VE270 Lecture 1

> Digitization: From **Analog** to **Digital**.

Signed Binary Numbers

Representation of Negative Numbers

1. Sign and Magnitude

MSB, or the left most digit is the signed bit, $0 \to$ positive, $1 \to$ negative.

2. 1's complement representation of $-N$

Negation of every bit of N.

$(3)_{10} = (0011)_2$, then $(-3)_{10} = (1100)_2$.

3. 2's complement representation of $-N$

Negation of every bit of N, then plus 1.

$(3)_{10} = (0011)_2$, then $(-3)_{10} = (1100)_2 + (1)_2 = (1101)_2$.

Signed 2's complement Number

If there are n digits, the range is from -2^{n-1} to $2^{n-1} - 1$.

Overflow

> If an n**-bit** 2's complement number is greater than $2^{n-1} - 1$ or less than -2^{n-1}, we say there is an overflow.

- a, b with sign bits same but result's sign bit is not the same **is overflow**.
- a, b with different sign bit **can't overflow**.

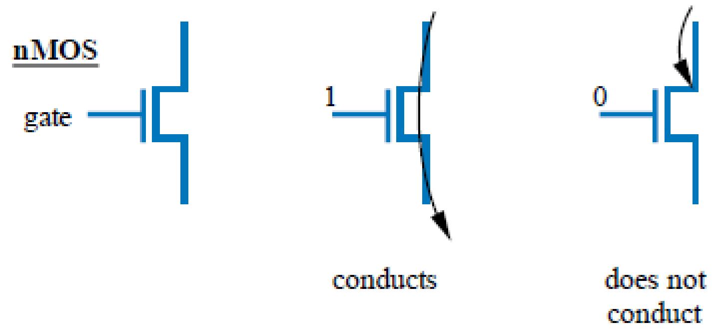
Trick

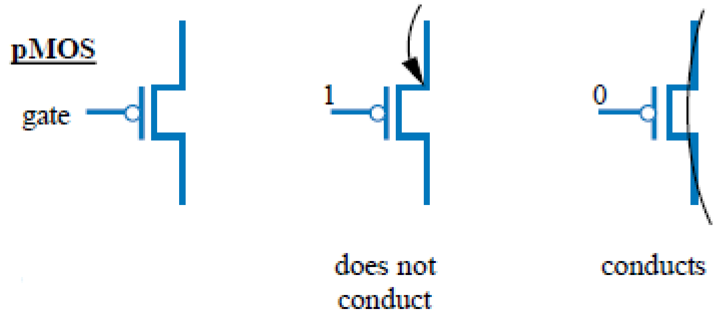
- Addition with **signed 2's complement number** should only in these situations:
  - a, b are all positive.
  - a, b are positive and negative.

  So if a, b are all negative **will certainly cause a overflow**.

VE270 Lecture 2

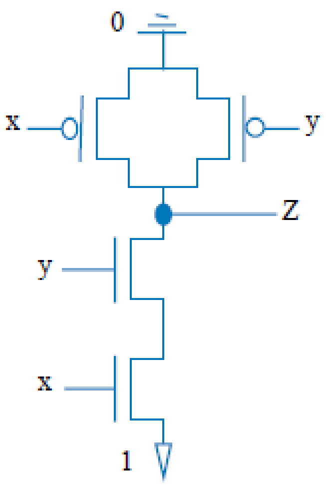
CMOS Transistor

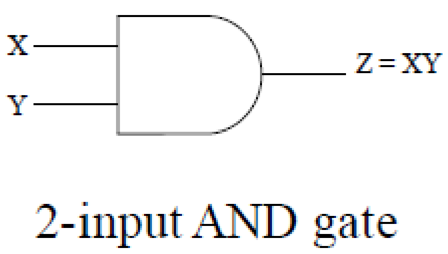
- nMOS: 

- pMOS: 

AND Logic

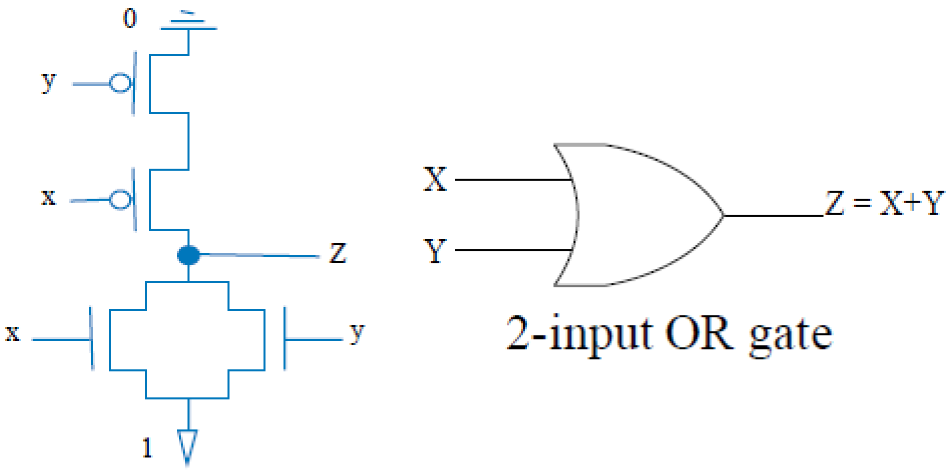
$Z = X \cdot Y$ and X, Y, Z are all **variables** with $\cdot$ as **AND operator**.





