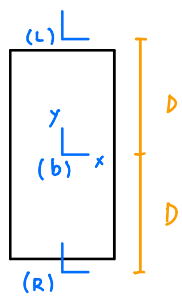
Tianyu Li , ME495 HW2 , Kinematics



$T_{bL}(0, 0, D)$

$T_{bR}(0, 0, -D)$

Adjoints

$$A_{bL} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A_{bR} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse Adjoints

$$A_{Lb} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A_{Rb} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Twist

$$V_b = \begin{bmatrix} \dot\theta \\ V_x \\ V_y \end{bmatrix} \qquad V_L = \begin{bmatrix} \dot\theta \\ V_{xL} \\ V_{yL} \end{bmatrix} \qquad V_R = \begin{bmatrix} \dot\theta \\ V_{xR} \\ V_{yR} \end{bmatrix}$$

$$V_i = A_{ib} V_b$$

Left Wheel velocities

$$V_L = A_{Lb} V_b$$

$$\begin{bmatrix} \dot\theta \\ V_{xL} \\ V_{yL} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ V_x \\ V_y \end{bmatrix} = \begin{bmatrix} \dot\theta \\ -D\dot\theta + V_x \\ V_y \end{bmatrix}$$

$$\begin{bmatrix} V_{xL} \\ V_{yL} \end{bmatrix} = \begin{bmatrix} r\dot{\phi}_L \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_L \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ -D\dot{\theta} + V_x \\ V_y \end{bmatrix} \Rightarrow \begin{aligned} r\dot{\phi}_L &= -D\dot{\theta} + V_x \\ \dot{\phi}_L &= -\frac{D}{r}\dot{\theta} + \frac{1}{r}V_x \end{aligned}$$

$$\begin{bmatrix} \dot{\phi}_L \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{D}{r} & \frac{1}{r} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix} \qquad (1)$$

Right Wheel velocities

$$V_R = A_{Rb} V_b$$

$$\begin{bmatrix} \dot{\theta} \\ V_{xR} \\ V_{yR} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ D & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ D\dot{\theta} + V_x \\ V_y \end{bmatrix}$$

$$\begin{bmatrix} V_{xR} \\ V_{yR} \end{bmatrix} = \begin{bmatrix} r\dot{\phi}_R \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} \dot{\theta} \\ r\dot{\phi}_R \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{\theta} \\ D\dot{\theta} + V_x \\ V_y \end{bmatrix} \Rightarrow \begin{aligned} r\dot{\phi}_R &= D\dot{\theta} + V_x \\ \dot{\phi}_R &= \frac{D}{r}\dot{\theta} + \frac{1}{r}V_x \end{aligned}$$

$$\begin{bmatrix} \dot{\phi}_R \\ 0 \end{bmatrix} = \begin{bmatrix} \frac{D}{r} & \frac{1}{r} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix} \qquad (2)$$

Odometry

$$\dot{\phi}_i = u_i \quad , \quad \dot{\phi}_i \approx \frac{\Delta\phi_i}{\Delta t}$$ constant velocity between time interval

From equation (1) and (2)

$$\begin{bmatrix} \dot{\phi}_L \\ \dot{\phi}_R \end{bmatrix} = \begin{bmatrix} -\frac{D}{r} & \frac{1}{r} & 0 \\ \frac{D}{r} & \frac{1}{r} & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ V_x \\ V_y \end{bmatrix}$$

$$\Downarrow$$

$$\begin{cases} \dot{\phi}_L = -\frac{D}{r}\dot{\theta} + \frac{1}{r}V_x \\ \dot{\phi}_R = \frac{D}{r}\dot{\theta} + \frac{1}{r}V_x \end{cases}$$

Solve for $\dot{\theta}$ and V_x in body twist

$$\begin{cases} \dot{\theta} = \frac{r}{2D}(\dot{\phi}_R - \dot{\phi}_L) & (3) \\ V_x = \frac{r}{2}(\dot{\phi}_R + \dot{\phi}_L) & (4) \end{cases}$$