Recurrent Neural Networks

- Vanilla Neural Network
- Applications
- LSTM
- GRU

source from http://cs231n.stanford.edu

WHY RNN (SEQUENCE-TO-SEQUENCE) ?

Image Classification: A core task in Computer Vision

(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

cat

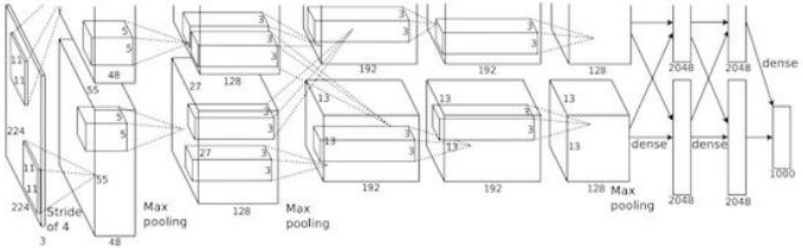
Image features vs ConvNets

Feature Extraction

f

10 numbers giving scores for classes

training

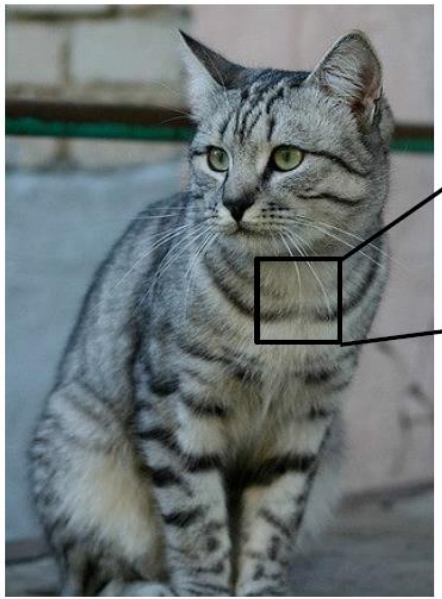


Krizhevsky, Sutskever, and Hinton, "Imagenet classification with deep convolutional neural networks", NIPS 2012. Figure copyright Krizhevsky, Sutskever, and Hinton, 2012. Reproduced with permission.

10 numbers giving scores for classes

training

The Problem: Semantic Gap




```
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 [ 91  98 102 106 104  79  98 103  99 105 123 136 110 105  94  85]
 [ 76  85  90 105 120 105  87  96  95  99 115 112 106 103  99  85]
 [ 99  81  81  93 120 131 127 100  95  98 102  99  96  93 101  94]
 [106  91  61  64  69  91  88  85 101 107 109  98  75  84  96  95]
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 [115 114 109 123 150 148 131 118 113 109 100  92  74  65  72  78]
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 [ 62  65  82  89  78  71  80 101 124 126 119 101 107 114 131 119]
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 [130 128 134 161 139 100 109 118 121 134 114  87  65  53  69  86]
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 [123 107  96  86  83 112 153 149 122 109 104  75  80 107 112  99]
 [122 121 102  80  82  86  94 117 145 148 153 102  58  78  92 107]
 [122 164 148 103  71  56  78  83  93 103 119 139 102  61  69  84]]
```
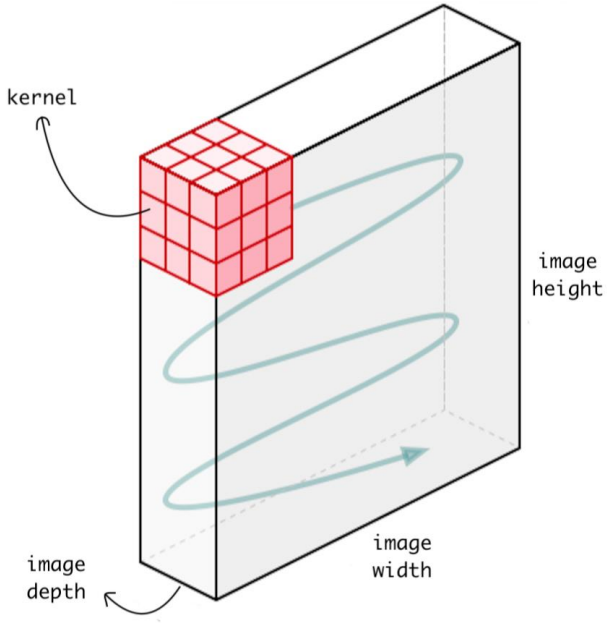

What the computer sees

An image is just a big grid of numbers between [0, 255]:

e.g. 800 x 600 x 3
(3 channels RGB)

Convolutions in 2D



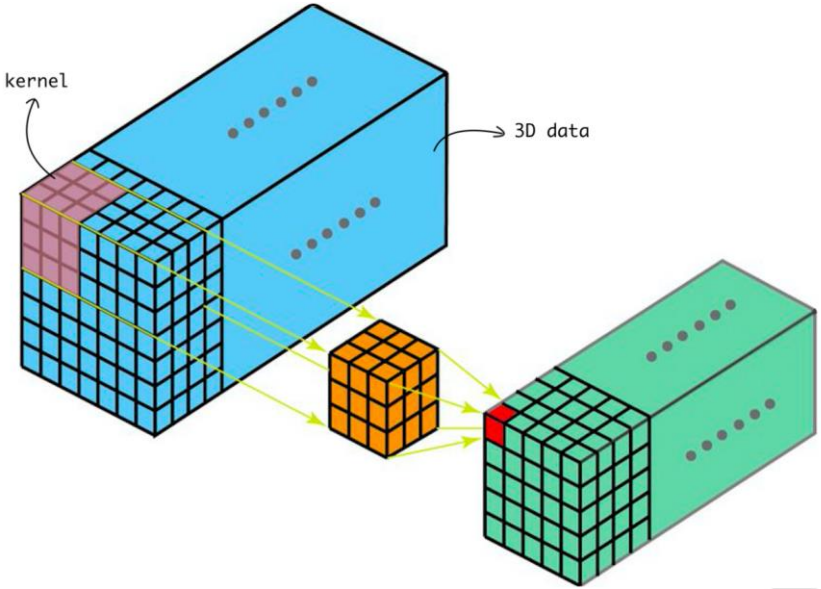
7x7x3 * 3x3x3

→5x5x**16** ← **16** filters

5x5x16 * 3x3x16

→ 3x3x**32** ← 32 filters

3D Convolution



channel

7x7x14x1 * 3x3x3x1

→5x5x12x16 ← **16** filters

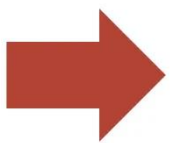
5x5x12x16 * 3x3x3x16

→ 3x3x10x32 ← **32** filters

One-hot encoding (or 1-of-N encoding)

How to represent a word?

Vocabulary:
Man, woman, boy, girl, prince, princess, queen, king, monarch



	1	2	3	4	5	6	7	8	9
man	1	0	0	0	0	0	0	0	0
woman	0	1	0	0	0	0	0	0	0
boy	0	0	1	0	0	0	0	0	0
girl	0	0	0	1	0	0	0	0	0
prince	0	0	0	0	1	0	0	0	0
princess	0	0	0	0	0	1	0	0	0
queen	0	0	0	0	0	0	1	0	0
king	0	0	0	0	0	0	0	1	0
monarch	0	0	0	0	0	0	0	0	1

Each word gets a 1x9 vector representation

One-hot encoding (or 1-of-N encoding)

How to represent a word?

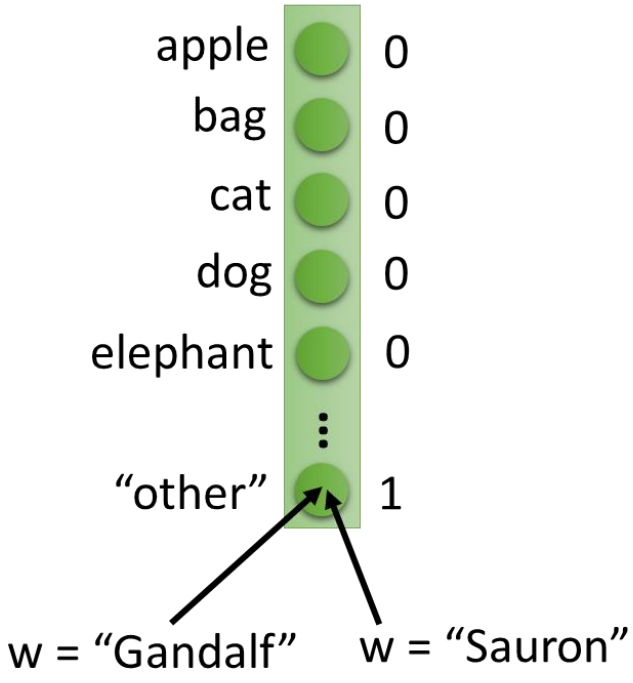
	1	2	3	4	5	6	7	8	9
man	1	0	0	0	0	0	0	0	0
woman	0	1	0	0	0	0	0	0	0
boy	0	0	1	0	0	0	0	0	0
girl	0	0	0	1	0	0	0	0	0
prince	0	0	0	0	1	0	0	0	0
princess	0	0	0	0	0	1	0	0	0
queen	0	0	0	0	0	0	1	0	0
king	0	0	0	0	0	0	0	1	0
monarch	0	0	0	0	0	0	0	0	1

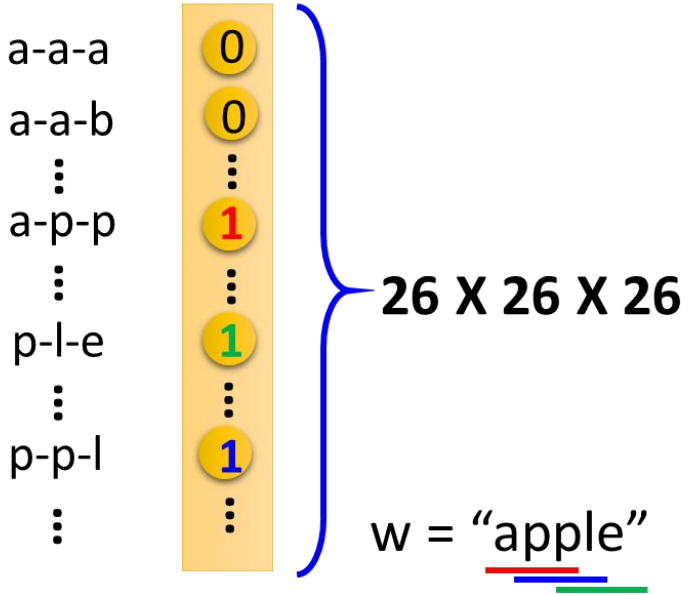
Problem:

- Dimensions increase linearly with the vocabulary. As such, **memory** use is prohibitively large.
- The embedding matrix is very **sparse**, mainly made up of zeros.
- **No shared information** between words and no commonalities between similar words.

Custom Encoding

try to reduce the dimensionality of the encoding



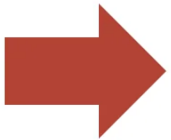


台大李宏毅教授

Word embedding

try to reduce the dimensionality of the encoding

Vocabulary:
Man, woman, boy, girl, prince, princess, queen, king, monarch



	Femininity	Youth	Royalty
Man	0	0	0
Woman	1	0	0
Boy	0	1	0
Girl	1	1	0
Prince	0	1	1
Princess	1	1	1
Queen	1	0	1
King	0	0	1
Monarch	0.5	0.5	1

Each word gets a 1x3 vector

Similar words… similar vectors

Word Embedding

SPORTS • BASEBALL

Missing Baseball? Taiwanese Games Are Now Broadcasting in English. Here's How to Watch Live

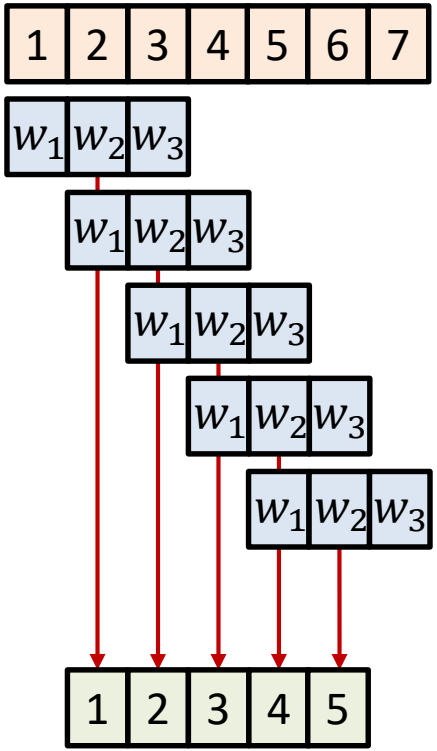
Infielder Wei Chen Wang (R) #9 of CTBC Brothers is tagged out during the CPBL game between CTBC Brothers and Uni-President Lions at Taichung Intercontinental Baseball Stadium on April 12, 2020 in Taichung, Taiwan. Taiwan's Chinese Professional Baseball League started April 11 2020. Due to COVID-19, only staff and members of the media were allowed to attend. Gene Wang–Getty Images

number of words

Word vector

1	1	0	0	0	0	0	0	0	0
1	0	0	1	0	0	1	1	1	0
1	0	0	0	1	0	1	1	1	1
1	0	0	1	1	1	0	1	0	1
0	0	0	1	1	1	1	0	0	0
.									
.									
.									
1	0	0	1	1	1	0	0	1	0

Convolutions in 1D



The length result of convolution is:

length – width + 1 = $7 - 3 + 1 = 5$

Example:

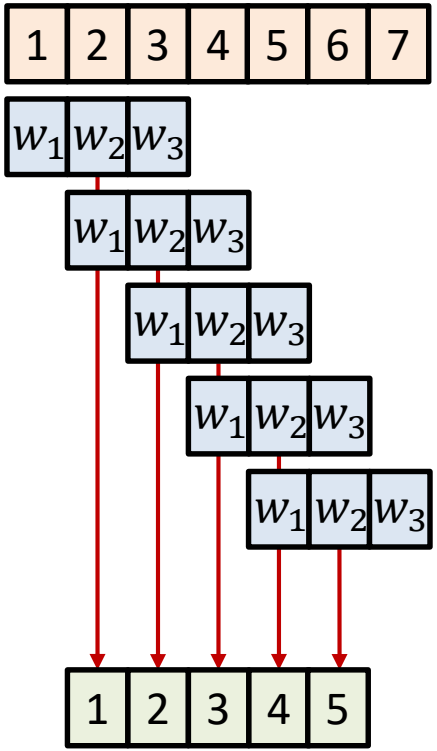
7x1 * 3x1

→5x**16** ← **16** filters

5x16 * 3x16

→ 3x**32** ← 32 filters

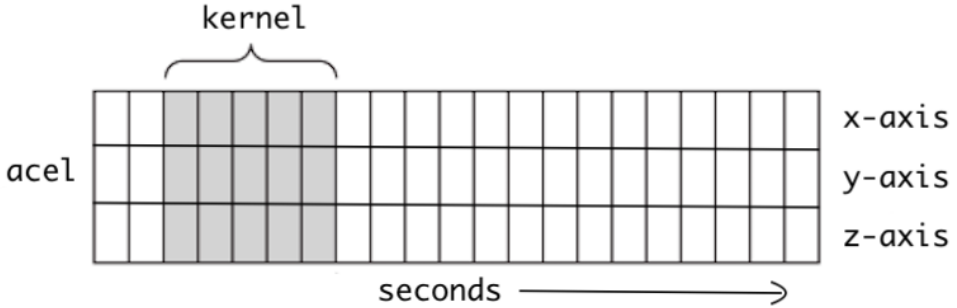
Convolutions in 1D



The length result of convolution is:

length – width + 1 = $8 - 3 + 1 = 6$

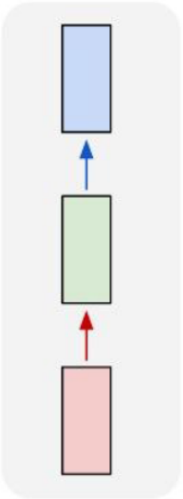
Extension



The output will be (3, …)

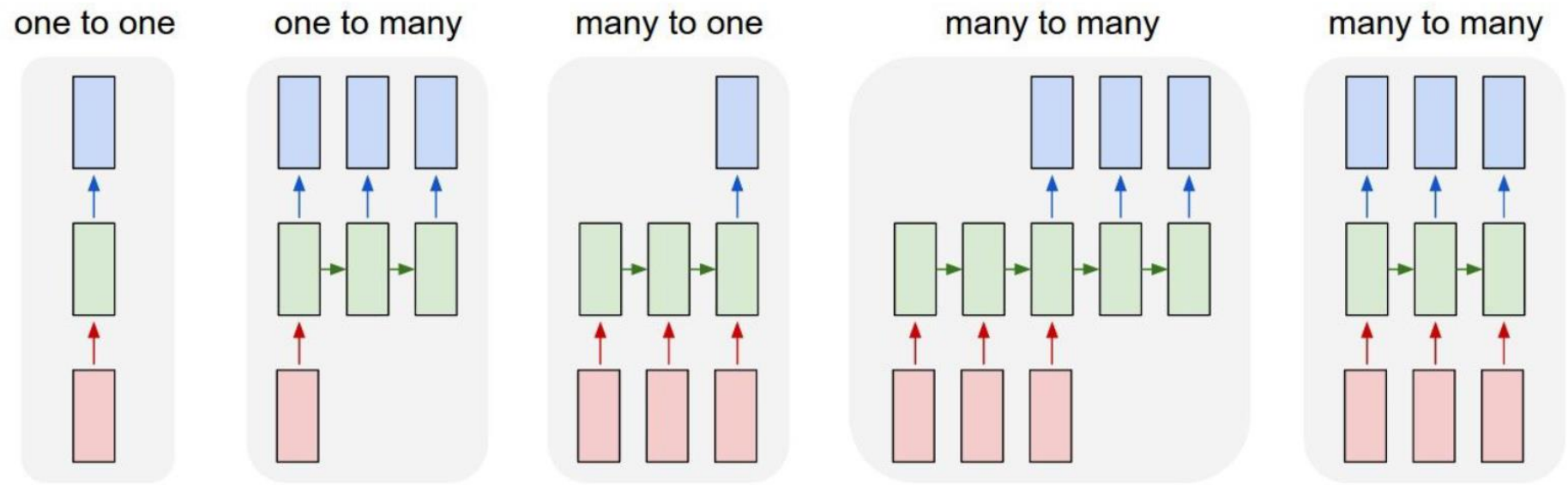
“Vanilla” Neural Network

one to one



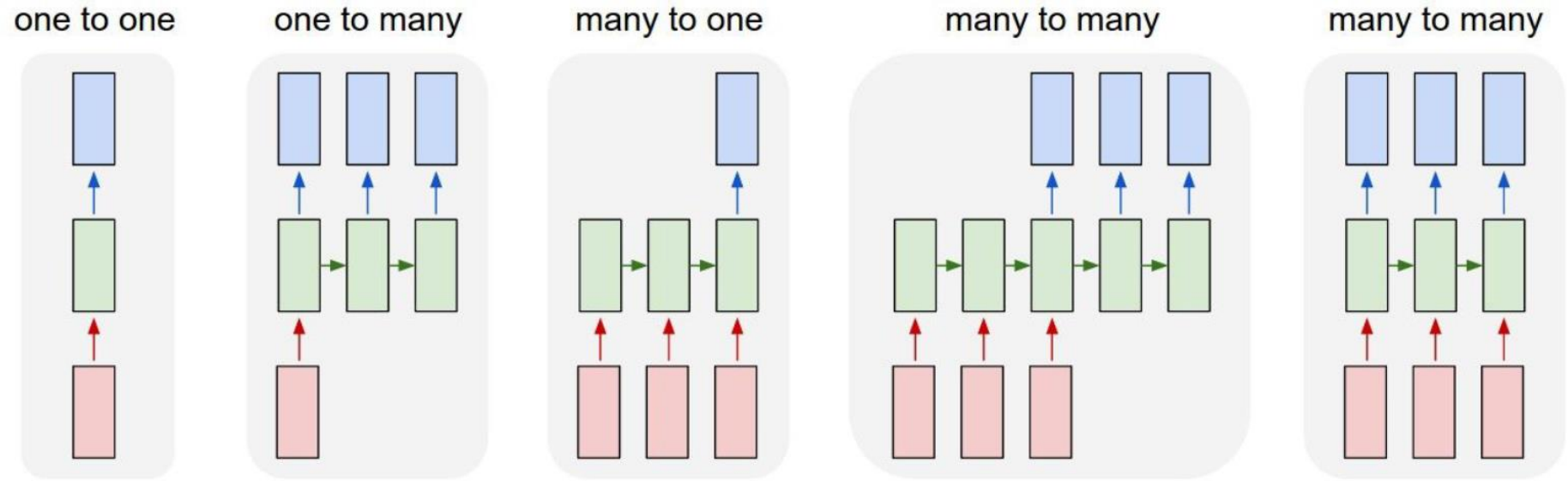
Vanilla Neural Networks

Recurrent Neural Networks: Process Sequences



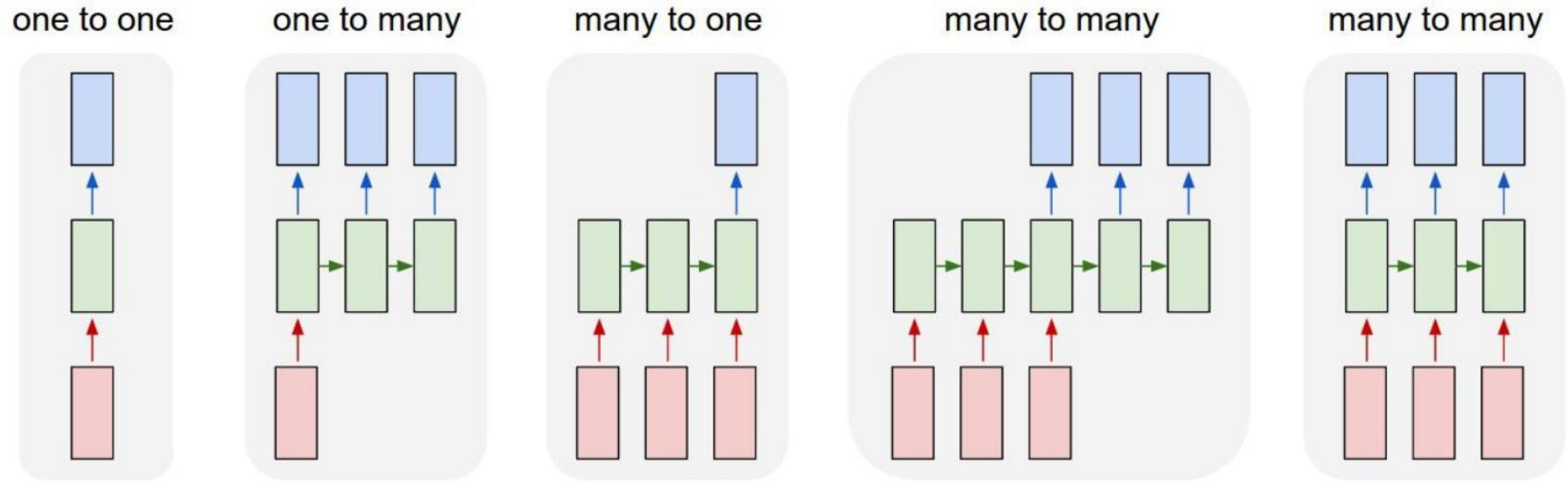
e.g. **Image Captioning**
image -> sequence of words

