

## Problem 1

$$(i) \quad A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -1 & 1 \end{pmatrix} \quad \vec{U} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

$$A^T A = \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}$$

$$\det(A^T A) = 3 \cdot 3 - (-1) \cdot (-1) = 9 - 1 = 8$$

↙  
Invertible

$$(A^T A)^{-1} = \frac{1}{8} \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

$$p = A(A^T A)^{-1} A^T U = \begin{pmatrix} 1 & 1 \\ 1 & -1 \\ -1 & 1 \end{pmatrix} \frac{1}{8} \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$$

$$= \frac{1}{8} \begin{pmatrix} 3+1 & 1+3 \\ 3-1 & 1-3 \\ -3+1 & -1+3 \end{pmatrix} \begin{pmatrix} 2+2-1 \\ 2-2+1 \end{pmatrix} = \frac{1}{8} \begin{pmatrix} 4 & 4 \\ 2 & -2 \\ -2 & 2 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$= \frac{1}{8} \begin{pmatrix} 12+4 \\ 6-2 \\ -6+2 \end{pmatrix} = \frac{1}{8} \begin{pmatrix} 16 \\ 4 \\ -4 \end{pmatrix} = \begin{pmatrix} 2 \\ 1/2 \\ -1/2 \end{pmatrix}$$

Page 2

(ii)

$$\vec{e} = \vec{u} - \vec{p} = (2, 2, 1) - (2, 1/2, -1/2)$$

$$= (0, 3/2, 3/2)$$

$$\vec{e} \cdot \vec{a} = (0, 3/2, 3/2) \cdot (1, 1, -1)$$

$$= 0 + 3/2 - 3/2 = 0$$

$$\vec{e} \cdot \vec{b} = (0, 3/2, 3/2) \cdot (1, -1, 1)$$

$$= 0 - 3/2 + 3/2 = 0$$

Problem 2

(i) For  $-3 \leq x \leq -2$

$$\int_{-3}^{-2} dx x (cx + 3) = \left[ c \frac{x^4}{4} + \frac{3x^2}{2} \right]_{-3}^{-2} =$$

$$= \left[ c \frac{16 - 81}{4} + \frac{3(4 - 9)}{2} \right]$$

$$= \left( \frac{-65c}{4} + \frac{-15}{2} \right) = 0 \Rightarrow c = \frac{15}{2} \cdot \frac{4}{65} \Rightarrow$$

$$\Rightarrow c = \frac{60}{130} \Rightarrow c = \frac{6}{13}$$



For  $2 \leq x \leq 3$

$$\int_2^3 2x(3-x) = \left[ \frac{3x^2}{2} - \frac{x^3}{3} \right]_2^3$$

$$= \left[ \frac{3(9-4)}{2} - \left( \frac{27-8}{3} \right) \right]$$

$$= \frac{15}{2} - \left( \frac{19}{3} \right) = 0 \Rightarrow c = -\frac{15}{2} \cdot \frac{3}{19}$$

$$\Rightarrow c = -\frac{45}{38}$$

⑥ The calculations I made for  $c$  are not correct so I can't calculate the 6 sub-question.

The cdf will be calculated by replacing it in the function and add-up as the  $x$  increases. For  $x < -3$ ,  $x > 3$  and  $-2 < x < 2$  the function will be zero. For other values it will take a number. It will be of the type:

$$f = \begin{cases} 0 & x < -3 \\ \text{number} & -3 \leq x \leq -2 \\ 0 & -2 < x < 2 \\ \text{number} & 2 \leq x \leq 3 \\ 0 & 3 < x \end{cases}$$