DAMA 50 WRITTEN ASSIGNMENT 2

Problem 7

a. Vectors a and b are orthogonal.

```
In [38]: # 7.a

In [40]: A = [vector([1,1,2]), vector([1,-1,0]), vector([1,0,4])]
         from sage.modules.misc import gram_schmidt
         B, mu = gram_schmidt(A)

In [41]: A

Out[41]: [(1, 1, 2), (1, -1, 0), (1, 0, 4)]

In [42]: A[0]*A[1]

Out[42]: 0

In [43]: A[0]*A[2]

Out[43]: 9

In [44]: A[1]*A[2]

Out[44]: 1

In [45]: mu

Out[45]: [  0   0   0]
         [  0   0   0]
         [3/2 1/2   0]
```

b.

```
In [8]: # 7.b

In [3]: a = vector([1,1,2])
        a1 = a.normalized()
        a1

Out[3]: (1/6*sqrt(6), 1/6*sqrt(6), 1/3*sqrt(6))

In [4]: b = vector([1,-1,0])
        b1 = b.normalized()
        b1

Out[4]: (1/2*sqrt(2), -1/2*sqrt(2), 0)

In [5]: c = vector([1,0,4])
        c1 = c.normalized()
        c1

Out[5]: (1/17*sqrt(17), 0, 4/17*sqrt(17))
```

DAMA 50 WRITTEN ASSIGNMENT 2

c.

In [6]:
```
# 7.c
```

In [7]:
```
V = QQ^5
d = vector([1,1,1])
B = [a1, b1, c1, d]
C = column_matrix(B).augment(identity_matrix(3), subdivide = True)
C
C.rref()
```

Out[7]:
```
[            1             0             0   7/6*sqrt(6)|  2/3*sqrt(6)   2/3*sqrt(6)  -1/6*sqrt(6)]
[            0             1             0   1/6*sqrt(2)|  2/3*sqrt(2)  -1/3*sqrt(2)  -1/6*sqrt(2)]
[            0             0             1 -1/3*sqrt(17)|-1/3*sqrt(17) -1/3*sqrt(17)  1/3*sqrt(17)]
```

In [62]:
```
C.pivots()
```

Out[62]:
```
(0, 1, 2)
```

Problem 8

a.

In [ ]:
```
# 8.a
```

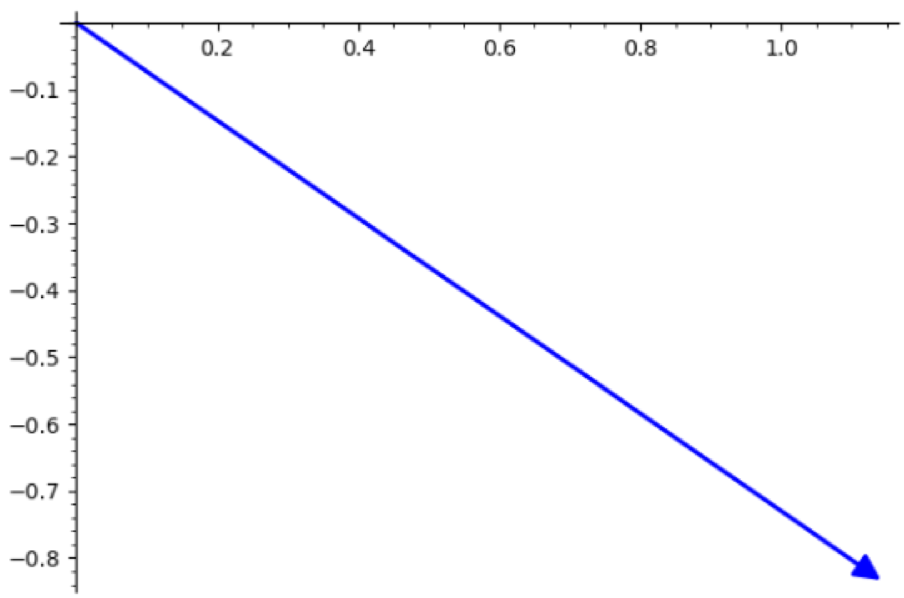
In [54]:
```
def vector_rotating(a, b):
    v = vector([a[0]*cos(b) - a[1]*sin(b), a[0]*sin(b) + a[1]*cos(b)])
    return v
```

b.

In [55]:
```
# 8.b
```

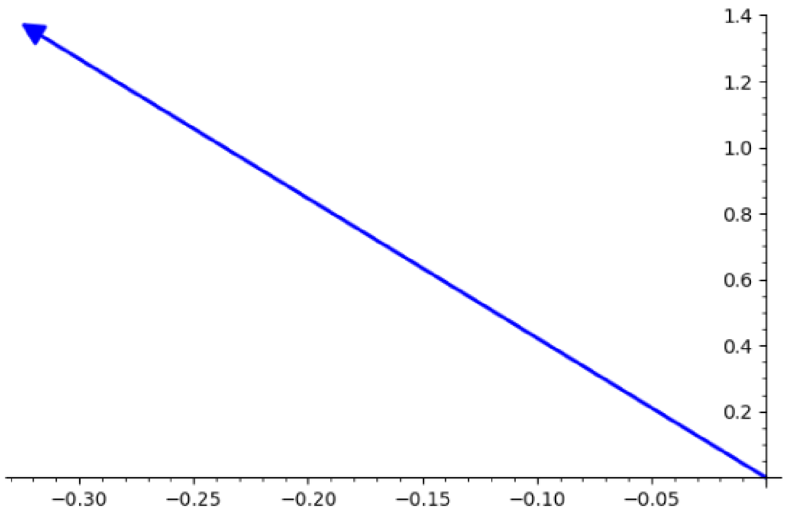
In [56]:
```
a = vector([1,1])
a1 = vector_rotating(a, 30)
plot(a1)
```