Recurrent Neural Networks: Process Sequences



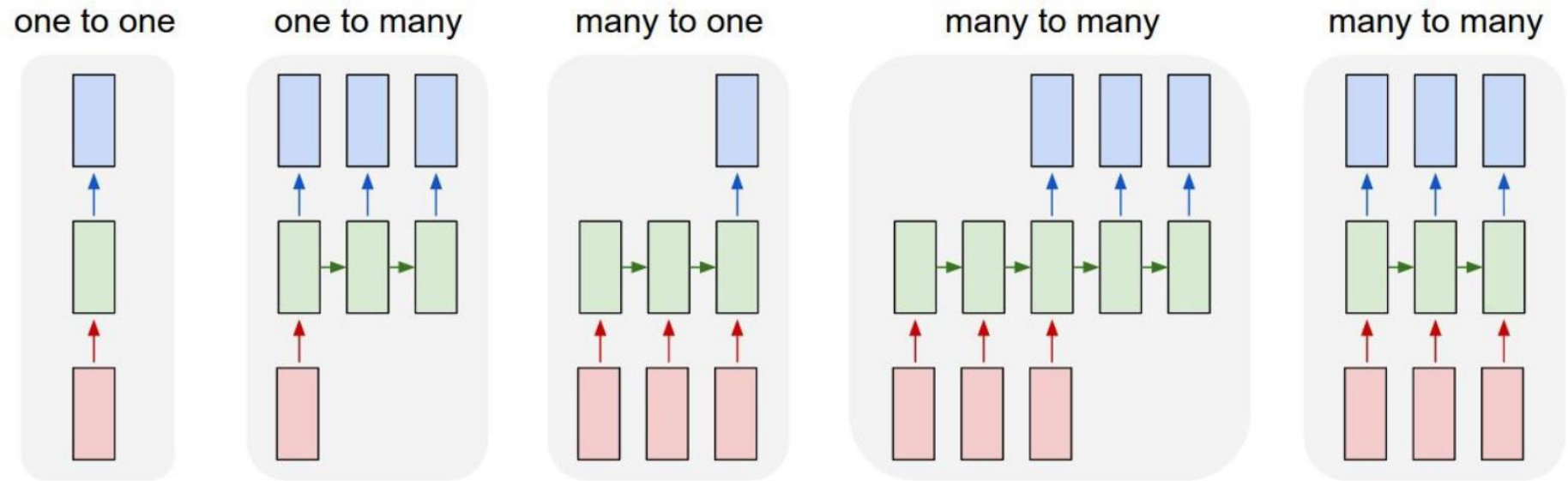
e.g. **Sentiment Classification**
sequence of words -> sentiment

Recurrent Neural Networks: Process Sequences



e.g. **Machine Translation**
seq of words -> seq of words

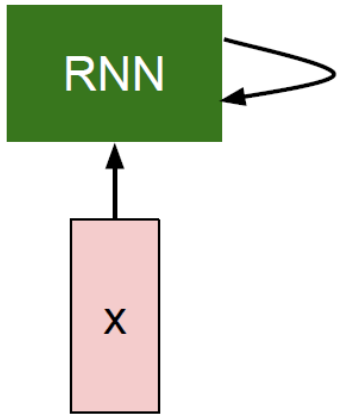
Recurrent Neural Networks: Process Sequences



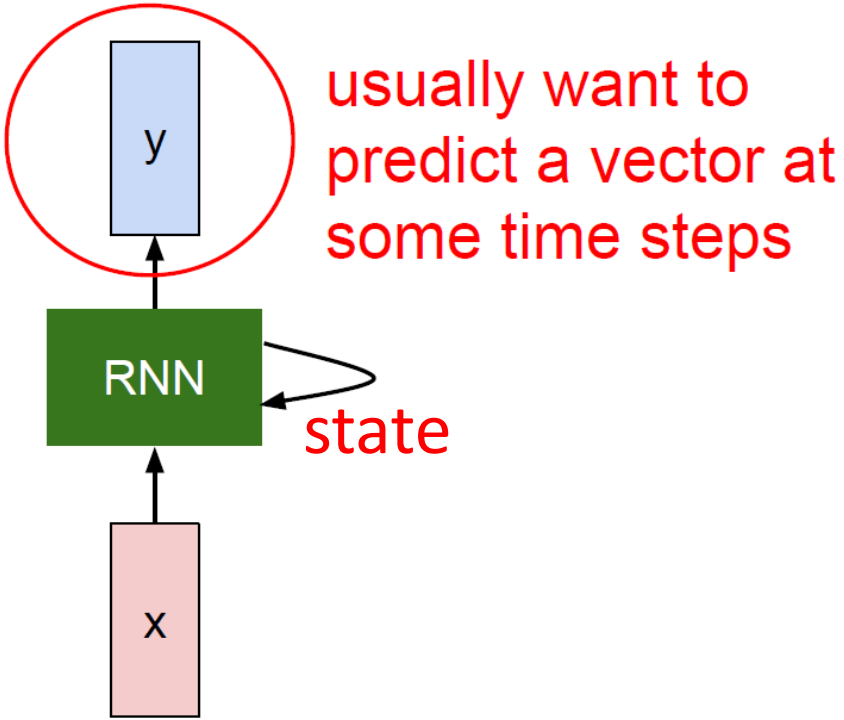
e.g. **Video classification on frame level**

VANILLA NEURAL NETWORK

Recurrent Neural Network



Recurrent Neural Network



Recurrent Neural Network

We can process a sequence of vectors **x** by applying a **recurrence formula** at every time step:

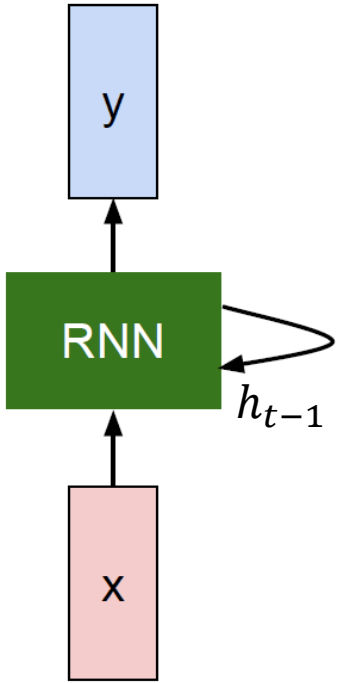
$$h_t = f_W(h_{t-1}, x_t)$$

new state

some function with parameters W

old state

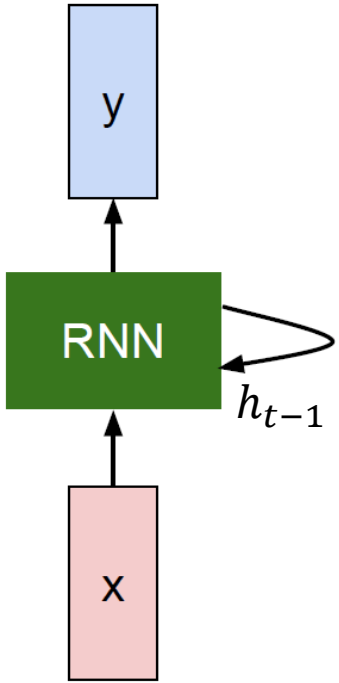
input vector at some time step



Recurrent Neural Network

We can process a sequence of vectors **x** by applying a **recurrence formula** at every time step:

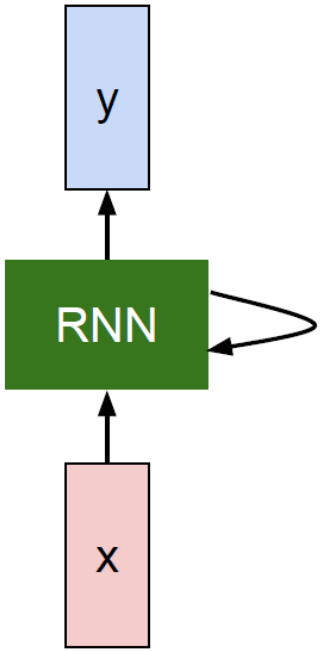
$$h_t = f_W(h_{t-1}, x_t)$$



Notice: the same function and the same set of parameters are used at every time step.

(Simple) Recurrent Neural Network

The state consists of a single *"hidden"* vector **h**:



$$h_t = f_W(h_{t-1}, x_t)$$

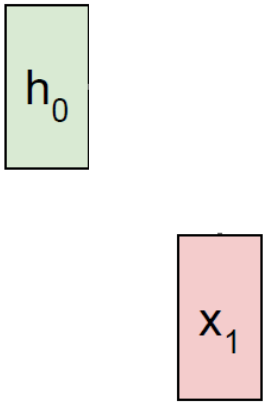
$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

$$y_t = W_{hy}h_t$$

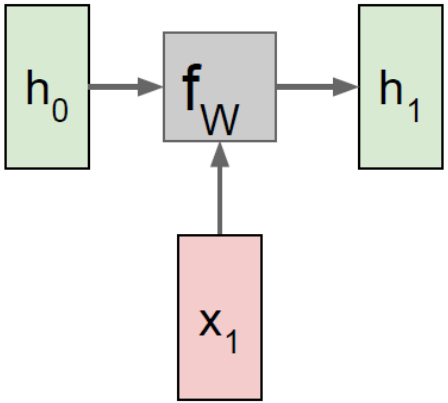
Sometimes called a "Vanilla RNN" or an "Elman RNN" after Prof. Jeffrey Elman

RNN: Computational Graph

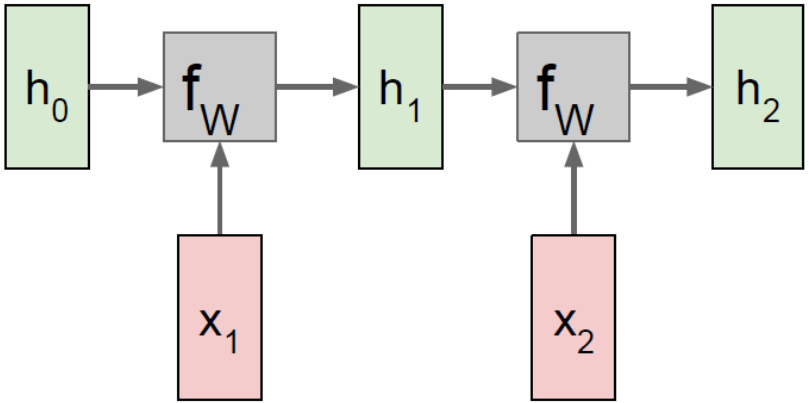
Initial hidden state
Either set to all 0,
Or learn it



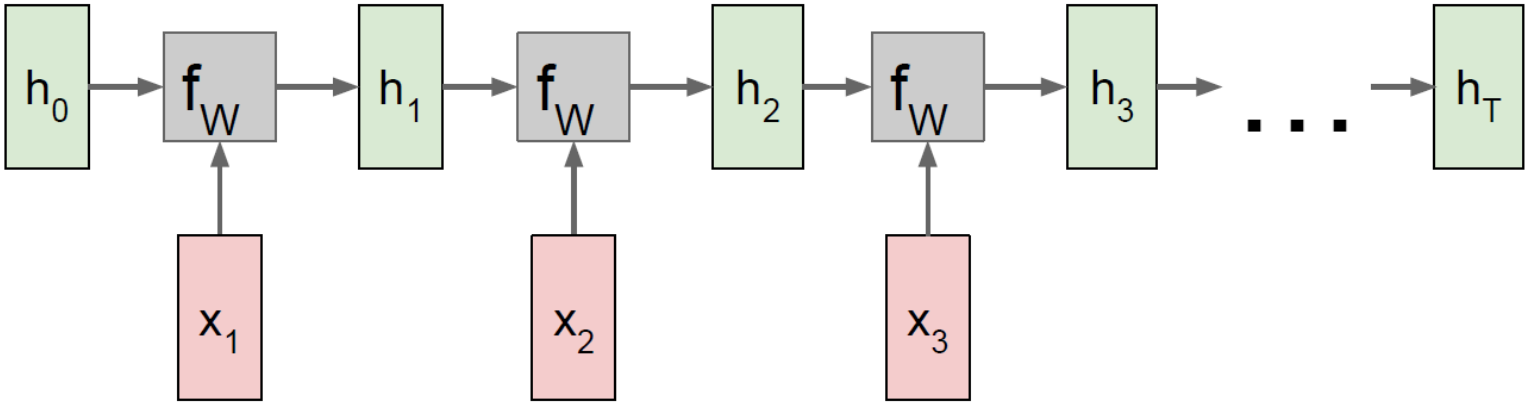
RNN: Computational Graph



RNN: Computational Graph

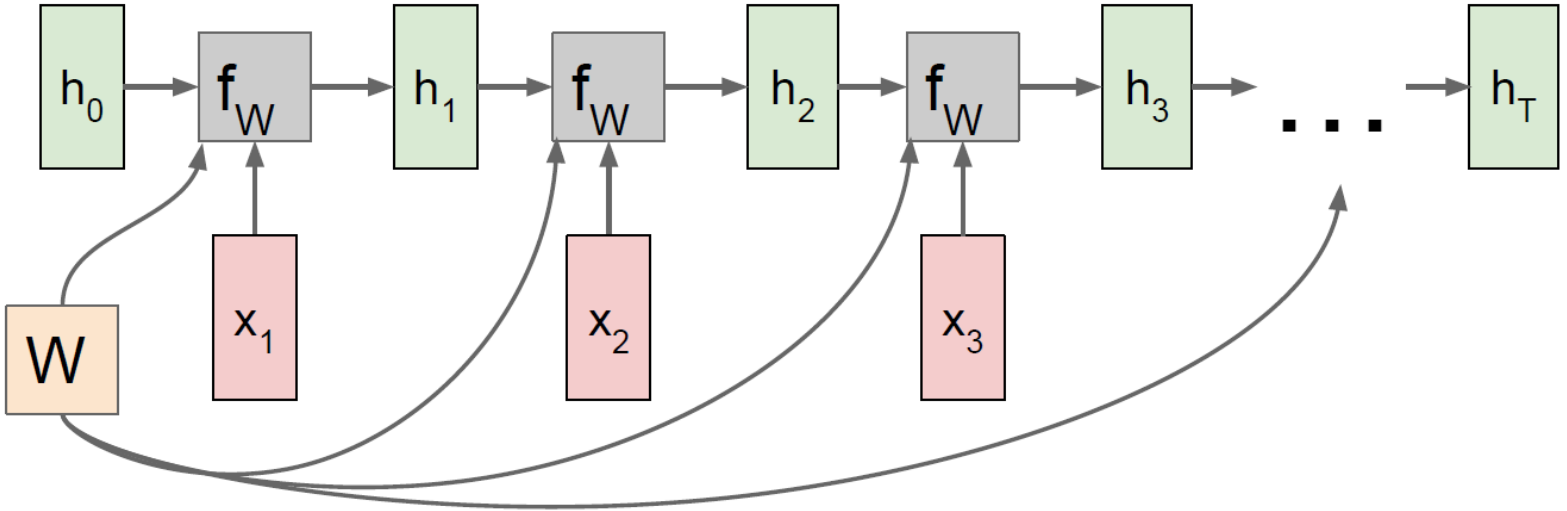


RNN: Computational Graph

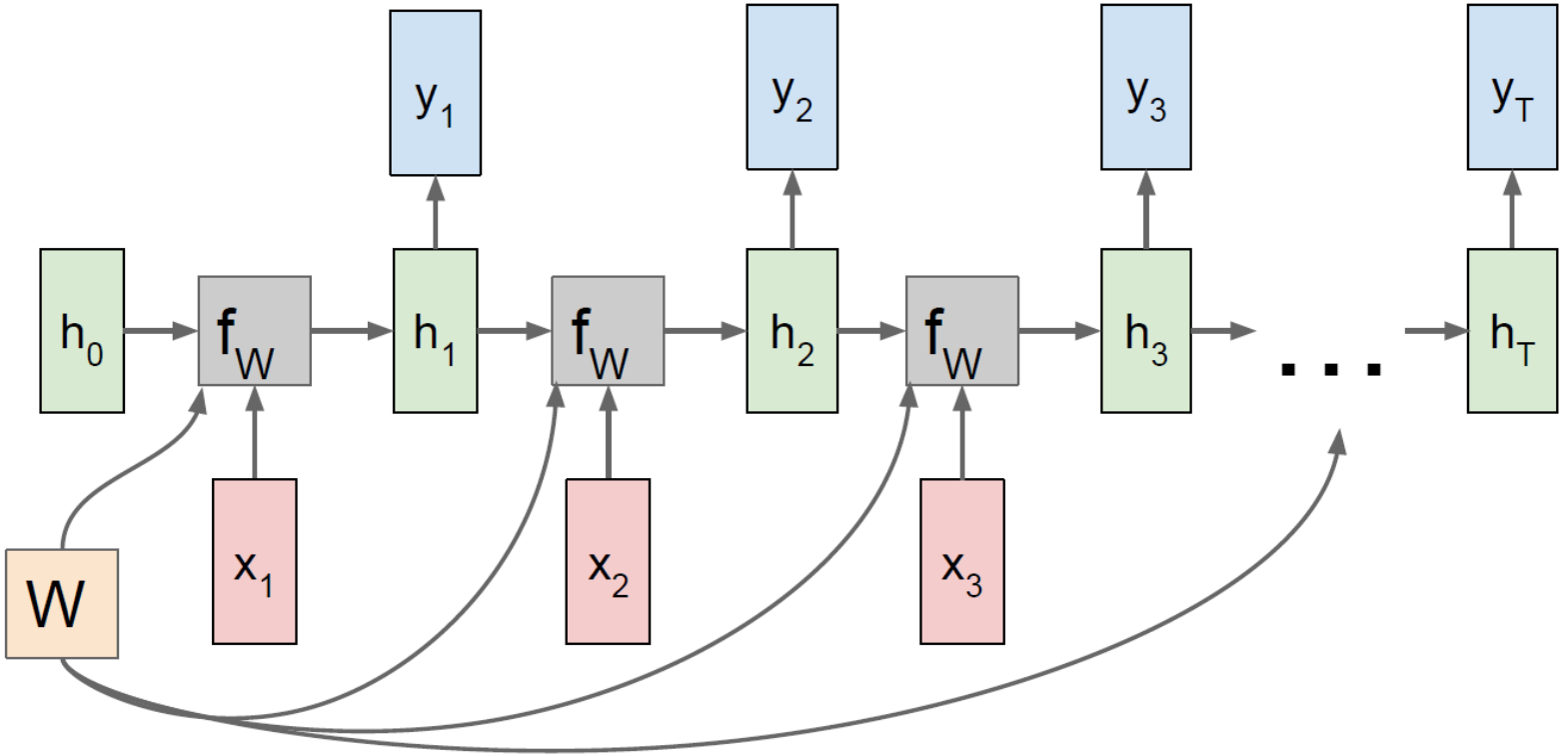


RNN: Computational Graph

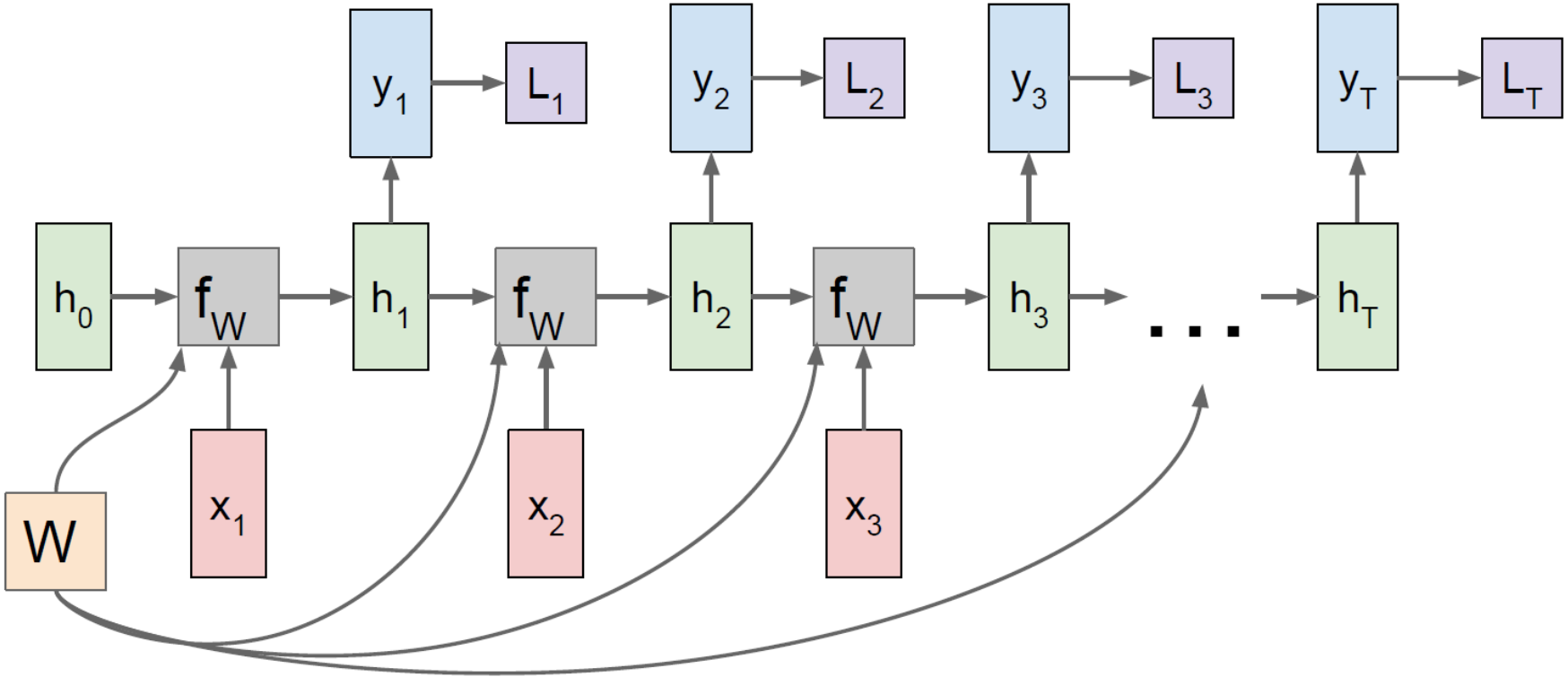
Re-use the same weight matrix at every time-step



