

# Contents

$$\begin{bmatrix} A_1^T \\ A_2^T \end{bmatrix} V_{n,re} = \begin{bmatrix} V_{b,1,re} \\ V_{b,2,re} \end{bmatrix} \quad (1)$$

$$\begin{bmatrix} A_1^T \\ A_2^T \end{bmatrix} V_{n,im} = \begin{bmatrix} V_{b,1,im} \\ V_{b,2,im} \end{bmatrix} \quad (2)$$

$$\begin{bmatrix} A_1 & A_2 \end{bmatrix} \begin{bmatrix} I_{b,1,re} \\ I_{b,2,re} \end{bmatrix} = 0 \quad (3)$$

$$\begin{aligned} AI_{b,re} &= 0 \\ AI_{b,im} &= 0 \end{aligned} \quad (4)$$

branch constraint

$$(G + jB)(V_{b,re} + jV_{b,im}) = I_{b,re} + jI_{b,im} \quad (5)$$

$$GV_{b,re} - BV_{b,im} = I_{b,re} \quad (6)$$

$$GV_{b,im} + BV_{b,re} = I_{b,im} \quad (7)$$

$$\begin{bmatrix} G & -B \\ B & G \end{bmatrix} \begin{bmatrix} V_{b,1,re} \\ V_{b,1,im} \end{bmatrix} = \begin{bmatrix} I_{b,1,re} \\ I_{b,1,im} \end{bmatrix} \quad (8)$$

$$\begin{bmatrix} G & -B \\ B & G \end{bmatrix} \begin{bmatrix} A_1^T V_{n,re} \\ A_1^T V_{n,im} \end{bmatrix} = \begin{bmatrix} I_{b,1,re} \\ I_{b,1,im} \end{bmatrix} \quad (9)$$

$$\begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix} \begin{bmatrix} G & -B \\ B & G \end{bmatrix} \begin{bmatrix} A_1^T V_{n,re} \\ A_1^T V_{n,im} \end{bmatrix} = \begin{bmatrix} A_1 & 0 \\ 0 & A_1 \end{bmatrix} \begin{bmatrix} I_{b,1,re} \\ I_{b,1,im} \end{bmatrix} \quad (10)$$

$$\begin{bmatrix} A_1 G A_1^T & -A_1 B A_1^T \\ A_1 B A_1^T & A_1 G A_1^T \end{bmatrix} \begin{bmatrix} V_{n,re} \\ V_{n,im} \end{bmatrix} = - \begin{bmatrix} A_2 I_{b,2,re} \\ A_2 I_{b,2,im} \end{bmatrix} \quad (11)$$

The linear jacobian:

$$\begin{bmatrix} A_1 G A_1^T & -A_1 B A_1^T \\ A_1 B A_1^T & A_1 G A_1^T \end{bmatrix} \begin{bmatrix} V_{n,re} - V_{n,re}^0 \\ V_{n,im} - V_{n,im}^0 \end{bmatrix} = - \begin{bmatrix} A_2 (I_{b,2,re} - I_{b,2,re}^0) \\ A_2 (I_{b,2,im} - I_{b,2,im}^0) \end{bmatrix} \quad (12)$$

$$f(V_{n,re}, V_{n,im}, I_{b,2,re}, I_{b,2,im}) = 0 \quad (13)$$

The deviration:

$$f = f_0 + \frac{\partial f}{\partial V_{n,re}}(V_{n,re} - V_{n,re}^0) \quad (14)$$

$$\begin{bmatrix} \frac{\partial f}{\partial V_{n,re}} & \frac{\partial f}{\partial V_{n,im}} & \frac{\partial f}{\partial I_{b,2,re}} & \frac{\partial f}{\partial I_{b,2,im}} \end{bmatrix} \begin{bmatrix} V_{n,re} - V_{n,re}^0 \\ V_{n,im} - V_{n,im}^0 \\ I_{b,2,re} - I_{b,2,re}^0 \\ I_{b,2,im} - I_{b,2,im}^0 \end{bmatrix} = -f_0 \quad (15)$$

The total equation:

$$\begin{bmatrix} A_1 G A_1^T & -A_1 B A_1^T & A_2 & 0 \\ A_1 B A_1^T & A_1 G A_1^T & 0 & A_2 \\ \frac{\partial f}{\partial V_{n,re}} & \frac{\partial f}{\partial V_{n,im}} & \frac{\partial f}{\partial I_{b,2,re}} & \frac{\partial f}{\partial I_{b,2,im}} \end{bmatrix} \begin{bmatrix} V_{n,re} - V_{n,re}^0 \\ V_{n,im} - V_{n,im}^0 \\ I_{b,2,re} - I_{b,2,re}^0 \\ I_{b,2,im} - I_{b,2,im}^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -f_0 \end{bmatrix} \quad (16)$$