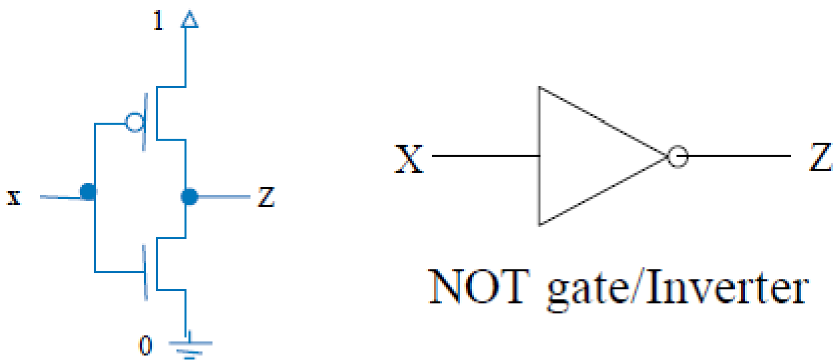
OR Logic

$Z = X + Y$ and $Z = 1$ if either $X = 1$ or $Y = 1$, or both; with $+$ as **OR operator**.

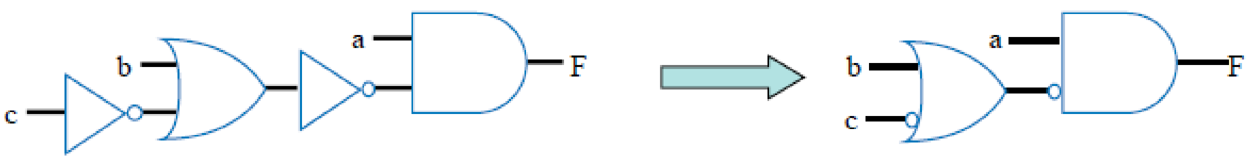


NOT Logic

$Z = X'$ or $Z = \overline{X}$ means $Z = 1 - X$ with Z and X either 0 or 1.



Logic Gates Convert

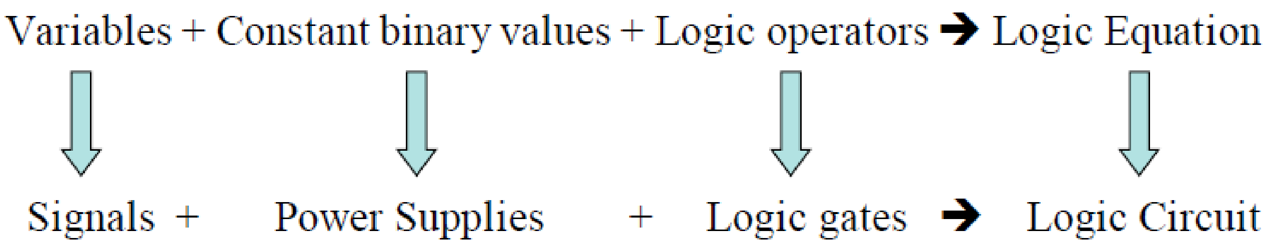


(We can omit **NOT**'s triangle with only a little circle on the diagram.)

Priority of Logic Operations

NOT > AND > OR

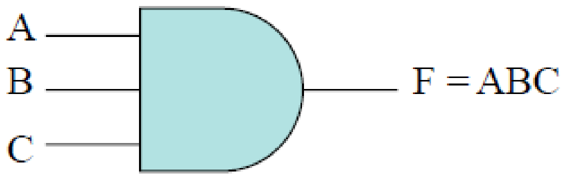
From Logic Equation to Logic Circuit



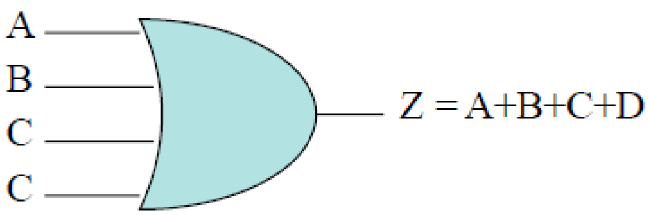
Timing Diagram

Show the response to change on a signal in voltage levels with time.