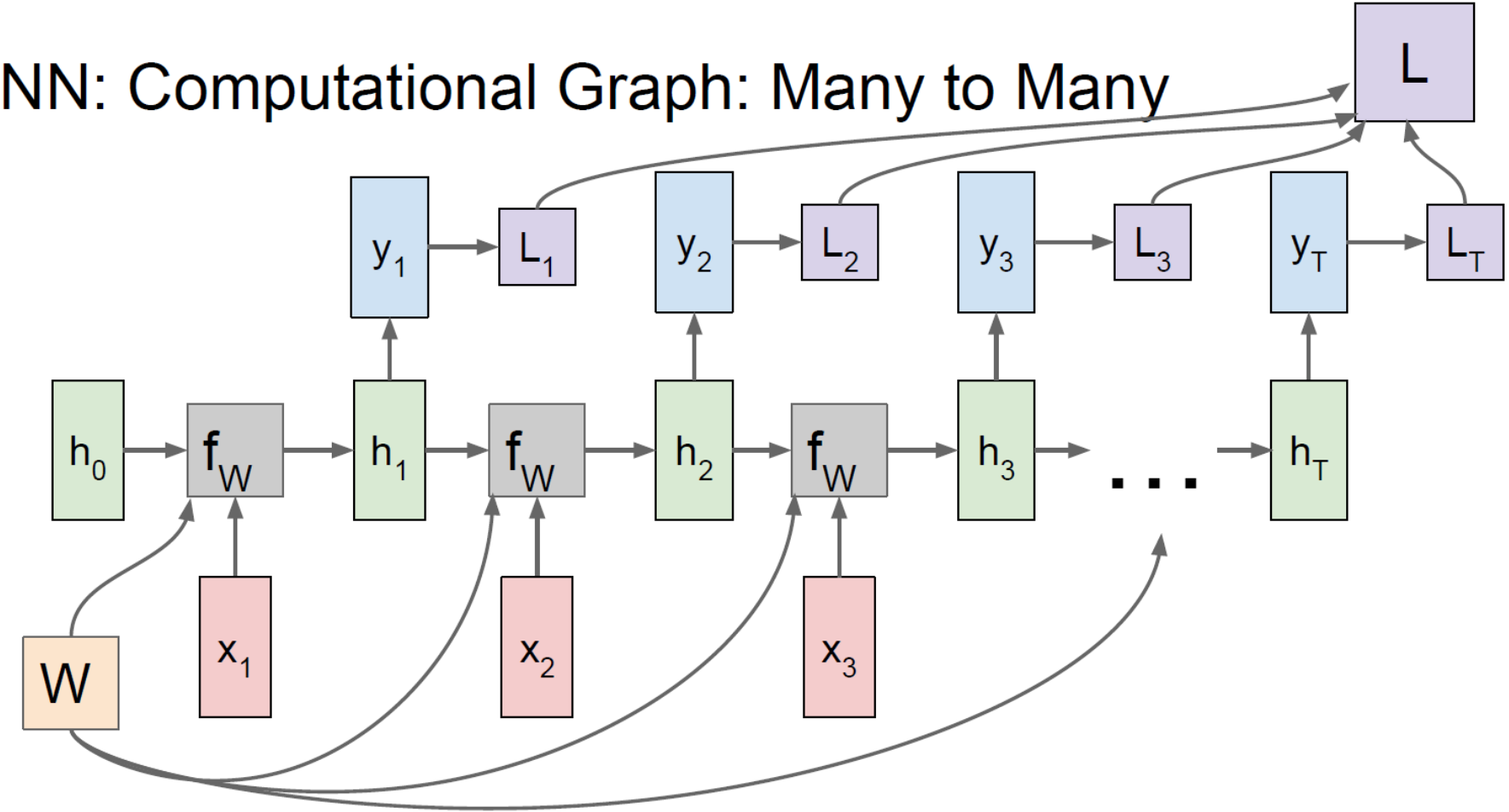
RNN: Computational Graph: Many to Many



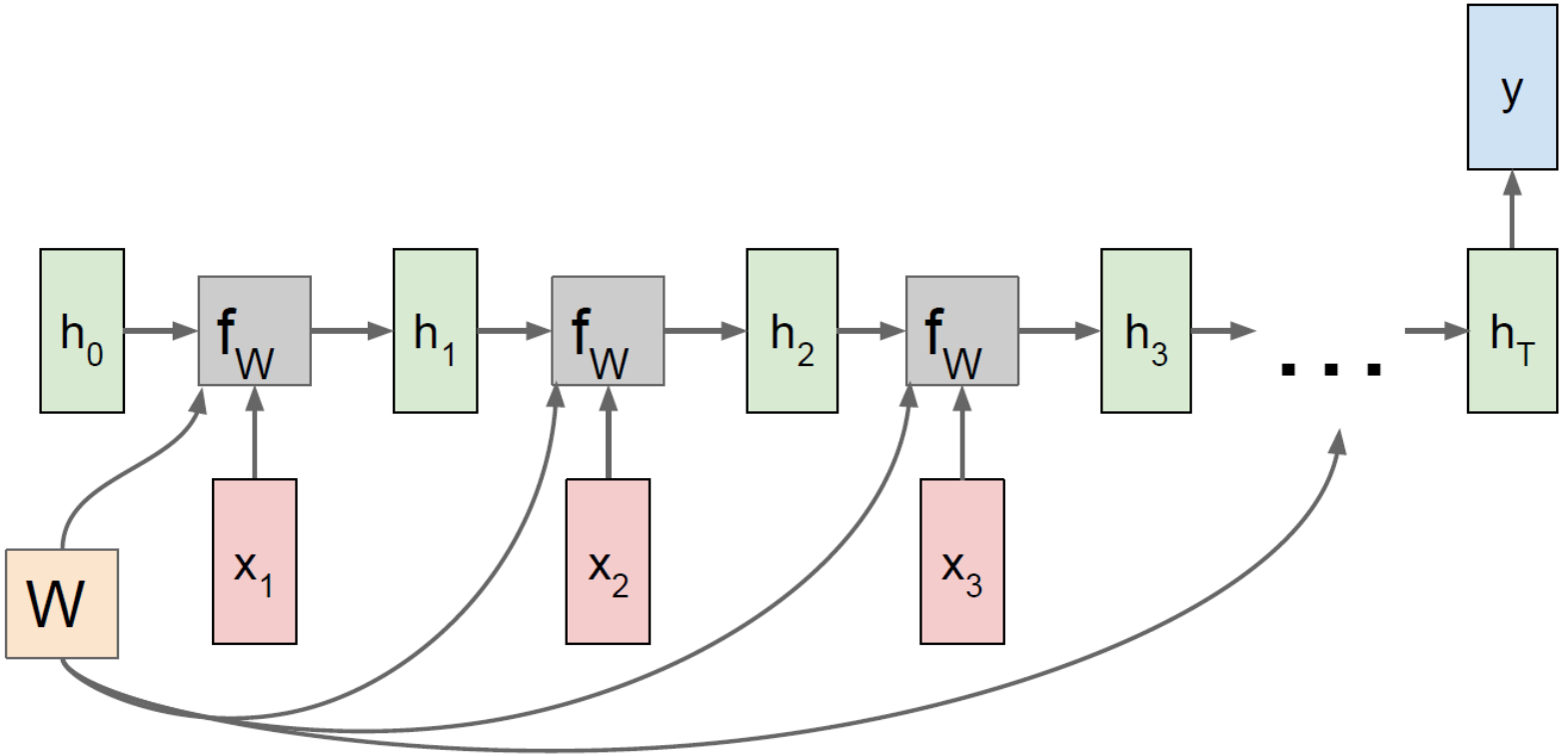
RNN: Computational Graph: Many to Many



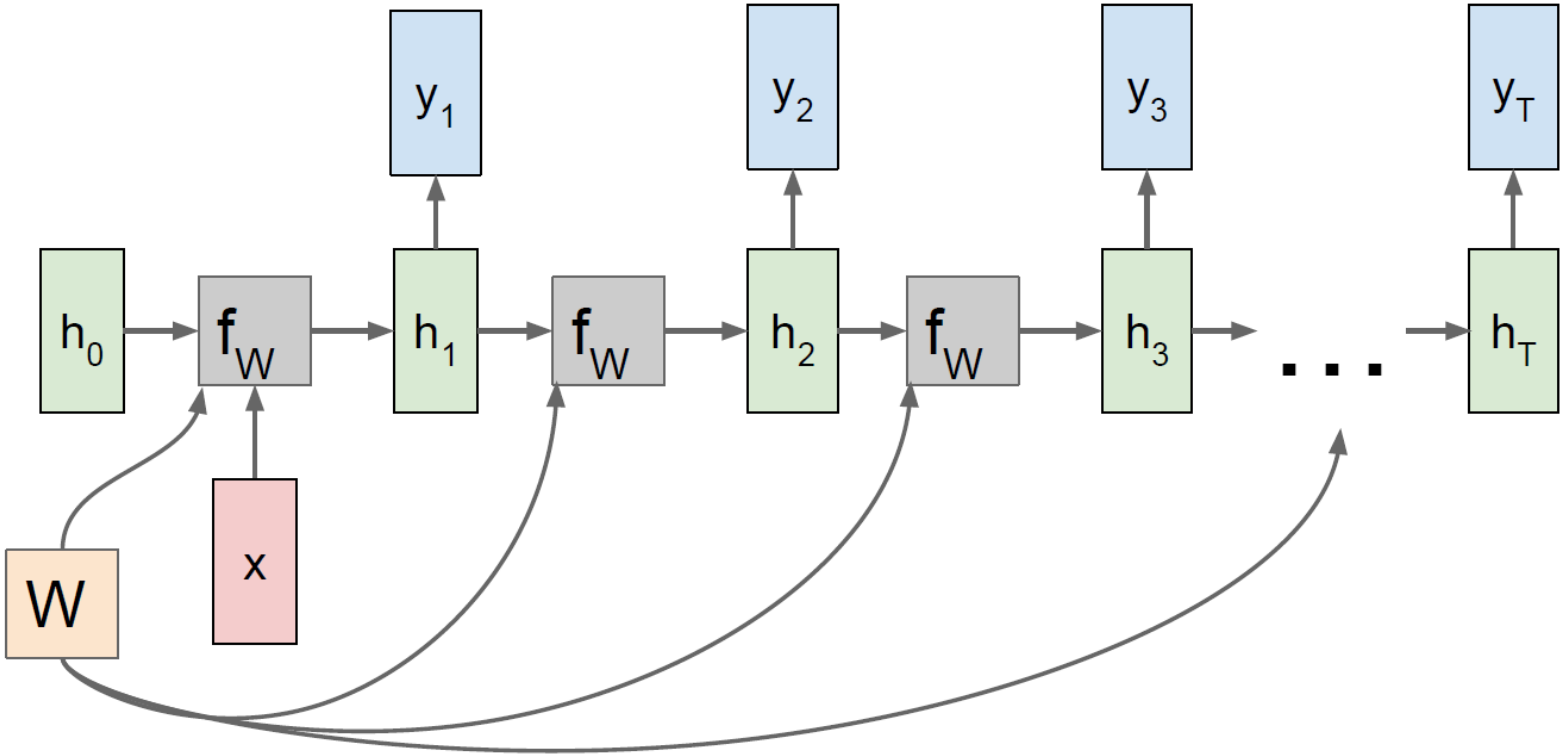
RNN: Computational Graph: Many to Many



RNN: Computational Graph: Many to One

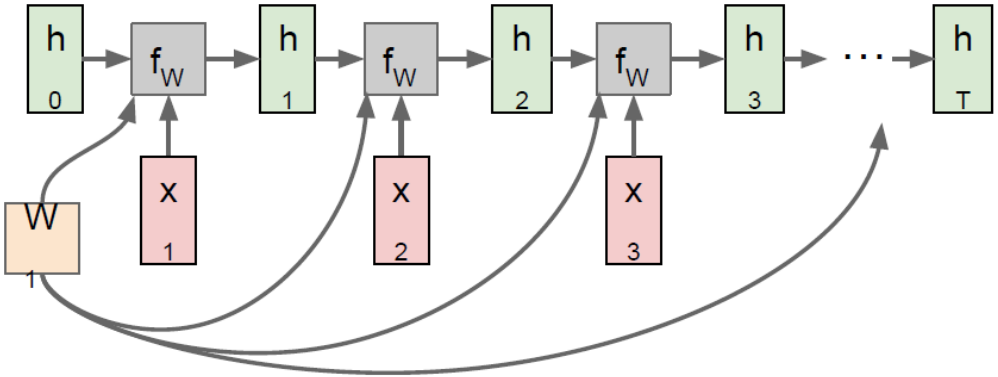


RNN: Computational Graph: One to Many



Sequence to Sequence: Many-to-one + one-to-many

Many to one: Encode input sequence in a single vector

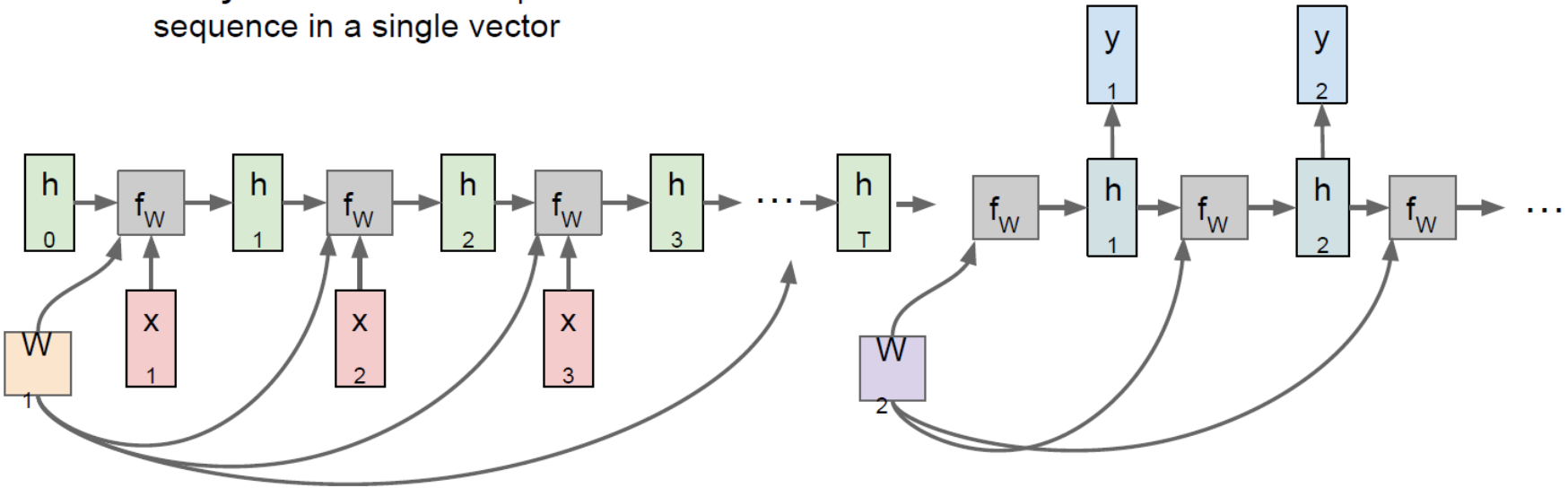


Sutskever et al, "Sequence to Sequence Learning with Neural Networks", NIPS 2014

Sequence to Sequence: Many-to-one + one-to-many

Many to one: Encode input sequence in a single vector

One to many: Produce output sequence from single input vector



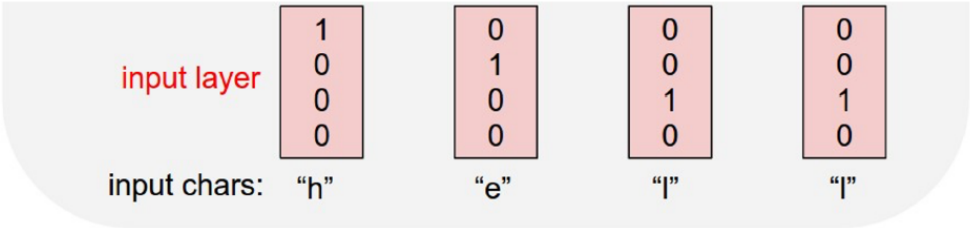
Sutskever et al, "Sequence to Sequence Learning with Neural Networks", NIPS 2014

Example: Language Modeling

Given characters 1, 2, …, t-1,
model predicts character t

Training sequence: ”hello”

Vocabulary: [h, e, l, o]



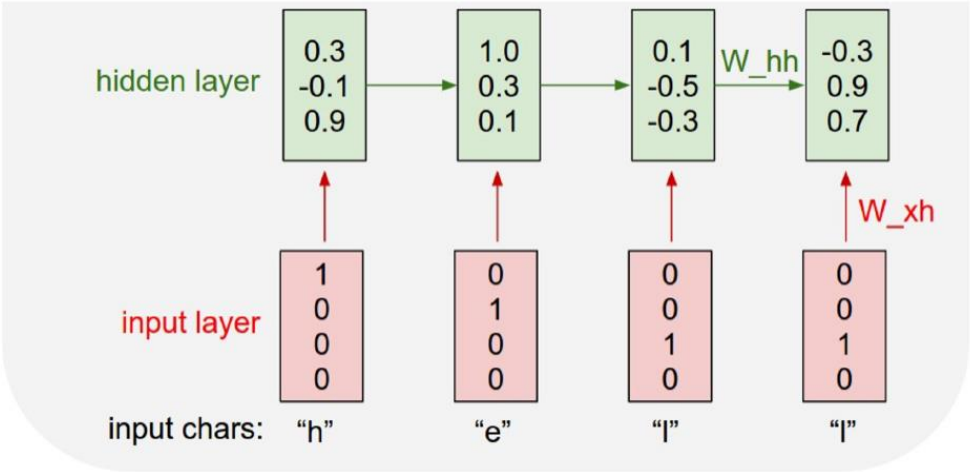
Example: Language Modeling

Given characters 1, 2, …, t-1,
model predicts character t

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Training sequence: ”hello”

Vocabulary: [h, e, l, o]



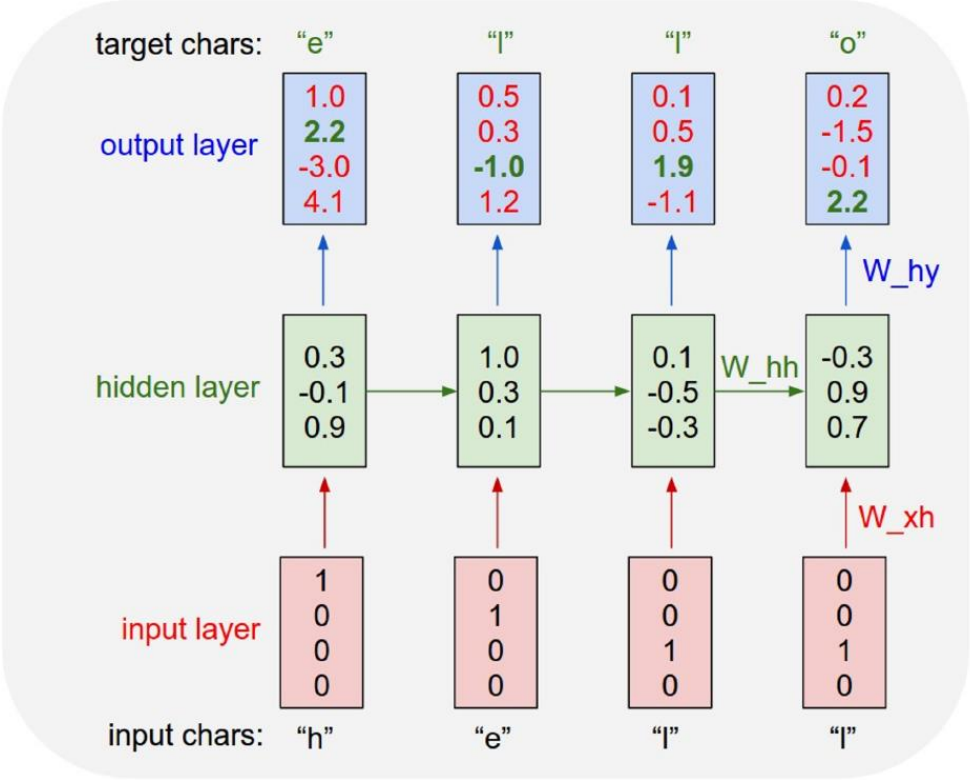
Example: Language Modeling

Given characters 1, 2, …, t-1, model predicts character t

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Training sequence: ”hello”

