Homework 3

ISyE 6420
Fall 2019

> Due September 29, 2019, 11:55pm. HW3 is not time limited except the due date. Late submissions will not be accepted.

> Use of all available electronic and printed resources is allowed except direct communication that violates Georgia Tech Academic Integrity Rules.

Estimating the Precision Parameter of a Rayleigh Distribution. If two random variables X and Y are independent of each other and normally distributed with variances equal to σ^2, then the variable $R = \sqrt{X^2 + Y^2}$ follows the Rayleigh distribution. Parameterized with precision parameter $\xi = \frac{1}{\sigma^2}$, the Rayleigh random variable R has a density

$$f(r) = \xi r \exp\left\{-\frac{\xi r^2}{2}\right\}, \quad r \geq 0, \ \xi > 0.$$

An example of such random variable would be the distance of darts from the target center in a dart-throwing game where the deviations in the two dimensions of the target plane are independent and normally distributed.

(a) Assume that the prior on ξ is exponential with the rate parameter λ. Show that the posterior is gamma $\mathcal{G}a\left(2, \lambda + \frac{r^2}{2}\right)$.

(b) Assume that $R_1 = 3, R_2 = 4, R_3 = 2$, and $R_4 = 5$ are Rayleigh-distributed random observations representing the distance of a dart from the center. Find the posterior in this case for the same prior form (a), and give a Bayesian estimate of ξ.

(c) For $\lambda = 1$, numerically find 95% Credible Set for ξ .

Hint: In (b) show that if $r_1, r_2, \ldots, r_n$ are observed, and the prior on ξ is exponential $\mathcal{E}(\lambda)$, then the posterior is gamma $\mathcal{G}a\left(n + 1, \lambda + \frac{1}{2}\sum_{i=1}^{n} r_i^2\right)$.

2. Estimating Chemotherapy Response Rates. An oncologist believes that 90% of cancer patients will respond to a new chemotherapy treatment and that it is unlikely that this proportion will be below 80%. Elicit a beta prior on proportion that models the oncologist's beliefs.

Hint: For elicitation of the prior use $\mu = 0.9$, $\mu - 2\sigma = 0.8$ and expressions for μ and σ for beta.

During a trial, in 30 patients treated, 22 responded.

(a) What are the likelihood and posterior distributions? What is the Bayes estimator of the proportion?

(b) Using Octave, R, or Python, find 95% Credible Set for p.

(c) Using Octave, R, or Python, test the hypothesis $H_0 : p \geq 2/3$ against the alternative $H_1 : p < 2/3$.

(d) Using WinBUGS, find the Bayes estimator and Credible Set and conduct the test. Compare WinBUGS results with (a-c).