$H_{v2} = \{x \in \mathbb{R}^2 \mid v_2 \cdot x \leq 1\}$ and $H_{v3} = \{x \in \mathbb{R}^2 \mid v_3 \cdot x \leq 1\}$

let $x = [x_1, x_2]^T$

say choose the following:

$v_1 = \{1, 0\} \Rightarrow H_{v1} = x_1 \leq 1$

$v_2 = \{0, 1\} \Rightarrow H_{v2} = x_2 \leq 1$

$v_3 = \{1, 1\} \Rightarrow H_{v3} = x_1 + x_2 \leq 1$

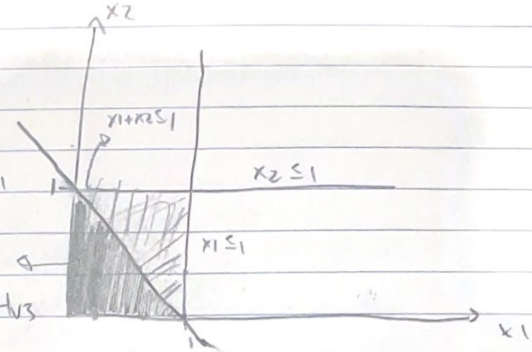
check bounded

$|v_1| = 1 \leq 3$

$|v_2| = 1 \leq 3$

$|v_3| = \sqrt{2} \leq 3$ → so C is bounded

This is $C = H_{v1} \cap H_{v2} \cap H_{v3}$



4) Let $C \subset \mathbb{R}^n$ be given set, $C \neq \emptyset$. Let $f : \mathbb{R}^n \to \mathbb{R}$ be given function

(Prob1) $\max f(x)$

s.t. $x \in C$

Define $F_r = \{x \in \mathbb{R}^n \mid f(x) \geq r\}$

(Prob2) $\max r$

$C \cap F_r \neq \emptyset$ (condition on r)

Assume optimal sol of (Prob1) and optimal sol of (Prob2) exist.
Find and explain relation between their optimal sol.

$n = 2$

$C \subset \mathbb{R}^2, C \neq \emptyset$

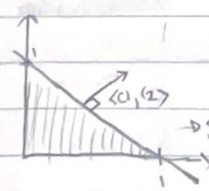
prob1) $\max f(x)$

s.t. $x \in C$

and f is linear

$\Rightarrow f(x) = x_1 + x_2$

s.t. $x_1 + x_2 \leq 1 \in C$



→ So max $f(x)$ occurs on here

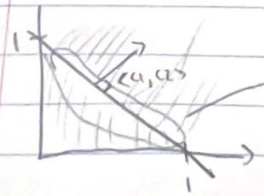
prob2) $\max r$

s.t. $C \cap F_r \neq \emptyset$

And $F_r = \{x \in \mathbb{R}^2 \mid f(x) \geq r\}$

$F_r = \{x \in \mathbb{R}^2 \mid x_1 + x_2 \geq r\}$

let $r = 1$



→ Then $x_1 + x_2 = 1$ is constraint and that $C \cap F_r \neq \emptyset$ $r \in C$ and $f(x) \geq r$

so if we generalize this to n, f and C

Then this means we need an r that is max value within C, and also is $\leq f(x)$, whereas x is also in C.

so optimal sol of prob1

$\Rightarrow$ $x \in C$, such $f(x) = \max$

And optimal sol of prob2

$\rightarrow$ max

is $r \in C \cap F_r$, where $f(x) \geq r$

And by the graphical method earlier, this happens when $\max f(x) = r$

s.t. $x, r \in C$

and $r \in C \cap F_r \neq \emptyset$