Vocabulary: [h, e, l, o]



Example: Language Modeling

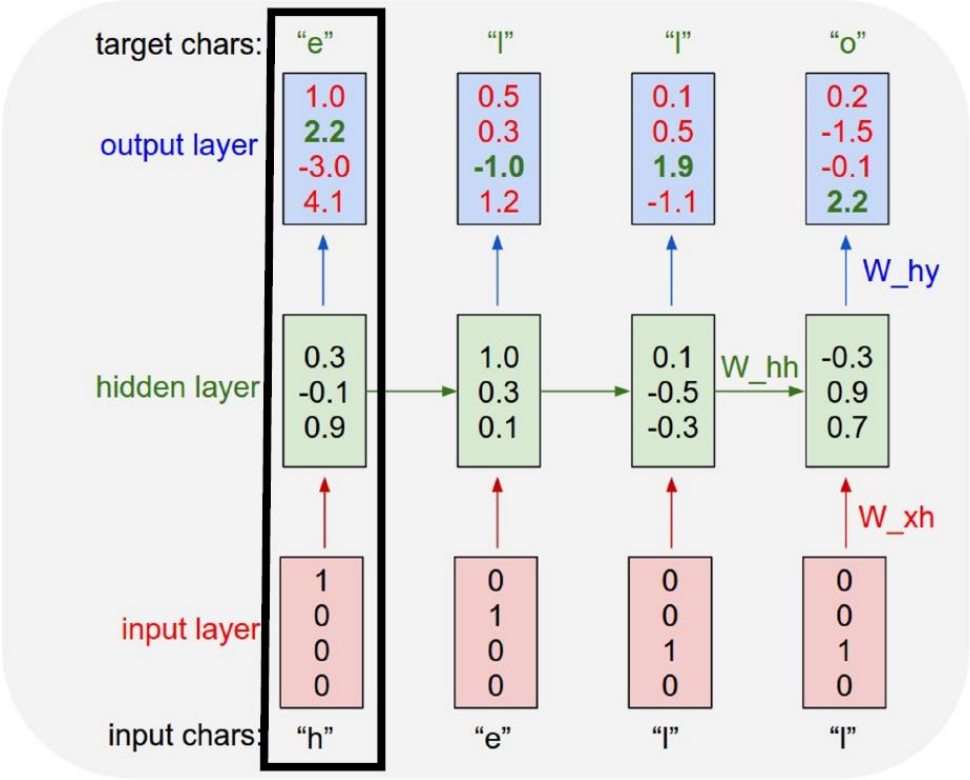
Given “h”, predict “e”

Given characters 1, 2, …, t-1, model predicts character t

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Training sequence: ”hello”

Vocabulary: [h, e, l, o]



Example: Language Modeling

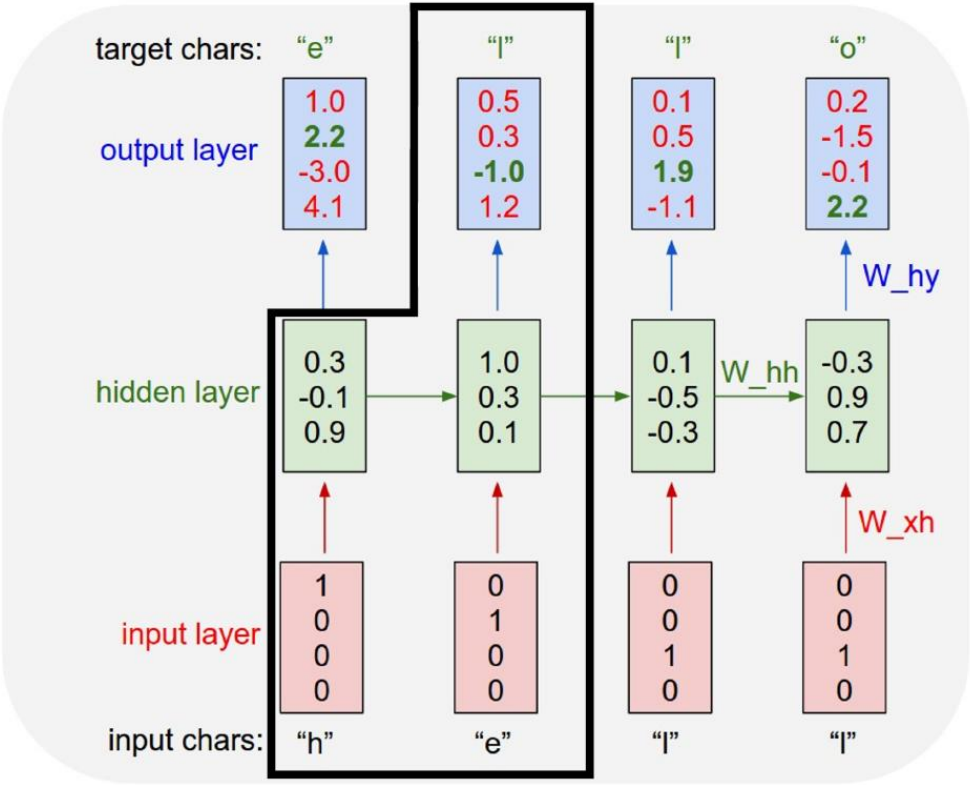
Given “he”, predict “l”

Given characters 1, 2, …, t-1, model predicts character t

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Training sequence: ”hello”

Vocabulary: [h, e, l, o]



Example: Language Modeling

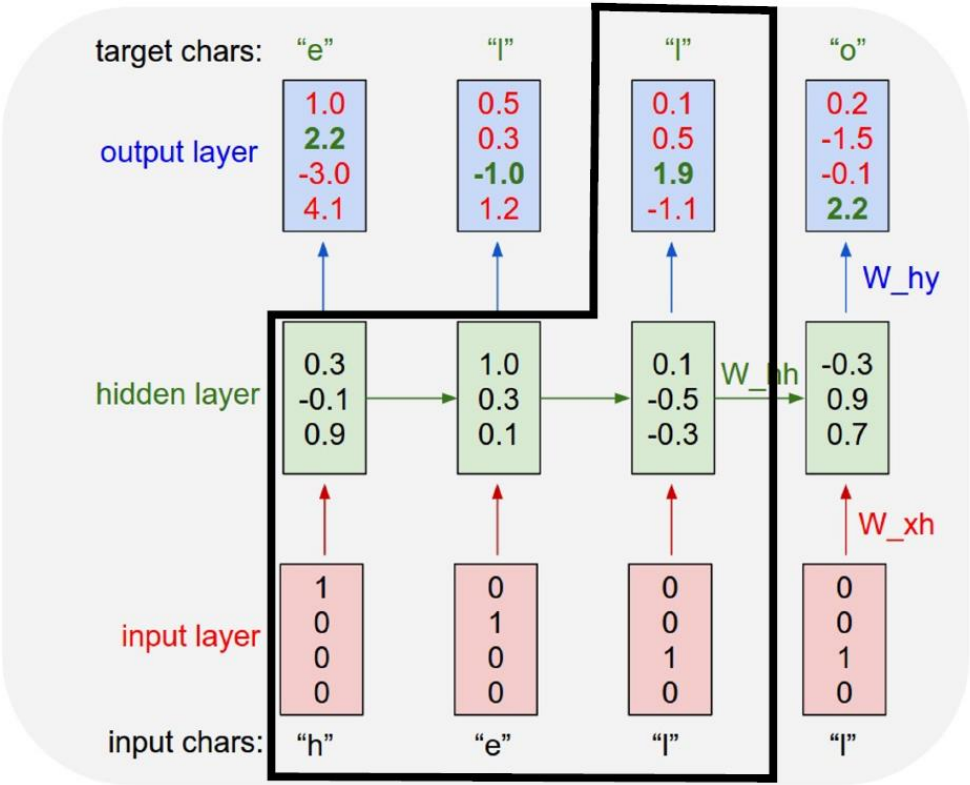
Given “hel”, predict “l”

Given characters 1, 2, …, t-1, model predicts character t

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Training sequence: ”hello”

Vocabulary: [h, e, l, o]



Example: Language Modeling

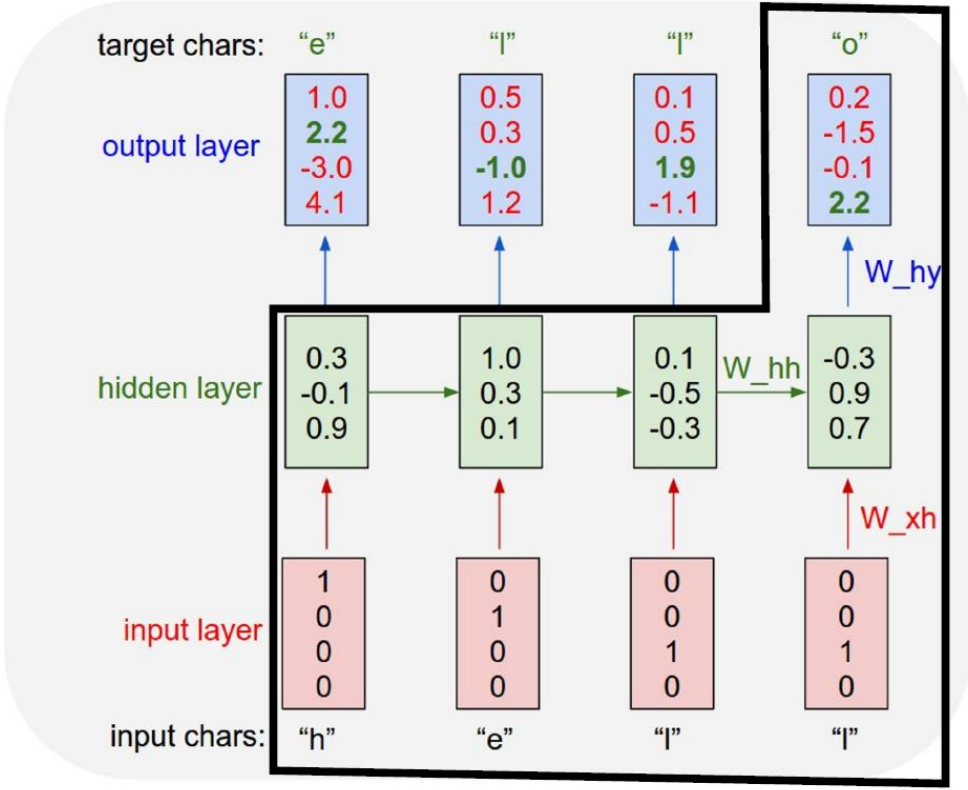
Given “hell”, predict “o”

Given characters 1, 2, …, t-1, model predicts character t

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$

Training sequence: ”hello”

Vocabulary: [h, e, l, o]

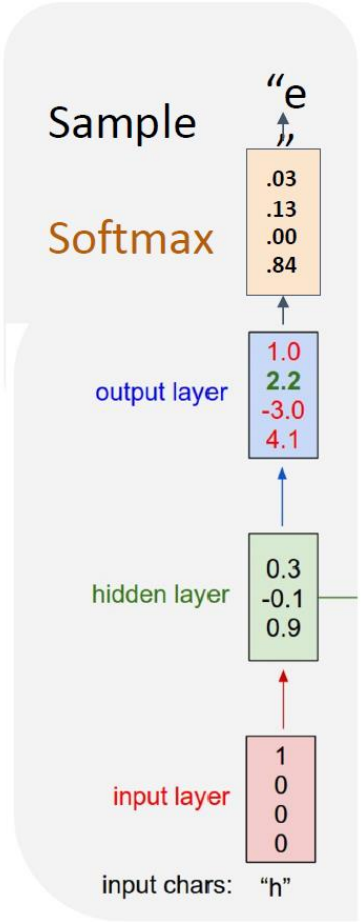


Example: Language Modeling

At test-time, **generate** new text: sample characters one at a time, feed back to model

Training sequence: ”hello”

Vocabulary: [h, e, l, o]



Example: Language Modeling

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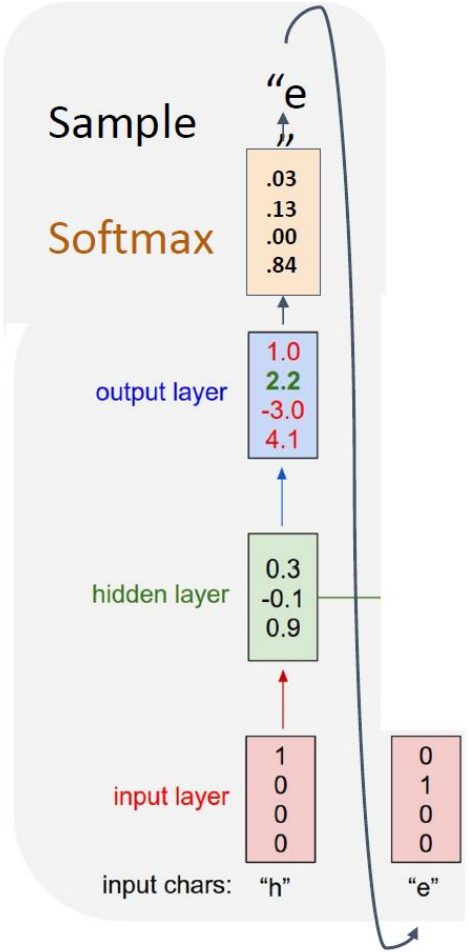
Training sequence: ”hello”

Vocabulary: [h, e, l, o]



Example: Language Modeling

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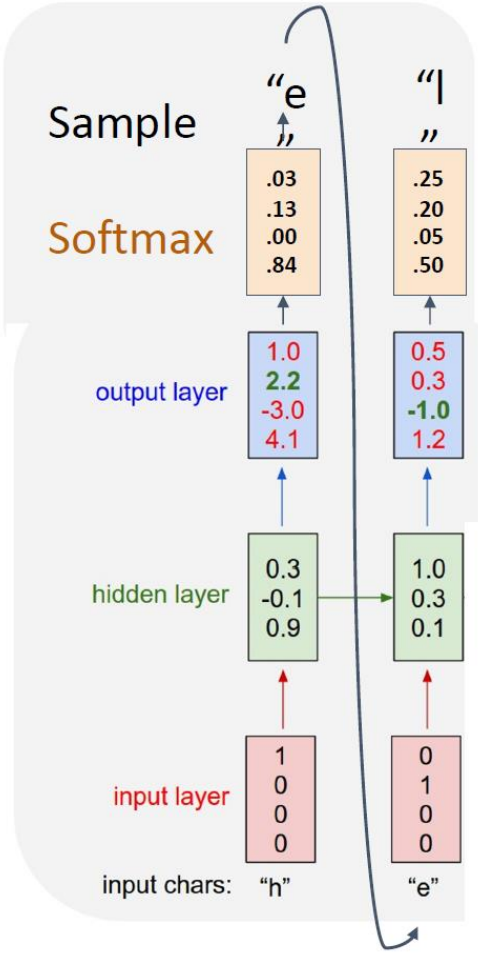

Example: Language Modeling

At test-time, **generate** new text: sample characters one at a time, feed back to model

Training sequence: ”hello”

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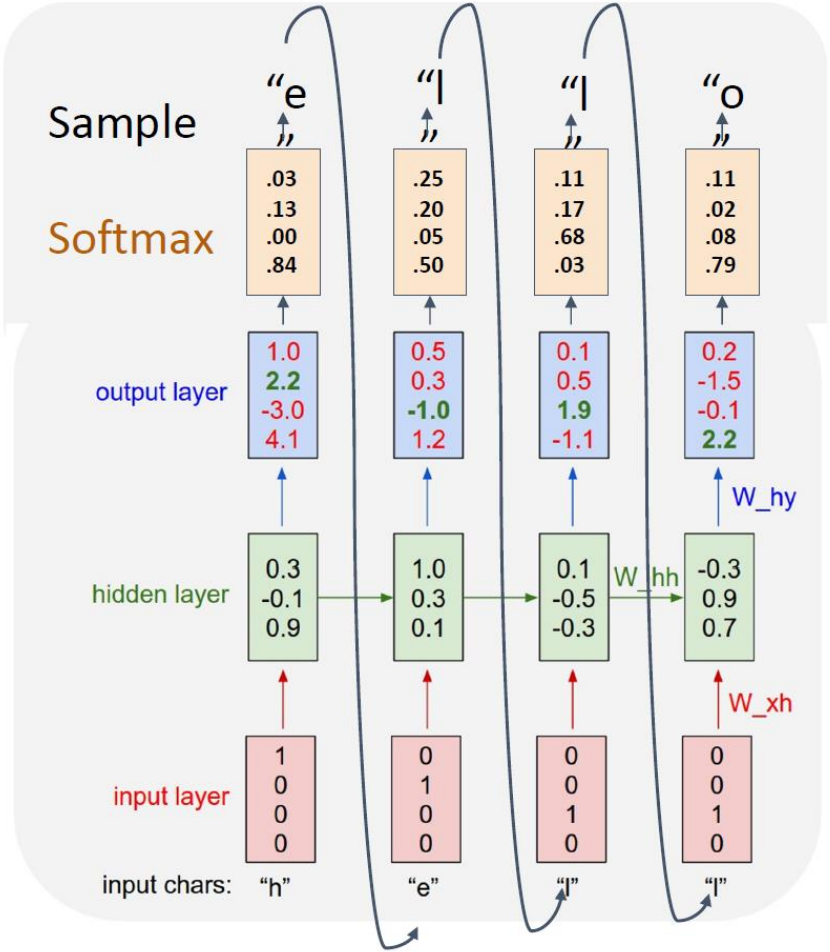
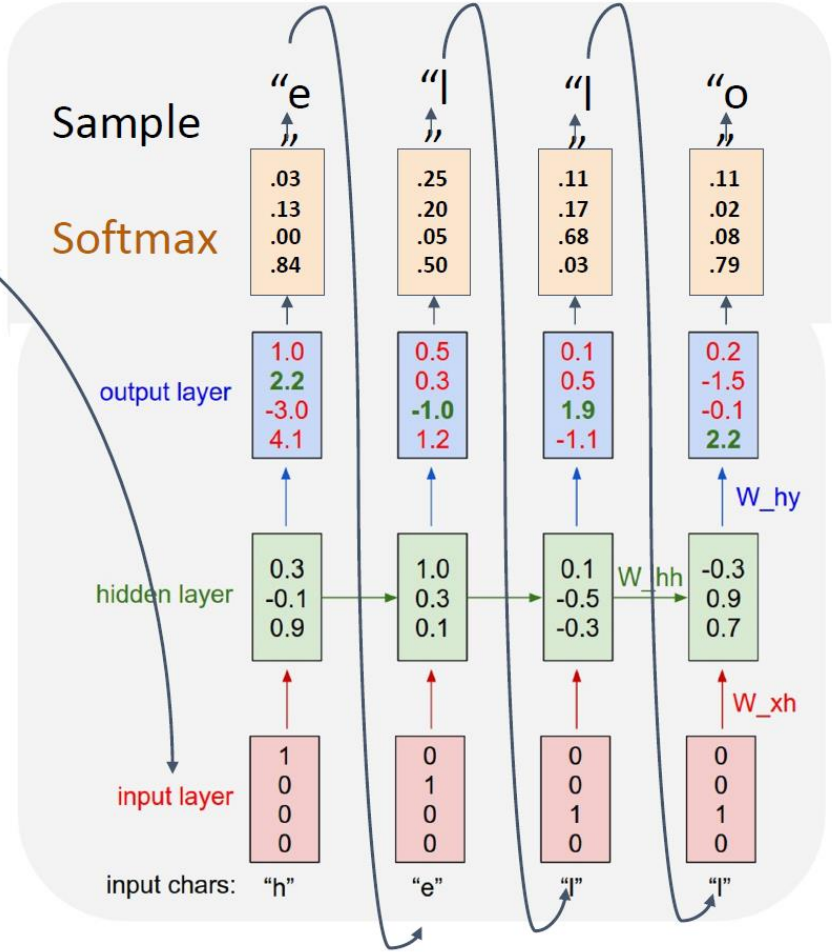


Example: Language Modeling

So far: encode inputs as **one-hot-vector**

$$\begin{bmatrix} w_{11} & w_{12} & w_{13} & w_{14} \\ w_{21} & w_{22} & w_{23} & w_{14} \\ w_{31} & w_{32} & w_{33} & w_{14} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} w_{11} \\ w_{21} \\ w_{31} \end{bmatrix}$$

Matrix multiply with a one-hot vector just extracts a column from the weight matrix. Often extract this into a separate **embedding** layer



