VE270 Lecture 4 Logic Optimization

Simplification and Optimization

Care for two things:

- Delay (Gate delay)
- Size (transistors)

For quick estimation, assume:

- every gate need a "1 **gate-delay**"
- every **gate input** require **2 transistors**
- ignore **inverters** for simplicity

Two-level Logic Optimization

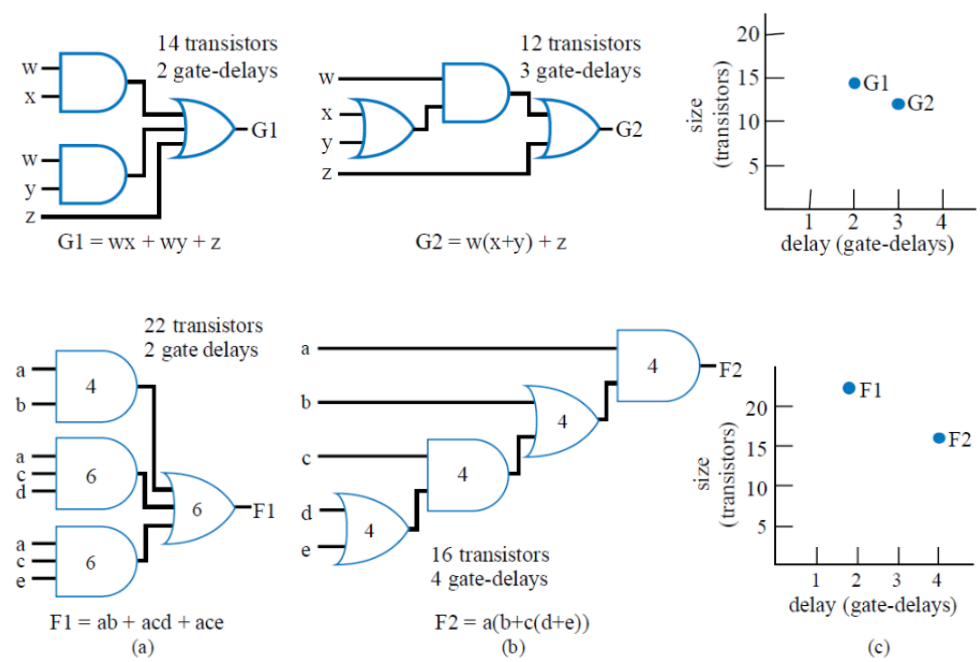
We use **algebraic method**.

Goal: we change circuit into two levels (AND-OR network) with minimum transistors.

Sum-of-Products yields two levels. (e.g. $F = abc + a'b'c'$)

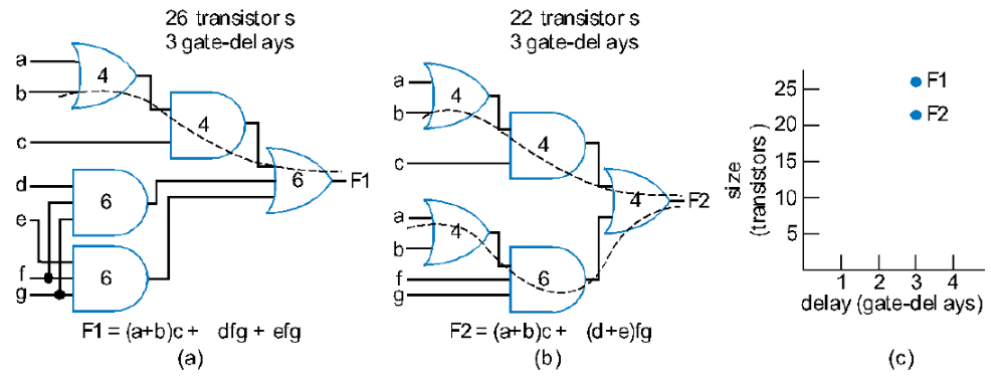
Multi-level Logic Optimization

tradeoff exists, we improve some, worsen some.



Critical Path

The **longest delay path** for an input to output in the circuit.



