Distributed Opportunistic Scheduling For Ad-Hoc Communications: An Optimal Stopping Approach

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Abstract

Due to the distributed nature of ad-hoc communications, channel-aware distributed scheduling is very challenging in wireless ad-hoc networks. In this paper, we study distributed opportunistic scheduling (DOS) for an ad-hoc network model, where many links contend for the same channel using random access. In such a network, DOS involves a process of joint channel probing and distributed scheduling. Due to channel fading, the link condition corresponding to a successful channel probing could be either good or poor. In the latter case, further channel probing, although at the cost of additional delay, may lead to better channel conditions and hence yield higher throughput. The desired tradeoff boils down to judiciously choosing the optimal stopping rule for channel probing and distributed scheduling. In this study, we pursue a rigorous characterization of the optimal strategies from two perspectives, namely, a network-centric perspective and a user-centric perspective.

We first consider DOS from a network-centric point of view, where links cooperate to maximize the overall network throughput. Using optimal stopping theory, we show that for a homogenous network, the optimal scheme for distributed opportunistic scheduling turns out to be a *pure threshold policy*, where the rate threshold can be obtained by solving a fixed point equation. We then generalize the above study to the heterogeneous case. Somewhat surprisingly, the optimal rate threshold turns out to be the same across all links regardless of the channel statistics and transmission probabilities. We further devise iterative algorithms for computing the threshold. We also generalize the studies to take into account fairness requirements.

Next, we explore distributed opportunistic scheduling from a user-centric perspective, where each link seeks to maximize its own throughput in a selfish manner. We treat the problem of threshold selection across different links as a non-cooperative game. To this end, we first characterize each user's individual throughput and introduce the notion of "effective channel probing time". We explore the existence and uniqueness of the Nash equilibrium, and show that the Nash equilibrium can be approached by the best response strategy. Since the best response strategy requires global information, we then develop an online stochastic iterative algorithm based on local observations only, and establish its convergence to the Nash equilibrium under some regularity conditions by using recent results on asynchronous stochastic approximation algorithms. Since there is an efficiency loss at the Nash equilibrium, we then study pricing-based mechanisms to mitigate the loss.

Our results reveal that rich PHY/MAC diversities are available for exploitation in ad-hoc networks. We believe that these initial steps open a new avenue for channel-aware distributed scheduling.

Index Terms: Distributed Opportunistic Scheduling, Game Theory, Ad-Hoc Networks, Optimal Stopping, Threshold Policy

1 Introduction

1.1 Motivation

Wireless ad hoc networks have emerged as a promising solution that can facilitate communications between wireless devices without a planned fixed infrastructure. Different from its wireline counterpart, the design of wireless ad hoc networks faces a number of unique challenges in wireless communications, including 1) co-channel interference among active links in a neighborhood; and 2) time varying channel conditions over fading channels. The traditional wisdom for wireless network design is to separate link losses caused by fading from those by interference. That is, the PHY layer addresses the issues of fading, and the MAC layer addresses the issue of contention. However, as shown in [2] [13], fading can often adversely affect the MAC layer in many realistic scenarios. The coupling between the timescales of fading and MAC calls for a unified PHY/MAC design for wireless ad-hoc networks, in order to achieve optimal throughput and latency.

Notably, there has recently been a surge of interest in channel-aware scheduling and channel-aware access control. Channel aware opportunistic scheduling was first developed for the downlink transmissions in multiuser wireless networks (see, e.g., [4], [11], [23], [25], [34], [36]). Opportunistic scheduling originates from a holistic view: roughly speaking, in a multiuser wireless network, at each moment it is likely that there exists a user with good channel conditions; and by picking the instantaneous "on-peak" user for data transmission, opportunistic scheduling can utilize the wireless resource more efficiently. *A key assumption in these studies is that the scheduler has knowledge of the instantaneous channel conditions for all links, and therefore the scheduling is centralized.*

Channel aware random access has been investigated for the uplink transmissions in a many-to-one network, where channel probing can be realized by broadcasting pilot signals from the base station. Notably, [1, 30] study opportunistic ALOHA under a collision model, with a basic idea being that in every slot each user transmits with a probability based on its own channel condition. While recent work [22] does not assume a base station in a wireless LAN, the transmitter node still needs to collect channel information from potential receivers, thereby serving as a tentative "virtual" base station. A key observation is that in the existing work on rate adaptation for ad hoc communications (see, e.g., [20, 29, 32]), a link continues transmission after a successful channel contention, no matter whether the channel condition is good or poor. Clearly, this leaves much room for improvement by devising channel-aware scheduling.

Unfortunately, little work has been done on developing channel-aware distributed scheduling to reap rich diversity gains for enhancing ad hoc communications. This is perhaps due to the fact that channel-aware distributed scheduling is indeed challenging, since *the distributed nature of ad hoc communications dictates that each link has no knowledge of others' channel conditions* (in fact, even its own channel condition is unknown before channel probing). A principal goal of this study is to fill this void, and obtain a rigorous understanding of distributed opportunistic scheduling (DOS) for ad-hoc communications.

In this paper, we take some initial steps in this direction and consider a single-hop ad-hoc network where all links can hear others' transmissions. In such a network, links contend for the same channel using random access, and a collision model is assumed which indicates that at most one link can transmit successfully at each time. We assume that after a successful contention, the channel condition of the successful link is measured (e.g., by using some pilot signals embedded in the probing packets). Due to channel fading, the link condition corresponding to this successful channel probing can be either good or poor. In the latter case, data packets have to be transmitted at low rates, leading to possible throughput degradation. A plausible alternative is to let this link give up this transmission opportunity, and allow all the links re-contend for the channel, in the hope that some link with a better channel condition can transmit after the re-contention. Intuitively speaking, because different links at different time slots experience different channel conditions, it is likely that after further probing, the channel can be taken by a link with a better channel condition, resulting in possible higher throughput. In this way, the multiuser diversity across links and the time diversity across slots can be exploited in a joint opportunistic manner. It is in this sense that we call this process of joint probing and scheduling "distributed opportunistic scheduling". We should caution that on the other hand, each channel probing comes with a cost in terms of the contention time, which could be used for data transmission.

Clearly, there is a *tradeoff* between the throughput gain from better channel conditions and the cost for further channel probing. The desired tradeoff boils down to judiciously choosing the optimal stopping rule for channel probing, in order to maximize the throughput. In this paper, we obtain a systematic characterization of this tradeoff by appealing to optimal stopping theory [15, 18], and explore channel-aware distributed scheduling to exploit multiuser diversity and time diversity for wireless ad-hoc networks in an opportunistic manner. We shall tackle this problem from the following two perspectives: 1) a network-centric perspective in which all links "cooperate" to maximize the overall network throughput; and 2) a user-centric view where each link seeks to maximize its own throughput selfishly.

1.2 Summary of Main Results

The common theme of the first thrust is distributed opportunistic scheduling from a network-centric perspective. We start with the basic case where all links have the same channel statistics. Recall that when a link discovers that its channel condition is relatively poor after a successful channel contention, it can skip the transmission opportunity so that some link with a better condition would have the chance to transmit in the next round channel probing. We should point out that there is no guarantee for this to happen due to the stochastic nature of random contention and time varying channel conditions. Nevertheless, as channel probing continues, the likelihood of reaching a better channel condition increases. In a nutshell, distributed opportunistic scheduling boils down to a process of joint channel probing and scheduling.

Mathematically speaking, we treat distributed opportunistic scheduling as a team game. Building on optimal stopping theory [15, 18], we cast the problem as a *maximal rate of return* problem, where the rate of return refers to the average throughput. As noted above, since the cost, in terms of the contention duration, is random, we use the Maximal Inequality to establish the existence of the optimal stopping rule. Then, we develop the optimal strategy for distributed opportunistic scheduling, by characterizing the optimal stopping rule to "control" the channel probing process and hence to maximize the overall throughput. In particular, we show that the optimal strategy is a *pure threshold policy*, [1] in the sense that the decision on further channel probing or data transmission is based on the local channel condition only, and the threshold is invariant in time. Therefore, it is amenable to easy distributed implementation. Furthermore, it turns out that the optimal threshold can be chosen to be the maximum network throughput, which can be obtained by solving a fixed point equation. We then generalize the above study to the case with heterogeneous links, where different links may have different channel statistics. Due to the channel heterogeneity, the channel conditions corresponding to consecutive successful channel probings may follow different distributions. Again, we show that the optimal strategy for joint channel probing and distributed scheduling is a pure threshold policy. Somewhat surprisingly, the optimal thresholds turn out to be the same across all the links regardless of the channel statistics and contention probabilities. We further devise an iterative algorithm to compute the optimal threshold. We note that the proof for the convergence of the iterative algorithm is nontrivial, and the standard techniques (e.g., contraction mapping [7]) are not applicable here. Instead, we use a novel approach based on the monotonicity of the iterates to establish the convergence. We also generalize the studies to take into account fairness requirements.

[1]A threshold policy is called pure if the threshold is invariant in time.

In the second thrust, we focus on DOS from the user-centric perspective, where each link seeks to maximize its own throughput in a selfish manner. We treat the rate threshold selection problem across different links as a non-cooperative game. Needless to say, game theory is a powerful tool to describe complex interactions among players, and predict their choices of strategies. In a non-cooperative game, each player seeks to maximize some utility function (payoff function) in a distributed manner by choosing its strategy from a strategy set. The game settles at an equilibrium point if one exists. Due to the selfish nature of the players, the equilibrium is not necessarily the optimal point that maximizes the social utility.

More specifically, we first characterize each link's individual throughput as its payoff function. We establish the existence of the Nash equilibrium for the non-cooperative game, and show the uniqueness of the Nash equilibrium under some sufficient conditions. Based on the best response strategy, we devise a distributed iterative algorithm, and establish its convergence to the Nash equilibrium for any non-negative initial threshold values. It is worth noting that the proof for the convergence of the best response strategy is nontrivial, and the standard approaches (e.g., using contraction mapping [7] or standard conditions in [37]) are not applicable here. Indeed, the proof is constructive, and involves a sandwich argument. Observing that the best response strategy requires global information, we then develop an online stochastic iterative algorithm based on local observations only. In light of the asynchronous feature of the online algorithm, we appeal to recent results on asynchronous stochastic approximation algorithms [9] and establish its convergence under some regularity conditions. Finally, observe that due to the selfish nature of the players, the equilibrium point does not necessarily maximize the social utility. We examine the efficiency loss in terms of the throughput in the non-cooperative game, compared to the network-centric case, and explore pricing-based mechanisms to mitigate the loss.

In summary, the study in this paper on distributed opportunistic scheduling, for both the network-centric case and the user-centric case, reveals that rich PHY/MAC diversities are available for exploitation in ad-hoc communications. We believe that these initial steps open a new avenue for channel-aware distributed scheduling, and are useful for enhancing MAC protocol design for wireless LANs and wireless mesh networks.

1.3 Related Work and Organization

As noted above, there has been much work on centralized opportunistic scheduling (e.g., [4], [11], [12], [23], [25], [34], [36]), channel-aware ALOHA (e.g., [1, 30]) and MAC design with rate adaptation (e.g., [20, 29, 32]). Most relevant to our study are perhaps (e.g., [1, 20, 30, 32]). The main differences between this study and the studies [1, 30] lie in the following two aspects: 1) We consider ad hoc communications assuming no centralized coordination, and the transmission scheduling is done distributively; and 2) the transmitter nodes have no knowledge of other links' channel conditions, and even their own channel conditions are not available before contention. These limitations, dictated by the distributed nature of ad hoc communications, pose great challenges for exploiting channel diversity in distributed scheduling. A major difference between our study and the studies in [20, 32] is that our scheme allows links to opportunistically utilize the channel whereas in the schemes in [20, 32] the transmission rate is adapted based on the current channel condition, regardless of whether the channel condition is poor or good.

Along a different avenue, opportunistic channel probing for single-user multichannel systems has been studied in [19], where the basic idea is to opportunistically probe and select a transmission channel among multiple channels between the transmitter node and the receiver node. In contrast, in this study, we consider multiple links (each with its own transmitter and receiver) sharing one single channel and explore distributed scheduling, assuming that each link has no knowledge of other links' channel conditions.

There has been a surge of interests in using game theory to study wireless networks (see, e.g., [17], [21], [27], [33]). We note that a game theoretic formulation on random access protocols has been investigated, and the most relevant to our study are perhaps [3, 10, 35], with one major difference being that none of these works exploit time-varying channel conditions for scheduling.

The rest of the paper is organized as follows. In Section 2, we introduce some background knowledge on optimal stopping theory. Section 3 presents the model for random-access based channel probing and scheduling. In Section 4 and Section 5, we formulate the problem of joint channel probing and scheduling from the network-centric perspective and the user-centric perspective, respectively. Section 6 investigates the efficiency loss of the non-cooperative game, compared to that of the team game, and proposes a pricing mechanism to mitigate the price of anarchy. In Section 7, we provide numerical examples to corroborate the theoretic results. Finally, Section 8 concludes the paper.

2 A Preliminary on Optimal Stopping Theory

As noted above, in an ad-hoc network with many links, when a link discovers that its channel condition is "relatively poor" after a successful channel contention, it can either transmit or skip this opportunity so that in the next round some link with a better condition would have the chance to transmit. This is intimately related to the optimal stopping strategy in sequential analysis [18].

Simply put, an optimal stopping rule is a strategy for deciding when to take a given action based on the past events in order to maximize the average return, where the return is the net gain (the difference between the reward and the cost) [15, 18]. More specifically, let $\{Z_1, Z_2, \ldots\}$ denote a sequence of random variables, and $\{y_0, y_1(z_1), y_2(z_1, z_2), \ldots, y_\infty(z_1, z_2, \ldots)\}$ a sequence of real-valued reward functions. The reward is $y_n(z_1, \ldots, z_n)$ if the strategy chooses to stop at time n. The theory of optimal stopping is concerned with determining the stopping time N to maximize the expected reward $E[Y_N]$; and in general N is called a *stopping time* if $\{N = n\} \in \mathcal{F}_n$, where $\mathcal{F}_n$ is the $\sigma-$algebra generated by $\{Z_j, j \leq n\}$.