Out[56]:



DAMA 50 WRITTEN ASSIGNMENT 2

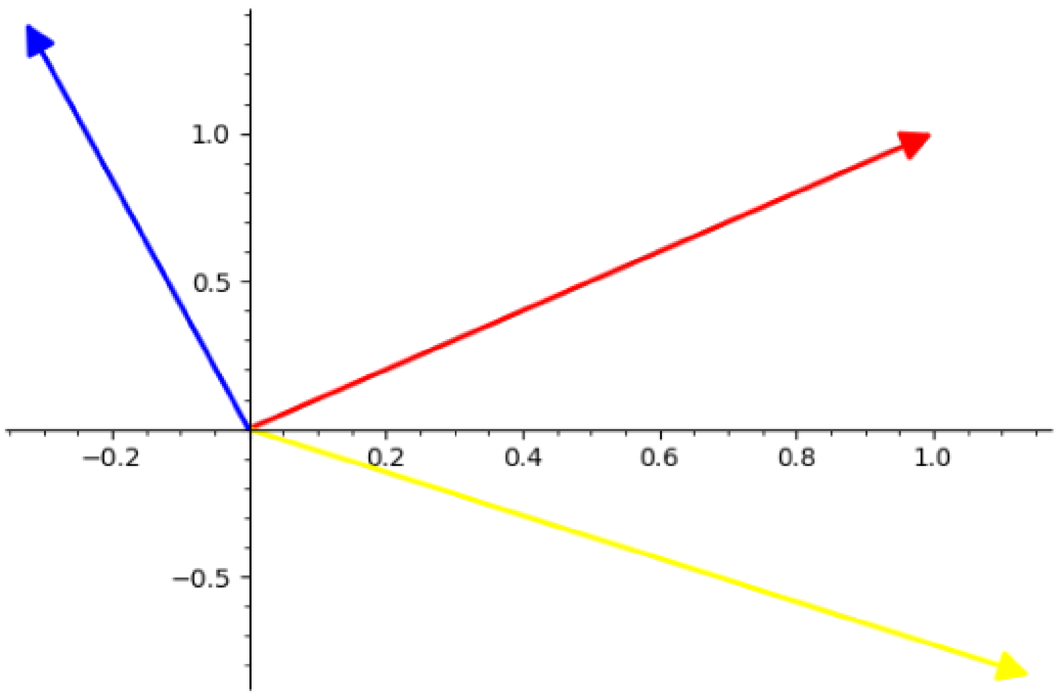
In [57]:
```
a2 = vector_rotating(a, 45)
plot(a2)
```
Out[57]:



c.

In [58]:
```
# 8.c
```

In [59]:
```
plot(a, color='red') + plot(a1, color='yellow') + plot(a2, color='blue')
```
Out[59]:



d.

```
In [60]: # 8.d

In [81]: i = vector(a)
         j = vector(a2)
         dp = i.dot_product(j)
         s = dp/(norm(i) * norm(j))
         cos(s)

Out[81]: cos(sqrt(2)*cos(45)/sqrt((cos(45) + sin(45))^2 + (cos(45) - sin(45))^2))
```

Problem 9

9

$$\vec{x}^T A \vec{x} = [x_1\ x_2\ x_3]\, A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = [x_1\ x_2\ x_3] \begin{bmatrix} 2x_1 - 1x_2 + 0 \\ -x_1 + 2x_2 - x_3 \\ 0 - x_2 + 2x_3 \end{bmatrix}$$

$$= 2x_1^2 - x_1x_2 + 0 - x_1x_2 + 2x_2^2 - x_2x_3 + 0 - x_2x_3 + 2x_3^2$$

$$= 2x_1^2 - 2x_1x_2 + 2x_2^2 - 2x_2x_3 + 2x_3^2$$

The matrix is symmetric but not positive definite because there can be x values that can make it be less than zero. So it is not positive semi-definite either.

