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A Note on

A toy model of unemployment

Content of the Handout

1. Overview

The existence of involuntary unemployment has long been recognized as one of the main challenges in modern industrialized economies, making Keynesian economics relevant for analyzing and addressing unemployment until the 1970s. Keynes (1937) argued that unemployment was the result of weak aggregate demand: radical uncertainty about future states of the economy leads entrepreneurs and households to hold precautionary savings, rendering aggregate consumption and investment too weak for the economy to reach full employment.

At the end of the 1960s, Phelps (1967), along with Milton Friedman (among others), questioned the way the Phillips curve was designed, particularly criticizing the absence of expectations in the price and inflation formation process. Oil price shocks in the 1970s demonstrated the flaws in some Keynesian model assumptions, particularly regarding price rigidity and the absence of expectations. Lucas (1976) contributed to building a new framework where involuntary unemployment was absent. Despite the central role of unemployment in policy debates, this variable has been largely absent from this new generation of models, now the workhorse for the analysis of monetary policy, known as New Keynesian models.

Meanwhile, one strand of economics analyzed unemployment as a search and matching process between households and firms. Early contributions to the current vintage of search and matching models include Diamond (1982a), Diamond (1982b), Mortensen (1982), and Pissarides (1984). These models offer a central role to the interactions between firms and households in terms of job matching and wage negotiating. However, these contributions were partial equilibrium and were not yet integrated into a dynamic macroeconomic model. This literature is referred to as the Diamond-Mortensen-Pissarides (DMP) models.

Since the 2000s, economists have started to incorporate the DMP model into real business cycle models and New Keynesian models, as seen in Shimer (2005) and Hall (2005). This note aims to present the analytics of such a model with a low set of frictions.

2. The model

2.1 Households

Suppose a welfare maximizing household enjoying current and future consumption and dislike working with respective curvatures σ_C, $\sigma_H > 0$. The j-th household maximizes the following stream of utilities discounted at a fixed rate β:

$$\max_{\{C_{jt},d_t,n_{jt}\}} \sum_{\tau=0}^{\infty} \beta^{\tau} E_t \left\{ \frac{\varepsilon^C_{t+\tau} C^{1-\sigma_C}_{jt+\tau}}{1-\sigma_C} - \chi n_{jt+\tau} \right\} \tag{1}$$

where C_{jt} is consumption, n_{jt} is employment rate and $\chi > 0$ is a shift parameter allowing to pin down the amount the employment rate in steady state. Consumption is subject to an AR(1) preference shock denoted ε^C_t that increase the amount of utility obtained from one unit of consumption. Assuming that the size of the working age population is one, unemployment (rate) is given by:

$$n_{jt} = 1 - u_{jt} \tag{2}$$

The law of motion of employment is given by:

$$n_{jt} = (1-\rho)\, n_{jt-1} + x_t u_{t-1} \tag{3}$$

where $\rho \in [0,1]$ is the separation rate with the term ρn_{jt-1} that can be interpreted as the frictional unemployment. The term $x_t u_{t-1}$ is the number of unemployed workers who have found a job between t-1 and t, while (1-x_t)u_{jt-1} is the fraction who stays unemployed between $t-1$ and t. Here, the hiring rate x_t is decided by firms, not by the household this is why there is no j subscript. Similarly, household j is so small that its own employment decisions has no implication on the unemployemnt rate u_{t-1}.

The budget constraint in real terms reads as:

$$C_{jt} + d_{jt} = \frac{R_{t-1}}{\pi_t} d_{jt-1} + n_{jt} w_t + (1-n_{jt})\, b - T_{jt}, \tag{4}$$

where d_{jt} is the amount of real deposits, π_t is the inflation rate given by $\pi_t = P_t/P_{t-1}$, R_{t-1} is the pre-determined nominal interest rate of real deposits d_{jt}, w_t the real wage is computed as $w_t = W_t/P_t$ and a lump-sum tax financing the government T_{jt}. Here, the fraction of the family in employment is remunerated by the real wage w_t while the fraction of unemployed $1-n_{jt}$ obtains an unemployment insurance b.

The Lagrangian problem reads as:

$$\begin{aligned}
\mathcal{L}^H_t = & \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\frac{\varepsilon^C_{t+\tau} C^{1-\sigma_C}_{jt+\tau}}{1-\sigma_C} - \chi n_{jt+\tau} \right] \\
& + \beta^{\tau} \lambda^C_{t+\tau} \left[\frac{R_{t-1+\tau}}{\pi_{t+\tau}} d_{t-1+\tau} + H_{t+\tau} w_{t+\tau} - C^H_{t+\tau} - d_{t+\tau} - T_{t+\tau} \right] \\
& + \beta^{\tau} \lambda^C_{t+\tau} V^H_{t+\tau} \left[(1-\rho)\, n_{jt-1+\tau} + x_{t+\tau} u_{t-1+\tau} - n_{jt+\tau} \right]
\end{aligned}$$

where λ^H_t and V^H_t are lagrangian multipliers on each constraints. V^H_t is pre-multiplied by the marginal utility of consumption to express unemployment in terms of current consumption.