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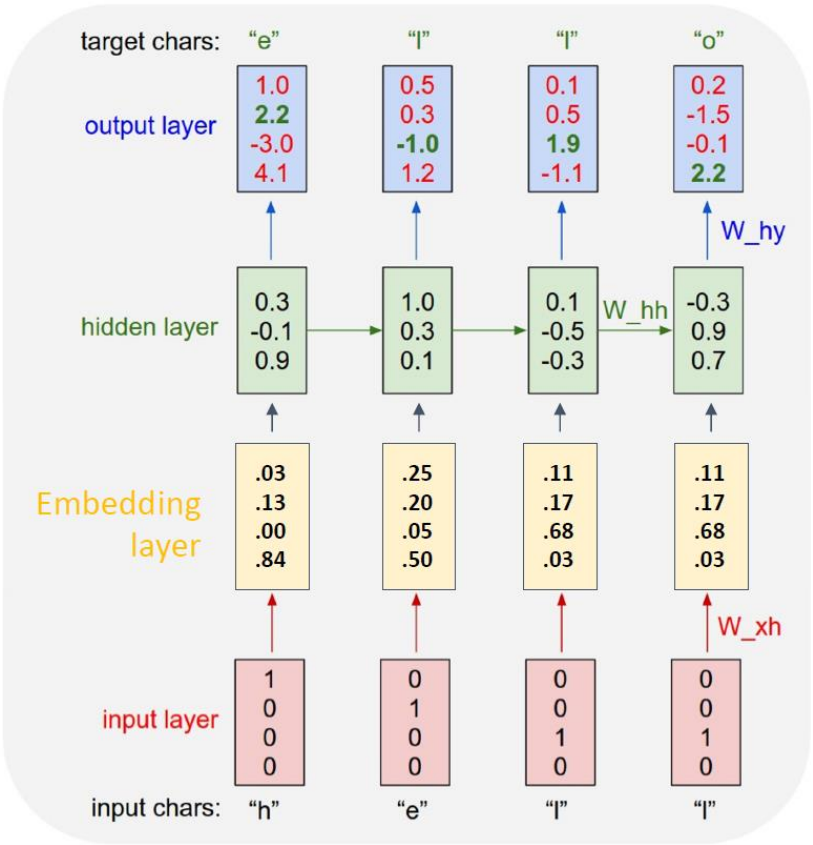
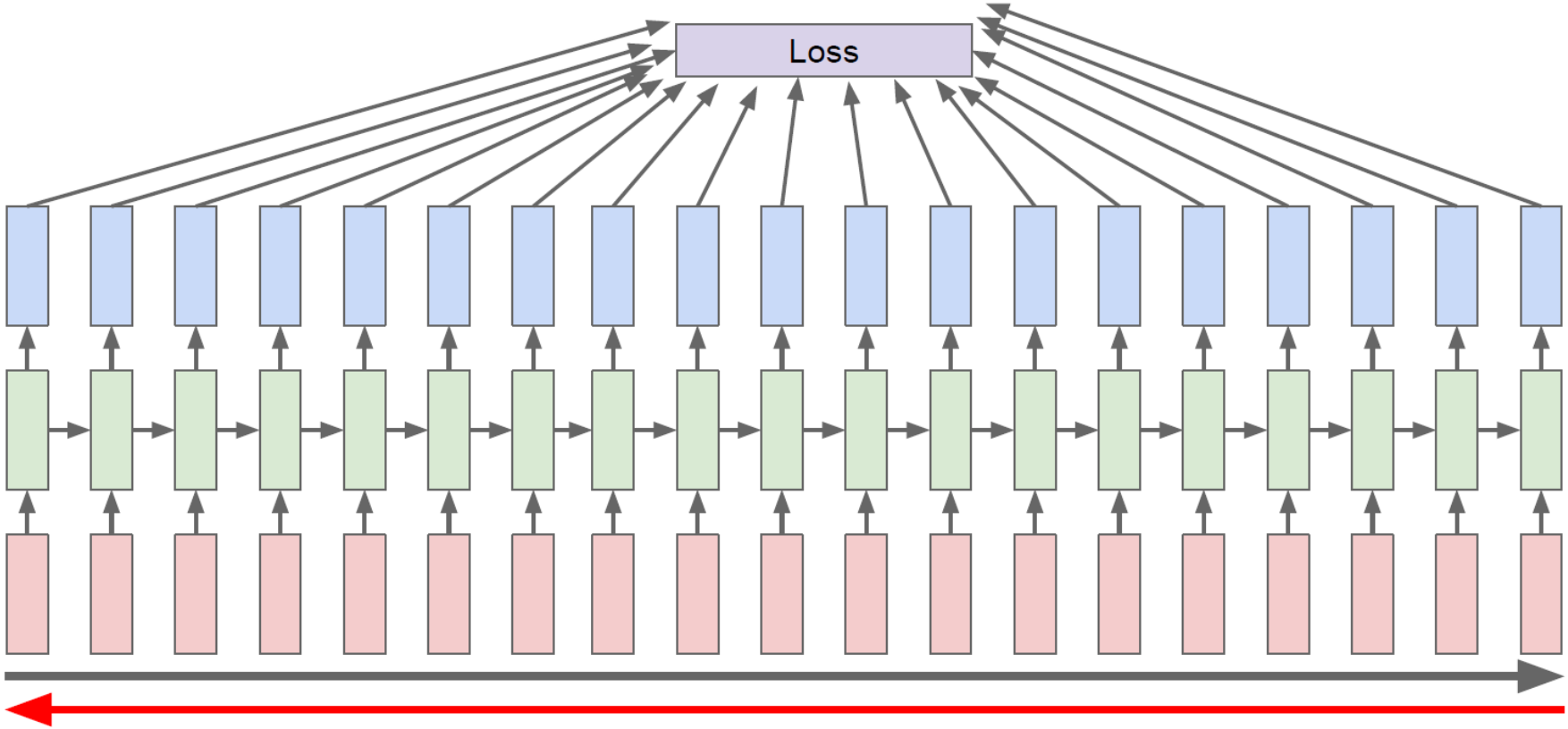


Backpropagation through time

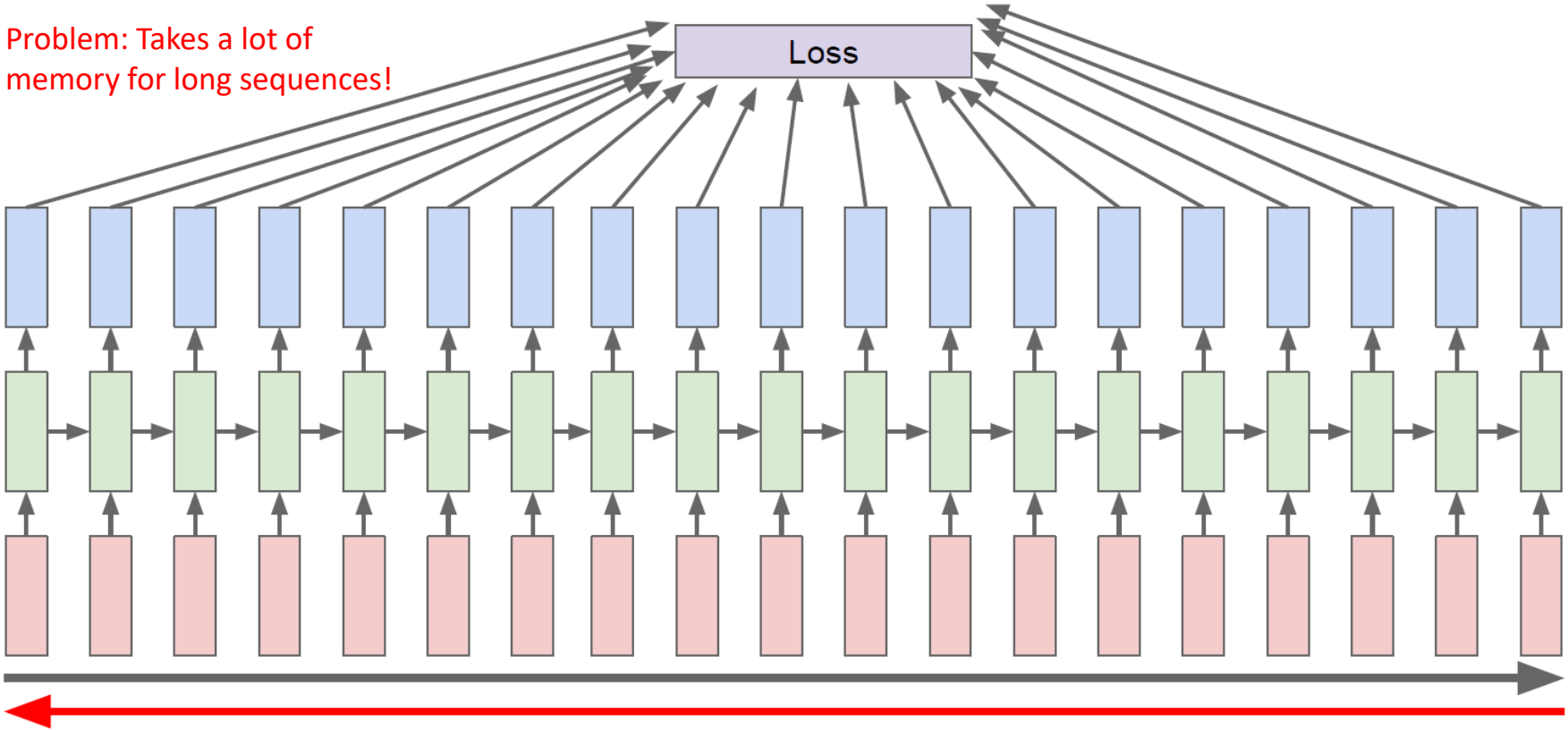
Forward through entire sequence to compute loss, then backward through entire sequence to compute gradient



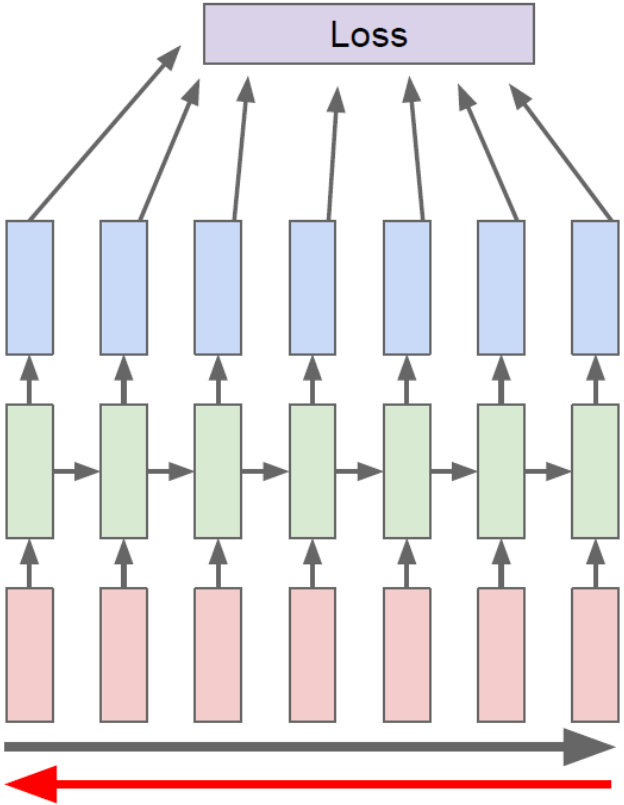
Backpropagation through time

Forward through entire sequence to compute loss, then backward through entire sequence to compute gradient

Problem: Takes a lot of memory for long sequences!

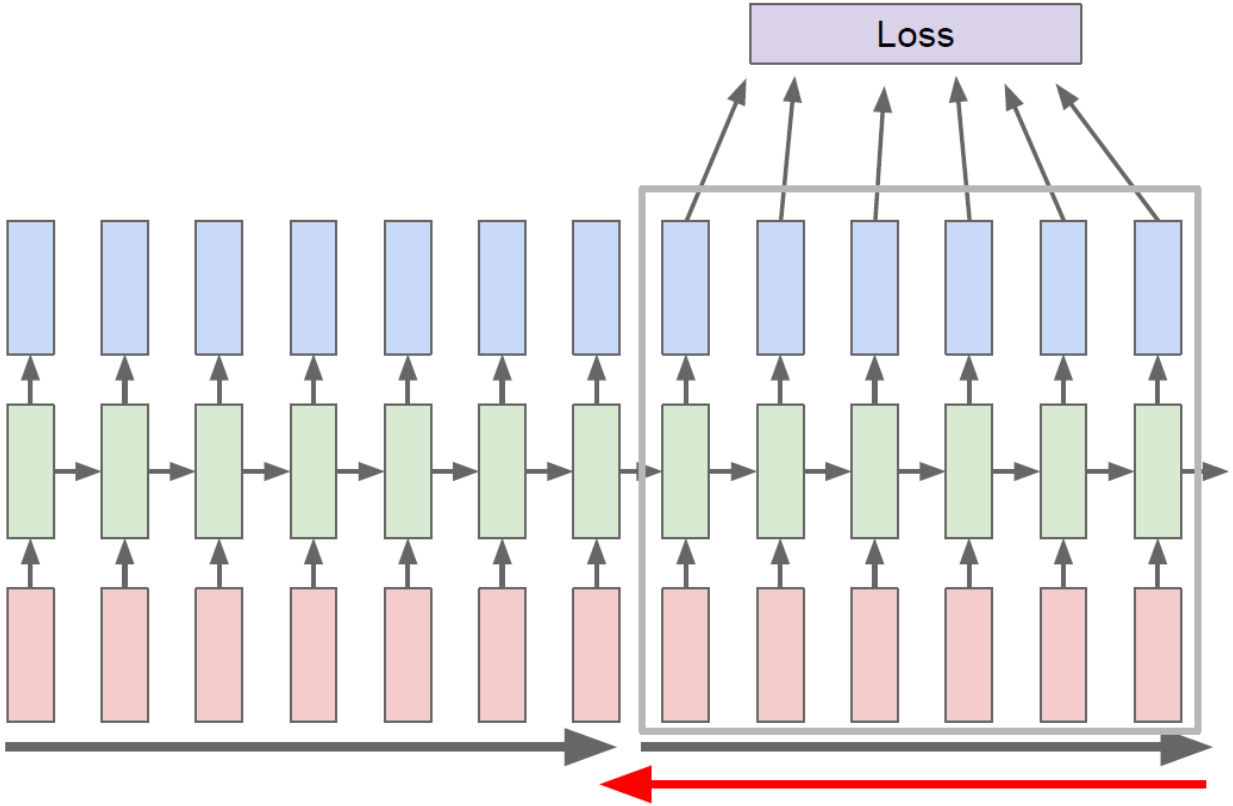


Truncated Backpropagation through time



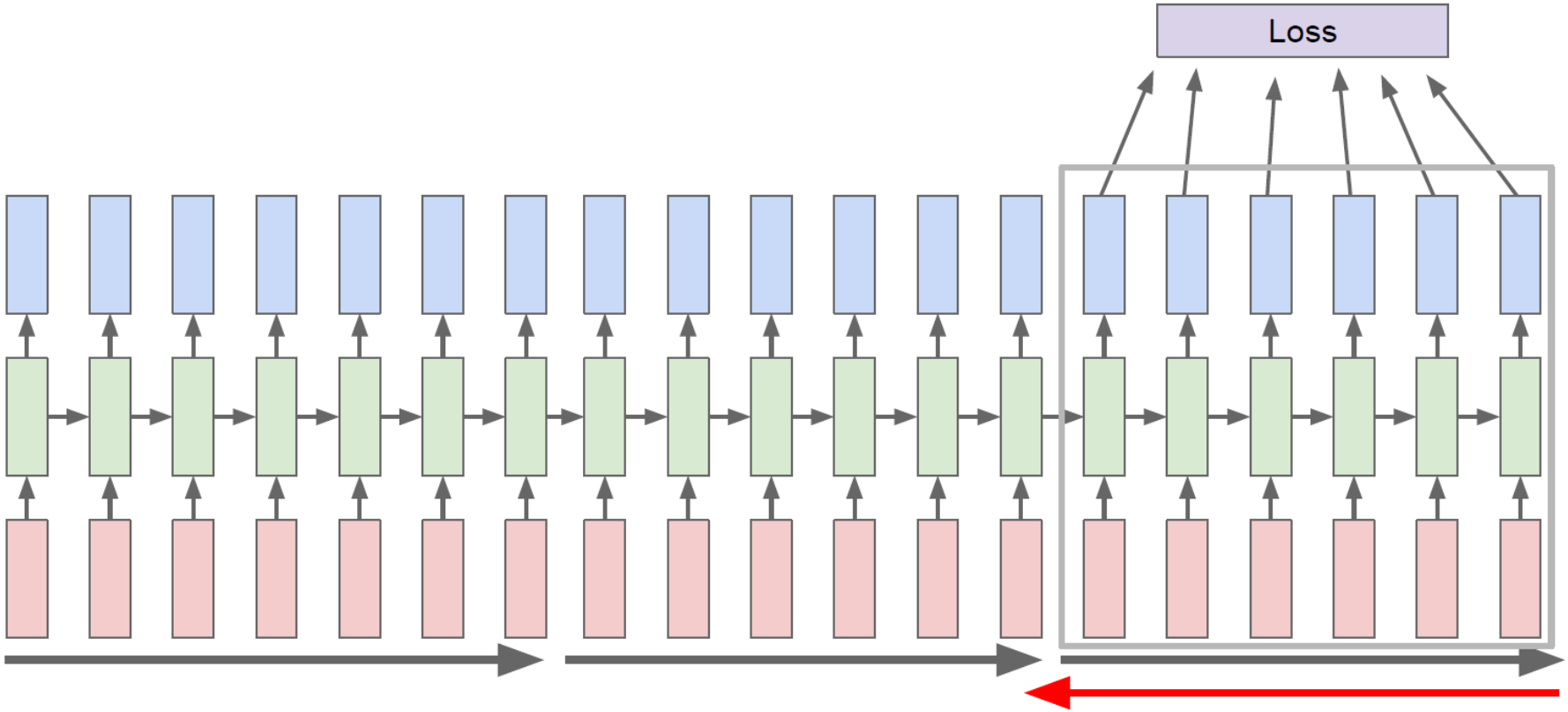
Run forward and backward through chunks of the sequence instead of whole sequence

Truncated Backpropagation through time



Carry hidden states forward in time forever, but only backpropagate for some smaller number of steps

Truncated Backpropagation through time



min-char-rnn.py gist: 112 lines of Python

```
"""
Minimal character-level Vanilla RNN model. Written by Andrej Karpathy (@karpathy)
BSD License
"""
import numpy as np

# data I/O
data = open('input.txt', 'r').read() # should be simple plain text file
chars = list(set(data))
data_size, vocab_size = len(data), len(chars)
print 'data has %d characters, %d unique.' % (data_size, vocab_size)
char_to_ix = { ch:i for i,ch in enumerate(chars) }
ix_to_char = { i:ch for i,ch in enumerate(chars) }

# hyperparameters
hidden_size = 100 # size of hidden layer of neurons
seq_length = 25 # number of steps to unroll the RNN for
learning_rate = 1e-1

# model parameters
Wxh = np.random.randn(hidden_size, vocab_size)*0.01 # input to hidden
Whh = np.random.randn(hidden_size, hidden_size)*0.01 # hidden to hidden
Why = np.random.randn(vocab_size, hidden_size)*0.01 # hidden to output
bh = np.zeros((hidden_size, 1)) # hidden bias
by = np.zeros((vocab_size, 1)) # output bias

def lossFun(inputs, targets, hprev):
  """
  inputs,targets are both list of integers.
  hprev is Hx1 array of initial hidden state
  returns the loss, gradients on model parameters, and last hidden state
  """
  xs, hs, ys, ps = {}, {}, {}, {}
  hs[-1] = np.copy(hprev)
  loss = 0
  # forward pass
  for t in xrange(len(inputs)):
    xs[t] = np.zeros((vocab_size,1)) # encode in 1-of-k representation
    xs[t][inputs[t]] = 1
    hs[t] = np.tanh(np.dot(Wxh, xs[t]) + np.dot(Whh, hs[t-1]) + bh) # hidden state
    ys[t] = np.dot(Why, hs[t]) + by # unnormalized log probabilities for next chars
    ps[t] = np.exp(ys[t]) / np.sum(np.exp(ys[t])) # probabilities for next chars
    loss += -np.log(ps[t][targets[t],0]) # softmax (cross-entropy loss)
  # backward pass: compute gradients going backwards
  dWxh, dWhh, dWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
  dbh, dby = np.zeros_like(bh), np.zeros_like(by)
  dhnext = np.zeros_like(hs[0])
  for t in reversed(xrange(len(inputs))):
    dy = np.copy(ps[t])
    dy[targets[t]] -= 1 # backprop into y. see http://cs231n.github.io/neural-networks-case-study/#grad if c
    dWhy += np.dot(dy, hs[t].T)
    dby += dy
    dh = np.dot(Why.T, dy) + dhnext # backprop into h
    dhraw = (1 - hs[t] * hs[t]) * dh # backprop through tanh nonlinearity
    dbh += dhraw
    dWxh += np.dot(dhraw, xs[t].T)
    dWhh += np.dot(dhraw, hs[t-1].T)
    dhnext = np.dot(Whh.T, dhraw)
  for dparam in [dWxh, dWhh, dWhy, dbh, dby]:
    np.clip(dparam, -5, 5, out=dparam) # clip to mitigate exploding gradients
  return loss, dWxh, dWhh, dWhy, dbh, dby, hs[len(inputs)-1]

def sample(h, seed_ix, n):
  """
  sample a sequence of integers from the model
  h is memory state, seed_ix is seed letter for first time step
  """
  x = np.zeros((vocab_size, 1))
  x[seed_ix] = 1
  ixes = []
  for t in xrange(n):
    h = np.tanh(np.dot(Wxh, x) + np.dot(Whh, h) + bh)
    y = np.dot(Why, h) + by
    p = np.exp(y) / np.sum(np.exp(y))
    ix = np.random.choice(range(vocab_size), p=p.ravel())
    x = np.zeros((vocab_size, 1))
    x[ix] = 1
    ixes.append(ix)
  return ixes

n, p = 0, 0
mWxh, mWhh, mWhy = np.zeros_like(Wxh), np.zeros_like(Whh), np.zeros_like(Why)
mbh, mby = np.zeros_like(bh), np.zeros_like(by) # memory variables for Adagrad
smooth_loss = -np.log(1.0/vocab_size)*seq_length # loss at iteration 0
while True:
  # prepare inputs (we're sweeping from left to right in steps seq_length long)
  if p+seq_length+1 >= len(data) or n == 0:
    hprev = np.zeros((hidden_size,1)) # reset RNN memory
    p = 0 # go from start of data
  inputs = [char_to_ix[ch] for ch in data[p:p+seq_length]]
  targets = [char_to_ix[ch] for ch in data[p+1:p+seq_length+1]]

  # sample from the model now and then
  if n % 100 == 0:
    sample_ix = sample(hprev, inputs[0], 200)
    txt = ''.join(ix_to_char[ix] for ix in sample_ix)
    print '----\n %s \n----' % (txt, )

  # forward seq_length characters through the net and fetch gradient
  loss, dWxh, dWhh, dWhy, dbh, dby, hprev = lossFun(inputs, targets, hprev)
  smooth_loss = smooth_loss * 0.999 + loss * 0.001
  if n % 100 == 0: print 'iter %d, loss: %f' % (n, smooth_loss) # print progress

  # perform parameter update with Adagrad
  for param, dparam, mem in zip([Wxh, Whh, Why, bh, by],
                                [dWxh, dWhh, dWhy, dbh, dby],
                                [mWxh, mWhh, mWhy, mbh, mby]):
    mem += dparam * dparam
    param += -learning_rate * dparam / np.sqrt(mem + 1e-8) # adagrad update

  p += seq_length # move data pointer
  n += 1 # iteration counter
```

https://gist.github.com/karpathy/d4dee566867f8291f086

APPLICATIONS

Sequential Processing of Non-Sequence Data

Classify images by taking a series of “glimpses”