Problem 10

DAMA 50 WRITTEN ASSIGNMENT 2

10

(a) $P_\pi = \dfrac{bb^T}{b^Tb} = \dfrac{1}{41}\begin{bmatrix}3\\4\\4\end{bmatrix}\begin{bmatrix}3 & 4 & 4\end{bmatrix} = \dfrac{1}{41}\begin{bmatrix}9 & 12 & 12\\12 & 16 & 16\\12 & 16 & 16\end{bmatrix}$

$$\pi_U(a) = P_\pi a = \frac{1}{41}\begin{bmatrix}9 & 12 & 12\\12 & 16 & 16\\12 & 16 & 16\end{bmatrix}\begin{bmatrix}2\\2\\1\end{bmatrix} = \frac{1}{41}\begin{bmatrix}18+24+24\\24+32+32\\12+16+16\end{bmatrix} =$$

$$= \frac{1}{41}\begin{bmatrix}66\\88\\44\end{bmatrix} \in span\left[\begin{bmatrix}2\\2\\1\end{bmatrix}\right]$$

(b) $U = span\left[\begin{bmatrix}2\\2\\1\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix}\right]$

We turn that to a matrix $B = \begin{bmatrix}2 & 1\\2 & 0\\1 & 0\end{bmatrix}$

$$B^TB = \begin{bmatrix}2 & 2 & 1\\1 & 0 & 0\end{bmatrix}\begin{bmatrix}2 & 1\\2 & 0\\1 & 0\end{bmatrix} = \begin{bmatrix}4+4+1 & 2\\2 & 1\end{bmatrix} = \begin{bmatrix}9 & 2\\2 & 1\end{bmatrix}$$

$$B^Tb = \begin{bmatrix}2 & 2 & 1\\1 & 0 & 0\end{bmatrix}\begin{bmatrix}3\\4\\4\end{bmatrix} = \begin{bmatrix}6+8+4\\3\end{bmatrix} = \begin{bmatrix}18\\3\end{bmatrix}$$

$$\begin{bmatrix}9 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}\lambda_1\\\lambda_2\end{bmatrix} = \begin{bmatrix}18\\3\end{bmatrix} \Rightarrow \begin{aligned}9\lambda_1 + 2\lambda_2 &= 18\\ -2(2\lambda_1 + \lambda_2 &= 3)\\ \hline 5\lambda_1 = 12 \Rightarrow \lambda_1 &= \frac{12}{5},\ \lambda_2 = -\frac{9}{5}\end{aligned}$$

$$\lambda = \begin{bmatrix}12/5\\-9/5\end{bmatrix}$$

$$\pi_U(b) = B\lambda = \begin{bmatrix}2 & 1\\2 & 0\\1 & 0\end{bmatrix}\begin{bmatrix}12/5\\-9/5\end{bmatrix} = \begin{bmatrix}33/5\\24/5\\12/5\end{bmatrix}$$

DAMA 50 WRITTEN ASSIGNMENT 2

(c) (i) $|n_u(\vec{b}) - \vec{\alpha}| = \frac{1}{41}\begin{bmatrix}66\\88\\44\end{bmatrix} - \begin{bmatrix}2\\2\\1\end{bmatrix} = \frac{1}{41}\begin{bmatrix}-16\\6\\3\end{bmatrix}$

$\langle |n_u(\vec{b}) - \vec{\alpha}|, \vec{\alpha} \rangle = \frac{1}{41}\begin{bmatrix}-16\\6\\3\end{bmatrix}\begin{bmatrix}2\\2\\1\end{bmatrix} = -\frac{32}{41} + \frac{12}{41} + \frac{3}{41} = -\frac{17}{41}$

(ii) $\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & 2 & 1\\ 1 & 0 & 0\end{vmatrix} \Rightarrow \vec{i}\begin{vmatrix}2 & 0\\ 1 & 0\end{vmatrix} - \vec{j}\begin{vmatrix}2 & 1\\ 1 & 0\end{vmatrix} + \vec{k}\begin{vmatrix}2 & 2\\ 1 & 0\end{vmatrix} =$

$= \vec{i}(2\cdot 0 - 1\cdot 0) - \vec{j}(2\cdot 0 - 1) + \vec{k}(2\cdot 0 - 2)$

$= \vec{j} - 2\vec{k} \Rightarrow \vec{v} = (0, 1, -2)$

i. The product is not 0 so they're not perpendicular.

ii. The vector perpendicular to both $\vec{\alpha}$ and $\vec{c}$ that we found is not equal to $\vec{b}$ so they are also not perpendicular.

(d) $P_n = B(B^TB)^{-1}B^T = \begin{bmatrix}2 & 1\\ 2 & 0\\ 1 & 0\end{bmatrix}\left(\begin{bmatrix}9 & 2\\ 2 & 1\end{bmatrix}\right)^{-1}\begin{bmatrix}2 & 2 & 1\\ 1 & 0 & 0\end{bmatrix}$

$= \begin{bmatrix}2 & 1\\ 2 & 0\\ 1 & 0\end{bmatrix}\begin{bmatrix}1 & -2\\ -2 & 9\end{bmatrix}\begin{bmatrix}2 & 2 & 1\\ 1 & 0 & 0\end{bmatrix} = \begin{bmatrix}0 & 5\\ 2 & -4\\ 1 & -2\end{bmatrix}\begin{bmatrix}2 & 2 & 1\\ 1 & 0 & 0\end{bmatrix}$

$= \begin{bmatrix}5 & 0 & 0\\ 0 & 4 & 2\\ 0 & 2 & 1\end{bmatrix}$