**CSE 150 – Foundations of Computer Science: Honors** **Midterm Exam**
**Michael Bender** **Tuesday, November 1, 2016**

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# TEST VERSION: 1

Name: Tony Li ID #: ______________________

**INSTRUCTIONS:**

- Unless otherwise stated, your answers should be at most 1 or 2 sentences (excluding work.)
- This is a closed book, closed notes exam.
- Check to see that you have **??** pages including this cover and scratch pages.
- Read all the problems before starting work.
- Think before you write.
- If you leave a question blank or write just "I do not know," you get 25% automatically.
- Good luck!!

Academic integrity is expected of all students at all times, whether in the presence or absence of members of the faculty.
Understanding this, I declare that I shall not give, use, or receive unauthorized aid in this examination. I have been warned that if I am caught cheating (either receiving or giving unauthorized aid) I will get a "Q" grade for this course, and a letter will be sent to the Committee on Academic Standing and Appeals (CASA) requesting that an academic dishonesty notation be placed on my transcript. Further action against me may also be taken.

Signature [1] Tony Li

| Problem | Score | Maximum |
|---|---|---|
| signature | | 2 |
| 1 | | 48 |
| 2 | | 18 |
| 3 | | 12 |
| Total | | 80 |

[1] No "I dunno" points for leaving this blank. ☺

**Problem 1. (48 points)**

**Write your choice on the left of the question number.**

**Part A**

(In this part each question has a **single** correct answer. 4 points each.)

(1) Which of the following is a proposition:

A. A proposition.

B. Choose me.

C. Don't you want to choose the last option?

D. Surely you're joking.

E. Trust none of the above answers.

D

(2) Let $S$ be an infinite set and suppose we have $S = S_1 \times S_2 \times S_3 \times \cdots \times S_n$, where $n \in \mathbb{N}$. Which of the following is correct?

A. At least one of the sets $S_i$ is an infinite set.

B. At least one of the sets $S_i$ is an finite set.

C. At most one of the sets $S_i$ can be an infinite set.

D. At most one of the sets $S_i$ is a finite set.

E. None of the above.

A

(3) If $f(x) = \lfloor x \rfloor$, over which of the following domains and co-domains is it bijective? (Answers in the form (Domain, Co-domain).)[2]

A. $(\mathbb{R}, \mathbb{R})$

B. $(\mathbb{Z}, \mathbb{N})$

C. $(\mathbb{Z}, \mathbb{Z})$

D. $(\mathbb{Z}, \mathbb{Q})$

E. $(\mathbb{Q}, \mathbb{R})$

C

(4) Recall the block-unstacking problem from class. We start with a stack of $n$ blocks, and the objective is to unstack them fully. The *unstacking operation* takes a stack of size $s$, and splits it into two stacks, of size $x$ and $s - x$. The unstacking cost is $x(s - x) + 1$. What is the minimum cost to unstack all of the blocks?

A. $n^2 - 1$

B. $n(n - 2) + 2$

C. $n(n - 1)/2$

D. $n(n + 1)/2$

E. $(n + 2)(n - 1)/2$

E

[2] The notation $\lfloor x \rfloor$ (the "floor" of $x$) means the largest integer less than or equal to $x$. Thus, $\lfloor 2 \rfloor = 2$ and $\lfloor 1.9 \rfloor = 1$.

(5) If $a \equiv 0 \pmod{3}$, $a \equiv 1 \pmod{4}$, $a \equiv 2 \pmod{5}$, which of the following could $a$ equal?

A. 27

B. 256

C. 257

D. 357

E. 2357

D

(6) We say that a relation $R$ is ***coreflexive*** if for any $x$ and $y$:

$$(x, y) \in R \Rightarrow x = y.$$

Let $S = \{a, b, c\}$. Which of the following relations on $S$ is **both** reflexive and coreflexive?

A. $\emptyset$

B. $\{(a, c), (b, c), (c, a), (c, b)\}$

C. $\{(a, a), (b, b), (c, c)\}$

D. $\{(a, a), (b, b)\}$

E. $\{(a, a), (b, b), (c, c), (a, c), (a, b)\}$

C

(7) Which of the following formulae correctly represents "The set $A$ has exactly two elements"?

A. $\exists x, y \in A,\ x \neq y$.

B. $\forall x, y \in A,\ x \neq y$.

C. $\forall z \in A,\ \exists x, y \in A,\ (x \neq y) \wedge \big((z = x) \vee (z = y)\big)$.

D. $\exists x, y \in A,\ (x \neq y) \wedge \big(\forall z \in A,\ (z \neq x) \Rightarrow (z = y)\big)$.

E. $\exists x, y \in A,\ \forall z \in A,\ (z = x) \vee (z = y)$.

D

(8) Let $T$ be the set of all reflexive binary relations on $\mathbb{N}$, the set of natural numbers. Then $T$ is uncountable. We prove it by contradiction.

Assume we have a enumeration $T = \{R_1,\ R_2,\ \dots\}$. Then we create a relation $R$, such that $(i,i) \in R,\ \forall i \in \mathbb{N}$. And for all $i$ and $j$ where $i \neq j$, $(i,j) \in R \iff (i,j) \notin R_{|i-j|}$. Then $R \notin T$ but $R$ is reflexive. So $T$ is uncountable.

Which of the following judgments is correct?

A. The conclusion is wrong.

B. The proof is wrong because the assumed enumeration is not clear.

C. The proof is wrong because $R$ is not even reflexive.

D. The proof is wrong because $R$ could still equal some $R_k \in T$.

E. The proof is right.

E

## Part B

(In this part each question may have **one or more** correct answers. 4 points each.
If your answer is a **nonempty strict subset** of the correct answer, you get 2.5 points.)

