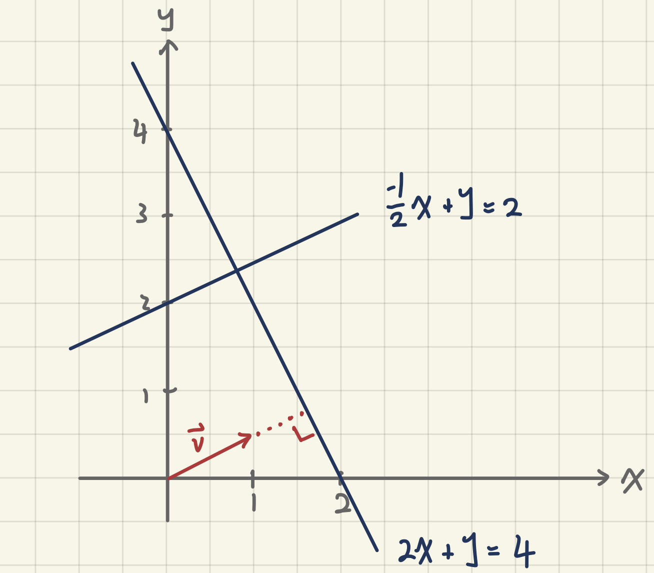
1.



(a) vector equation : $\vec{v} = \vec{P} + t \cdot \vec{S}$

$\vec{P}$ is point vector , since $\vec{v}$ start from origin $\Rightarrow \vec{P} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

t is an arbitrary constant

$\vec{S}$ is a slope vector

The slope vector of the $\begin{cases} \text{normal is } \begin{bmatrix} 2 \\ 1 \end{bmatrix} \\ \text{unit normal is } \begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix} \end{cases}$, $\|\vec{S}\| = 1$

$$\Rightarrow \vec{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} \frac{2}{\sqrt{5}} \\ \frac{1}{\sqrt{5}} \end{bmatrix} = \begin{bmatrix} \frac{2}{\sqrt{5}}t \\ \frac{1}{\sqrt{5}}t \end{bmatrix} \#$$

(b) The slope of the line $2x+y=4$ is -2

The slope of the perpendicular line is $\frac{1}{2} \Rightarrow \frac{-1}{2}x + y = a$

The perpendicular line pass through $P(0,2) \Rightarrow \frac{-1}{2}x + y = 2$ #

2.

$x \in R^m$, $b \in R^m$

$$f(x) = b^T x = [\, b_1 \;\; b_2 \;\; \cdots \;\; b_m \,] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \sum_{i=1}^{m} b_i x_i$$

$$\nabla_x f(x) = \begin{bmatrix} \frac{\partial f(x)}{\partial x_1} \\ \frac{\partial f(x)}{\partial x_2} \\ \vdots \\ \frac{\partial f(x)}{\partial x_m} \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} = b$$

$$\frac{\partial f(x)}{\partial x_k} = \frac{\partial \left(\sum_{i=1}^{m} b_i x_i\right)}{\partial x_k} = \frac{\partial \left(b_1 x_1 + b_2 x_2 + \cdots + b_k x_k + \cdots + b_m x_m\right)}{\partial x_k} = b_k$$

3.

$x \in R^m$, $A \in S^m$

$$f(x) = x^T A x = [x_1\ x_2 \cdots x_m] \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & & \vdots \\ \vdots & & \ddots & \\ A_{m1} & & \cdots & A_{mm} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = \sum_{i=1}^{m} \sum_{j=1}^{m} A_{ij} x_i x_j$$

$$A_{11} x_1 x_1 + A_{12} x_1 x_2 + \cdots + \boxed{A_{1k} x_1 x_k} + \cdots + A_{1m} x_1 x_m$$

$$A_{21} x_2 x_1 + A_{22} x_2 x_2 + \cdots + \boxed{A_{2k} x_2 x_k} + \cdots + A_{2m} x_2 x_m$$

$$\boxed{A_{k1} x_k x_1 + A_{k2} x_k x_2} + \cdots + \boxed{A_{kk} x_k x_k} + \cdots + \boxed{A_{km} x_k x_m}$$

$$+)\ A_{m1} x_m x_1 + A_{m2} x_m x_2 + \cdots + \boxed{A_{mk} x_m x_k} + \cdots + A_m x_m x_m$$

$$f(x)$$

$$\nabla_x f(x) = \begin{bmatrix} \frac{\partial f(x)}{\partial x_1} \\ \vdots \\ \frac{\partial f(x)}{\partial x_m} \end{bmatrix} = \begin{bmatrix} 2 R_1 x \\ \vdots \\ 2 R_m x \end{bmatrix} = 2Ax \quad \#$$

• R_1 is the first row of A

$$\frac{\partial f(x)}{\partial x_k} = \frac{\partial}{\partial x_k} \left[\sum_{i \neq k} \sum_{j \neq k} A_{ij} x_i x_j + \underline{\sum_{i \neq k} A_{ik} x_i x_k} + \underline{\sum_{j \neq k} A_{kj} x_k x_j} + \underline{A_{kk} x_k^2} \right]$$

$$= \sum_{i \neq k} A_{ik} x_i + \sum_{j \neq k} A_{kj} x_j + 2 A_{kk} x_k$$

$$= \sum_{i=1}^{m} A_{ik} x_i + \sum_{j=1}^{m} A_{kj} x_j$$

• A is symmetric $\Rightarrow A_{ik} = A_{ki}$

$$= 2 \sum_{j=1}^{m} A_{kj} x_j$$

$$= 2 R_k x$$

• R_k is the k-th row of A