This is a help document for loop analysis method. As we know: There are three kinds of branches in distribution network. Constant PQ branches whose current is considered as knowns in the circuit analysis belong to *2*. The other branches are voltage source branches belong to *Set 3*. The branch with constant impedance belong to *Set 1*

There are two kinds of loop, one loop is from PQ load to the voltage source belongs to *Set l1* There are two kinds of loop, one loop is not from PQ load to the voltage source belongs to *Set l2*

$$B_{l1,1}V_{b,1,re} + B_{l1,2}V_{b,2,re} + B_{l1,3}V_{b,3,re} = 0 \tag{1}$$

$$B_{l2,1}V_{b,1,re} + B_{l2,2}V_{b,2,re} + B_{l2,3}V_{b,3,re} = 0 \tag{2}$$

$$B_{l1,1}V_{b,1,im} + B_{l1,2}V_{b,2,im} + B_{l1,3}V_{b,3,im} = 0 \tag{3}$$

$$B_{l2,1}V_{b,1,im} + B_{l2,2}V_{b,2,im} + B_{l2,3}V_{b,3,im} = 0 \tag{4}$$

$$\begin{bmatrix} B_{l1,1} & B_{l1,2} \\ B_{l2,1} & B_{l2,2} \end{bmatrix} \begin{bmatrix} V_{b,1,re} \\ V_{b,2,re} \end{bmatrix} = \begin{bmatrix} -B_{l1,3}V_{b,3,re} \\ -B_{l2,3}V_{b,3,re} \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} B_{l1,1} & B_{l1,2} \\ B_{l2,1} & B_{l2,2} \end{bmatrix} \begin{bmatrix} V_{b,1,im} \\ V_{b,2,im} \end{bmatrix} = \begin{bmatrix} -B_{l1,3}V_{b,3,im} \\ -B_{l2,3}V_{b,3,im} \end{bmatrix} \tag{6}$$

$$I_{b,1,re} = B_{l1,1}^T I_{l1,re} + B_{l2,1}^T I_{l2,re} \tag{7}$$

$$I_{b,1,im} = B_{l1,1}^T I_{l1,im} + B_{l2,1}^T I_{l2,im} \tag{8}$$

$$I_{b,2,re} = B_{l1,2}^T I_{l1,re} + B_{l2,2}^T I_{l2,re} \tag{9}$$

$$I_{b,2,im} = B_{l1,2}^T I_{l1,im} + B_{l2,2}^T I_{l2,im} \tag{10}$$

$$\begin{bmatrix} I_{b,1,re} \\ I_{b,2,re} \end{bmatrix} = \begin{bmatrix} B_{l1,1}^T & B_{l2,1}^T \\ B_{l1,2}^T & B_{l2,2}^T \end{bmatrix} \begin{bmatrix} I_{l1,re} \\ I_{l2,re} \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} I_{b,1,im} \\ I_{b,2,im} \end{bmatrix} = \begin{bmatrix} B_{l1,1}^T & B_{l2,1}^T \\ B_{l1,2}^T & B_{l2,2}^T \end{bmatrix} \begin{bmatrix} I_{l1,im} \\ I_{l2,im} \end{bmatrix} \tag{12}$$

$$(r_b + jx_b)(I_{b,1,re} + jI_{b,1,im}) = V_{b,1,re} + jV_{b,1,im} \tag{13}$$

$$\begin{bmatrix} r_b & -x_b \\ x_b & r_b \end{bmatrix} \begin{bmatrix} I_{b,1,re} \\ I_{b,1,im} \end{bmatrix} = \begin{bmatrix} V_{b,1,re} \\ V_{b,1,im} \end{bmatrix} \tag{14}$$

We can derived that:

$$\begin{bmatrix} r_b & -x_b \\ x_b & r_b \end{bmatrix} \begin{bmatrix} B_{l1,1}^T I_{l1,re} + B_{l2,1}^T I_{l2,re} \\ B_{l1,1}^T I_{l1,im} + B_{l2,1}^T I_{l2,im} \end{bmatrix} = \begin{bmatrix} V_{b,1,re} \\ V_{b,1,im} \end{bmatrix} \tag{15}$$

both side multiplied by the equation:

$$\begin{bmatrix} B_{l2,1} & 0 \\ 0 & B_{l2,1} \end{bmatrix}$$

For PQ branch type 2, has no elements in $B_{l2,2}$,so $B_{l2,2} = 0$ $B_{l2,1}V_{b,1,re} + 0 + B_{l2,3}V_{b,3,re} = 0$.

$$\begin{bmatrix} B_{l2,1}r_b & -B_{l2,1}x_b \\ B_{l2,1}x_b & B_{l2,1}r_b \end{bmatrix} \begin{bmatrix} B_{l1,1}^T I_{l1,re} + B_{l2,1}^T I_{l2,re} \\ B_{l1,1}^T I_{l1,im} + B_{l2,1}^T I_{l2,im} \end{bmatrix} = \begin{bmatrix} B_{l2,1}V_{b,1,re} \\ B_{l2,1}V_{b,1,im} \end{bmatrix} \tag{16}$$

$$\begin{aligned} &B_{l2,1}r_b B_{l1,1}^T I_{l1,re} + B_{l2,1}r_b B_{l2,1}^T I_{l2,re} \\ &- B_{l2,1}x_b B_{l1,1}^T I_{l1,im} - B_{l2,1}x_b B_{l2,1}^T I_{l2,im} \\ &= -B_{l2,3}V_{b,3,re} \end{aligned} \tag{17}$$

$$\begin{aligned} &B_{l2,1}x_b B_{l1,1}^T I_{l1,re} + B_{l2,1}x_b B_{l2,1}^T I_{l2,re} \\ &+ B_{l2,1}r_b B_{l1,1}^T I_{l1,im} + B_{l2,1}r_b B_{l2,1}^T I_{l2,im} \\ &= -B_{l2,3}V_{b,3,im} \end{aligned} \tag{18}$$

And then we can calculate the $I_{l1,re}$,$I_{l1,im}$ with $I_{l2,re}, I_{l2,im}$ as known.

$$\begin{aligned} &B_{l2,1}r_b B_{l2,1}^T I_{l2,re} - B_{l2,1}x_b B_{l2,1}^T I_{l2,im} \\ &= -B_{l2,3}V_{b,3,re} - B_{l2,1}r_b B_{l1,1}^T I_{l1,re} + B_{l2,1}x_b B_{l1,1}^T I_{l1,im} \end{aligned} \tag{19}$$

$$\begin{aligned} &B_{l2,1}x_b B_{l2,1}^T I_{l2,re} + B_{l2,1}r_b B_{l2,1}^T I_{l2,im} \\ &= -B_{l2,3}V_{b,3,im} - B_{l2,1}x_b B_{l1,1}^T I_{l1,re} - B_{l2,1}r_b B_{l1,1}^T I_{l1,im} \end{aligned} \tag{20}$$

Suppose that the branch direction of PQ load is from load point to the ground, Suppose that the voltage source direction is from ground to grid connected node.
$B_{l1,1}$ is path-branch matrix,the tree branch direction is from source node to load node.
loop 1 direction is from load point to ground. The matrix. $B_{l2,3}$ can be zero.
$B_{l2,1}$ should form according to the loop. the loop 2 direction is the same with the loop edge.