VE216 Lecture 16

Fourier Series

- Analysis equation: $X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt$
- Synthesis equation: $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)x^{j\omega t} d\omega$

$E(\omega) = X(j\omega)$

Relation between Fourier and Laplace Transforms

Property	$x(t)$	$X(s)$	$X(j\omega)$
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	$aX_1(j\omega) + bX_2(j\omega)$
Time shift	$x(t - t_0)$	$e^{-st_0}X(s)$	$e^{-j\omega t_0}X(j\omega)$
Time scale	$x(at)$	$\frac{1}{|a|}X\left(\frac{s}{a}\right)$	$\frac{1}{|a|}X\left(\frac{j\omega}{a}\right)$
Differentiation	$\frac{dx(t)}{dt}$	$sX(s)$	$j\omega X(j\omega)$
Multiply by t	$tx(t)$	$-\frac{d}{ds}X(s)$	$-\frac{1}{j}\frac{d}{d\omega}X(j\omega)$
Convolution	$x_1(t) * x_2(t)$	$X_1(s) \times X_2(s)$	$X_1(j\omega) \times X_2(j\omega)$

Fourier Transform Property