Introduction to Statistical Method

Estimation

An **estimator** for a population parameter θ is a statistic and donated by $\hat{\theta}$.

Any given value of $\hat{\theta}$ is an **estimate**.

Desirable Properties of a Point Estimator

- The expected value of $\hat{\theta}$ should be θ.
- $\hat{\theta}$ should have **small variance** for **large sample size**.

Bias

- The difference $\theta - E[\hat{\theta}]$ is the bias of an estimator $\hat{\theta}$ for a population parameter θ.
- $E[\hat{\theta}] = \theta$ means $\hat{\theta}$ is unbiased.

Mean Square Error of Estimator

- The **mean square error** of $\hat{\theta}$ is defined as $\mathrm{MSE}(\hat{\theta}) := E[(\hat{\theta} - \theta)^2]$.
- The mean square error measures the **overall quality of an estimator**.
- we can derive MSE in another form:

$$\begin{aligned}
\mathrm{MSE}(\hat{\theta}) &= E[(\hat{\theta} - E[\hat{\theta}] + E[\hat{\theta}] - \theta)^2] \\
&= E[(\hat{\theta} - E[\hat{\theta}])^2 + (E[\hat{\theta}] - \theta)^2 + 2(\hat{\theta} - E[\hat{\theta}])(E[\hat{\theta}] - \theta)] \\
&= Var(\hat{\theta}) + (bias)^2 + 2E[(\hat{\theta} - E[\hat{\theta}])(E[\hat{\theta}] - \theta)] \\
&= Var(\hat{\theta}) + (bias)^2 + 2E[(\hat{\theta} - E[\hat{\theta}])](E[\hat{\theta}] - \theta) \\
&= Var(\hat{\theta}) + (bias)^2 + 2(E[\hat{\theta}] - E[\hat{\theta}])(E[\hat{\theta}] - \theta) \\
&= Var(\hat{\theta}) + (bias)^2
\end{aligned}$$