns have been demeaned using an expanding mean. The timing strategies are scaled to 10% unconditional volatility *ex-post* to make the alphas comparable across countries. For each country, the table shows the 2-factor alpha of BAB-timing wrt. the country-specific market and BAB strategy. t-statistics are corrected for heteroscedasticity. The sample is February 1931 to January 2016 for the U.S. and January 1989 to January 2016 for international equities.

	Alpha wrt. MKT and BAB			t-Statistics		
	1M	6M	12M	1M	6M	12M
Australia	0.50***	0.45***	0.31**	3.24	3.96	2.24
Austria	0.40**	0.71***	0.63***	2.40	4.46	4.16
Belgium	−0.10	0.04	−0.10	−0.66	0.28	−0.62
Canada	0.64***	0.34*	0.02	2.72	1.82	0.10
Switzerland	−0.06	0.04	0.08	−0.37	0.28	0.54
Germany	0.13	0.29*	0.39***	0.71	1.71	2.65
Denmark	0.37*	0.25	0.21	1.89	1.45	1.28
Spain	0.41***	0.42***	0.24*	2.93	3.08	1.81
Finland	0.07	0.20	0.05	0.45	1.37	0.36
France	0.43**	0.62***	0.45***	2.21	3.75	2.84
United Kingdom	0.98***	0.65***	0.38**	5.72	4.12	2.37
Greece	0.35**	0.56***	0.56***	2.05	3.34	3.58
Hong Kong	0.82***	0.90***	0.60***	4.17	4.66	3.54
Ireland	0.52***	0.44***	0.29*	3.03	2.82	1.85
Israel	0.50***	0.19	0.11	2.84	1.27	0.98
Italy	0.58***	0.57***	0.41***	3.56	3.51	2.64
Japan	0.73***	0.57***	0.31**	4.73	3.77	2.07
Netherlands	0.23	0.40***	0.42***	1.46	2.96	2.78
Norway	0.35**	0.29*	0.01	2.08	1.76	0.05
New Zealand	0.46***	0.45***	0.23	3.10	2.94	1.45
Portugal	0.13	0.10	−0.06	0.95	0.84	−0.47
Singapore	0.58***	0.82***	0.68***	3.22	4.65	4.20
Sweden	0.23	0.05	0.05	1.32	0.23	0.25
United States	0.59***	0.55***	0.31***	5.40	5.18	2.80
Average	0.44	0.45	0.29	2.57	2.85	1.91
Cap Weighted	1.09***	1.30***	0.91***	5.18	7.10	4.96

***,**, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 8: Alpha of Sign BAB-Timing Portfolios wrt. the Market and BAB

The table shows the alpha of BAB-timing portfolios wrt. the market and BAB. Alphas are in percent per month. The BAB-timing portfolios are constructed as

$$\text{BAB-Timing} = \text{BAB} \times \text{sign}(\text{Past } N\text{-month demeaned market return}), N = 1, 6, 12$$

where the BAB strategy has been scaled to *ex-ante* 10% vol and the market returns have been demeaned using an expanding mean. For each country, the table shows the 2-factor alpha of BAB-timing wrt. the country-specific market and BAB strategy. t-statistics are corrected for heteroscedasticity. The sample is February 1931 to January 2016 for the U.S. and January 1989 to January 2016 for international equities.

	Alpha wrt. MKT and BAB			t-Statistics		
	1M	6M	12M	1M	6M	12M
Australia	0.53***	0.18	0.11	3.13	1.19	0.74
Austria	0.39***	0.64***	0.55***	2.62	4.57	4.21
Belgium	0.02	0.13	−0.16	0.10	0.87	−1.03
Canada	0.52***	0.17	0.08	3.15	1.00	0.49
Switzerland	−0.08	−0.09	0.12	−0.55	−0.63	0.90
Germany	0.23	0.38**	0.60***	1.37	2.35	3.99
Denmark	0.21	0.17	0.05	1.34	1.05	0.33
Spain	0.33**	0.41***	0.14	2.11	2.89	1.00
Finland	0.21	0.07	−0.01	1.41	0.50	−0.09
France	0.46***	0.76***	0.54***	2.86	4.95	3.63
United Kingdom	0.86***	0.67***	0.46***	5.93	4.32	2.92
Greece	0.26	0.43**	0.46***	1.42	2.50	2.75
Hong Kong	0.55***	0.74***	0.39**	3.38	4.49	2.50
Ireland	0.53***	0.59***	0.20	3.35	3.89	1.28
Israel	0.50**	0.44**	0.32**	2.53	2.48	2.05
Italy	0.55***	0.57***	0.42**	3.31	3.55	2.57
Japan	0.73***	0.61***	0.46***	4.73	4.29	3.20
Netherlands	0.22	0.65***	0.45***	1.34	3.91	3.01
Norway	0.20	0.20	−0.03	1.30	1.34	−0.19
New Zealand	0.51***	0.39***	0.28*	3.47	2.67	1.90
Portugal	0.07	0.28*	0.04	0.45	1.72	0.27
Singapore	0.51***	0.71***	0.52***	3.12	4.73	3.32
Sweden	0.05	0.00	0.24	0.26	0.02	1.54
United States	0.56***	0.43***	0.29***	5.00	3.87	2.75
Average	0.40	0.43	0.31	2.53	2.77	2.03
Cap Weighted	0.99***	1.06***	0.90***	6.02	6.64	5.55