Gates with Multiple Inputs

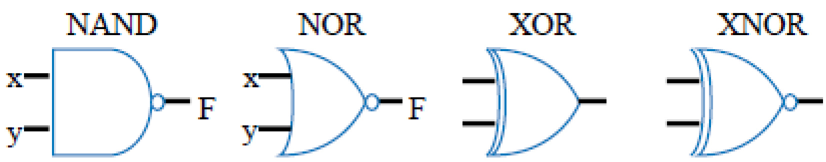


3-input AND gate



4-input OR gate

More Gates



x	y	F
0	0	1
0	1	1
1	0	1
1	1	0

x	y	F
0	0	1
0	1	0
1	0	0
1	1	0

x	y	F
0	0	0
0	1	1
1	0	1
1	1	0

x	y	F
0	0	1
0	1	0
1	0	0
1	1	1

VE270 Lecture 3

> Boolean Algebra & Optimization

Boolean Algebra Terminology

Example: $F(a, b, c) = a'bc + abc' + ab + c$

- **Variable**: Represents a value (0, 1)
  - Three variables: a, b, c
- **Literal**: Appearance of a variable, in true or complemented form

  (true form: a; complemented form: a')
  - Nine literals: a', b, c, a, b, c', a, b, c
- **Product term**: AND of literals
  - Four product terms: $a'bc$, abc', ab, c ($c = c \cdot 1$)
- **Sum term**: OR of literals
  - No sum terms

    (in $F = (a + b + c) \cdot d$, $a + b + c$ is sum term)
- **Sum of products**: Equation written as OR of product terms only
  - $F(a, b, c) = a'bc + abc' + ab + c$ is a sum of product form
  - $F(a, b, c, d) = (a + b) \cdot c + d$ is not

Basic Theorems of Boolean Algebra

Theorem1

$x + 0 = x$ and $x \cdot 0 = 0$.

x OR 0 is x, x AND 0 is 0.

Theorem2

$x + x' = 1$ and $x \cdot x' = 0$.

x OR $\neg x$ is 1, x AND $\neg x$ is 0.

Theorem3

$x + x = x$ and $x \cdot x = x$.

x OR x is only x, since $x = x$; x AND x is only x, since $x = x$.

Theorem4

$x + 1 = 1$ and $x \cdot 1 = x$.

x OR 1 is 1, x AND 1 is x.

Involution

$(x')' = x$, since $\neg(\neg x) = x$.

Commutative

$x + y = y + x$ and $x \cdot y = y \cdot x$.

x OR y is y OR x, x AND y is y AND x.

Associative

$x + (y + z) = (x + y) + z$ and $x \cdot (y \cdot z) = (x \cdot y) \cdot z$.

x OR $(y$ OR $z)$ = x OR y OR z = (x OR y) OR z,

x AND (y AND z) = (x AND y) AND z = x AND y AND z.

Distributive

$x \cdot (y + z) = x \cdot y + x \cdot z$ and $x + y \cdot z = (x + y) \cdot (x + z)$.

$x \cdot (y + z) = x \cdot y + x \cdot z$,

$$x + y \cdot z = x \cdot 1 + y \cdot z = x \cdot (1 + y) + y \cdot z$$
$$= x + x \cdot y + y \cdot z = x \cdot x + (x + z) \cdot y$$
$$= x \cdot (x + z) + (x + z) \cdot y = (x + y) \cdot (x + z)$$

Absorption

$x + x \cdot y = x$ and $x \cdot (x + y) = x$.

$x + x \cdot y = x \cdot (1 + y') + x \cdot y = x + x \cdot (y + y') = x + x = x$,

$x \cdot (x + y) = x + x \cdot y = x$.

Theorem5

$x \cdot y + x \cdot y' = x$ and $(x + y) \cdot (x + y') = x$.

$x \cdot y + x \cdot y' = x \cdot (y + y') = x$,

$(x + y) \cdot (x + y') = x + y \cdot y' + x \cdot y + x \cdot y' = x$.

Theorem6

$x + x' \cdot y = x + y$ and $x \cdot (x' + y) = x \cdot y$.

$x + x' \cdot y = x \cdot (1 + y) + x' \cdot y = x + (x' + x) \cdot y = x + y$,

$x \cdot (x' + y) = x \cdot y + 0 = x \cdot y$.