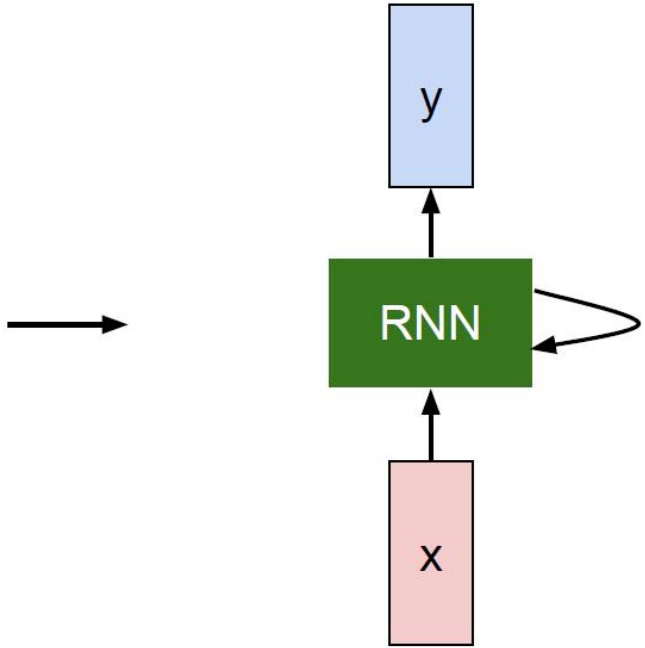
Ba, Mnih, and Kavukcuoglu, “Multiple Object Recognition with Visual Attention”, ICLR 2015.
Gregor et al, “DRAW: A Recurrent Neural Network For Image Generation”, ICML 2015
Figure copyright Karol Gregor, Ivo Danihelka, Alex Graves, Danilo Jimenez Rezende, and Daan Wierstra, 2015. Reproduced with permission.

THE SONNETS

by William Shakespeare

From fairest creatures we desire increase,
That thereby beauty's rose might never die,
But as the riper should by time decease,
His tender heir might bear his memory:
But thou, contracted to thine own bright eyes,
Feed'st thy light's flame with self-substantial fuel,
Making a famine where abundance lies,
Thyself thy foe, to thy sweet self too cruel:
Thou that art now the world's fresh ornament,
And only herald to the gaudy spring,
Within thine own bud buriest thy content,
And tender churl mak'st waste in niggarding:
    Pity the world, or else this glutton be,
    To eat the world's due, by the grave and thee.

When forty winters shall besiege thy brow,
And dig deep trenches in thy beauty's field,
Thy youth's proud livery so gazed on now,
Will be a tatter'd weed of small worth held:
Then being asked, where all thy beauty lies,
Where all the treasure of thy lusty days;
To say, within thine own deep sunken eyes,
Were an all-eating shame, and thriftless praise.
How much more praise deserv'd thy beauty's use,
If thou couldst answer 'This fair child of mine
Shall sum my count, and make my old excuse,'
Proving his beauty by succession thine!
    This were to be new made when thou art old,
    And see thy blood warm when thou feel'st it cold.



at first:

```
tyntd-iafhatawiaoihrdemot  lytdws  e ,tfti, astai f ogoh eoase rrranbyne 'nhthnee e
plia tklrgd t o idoe ns,smtt   h ne etie h,hregtrs nigtike,aoaenns lng
```

train more

```
"Tmont thithey" fomesscerliund
Keushey. Thom here
sheulke, anmerenith ol sivh I lalterthend Bleipile shuwy fil on aseterlome
coaniogennc Phe lism thond hon at. MeiDimorotion in ther thize."
```

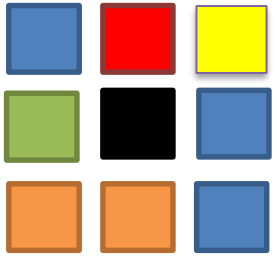
train more

```
Aftair fall unsuch that the hall for Prince Velzonski's that me of
her hearly, and behs to so arwage fiving were to it beloge, pavu say falling misfort
how, and Gogition is so overelical and ofter.
```

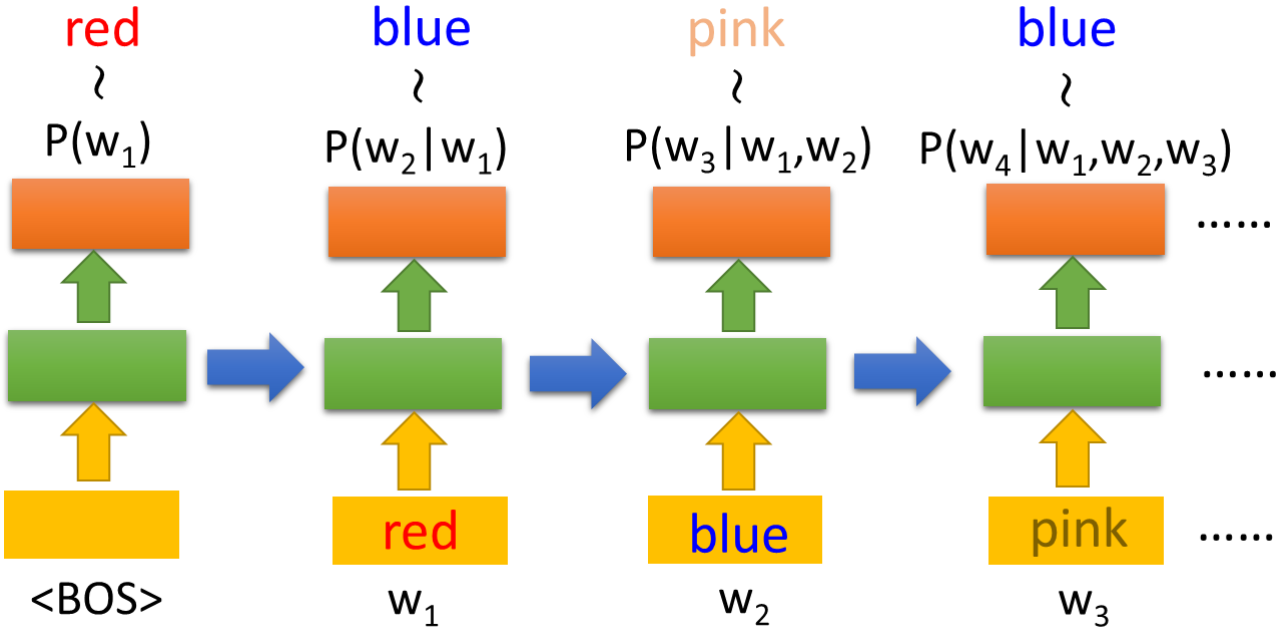
train more

```
"Why do what that day," replied Natasha, and wishing to himself the fact the
princess, Princess Mary was easier, fed in had oftened him.
Pierre aking his soul came to the packs and drove up his father-in-law women.
```

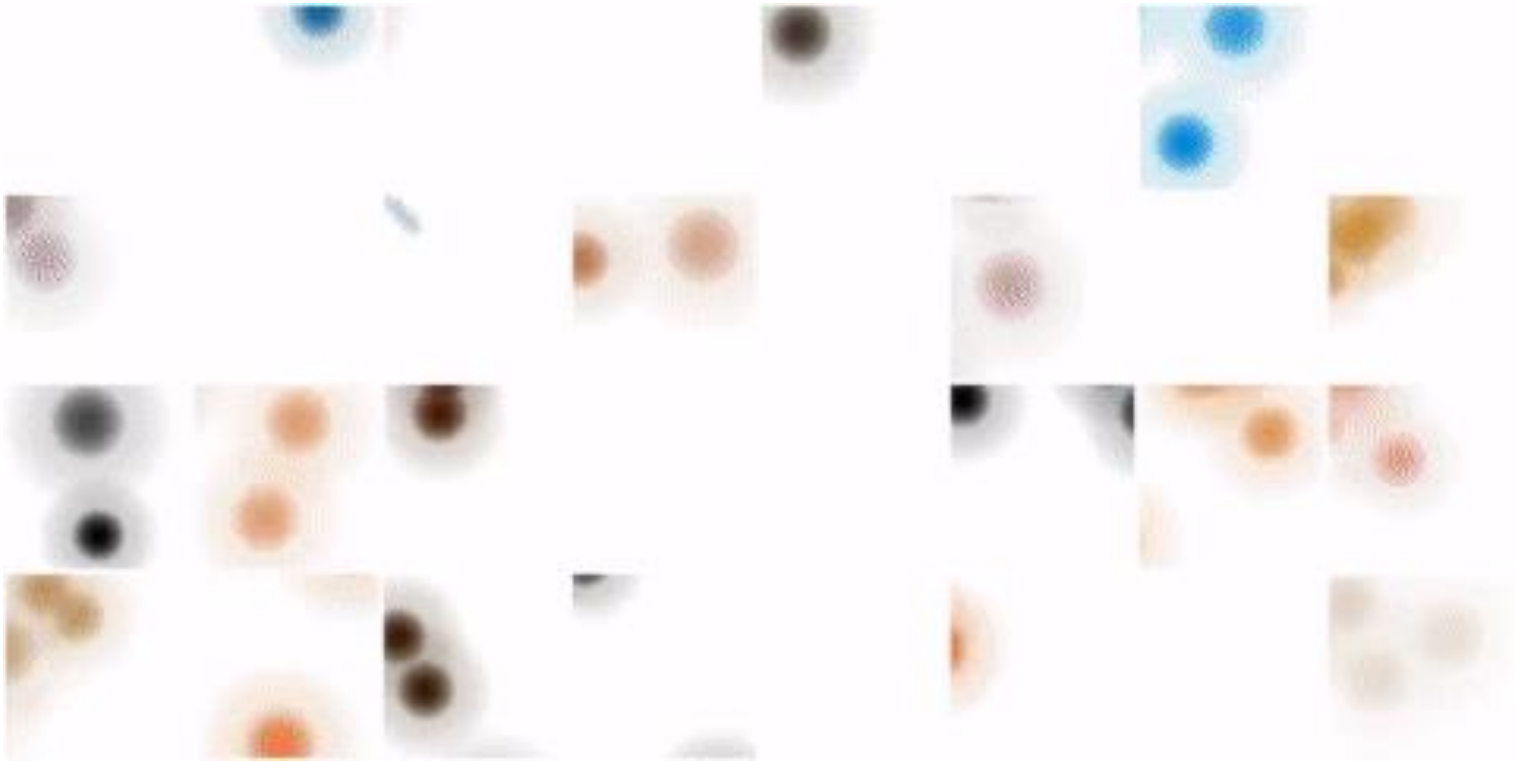
Generation



- Images are composed of pixels
  - Generating a pixel at each time by RNN



台大李宏毅教授



Synthesizing Programs for Images using Reinforced Adversarial Learning, ICML, 2018

Image Captioning

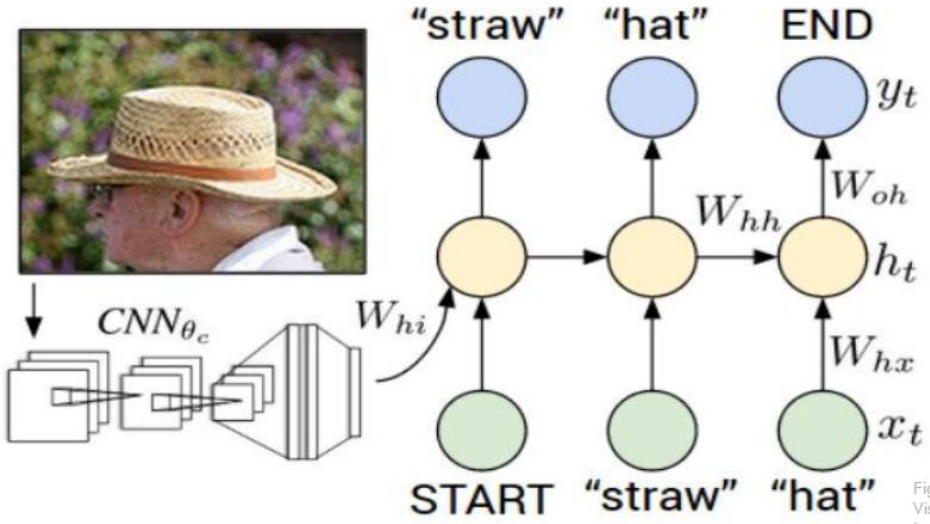
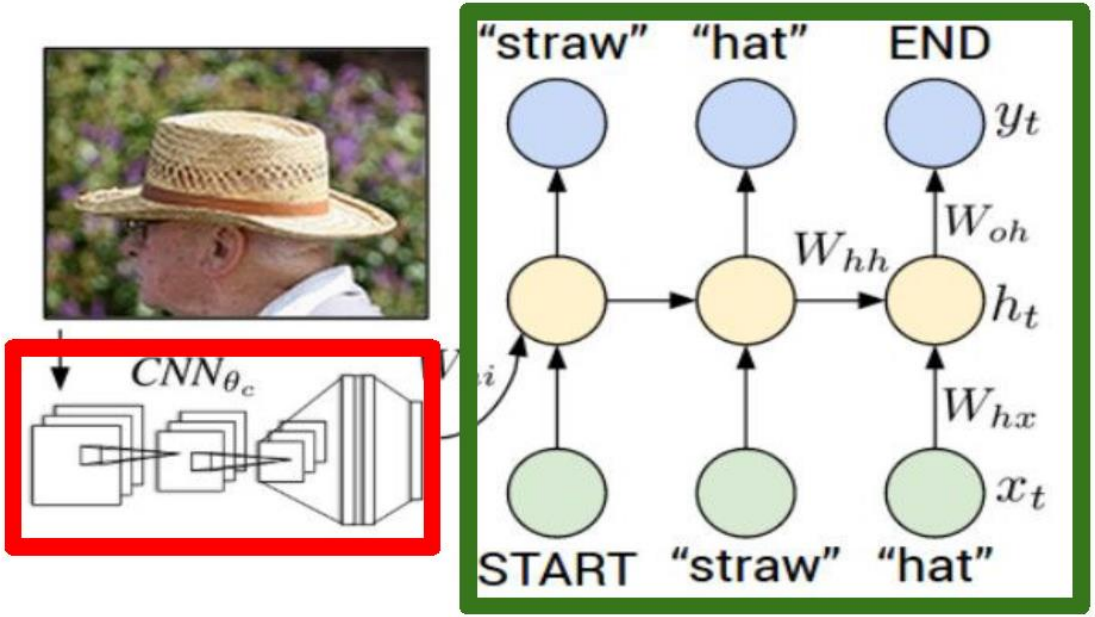


Figure from Karpathy et a, "Deep Visual-Semantic Alignments for Generating Image Descriptions", CVPR 2015; figure copyright IEEE, 2015.
Reproduced for educational purposes.

Explain Images with Multimodal Recurrent Neural Networks, Mao et al.
Deep Visual-Semantic Alignments for Generating Image Descriptions, Karpathy and Fei-Fei
Show and Tell: A Neural Image Caption Generator, Vinyals et al.
Long-term Recurrent Convolutional Networks for Visual Recognition and Description, Donahue et al.
Learning a Recurrent Visual Representation for Image Caption Generation, Chen and Zitnick

Recurrent Neural Network



Convolutional Neural Network

Transfer learning: Take CNN trained on ImageNet, chop off last layer

image

conv-64
conv-64
maxpool

conv-128
conv-128
maxpool

conv-256
conv-256
maxpool

conv-512
conv-512
maxpool

conv-512
conv-512
maxpool

FC-4096
FC-4096

x0

<START>

image

conv-64
conv-64
maxpool

conv-128
conv-128
maxpool

conv-256
conv-256
maxpool

conv-512
conv-512
maxpool

conv-512
conv-512
maxpool

FC-4096
FC-4096

Before:

$$h_t = \tanh(\boldsymbol{W_{hh}h_{t-1}} + \boldsymbol{W_{xh}x_t} + b_h)$$

Now:

$$\tanh(\boldsymbol{W_{hh}h_{t-1}} + \boldsymbol{W_{xh}x_t} + \boldsymbol{W_{ih}v} + b_h)$$

image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
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Before:

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

Now:

$$\tanh(W_{hh}h_{t-1} + W_{xh}x_t + W_{ih}v + b_h)$$

image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096

Before:

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

Now:

$$\tanh(W_{hh}h_{t-1} + W_{xh}x_t + W_{ih}v + b_h)$$

image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096

Before:

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

Now:

$$\tanh(W_{hh}h_{t-1} + W_{xh}x_t + W_{ih}v + b_h)$$

W_{ih}

man in straw

y0 y1 y2

h0 h1 h2

x0 x1 x2

<START> man in straw

Sample word and copy to input

image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096

Before:

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

Now:

$$\tanh(W_{hh}h_{t-1} + W_{xh}x_t + W_{ih}v + b_h)$$

W_{ih}

man in straw hat

y0 y1 y2 y3

h0 h1 h2 h3

x0 x1 x2 x3

<START> man in straw hat

Sample word and copy to input

image
conv-64
conv-64
maxpool
conv-128
conv-128
maxpool
conv-256
conv-256
maxpool
conv-512
conv-512
maxpool
conv-512
conv-512
maxpool
FC-4096
FC-4096

Before:

$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t + b_h)$$

Now:

$$\tanh(W_{hh}h_{t-1} + W_{xh}x_t + W_{ih}v + b_h)$$

W_{ih}

Stop after sampling <END> token

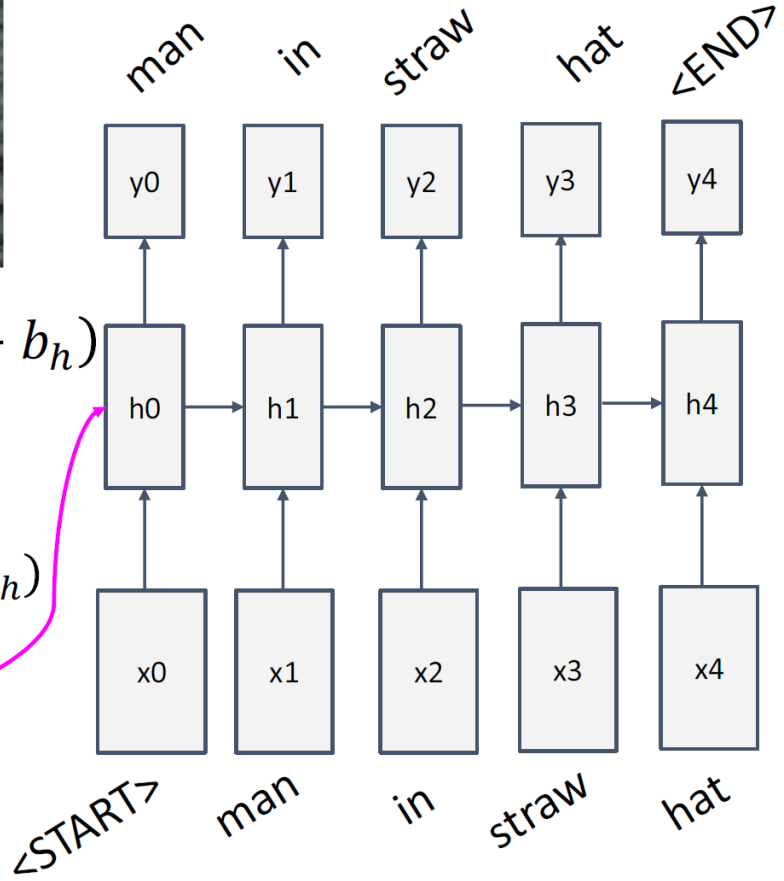


Image Captioning: Example Results

Captions generated using neuraltalk2
All images are CC0 Public domain:
cat suitcase, cat tree, dog, bear,
surfers, tennis, giraffe, motorcycle

A cat sitting on a suitcase on the floor

A cat is sitting on a tree branch

A dog is running in the grass with a frisbee

A white teddy bear sitting in the grass

Two people walking on the beach with surfboards

A tennis player in action on the court

Two giraffes standing in a grassy field

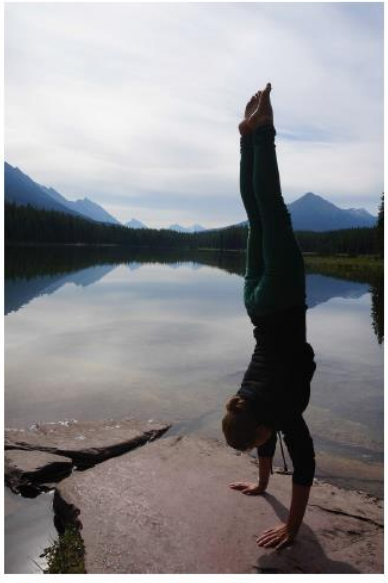
A man riding a dirt bike on a dirt track