***,**, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 9: Country BAB Returns Sorted on Past Market Returns

Each month, I sort the 24 countries in Table 1 into quintiles based on the past market return of the country. Within each group, I form an equal weighted portfolio of the BAB strategies for the countries in the group. The H-L column shows the performance of a portfolio that is long quintile 1 and short quintile 5. Excess returns and alphas are reported in percent per month, and volatilities and Sharpe ratios are annualized. 2-factor alphas are computed wrt. the global market and global BAB portfolios, and 5-factor alphas are computed wrt. the global MKT, SMB, HML, UMD, and BAB portfolios. t-statistics are reported in brackets. The sample is 1988 to 2016.

Panel A: BAB Returns Sorted on Past 1 Month Market Returns						
	Low				High	H-L
Excess Return	0.14	0.60***	0.57***	0.83***	1.00***	0.86***
t-stat/Wald F	[1.53]	[6.14]	[5.53]	[9.20]	[10.76]	[8.21]
2-Factor Alpha	−0.10	0.31***	0.27***	0.56***	0.74***	0.84***
t-stat/Wald F	[−1.23]	[3.87]	[2.92]	[7.08]	[9.23]	[7.39]
5-Factor Alpha	−0.11	0.31***	0.29***	0.56***	0.74***	0.85***
t-stat/Wald F	[−1.40]	[3.83]	[3.15]	[7.07]	[9.06]	[7.48]
Volatility	0.05	0.06	0.06	0.06	0.06	0.06
Sharpe Ratio	0.30	1.20	1.08	1.80	2.10	1.60
Panel B: BAB Returns Sorted on Past 6 Month Market Returns						
	Low				High	H-L
Excess Return	0.22**	0.61***	0.61***	0.76***	0.92***	0.70***
t-stat/Wald F	[2.24]	[6.06]	[6.32]	[8.17]	[10.25]	[6.11]
2-Factor Alpha	−0.04	0.31***	0.31***	0.51***	0.69***	0.73***
t-stat/Wald F	[−0.51]	[3.95]	[3.90]	[5.90]	[8.18]	[6.28]
5-Factor Alpha	−0.07	0.31***	0.31***	0.51***	0.72***	0.79***
t-stat/Wald F	[−0.78]	[3.92]	[3.90]	[5.95]	[8.61]	[6.75]
Volatility	0.06	0.06	0.06	0.06	0.05	0.07
Sharpe Ratio	0.44	1.18	1.24	1.60	2.00	1.19
Panel C: BAB Returns Sorted on Past 12 Month Market Returns						
	Low				High	H-L
Excess Return	0.29***	0.49***	0.67***	0.68***	0.99***	0.70***
t-stat/Wald F	[3.00]	[5.13]	[7.05]	[7.13]	[11.47]	[6.48]
2-Factor Alpha	0.02	0.20**	0.40***	0.41***	0.76***	0.74***
t-stat/Wald F	[0.20]	[2.48]	[4.95]	[5.10]	[9.63]	[6.78]
5-Factor Alpha	−0.01	0.19**	0.41***	0.41***	0.77***	0.77***
t-stat/Wald F	[−0.09]	[2.51]	[5.09]	[5.27]	[9.58]	[7.00]
Volatility	0.06	0.06	0.06	0.06	0.05	0.07
Sharpe Ratio	0.59	1.00	1.38	1.39	2.24	1.27

***,**, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 10: Long/Short BAB Cross-Country Selection

The table shows the performance of the cross-country BAB strategy based on the 24 countries in Table 1. The strategy goes long BAB in half of the countries and short BAB in half of the countries, based on the demeaned ranks of the countries' past N-month market returns. The 2-factor alpha is computed wrt. the global cap-weighted market and a global cap-weighted BAB strategy. The 5-factor alpha adds the global SMB, HML, and momentum portfolios as factors. t-statistics are reported in brackets and are corrected for heteroscedasticity. The sample is 1988 to 2016.