Application of Basic Theorems

- $(a + b) \cdot (c + b) \cdot (d' + b) \cdot (a \cdot c \cdot d' + e)$
  $= (a \cdot c \cdot d' + b) \cdot (a \cdot c \cdot d' + e)$
  $= (a \cdot c \cdot d' + b \cdot e)$
- $w \cdot x' \cdot y' + w \cdot x \cdot z' + w \cdot x' \cdot y \cdot z'$
  $= w \cdot x' \cdot (y' + y \cdot z') + w \cdot x \cdot z'$
  $= w \cdot x' \cdot (y' + z') + w \cdot x \cdot z'$
  $= w \cdot z' + w \cdot x' \cdot y'$

De Morgan's Law

$(x + y)' = x' \cdot y'$

$(x \cdot y)' = x' + y'$

Consensus Theorem

$x \cdot y + x' \cdot z + y \cdot z = x \cdot y + x' \cdot z$

$x \cdot y + x' \cdot z + y \cdot z$
$= x \cdot y + x' \cdot z + (x + x') \cdot y \cdot z$
$= x \cdot y + x' \cdot z + x \cdot y \cdot z + x' \cdot y \cdot z$
$= x \cdot y \cdot (1 + z) + x' \cdot z \cdot (1 + y)$
$= x \cdot y + x' \cdot z$

$$a'b' + ac + bc' + b'c + ab = a'b' + ac + bc'$$

XOR Properties

Theorem1

$x \oplus 0 = x, x \oplus 1 = x'$

Theorem2

$x \oplus x = 0, x \oplus x' = 1$

Theorem3

$x \oplus y' = x' \oplus y = (x \oplus y)'$

Commutative

$x \oplus y = y \oplus x$

Associative

$(x \oplus y) \oplus z = x \oplus (y \oplus z) = x \oplus y \oplus z$

Boolean Representation: Minterm and Maxterm

Minterm

A product of n literals in which each literal appears exactly once in either true or complemented form, but not both.

Represented by m_i.

Maxterm

A sum of n literals in which each literal appears exactly once in either true or complemented form, but not both.

Represented by M_i.

Subscription Example

x	y	z	Minterms Term	Minterms Designation	Maxterms Term	Maxterms Designation
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

From Truth Table to Minterm Expression

x con1	y con2	z con3	F result
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

Result would happen if con1 is **false** AND con2 is **false** AND con3 is **true**, **x'y'z**

Result would happen if con1 is **false** AND con2 is **true** AND con3 is **true**, **x'yz**

Result would happen if con1 is **true** AND con2 is **false** AND con3 is **false**, **xy'z'**

Result would happen if con1 is **true** AND con2 is **false** AND con3 is **true**, **xy'z**

Result would be true if any of these four conditions is true, implies OR logic, This relationship is expressed by:
F = x'y'z + x'yz + xy'z' + xy'z

- A Boolean Equation can be derived from a truth table and expressed as a sum-of-minterms (standard-sum-of-products).
- Each minterm corresponds to a 1 in the truth table in the sum-of-minterms expression.
- $F = x'y'z + x'yz + xy'z + xy'z = m_1 + m_3 + m_4 + m_5 = \sum m(1,3,4,5)$

Minterms and Maxterms Conversion

Complement of Minterm is corresponding Maxterm

$m_i = M_i$

e.g.: $m_0 = x'y'z', m_0' = (x'y'z')' = x + y + z = M_0$

Conversion between Standard Forms

The term numbers **missing from one form** will be the term number **used in the other form**.

e.g.: if all the terms are indexed by 0 ~ 7, then $F = \sum m(1, 2, 4, 7) = \prod M(0, 3, 5, 6)$.

another example:

Truth Table

x	y	z	F1
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Sum-of-minterms

$F1 = x'y'z + xy'z' + xy'z$
$\quad xyz' + xyz$

$F1 = m_1 + m_4 + m_5 + m_6 + m_7$

$F1 = \Sigma\,(1, 4, 5, 6, 7)$

Product-of-maxterms

$F1 = (x+y+z) \bullet (x+y'+z) \bullet$
$\quad (x+y'+z')$

$F1 = M_0 \bullet M_2 \bullet M_3$

$F1 = \Pi\,(0, 2, 3)$

Incompletely Specified Functions

In a circuit, some input conditions may never happen, then the output is not completely specified.

The corresponding output is set as `"x"`, meaning **don't care**.

The **don't care** output could be either 0 or 1.

$F = \sum m(1, 3, 4) + \sum d(2, 5)$:

x	y	z	F
0	0	0	0
0	0	1	1
0	1	0	X
0	1	1	1
1	0	0	1
1	0	1	X
1	1	0	0
1	1	1	0

Simplified Forms

The minterm and maxterm forms can be further simplified:

- contain less number of terms.
- have less literals.