A Proofs

A.1 Proof of Theorem 3.3

Proof. Problem (reg-LSDWPTSVM$''^k$) is equivalent to the following QP problem:

$$\min_{\boldsymbol{v}_k \in \mathbb{R}_{m_l+l}} \frac{1}{2}\boldsymbol{v}_k^T\left(\mathbf{E}_k\mathbf{E}_k^T + \delta_k\mathbf{M}_n\right)\boldsymbol{v}_k + \frac{C_k}{2}\|\mathbf{1}_{N_{\text{-}k}} - \mathbf{E}_{\text{-}k}^T\boldsymbol{v}_k\|_2^2. \tag{29}$$

Since $\mathbf{P}_k = \mathbf{E}_k\mathbf{E}_k^T + \delta_k\mathbf{M}_n + \mathbf{E}_{\text{-}k}\mathbf{E}_{\text{-}k}^T \succ 0$, (29) is a strictly convex QP problem and it has a unique solution $\boldsymbol{v}_k^* = C_k\mathbf{P}_k^{-1}\mathbf{E}_{\text{-}k}\mathbf{1}_{N_{\text{-}k}}$. Let $\boldsymbol{\xi}^{k^*} = \mathbf{1}_{N_{\text{-}k}} - \mathbf{E}_{\text{-}k}^T\boldsymbol{v}_k^* = \left(\mathbf{I}_{N_{\text{-}k}} - C_k\mathbf{E}_{\text{-}k}^T\mathbf{P}_k^{-1}\mathbf{E}_{\text{-}k}\right)\mathbf{1}_{N_{\text{-}k}}$, then $(\boldsymbol{v}_k^*, \boldsymbol{\xi}^{k^*})$ solves the problem (reg-LSDWPTSVM$''^k$).

□

A.2 Proof of Theorem 3.4

Recall the Sherman-Morrison-Woodbury formula as the following:

Lemma A.1 (Sherman-Morrison-Woodbury formula)**.** Given an invertible matrix $\mathbf{A} \in \mathbb{R}^{m\times m}$, $\mathbf{U} \in \mathbb{R}^{m\times n}$ and $\mathbf{V} \in \mathbb{R}^{n\times m}$, and let $\mathbf{B} = \mathbf{A} + \mathbf{UV}$. If matrix $\mathbf{I}_n + \mathbf{V}\mathbf{A}^{-1}\mathbf{U}$ is invertible, then $\mathbf{B}$ is invertible and

$$\mathbf{B} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{U}\left(\mathbf{I}_n + \mathbf{V}\mathbf{A}^{-1}\mathbf{U}\right)^{-1}\mathbf{V}\mathbf{A}^{-1}.$$

Then we provide the proof of Theorem 3.4 as follows.

Proof. Notice that, $\epsilon\mathbf{I}_{m_l+l} + \mathbf{Q}_k\mathbf{Q}_k^T$ is positive definite. Denote the optimal solution to the problem (reg-LSDWPTSVM$''^k_\epsilon$) as $(\boldsymbol{v}_k^*, \boldsymbol{\xi}^{k^*})$, by Theorem 3.3

$$\begin{aligned} \boldsymbol{v}_k^* &= C_k\left(\epsilon\mathbf{I}_{m_l+l} + \mathbf{Q}_k\mathbf{Q}_k^T\right)^{-1}\mathbf{E}_{\text{-}k}\mathbf{1}_{N_{\text{-}k}} \\ \boldsymbol{\xi}^{k^*} &= \left(\mathbf{I}_{N_{\text{-}k}} - C_k\mathbf{E}_{\text{-}k}^T\left(\epsilon\mathbf{I}_{m_l+l} + \mathbf{Q}_k\mathbf{Q}_k^T\right)^{-1}\mathbf{E}_{\text{-}k}\right)\mathbf{1}_{N_{\text{-}k}} \end{aligned} \tag{30}$$

By Lemma A.1,

$$\begin{aligned} \left(\epsilon\mathbf{I}_{m_l+l} + \mathbf{Q}_k\mathbf{Q}_k^T\right)^{-1} &= \frac{1}{\epsilon}\mathbf{I}_{m_l+l} - \frac{1}{\epsilon^2}\mathbf{Q}_k(\mathbf{I}_{N_k} + \frac{1}{\epsilon}\mathbf{Q}_k^T\mathbf{Q}_k)^{-1}\mathbf{Q}_k^T \\ &= \frac{1}{\epsilon}\left(\mathbf{I}_{m_l+l} - \mathbf{Q}_k(\epsilon\mathbf{I}_{N_k} + \mathbf{Q}_k^T\mathbf{Q}_k)^{-1}\mathbf{Q}_k^T\right). \end{aligned}$$

which yields (27).

□