	Past Market Return		
	1 Month	6 Months	12 Months
Excess Return	0.67***	0.52***	0.54***
t-stat	[8.55]	[6.16]	[6.56]
2 Factor Alpha	0.66***	0.56***	0.58***
t-stat	[7.59]	[6.39]	[6.73]
5 Factor Alpha	0.67***	0.59***	0.61***
t-stat	[7.60]	[6.81]	[6.94]
Volatility	4.87	5.31	5.18
Sharpe Ratio	1.64	1.18	1.26

***,**, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Table 11: BAB-Timing for Equity Indices

The table shows the performance of BAB and BAB-timing for international equity indices. The BAB-timing strategies go long or short BAB based on the past 1, 6, and 12 month market return, where the market returns are demeaned using an expanding mean. Finally, the BAB-timing strategies are scaled to an *ex-post* volatility of 10% to make the alphas comparable. The sample period is 1981 to 2015 for the developed region and 1991 to 2015 for the emerging region. t-statistics are corrected for heteroscedasticity.

Panel A: Performance of BAB

	Alpha wrt. MKT	t-Statistics
Developed	0.37***	2.83
Emerging	0.66***	4.47

Panel B: Performance of BAB-Timing

	Alpha wrt. MKT and BAB			t-Statistics		
	1M	6M	12M	1M	6M	12M
Developed	0.42***	0.19	0.10	3.29	1.44	0.69
Emerging	0.59***	0.64***	0.28*	3.78	3.61	1.74

***,**, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

A The Leverage Aversion Theory

In this appendix I link the empirical observation that past market returns forecast future BAB returns to the leverage aversion theory, thus showing that the empirical result is predicted by a general equilibrium model.

In the leverage aversion theory, the equilibrium expected return on the BAB strategy depends on investors' demand for leverage. The demand for leverage is in turn determined by the investors risk-aversion and the volatility of the assets. Asset volatility and risk-aversion only enter into the model through the product of asset variance and risk-aversion, and I thus only focus on the effect of the risk-aversion parameter since the effect of asset volatility is identical.

To illustrate the effect of changing risk-aversion, I numerically solve for the equilibrium expected BAB return in a simple version of the leverage aversion theory. I consider a simple case with only two assets, asset 1 and asset 2. To solve the model numerically, assume that both assets have an expected price of \$1 at $t+1$, such that $E_t(P^1_{t+1}) = E_t(P^2_{t+1}) = 1$. Further, I assume that there are 200 shares outstanding of asset 1 and 100 shares outstanding of asset 2. Thus, asset 1 makes up a larger fraction of the market, and the average investor will need to hold twice as many shares of asset 1 as he does of asset 2. For this reason, asset 1 will have a beta higher than 1.0 and asset 2 will have a beta lower than 1.0. The risk-free rate is 0 as in FP.[11] I assume the payoffs of both assets have a volatility of 0.2 and a correlation of 0.5. Next, assume there are two agents. Agent 1 is unconstrained, i.e., he can borrow as much as he wants. Agent 2, however, cannot borrow, such that $m_2 = 1$.

The importance of each investor in determining equilibrium prices is determined by the investor's absolute risk aversion, with a lower value making the investor more important (a lower absolute risk aversion is analogous to a larger wealth, holding relative risk aversion

[11] The risk-free rate can be endogenized by letting the constrained investors have a lower value of the discount rate parameter than the unconstrained investors.

constant). In addition, the wealth of the constrained investor matters, since he cannot borrow. I assume that the two investors have the same *relative risk aversion*. Note that the expected value of the total stock market at $t+1$ is \$300. I therefore assume the constrained investor has an initial wealth of \$270 such that he is large enough that he can distort prices relative to the unconstrained CAPM equilibrium prices. Since he cannot borrow, he will be constrained for reasonable levels of risk aversion. Finally, I assume that the unconstrained investor has an initial wealth of \$5.

