DAMA50 - 1st assignment

Exercise 7

In [5]:
```
m = matrix(QQ, 3, 4, lambda i,j: i + j - 1); show(m)
```
Out[5]:
$$\begin{pmatrix} -1 & 0 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

In [6]:
```
m.rref()
```
Out[6]:
```
[ 1  0 -1 -2]
[ 0  1  2  3]
[ 0  0  0  0]
```

In [7]:
```
m.rank()
```
Out[7]:
```
2
```

In [8]:
```
n = matrix(QQ, 3, 4, lambda i,j: (-1)^j); show(n)
```
Out[8]:
$$\begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}$$

In [9]:
```
n.rref()
```
Out[9]:
```
[ 1 -1  1 -1]
[ 0  0  0  0]
[ 0  0  0  0]
```

In [10]:
```
n.rank()
```
Out[10]:
```
1
```

Exercise 8

A.

In [23]:
```
var('p q r s λ');
λ = 0;
a1 = p + 2*q + 3*r + s == 1;
a2 = 2*p + q + 8*r + 2*s == 2;
a3 = p + 6*q -3*r + 5*s == -2;
a4 = 2*p - q + r - s == λ;
solve([a1,a2,a3,a4],p,q,r,s,)
```
Out[23]:
```
[[p == (-18/65), q == (17/65), r == (51/130), s == (-11/26)]]
```

B.

In [17]:
```
K = matrix(QQ, [[1,2,3,1],[2,1,8,2],[1,6,-3,5],[2,-1,1,-1]]); show(K)
```

Out[17]:

$$\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 1 & 8 & 2 \\ 1 & 6 & -3 & 5 \\ 2 & -1 & 1 & -1 \end{pmatrix}$$

In [20]:
```
K.inverse()*vector([1,2,-2,0])
```

Out[20]:
```
(-18/65, 17/65, 51/130, -11/26)
```

C.

In [24]:
```
var('p q r s λ');
a1 = p + 2*q + 3*r + s == 1;
a2 = 2*p + q + 8*r + 2*s == 2;
a3 = p + 6*q -3*r + 5*s == -2;
a4 = 2*p - q + r - s == λ;
solve([a1,a2,a3,a4],p,q,r,s,)
```

Out[24]:
```
[[p == 31/65*λ - 18/65, q == -4/65*λ + 17/65, r == -6/65*λ + 51/130, s == -1/13*λ - 11/26]]
```

D.

In [21]:
```
K.augment(vector([1,2,-2,0]))
```

Out[21]:
```
[ 1  2  3  1  1]
[ 2  1  8  2  2]
[ 1  6 -3  5 -2]
[ 2 -1  1 -1  0]
```

In [22]:
```
K.augment(vector([1,2,-2,0])).echelon_form()
```

Out[22]:
```
[      1       0       0       0  -18/65]
[      0       1       0       0   17/65]
[      0       0       1       0  51/130]
[      0       0       0       1  -11/26]
```

Exercise 9

$$\vec{A}x = \vec{b} \Rightarrow \begin{pmatrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \Rightarrow$$

$$\Rightarrow \left(\begin{array}{cccccc|c} 0 & 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & -1 & 1 \end{array}\right) \Rightarrow \quad R_3 = R_3 - R_1$$

$$\Rightarrow \left(\begin{array}{cccccc|c} 0 & 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 \end{array}\right) \quad R_2 = R_2 + R_3 \Rightarrow$$

$$\Rightarrow \left(\begin{array}{cccccc|c} 0 & 1 & 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 \end{array}\right) \quad R_1 = R_2 + R_3 \Rightarrow R_3 = -1R_3$$

$$\Rightarrow \left(\begin{array}{cccccc|c} 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 & 1 & -1 & 1 \end{array}\right) \Rightarrow \begin{aligned} x_2 + x_6 &= 1 \\ x_4 + x_6 &= -2 \\ x_5 - x_6 &= 1 \end{aligned}$$

For $x_6 = r$ we have

$x_2 = 1 - r$, $x_4 = -2 - r$, $x_5 = 1 + r$, $x_6 = r$

Exercise 10

$$\begin{bmatrix} 2 & 1 & 3 \\ -1 & 1 & -3 \\ 3 & -2 & 8 \end{bmatrix}$$

$$2\begin{vmatrix} 1 & -3 \\ -2 & 8 \end{vmatrix} - 1\begin{vmatrix} -1 & -3 \\ 3 & 8 \end{vmatrix} + 3\begin{vmatrix} -1 & 1 \\ 3 & -2 \end{vmatrix}$$

$$= 2\left(8\cdot 1-(-2)\cdot(-3)\right) - 1\left((-1)\cdot 8-(-3)\cdot 3\right) + 3\left((-1)\cdot(-2)-3\cdot 1\right)$$

$$= 2\cdot 2 - 1\cdot 1 + 3\cdot(-1) = 4 - 1 - 3 = 0$$

Since the determinant equals to zero the vectors are linearly independent

Quiz

Q1

In [1]:
```
C = matrix([[1,1,0,-1],[1,-1,1,0],[1,0,-1,1]]);
show(C)
```

Out[1]:

$$\begin{pmatrix} 1 & 1 & 0 & -1 \\ 1 & -1 & 1 & 0 \\ 1 & 0 & -1 & 1 \end{pmatrix}$$

In [2]:
```
C.rank()
```

Out[2]: 3

Q2

In [4]:
```
A = matrix([[1,3],[5,1]]); show(A)
```
Out[4]:
$$\begin{pmatrix} 1 & 3 \\ 5 & 1 \end{pmatrix}$$

In [6]:
```
show(A^(-1)*A.transpose())
```
Out[6]:
$$\begin{pmatrix} \frac{4}{7} & -\frac{1}{7} \\ \frac{1}{7} & \frac{12}{7} \end{pmatrix}$$

Q4

In [16]:
```
x1 = vector([1,-3,4]);
x2 = vector([-2,1,-1]);
x3 = vector([1,1,0]);
a = vector((1/2)*(x1 + 2*x2 -x3));
a
```
Out[16]: (-2, -1, 1)

Q6

In [7]:
```
A = matrix([[1,2],[3,4]]);show(A)
```
Out[7]:
$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

In [9]:
```
det(A^7)
```
Out[9]: -128