3) For non-empty set $S \subset \mathbb{R}^n$ and positive real number $r \in R$

$r > 0$ define set rS as follow:

$rS := \{ z \in \mathbb{R}^n \mid z = rx, x \in S \}$

rx is multiplication of vector $x \in \mathbb{R}^n$ by scalar $r \in \mathbb{R}$.

rS is set of all points by multiplying r with vectors $x \in S$

prove if S is convex, then rS is convex set as well

A set $S \subset \mathbb{R}^n$ is convex if $\forall \vec{x}, \vec{y} \in S$, $\forall t \in [0,1]$

s.t. $(1-\lambda)\vec{x} + \lambda\vec{y} = \vec{z} \in S$

Assume S is convex, then

$rS = z \in \mathbb{R}^n \mid z = rx, x \in S$

$\Rightarrow (1-\lambda)\vec{x} + t\vec{y} = r\vec{x}, \vec{x} \in S$

$= \vec{z} \Rightarrow \vec{z} \in S$

so if a non-empty set $S \subset \mathbb{R}^n$ is convex, then rS is also convex set

4) For given two non-empty sets $S_1, S_2 \subset \mathbb{R}^n$, define $S_1 + S_2$ as follows: $S_1 + S_2 = \{ z \in \mathbb{R}^n \mid \exists x \in S_1, \exists y \in S_2 \text{ s.t. } z = x + y \}$

a) $S_1 = \{ (x_1, x_2) \in R^2 \mid |x_1 - 1| \le 1 \; \& \; |x_2 - 2| \le 1 \}$ sketch $S_1 + S_2$

& $S_2 = \{ (x_1, x_2) \in R^2 \mid |x_1| \le 2 \; \& \; |x_2| \le 1 \}$

for S_1

$\Rightarrow |x_1 - 1| \le 1 \Rightarrow -1 \le x_1 - 1 \le 1 \Rightarrow 0 \le x_1 \le 2$

$\Rightarrow |x_2 - 2| \le 1 \Rightarrow -1 \le x_2 - 2 \le 1 \Rightarrow 1 \le x_2 \le 3$

for S_2

$|x_1| \le 2 \Rightarrow -2 \le x_1 \le 2$

$|x_2| \le 1 \Rightarrow -1 \le x_2 \le 1$

And $S_1 + S_2 \Rightarrow -2 \le x_1 \le 4$ for x_1 || $0 \le x_2 \le 4$ for x_2