3 System Model

Random access is widely used for medium access control in wireless ad hoc networks. Consider a single-hop ad-hoc network with M links (see Fig. 1), where link m contends for the channel with probability p_m, $m = 1, \ldots, M$. A collision model is assumed for the random access, where a channel contention of a link is said to be successful if no other links transmit at the same time. We assume that the local channel condition can be obtained after a successful channel contention. Accordingly, the overall successful channel probing probability in each slot, p_s, is then given by $\sum_{m=1}^{M}(p_m \prod_{i \neq m}(1 - p_i))$ [30]. (To avoid triviality, we assume that $p_s > 0$.)

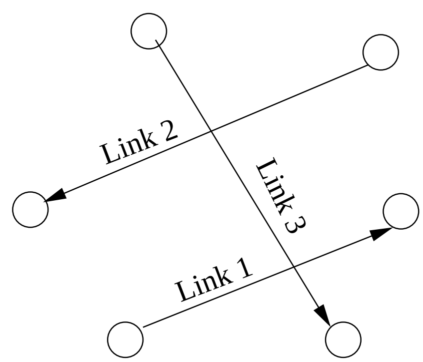

Figure 1: An example of a single-hop ad-hoc network.

For convenience, we call the random duration of achieving a successful channel contention as one round of channel probing. It is clear that the number of slots (denoted by K) for a successful channel contention (probing) is a Geometric random variable, i.e., $K \sim Geometric(p_s)$. Let τ denote the duration of mini-slot for channel contention. It follows that the random duration corresponding to one round of channel probing is $K\tau$, with expectation τ/p_s.

Let $s(n)$ denote the successful link in the n-th round of channel probing, and $R_{n,s(n)}$ denote the corresponding transmission rate. In wireless communications, $R_{n,s(n)}$ depends on the time varying channel condition, and hence is random. Following the standard assumption on the block fading channel in wireless communications [20, 32], we assume that $R_{n,s(n)}$ remains constant for a duration of T, where T is the data transmission duration and is no greater than the channel coherence time.

To get a more concrete sense of joint channel probing and distributed scheduling, we depict in Fig. 2 an example with N rounds of channel probing and one single data transmission. Specifically, suppose after the first round of channel probing with a duration of $K_1\tau$, the rate of link $s(1)$, $R_{1,s(1)}$, is small (indicating a poor channel condition); and as a result, $s(1)$ gives up this transmission opportunity and let all the links re-contend. Then, after the second round of channel probing with a duration of $K_2\tau$, link $s(2)$ also gives up the transmission because $R_{2,s(2)}$ is small. This continues for N rounds until link $s(N)$ transmits because $R_{N,s(N)}$ is good.

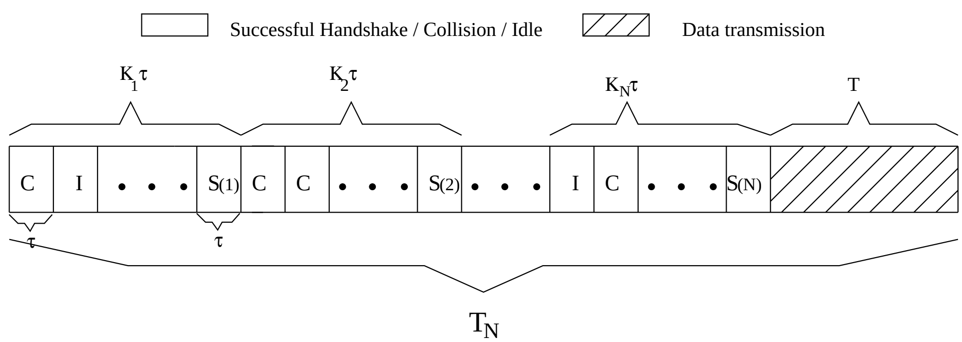

Figure 2: A sample realization of channel probing and data transmission

In this study, we provide a systematic study on distributed opportunistic scheduling by using optimal stopping theory. We first impose the following assumption on the transmission rates across different rounds of channel probing.

A1) $\{R_{n,s(n)}, n = 1, 2, \ldots\}$ are independent.

We note that the above condition holds in many practical scenarios of interest, and the rational behind is as follows: 1) in a multi-user wireless network, the likelihood of one link (say link m) achieving two consecutive successful channel probing, $p_m^2 \prod_{i \neq m}(1-p_i)^2$, is fairly small, especially when the number of links is large; and 2), even if this happens, it is reasonable to assume that the channel conditions corresponding to two successful channel probings are independent since the channel probing duration in between is comparable to the channel coherence time.

4 Distributed Opportunistic Scheduling: A Team Game View

In this section, we treat distributed opportunistic scheduling, namely, joint channel probing and distributed scheduling, as a team game in which all links collaborate to maximize the overall network throughput. In particular, building on optimal stopping theory, we cast the problem as *maximizing the rate of return*, where the rate of return refers to the average throughput [18]. For convenience, let $R_{(n)}$ denote the rate corresponding to the n-th round successful channel probing, i.e., $R_{(n)} = R_{n,s(n)}$. Without loss of generality, we assume that the second moment of $R_{(n)}$ exists.

As illustrated in Fig. 2, after one round of channel probing, a stopping rule N decides whether the successful link carries out data transmission, or simply skips this opportunity and let all the links re-contend. Suppose that this game on joint channel probing and transmission is carried out L times, and let $\{N_1, \ldots, N_L\}$ denote the corresponding stopping times, T_{N_l} the l-th realization of the duration for probing and data transmission. Then, appealing to the Renewal Theorem, we have that

$$x_L = \frac{\sum_{l=1}^{L} R_{(N_l)}T}{\sum_{l=1}^{L} T_{N_l}} \longrightarrow \frac{E[R_{(N)}T]}{E[T_N]} \quad a.s., \tag{1}$$

where $E[R_{(N)}T]/E[T_N]$ is the rate of return [18]. Clearly, $R_{(N)}$ and T_N are stopped random variables since N is a stopping time. Accordingly, the distributions of $R_{(N)}$ and T_N depend on that of the stopping time N. Define

$$Q \triangleq \{N : N \geq 1, E[T_N] < \infty\}. \tag{2}$$

It then follows that the problem of maximizing the long-term average throughput can be cast as a maximal-rate-of-return problem, in which a key step is to characterize the optimal stopping rule N^* and the optimal throughput x^*, as

$$N^* \triangleq \arg\max_{N \in Q} \frac{E[R_{(N)}T]}{E[T_N]}, \quad x^* \triangleq \sup_{N \in Q} \frac{E[R_{(N)}T]}{E[T_N]}. \tag{3}$$

4.1 Optimal Stopping Rule for Channel Probing

We now exploit optimal stopping theory [15, 18] to solve the problem in (3).

4.1.1 The Case with Homogeneous Links

For ease of exposition, we first consider a network with homogenous links where all links have the same channel statistics with the same distribution $F_R(r)$. By **A1**, $\{R_{(n)}, n = 1, 2, \ldots\}$ is a sequence of i.i.d. random variables with distribution $F_R(r)$.

Observe that different from standard optimal stopping problems, the cost in terms of the probing duration is random due to the stochastic nature of channel probing. In light of this, we use the Maximal Inequality to establish the existence of the optimal stopping rule. We have the following proposition.

Proposition 4.1 *a) The optimal stopping rule N^* for channel probing exists, and is given by*

$$N^* = \min\{n \geq 1 : R_{(n)} \geq x^*\}. \tag{4}$$

b) The maximum throughput x^* is an optimal threshold, and is the unique solution to

$$E(R_{(n)} - x)^+ = \frac{x\tau}{p_s T}. \tag{5}$$

The proof is relegated in Appendix A.

Remarks: 1) Proposition 4.1 reveals that the optimal stopping rule N^* for channel probing is a pure threshold policy, and the stopping decision can be made based on the current rate only. Accordingly, the optimal channel probing and scheduling strategy takes the following simple form: If the successful link discovers that the current rate $R_{(n)}$ is higher than the threshold x^*, it transmits the data with rate $R_{(n)}$; otherwise, it skips this transmission opportunity (e.g., by skipping CTS), and then the links re-contend.

2) We note that the maximum throughput x^* is unique, but the optimal threshold in (4) may not be unique in general. It is not difficult to show the uniqueness of the optimal threshold in the continuous rate case with $f(r) > 0,\ \forall\ r > 0$. In contrast, in the discrete rate case, changing the threshold in between two adjacent quantization levels would not affect its optimality since the new threshold policy achieves the same throughput. (In what follows, for the discrete rate case, we treat the thresholds in between two adjacent quantization levels "effectively" the same.)

3) It can be shown that

$$E[T_N] = \frac{\tau}{p_s} E[N] + T. \tag{6}$$

Based on (6) and the proof of Theorem 4.1, it can also be shown that if the random contention time $K\tau$ is replaced with a constant probing time τ/p_s, the optimal stopping rule (4) and the optimal throughput remain the same.

Based on the structure of the optimal stopping rule N^* in (4), we have the following corollary.

Corollary 4.1 *a) The stopping time N^* is geometrically distributed with parameter $1 - F_R(x^*)$.*

b) The stopped random variable R_{N^*} has the following distribution:

$$F_{R_{N^*}}(r) = \begin{cases} \frac{F_R(r) - F_R(x^*)}{1 - F_R(x^*)}, & r \geq x^*; \\ 0, & \textit{otherwise}. \end{cases}$$

c) The stopped random variable $\frac{T_{N^*}-T}{\tau}$ is geometrically distributed with parameter $p_s[1-F_R(x^*)]$.

Part a) of Corollary 4.1 indicates that the channel probing process would stop in a finite time almost surely. It follows from part b) and c) of Corollary 4.1 that

$$\frac{E[R_{N^*}T]}{E[T_{N^*}]} = \frac{\int_{x^*}^{\infty} r dF_R(r)}{\frac{\delta}{p_s} + 1 - F_R(x^*)}, \tag{7}$$

where $\delta = \tau/T$.

We note that the maximum throughput x^* is obtained by solving the fixed point equation (5), which in general does not admit a closed-form solution. In what follows, we derive a lower bound and an upper bound on x^*. We have the following proposition.

Proposition 4.2

$$x^L \leq x^* \leq x^U,$$

where x^L and x^U are given by

$$x^L \triangleq \frac{E[R]}{\frac{\delta}{p_s}+1}, \qquad x^U \triangleq \sqrt{\frac{E[R^2]}{2\frac{\delta}{p_s}}}. \tag{8}$$

The proof is relegated to Appendix B.

Remarks: 1) Observe that x^L is the throughput of the OAR scheme in [32], which can be viewed as a degenerated stopping algorithm with zero threshold.

2) Note that $\sqrt{\frac{E[R^2]}{2\frac{\delta}{p_s}}}$ is the maximum throughput corresponding to the optimal genie-aided scheduling when channel realizations are known *a priori*. Indeed, this can be seen from the proof of Proposition 4.2.

4.1.2 The Case with Heterogeneous Links

In the above, it is assumed that all links have the same channel statistics. As a result, $R_{n,s(n)}$ follows the same distribution $F_R(r)$. In many practical scenarios, it is likely that different links may have different channel statistics. As a result, if $s(n+1) \neq s(n)$, $R_{n,s(n)}$ and $R_{n+1,s(n+1)}$ may follow different distributions. Nevertheless, we can treat $R_{n,s(n)}$ as a compound random variable. Accordingly, a key step is to characterize the distribution of $R_{n,s(n)}$ for the heterogeneous case.

To this end, let $F_m(\cdot)$ denote the distribution for each link $m \in \{1, 2, \ldots, M\}$. It can be shown that

$$P(R_{(n)} \leq r) = E\left[P(R_{n,m} \leq r) | s(n) = m\right] = \sum_{m=1}^{M} \frac{p_{s,m}}{p_s} F_m(r), \tag{9}$$

where $p_{s,m} \triangleq p_m \prod_{i \neq m}(1-p_i)$ is the successful probing probability of user m. Based on (9), it is clear that $R_{(n)}$ is a compound random variable whose distribution is a "mixed" version of that across the links. We have the following proposition regarding the optimal threshold policy.

Proposition 4.3 *The maximum throughput x^* in the heterogeneous case is an optimal threshold, and is the unique solution to the following equation:*

$$x = \frac{\sum_{m=1}^{M} p_{s,m} \int_x^\infty r dF_m(r)}{\delta + \sum_{m=1}^{M} p_{s,m} (1 - F_m(x))}. \tag{10}$$

Remarks: For the heterogeneous case, *a priori*, it is not clear that different links would have different thresholds or not since their channel statistics are different. However, Proposition 4.3 indicates that in the optimal strategy the threshold is the same for all the links (again, for the discrete rate case, we treat the thresholds in between two adjacent quantization levels "effectively" the same). Our intuition is as follows: When all the links have the same threshold, links with better channel conditions would have a higher likelihood to transmit accordingly.

4.2 Iterative Computation Algorithm for x^*

In the following, we devise an iterative algorithm to compute x^*. To this end, rewrite (10) as $x = \Phi(x)$, with

$$\Phi(x) \triangleq \frac{\sum_{m=1}^{M} p_{s,m} \int_x^\infty r dF_m(r)}{\delta + \sum_{m=1}^{M} p_{s,m} (1 - F_m(x))}. \tag{11}$$

Accordingly, we propose the following iterative algorithm for computing x^*.

$$x_{k+1} = \Phi(x_k), \text{ for } k = 0, 1, 2, \ldots, \tag{12}$$

where x_0 is the initial value. We have the following proposition on the convergence of the above iterative algorithm.

Proposition 4.4 *The iterates generated by algorithm in (12) converge to x^* for any positive initial value x_0.*

A standard approach for establishing the convergence of iterative fixed point algorithms is via the Contraction (or Pseudo-Contraction) Mapping Theorem [7], which is unfortunately not applicable here since $\Phi(x)$ is not a pseudo-contraction mapping in some cases. For instance, suppose for any m, $f_m(r)$ is given by

$$f_m(r) = \begin{cases} 0, & r < 0 \\ 0.01, & 0 \le r < 96 \\ 0.005(r-94), & 96 \le r < 98 \\ 0.02(r-97)^{-3}, & r \ge 98 \end{cases} \tag{13}$$

Let $p_{s,m} = 0.99/\mathrm{M}$ and $\delta = 0.05$. The corresponding optimal point $x^* = 72.82$. However,

$$|\Phi(95.5) - x^*| = |45.88 - 72.82| > |95.5 - 72.82|,$$

which violates the condition for pseudo-contraction mapping.

Remarks: 1) In light of the above observation, we provide in Appendix C a new proof for the convergence of iterative algorithm in (12).

2) Observe that computing the optimal throughput x^* requires the knowledge of the channel information of all links. Alternatively, x^* can be computed online by using a distributed iterative algorithm, in which each link independently computes its threshold based on local information only. Indeed in Section 5.5, we propose a distributed online algorithm for the non-cooperative game case.

4.3 Optimal Stopping Rule for Channel Probing Under Fairness Constraints

In the above studies, the optimal distributed scheduling is aimed at maximizing the overall network throughput. We next generalize the studies to take into account fairness requirements. Under fairness constraints, the objective of distributed opportunistic scheduling boils down to maximizing the total network utility function, where user m's utility is a function of its rate and serves as a measure of satisfaction that user m has from sharing the channel. For example, the reward function (utility function), denoted $\{U_m(r),\ \forall\ m\}$ can take the following form [26]:

$$U_{m,\kappa}(r) = \begin{cases} w_m \log r, & \text{if } \kappa = 1 \\ w_m(1-\kappa)^{-1} r^{1-\kappa}, & \kappa \ge 0, \kappa \ne 1. \end{cases} \tag{14}$$

Then, the optimal strategy for distributed opportunistic scheduling is to characterize the optimal stopping rule N_U^* for maximizing the return rate of the total network utility, i.e.,

$$N_U^* \triangleq \arg\max_{N \in Q} \frac{E[U(R_N)T]}{E[T_N]}, \tag{15}$$

Interestingly, when $\kappa = 0$, the above problem degenerates to the problem of maximizing the overall throughput. Furthermore, when $\kappa = 1$, *proportional fairness* is achieved by N_U^*.

It is not difficult to see that the optimal stopping rule N_U^* can be derived, along the same line as in Proposition 4.1 and 4.3. We note that this study can be further extended to incorporate more complicated fairness constraints.

5 Distributed Opportunistic Scheduling: A Non-Cooperative Game Perspective

5.1 Formulation of the Rate Threshold Selection Problem as A Non-Cooperative Game

In the above sections, we formulate distributed opportunistic scheduling, namely joint channel probing and distributed scheduling, as a team game in which links collaborate together to optimize the network throughput. In this section, we treat joint channel probing and distributed scheduling as a non-cooperative game, where links seek to maximize their own throughput by choosing the threshold $\{x_m, m = 1, 2, \cdots, M\}$ in a selfish manner. Towards this end, first recall from the above sections that the optimal strategy for distributed opportunistic scheduling is a pure threshold policy, and the scheduling decision can be made based on the current rate only. Accordingly, for a given set of thresholds across links, the average throughput for each link can be obtained as follows.

Lemma 5.1 *Let $F_m(r)$ denote the rate distribution for each link $m \in \{1, 2, \cdots, M\}$. Assume that the threshold for link m is x_m. Then, the average throughput of link m is given by*

$$\phi_m(\mathbf{x}) = \frac{p_{s,m} \int_{x_m}^{\infty} r dF_m(r)}{\delta + \sum_{i=1}^{M} p_{s,i} \left(1 - F_i(x_i)\right)}, \tag{16}$$

where $\delta = \tau/T$, τ is the duration of mini-slot channel contention and T is the duration of data transmission.

The proof follows directly from ergodicity. Specifically, assume there are total L number of events including collisions, idle events, successful channel probings of each user and their transmissions. Therefore, the average throughput of link m can be calculated as follows:

$$\bar{\phi}_m = \frac{L p_{s,m} (1 - F_m(x_m)) \frac{\int_{x_m}^{\infty} r dF_m(r)}{1 - F_m(x_m)} T}{\sum_{i=1}^{M} L p_{s,i} (1 - F_i(x_i))(\tau + T) + L\left(1 - \sum_{i=1}^{M} p_{s,i}(1 - F_i(x_i))\right)\tau}. \tag{17}$$

Let $L \longrightarrow \infty$, $\bar{\phi}_m \rightarrow \phi_m$ and we obtain the desired result.

To get a more concrete understanding of $\phi_m(\mathbf{x})$, we rewrite (16) as follows:

$$\phi_m(\mathbf{x}) = \frac{\frac{\int_{x_m}^{\infty} r dF_m(r)}{1 - F_m(x_m)} T}{\frac{\tau + \sum_{i \neq m} p_{s,i}(1 - F_i(x_i)) T}{p_{s,m}(1 - F_m(x_m))} + T}. \tag{18}$$

It can be seen that the numerator in (18) is the expected throughput of user m, whereas the denominator can be decomposed into two parts: 1) the expected channel probing time $\frac{\tau + \sum_{i \neq m} p_{s,i}(1 - F_i(x_i)) T}{p_{s,m}(1 - F_m(x_m))}$, and 2) the data transmission time T. Furthermore, in the expected channel probing time, $p_{s,m}(1 - F_m(x_m))$ is the successful probing and transmission probability of user m, while $\tau + \sum_{i \neq m} p_{s,i}\,(1 - F_i(x_i))\,T$ can be viewed as the *effective channel probing time* for user m, consisting of the constant probing time τ and the average transmission time of other users $\sum_{i \neq m} p_{s,i}\,(1 - F_i(x_i))\,T$.

Next, we cast the threshold selection problem across different links as a non-cooperative game, in which each individual link chooses its threshold x_m to maximize its own throughput ϕ_m in a selfish manner. Specifically, let $\mathbf{G} = [\{1, 2, \ldots, M\}, \times_{m \in \{1,2,\ldots,M\}} A_m, \{\phi_m, m \in \{1, 2, \ldots, M\}\}]$ denote the non-cooperative threshold selection game, where the links in $\{1, 2, \ldots, M\}$ are the players of the game, $A_m = \{x_m | 0 \leq x_m < \infty\}$ is the action set of player m, and ϕ_m is treated as the utility (payoff) function for player m. Formally, the non-cooperative game is expressed as

$$(\mathbf{G}) \quad \max_{x_m \in A_m} \phi_m(\mathbf{x}) \quad \forall\, m = 1, 2, \cdots, M. \tag{19}$$

5.2 Nash Equilibrium for Non-Cooperative Game

Treating the rate threshold selection problem as a non-cooperative game, we investigate the corresponding Nash equilibrium [28].

Definition 5.1 A threshold vector $\mathbf{x}^* = \{x_1^*, x_2^*, \cdots, x_M^*\}$ is said to be a Nash equilibrium of game $\mathbf{G}$, if for every link m

$$\phi_m(x_m^*, \mathbf{x}_{-m}^*) \geq \phi_m(x_m, \mathbf{x}_{-m}^*), \quad \forall\, x_m \in A_m, \tag{20}$$

where $\mathbf{x}_{-m} \triangleq [x_1, \cdots, x_{m-1}, x_{m+1}, \cdots, x_M]^T$.

In other words, at the Nash equilibrium, no link can increase its throughput by unilaterally deviating its threshold from the equilibrium, given the thresholds of other links.

We first examine the existence of Nash equilibrium in game $\mathbf{G}$. Based on [28, Proposition 20.3], by showing that $\phi_m(\mathbf{x})$ is a quasi-concave function of x_m, we have the following proposition on the existence of the Nash equilibrium for the threshold selection game.

Proposition 5.1 *There exists a Nash equilibrium in the threshold selection game G, which satisfies the following set of equations: for $m = 1, 2, \ldots, M$,*

$$x_m^* = \phi_m(x_m^*, \mathbf{x}_{-m}^*) = \frac{p_{s,m} \int_{x_m^*}^{\infty} r dF_m(r)}{\delta + \sum_{i=1}^{M} p_{s,i}\,(1 - F_i(x_i^*))}. \tag{21}$$

The proof is relegated to Appendix D.

We further have the following proposition on the property of the Nash equilibria that satisfy (21).

Proposition 5.2 [Componentwise monotonicity]: *Suppose* $\mathbf{x}^*$ *and* $\mathbf{y}^*$ *are two Nash equilibrium points satisfying (21). If there exists* $k \in \{1, 2, \ldots, M\}$ *such that* $x_k^* < y_k^*$*, then* $x_m^* < y_m^*$*,* $\forall\, m \in \{1, 2, \ldots, M\}$*, i.e.,* $\mathbf{x}^* < \mathbf{y}^*$*.*

Proof: It is clear that

$$\varphi_m(x) \triangleq \frac{p_{s,m} \int_x^\infty r dF_m(r)}{x}, \tag{22}$$

is strictly decreasing in $x \in (0, \infty)$, $\forall\, m \in \{1, 2, \cdots, M\}$. Rewrite (21) as

$$\delta + \sum_{i=1}^{M} p_{s,i} (1 - F_i(x_i^*)) = \varphi_m(x_m^*),\ \forall\, m \in \{1, 2, \ldots, M\}. \tag{23}$$

Since $x_k^* < y_k^*$, it follows that

$$\varphi_k(x_k^*) = \delta + \sum_{i=1}^{M} p_{s,i} (1 - F_i(x_i^*)) > \delta + \sum_{i=1}^{M} p_{s,i} (1 - F_i(y_i^*)) = \varphi_k(y_k^*). \tag{24}$$

Combining (24) and (23) for $m \neq k$ yields that

$$\varphi_m(x_m^*) > \varphi_m(y_m^*),\ \forall\, m \neq k. \tag{25}$$

As a result, we conclude that $x_m^* < y_m^*$ for all $m \in \{1, 2, \ldots, M\}$. ∎

Remarks: Proposition 5.2 suggests that the Nash equilibrium points can be sorted in an ascending order. Let $\mathbf{x}^*(n), n = 1, 2, \ldots$ denote the sorted sequence, i.e., $\mathbf{x}^*(1) < \mathbf{x}^*(2) < \cdots$. Clearly, $\limsup_n \mathbf{x}^*(n)$ is bounded, and is also a Nash equilibrium point. Needless to say, $\mathbf{x}^*_{\max} \triangleq \max_n \mathbf{x}^*(n)$ is the most desired Nash equilibrium point in terms of the throughput efficiency. That is, the Nash equilibrium $\mathbf{x}^*_{\max}$ is the Pareto-dominant equilibrium, i.e., $\phi_m(\mathbf{x}^*_{\max}) \geq \phi_m(\mathbf{x}^*(n))$, $\forall\, m, n$.

5.3 Uniqueness of Nash Equilibrium

Needless to say, the uniqueness of Nash equilibrium is of great interest for a non-cooperative game. Unfortunately, in general, the Nash equilibrium that satisfies (21) is not necessarily unique, as illustrated by the following example. Suppose that there are two links in the network, with the same rate distribution as

$$R(r) = \begin{cases} \text{2Mbps}, & \text{w.p. } 0.5, \\ \text{12Mbps}, & \text{w.p. } 0.5. \end{cases} \tag{26}$$

Let $p_{s,1} = p_{s,2} = 0.2$ and $\delta = 0.35$. Then, there exist two Nash equilibria at $\mathbf{x} = (1.867, 1.867)$ and $\mathbf{x} = (2.18, 2.18)$ that satisfy (21).

In what follows, we provide some sufficient conditions for establishing the uniqueness of Nash equilibrium. Consider a network with homogeneous links, where all links have the same channel statistics $F(r)$ and the same contention probability p. Then, (21) boils down to

$$x_m^* = \phi_m(x_m^*, \mathbf{x}_{-m}^*) = \frac{\frac{p_s}{M}\int_{x_m^*}^{\infty} r dF(r)}{\delta + \frac{p_s}{M}\sum_{i=1}^{M}(1 - F(x_i^*))}, \tag{27}$$

where $p_s = Mp(1-p)^{M-1}$.

Proposition 5.3 *In homogeneous networks, the Nash equilibrium is unique if and only if the equation $x = \phi_m(x, x, \cdots, x)$ has unique solution.*

The proof is relegated to Appendix E.

Rewrite $x = \phi_m(x, x, \cdots, x)$ as

$$d(x) \triangleq \delta x/p_s + x(1 - F(x)) - \frac{1}{M}\int_x^{\infty} r dF(r) = 0. \tag{28}$$

Then, the problem boils down to showing that the solution of $d(x) = 0$ is unique.

5.3.1 Continuous Rate over Rayleigh Fading

We first consider the case where the transmission rate is given by the Shannon channel capacity:

$$R(h) = \log(1 + \rho h) \text{ nats/s/Hz}, \tag{29}$$

where ρ is the normalized average SNR, and h is the channel gain corresponding to Rayleigh fading.

Proposition 5.4 *The Nash equilibrium of the threshold selection game* **G** *is unique under the rate model in (29).*

The proof is relegated to Appendix F.

5.3.2 General Continuous Rate Case

Consider a homogenous network where the transmission rate follows a general continuous distribution with pdf $f(r) \geq 0,\ \forall\ r > 0$. We have the following sufficient condition regarding the uniqueness of Nash equilibrium.

Proposition 5.5 *The Nash equilibrium of the threshold selection game* **G** *is unique if* $rf(r) < \frac{M\delta}{p_s(M-1)},\ \ \forall\ r > 0.$

Proof: The derivative of $d(x)$ is given by

$$d'(x) = \delta/p_s + (1 - F(x)) - \frac{(M-1)}{M} x f(x) \tag{30}$$

If $xf(x) < \frac{M\delta}{p_s(M-1)}$, for all $x > 0$, then $d(x)$ is monotonically increasing for $x \geq 0$. Combining this with $d(0) < 0$ and $d(\infty) > 0$, we conclude that $d(x) = 0$ has unique solution. ∎

5.4 Best Response Strategy

Based on the structure of game $\mathbf{G}$, we can use the following *best response strategy* to iteratively compute the Nash equilibrium: $\forall\, m \in \{1, 2, \cdots, M\}$,

$$x_m(k+1) = x_m^*(k), \text{ for } k = 0, 1, 2, \cdots \tag{31}$$

where $x_m^*(k)$ is the unique solution to the equation

$$x_m = \phi_m(x_m, \mathbf{x}_{-m}(k)).$$

Remarks: 1) The algorithm in (31) is a two time-scale iterative algorithm: On the smaller time-scale, each link can use an iterative algorithm to compute $x_m^*(k)$, which is the *best response* for link m at iteration k; and on the larger time-scale, each link updates its threshold based on (31).

Proposition 5.6 *Suppose that the Nash equilibrium is unique. Then, for any non-negative initial value* $\mathbf{x}(0)$*, the sequence* $\{\mathbf{x}(k)\}$ *generated by the iterative algorithm in (31) converge to the Nash equilibrium* $\mathbf{x}^*$*, as* $k \to \infty$.

We note that standard techniques for establishing the convergence of the best response strategy (e.g., contraction mapping [7] and standard conditions [37]) are unfortunately not applicable here. Instead, we provide in Appendix G a constructive proof using sandwich argument.

Note that the convergence of the above iterative algorithm assumes that the Nash equilibrium of Game $\mathbf{G}$ is unique. In what follows, we devise a different iterative algorithm to compute the Nash equilibria for cases where this assumption does not hold. Specifically, suppose that link m updates its threshold as

$$x_m(k+1) = \phi_m(x_m(k), \mathbf{x}_{-m}(k)) = \frac{p_{s,m} \int_{x_m(k)}^{\infty} r dF_m(r)}{\delta + \sum_{i=1}^{M} p_{s,i} \left(1 - F_i(x_i(k))\right)},\ \forall\, m \in \{1, 2, \ldots, M\}. \tag{32}$$

The convergence of the above iterative algorithm is established in the following proposition.

Proposition 5.7 *Starting with all zero initial value, i.e.,* $\mathbf{x}(0) = \mathbf{0}$ *componentwise, the sequence* $\{\mathbf{x}(k)\}$ *generated by the iterative algorithm in (32) converge to one of the Nash equilibria that satisfy (21), as* $k \to \infty$.

The proof is relegated to Appendix H.

Remarks: Compared to the best response strategy in (31), the iterative algorithm corresponding to (32) is a single time-scale algorithm, and the complexity is lower. However, the updates given by (32) is not necessarily the best response, and as a consequence, it would take longer to converge. It is in this sense that we call it a "pseudo-best" response strategy. We shall illustrate this by numerical examples in Section 7.

5.5 Online Algorithm for Computing Nash Equilibrium

Observe that in both (31) and (32), computing x_m^* requires the knowledge of $\sum_{i=1}^{M} p_{s,i}(1 - F_i(x_i^*))$, which involves the channel information of all links. In this section, a distributed asynchronous iterative algorithm is proposed in which each link independently computes the optimal threshold $x_m^*, \quad \forall\, m \in \{1, 2, \cdots, M\}$, based on local observations only.

Rewrite (21) as

$$x_m^* = \frac{p_{s,m} \int_{x_m^*}^{\infty} r dF_m(r) - x_m^* \delta}{\sum_{i=1}^{M} p_{s,i}(1 - F_i(x_i^*))}, \quad \forall\, m.$$

Define

$$g_m(\mathbf{x}) \triangleq \frac{p_{s,m} \int_{x_m}^{\infty} r dF_m(r) - x_m \delta}{\sum_{i=1}^{M} p_{s,i}(1 - F_i(x_i))} - x_m.$$

If the Nash equilibrium is unique, then $\mathbf{x}^*$ is the unique root to the equation $g(\mathbf{x}) = 0$.

5.5.1 An Asynchronous Distributed Stochastic Approximation Algorithm

Recall that the collision model is assumed for channel contention, indicating that at most one link can successfully occupy the channel each time. As a result, only the successful link can update its threshold. Clearly, the updating is *asynchronous* across the links.

As illustrated in Fig. 3, let $v(k)$ denote the duration of channel probing in between the $(k-1)$th transmission and the kth transmission, which can be observed locally. It can be shown that $v(k)$ is a local "unbiased estimation" of $1/\sum_{i=1}^{M} p_{s,i}(1 - F_i(x_i(k)))$. Define

$$\widetilde{g_m}(k) \triangleq v(k) \left[p_{s,m} \int_{x_m(k)}^{\infty} r dF_m(r) - x_m(k)\delta \right] - x_m(k). \tag{33}$$

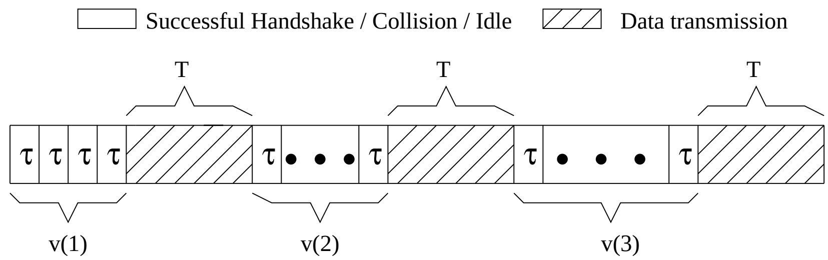

Figure 3: A sample realization of $v(k)$

It is clear that $\widetilde{g_m}(k)$ involves local information only. Let N^m be an infinite subset of $\mathcal{N}$ indicating the set of times at which an update of x_m is performed. Based on stochastic approximation theory, the distributed iterative algorithm can be written as

$$x_m(k+1) = \left[x_m(k) + a_m(k)\left[\widetilde{g_m}(\mathbf{x}(k))\right] I\{k \in N^m\}\right]_0^b, \tag{34}$$

where $a_m(n)$ is the stepsize, $I\{\cdot\}$ is the indicating function, and $[\cdot]_0^b$ is the projection between 0 and b, with $[x]_0^b = \min(b, \max(x, 0))$. The algorithm in (34) is a distributed asynchronous algorithm with stochastic perturbation. The truncation is due to the fact that x_m^* is bounded above by $\frac{p_{s,m}}{\delta}\int_0^\infty r dF_m(r)$.

5.5.2 Basic Properties of Online Algorithm

Let $\{\mathcal{F}_k, k = 0, 1, \cdots\}$ be a family of non-increasing $\sigma-$algebras defined on the probability space $(\Omega, \mathcal{F}, \mathcal{P})$, and $v(k)$ be measurable with respect to $\mathcal{F}_k$. Observe that at the kth iteration, link m transmits with probability $p_{s,m}(1 - F_m(x_m(k)))$. As a result, the probability of successful transmission is given by $\sum_{i=1}^M p_{s,i}(1 - F_i(x_i(k)))$. The number of mini-slots required for a successful channel probing is a geometrical random variable with parameter $\sum_{i=1}^M p_{s,i}(1 - F_i(x_i(k)))$. It follows that the average probing time is given by

$$E[v(k)|\mathcal{F}_k] = \frac{1}{\sum_{i=1}^M p_{s,i}(1 - F_i(x_i(k)))}. \tag{35}$$

(35) reveals that $v(k)$ is a local unbiased estimation of $1/\sum_{i=1}^M p_{s,i}(1 - F_i(x_i(k)))$. We then have the following lemmas:

Lemma 5.2 *For all $k \geq 0$, define $w_m(k) = \widetilde{g_m}(k) - g_m(k)$, then*

$$E[w_m(k)|\mathcal{F}(k)] = 0, \quad \text{and} \quad \sum_{k=1}^{\infty} a_m(k) w_m(k) < \infty, \quad w.p.1. \tag{36}$$

The proof is relegated to Appendix I.

Lemma 5.3 *There exists a deterministic constant $\Delta > 0$ such that for any $m \in \{1, 2, \cdots, M\}$,*

$$\lim_{k\to\infty} \inf \frac{1}{k} \sum_{j=0}^{k-1} I\{j \in N^m\} \geq \Delta \qquad w.p.1. \tag{37}$$

Proof: It is clear that for any time index k,

$$\begin{aligned} Pr(k \in N^m) &= \frac{p_{s,m}[1 - F_m(x_m(k))]}{\sum_{i=1}^{M} p_{s,i}[1 - F_i(x_i(k))]} \\ &\geq p_{s,m}(1 - F_m(b)). \end{aligned} \tag{38}$$

It follows that

$$\lim_{k\to\infty} \inf \frac{1}{k} \sum_{j=0}^{k-1} I\{j \in N^m\} \geq p_{s,m}(1 - F_m(b)) > 0 \qquad w.p.1. \tag{39}$$

∎

Lemma 5.3 guarantees that even the updating is asynchronous across links, each link eventually has the chance to update.

Lemma 5.4 *$g(\cdot)$ is Lipschitz; i.e., for some $K > 0$,*

$$||g(x) - g(y)|| \leq K||x - y||, \quad \forall\ x, y \in [0, b]. \tag{40}$$

This lemma follows from the fact that the derivative of $g(\cdot)$ is bounded.

5.5.3 Stochastic Convergence of the Algorithm

Define the stepsize as

$$a_i(k) = a(i, \sum_{l=1}^{k} I\{k \in N^i\}).$$

Based on [5, 9], we impose the following conditions.

B1) The sequence $\{a(i, k)\}$ satisfy

$$\sum_{k=1}^{\infty} a(i, k) = \infty, \quad \text{and} \quad \sum_{k=1}^{\infty} a(i, k)^2 < \infty.$$

and for $\beta \in (0, 1)$, $\forall\ i, j$

$$\lim_{k\to\infty} \frac{\sum_{l=1}^{\lfloor \beta k \rfloor} a(i, l)}{\sum_{l=1}^{k} a(i, l)} = 1, \quad \text{and} \quad \lim_{k\to\infty} \frac{\sum_{l=1}^{k} a(i, l)}{\sum_{l=1}^{k} a(j, l)} > 0.$$

B2) The Nash equilibrium defined in (21) is unique.

Theorem 5.1 *Under Conditions* **B1** *and* **B2***, for any non-negative initial value* $\mathbf{x}(0)$*, the sequence* $\{\mathbf{x}(k)\}$ *generated by (34) converge to the Nash equilibrium* $\mathbf{x}^*$ *almost surely, as* $k \to \infty$*.*

The proof is relegated to Appendix J.

6 The Price of Anarchy

In this section, we study the efficiency loss of the non-cooperative game, compared to that of the team game.

6.1 Efficiency Loss of Non-cooperative Game

Let x_{co}^* denote the optimal network throughput in the team game case. Recall $x_{co}^* = x^*$, where x^* is the root to the equation

$$x = \Phi(x) = \frac{\sum_{m=1}^{M} p_{s,m} \int_{x}^{\infty} r dF_m(r)}{\delta + \sum_{m=1}^{M} p_{s,m} \left(1 - F_m(x)\right)}. \tag{41}$$

Let x_{nco}^* denote the network throughput at the Nash equilibrium point $\mathbf{x}^*$ for the non-cooperative case, and that $x_{nco}^* = \sum_{m=1}^{M} \phi_m(\mathbf{x}^*)$. Clearly, the optimal network throughput in the team game is no less than the network throughput at the Nash equilibrium in the non-cooperative game, i.e., $x_{co}^* \geq x_{nco}^*$. We have the following result regarding the efficiency of the two different games.

Proposition 6.1 *If* $M \geq 2$ *and* $f_m(r) > 0$, $\forall\ m, r$, *then the optimal network throughput in the team game is always larger than that at the Nash equilibrium in the non-cooperative game, i.e.,* $x_{co}^* > x_{nco}^*$*.*

The proof is relegated to Appendix K.

As expected, Proposition 6.1 implies that for $M \geq 2$, the efficiency η defined as $\eta \triangleq x_{nco}^*/x_{co}^*$ is strictly less than one [31].

6.2 Non-Cooperative Game with Pricing

The Nash equilibrium is a solution to the non-cooperative game, where no link can improve its throughput any further through individual effort. Clearly, the non-cooperative game approach is inefficient due to the selfish decisions made by individual links, and this is the so-called *price of anarchy* [31].

The price of anarchy can be mitigated by introducing a pricing-based mechanism, in which users are "encouraged" to adopt a social behavior. In the above study, each link aims to maximize its own throughput $\phi_m(\mathbf{x})$ by adjusting its threshold x_m, but the overhead it imposes on other links is ignored. In order to mitigate the overhead, a plausible pricing function is given by $c_m(\mathbf{x}) = c\alpha_m(\mathbf{x})$, where c is a preset-parameter for all links and $\alpha_m(\cdot)$ is defined as

$$\alpha_m(\mathbf{x}) \triangleq \frac{p_{s,m}(1 - F_m(x_m))}{\delta + \sum_{i=1}^{M} p_{s,i}(1 - F_i(x_i))}, \tag{42}$$

which points to the portion of time link m transmits. It is a usage-based pricing policy, where the cost(charge) is proportional to the amount of services consumed by the link [33]. Accordingly, define the utility function as $u_m(\mathbf{x}) \triangleq \phi_m(\mathbf{x}) - c_m(\mathbf{x})$. Then, the "new" non-cooperative game is as follows:

$$(\tilde{\mathbf{G}}) \quad \max_{x_m \in A_m} \ u_m(\mathbf{x}), \ m = 1, 2, \ldots, M. \tag{43}$$

Note that Game $\tilde{\mathbf{G}}$ is the same game as the original Game $\mathbf{G}$ with different payoff functions. Next, we establish the existence of Nash equilibrium for the new Game $\tilde{\mathbf{G}}$.

Proposition 6.2 *For some $c > 0$, there exists a Nash equilibrium $\tilde{\mathbf{x}}^*$ in the new game $\tilde{\mathbf{G}}$, which outperforms the one without pricing mechanism, i.e., $\tilde{x}^*_{pricing} \triangleq \sum_{m=1}^{M} \phi_m(\tilde{\mathbf{x}}^*) \geq x^*_{nco}$.*

The proof follows the same line as that of Proposition 5.1.

In Section 7, we compare the results in games with and without pricing, and show the price of anarchy could be reduced by the pricing-based mechanism.

7 Numerical Results

7.1 Numerical Examples for the Team Game

Needless to say, a key performance metric is the throughput gain of distributed opportunistic scheduling over the approaches without using optimal stopping. For convenience, define the throughput gain as

$$g \triangleq \frac{x^* - x^L}{x^L},$$

where x^L is the average throughput of the OAR scheme [32] without using optimal stopping, and $x^L = \Phi(0)$.

We consider the following two cases: 1) the continuous rate case based on Shannon capacity, and 2) the discrete rate case based on IEEE 802.11b.

7.1.1 Example 1: The Continuous Rate Case for Homogeneous Networks

Consider the case that the transmission rate is given by the Shannon channel capacity:

$$R(h) = \log(1 + \rho h) \text{ nats/s/Hz},$$

where ρ is the normalized average SNR, and h is the random channel gain corresponding to Rayleigh fading. It follows from (5) that

$$x^* = \Phi(\rho, x^*) = \frac{x^* \exp\left(-\frac{\exp(x^*)}{\rho}\right) + E_1\left(\exp(x^*)/\rho\right)}{\frac{\exp(-1/\rho)\delta}{p_s} + \exp\left(-\frac{\exp(x^*)}{\rho}\right)}, \tag{44}$$

where $E_1(x)$ is the *exponential integral function* defined as

$$E_1(x) \triangleq \int_x^\infty \frac{\exp(-t)}{t} dt.$$

Note that (44) can be further simplified as

$$x^* = \frac{p_s}{\delta} \exp\left(\frac{1}{\rho}\right) E_1\left(\frac{\exp(x^*)}{\rho}\right). \tag{45}$$

We have the following results on the optimal throughput x^* and the throughput gain $g(\rho)$.

Proposition 7.1 *a) The optimal throughput x^* is an increasing function of the average SNR ρ.*

b) The throughput gain $g(\rho)$ is maximized when $\rho \longrightarrow 0$, and

$$g(\rho) \longrightarrow \left(1 + \frac{\delta}{p_s}\right) \left.\frac{dx^*(\rho)}{d\rho}\right|_{\rho=0} - 1, \quad \textit{as } \rho \longrightarrow 0, \tag{46}$$

where $\left.\frac{dx^*(\rho)}{d\rho}\right|_{\rho=0}$ *is the root of*

$$x \exp(x) \quad = \quad \frac{p_s}{\delta}. \tag{47}$$

The proof is relegated in Appendix L.

Remarks: Proposition 7.1 reveals that the maximum gain is achieved in the low SNR region. In the extreme case when $\rho \to 0$, the gain is determined by the system parameters δ and p_s only. From (46) and (47), it is not difficult to see that the throughput gain increases as δ decreases or p_s increases. This is because a smaller δ or a larger p_s indicates that the probing cost is relatively insignificant.

We provide numerical examples to illustrate the above results. Unless otherwise specified, we assume that τ, T, p_s, and M are chosen such that $\delta = 0.1, p_s = \exp(-1)$.

Fig. 4 depicts $\Phi(\rho, x)$ as a function of x, for different ρ. It can be seen that the optimal average throughput x^* is strictly increasing in ρ. In Table 1, we examine the convergence of the iterative algorithm in (12). It can be seen that the convergence speed of the iterative algorithm in (12) is fast, and the iterates approaches x^* usually within three or four iterations indifferent to the initial value x_0.

Table 2 illustrates that $g(\rho)$ is more significant in the low SNR region, and is a decreasing function of ρ. In Table 3, we present the maximum throughput gain $g(0)$ as a function of δ/p_s. It can be observed that

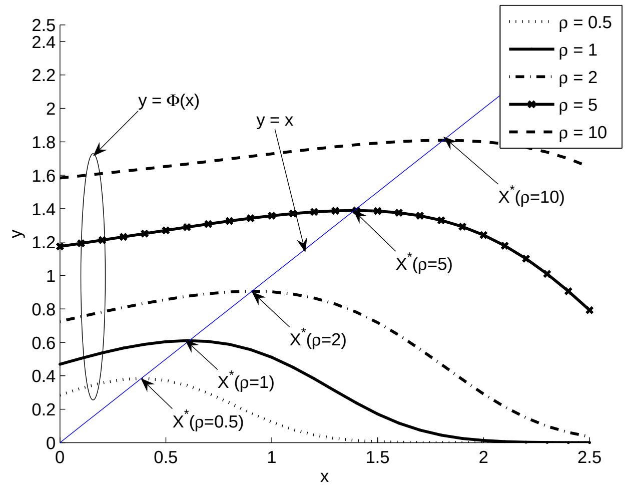

Figure 4: $\Phi(x)$ vs. x

Table 1: Convergence behavior of the iterative algorithm in (12)

ρ	x_0	x_1	x_2	x_3	x^*
0.5	0.5	0.372213	0.384157	0.384283	0.384
1	0.5	0.603993	0.610418	0.610442	0.610
2	1.0	0.902320	0.906009	0.906014	0.906
5	1.0	1.357985	1.389121	1.389379	1.389
10	1.0	1.728041	1.807727	1.809031	1.809

$g(0)$ increases as the value of δ/p_s decreases. Intuitively speaking, a smaller value of δ indicates that the channel probing incurs less overhead; and a larger value of p_s implies that the random access scheme yields higher throughput.

Table 2: Throughput gain

ρ	0.5	1	2	5	10
x^*	0.40	0.60	0.90	1.40	1.80
x^L	0.28	0.47	0.73	1.17	1.58
$g(\rho)$	42.8%	27.7%	23.3%	19.7%	13.9 %

7.1.2 Example 2: The Discrete Rate Case for Homogeneous Networks

Next, we study an example based on IEEE 802.11b, in which the transmission rates can be 2Mbps, 5.5Mbps and 11Mbps, with the following distribution:

$$R(h) = \begin{cases} 2 & \text{w.p. } p_2 = \frac{P(\gamma_2 \leq \rho h < \gamma_{5.5})}{P(\rho h \geq \gamma_2)} \\ 5.5 & \text{w.p. } p_{5.5} = \frac{P(\gamma_{5.5} \leq \rho h < \gamma_{11})}{P(\rho h \geq \gamma_2)} \\ 11 & \text{w.p. } p_{11} = \frac{P(\gamma_{11} \leq \rho h)}{P(\rho h \geq \gamma_2)}, \end{cases} \tag{48}$$

where $\gamma_2, \gamma_{5.5}$ and γ_{11} are the minimum SNR thresholds to support transmission rates of 2Mbps, 5.5Mbps and 11Mbps respectively.

Needless to say, the optimal throughput can be computed by using the general iterative algorithm presented in (12). However, since the number of quantization levels is small (i.e., 3 in this case), we can use "trial and error" to obtain the optimal throughput x^* as:

$$x^*(\rho) = x^L \mathbf{I}\left(x^L < 2\right) + \frac{5.5p_{5.5} + 11p_{11}}{\frac{\delta}{p_s} + 1 - p_2} \mathbf{I}\left(2 \leq \frac{5.5p_{5.5} + 11p_{11}}{\frac{\delta}{p_s} + 1 - p_2} < 5.5\right) + \frac{11p_{11}}{\frac{\delta}{p_s} + p_{11}} \mathbf{I}\left(5.5 \leq \frac{11p_{11}}{\frac{\delta}{p_s} + p_{11}} < 11\right),$$

Table 3: Maximum throughput gain

δ/p_s	0.136	0.271	0.544	1.359	2.718
g (numerical)	76.4%	47.0%	25.7%	9.2%	3.5%
g (by (46))	76.6%	47.2%	25.7%	9.2%	3.5%

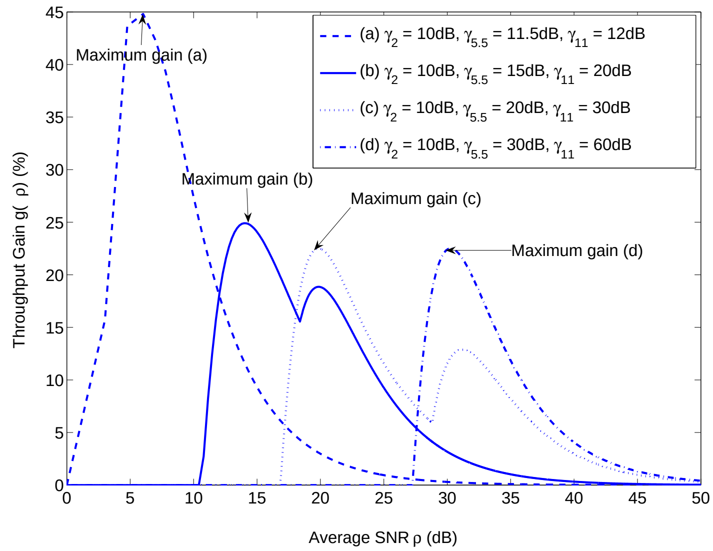

Figure 5: Throughput gain $g(\rho)$ as a function of average SNR ρ

where x^L is given by Proposition 4.2, $p_2, p_{5.5}$ and p_{11} can be computed from (48), and $\mathbf{I}(\cdot)$ is the indicator function.

As in centralized opportunistic scheduling, significant multiuser diversity gain can be achieved if the rate exhibits enough variation. Indeed, this can be observed in Fig. 5, in which we plot the throughput gain of distributed opportunistic scheduling for different sets of thresholds $\{\gamma_2, \gamma_{5.5}, \gamma_{11}\}$.

7.1.3 Example 3: The Continuous Rate Case for Heterogeneous Networks

Based on (10), it can be shown that the optimal threshold for the heterogeneous case, x^*, satisfies the following equation:

$$x^* = \frac{1}{\delta} \sum_{m=1}^{M} p_{s,m} \exp\left(\frac{1}{\rho_m}\right) E_1\left(\frac{\exp(x^*)}{\rho_m}\right). \tag{49}$$

Note that the average throughput without using optimal stopping rule is given by

$$x^L = \frac{\sum_{m=1}^{M} p_{s,m} \exp(1/\rho_m) E_1\left(1/\rho_m\right)}{\delta + p_s}. \tag{50}$$

In the following example, we consider a heterogeneous network model with 5 users, each with different transmission probabilities and channel statistics. The performance of the iterative algorithm in (12) is examined in Table 4. Clearly, the iterative algorithm in (12) exhibits fast convergence rate.

As is clear in (49), the optimal threshold x^* (namely the maximum throughput) depends on all SNR parameters $\{\rho_m, \forall\, m\}$ across links, and is monotonically increasing in each ρ_m. However, different from the homogeneous case, the gain g is no longer monotonically decreasing in each individual SNR. To get a more concrete sense, we plot in Fig. 6 the relationship between g and ρ_1, with other SNR parameters fixed. As illustrated in the figure, g decreases as ρ_1 increases from -10dB to 10dB. This is because when ρ_1 is small, the optimal throughput x^* is determined mainly by other SNR parameters and remains almost constant, whereas the throughput without using optimal stopping strategy (x^L) always increases. Furthermore, g increases when ρ_1 exceeds 10dB. Our intuition is that in this SNR regime user 1 becomes the dominating user in the system, and therefore x^* increases much faster than x^L.

Table 4: Convergence behavior of the iterative algorithm in (12)

ρ (dB)	x_0	x_1	x_2	x_3	x^*
[0 10 10 8.5 6]	0.684	1.259	1.382	1.385	1.385
[10 10 10 8.5 6]	0.026	1.620	1.877	1.892	1.892
[20 10 10 8.5 6]	0.777	2.695	3.054	3.073	3.073

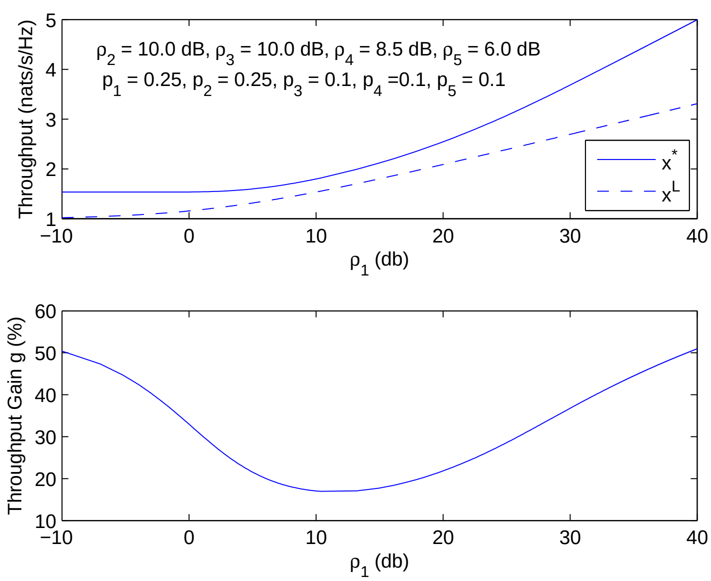

Figure 6: Throughput gain $g(\rho)$ as a function of average SNR ρ_1

7.2 Numerical Examples for the Non-Cooperative Game

In this section, we provide numerical examples to illustrate the above results. Unless otherwise specified, we assume that τ, T, p_s and M are chosen such that $\delta = 0.1$, $p_s = \exp(-1)$, and $p_{s,m} = p_s/M$.

Table 5 illustrates the convergence behavior of the best response strategy in (31), for 2 links randomly picked from the 5 links. It can be seen that with the knowledge of global information, the threshold converges to the optimal point within a few iterations. For comparison, Table 6 shows the convergence behavior of the "pseudo-best" response strategy in (32), which takes more iterations to converge. Fig. 7 depicts the convergence behavior of the online algorithm for computing Nash Equilibrium. As expected, it takes hundreds of iterations for the proposed asynchronous distributed stochastic algorithm to converge. Moreover, all three algorithms converge to the same equilibrium point.

Table 5: Convergence behavior of the best response strategy

Link index	x_0	x_1	x_2	x_3	x^*
Link 1 ($\rho_1 = 3$dB)	1.00	0.267	0.298	0.300	0.30
Link 2 ($\rho_2 = 5$dB)	1.00	0.175	0.389	0.390	0.39

Table 6: Convergence behavior of the "pseudo-best" response strategy

Link index	x_0	x_1	x_2	x_3	x_4	x_5	x^*
Link 1 ($\rho_1 = 3$dB)	1.00	0.360	0.293	0.299	0.300	0.300	0.30
Link 2 ($\rho_2 = 5$dB)	1.00	0.108	0.386	0.388	0.388	0.390	0.39

7.3 Numerical Examples for Price of Anarchy

We also present in Table 7 the efficiency loss due to the selfish behavior of individual links, i.e., the price of anarchy. It can be seen from Table 7 that the efficiency is strictly less than 1 when two or more links exist in the network, which corroborates the conclusion of Proposition 6.1. Moreover, the efficiency η decreases as the number of links M increases, as illustrated in Fig. 8. When M goes to ∞, the total throughput for non-cooperative game x^*_{nco} converges to x^L, which implies that every link transmits with threshold $x^*_m = 0$. Furthermore, η approaches to $1/(1+g)$, where g is the throughput gain. Our intuition is as follows: In the non-cooperative game, when the number of links increases, the effective channel probing time in (18) increases as well. As a result, the thresholds across links decrease and approach zero.

In Table 7, we also present the efficiency improvement by using the pricing mechanism. Let $x^*_{pricing}$ denote the network throughput at the Nash equilibrium for the non-cooperative game with pricing $\tilde{\mathbf{G}}$ defined in

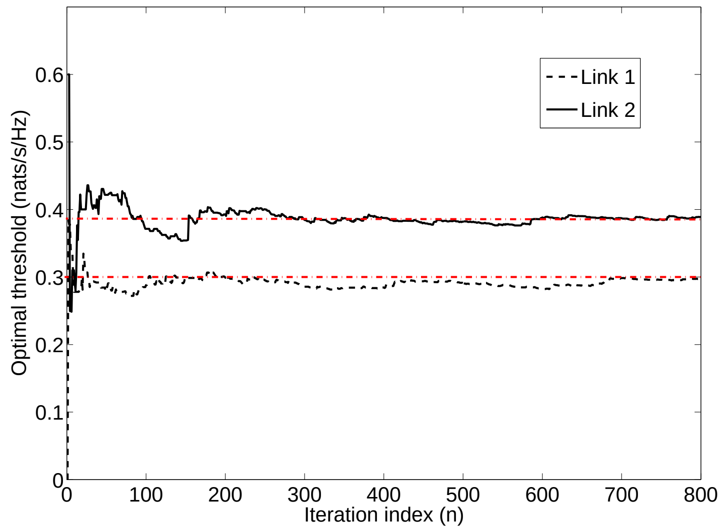

Figure 7: Convergence behavior of the online algorithm for computing Nash Equilibrium

(43). The efficiency η' is defined as $\eta' = x^*_{pricing}/x^*_{co}$. It can be seen from Table 7 that by carefully choosing the parameter c, the efficiency loss can be significantly reduced. However, it is still unable to achieve the optimal throughput in the team game case (the social optimum).

8 Conclusions

In this study, we considered an ad-hoc network model where many links contend for the channel using random access, and studied distributed opportunistic scheduling (DOS) to resolve collisions therein while exploiting

Table 7: The Price of Anarchy

Number of links	1	2	3	4	5
x^*_{co}	0.586	0.664	1.085	1.217	1.364
x^*_{nco}	0.586	0.624	0.994	1.043	1.127
η	100%	94.0%	91.6%	85.7%	82.6%
$x^*_{pricing}$	0.586	0.650	1.055	1.170	1.293
η'	100%	97.9%	97.2%	96.1%	94.8%

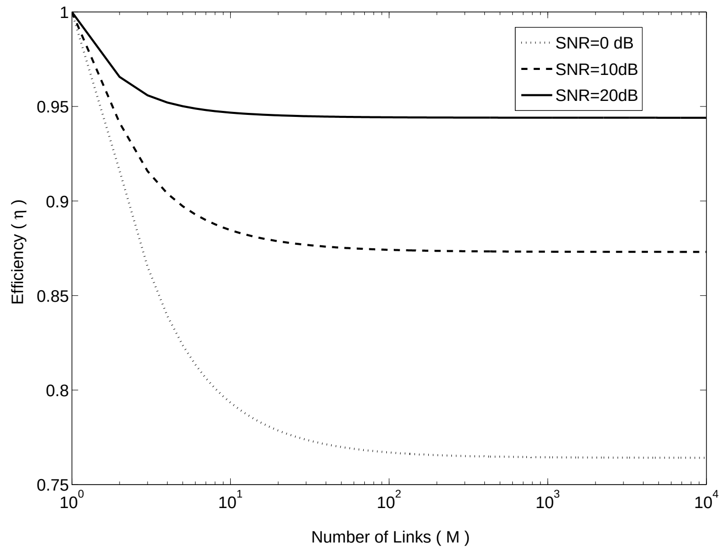

Figure 8: The efficiency η as the number of links M

channel variation for data transmission. In such a network, distributed opportunistic scheduling boils down to a process of joint channel probing and distributed scheduling. We first investigated DOS from a network-centric point of view, where links cooperate to maximize the overall network throughput. Specifically, we treated the joint process of channel probing and scheduling as a maximal-rate-of-return problem, and characterized the optimal strategies, for both homogenous networks and heterogeneous networks. We showed that the optimal DOS strategy is a pure threshold policy, where the threshold is the solution to a fixed point equation. Furthermore, we devised iterative algorithms to compute it.

Next, we studied distributed opportunistic scheduling from a user-centric perspective, where links seek to maximize their own throughput in a selfish manner. We treated the problem of threshold selection across different links as a non-cooperative game. Then, we explored the existence and uniqueness of the Nash equilibrium, and showed that the Nash equilibrium can be approached by the best response strategy. We then developed an online stochastic iterative algorithm based on local observations only, and we established its convergence under some regularity conditions, using recent results on asynchronous stochastic approximation algorithms. As expected, we observed an efficiency loss at the Nash equilibrium, and we proposed a pricing-based mechanism to mitigate the efficiency loss.

In summary, in this paper, we took some initial steps towards studying channel-aware distributed scheduling in ad-hoc networks from two different perspectives, namely, a network-centric perspective and a user-centric perspective. In particular, building on optimal stopping theory, we characterized the fundamental tradeoff between the throughput gain from better channel conditions and the cost for further channel probing, and explored channel-aware distributed scheduling to exploit multiuser diversity and time diversity in an opportunistic manner. Our findings in this study reveal that rich PHY/MAC diversities can be achieved by devising channel-aware scheduling in ad-hoc networks.

Clearly, the coupling between the time-scales of fading and MAC calls for unified PHY/MAC optimization. It is of great interest to generalize this study to multi-hop ad-hoc networks, and develop channel-aware scheduling for real-time traffic. We are currently pursuing a theoretic foundation of channel-aware distributed scheduling along these avenues.

9 Acknowledgements

We thank Prof. R. Srikant for his insightful suggestions on introducing pricing mechanisms for studying the user-centric case.

Appendices

A Proof of Proposition 4.1

The proof of Proposition 4.1 hinges heavily on the tools in optimal stopping theory [18]. More specifically, based on Theorem 1 in [18, Chapter 6], in order to maximize the average throughput $\frac{E[R_{(N)}T]}{E[T_N]}$, a key step is to find an optimal stopping algorithm $N(x)$ such that

$$V^*(x) = E[R_{(N(x))}T - xT_{N(x)}] = \sup_{N \in Q} E[R_{(N)}T - xT_N].$$

It then follows from Theorem 1 in [18, Chapter 3] that $N(x)$ exists if the following conditions are satisfied:

$$E\sup_{n} Z_n < \infty, \quad \text{and} \quad \limsup_{n \longrightarrow \infty} Z_n = -\infty \ a.s., \tag{51}$$

where $Z_n \triangleq R_{(n)}T - xT_n$, $T_n \triangleq \sum_{j=1}^{n} K_j\tau + T$, and $K_j, j = 1, 2, \ldots, n$, denote the number of contentions during the jth channel probing.

The rest of the proof has two main steps. Step 1: we establish the existence of the optimal stopping rule $N(x)$; Step 2: we characterize the optimal strategy N^*.

Step 1: It is clear that $\limsup_{n\to\infty} Z_n \longrightarrow -\infty$.

Observe that $E[\sup_n Z_n]$ is bounded above by

$$E[\sup_n Z_n] \leq E\left[\sup_n \left\{R_{(n)}T - nx\tau\left(\frac{1}{p_s} - \epsilon\right)\right\}\right] - Tx + E\left[\sup_n \sum_{j=1}^{n} x\tau\left(\frac{1}{p_s} - \epsilon - K_j\right)\right], \tag{52}$$

where ϵ is chosen such that $0 < \epsilon < 1/p_s$. It then follows from the Maximal Inequalities in Theorem 1 and Theorem 2 in [18, Chapter 4] that the first term and the last term of the right hand side of (52) are both finite, and hence $E[\sup_n Z_n] < \infty$.

Step 2: Next, we characterize $N(x)$ and N^*. It can be shown that $N(x)$ is given by

$$N(x) = \min\{n \geq 1 : R_{(n)}T \geq V^*(x) + xT\}, \tag{53}$$

and $V^*(x)$ satisfies the following *optimality equation*:

$$E[\max(R_{(n)}T - xT - Kx\tau, V^*(x) - Kx\tau)] = V^*(x). \tag{54}$$

Note that $V^*(x^*) = 0$ from Theorem 1 in [18, Chapter 6] and (54) becomes $E[R_{(n)} - x^*]^+ = \frac{x^*\tau}{p_s T}$ since $E[K] = 1/p_s$. The optimal stopping rule (53) now becomes $N^* = \min\{n \geq 1 : R_{(n)} \geq x^*\}$.

Next we show that (5) has a unique solution. We first note that $f(x) \triangleq E[R_{(n)} - x]^+$ is continuous in x. To see this, let $\{x_l, l = 1, 2, \ldots\}$ be a sequence of real positive numbers, and $\lim_{l\to\infty} x_l = x$, then $R_{(n)} - x_l \to R_{(n)} - x$ almost surely. Since $|R_{(n)} - x_l| \leq R_{(n)}$, we have that $f(x_l) \to f(x)$ by using Dominated Convergence Theorem [15]. Since $f(x)$ decreases from $E[R_{(n)}]$ to 0 and the right hand side of (5) strictly increases from 0 to ∞ as x grows, it follows that (5) has a unique finite solution. ∎

B Proof of Proposition 4.2

It is clear that x^L is achieved by a special stopping algorithm (which stops at the very first time). Therefore, by the definition of x^*, $x^L \leq x^*$.

To show that x^U is an upper-bound on x^*, recall that from Remark 3) for Prop. 4.1, replacing the random contention period, $K\tau$, with a constant access time, τ/p_s, would yield the same optimal long-term average rate x^*. Accordingly, the upper-bound derived for the constant access time case also serves an upper-bound on x^*.

Observe that for any constant x,

$$E\left[\sup_n \left\{R_{(n)}T - x \cdot \left(\frac{\tau}{p_s}n + T\right)\right\}\right] = E\left[\sup_n \left(R_{(n)}T - x\frac{\tau}{p_s}n\right)\right] - xT \leq \frac{E[T^2R^2]}{2\frac{x\tau}{p_s}} - xT. \tag{55}$$

Plugging $x = \sqrt{\frac{E[R^2]}{2\frac{\delta}{p_s}}}$ yields that

$$E\left[\sup_n \left\{ R_{(n)}T - \sqrt{\frac{E[R^2]}{2\frac{\delta}{p_s}}} \cdot \left(\frac{\tau}{p_s}n + T\right)\right\}\right] \leq 0. \tag{56}$$

Furthermore, we have that

$$E\left[R_N^* T - x^* \cdot \left(\frac{\tau}{p_s}N^* + T\right)\right] = 0. \tag{57}$$

Combining (56) and (57), we have that

$$\begin{aligned} E\left[\sup_n \left\{ R_n T - \sqrt{\frac{E[R^2]}{2\frac{\delta}{p_s}}} \cdot \left(\frac{\tau}{p_s}n + T\right)\right\}\right] &\leq E\left[R_{N^*}T - x^* \cdot \left(\frac{\tau}{p_s}N^* + T\right)\right] \\ &\overset{(a)}{=} \sup_{N \in Q} E\left[R_N T - x^* \cdot \left(\frac{\tau}{p_s}N + T\right)\right] \\ &\overset{(b)}{\leq} E\left[\sup_n \left\{ T R_n - x^* \cdot \left(\frac{\tau}{p_s}n + T\right)\right\}\right], \end{aligned} \tag{58}$$

where (a) is by the definition of N^*, and (b) can be obtained using the same technique as in Fatou's Lemma [8].

It is clear that for any $x_1 \leq x_2$,

$$E\left[\sup_n \left\{ R_{(n)}T - x_1 \cdot \left(\frac{\tau}{p_s}n + T\right)\right\}\right] \geq E\left[\sup_n \left\{ R_{(n)}T - x_2 \cdot \left(\frac{\tau}{p_s}n + T\right)\right\}\right].$$

It follows from (58) that $x^* \leq \sqrt{\frac{E[R^2]}{2\frac{\delta}{p_s}}}$. ∎

C Proof of Proposition 4.4

It can be shown that $\Phi(x)$ is the average network throughput under the following stopping rule:

$$N = \min\{n \geq 1 : R_{(n)} \geq x\}.$$

It then follows from Proposition 4.1 that x^* is a global maximum point of $\Phi(x)$, i.e.,

$$x^* = \max_x \Phi(x). \tag{59}$$

From (59) and Proposition 4.3, it is clear that $y = \Phi(x)$ only intersects $y = x$ at the point x^*. See Fig. 9 for a pictorial illustration. This, together with the fact that $\Phi(0) > 0$, yields that

$$\Phi(x) \geq x, \forall\ x \leq x^*; \Phi(x) \leq x, \forall\ x > x^*. \tag{60}$$

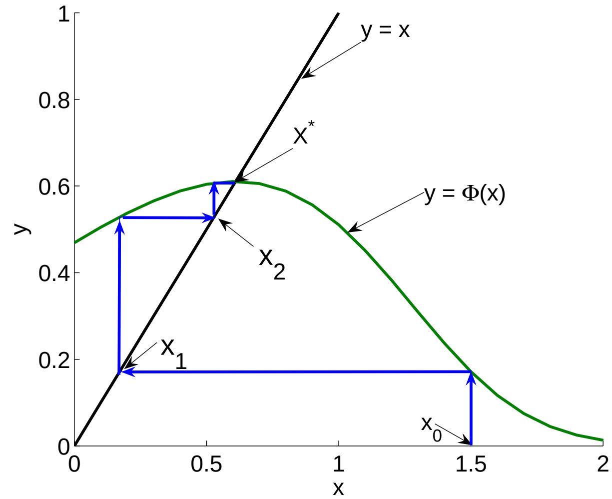

Figure 9: Convergence of the iterative algorithm (12).

Without loss of generality, we can assume that $x_0 \le x^*$ (we note that if $x_0 > x^*$, $x_1 = \Phi(x_0) \le \Phi(x^*) = x^*$ according to (59)). Next, suppose that $x_k \le x^*$. From (60), we obtain that $x_k \le \Phi(x_k) = x_{k+1} \le x^*$, where the last inequality is due to the fact that $\Phi(x_k) \le \Phi(x^*) = x^*$ from (59). Since $0 < x_0 \le x^*$, it follows that $\{x_k, k = 1, 2, \ldots\}$ is a monotonically increasing positive sequence with an upper-bound x^*. As a result, the sequence $\{x_k, k = 1, 2, \ldots\}$ converge to a limit, denoted as x_∞.

To show that $x_\infty = x^*$, we rewrite $x_{k+1} = \Phi(x_k)$ as

$$E[R_{(n)} - x_k]^+ - x_k \frac{\delta}{p_s} = (x_{k+1} - x_k) \left(\frac{\delta}{p_s} + \sum_{m=1}^{M} \frac{p_{s,m}}{p_s} (1 - F_m(x_k)) \right). \qquad (61)$$

Observe that $E[R_{(n)} - x]^+$ is continuous in x (see the proof of Proposition 4.1), $x_{k+1} - x_k \to 0$ as $k \to \infty$, and $\frac{\delta}{p_s} + \sum_{m=1}^{M} \frac{p_{s,m}}{p_s} (1 - F_m(x_k)) \le \frac{\delta}{p_s} + 1 < \infty$. Therefore, taking limits on both sides of (61) yields that $E[R_{(n)} - x_\infty]^+ - \delta x_\infty / p_s = 0$. It follows from Proposition 4.3 that $E[R_{(n)} - x]^+ = x\frac{\delta}{p_s}$ has a unique solution, we conclude that $x_\infty = x^*$. ∎

D Proof of Proposition 5.1

To establish the existence of a Nash equilibrium for the threshold selection game, we apply [28, Proposition 20.3], which requires that the action set A_m is a non-empty compact convex set for any m and the utility function $\phi_m(\cdot)$ is quasi-concave on A_m. Recall that a function $f : \mathscr{R}^n \to [0, \infty)$ is quasi-concave if the sublevel sets $S_c = \{x | f(x) \geq c\}$ are convex for all c [6]. To this end, rewrite $\phi_m(x, \mathbf{x}_{-m}) \geq c$ as

$$\hat{\phi}_m(x) \triangleq p_{s,m} \int_x^{\infty} (r-c) dF_m(r) - c\delta - c \sum_{j \neq m} p_{s,j} \left(1 - F_j(x_j)\right) \geq 0. \tag{62}$$

Then, it suffices to show that for any given c, $\hat{\phi}_m(x)$ is quasi-concave in $[0, \infty)$.

Observe that for any given c, $\hat{\phi}_m(x)$ is non-decreasing in $[0, c]$, and is non-increasing in $[c, \infty)$. It follows from [6] that $\hat{\phi}_m(x)$ is a quasi-concave function in $[0, \infty)$, which implies that for any c, the sublevel set $S_c = \{x | \phi_m(x, \mathbf{x}_{-m}) \geq c\}$ is convex. Thus, the Nash equilibrium for the non-cooperative game $\mathbf{G}$ exists.

Observe from Proposition 4.3 that $\phi_m(x_m, \mathbf{x}^*{}_{-m})$ is maximized at x_m^*, which is the unique solution to the equation $x_m = \phi_m(x_m, \mathbf{x}_{-m}^*)$. By the definition of Nash equilibrium, $\mathbf{x}^*$ is the Nash equilibrium, thereby concluding the proof. ∎

E Proof of Proposition 5.3

We need the following lemma first.

Lemma E.1 *If $\mathbf{x} = [x_1, x_2, \cdots, x_M]^T$ is a Nash equilibrium, then all its elements are equal, i.e., $x_1 = x_2 = \cdots = x_M$.*

Proof: Assume that there exists a Nash Equilibrium with unequal elements. Without loss of generality, assume that $x_1 > x_2$. It follows from (27) that

$$\begin{aligned} x_1 &= \phi_1(x_1, \mathbf{x_{-1}}) = \phi_2(x_1, \mathbf{x_{-1}}), \\ x_2 &= \phi_2(x_2, \mathbf{x_{-2}}) = \phi_1(x_2, \mathbf{x_{-2}}), \end{aligned} \tag{63}$$

which indicates that $\mathbf{x}' = [x_2, x_1, x_3 \cdots, x_M]^T$ is also a Nash equilibrium. This contradicts the component-wise monotonicity of Nash equilibria, as shown in Proposition 5.2. ∎

Lemma E.1 indicates that Nash equilibrium satisfies the equation $x = \phi_m(x, x, \cdots, x)$. Conversely, based on Proposition 5.1, the solutions of the equation $x = \phi_m(x, x, \cdots, x)$ are Nash equilibria. ∎

F Proof of Proposition 5.4

It suffices to show that the equation $d(x) = 0$ has unique solution. To this end, rewrite $d(x)$ as

$$d(x) = \delta x / p_s + \frac{M-1}{M} x e^{((1-e^x)/\rho)} - \frac{1}{M} e^{(1/\rho)} E_1(e^x/\rho), \tag{64}$$

where $E_1(x)$ is the *exponential integral function* defined as

$$E_1(x) \triangleq \int_x^\infty \frac{\exp(-t)}{t} dt. \tag{65}$$

Then, the derivative of $d(x)$ is given by

$$d'(x) = \delta/p_s + e^{((1-e^x)/\rho)} - \frac{M-1}{M} x e^{((1-e^x)/\rho)} e^x/\rho. \tag{66}$$

To show that $d(x) = 0$ has unique solution, we need the following lemmas.

Lemma F.1 $\frac{e^{-x}}{x} > E_1(x), \quad \forall\ x > 0$

Proof: It is clear that

$$\left(\frac{e^{-x}}{x} - E_1(x)\right)' = -e^{-x}/x^2 < 0, \quad \forall\ x > 0 \tag{67}$$

Moreover,

$$\left(\frac{e^{-x}}{x} - E_1(x)\right) \to 0, \text{ as } x \to \infty, \tag{68}$$

indicating that $\frac{e^{-x}}{x} > E_1(x), \quad \forall\ x > 0.$ ∎

Lemma F.2 $d(x) > 0, \ \forall\ x \in \{x > 0 | d'(x) = 0\} > 0.$

Proof:

It follows from (66) that if $d'(x) = 0$, then

$$(M-1)xe^x/\rho > M. \tag{69}$$

It follows that

$$\begin{aligned} d(x) &\overset{(a)}{>} \frac{(M-1)}{M} x e^{((1-e^x)/\rho)} - \frac{e^{(1/\rho)}}{M} E_1(e^x/\rho) \\ &= \frac{e^{(1/\rho)}}{M}[(M-1)xe^{(-e^x/\rho)} - E_1(e^x/\rho)] \\ &\overset{(b)}{>} \frac{e^{(1/\rho)}}{M}\left[M\frac{e^{(-e^x/\rho)}}{e^x/\rho} - E_1(e^x/\rho)\right] \\ &\overset{(c)}{>} 0, \end{aligned} \tag{70}$$

where (a) follows from $\delta x/p_s > 0$, (b) from equation (69), and (c) from Lemma F.1. ∎

Clearly, $d(0) < 0$ and $d(\infty) > 0$ implies that the solution to $d(x) = 0$ exists. Next, suppose that the equation $d(x) = 0$ has more than one solutions. Then, it can be shown that there exists an $x_0 > 0$ such that $d'(x_0) = 0$ and $d(x_0) \leq 0$, which contradicts Lemma F.2, thereby concluding the proof. ∎

G Proof of Proposition 5.6

Let $\psi_m(\mathbf{x})$ denote the unique solution to the fixed point equation $x = \phi_m(x, \mathbf{x}_{-m})$, for $m = 1, 2, \cdots, M$, and $\Psi(\cdot) = [\psi_1(\cdot), \psi_2(\cdot), \cdots, \psi_M(\cdot)]^T$. Note that $\Psi(\mathbf{x})$ is monotonically increasing on $\mathbf{x}$ since for any $\mathbf{x}^1 > \mathbf{x}^2 \geq \mathbf{0}$, we have that

$$\psi_m(\mathbf{x}^1) = \max_x \phi_m(x, \mathbf{x}^1_{-m}) \geq \max_x \phi_m(x, \mathbf{x}^2_{-m}) = \psi_m(\mathbf{x}^2).$$

Given any non-negative initial value $\mathbf{x}(0)$, it follows that

$$\begin{array}{rcl}
\mathbf{0} \leq & \mathbf{x}(0) & \leq \infty \\
\Psi(\mathbf{0}) \leq & \Psi(\mathbf{x}(0)) = \mathbf{x}(1) & \leq \Psi(\infty) \\
\Psi^2(\mathbf{0}) \triangleq \Psi(\Psi(\mathbf{0})) \leq & \Psi(\mathbf{x}(1)) = \mathbf{x}(2) & \leq \Psi(\Psi(\infty)) \triangleq \Psi^2(\infty) \\
\vdots & \vdots & \vdots \\
\Psi^k(\mathbf{0}) \leq & \Psi(\mathbf{x}(k-1)) = \mathbf{x}(k) & \leq \Psi^k(\infty)
\end{array} \tag{71}$$

It is clear that the sequence $\{\Psi^k(\mathbf{0}),\ k = 1, 2, \cdots\}$ are monotonically increasing and bounded above, and as a result, the sequence converge to a limit, denoted as $\Psi^\infty(\mathbf{0})$. To show that $\Psi^\infty(\mathbf{0}) = \mathbf{x}^*$, by definition, we have that

$$\psi^\infty_m(\mathbf{0}) = \phi_m(\psi^\infty_m(\mathbf{0}), \Psi^\infty_{-m}(\mathbf{0})),\ \forall\ m = 1, 2, \cdots, M,$$

which indicates that $\Psi^\infty(\mathbf{0})$ satisfies (21), and thus it is a Nash equilibrium. By the assumption on uniqueness of the Nash equilibrium, we have that $\Psi^\infty(\mathbf{0}) = \mathbf{x}^*$.

Similarly, we can show that the sequence $\{\Psi^k(\infty),\ k = 1, 2, \cdots\}$ are monotonically decreasing and bounded below, and thus also converge to $\mathbf{x}^*$. Using a sandwich argument, let $k \to \infty$ in (71), it follows that the sequence $\{\mathbf{x}(k),\ k = 1, 2, \cdots\}$ converge to $\mathbf{x}^*$. ∎

H Proof of Proposition 5.7

We first show by induction that $\{\mathbf{x}(k)\}$ converges. Define $x^*_{m,k} \triangleq \max_x \phi_m(x, \mathbf{x}_{-m}(k))$ Given the initial value $x_m(0) = 0, \forall\ m$, we have that

$$0 < x_m(1) = \phi_m(0, \mathbf{0}) \leq x^*_{m,0}. \tag{72}$$

Next, suppose that $x_m(k) \geq x_m(k-1)$, and $x_m(k) \leq x^*_{m,k-1}$ $\forall$ m. Then, observe that

$$\begin{aligned} x_m(k+1) &= \phi_m(x_m(k), \mathbf{x}_{-m}(k)) \\ &\geq \phi_m(x_m(k), \mathbf{x}_{-m}(k-1)) \\ &\geq \phi_m(x_m(k-1), \mathbf{x}_{-m}(k-1)) \\ &= x_m(k). \end{aligned} \tag{73}$$

Moreover,

$$x_m(k+1) = \phi_m(x_m(k), \mathbf{x}_{-m}(k)) \leq \max_x \phi_m(x, \mathbf{x}_{-m}(k)) = x^*_{m,k}. \tag{74}$$

It follows that $\forall$ m, $\{x_m(k),\ k = 1, 2, \cdots\}$ is a monotonically increasing sequence with an upper-bound $x^U_m = \frac{p_{s,m}}{\delta} \int_0^\infty r dF_m(r)$. As a result, the sequence $\{\mathbf{x}(k),\ k = 1, 2, \cdots\}$ converge to a limit, denoted as $\mathbf{x}(\infty)$.

To show that $\mathbf{x}(\infty)$ is a Nash equilibrium, using the similar argument as in the proof of Proposition 4.3, we can take limits on both sides of (32) yields that

$$x_m(\infty) = \phi_m(x_m(\infty), \mathbf{x}_{-m}(\infty)),\ \forall\ m, \tag{75}$$

which indicates that $\mathbf{x}(\infty)$ satisfies (21), thereby concluding the proof. ∎

I Proof of Lemma 5.2

Since $x_m(k)$ is $\mathcal{F}_k$ measurable, it follows that

$$\begin{aligned} & E[w_m(k)|\mathcal{F}(k)] \\ = \ & E\left[v(k) \left[p_{s,m} \int_{x_m(k)}^\infty r dF_m(r) - \delta x_m(k) \right] - \frac{p_{s,m} \int_{x_m(k)}^\infty r dF_m(r) - \delta x_m(k)}{\sum_{i=1}^M p_{s,i}(1 - F_i(x_i(k)))} \middle| \mathcal{F}(k) \right] \\ = \ & \left[p_{s,m} \int_{x_m(k)}^\infty r dF_m(r) - \delta x_m(k) \right] \left[E[v(k)|\mathcal{F}_n] - \frac{1}{\sum_{i=1}^M p_{s,i}(1 - F_i(x_i(k)))} \right] \\ = \ & 0. \end{aligned} \tag{76}$$

Define $\psi_m(k) \triangleq \sum_{t=1}^{k-1} a_m(t) w_m(t)$, then $\psi_m(k)$ is a martingale, since

$$E[\psi_m(k+1)|\mathcal{F}(k)] = E\left[a_m(k) w_m(k)|\mathcal{F}(k)\right] + E[\psi_m(k)|\mathcal{F}(k)] = \psi_m(k). \tag{77}$$

Moreover,

$$E\left[\sup_{k \geq 1} a_m(k) w^+_m(k) \right] < \infty, \tag{78}$$

where $w^+(\cdot) \triangleq \max(w(\cdot), 0)$. By Theorem 7.4.4 in [16], $\psi(n)$ converges w.p.1. ∎

J Proof of Theorem 5.1

Proof: The proof has two main steps. Step 1: we study the process corresponding to interpolating the sequence $\{\mathbf{x}(k)\}$, and show that the effect of the asynchronism term and the unbiased estimation term would diminish as $t \to \infty$. Step 2: we establish the convergence of $\{\mathbf{x}(t)\}$ by appealing to the mean ordinary differential equation (ODE) method. The convergence of the original sequence $\{\mathbf{x}(k)\}$ then follows from the similar argument as in Section 1.3 in [14].

Towards this end, define

$$\begin{aligned} D_m(k) &\triangleq [x_m(k) + a_m(k)\widetilde{g_m}(\mathbf{x}(k))I\{n \in N^m\}]_0^b \\ &\quad - [x_m(k) + a_m(k)\widetilde{g_m}(\mathbf{x})I\{k \in N^m\}]\,. \\ S_m(k) &\triangleq g_m(\mathbf{x}). \end{aligned} \tag{79}$$

(34) can be rewritten as

$$\begin{aligned} x_m(k+1) = x_m(k) &+ D_m(k) \\ &+ a_m(k)S_m(k)I\{k \in N^m\} \\ &+ a_m(k)w_m(\mathbf{x}(k))I\{k \in N^m\}, \end{aligned} \tag{80}$$

where projection operation is contained by $D_m(k)$. The effects of asynchronism and observation error are contained by the last two terms, which would diminish when k goes to infinity, the problem will be converted to a classical projected ordinary differential equation problem. It follows from (80) that

$$\begin{aligned} x_m(k+n) = x_m(k) &+ \sum_{j=k}^{n+k-1} D_m(j) \\ &+ \sum_{j=k}^{n+k-1} a_m(j)S_m(j)I\{j \in N^m\} \\ &+ \sum_{j=k}^{n+k-1} a_m(j)w_m(\mathbf{x}(j))I\{j \in N^m\}, \end{aligned} \tag{81}$$

Lemma J.1 *For all $m \in \{1, 2, \cdots, M\}$*

$$\lim_{k\to\infty} \sum_{j=k}^{\infty} a_m(j)w_m(\mathbf{x}(j))I\{j \in N^m\} = 0, \quad a.s. \tag{82}$$

This lemma follows from Lemma 5.2.

For convenience, define

$$a(k) \triangleq \sum_{i=1}^{M} a_i(k) I\{i \in N^i\}, \tag{83}$$

and the time epochs

$$t_n \triangleq \sum_{k=0}^{n} a(k), \quad t_0 = 0. \tag{84}$$

Define the function $n(t)$ as

$$n(t) \triangleq \max\{n : t_n \leq t\} \tag{85}$$

Accordingly, define the linear interpolating function

$$\begin{aligned} x_m^0(t_n) &= x_m(n) \\ x_m^0(t) &= x_m^0(t_{n(t)} + (x_m^0(t_{n(t)+1}) - x_m^0(t_{n(t)})) \frac{t - t_{n(t)}}{t_{n(t)+1} - t_{n(t)}} \\ x_m^n(t) &= x_m^0(t + t_n). \end{aligned} \tag{86}$$

and

$$\begin{aligned} q_m^0(t_n) &= \sum_{k=0}^{n-1} a_m(k) w_m(k), \\ D_m^0(t_n) &= \sum_{k=0}^{n-1} D_m(k). \end{aligned} \tag{87}$$

Then, the corresponding linear interpolating function $q_m^n(t)$ and $D_m^n(t)$ are defined by (86) with $x_m^0(n)$ replaced by $q_m^0(n)$ and $D_m^0(n)$, respectively. Define

$$\gamma_m^0(s) = \begin{cases} 1, & I\{n(s) = m\} \\ 0, & \text{otherwise} \end{cases} \tag{88}$$

and

$$\gamma_m^k(s) = \gamma_m(t_k + s). \tag{89}$$

Then, it follows that the update algorithm in (81) can be rewritten as

$$x_m^n(t) = x(0) + \int_0^t S_m(\bar{x}(s)) \gamma_m^k(s) ds + q_m^n(t) + D_m^n(t), \tag{90}$$

where $\bar{x}_t = x(n),\ t \in [t_{n(t)}, t_{n(t)+1})$. By Lemma J.1, for finite t

$$\lim_{n\to\infty} q_m^n(t) = 0, \quad \forall\, m \in \{1, 2, \cdots, M\}, \tag{91}$$

which indicates that the term $q_m^n(t)$ has diminishing contributions as $n \to \infty$ and can be ignored in the limiting integral equation. Next, by appealing to the Arzela-Ascoli Theorem (Proposition 1.3.1 in [14]), we shall show that the sequence of $x_m^n(t)$ has convergent subsequence. Note that $x_m^n(t)$ and $D_m^n(t), \quad n = 0, 1, 2, \cdots$ are Lipschitz with the same Lipschitz constant, by Arzela-Ascoli Theorem, it follows that there exists a subsequence of $D_m^n(t)$ along which a limit exists. Similarly, we can pick a subsequence along which $x_m^n(t)$ converges to $x(t)$.

In order to show that the limiting function $x(t)$ has a deterministic integral equation description, the random term $\gamma_m^n(t)$ need to be removed.

Lemma J.2 *Given Condition* **B1, B2** *and Lemmas 5.3, 5.4, there exists* $\gamma_m^* > 0$ *such that*

$$\lim_{n\to\infty} \int_0^t f_m(x)\gamma_m^n(x)dx = \int_0^t f_m(x)\gamma_m^* dx. \tag{92}$$

The proof can be found in [9].

In summary

$$\lim_{n\to\infty} x_m^n(t) = \lim_{n\to\infty} x_m(0) + \lim_{n\to\infty} \int_0^t S_m(\bar{x}(s))\gamma_m^n(s)ds + \lim_{n\to\infty} q_m^n(t) + \lim_{n\to\infty} D_m^n(t).$$

By Lemma J.1 and J.2, we have the following deterministic integral equation for $x_m(t)$

$$x_m(t) = x_m(0) + \int_0^t S_m(x(s))\gamma_i^* ds + D_m(t). \tag{93}$$

It follows that the above integral equation has the equivalent differential equation representation as [14]

$$\frac{dx_m(t)}{dt} = \lim_{\Delta\to 0} \frac{[x_m(t) + \Delta\gamma_m^* S_m(x(t))]_0^b - x_m(t)}{\Delta}, \quad \forall\, m \in \{1, 2, \cdots, M\}. \tag{94}$$

It follows from the uniqueness of the Nash equilibrium and the similar argument as in Section 5 in [24] that $\mathbf{x}(k)$ converge to the Nash equilibrium $\mathbf{x}^*$, as $k \to \infty$. ∎

K Proof of Proposition 6.1

It is clear from Proposition 4.1 that $x_{co}^* \geq x_{nco}^*$. We next examine the efficiency loss due to non-cooperativity. We first present the following lemma.

Lemma K.1 *Consider the following nonlinear optimization problem:*

$$\mathbf{\Xi}: \quad \max_{\{0 \le x_m < \infty, m=1,2,\ldots,M\}} \quad \sum_{m=1}^{M} \phi_m(\mathbf{x}), \tag{95}$$

where ϕ_m is defined in (16). Then the optimal solution to Problem $\mathbf{\Xi}$ in (95) is $x^*\mathbf{u}$, where $\mathbf{u} = [1, \ldots, 1]$, and x^* is the unique solution to (10).

Proof: First, take derivative of the objective function in $\mathbf{\Xi}$ with respect to $\{x_m\}$. After some algebra, it turns out that

$$\sum_m \phi_m(\mathbf{x}) = x_m, \quad m = 1, 2, \ldots, M, \tag{96}$$

which indicates all x_m^* are the same at the optimal point. Let $x_t = x_m^*, \forall\, m$. It follows from (96) that x_t is the solution of the following fixed point equation

$$x = \frac{\sum_{m=1}^{M} p_{s,m} \int_x^\infty r dF_m(r)}{\delta + \sum_{m=1}^{M} p_{s,m}\left(1 - F_m(x)\right)}, \tag{97}$$

which is exactly (10). Since the solution of (10) is unique, we have that $x_t = x^*$. ∎

Clearly, $x_{co}^* \ge x_{nco}^*$. Next, we prove the second part of Proposition 6.1 by showing that the equality cannot be achieved. To this end, it is sufficient to examine the following two cases: 1) if the components of $\mathbf{x}^*$ are not the same, then Lemma K.1 implies that $x_{co}^* > x_{nco}^*$. 2) If the components of $\mathbf{x}^*$ are the same, say $\mathbf{x}^* = x_c\mathbf{u}$, combining (41) and (21), it is not difficult to see that $x_c \ne x^*$. Accordingly, $x_{co}^* > x_{nco}^*$. ∎

L Proof of Proposition 7.1

It is not difficult to show that $\frac{dx^*(\rho)}{d\rho} > 0$ for any $\rho > 0$. Therefore, $x^*(\rho)$ is strictly increasing in ρ. Similarly, we can show that $g(\rho)$ is a decreasing function of ρ, and $g(\rho) \longrightarrow 0$ as $\rho \longrightarrow \infty$.

To examine the extreme case when $\rho \longrightarrow 0$, write $g(\rho)$ as follows using (45):

$$g(\rho) = \left(1 + \frac{p_s}{\delta}\right) \frac{E_1\left(\frac{\exp(x^*)}{\rho}\right)}{E_1\left(\frac{1}{\rho}\right)} - 1. \tag{98}$$

Using L'Hospital's rule yields that

$$g(\rho) \longrightarrow \left(1 + \frac{p_s}{\delta}\right) \exp\left(- \left.\frac{dx^*(\rho)}{d\rho}\right|_{\rho=0}\right) - 1, \text{ as } \rho \longrightarrow 0. \tag{99}$$

Next, we characterize $\left.\frac{dx^*(\rho)}{d\rho}\right|_{\rho=0}$. Rewrite (45) as follows

$$\frac{\delta}{p_s} x^* \exp\left(-\frac{1}{\rho}\right) = E_1\left(\frac{\exp(x^*)}{\rho}\right). \tag{100}$$

Taking derivative with respect to ρ on both sides of (100) and rearranging the terms yield that

$$\frac{\delta}{p_s}\frac{dx^*}{d\rho}\exp(x^*)\rho + \frac{\delta}{p_s}\frac{x^*}{\rho}\exp(x^*) = \exp\left(-\frac{\exp(x^*)-1}{\rho}\right)\left(1-\frac{dx^*}{d\rho}\rho\right)\exp(x^*). \tag{101}$$

Let $\rho \longrightarrow 0$ in (101). Using the facts that $x^*(\rho) \longrightarrow 0$, $\frac{x^*(\rho)}{\rho} \longrightarrow \frac{dx^*}{d\rho}$ and $\frac{\exp(x^*(\rho))-1}{\rho} \longrightarrow \left.\frac{dx^*(\rho)}{d\rho}\right|_{\rho=0}$ as $\rho \longrightarrow 0$, it follows that $\left.\frac{dx^*(\rho)}{d\rho}\right|_{\rho=0}$ is the root of $x\exp(x) = p_s/\delta$. The proposition follows from (99). ∎

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