

# Hashing

**Dirk Van Gucht**

Based on slides by Hector Garcia-  
Molina

# Hashing: hash function

- Let  $K$  be a domain of key values ( $K$  can be very large)
- Let  $R = [0, m)$  (usually  $m \ll |K|$ ) be a range of values
- A **hash function**  $h$  maps  $K$  to  $R$

$$h: K \rightarrow R$$

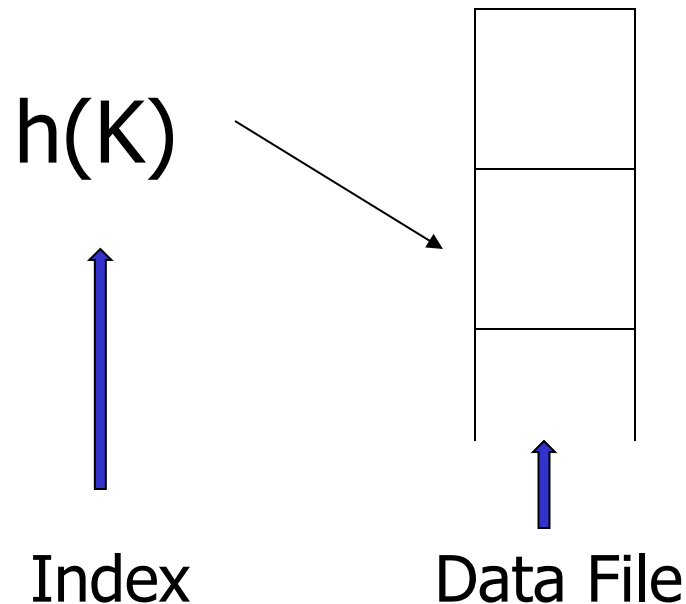
Example:  $h(k) = k \bmod m$

- Given a record  $r$  with key value  $k$ ,  $h(k)$  provides an address (name) of a **bucket** in which to store  $r$
- A bucket is stored in secondary memory as a block or a list of blocks
- Retrieving a bucket (and the records therein) can be done in  $O(n)$  where  $n$  is the number of blocks that store the bucket.

# Hash function: collision

- Let  $r_1$  and  $r_2$  be two records with key values  $k_1$  and  $k_2$
- We permit that  $k_1 = k_2$
- We say that  $h$  has a collision for  $r_1$  and  $r_2$  if  $h(k_1) = h(k_2)$   
Consequence:  $r_1$  and  $r_2$  will be stored in the same bucket
- If  $K$  is the domain of a primary key, a collision will store records with different key values in same bucket (not desirable)
- If  $K$  is not the domain of a primary key, different records with the same key values will be placed in the same bucket (desirable). Partitioning.
- The latter property is exploited in key-value stores since records with the same key value will be sent to the same reducer

Next: example to illustrate  
inserts, overflows, deletes



# EXAMPLE 2 records/bucket

(For simplicity, we identify a record with its key value)

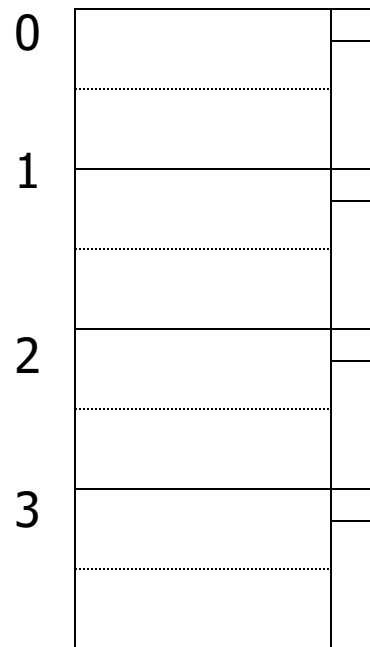
INSERT:

$$h(a) = 1$$

$$h(b) = 2$$

$$h(c) = 1$$

$$h(d) = 0$$



## EXAMPLE 2 records/bucket

INSERT:

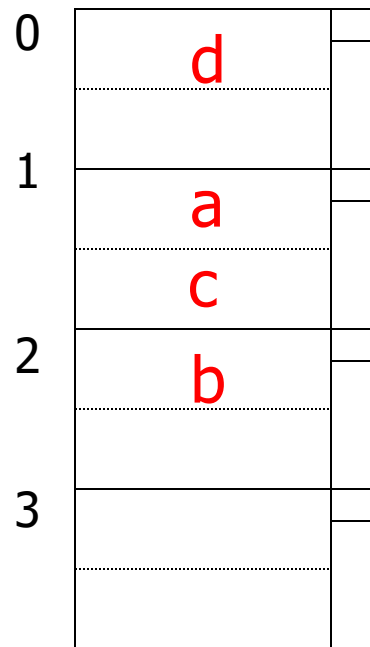
$h(a) = 1$

$h(b) = 2$

$h(c) = 1$

$h(d) = 0$

$h(e) = 1$



## EXAMPLE 2 records/bucket

INSERT:

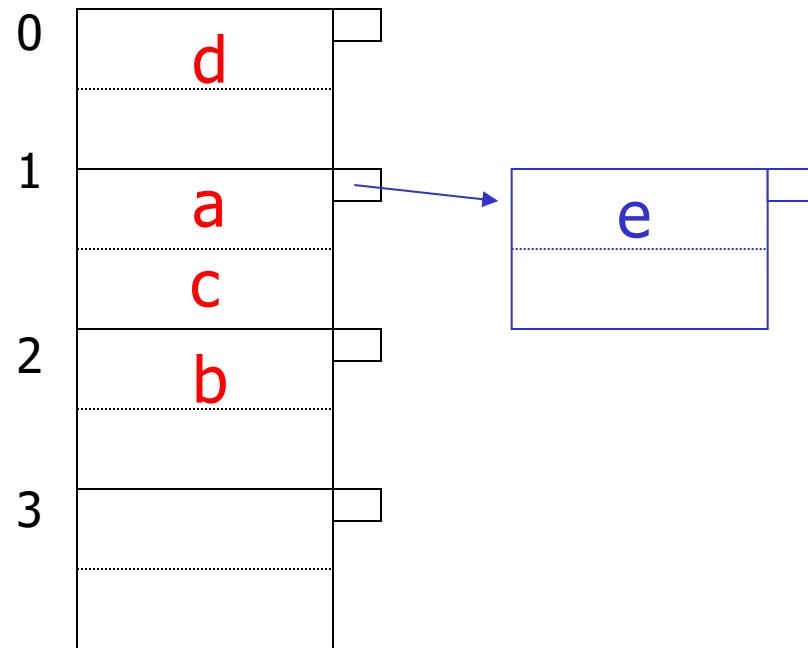
$h(a) = 1$

$h(b) = 2$

$h(c) = 1$

$h(d) = 0$

$h(e) = 1$

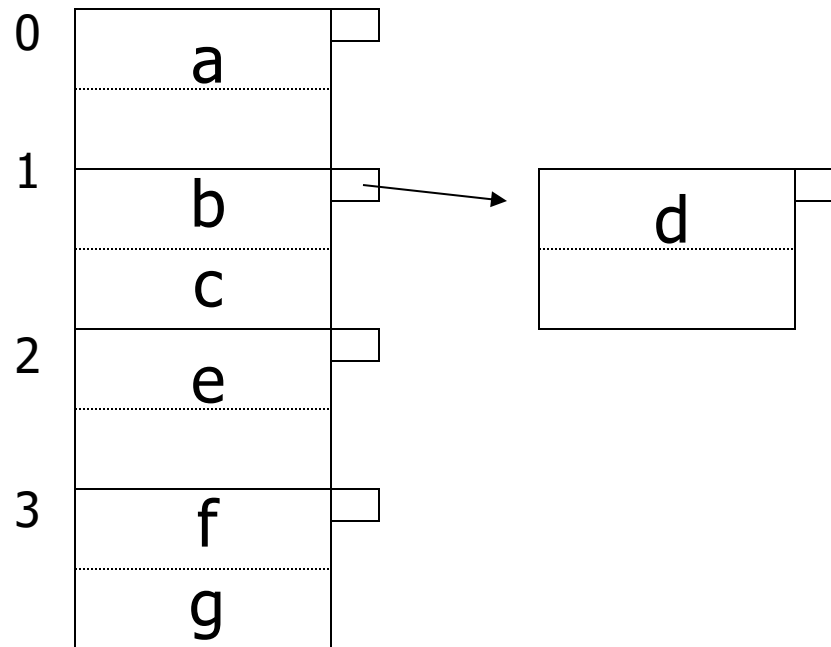


# EXAMPLE: deletion

Delete:

e

f





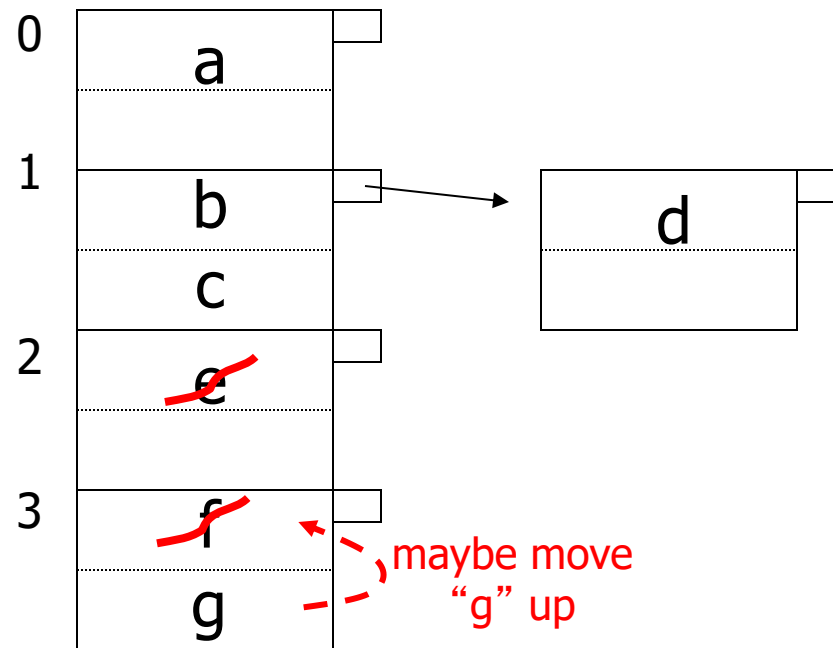
# EXAMPLE: deletion

Delete:

e

f

c



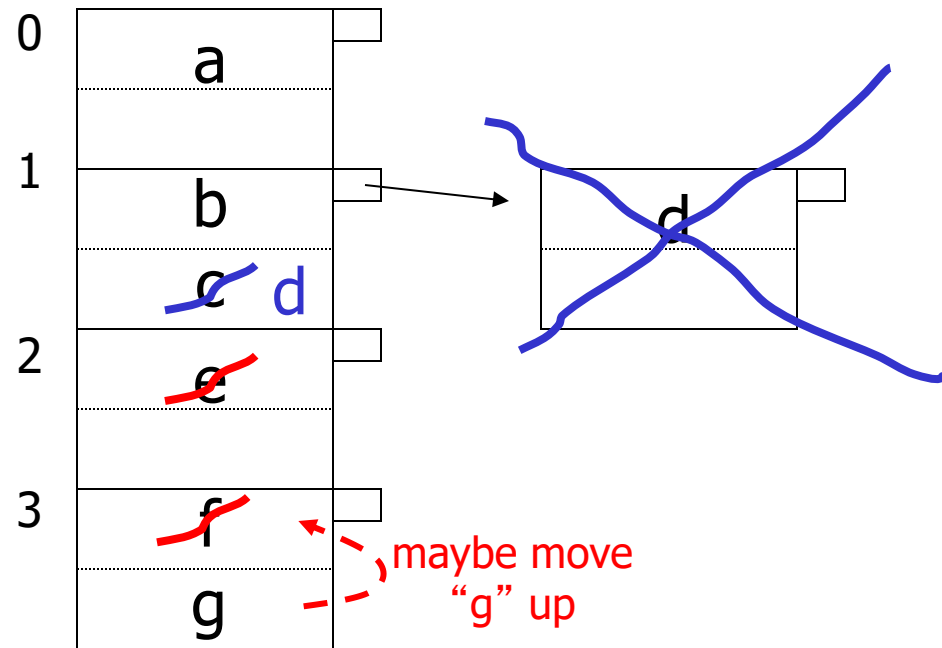
# EXAMPLE: deletion

Delete:

e

f

c

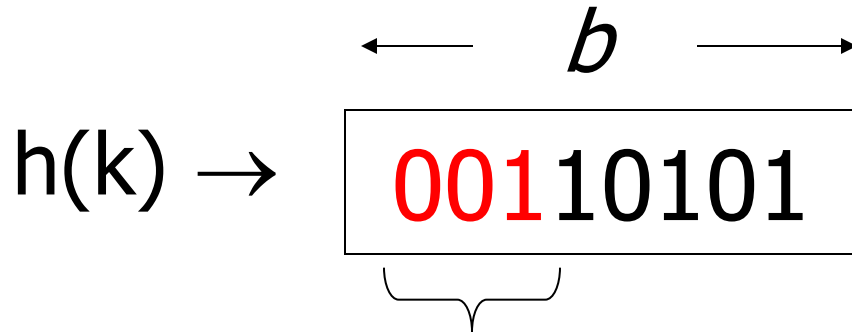


# How do we cope with growth?

- Overflows and reorganizations  
Reorganization can be done by enlarging the range  $R$  and changing the hash function  
Reorganization requires complete rehashing and is linear in the  $|Data\ file|$
- Dynamic hashing (extensible hashing)

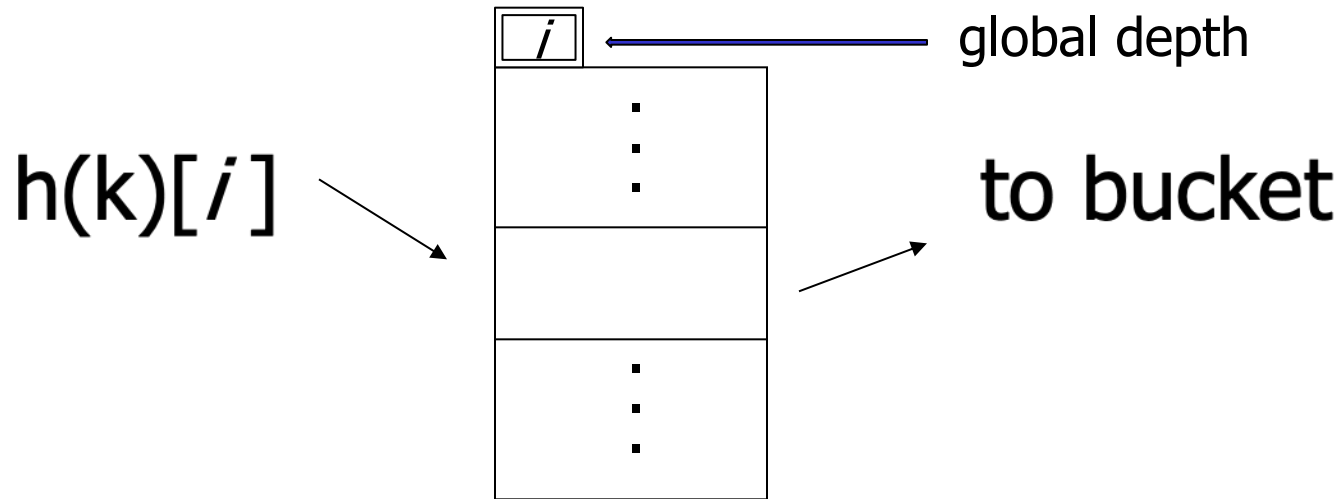
## Extensible hashing: two ideas

(a) Use  $i$  of  $b$  bits output by hash function



use  $i \rightarrow$  grows/shrinks over time....

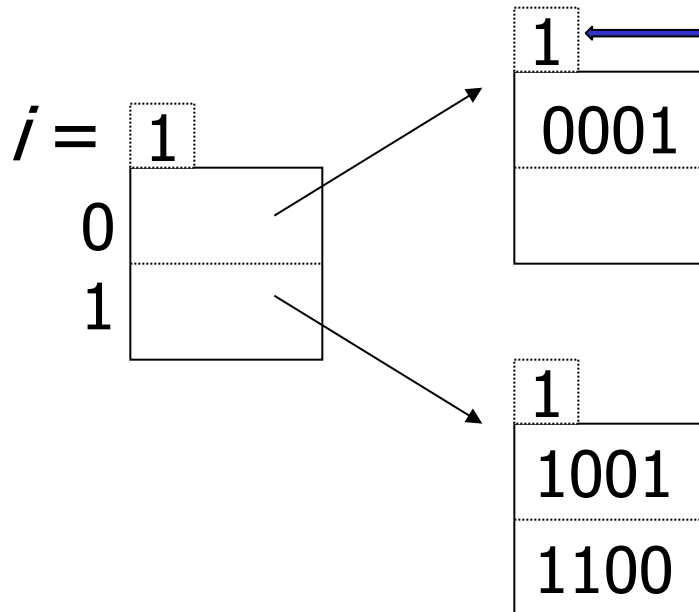
(b) Use directory of size  $2^i$



$h(k)$  has  $b$  bits, but we will only look at its first  $i$  bits

$h(k)[i]$  consists of the first  $i$  bits of  $h(k)$   
these  $i$  bits specify the position for  $k$  the  
directory

