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**CSE 150 – Honors Foundations of Computer Science – Fall 2016**

Homework 2

Part A: Due Oct 10th at midnight
Part B: Due Oct 17th at midnight

You should hand in your problem set *online* using blackboard. Write your solutions using latex. *No late problem sets accepted.*

## Part A

Problem 1

For each of the following statements about sets determine whether it is always true (also provide an example), or only sometimes true (also provide an example and counterexample). Please provide an explanation.

1. $A \in P(A)$
2. $A \subseteq P(A)$
3. $(\,|A| \leq |B|\,) \Rightarrow (A \subseteq B\,)$
4. $(A \subseteq B\,) \Rightarrow (\,|A| \leq |B|\,)$

Find the smallest two finite sets $A$ and $B$ for each of the four conditions.

*Note*: The smallest sets may not be unique.

Problem 2

1. $A \in B$, $A \subseteq B$, and $P(A) \subseteq B$.
2. $(\mathbb{N} \bigcap A) \in A$, $B \subset A$, and $P(B) \subseteq A$.
3. $A \subseteq (P(P(B)) - P(A))$.
4. $A \supseteq (P(P(B)) - P(A))$.

Problem 3

Prove or disprove (by providing a counterexample) each of the following properties of binary relations:

Let $S(A)$ be the symmetric closure of set $A$. Let $T(A)$ be the transitive closure of set $A$.

For every binary relation $R$,

1. $T(S(R)) \subseteq S(T(R))$
2. $S(T(R)) \subseteq T(S(R))$

Problem 4

How many reflexive binary relations are there on $S \times S$? How many symmetric relations? Explain. *Bonus*: How many equivalence relations are there on $S \times S$? Explain.

## Part B

Problem 5

Show that each function $f : \mathbb{N} \to \mathbb{N}$ has the listed properties.

1. $f(x) = 2x$ (one-to-one but not onto)
2. $f(x) = x + 1$ (one-to-one but not onto)
3. $f(x) =$ if $x$ is odd then $x - 1$ else $x + 1$ (bijective)

Problem 6

Show that the product $(a + bi)(c + di)$ of two complex numbers can be evaluated using just three real-number multiplications. You may use a few extra additions.

Problem 7

Given a function $f : A \to A$. An element $a \in A$ is called a *fixed point* of $f$ if $f(a) = a$. Find the set of fixed points for each of the following functions.

1. $f : A \to A$ where $f(x) = x$.
2. $f : \mathbb{N} \to \mathbb{N}$ where $f(x) = x + 1$.
3. $f : \mathbb{N}_6 \to \mathbb{N}_6$ where $f(x) = 2x \mod 6$.
4. $f : \mathbb{N}_6 \to \mathbb{N}_6$ where $f(x) = 3x \mod 6$.

Problem 8

Let $f(x) = x^2$ and $g(x, y) = x + y$. Find compositions that use the functions $f$ and $g$ for each of the following expressions.

1. $(x + y)^2$
2. $x^2 + y^2$
3. $(x + y + z)^2$
4. $x^2 + y^2 + z^2$