Optimization:

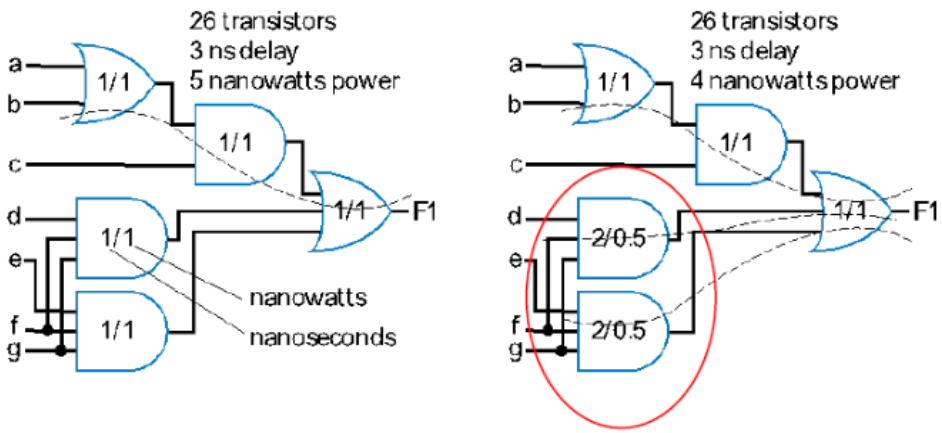
- Reduce overall delay by shortening length of **critical path**
- Reduce the size by using **multi-level** on non-critical path
    - but may make **non-critical path** become **critical path**.

Power Optimization

CMOS power consuming: $P = K \cdot CV^2 \cdot f$

- K constant
- C capacitance of wire
- V voltage
- f switching frequency

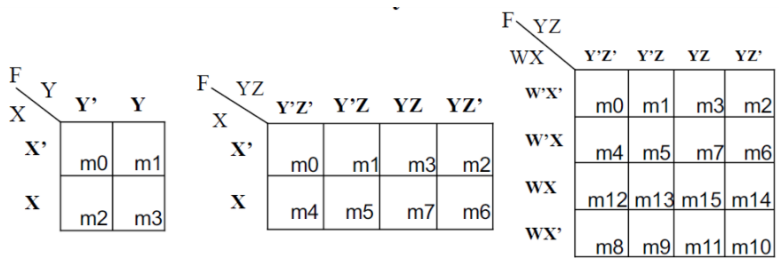
Low-Power Gate on Non-critical Path



- Fast/high-power
- Slow/low-power

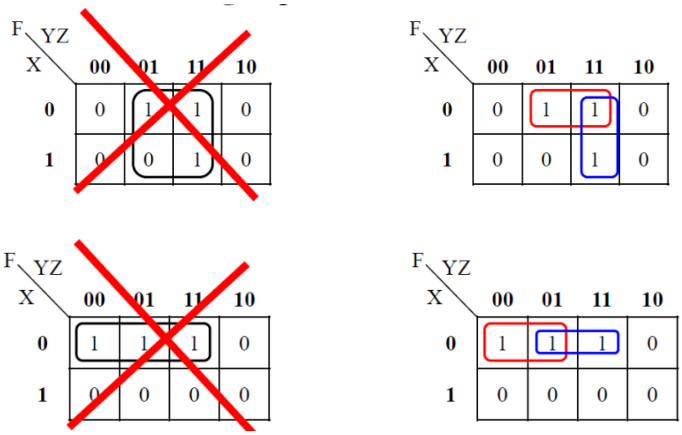
So use slow/low-power gates on non-critical path reduces power without increase delay.

K-Map Logic Optimization

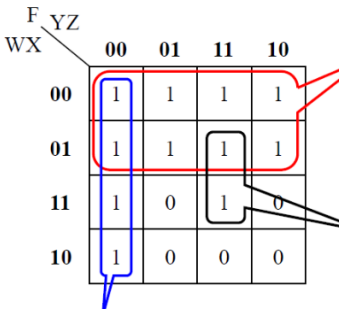


Simplifying: Grouping and Canceling

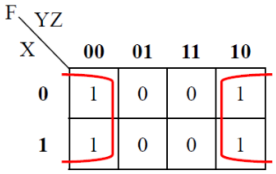
- Group is in shape of rectangle or square
- Group the adjacent 1's until all the 1's are grouped
- The number of 1's in the group should be $2^N, N = 0, 1, 2, \cdots$
- No zeros in group