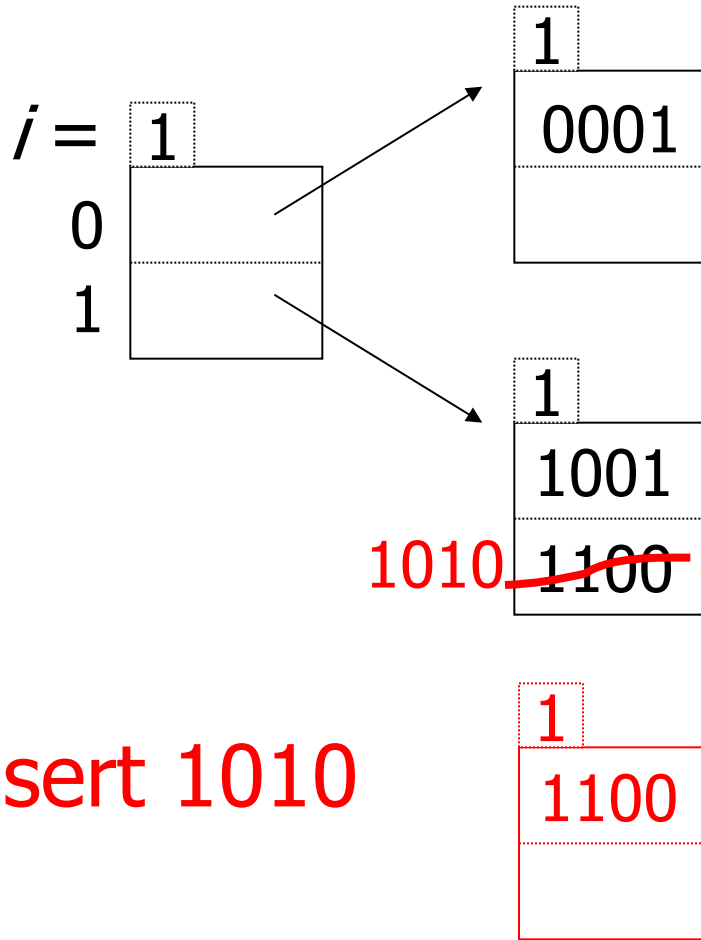
Example:  $h(k)$  is 4 bits; 2 keys/bucket



local depth of bucket  $\leq i$

Insert 1010

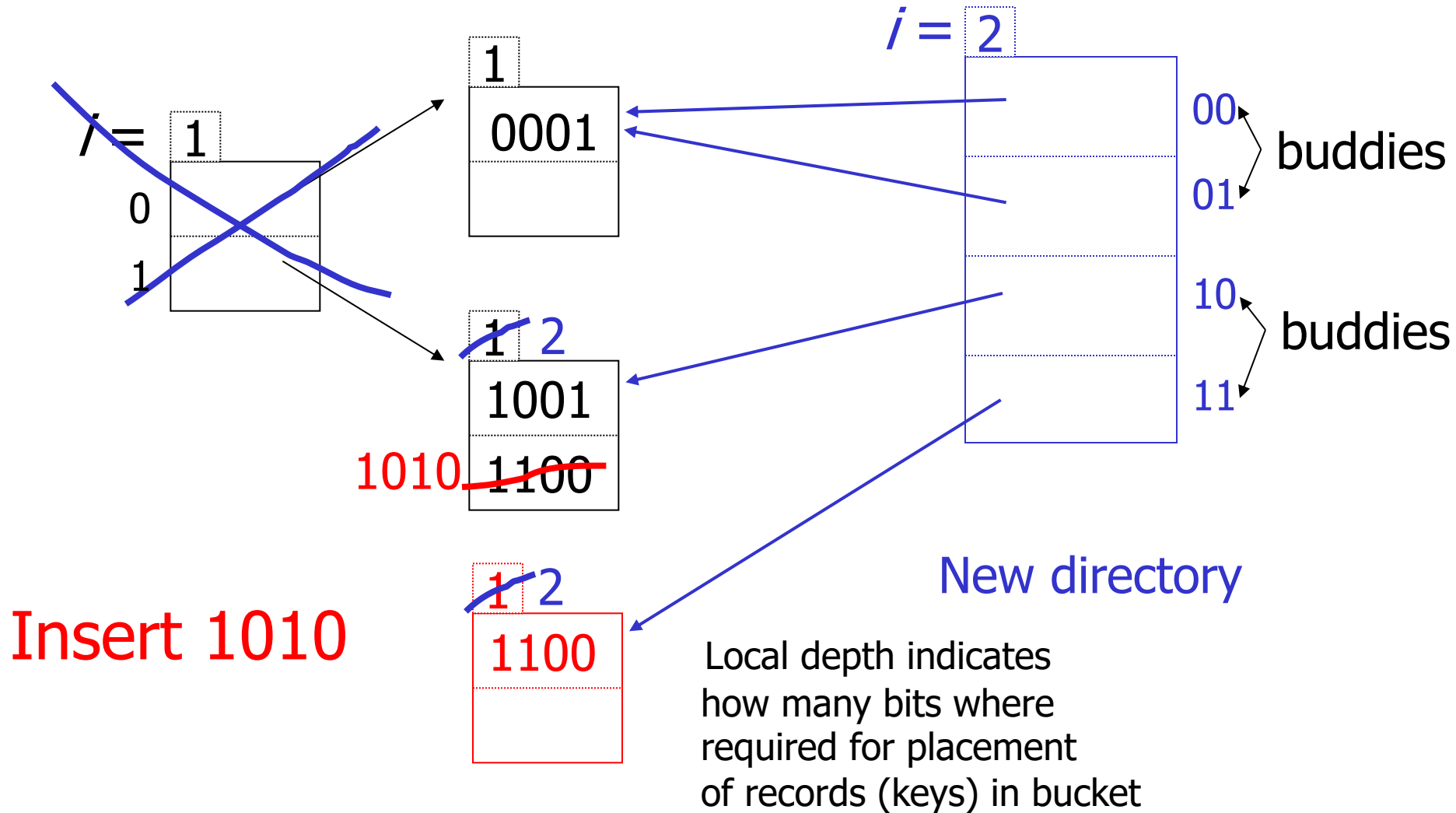
# Example: $h(k)$ is 4 bits; 2 keys/bucket