I vary relative risk aversion from 0 to 5 and solve numerically for equilibrium prices and returns. In Figure 3 I plot the equilibrium expected return on the BAB factor as a function of the equilibrium expected return on the market. As seen in Figure 3, when risk-aversion is high, the demand for equities is low, the expected return on the market is high, and the constraint does not bind for the investor who cannot apply leverage. Both investors hold the market portfolio and the BAB premium is zero. As risk-aversion decreases, the investors increase their demand for equities, this drives up prices and drives down the expected return on the market. The investor who cannot apply leverage becomes constrained and starts to overweight the high-beta asset which generates a positive BAB premium. As risk-aversion continues to decline, investors continue to drive up prices and the constrained investor increases his overweight of the high-beta asset. However, since expected returns are compressed toward zero, the advantage of begin able to apply leverage declines and the BAB premium declines as well. In the limit when risk-aversion is zero, the expected return on the market is zero, and the BAB premium is zero as well.

[Figure 3 about here.]

B Construction of BAB-factors

To construct a BAB portfolio of the country indices, I follow FP and estimate the beta for country i as

$$\hat{\beta}_i^{TS} = \hat{\rho}\frac{\hat{\sigma}_i}{\hat{\sigma}_m}, \tag{9}$$

where $\hat{\sigma}_i$ and $\hat{\sigma}_m$ are the estimated volatilities for the security and the market and $\hat{\rho}$ is their correlation. I use a one-year rolling standard deviation for volatilities and a five-year horizon to estimate the correlation. Further, I use one-day log returns to estimate volatilities and overlapping three-day log returns for correlations to control for non-synchronous trading, since the indices are traded on different exchanges in different countries. I require at least 6 months (132 trading days) of non-missing data to estimate volatilities and at least three years (792 trading days) of non-missing data for correlations. As in Frazzini and Pedersen (2014), I shrink the estimated betas for individual securities toward 1:

$$\hat{\beta}_i = 0.6\hat{\beta}_i^{TS} + 0.4. \tag{10}$$

The shrinkage does not affect how securities are sorted, but it does affect the construction of the BAB-factor since the BAB-factor is scaled to be ex-ante beta-neutral.

Securities in an asset class are ranked in ascending order based on their estimated betas. The low- (high-) beta portfolio is composed of all securities with a beta below (above) its asset class median. In each portfolio, securities are weighted by their ranked beta, such that lower-beta securities have larger weights in the low-beta portfolio and higher-beta securities have higher weights in the high-beta portfolio. The portfolios are rebalanced at the end of each month.

If r_{t+1}^L is the return on the low-beta portfolio with ex-ante beta β_t^L and r_{t+1}^H is the return on the high-beta portfolio with ex-ante beta β_t^H, the BAB portfolio is the self-financing zero-beta portfolio that is long the low-beta portfolio and short the high-beta portfolio:

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L}\left(r_{t+1}^L - r^f\right) - \frac{1}{\beta_t^H}\left(r_{t+1}^H - r^f\right). \tag{11}$$

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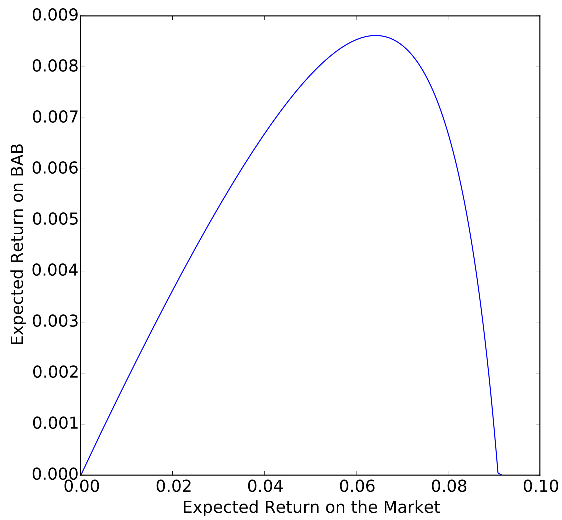

Figure 3: Effect of Discount Rates in the Leverage Aversion Theory

The figure shows the effect of changing risk-aversion and volatility in the leverage aversion model. I numerically solve for the BAB-premium in a model with two investors and two assets. One investor is unconstrained and the other cannot apply leverage. Asset 1 has a larger market share than asset 2, and hence asset 1 has a beta larger than 1 and asset 2 has a beta lower than 1. When discount rates are very high (to the right) prices are low, agents are unconstrained, the CAPM holds, and the BAB-premium is zero. As discount rates decrease, agents increase their demand for equities and drive up prices in equilibrium. The constraint starts to bind for the constrained investor and he starts to over-weight the high-beta stock relative to the market portfolio. The CAPM no longer holds, and the BAB-premium increases. As discount rates further decreases, prices increase, expected returns decline, and betas are compressed towards 1. The advantage of being able to apply leverage decreases, and at some point the constrained agent become less constrained and the BAB-premium starts to decrease.