Image Captioning: Failure Cases

Captions generated using neuraltalk2
All images are CC0 Public domain: fur coat, handstand, spider web, baseball

A woman is holding a cat in her hand

A person holding a computer mouse on a desk



A woman standing on a beach holding a surfboard

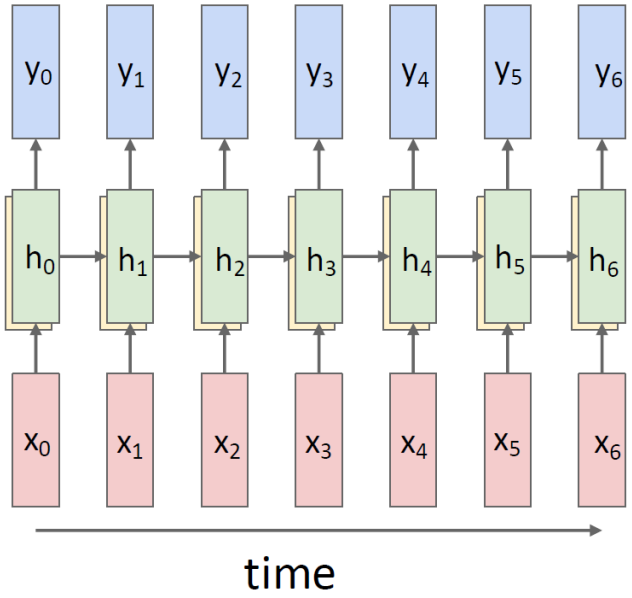
A bird is perched on a tree branch

A man in a baseball uniform throwing a ball

LSTM & GRU

Single-Layer RNNs

$$h_t = \tanh\left(W\begin{pmatrix}h_{t-1}\\ x_t\end{pmatrix} + b_h\right)$$

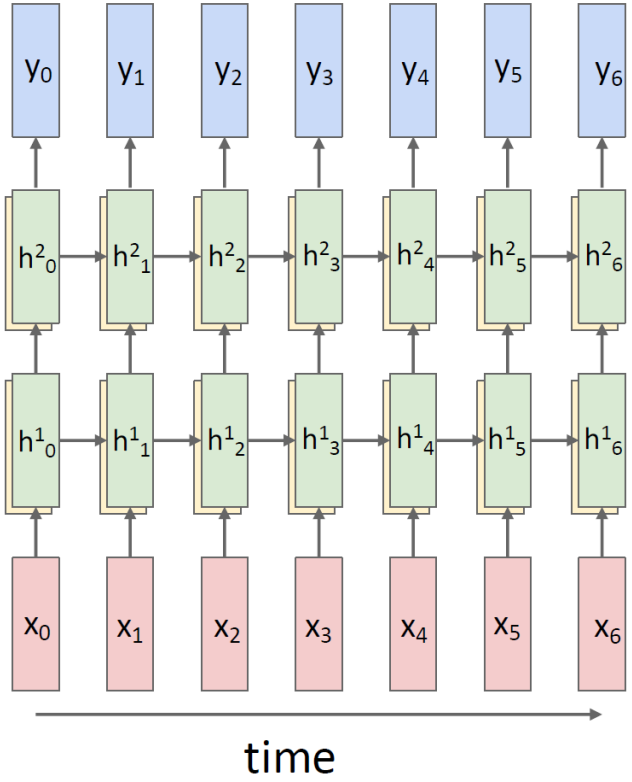


Mutilayer RNNs

$$h_t^{\ell} = \tanh\left(W \begin{pmatrix} h_{t-1}^{\ell} \\ h_t^{\ell-1} \end{pmatrix} + b_h^{\ell}\right)$$

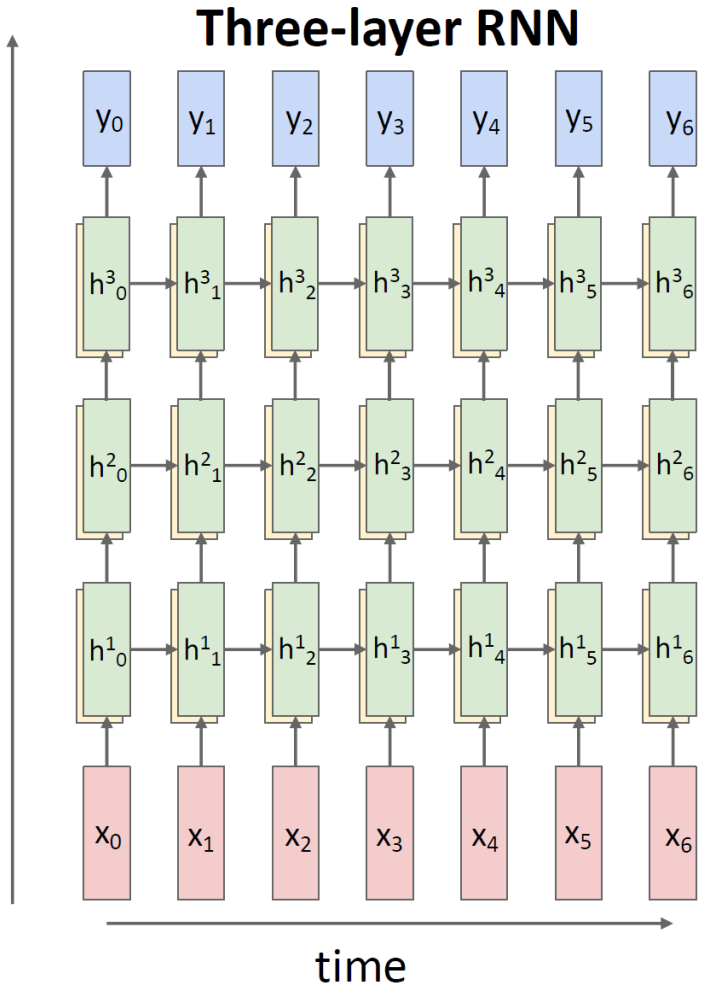
depth

Two-layer RNN: Pass hidden states from one RNN as inputs to another RNN



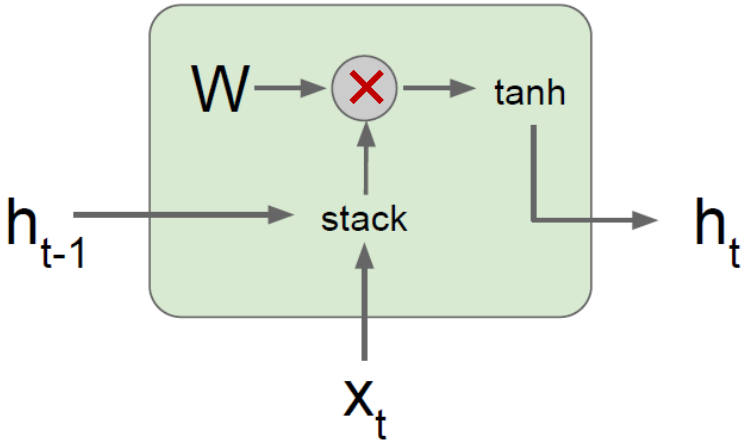
Mutilayer RNNs

$$h_t^{\ell} = \tanh\left(W \begin{pmatrix} h_{t-1}^{\ell} \\ h_t^{\ell-1} \end{pmatrix} + b_h^{\ell}\right)$$



Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013

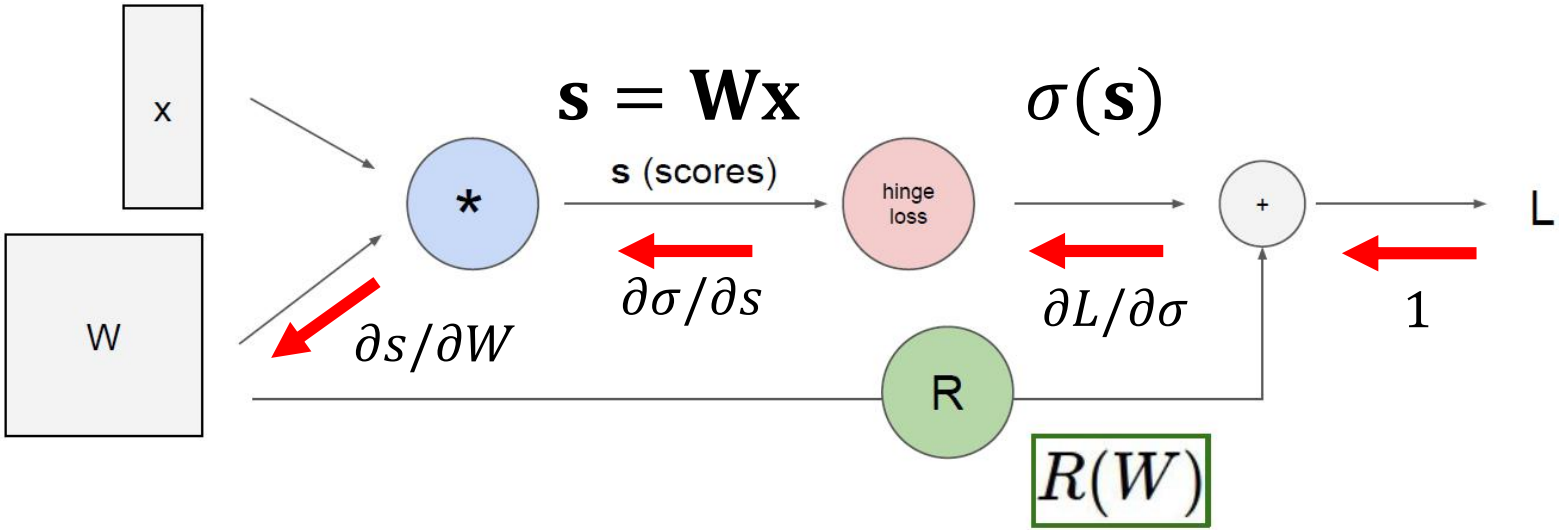


$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$
$$= \tanh\left((W_{hh} \;\; W_{xh}) \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right)$$
$$= \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix} \right)$$

Recall

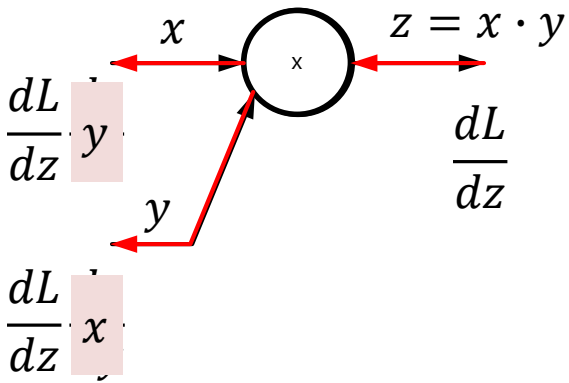
Computational graphs

$$\frac{\partial L}{\partial W} = 1 \cdot \frac{\partial L}{\partial \sigma} \cdot \frac{\partial \sigma}{\partial s} \cdot \frac{\partial s}{\partial W}$$



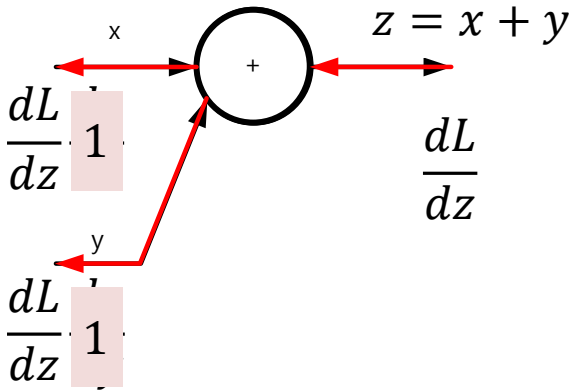
Recall (Multiplier)

$$\frac{dz}{dx} = y \qquad \frac{dz}{dy} = x$$



Recall (Adder)

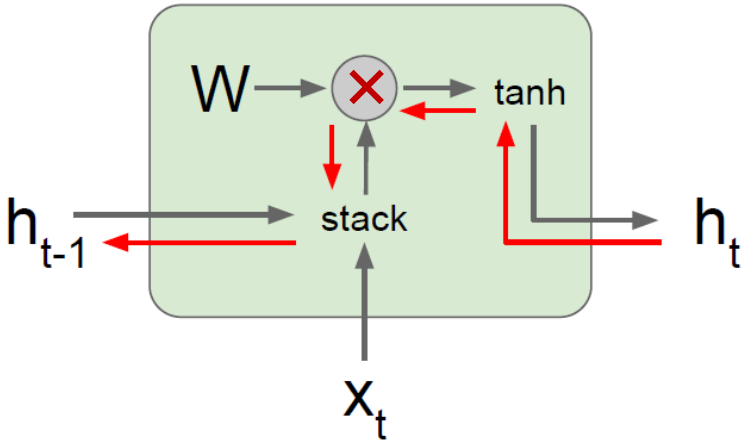
$$\frac{dz}{dx} = 1 \qquad \frac{dz}{dy} = 1$$



Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013

Backpropagation from h_t to h_{t-1} multiplies by W (actually W_{hh}^T)



$$h_t = \tanh(W_{hh}h_{t-1} + W_{xh}x_t)$$
$$= \tanh\left((W_{hh} \quad W_{hx})\begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$
$$= \tanh\left(W\begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
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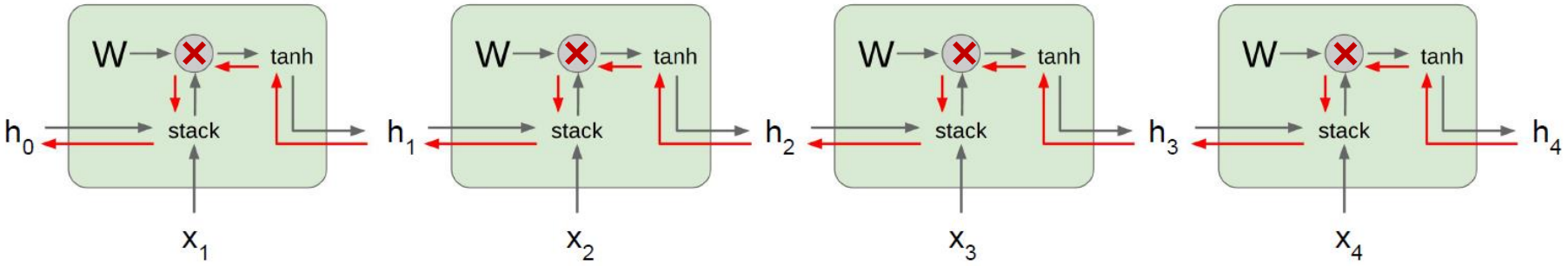
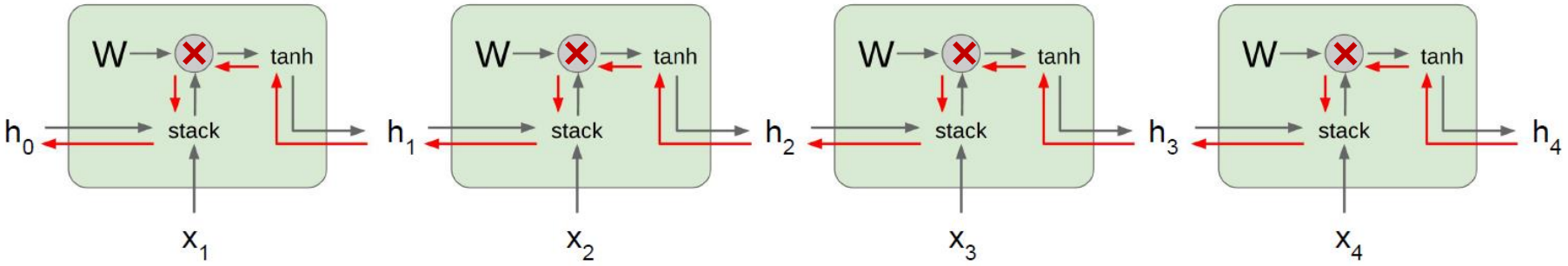


Computing gradient of h_0 involves many factors of W (and repeated tanh)

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



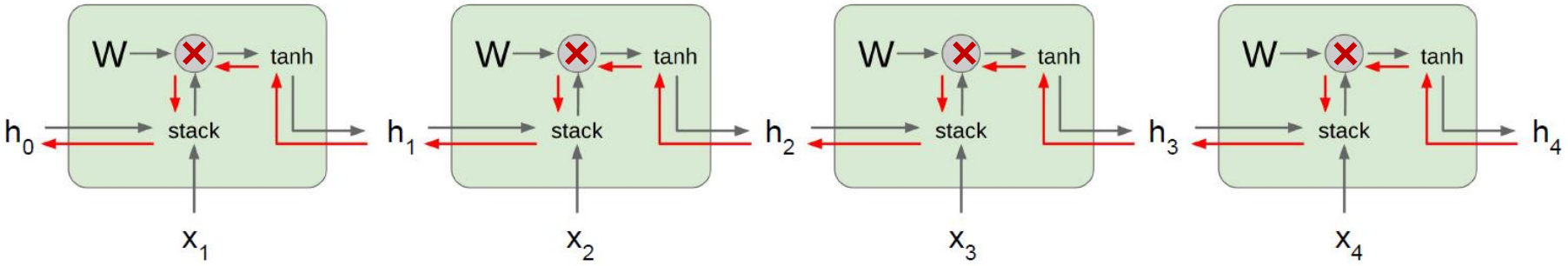
Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

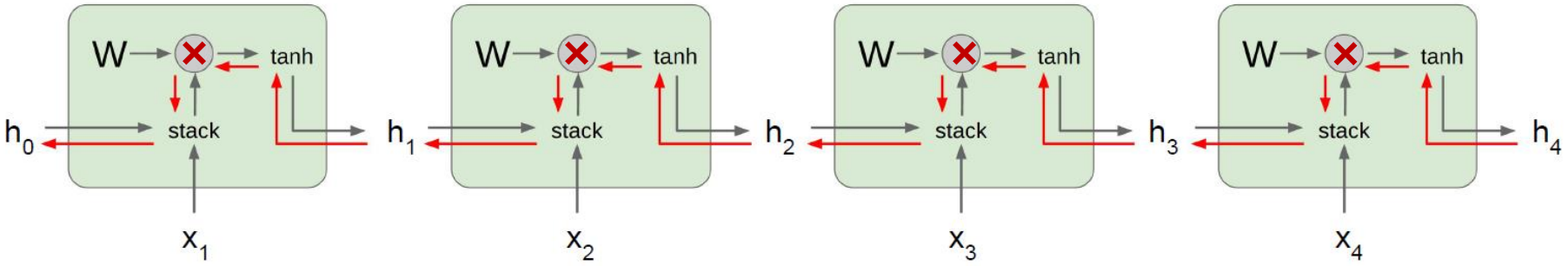
→ **Gradient clipping**: Scale gradient if its norm is too big

```
grad_norm = np.sum(grad * grad)
if grad_norm > threshold:
  grad *= (threshold / grad_norm)
```

Largest singular value < 1:
Vanishing gradients

Vanilla RNN Gradient Flow

Bengio et al, "Learning long-term dependencies with gradient descent is difficult", IEEE Transactions on Neural Networks, 1994
Pascanu et al, "On the difficulty of training recurrent neural networks", ICML 2013



Computing gradient of h_0 involves many factors of W (and repeated tanh)

Largest singular value > 1:
Exploding gradients

Largest singular value < 1:
Vanishing gradients

→ Change RNN architecture

Long Short Term Memory (LSTM)

Vanilla RNN

$$h_t = \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$

LSTM

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Hochreiter and Schmidhuber, "Long Short Term Memory", Neural Computation 1997

Long Short Term Memory (LSTM)

Vanilla RNN

$$h_t = \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$

LSTM

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Two state vectors at each timestep:

Cell state

Hidden state

Hochreiter and Schmidhuber, "Long Short Term Memory", Neural Computation 1997

Long Short Term Memory (LSTM)

Vanilla RNN

$$h_t = \tanh\left(W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}\right)$$

LSTM

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Compute four "gates" per timestep:

Input gate

Forget gate

Output gate

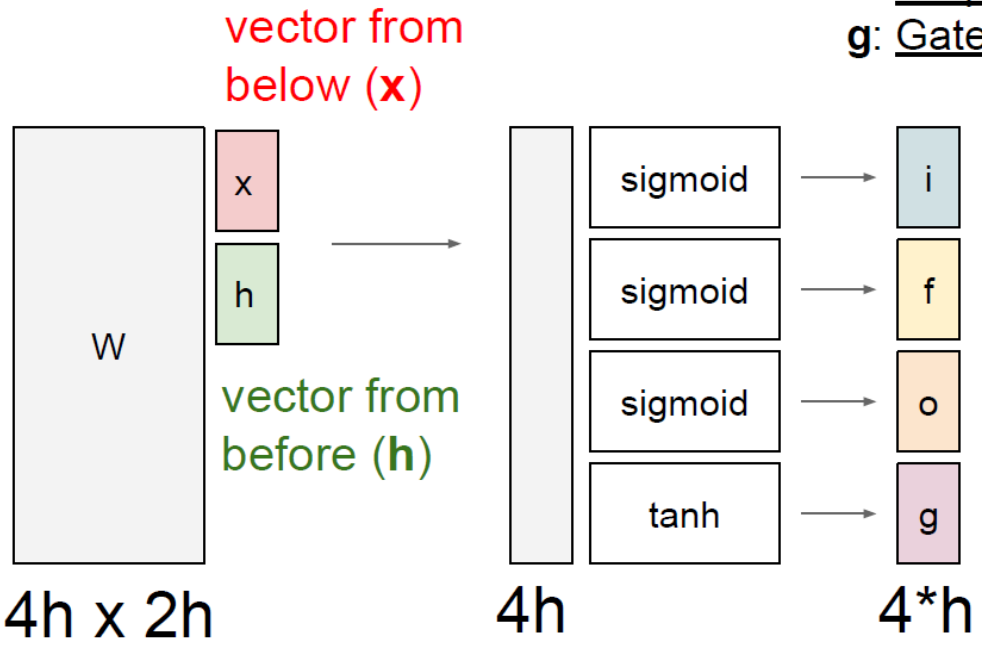
"Gate" gate

Hochreiter and S
1997

Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]

i: Input gate, whether to write to cell
f: Forget gate, Whether to erase cell
o: Output gate, How much to reveal cell
g: Gate gate (?), How much to write to cell



