Aggregate Functions and Data Partitioning

Collections and aggregate functions

- A collection is a grouping of some variable number of data items (possibly zero)
- Usually the data items in a collection are of the same type
- Aggregate functions are functions that apply to collections, i.e., they consider all these data items in these collections
- Applied to a collection, an aggregate function returns a single value

Examples of collections

- sets, multisets, dictionaries (maps), relations
- vectors, lists, arrays, series
- data structures: stacks, queues, hash tables, trees, graphs

Aggregate functions on unordered collections

- We will restrict ourselves to aggregate functions on sets, bags, and relations
- COUNT (we will often use the notation |A| instead of COUNT(A))
- SUM, AVERAGE, MIN, and MAX
- COUNT({a,b,c}) = 3; COUNT({{a,c,c,b,b,b}}) = 6
- SUM({1, 4, 7}) = 12; SUM({{1,1,1,1,1}}) = 5
- AVG({1,4,7}) = 4; AVG({{1,1,1,1,1}}) = 1
- MIN({1,4,7}) = MIN({{1,1,1,1,1}}) = 1
- MAX({1,4,7}) = 7; MAX({{1,1,1,1,1}}) = 1
- MIN({John,Eric,Ann}) = Ann
- These aggregate functions are supported in SQL

Applications of aggregate functions

- Data analytics
- Formulating complex queries
- Facilitating efficient query evaluation

Counting the size of a set in SQL

- Let R(A1,…,An) be a relation.
- Then |R| can be obtained as follows:

```
SELECT COUNT(*)
FROM    R r;
```

- Alternatively,

```
SELECT COUNT(1)
FROM    R r;
```

- Of course we can restrict the COUNT function to apply to a subset of R by applying a WHERE clause.

COUNT examples

- "Find the number of courses in which the student with sid 10 is enrolled."

```
SELECT COUNT(*)
FROM   Enroll E
WHERE E.sid = 's10';
```

- "Find the number of students who are not enrolled in any CS course."

```
SELECT COUNT(*)
FROM   Student S
WHERE S.Sid NOT IN (SELECT E.Sid
                    FROM   Enroll E, Course C
                    WHERE E.Cno = C.Cno AND C.Dept = 'CS');
```

COUNT Example

- Let R and S be two relations, then the following query will return $|R \times S| = |R||S|$, i.e. the size of the cartesian (cross) product of R and S.

```
SELECT COUNT(*)
FROM R r, S s;
```

- $R \times S = \{(r, s) \mid r \in R \wedge s \in S\}$

COUNT DISTINCT

R

A	B
a	1
a	2
b	1

```
SELECT COUNT(r1.A) AS Total
FROM    R r1, R r2
```

Total
9

```
SELECT COUNT(DISTINCT r1.A) AS Total
FROM    R r1, R r2
```

Total
2

Simulating COUNT with SUM

- The following SQL query uses the SUM aggregate function to determine the size of R, provided $R \neq \emptyset$

```
SELECT SUM(1)
FROM   R r;
```

- The bag that is generated by the query is {{1,…,1}} containing as many 1’s as there are tuples r in R

Example

A
a
b
c

- Consider the relation R
- |R| = 3

- COUNT applies to {{a,b,c}}
- SUM applies to {{1, 1, 1}}
- If you write the SQL query

```
SELECT SUM(2)
FROM   R r;
```

the result will be SUM({{2,2,2}}) = 6.

Caveat: empty collection

- Consider the relation R
- |R| = 0

A

- COUNT applied to {} gives 0
- SUM applied to {} gives NULL

```
SELECT COUNT(1)
FROM   R r;
```

0

```
SELECT SUM(1)
FROM   R r;
```

NULL

MIN and MAX aggregate functions

- MIN returns the smallest data item in the bag to which it applies.
- MAX returns the largest data item in the bag to which it applies.
- Data items can come from any ordered basic domain: integer, float, text
- A more general MIN function can be simulated using <= ALL

```
SELECT DISTINCT r.A1,...,r.An
FROM   R r
WHERE (r.A1,...,r.An) <= ALL (SELECT r1.A1,...,r1.An
                              FROM   R r1);
```

- MAX can be simulated using >= ALL
- However, there is a problem if MIN (MAX) is applied to an empty set.

CAVEAT: aggregate functions on empty set

- Except for COUNT, SQL aggregate functions return a NULL value when applied to an empty set (or bag).
- Assume R is the empty relation

A

- Then SELECT MIN(r.A) AS smallest FROM R r

  returns the relation

smallest
NULL

- However SELECT r.A AS smallest FROM R r WHERE r.A <= ALL(SELECT r1.A FROM R r1)

  returns the empty relation

smallest

Partitioning and counting

- "Determine for each student the number of courses taken by that student."

Enroll

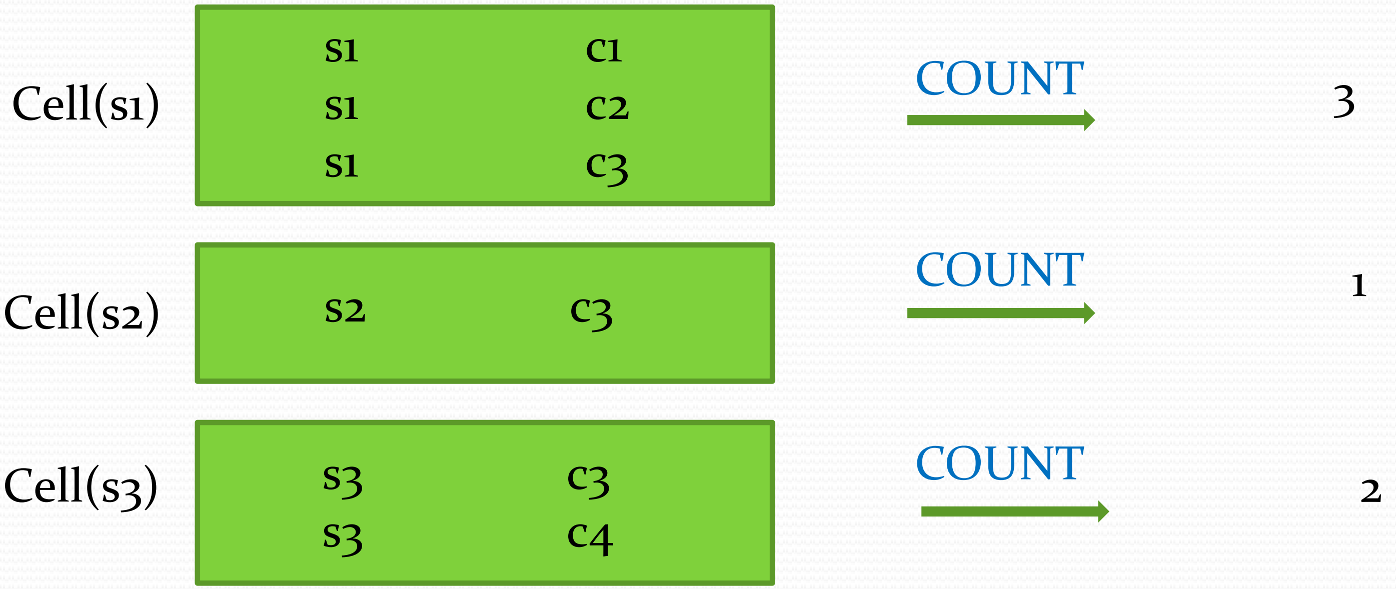
Sid	Cno
s1	c1
s2	c1
s1	c2
s3	c3
s3	c4
s1	c3

→

Sid	No_Courses
s1	3
s2	1
s3	2

Partition and map count function

- (1) First, Partition the Enroll table into cells (blocks) wherein each cell contains all the tuples that have a common sid value.
- (2) Next, Map the COUNT function over these cells.



Partition and map COUNT in SQL

- (1) The GROUP BY map COUNT method
- (2) The user-defined COUNT FUNCTION method
- (3) The SELECT COUNT-expression method

The GROUP BY map COUNT method

Map COUNT phase → SELECT E.Sid, COUNT(*) AS No_Courses
FROM Enroll E
Partition phase → GROUP BY(E.Sid)

Partition phase: the GROUP BY operator places each tuple E into the cell identified by its E.Sid value

Map COUNT phase: the COUNT function is mapped over the cells identified by the different possible E.sid values

Group by visualization

```
SELECT State, COUNT(*)
FROM facebook
GROUP BY State
```

facebook

Name	State	# of Friends
Matt	CA	300
Lisa	CA	500
Jeff	CA	600
Sarah	FL	400

final table

State	COUNT(*)
CA	0
FL	0

https://dataschool.com/how-to-teach-people-sql/how-sql-aggregations-work/

The user-defined COUNT FUNCTION method

- “Define a function with input parameter a student sid and as output the number of courses taken by that student.

```
CREATE FUNCTION NumberOfCourses (s TEXT) RETURNS bigint
AS $$
      SELECT COUNT(*)                              Map COUNT phase
      FROM    Enroll E
      WHERE E.Sid = s;
   $$ LANGUAGE SQL;
```

- Then execute the SQL query

```
SELECT  S.Sid, NumberOfCourses(S.Sid) AS No_Courses    Partition phase
FROM    Student S;
```

Student

Sid
s1
s2
s3
s4

Enroll

Sid	Cno
s1	c1
s1	c2
s1	c3
s2	c3
s3	c3
s3	c4

```
SELECT S.Sid, NumberOfCourses(S.Sid) AS No_Courses
FROM    Student S;
```

Sid	No_Courses
s1	3
s2	1
s3	2
s4	0

student s4 takes no courses

The SELECT COUNT-expression method

```
SELECT S.sid, (SELECT COUNT(E.Cno) AS NumberCourses
                 FROM    Enroll E
                 WHERE E.Sid = S.Sid)
FROM    Student S
```

Map phase

Partition phase

- Observe that the subquery identified by S.Sid appears in the outer SELECT clause.
- The COUNT of the result of this subquery is then delivered as a value in the outer SELECT clause.
- Notice that this expression must appear between parentheses.
- The output of this query is the same as that on the previous slide.

Example query

- “Find the sid of each student who takes the most courses.”
- Using the COUNT FUNCTION method:

```
SELECT  S.sid
FROM    Student S
WHERE NumberOfCourses(S.sid) >= ALL (SELECT NumberOfCourses(S1.Sid)
                                     FROM   Student S1);
```

Example query

- "Find the sid of each student who take the most courses."
- Using the GROUP BY method:

```
WITH
  NumberOfCoursesbyStudent AS (SELECT E.Sid, COUNT(E.Cno) As NumberOfCourses
                               FROM   Enroll E
                               GROUP BY(E.Sid))
  SELECT P.sid
  FROM   NumberOfCoursesbyStudent P
  WHERE  P.NumberOfCourses >= ALL (SELECT P1.NumberOfCourses
                                   FROM   NumberOfCoursesbyStudent P1);
```

Example query

- "Find the sid of each student who takes the most courses."
- Using the COUNT expression method:

```
SELECT   S.Sid
FROM     Student S
WHERE    (SELECT COUNT(E.cno)
          FROM    Enroll E
          WHERE   E.sid = S.sid) >= ALL (SELECT (SELECT COUNT(E.cno)
                                                  FROM   Enroll E
                                                  WHERE  E.sid = S1.sid)
                                         FROM   Student S1);
```

The COUNT-bug of GROUP BY

- The result of the following 2 queries is the same. Notice that there is a bug since, if a student sid takes no courses, then (sid, 0) does not appear in the output.

```
SELECT E.Sid, COUNT(E.Cno)
FROM   Enroll E
GROUP BY (E.Sid)
```

```
SELECT S.Sid, Count(E.Cno)
FROM   Student S, Enroll E
WHERE S.Sid = E.Sid
GROUP BY(S.Sid)
```

- These two queries give the same result and exhibit the COUNT bug: the tuple (s4,0) does not appear in the result.

Fixing the COUNT-bug

- To fix the COUNT-bug we need to add the (s,0) pair if student with sid s takes no courses. This can be done with the UNION operator.

```
(SELECT E.Sid, COUNT(E.Cno) AS No_Courses
 FROM   Enroll E
 GROUP BY (E.Sid))
UNION
(SELECT S.Sid, 0 AS No_Courses
  FROM   Student S
  WHERE S.Sid NOT IN (SELECT E.Sid
                                          FROM   Enroll E))
```

Partitioning on different dimensions

Enroll

Sid	Cno
s1	c1
s1	c2
s1	c3
s2	c3
s3	c3
s3	c4

SELECT COUNT(*)
FROM Enroll
GROUP BY ()

6

SELECT E.Sid, COUNT(*)
FROM Enroll E
GROUP BY (E.Sid)

E.sid	
s1	3
s2	1
s3	2

SELECT E.Sid, E.Cno, COUNT(*)
FROM Enroll E
GROUP BY (E.Sid, E.Cno)

E.Sid	E.Cno	
s1	c1	1
s1	c2	1
s1	c3	1
s2	c3	1
s3	c3	1
s3	c4	1

What can appear in the GROUP BY clause?

- Answer: any valid expression over the tuples in the FROM clause.

```
SELECT …
FROM R1 t1,…,Rn tn
WHERE …
GROUP BY(expression(t1,…,tn))
```

- Partition: there will be as many cells in the partition as there are different values for expression(t1,…,tn)

Example: expressions in GROUP BY

S

X	Y
2	3
1	3
2	1
0	3

```
SELECT s.x + s.y AS sum, COUNT(*) as cell_size
FROM   S s
GROUP BY (s.x + s.y)
```

sum	cell_size
3	2
4	1
5	1

Notice that there are 4 tuples in R assigned to variable s but only 3 different s.x+s.y values: 3, 4, 5. Thus there are only 3 cells in the partition.

Example: expression in GROUP BY

Person

Pid	Age
p1	10
p2	9
p3	12
p3	9

```
SELECT   p.age > 10 AS OlderThanTen, COUNT(*)
FROM     Person p
GROUP BY (p.age > 10);
```

OlderThanTen	Count
f	3
t	1

Restrictions on SELECT clause in GROUP BY query

- In a GROUP BY query, the SELECT clause may only contain aggregate expressions that returns a single value for each cell of the partition induced by the GROUP BY clause.

```
SELECT   AggregateExpression(cell(expression(t1,...,tn)), ...
FROM     R t1,...,tn
WHERE    condition(t1,...,tn)
GROUP BY (expression(t1,...,tn))
```

Aggregate expressions in SELECT clause

S

X	Y
2	3
1	3
2	1
0	3

```
SELECT s.x+s.y AS sum,
       SUM(s.x*s.y) AS
            sum_of_products
FROM   S s
GROUP BY (s.x+s.y)
```

sum	sum_of_products
3	2 = $(2 * 1 + 0 * 3)$
4	3 = (1*3)
5	6 = (2*3)

Aggregate expressions in SELECT clause

- The following query will raise an error since s.x is not necessarily unique in a cell defined by s.x+s.y values

S

X	Y
2	3
1	3
2	1
0	3

```
SELECT s.x
FROM   S s
GROUP BY (s.x+s.y)
```

The HAVING clause in GROUP BY queries

- The HAVING clause in a GROUP BY selects those cells from the partition induced by the GROUP BY clause that satisfy an Aggregate Condition.
- Only those cells are passed onto the SELECT clause.

```
SELECT AggregateExpression(Cell(expression(t1,…,tn)), …
FROM    R1 t1, …, R tn
WHERE  condition(t1,…,tn)
GROUP BY  (expression(t1,…,tn))
HAVING AggregateCondition(Cell(expression(t1,…,tn)))
```

Example: HAVING clause

- "For each student who majors in CS determine the number of courses taken by that student, provided that this number is at least 2."

```
SELECT E.Sid, COUNT(E.Cno)
FROM   Enroll E, Student S
WHERE E.Sid = S.Sid AND S.Major = 'CS'
GROUP BY (E.Sid)
HAVING COUNT(E.Cno) ≥ 2;
```

Simulating HAVING clause with user-defined functions in WHERE

- "For each student who majors in CS, determine the number of courses taken by that student, provided that this number is at least 3."
- The HAVING condition can be simulated in the WHERE clause with user-defined functions.

```
SELECT  S.Sid AS Sid, NumberOfCourses(S.Sid)
FROM    Student S
WHERE   S.major = 'CS' AND NumberOfCourses(S.Sid) ≥ 3
```

Spreadsheet (Data Cube)

Sid	Cno
s1	c1
s1	c2
s1	c3
s2	c1
s2	c2
s3	c2
s4	c1

	c1	c2	c3		sum(sid)	
s1	1	1	1		3	
s2	1	1	0		2	
s3	0	1	0		1	
s4	1	0	0		1	
sum(cno)	3	3	1			
					7	sum()

GROUPING sets

- It may be desirable to simultaneously generate different partitions and then apply an aggregation
- This is supported in SQL via GROUPING sets.

Sid	Cno
S1	C1
S1	C2
S2	C1
S2	C2
S3	C1

Sid	Cno	Count
S1	*	2
S2	*	2
S3	*	1
*	C1	3
*	C2	2

```
SELECT Sid, Cno, COUNT(*)
FROM    Enroll
GROUP BY
  GROUPING SETS((Sid),(Cno))
```