ISYE 6420: Homework 3

aelhabr3

1. Metropolis: The Bounded Normal Mean.

Instructions

Response

2. Gibbs Sampler and High/Low Protein Diet in Rats.

Instructions

Response

```
library(tidyverse)
compute_mu_new <- function(mu_i, tau_i, mu_0, tau_0, y_sum, n_obs) {
   mu_tau_0 <- mu_0 * tau_0
   mu_rnorm_num <- tau_i * y_sum + mu_tau_0
   mu_rnorm_den <- tau_0 + n_obs * tau_i
   mu_rnorm <- mu_rnorm_num / mu_rnorm_den
   sigma2_rnorm <- 1 / (tau_0 + n_obs * tau_i)
   sigma_rnorm <- sqrt(sigma2_rnorm)
   rnorm(1, mu_rnorm, sigma_rnorm)
}

compute_tau_new <- function(mu_new, y, a_0, b_0, n_obs) {
   shape_rgamma <- a_0 + 0.5 * n_obs
   rate_rgamma  <- b_0 + 0.5 * sum((y - mu_new) ^ 2)
   rgamma(1, shape = shape_rgamma, rate = rate_rgamma)
}

# Constants.
n_mcmc <- 10000
n_burnin <- 500
idx_final <- (n_burnin + 1):n_mcmc

# Data.
y_1 <- c(134, 146, 104, 119, 124, 161, 107, 107, 83, 113, 129, 97, 123)
y_2 <- c(70, 118, 101, 85, 107, 132, 94)
y_1_sum <- sum(y_1)
y_2_sum <- sum(y_2)
n_1_obs <- length(y_1)
n_2_obs <- length(y_2)

# Hypterparamters.
theta_1_0 <- 110
theta_2_0 <- theta_1_0
tau_1_0 <- 1 / 100
tau_2_0 <- tau_1_0
a_1_0 <- 0.01
a_2_0 <- a_1_0
b_1_0 <- 4
b_2_0 <- b_1_0

# Initial values.
theta_1_i <- mean(y_1) # theta_1_0
theta_2_i <- mean(y_2) # theta_1_i
tau_1_i <- 1 / sd(y_1) # tau_1_0
tau_2_i <- 1 / sd(y_2) # tau_1_i

# Output.
vec_num_n <- vector(mode = 'numeric', length = n)
# theta_1 <- vec_num_n
# theta_2 <- vec_num_n
# tau_1 <- vec_num_n
```

```
# tau_2 <- vec_num_n
mat_mcmc <- matrix(nrow = n_mcmc, ncol = 4)
colnames(mat_mcmc) <- c('theta_1', 'theta_2', 'tau_1', 'tau_2')
for (i in 1:n_mcmc) {
  theta_1_new <-
    compute_mu_new(
      mu_i = theta_1_i,
      tau_i = tau_1_i,
      mu_0 = theta_1_0,
      tau_0 = tau_1_0,
      y_sum = y_1_sum,
      n_obs = n_1_obs
    )

  theta_2_new <-
    compute_mu_new(
      mu_i = theta_2_i,
      tau_i = tau_2_i,
      mu_0 = theta_2_0,
      tau_0 = tau_2_0,
      y_sum = y_2_sum,
      n_obs = n_2_obs
    )

  tau_1_new <-
    compute_tau_new(
      mu_new = theta_1_new,
      y = y_1,
      a_0 = a_1_0,
      b_0 = b_1_0,
      n_obs = n_1_obs
    )

  tau_2_new <-
    compute_tau_new(
      mu_new = theta_2_new,
      y = y_2,
      a_0 = a_2_0,
      b_0 = b_2_0,
      n_obs = n_2_obs
    )

  # theta_1[i] <- theta_1_new
  # tau_1[i] <- tau_1_new
  # theta_2[i] <- theta_2_new
  # tau_2[i] <- tau_2_new
  mat_mcmc[i, ] <- c(theta_1_new, theta_2_new, tau_1_new, tau_2_new)

  theta_1_i <- theta_1_new
  tau_1_i <- tau_1_new
  theta_2_i <- theta_2_new
  tau_2_i <- tau_2_new
```

```
  if(i == n_mcmc) {
    res <- tibble::as_tibble(mat_mcmc)
    # res <- tibble::tibble(
    #   theta_1 = theta_1,
    #   theta_2 = theta_2,
    #   tau_1 = tau_1,
    #   tau_2 = tau_2
    # )
  }
}

res_final <- res %>% dplyr::slice(idx_final)
res_final
```

```
## # A tibble: 9,500 x 4
##    theta_1 theta_2    tau_1     tau_2
##      <dbl>   <dbl>    <dbl>     <dbl>
##  1   114.9  105.9  0.002296 0.003779
##  2   114.2  109.5  0.001765 0.001548
##  3   119.9   91.39 0.003425 0.004342
##  4   116.7  108.4  0.001158 0.002037
##  5   126.4  117.4  0.001990 0.002429
##  6   115.9   99.90 0.001422 0.0004156
##  7   119.6  106.4  0.002825 0.001361
##  8   117.7  103.7  0.002988 0.004247
##  9   113.7  101.9  0.002247 0.003192
## 10   120.7   99.81 0.001591 0.001153
## # ... with 9,490 more rows
```

```
summ_final <- res_final %>% summarise_all(mean)
summ_final
```

```
## # A tibble: 1 x 4
##   theta_1 theta_2    tau_1    tau_2
##     <dbl>   <dbl>    <dbl>    <dbl>
## 1   116.6   104.7 0.002333 0.002484
```

```
summ_final %>% mutate_at(vars(matches('tau')), list(sigma = ~1/.))
```

```
## # A tibble: 1 x 6
##   theta_1 theta_2    tau_1    tau_2 tau_1_sigma tau_2_sigma
##     <dbl>   <dbl>    <dbl>    <dbl>       <dbl>       <dbl>
## 1   116.6   104.7 0.002333 0.002484       428.6       402.6
```