Main idea:  
we need 2 bits to  
distinguish

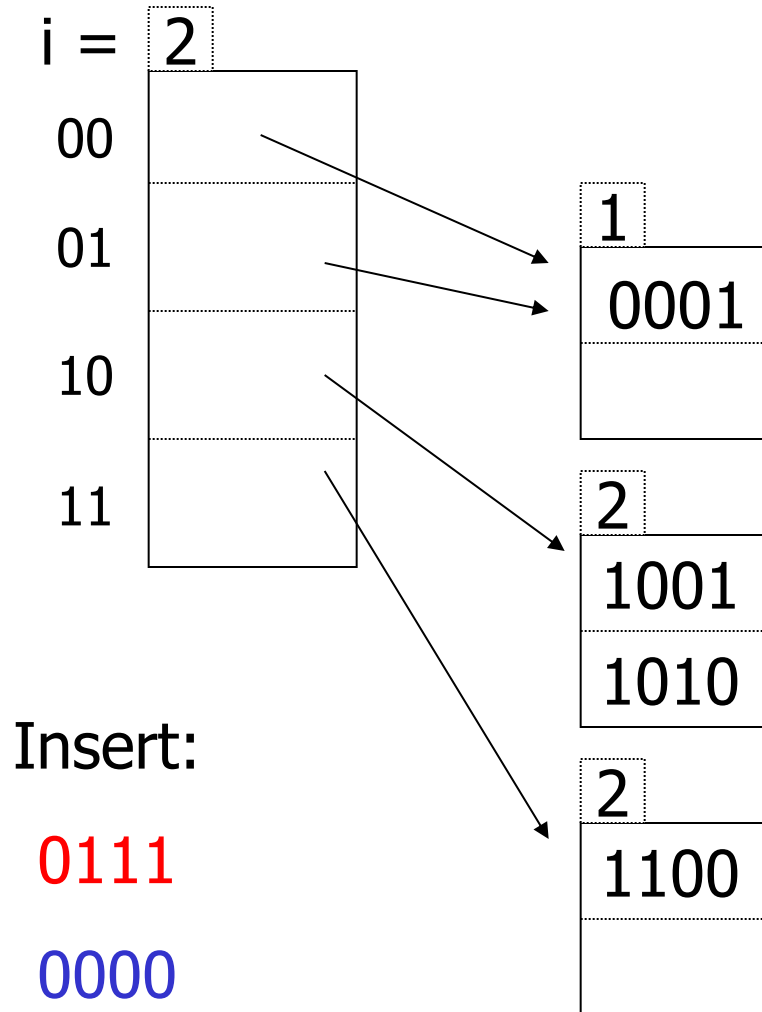
1001  
1100  
1010

# Example: $h(k)$ is 4 bits; 2 keys/bucket

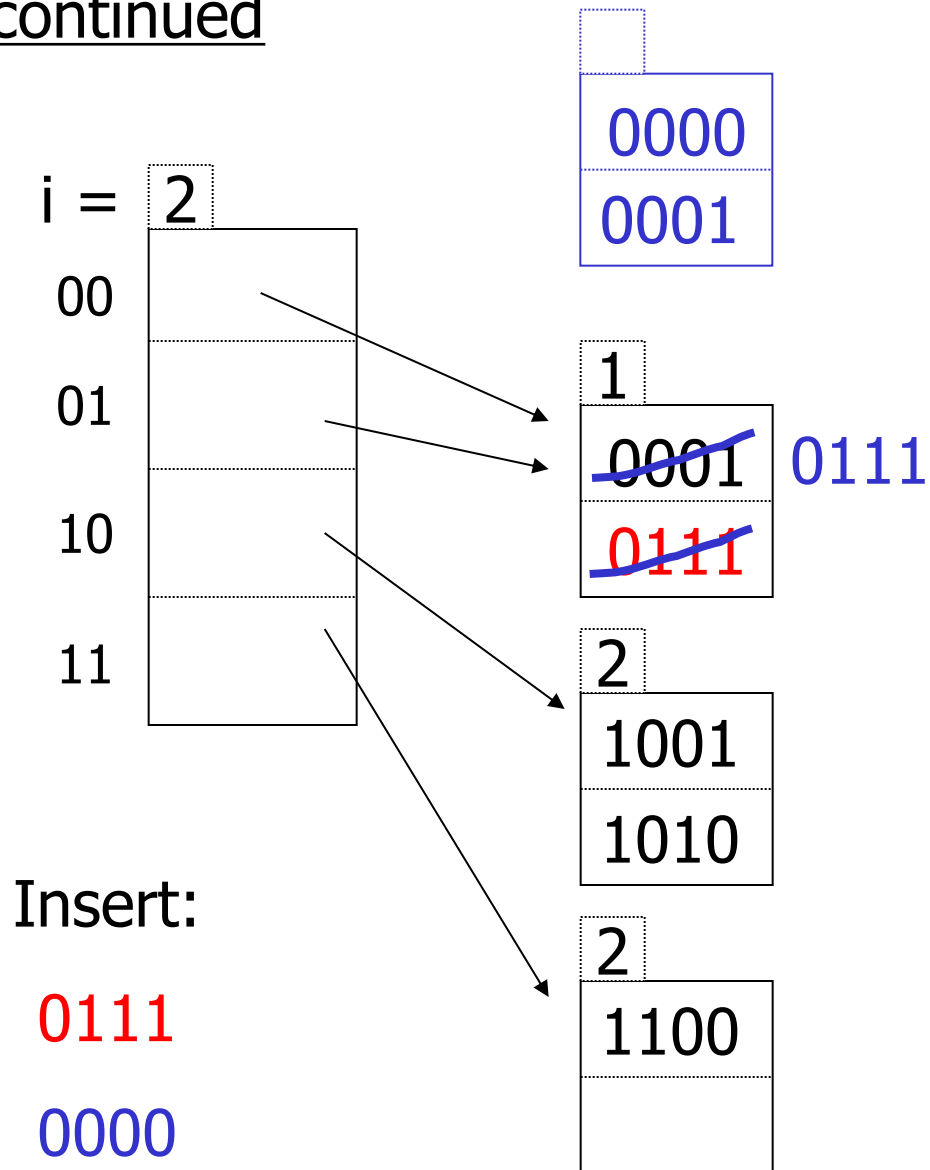




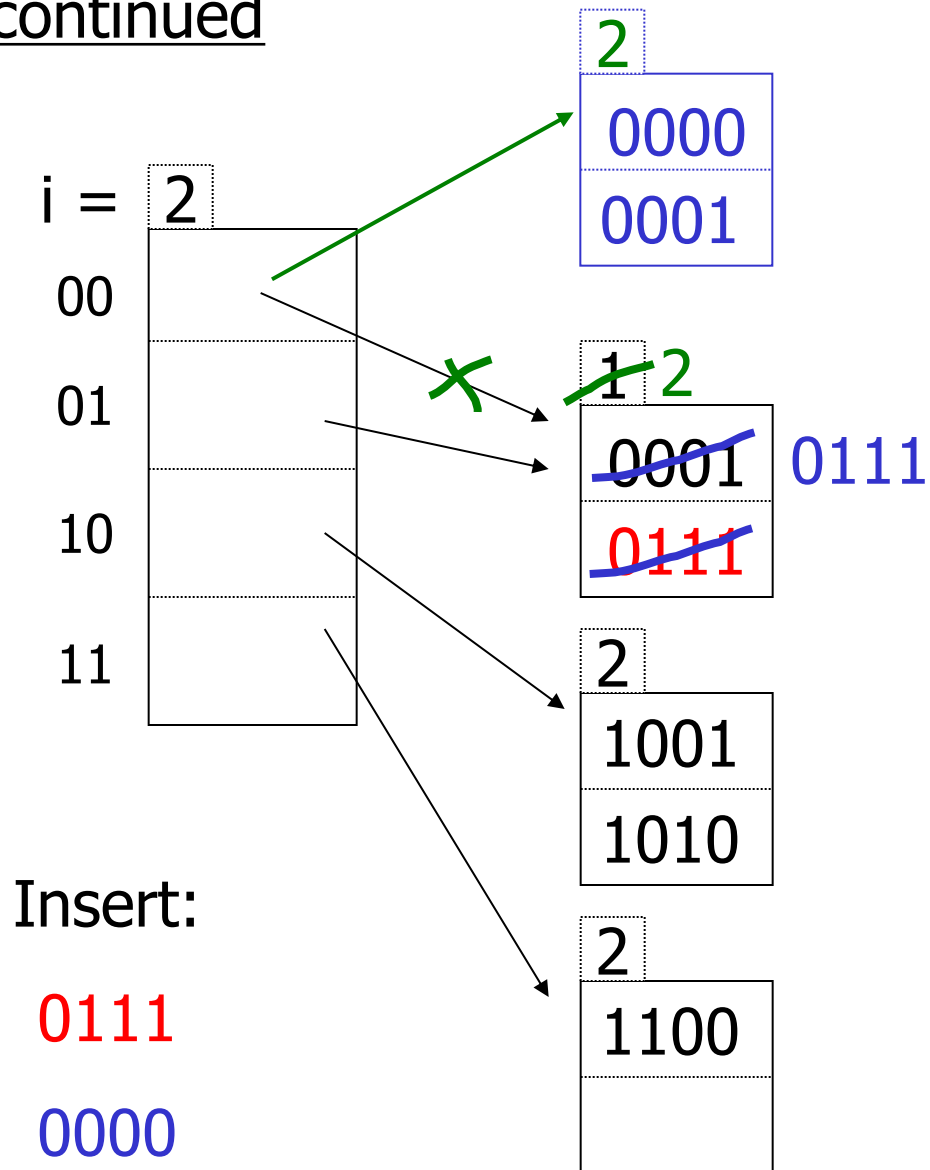
## Example continued



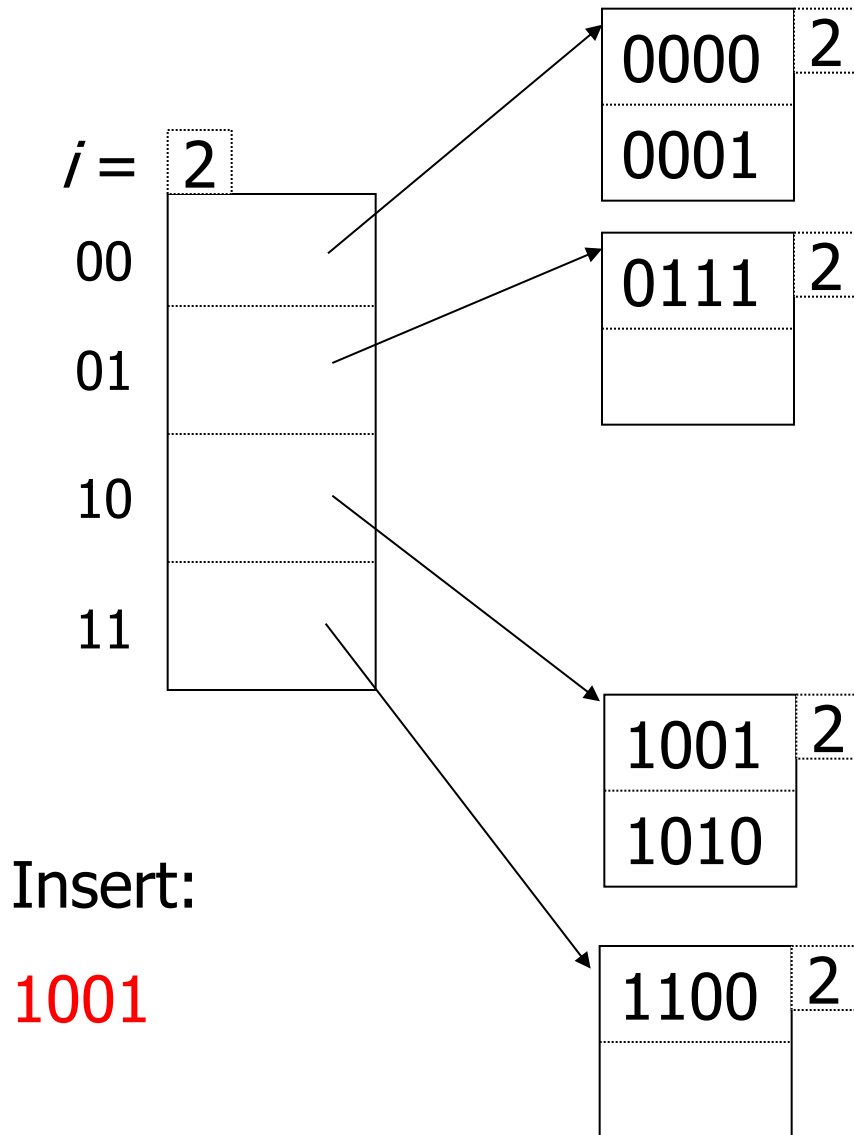
## Example continued



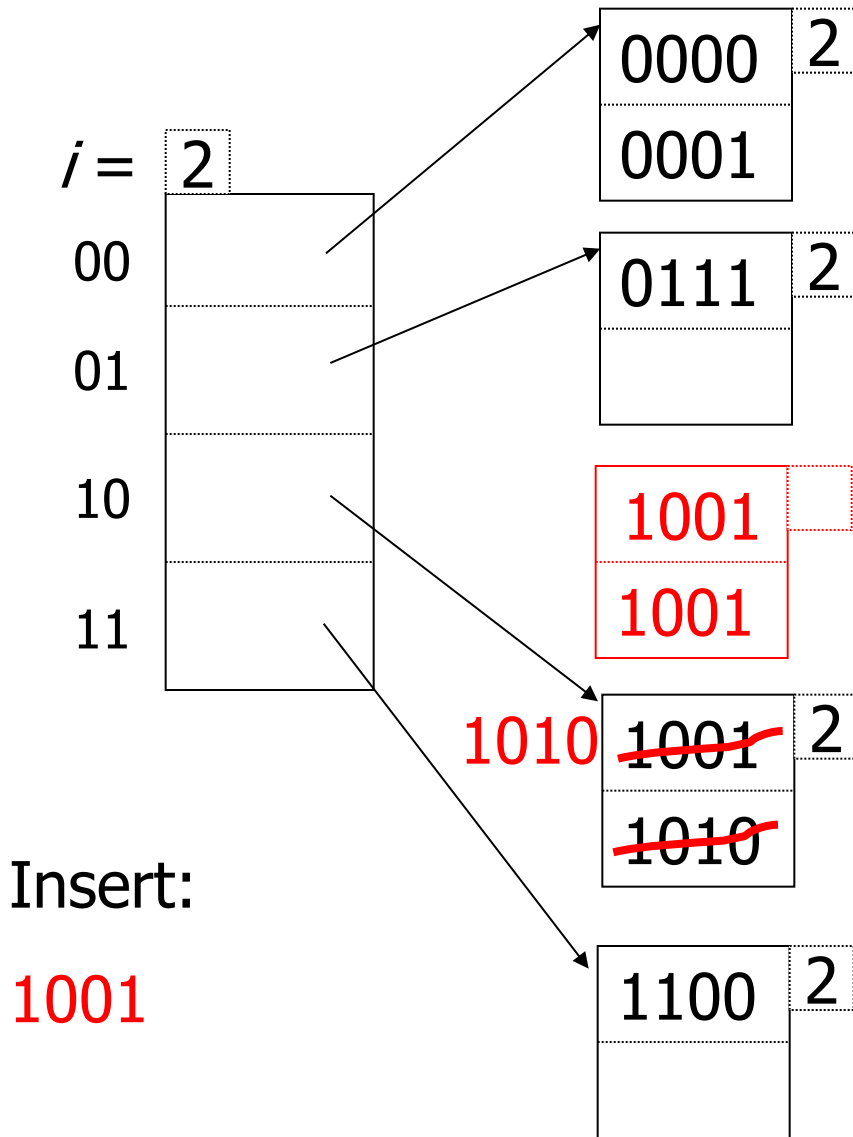