First order conditions are:

$$C_t : \varepsilon^C_t (C_t)^{-\sigma_C} - \lambda^H_t = 0 \tag{5}$$

$$d_t : \beta E_t \left\{ \lambda^C_{t+1} \right\} R_t / E_t \left\{ \pi_{t+1} \right\} - \lambda^C_t = 0 \tag{6}$$

$$n_t : -\chi + \lambda^C_t (w_t - b) - \lambda^C_t V^H_t + \beta \lambda^C_{t+1} (1-\rho)\, V^H_{t+1} \tag{7}$$

2.2 Final firms

Final firms competitively pack different types of good varieties i drawn on an unitary continuum to produce an homogeneous good sold to households. This firm is necessary when introducing monopolistic competition as they allows to create an elastic demand curve that intermediate firms exploits to generate positive profits. Production of final firms employs the following CES packing technology:

$$Y_t = \left(\int_0^1 y_{it}^{(\epsilon-1)/\epsilon} \mathrm{d}i \right)^{\epsilon/(\epsilon-1)} \tag{8}$$

where $\epsilon > 1$ is the perfect substitutability between different types of varieties i. The economy converges to a competitive equilibrium for $\epsilon \to 1$, while for any $\epsilon > 1$, goods are imperfect substitutes which allows firms to rise their selling price above their marginal cost.

Final firms are perfectly competitive and maximize profits, $P_t Y_t - p_{it} y_{it},$, under their technology constraint in Equation 8 to obtain the optimal demand constraint for the i-th good:

$$y_{it} = \left(\frac{p_{it}}{P_t} \right)^{-\epsilon} Y_t. \tag{9}$$

While the zero profit condition reads as: $P_t = (\int_0^1 p_{it}^{1-\epsilon} \mathrm{d}i)^{1/(1-\epsilon)}$.

2.3 Intermediate firms

Firms technology is given by the following production function

$$y_{it} = A \varepsilon_t^A k_{it-1}^{\alpha} n_{it}^{1-\alpha}, \tag{10}$$

where ε_t^A is the a technology shock, k_{it-1} is the physical stock of capital with fixed intensity of $\alpha \in [0;1]$ and n_{it} is the employment demand and A is a scaling constant.

As for households, the law of motion of employment:

$$n_{it} = (1-\rho)\, n_{it-1} + x_{it} u_t, \tag{11}$$

where x_{it} is the hiring rate of the i-th firm that premultiply the stock of unemployed.

Following **?**, emissions are given by:

$$e_{it} = \psi \left(1 - \mu_{it}\right) y_{it}^{1-\phi}$$

where e_{it} is the flow of CO2 emissions, with $\phi \in [0;1]$ the elasticity of emissions to GDP, $\psi \geq 0$ the carbon intensity and $0 \leq \mu_{it} \leq 1$ the fraction of emission avoided.

Profits reads as:

$$\Pi_{it} = y_{it} - w_t n_{it} - z_t k_{it-1} - G\left(x_{it}\right) u_t - f\left(\mu_{it}\right) y_{it} - \tau_t e_{it} \tag{12}$$

where abatement expenditures $f\left(\mu_{it}\right) = \theta_1 \mu_{it}^{\theta_2}$ with $\theta_1 \in [0,1]$ and $\theta_2 > 1$, the cost of increasing hirings reads as in Galí (2010) and Blanchard and Gali (2010):

$$G\left(x_{it}\right) = \frac{\Gamma}{1+\phi} x_{it}^{1+\phi}$$

where ϕ is the elasticity of hiring costs with respect to the hiring rate, and Γ is a constant which allows to pin down the total cost of hiring to one percent of total production as in Blanchard and Gali (2010).

Variable x_{it} is the probability of hiring a new worker, the associated costs can be interpreted as monitoring cost paid by firms to find a potential new worker.

