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CSE150
HW02a

Problem 1

For each of the following statements about sets determine whether it is always true (also provide an example), or only sometimes true (also provide an example and counterexample). Please provide an explanation.

1. $A \in P(A)$

   Always true.

   True Example: $\emptyset \in P(\emptyset) \iff \emptyset \in \{\emptyset\}$

2. $A \subseteq P(A)$

   Always true.

   True Example: $\emptyset \subseteq P(\emptyset) \iff \emptyset \subseteq \{\emptyset\}$

3. $(|A| \leq |B| \Rightarrow (A \subseteq B))$

   Sometimes True.

   True Example: $|\{4,5,6\}| \leq |\{4,5,6,7\}|$ and $\{4,5,6\} \subseteq \{4,5,6,7\}$ is true.
   False Example: $|\{1,2,3\}| \leq |\{4,5,6,7\}|$ and $\{1,2,3\} \subseteq \{4,5,6,7\}$ is not true.

4. $(A \subseteq B) \Rightarrow (|A| \leq |B|)$ Always true.

   True Example: $\{1,2\} \subseteq \{1,2,3\}$ and $|\{1,2\}| \leq |\{1,2,3\}|$

Problem 2

Find the smallest two finite sets A and B for each of the four conditions.

Note: The smallest sets may not be unique.

1. $A \in B, A \subseteq B$, and $P(A) \subseteq B$

   $A = \emptyset, B = \{\emptyset\}$

2. $(\mathbb{N} \cap A) \in A, B \subset A$, and $P(B) \subseteq A$.

   $A = \{\emptyset, \{\emptyset\}\}, B = \emptyset$

3. $A \subseteq (P(P(B)) - P(A)$.

   $A = \emptyset, B = \emptyset$

4. $A \supseteq (P(P(B)) - P(A)$.

   $A = \{\emptyset\}, B = \emptyset$

Problem 3

Prove or disprove (by providing a counterexample) each of the following properties of binary relations:

Let S(A) be the symmetric closure of set A. Let T(A) be the transitive closure of set A. For every binary relation R,

1. $T(S(R)) \subseteq S(T(R))$
2. $S(T(R)) \subseteq T(S(R))$