VE216 Lecture 17

> Discrete Time Frequency Representations

Complex Geometric Sequences

For the DT LTI $h[n]$ with input $x[n] = z^n$, we get

$$y[n] = (h * x)[n] = \sum_{k=-\infty}^{\infty} h[k] z^{n-k} = z^n \sum_{k=-\infty}^{\infty} h[k] z^{-k} = z^n H(z)$$

So $H(z) = \sum_{k=-\infty}^{\infty} h[k] z^{-k}$, this is for DT Transform.

Remember CT Transform $H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt$

Rational System Functions

We can derive $\sum z^{-k} Y = \sum z^{-p} Z$, then we can derive the $H(z)X = Y$ here.

DT Vector Diagram

$H(z_0) = K \frac{\sum_{k=0}^{k_q} (z_0 - q_k)}{\sum_{r=0}^{r_p} (z_0 - p_r)}$, very similar to CT Vector diagrams.

$|H(z_0)| = |K| \frac{\sum_{k=0}^{k_q} |z_0 - q_k|}{\sum_{r=0}^{r_p} |z_0 - p_r|}$ and $\angle H(z_0) = \angle K + \sum_{k=0}^{k_q} \angle (z_0 - q_k) - \sum_{r=0}^{r_p} \angle (z_0 - p_r)$.

DT Frequency Response

$$\begin{aligned} x[n] = \frac{1}{2}(e^{j\Omega_0 n} + e^{-j\Omega_0 n}) \leftrightarrow y[n] &= \frac{1}{2}(H(e^{j\Omega_0}) e^{j\Omega_0 n} + H(e^{-j\Omega_0}) e^{-j\Omega_0 n}) \\ &= \mathrm{Re}\{H(e^{j\Omega_0}) e^{j\Omega_0 n}\} \\ &= \mathrm{Re}\{|H(e^{j\Omega_0})| e^{j\angle H(e^{j\Omega_0})} e^{j\Omega_0 n}\} \\ &= |H(e^{j\Omega_0})| \cos(\Omega_0 n + \angle H(e^{j\Omega_0})) \end{aligned}$$

CT DT Frequency Responses Difference

- CT Frequency Response: $H(s)$ on imaginary axis, $s = j\omega$
- DT Frequency Response: $H(z)$ on unit circle, $z = e^{j\Omega}$

DT Periodicity

Since $e^{j\Omega_2} = e^{j(\Omega_1 + 2\pi k)} = e^{j\Omega_1}$, then the "highest" DT frequency $\Omega = \pi$