The dynamic Lagrangian problem reads as:

$$\begin{aligned}\mathcal{L}_t &= E_t\textstyle\sum_{\tau=0}^{\infty}\beta^s\dfrac{\lambda_{t+\tau}^C}{\lambda_t^C}\left[mc_{t+\tau}y_{it+\tau} - w_{t+\tau}n_{it+\tau} - z_{t+\tau}k_{it-1+\tau} - G\left(x_{it+\tau}\right)u_{t+\tau} - f\left(\mu_{it+\tau}\right)y_{it+\tau} - \tau_{t+\tau}e_{it+\tau}\right] \\ &+ E_t\textstyle\sum_{\tau=0}^{\infty}\beta^s\dfrac{\lambda_{t+\tau}^C}{\lambda_t^C}V_{it+\tau}^P\left[\left(1-\rho\right)n_{it-1+\tau} + x_{it+\tau}u_{t+\tau} - n_{it+\tau}\right] \\ &+ E_t\textstyle\sum_{\tau=0}^{\infty}\beta^s\dfrac{\lambda_{t+\tau}^C}{\lambda_t^C}\varrho_{it+\tau}\left[A\varepsilon_{t+\tau}^A k_{it-1+\tau}^{\alpha}n_{it+\tau}^{1-\alpha} - y_{it+\tau}\right]\end{aligned}$$

Note that $\beta^s\lambda_{t+\tau}^C/\lambda_t^C$ is the stochastic discount factor. As households are shareholders of firms because they own the physical capital, firms has the same discounting scheme as households. Also note that the firm takes as given the labor market situation (here measured as the unemployment rate) u_t, this is because each firm is so small that it is assumed that they cannot influence individually the aggregate unemployement rate.

First order conditions reads as follows:

$$\begin{aligned} n_{it} &: \varrho_{it}(1-\alpha)\frac{y_{it}}{n_{it}} - w_{it} - V_t^P + \beta\frac{\lambda_{t+1}^C}{\lambda_t^C}\left(1-\rho\right)V_{it+1}^P = 0 \\ x_{it} &: -G'\left(x_{it}\right)u_t + V_{it}^P u_t = 0 \\ k_{it-1} &: \varrho_{it}\alpha\frac{y_{it}}{k_{it-1}} - z_t = 0 \\ \mu_{it} &: -f'\left(\mu_{it}\right) + \tau_t\psi y_{it}^{1-\phi} = 0 \\ y_{it} &: mc_{it} - f\left(\mu_{it}\right) - \tau_t\left(1-\phi\right)\psi\left(1-\mu_{it}\right)y_{it}^{-\phi} - \varrho_{it} = 0 \end{aligned}$$

The standard hiring decision rate is given by this standard no arbitrage condition:

$$\underbrace{G'\left(x_{it}\right)}_{\text{Cost of hiring}} = \underbrace{\varrho_{it}(1-\alpha)\frac{y_{it}}{n_{it}} - w_{it}}_{\text{marginal profit of hiring}} + \underbrace{\beta\frac{\lambda_{t+1}^C}{\lambda_t^C}\left(1-\rho\right)G'\left(x_{it+1}\right)}_{\text{Future gains from keeping the worker employed}}$$

2.4 Price Setting

Once the marginal cost of goods has been determined, we can now deduce their markup over the marginal to determine their selling price. However, firms face price adjustment costs paid in terms of home goods P_t using a mechasim à la Rotemberg (1982). These are quadractic adjustment costs are given by:

$$\Delta_{it}^P = \frac{\xi}{2}\left(\frac{p_{it}}{p_{it-1}} - \bar{\pi}\right)^2 \tag{13}$$

where $\psi \geq 0$ is the cost of adjusting price above or below the steady state inflation rate, denoted $\bar{\pi}$.

The profit optimization becomes dynamic, intermediate firms set prices facing nominal rigidities by maximizing the discount sum of real profits:

$$\max_{\{p_{it}\}} E_t\sum_{\tau=0}^{\infty}\beta^{\tau}\frac{\lambda_{t+\tau}^C}{\lambda_t^C}\left[\frac{p_{it+\tau}}{P_{t+\tau}}y_{it+\tau} - mc_{t+\tau}y_{it+\tau} - Y_{t+\tau}\Delta_{it+\tau}^P\right] \tag{14}$$

where mc_t is the real marginal cost. Replacing the demand function for each type of goods, $y_{it} = (P_{it}/P_t)^{-\epsilon} Y_t$, the profits become:

$$\max_{\{p_{it}\}} E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \frac{\lambda^C_{t+\tau}}{\lambda^C_t} \left[\frac{p_{it+\tau}}{P_{t+\tau}} \left(\frac{p_{it+\tau}}{P_{t+\tau}} \right)^{-\epsilon} - mc_{t+\tau} \left(\frac{p_{it+\tau}}{P_{t+\tau}} \right)^{-\epsilon} - \Delta^P_{it+\tau} \right] Y_{t+\tau} \tag{15}$$