## Example continued



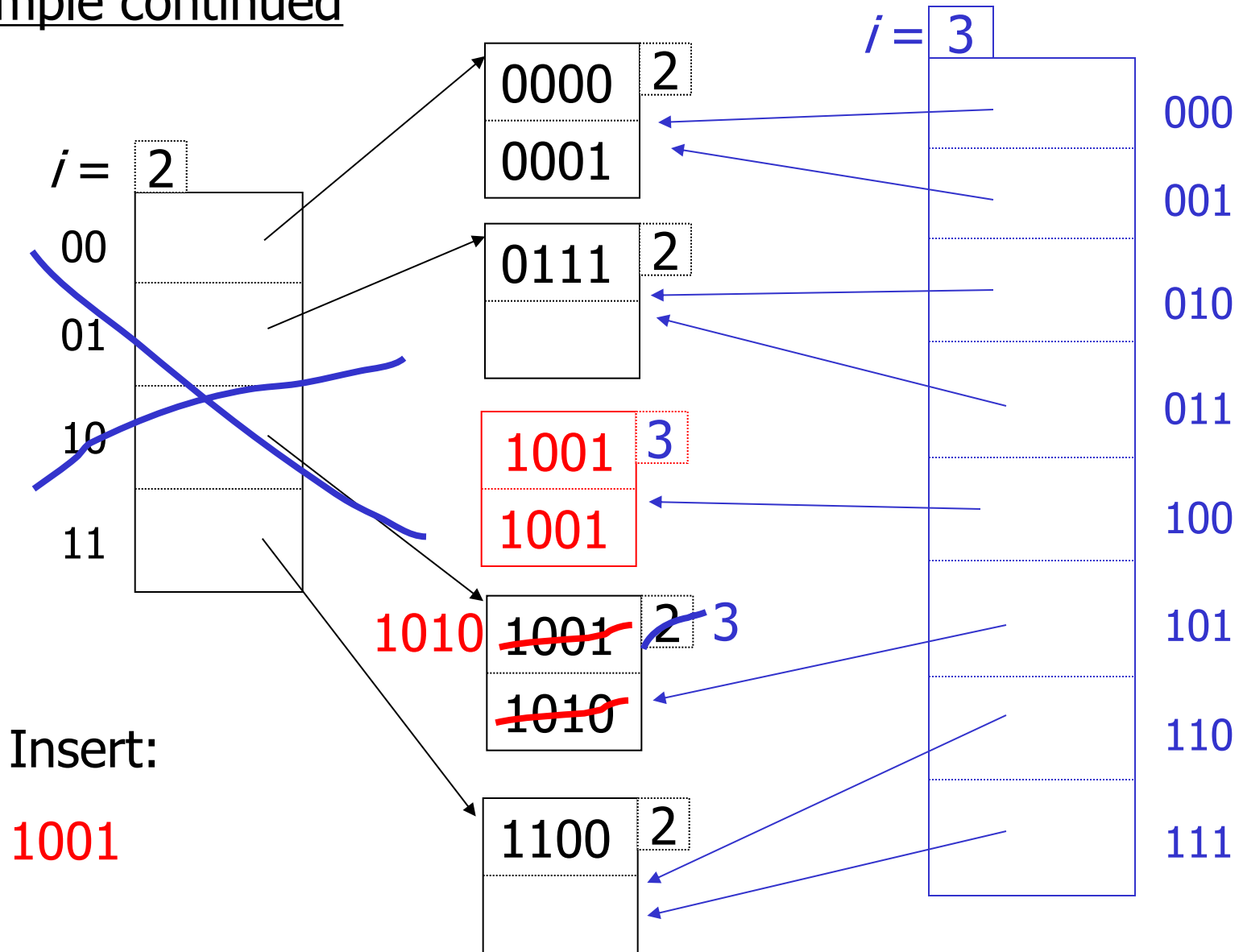
## Example continued



## Example continued



## Example continued



# Extensible hashing: deletion

- Merge blocks and cut directory if possible  
(Reverse insert procedure)
- Two blocks can be merged after a deletion if all records in these blocks can fit in a single block