$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

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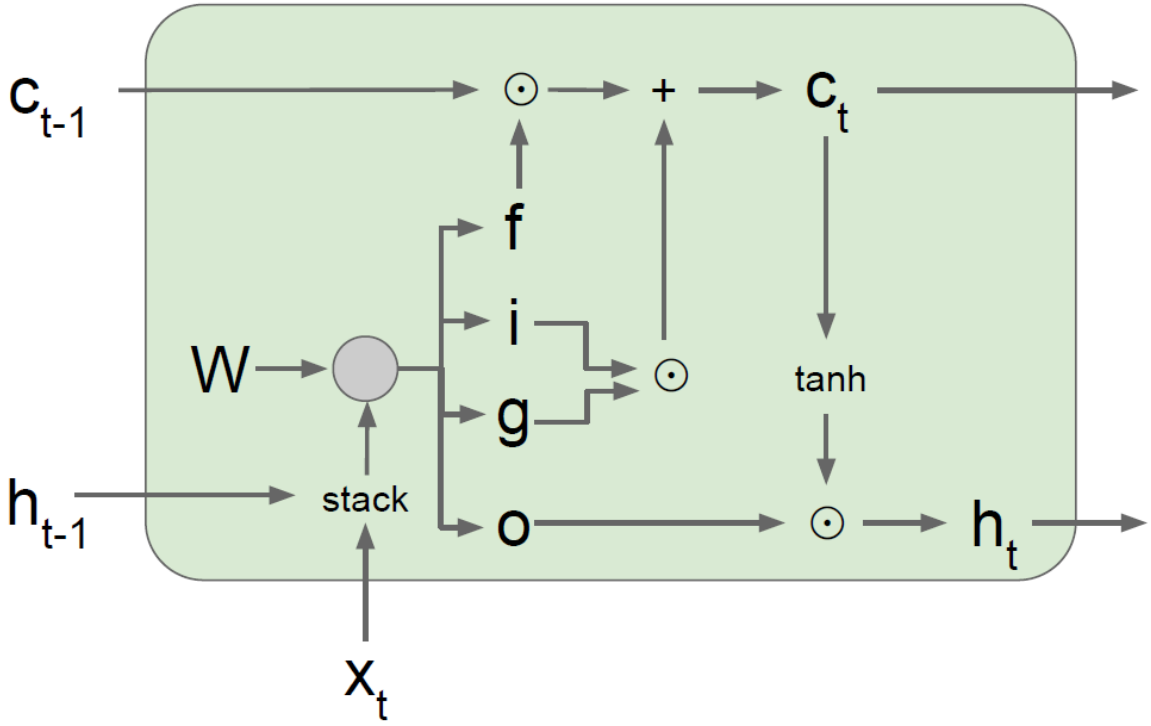
Long Short Term Memory (LSTM)

[Hochreiter et al., 1997]



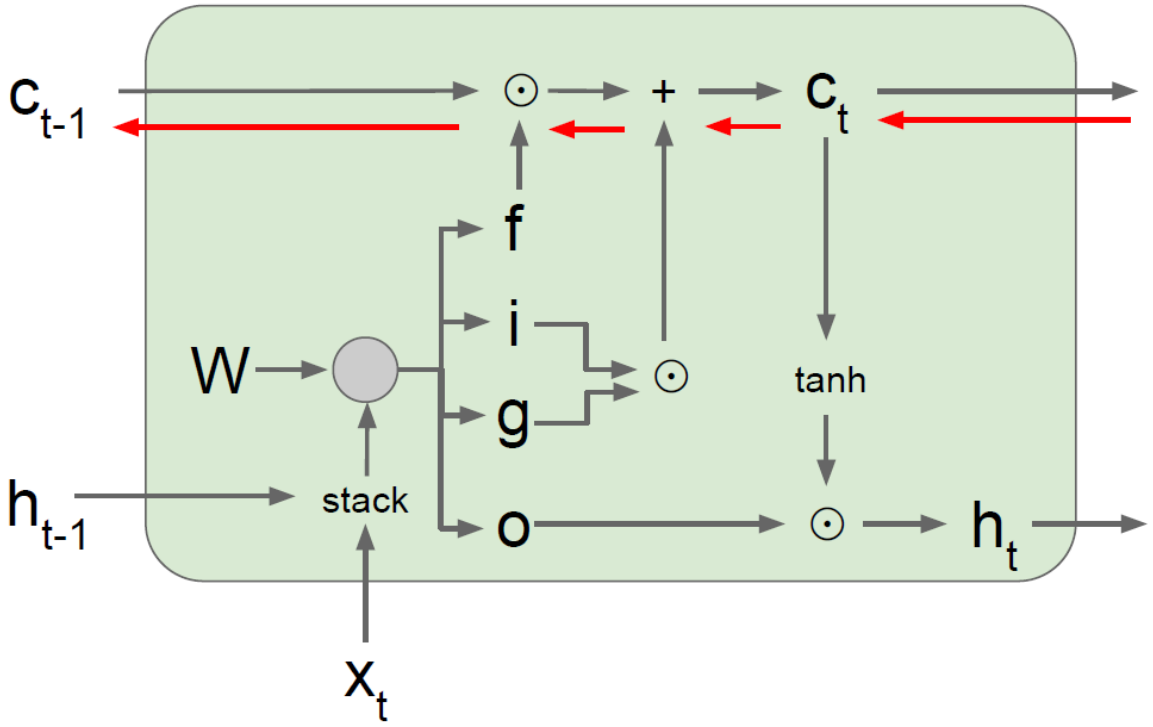
$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]



Backpropagation from c_t to c_{t-1} only elementwise multiplication by f, no matrix multiply by W

$$\begin{pmatrix} i \\ f \\ o \\ g \end{pmatrix} = \begin{pmatrix} \sigma \\ \sigma \\ \sigma \\ \tanh \end{pmatrix} W \begin{pmatrix} h_{t-1} \\ x_t \end{pmatrix}$$

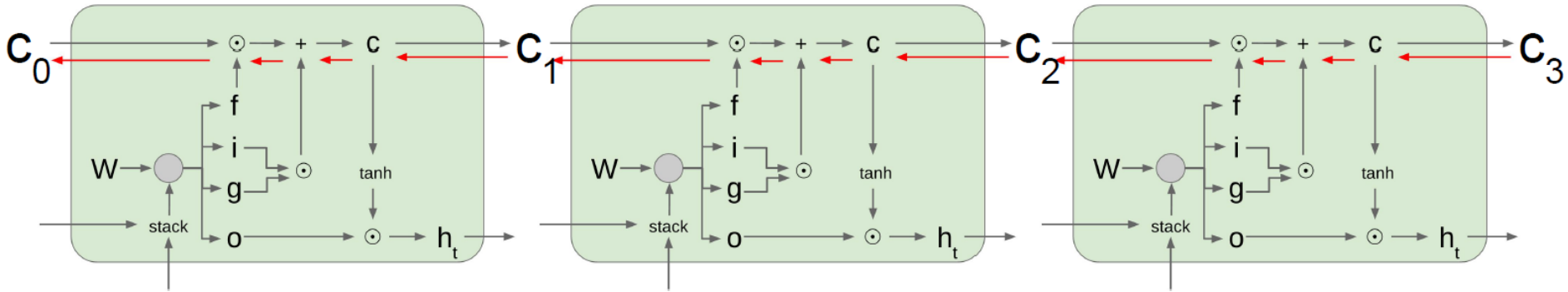
$$c_t = f \odot c_{t-1} + i \odot g$$

$$h_t = o \odot \tanh(c_t)$$

Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

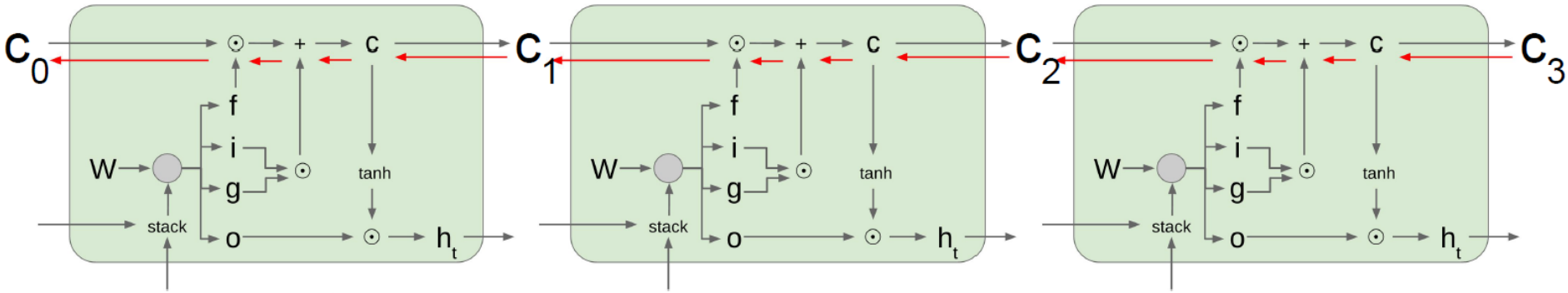
Uninterrupted gradient flow!



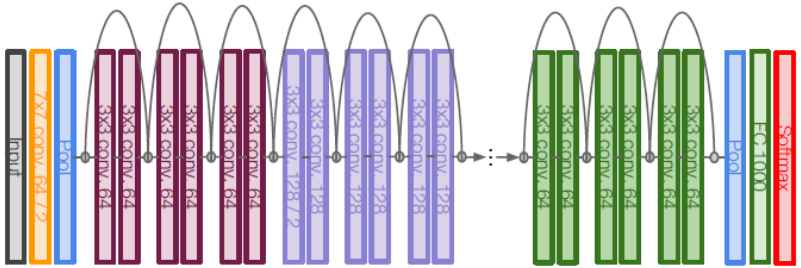
Long Short Term Memory (LSTM): Gradient Flow

[Hochreiter et al., 1997]

Uninterrupted gradient flow!



Similar to ResNet!



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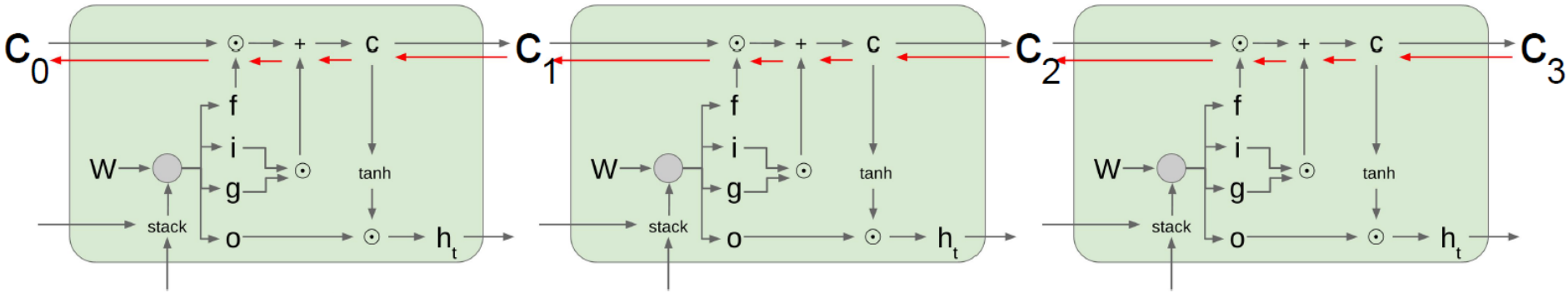
[Hochreiter et al., 1997]

Uninterrupted gradient flow!



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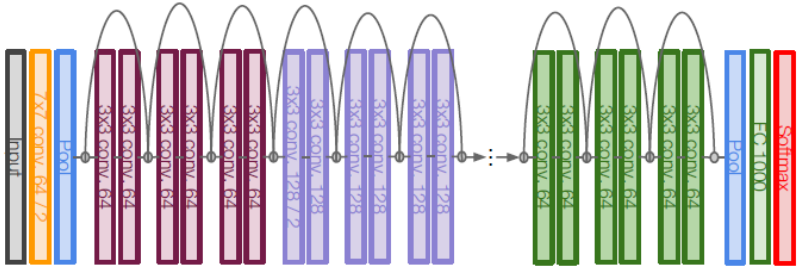


In between:

Highway Networks

$$g = T(x, W_T)$$

$$y = g \odot H(x, W_H) + (1 - g) \odot x$$

Srivastava et al, "Highway Networks", ICML DL Workshop 2015

Mutilayer RNNs

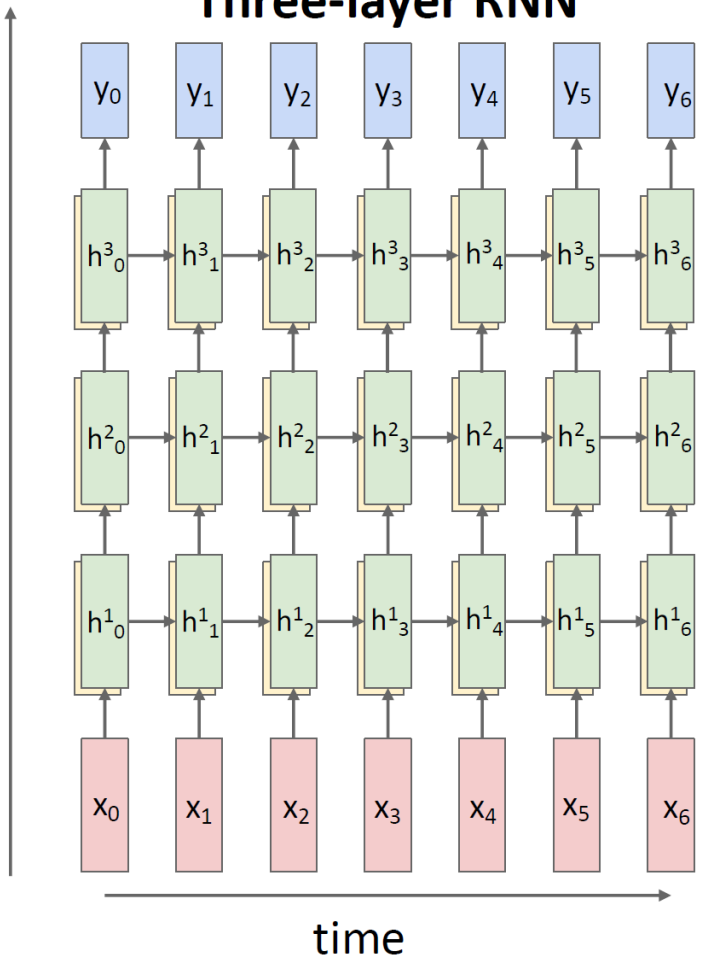
$$h_t^\ell = \tanh\left(W\begin{pmatrix}h_{t-1}^\ell \\ h_t^{\ell-1}\end{pmatrix} + b_h^\ell\right)$$

LSTM:

$$\begin{pmatrix}i_t^\ell \\ f_t^\ell \\ o_t^\ell \\ g_t^\ell\end{pmatrix} = \begin{pmatrix}\sigma \\ \sigma \\ \sigma \\ \tanh\end{pmatrix}\left(W\begin{pmatrix}h_{t-1}^\ell \\ h_t^{\ell-1}\end{pmatrix} + b_h^\ell\right)$$

$$c_t^\ell = f_t^\ell \odot c_{t-1}^\ell + i_t^\ell \odot g_t^\ell$$

$$h_t^\ell = o_t^\ell \odot \tanh\left(c_t^\ell\right)$$



Other RNN Variants

Gated Recurrent Unit (GRU)

Cho et al "Learning phrase representations using RNN encoder-decoder for statistical machine translation", 2014

$$r_t = \sigma(W_{xr}x_t + W_{hr}h_{t-1} + b_r)$$
$$z_t = \sigma(W_{xz}x_t + W_{hz}h_{t-1} + b_z)$$
$$\tilde{h}_t = \tanh(W_{xh}x_t + W_{hh}(r_t \odot h_{t-1}) + b_h)$$
$$h_t = z_t \odot h_{t-1} + (1 - z_t) \odot \tilde{h}_t$$

Other RNN Variants

Gated Recurrent Unit (GRU)

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$$r_t = \sigma(W_{xr}x_t + W_{hr}h_{t-1} + b_r)$$
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$$\tilde{h}_t = \tanh(W_{xh}x_t + W_{hh}(r_t \odot h_{t-1}) + b_h)$$
$$h_t = z_t \odot h_{t-1} + (1 - z_t) \odot \tilde{h}_t$$

10,000 architectures with evolutionary search:

Jozefowicz et al, "An empirical exploration of recurrent network architectures", ICML 2015

MUT1:

$$\begin{aligned} z &= \text{sigm}(W_{\text{xz}}x_t + b_{\text{z}}) \\ r &= \text{sigm}(W_{\text{xr}}x_t + W_{\text{hr}}h_t + b_{\text{r}}) \\ h_{t+1} &= \tanh(W_{\text{hh}}(r \odot h_t) + \tanh(x_t) + b_{\text{h}}) \odot z \\ &+ h_t \odot (1 - z) \end{aligned}$$

MUT2:

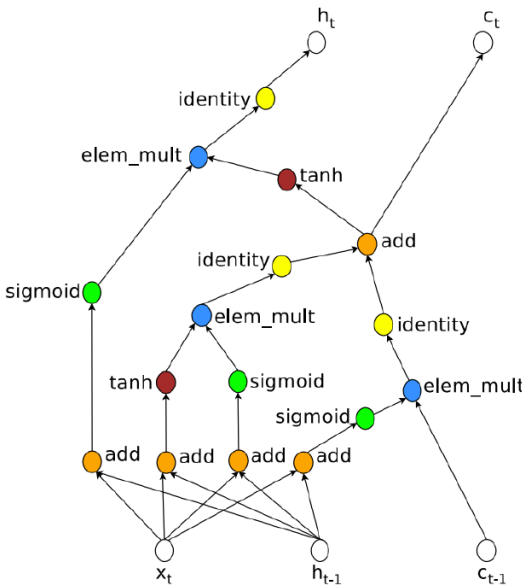
$$\begin{aligned} z &= \text{sigm}(W_{\text{xz}}x_t + W_{\text{hz}}h_t + b_{\text{z}}) \\ r &= \text{sigm}(x_t + W_{\text{hr}}h_t + b_{\text{r}}) \\ h_{t+1} &= \tanh(W_{\text{hh}}(r \odot h_t) + W_{xh}x_t + b_{\text{h}}) \odot z \\ &+ h_t \odot (1 - z) \end{aligned}$$

MUT3:

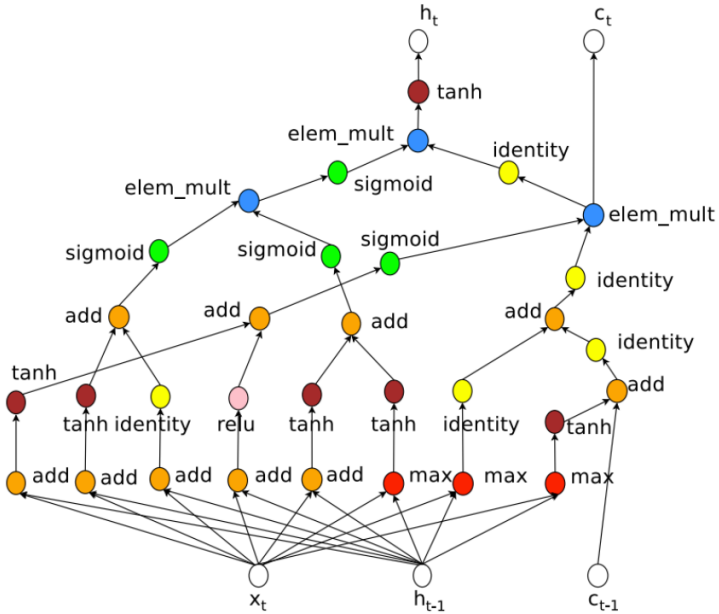
$$\begin{aligned} z &= \text{sigm}(W_{\text{xz}}x_t + W_{\text{hz}}\tanh(h_t) + b_{\text{z}}) \\ r &= \text{sigm}(W_{\text{xr}}x_t + W_{\text{hr}}h_t + b_{\text{r}}) \\ h_{t+1} &= \tanh(W_{\text{hh}}(r \odot h_t) + W_{xh}x_t + b_{\text{h}}) \odot z \\ &+ h_t \odot (1 - z) \end{aligned}$$

RNN Architectures: Neural Architecture Search

LSTM



Learned Architecture



Zoph and Le, "Neural Architecture Search with Reinforcement Learning", ICLR 2017

Summary

- RNNs allow a lot of flexibility in architecture design
- Vanilla RNNs are simple but don’t work very well
- Common to use LSTM or GRU: their additive interactions improve gradient flow
- Backward flow of gradients in RNN can explode or vanish. Exploding is controlled with gradient clipping. Vanishing is controlled with additive interactions (LSTM)
- Better/simpler architectures are a hot topic of current research
- Better understanding (both theoretical and empirical) is needed.