The first order condition solving the optimization problem is given by:

$$\left[(1-\epsilon) \frac{Y_t}{P_t} - \epsilon \frac{mc_t}{P_{it}} - \frac{\xi}{p_{it}} (\pi_{it} - \bar{\pi}) \right] Y_t + \frac{\xi}{p_{it}} E_t \left\{ \beta \frac{\lambda^C_{t+1}}{\lambda^C_t} (\pi_{it+1} - \bar{\pi}) \pi_{it+1} Y_{t+1} \right\} = 0$$

Assuming symmetry between firms in their price setting decisions ($i.e.$ $P_t = p_{it}$), the new keynesian phillips curve emerges:

$$(1-\epsilon) + \epsilon mc_t - \xi \pi_t (\pi_t - \bar{\pi}) + E_t \left\{ \beta \frac{\lambda^C_{t+1}}{\lambda^C_t} \frac{Y_{t+1}}{Y_t} \xi \pi_{t+1} (\pi_{t+1} - \bar{\pi}) \right\} = 0 \tag{16}$$

2.5 Wage bargaining

Bilateral bargaining between firms and workers. Following form:

$$S_t = \underbrace{\left(V^H_t \right)}_{\text{household marginal utility}}{}^{\eta} \underbrace{\left(V^P_t \right)}_{\text{firm marginal profit}}{}^{1-\eta}$$

where η is the relative bargaining power of households. Period-by-period Nash bargaining implies that the firm and each of its workers determine the wage in period t by solving the problem:

$$\max_{\{w_t\}} S_t \tag{17}$$

The solution to that problem implies the following constant share rule:

$$\begin{aligned} \frac{\partial S_t}{\partial w_t} = 0 \Leftrightarrow \eta \frac{\partial V^H_t}{\partial w_t} \frac{S_t}{V^H_t} + (1-\eta) \frac{\partial V^P_t}{\partial w_t} \frac{S_t}{V^P_t} = 0 \\ \Leftrightarrow \eta V^P_t = (1-\eta) V^H_t \end{aligned}$$

To illustrate how this equations determine the wage, we can rewrite substitute V^P_t and V^H_t:

$$w_t = \eta \left[\varrho_t (1-\alpha) \frac{y_{it}}{n_{it}} + m_{t,t+1} V^P_{t+1} \right] + (1-\eta) \left[-\chi / \lambda^C_t - b + m_{t,t+1} V^H_{t+1} \right]$$

where $m_{t,t+1} = \beta \lambda^C_{t+1} / \lambda^C_t (1-\rho)$. It is straightforward to see that the wage is the result between the desired wage for firms and household weighted by the bargaining power of each agent.

2.6 Authorities

We use a Taylor (1993) rule-type to model the setting of the nominal interest rate by the monetary authority:

$$R_t = R^{\rho}_{t-1} \left(\bar{R} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \right)^{1-\rho} \tag{18}$$

where $\rho \in [0, 1)$ is the smoothing coefficient of the monetary policy rule, ϕ_π the reaction to the inflation gap and ϕ_y the reaction to the ouput gap. Steady states for interest rates, inflation and output are respectively $\bar{R}$, $\bar{\pi}$ and $\bar{Y}$.

In addition, there is a government which consumes domestic goods. This public spending G_t is exogenous, as it follows an AR(1) shock process ε_t^G, which is expressed as a constant portion of output:

$$G_t = g^y \bar{Y} \varepsilon_t^G.$$

The CO2 tax policy is given by:

$$\tau_t = \tau_0 \varepsilon_t^\tau$$

where τ_0 is the steady state CO2 tax (trillions $ per Gt CO2), ε_t^τ is a policy shock.

The balance sheet reads as:

$$\frac{r_{t-1}}{\pi_t} d_{t-1} + G_t + b u_t = d_t + T_t + \tau_t E_t,$$

where b_t are government bonds, $T_t = \int_0^1 T_{jt} \mathrm{d}j$ and $E_t = \int_0^1 e_{it} \mathrm{d}i$.

2.7 Equilibrium conditions

And total demand is given by:

$$Y_t = C_t + I_t + G_t + \Delta_t^P Y_t + G\left(x_t\right) u_t + f\left(\mu_t\right) y_t. \tag{19}$$

Unemployment rate:

$$u_t = 1 - n_t \tag{20}$$

3. Measurement equations and data

Measurements can be set as follows (do not use all):

$$\begin{aligned} g_{\mathcal{Y},t} &= \Delta \log Y_t \\ g_{\mathcal{C},t} &= \Delta \log C_t \\ g_{\mathcal{I},t} &= \Delta \log I_t \\ \mathcal{U}_t &= u_t - \bar{u} \\ g_{\mathcal{P},t} &= \pi_t - \bar{\pi} \\ \mathcal{R}_t &= R_t - \bar{R} \end{aligned}$$

The model can be estimated on standard variables (output, consumption, investment, inflation, nominal rate) and specific variable that is here unemployment rate.

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