- One of these two blocks can be removed and the local depth of the other block one can be decreased by 1
- If **all** local depths are strictly smaller than the global depth  $i$ , then the directory can be cut in half and the

## Deletion example:

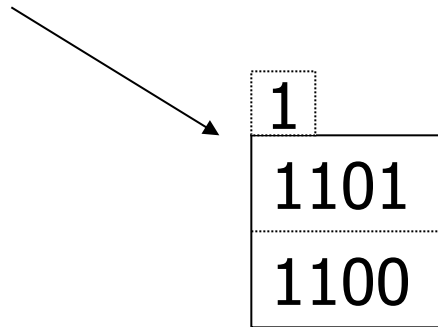
- Run through insert example in reverse!



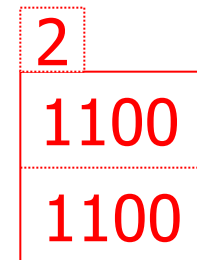
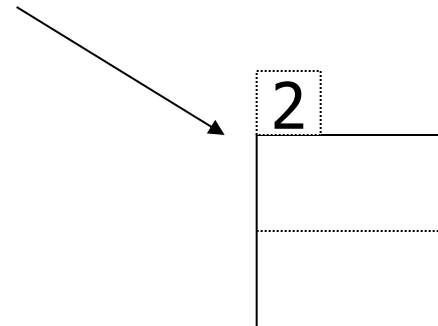
# Note: Still need overflow chains

- Example: many records with duplicate keys

insert 1100

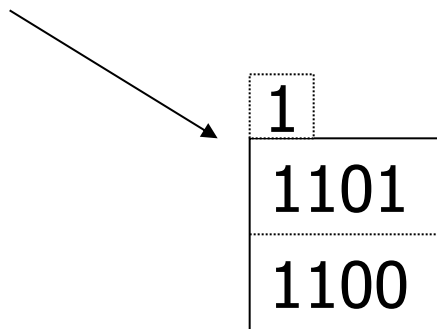


if we split:



# Solution: overflow chains

insert 1100



add overflow block:

