Contents

$$\begin{bmatrix} A_1^T \\ A_2^T \end{bmatrix} V_{n,re} = \begin{bmatrix} V_{b,1,re} \\ V_{b,2,re} \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} A_1^T \\ A_2^T \end{bmatrix} V_{n,im} = \begin{bmatrix} V_{b,1,im} \\ V_{b,2,im} \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} A_1 & A_2 \end{bmatrix} \begin{bmatrix} I_{b,1,re} \\ I_{b,2,re} \end{bmatrix} = 0 \tag{3}$$

sparse tabular analysis:

$$\begin{bmatrix} A_1^T & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ A_2^T & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 & 0 \\ 0 & A_1^T & 0 & -I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_2^T & 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_1 & 0 & A_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & A_1 & 0 & A_2 \\ 0 & 0 & G & -B & 0 & 0 & -I & 0 & 0 & 0 \\ 0 & 0 & B & G & 0 & 0 & 0 & -I & 0 & 0 \end{bmatrix} \begin{bmatrix} U_{n,re} \\ U_{n,im} \\ U_{b,1,re} \\ U_{b,1,im} \\ U_{b,2,re} \\ U_{b,2,im} \\ I_{b,1,re} \\ I_{b,1,im} \\ I_{b,2,re} \\ I_{b,2,im} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4}$$

$$\begin{aligned} AI_{b,re} &= 0 \\ AI_{b,im} &= 0 \end{aligned} \tag{5}$$

branch constraint

$$(G + jB)(V_{b,re} + jV_{b,im}) = I_{b,re} + jI_{b,im} \tag{6}$$

$$GV_{b,re} - BV_{b,im} = I_{b,re} \tag{7}$$

$$GV_{b,im} + BV_{b,re} = I_{b,im} \tag{8}$$

$$\begin{bmatrix} G & -B \\ B & G \end{bmatrix} \begin{bmatrix} V_{b,1,re} \\ V_{b,1,im} \end{bmatrix} = \begin{bmatrix} I_{b,1,re} \\ I_{b,1,im} \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} G & -B \\ B & G \end{bmatrix} \begin{bmatrix} A_1^T V_{n,re} \\ A_1^T V_{n,im} \end{bmatrix} = \begin{bmatrix} I_{b,1,re} \\ I_{b,1,im} \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix} \begin{bmatrix} G & -B \\ B & G \end{bmatrix} \begin{bmatrix} A_1^T V_{n,re} \\ A_1^T V_{n,im} \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix} \begin{bmatrix} I_{b,1,re} \\ I_{b,1,im} \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} A_1 G A_1^T & -A_1 B A_1^T \\ A_1 B A_1^T & A_1 G A_1^T \end{bmatrix} \begin{bmatrix} V_{n,re} \\ V_{n,im} \end{bmatrix} = - \begin{bmatrix} A_2 I_{b,2,re} \\ A_2 I_{b,2,im} \end{bmatrix} \tag{12}$$

The linear jacobian:

$$\begin{bmatrix} A_1 G A_1^T & -A_1 B A_1^T \\ A_1 B A_1^T & A_1 G A_1^T \end{bmatrix} \begin{bmatrix} V_{n,re} - V_{n,re}^0 \\ V_{n,im} - V_{n,im}^0 \end{bmatrix} = - \begin{bmatrix} A_2 (I_{b,2,re} - I_{b,2,re}^0) \\ A_2 (I_{b,2,im} - I_{b,2,im}^0) \end{bmatrix} \tag{13}$$

$$f(V_{n,re}, V_{n,im}, I_{b,2,re}, I_{b,2,im}) = 0 \tag{14}$$

The deviration:

$$f = f_0 + \frac{\partial f}{\partial V_{n,re}} (V_{n,re} - V_{n,re}^0) \tag{15}$$

$$\begin{bmatrix} \frac{\partial f}{\partial V_{n,re}} & \frac{\partial f}{\partial V_{n,im}} & \frac{\partial f}{\partial I_{b,2,re}} & \frac{\partial f}{\partial V_{b,2,im}} \end{bmatrix} \begin{bmatrix} V_{n,re} - V_{n,re}^0 \\ V_{n,im} - V_{n,im}^0 \\ I_{b,2,re} - I_{b,2,re}^0 \\ I_{b,2,im} - I_{b,2,im}^0 \end{bmatrix} = -f_0 \tag{16}$$

The total equation:

$$\begin{bmatrix} A_1 G A_1^T & -A_1 B A_1^T & A_2 & 0 \\ A_1 B A_1^T & A_1 G A_1^T & 0 & A_2 \\ \frac{\partial f}{\partial V_{n,re}} & \frac{\partial f}{\partial V_{n,im}} & \frac{\partial f}{\partial I_{b,2,re}} & \frac{\partial f}{\partial V_{b,2,im}} \end{bmatrix} \begin{bmatrix} V_{n,re} - V_{n,re}^0 \\ V_{n,im} - V_{n,im}^0 \\ I_{b,2,re} - I_{b,2,re}^0 \\ I_{b,2,im} - I_{b,2,im}^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -f_0 \end{bmatrix} \tag{17}$$