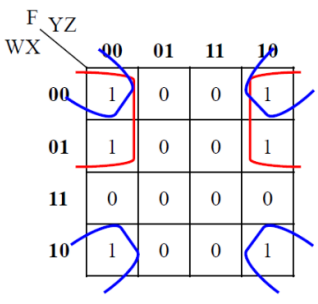
- Group as much adjacent 1's as possible



- Edges wrap around



F = Z'



F = W'Z' + X'Z'

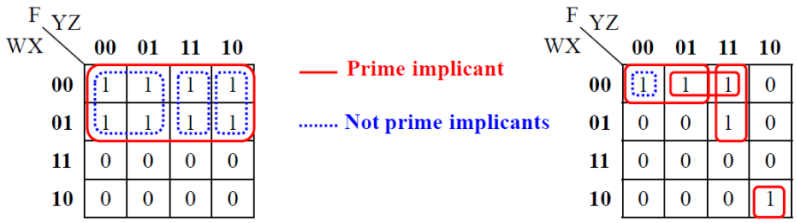
Group Patterns

- If a letter with both primed and unprimed form happen in a group, the letter canceled.
- The simplified result will be a sum-of-product form, the number of product terms is decided by the number of the groups.
- A group of all cells gives logic 1.
- A group of 2^n cells gives a term of $N-n$ literals. 2^N is total cell number.

Prime Implicants

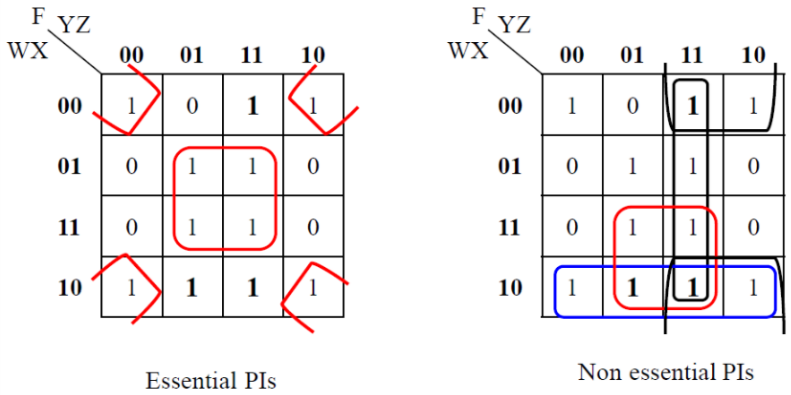
Implicant: a product term

Prime Implicant (PI): a group that cannot entirely contained by another implicant.



Essential Prime Implicants

Essential Prime Implicant: if the PI contain a cell can **ONLY** covered by this PI.



Essential PIs

Non essential PIs

Alternate Method: Product-of-Sum Simplification

- Redraw K-map for F' (switching 1's and 0's)
- then with DeMorgan's Law to change the F' sum-of-product into product-of-sum

Simplify any standard sum-of-product form

Method 1: fill out table directly

F = A'C + A'BD + AB'C + BCD

A	B	C	D	F	
0	0	0	0	0	m0
0	0	0	1	0	m1
0	0	1	0	1	m2
0	0	1	1	1	m3
0	1	0	0	0	m4
0	1	0	1	1	m5
0	1	1	0	1	m6
0	1	1	1	1	m7
1	0	0	0	0	m8
1	0	0	1	0	m9
1	0	1	0	1	m10
1	0	1	1	1	m11
1	1	0	0	0	m12
1	1	0	1	0	m13
1	1	1	0	0	m14
1	1	1	1	1	m15

F: AB \ CD	00	01	11	10
00	0	0	1	1
01	0	1	1	1
11	0	0	1	0
10	0	0	1	1

Method 2: convert all forms into sum-of-minterm

$$F = A'C + A'BD + AB'C + BCD = \sum m(2, 3, 5, 6, 7, 10, 11, 15)$$

Then put into K-map for simplification.

Don't Care Conditions

Construct Essential Prime Implicants with "Don't Care Conditions" if needed.

Then we apply same K-map simplification method.

Seven-Segment Display

Each LED has one anode (+) and one cathode (-).

Anode high-level-voltage, Cathode low-level-voltage, LED light on.

On Nexys2, we have seven segment display with common anode.