CORPORATE FINANCE: MT 2024 — EQUATIONS

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1 Basic maths

$$\rho(\tilde{C}, r_M) = \frac{Cov(\tilde{C}, r_M)}{\sqrt{\sigma^2_{\tilde{c}}\,\sigma^2_{r_M}}} = \frac{Cov(\tilde{C}, r_M)}{\sigma_{\tilde{c}}\sigma_{r_M}}$$

2 NPV

$$\begin{aligned} \text{NPV} &= \Delta(\text{firm value}) - \text{Financing cost} - \text{Required capital} \\ &= (V_1 - V_0) - [V^N - (I - K)] - I \end{aligned}$$

In a competitive market, financing cost $= 0$, so:

$$\text{NPV}_{PC} = (V_1 - V_0) - I$$

3 Perpetuity

$$V_p = \frac{x + V_p}{1 + R}$$
$$\frac{x}{R}$$

4 CAPM

Discount the unadjusted $\tilde{C}$s at their required return:

$$\begin{aligned} \text{Est Returns}: &\quad \tilde{R} = \frac{\tilde{C} - I}{I} = \frac{\tilde{C}}{I} - 1 \\ \text{CAPM}: &\quad \mathbb{E}R = r_F + \beta(\mu_M - r_F) \\ &\qquad\qquad \text{where} \quad \beta = \frac{Cov(\tilde{R}, r_M)}{\sigma^2_M} = \frac{Cov(\tilde{C}, r_M)}{I * \sigma^2_M} \\ \text{PV}: &\quad PV(\tilde{C}) = \sum_{t=1}^{n} \frac{\mathbb{E}\tilde{C}_t}{(1+R)^t} \\ \text{NPV}: &\quad NPV = PV - 1 \end{aligned}$$

Gives the right decision but a wrong valuation.

5 CE-CAPM

Discount adjusted $\tilde{C}$s at the r_F:

$$\text{Returns}: \quad \tilde{R} = \frac{\tilde{C} - PV(\tilde{C})}{PV(\tilde{C})} = \frac{\tilde{C}}{PV(\tilde{C})} - 1$$
$$\mathbb{E}\tilde{R} = \frac{\mathbb{E}\tilde{C}}{PV(\tilde{C})} - 1$$

For CE-CAPM, substitute $\mathbb{E}\tilde{R}$ into CAPM and $\tilde{R}$ into β and rearrange:

$$PV(\tilde{C}) = \frac{\mathbb{E}\tilde{C} - \lambda\, Cov(\tilde{C}, \tilde{r}_M)}{1 + r_F}, \quad \text{where} \quad \lambda = \frac{\mu_M - r_F}{\sigma^2_M}$$
$$NPV = \sum_{i=1}^{n} PV(\tilde{C}) - I$$