(1) Let $R_1$ and $R_2$ be two binary relations.

$R_1$ is said to be ***contained*** in $R_2$ if for any $x$ and $y$, $(x, y) \in R_1 \Rightarrow (x, y) \in R_2$.

The relation "is contained in" (which is a relation on relations) is:

A. Reflexive

B. Symmetric

C. Transitive

A, C

(2) Let $A = \Big\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\Big\}$. Which of the following are (is) true?

A. $\Big\{\{\{\emptyset\}\}\Big\} \in P(A)$.

B. $A \times \{\emptyset\} = \{\emptyset\}$.

C. $A \times \emptyset = \emptyset$.

C

(3) Let $A$ be an infinite set and $B$ be a countable set. Which pair(s) of sets of the following must have a bijection?

A. $A$ and $B$

B. $A$ and $P(B)$

C. $A$ and $A \cap B$

D. $A$ and $A \cup B$

E. $A$ and $A \times B$

D

(4) Which of the following statements are (is) true?

A. $\mathbb{N} \times \mathbb{Q}$ is countable.

B. The set of all binary relations on a countably infinite set is countable.

C. $P(\mathbb{Q})$ is countable.

D. The union of countably many countable sets can be uncountable.

E. The intersection of finitely many uncountable sets can be countably infinite.

A, E

**Problem 2. (18 points)**

For each of the following statements about sets, state whether it is **always** true (provide an **example**), **sometimes** true (provide an **example** and **counterexample**), or **never** true (provide a **counterexample**).

(1) $S \in P(P(S))$ **always** **sometimes** **never**

**Example**: $S = \emptyset$

**Counterexample**: $S = \{1, 2\}$

(2) $P(S \cap T) = P(S) \cap P(T)$ **always** **sometimes** **never**

**Example**: $S = \emptyset$

$T = \emptyset$

**Counterexample**: $S =$ ____________________

$T =$ ____________________

(3) $P(S - T) = P(S) - P(T)$ **always** **sometimes** **never**

**Example**: $S =$ ____________________________

$T =$ ____________________________

**Counterexample**: $S = \emptyset$

$T = \emptyset$

**Problem 3. (12 points)**

Let's do some counting. Given a finite set $S$, where $|S| = n$, fill in the blanks below and give a **one-sentence** justification for each.

(1) Total number of binary relations on $S$: $2^{n^2}$
Number of binary relations $= |P(SxS)| = 2^{|SxS|} = 2^{|S|^2} = 2^{n^2}$

(2) Number of reflexive binary relations on $S$: $2^{n^2-n}$

Number of reflexive binary relations $P(SxS$ - set containing binary relations (a,b) where (a,b) $\in SxS$ and $a = b)|$

| X | 0 | 1 | 2 | 3 | ... | n - 1 | n |
|---|---|---|---|---|---|---|---|
| 0 | (0, 0) | (0, 1) | (0, 2) | (0, 3) | (0, ...) | (0, n - 1) | (0, n) |
| 1 | (1, 0) | (1, 1) | (1, 2) | (1, 3) | (1, ...) | (1, n - 1) | (1, n) |
| 2 | (2, 0) | (2, 1) | (2, 2) | (2, 3) | (2, ...) | (2, n - 1) | (2, n) |
| 3 | (3, 0) | (3, 1) | (3, 2) | (3, 3) | (3, ...) | (3, n - 1) | (3, n) |
| ... | (..., 0) | (..., 1) | (..., 2) | (..., 3) | (..., ...) | (..., n - 1) | (..., n) |
| n - 1 | (n - 1, 0) | (n - 1, 1) | (n - 1, 2) | (n - 1, 3) | (n - 1, ...) | (n - 1, n - 1) | (n - 1, n) |
| n | (n, 0) | (n, 1) | (n, 2) | (n, 3) | (n, ...) | (n, n - 1) | (n, n) |

(3) Number of symmetric binary relations on $S$: $2^{\frac{n(n+1)}{2}}$

Number of symmetric relations = the size of the power set of all binary relations $(a, b)$ in $SxS$ that have a $(b, a)$ in $SxS$

| X | 0 | 1 | 2 | 3 | ... | n - 1 | n |
|---|---|---|---|---|---|---|---|
| 0 | (0, 0) | (0, 1) | (0, 2) | (0, 3) | (0, ...) | (0, n - 1) | (0, n) |
| 1 | (1, 0) | (1, 1) | (1, 2) | (1, 3) | (1, ...) | (1, n - 1) | (1, n) |
| 2 | (2, 0) | (2, 1) | (2, 2) | (2, 3) | (2, ...) | (2, n - 1) | (2, n) |
| 3 | (3, 0) | (3, 1) | (3, 2) | (3, 3) | (3, ...) | (3, n - 1) | (3, n) |
| ... | (..., 0) | (..., 1) | (..., 2) | (..., 3) | (..., ...) | (..., n - 1) | (..., n) |
| n - 1 | (n - 1, 0) | (n - 1, 1) | (n - 1, 2) | (n - 1, 3) | (n - 1, ...) | (n - 1, n - 1) | (n - 1, n) |
| n | (n, 0) | (n, 1) | (n, 2) | (n, 3) | (n, ...) | (n, n - 1) | (n, n) |

**Scratch Paper**

**Scratch Paper**

**Scratch Paper**

**Scratch Paper**