

Image Classification

- K-Nearest Neighbor
- Linear classifiers



source from <http://cs231n.stanford.edu>

Image Classification: A core task in Computer Vision



(assume given set of discrete labels)
{dog, cat, truck, plane, ...}

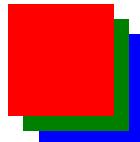
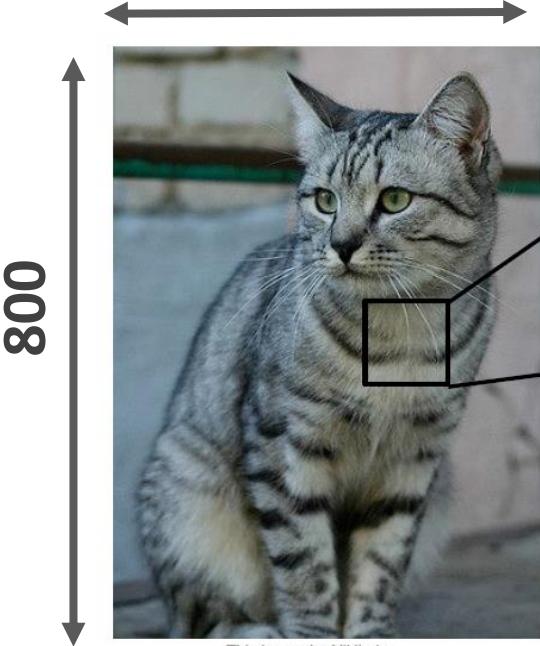


cat

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The Problem: Semantic Gap

600



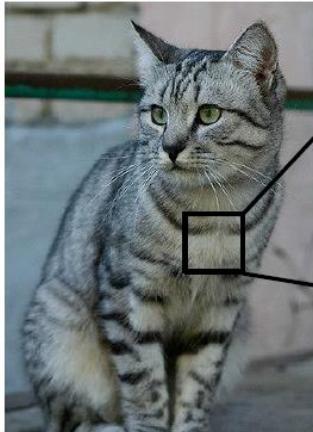
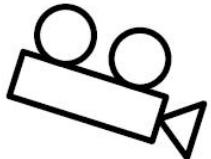
[[105 112 108 111 104 99 106 99 96 103 112 119 104 97 93 87] [91 98 102 106 104 79 98 103 99 105 123 136 118 105 94 85] [76 85 90 105 128 105 87 96 95 90 115 112 106 103 99 85] [99 81 81 93 120 131 127 100 95 98 102 99 96 93 101 94] [106 91 61 64 69 91 88 85 101 107 109 98 75 84 96 95] [114 108 85 55 55 69 64 54 64 87 112 129 98 74 84 91] [133 137 147 103 65 81 88 65 52 54 74 84 102 93 85 82] [128 137 144 140 109 95 86 70 62 65 63 63 60 73 86 101] [125 133 148 137 119 121 117 94 65 79 80 65 54 64 72 98] [127 125 131 147 133 127 126 131 111 96 89 75 61 64 72 84] [115 114 109 123 158 148 131 118 113 109 100 92 74 65 72 78] [89 93 98 97 108 147 131 118 113 114 113 109 106 95 77 80] [63 77 86 81 77 79 102 123 117 115 117 125 125 138 115 87] [62 65 82 89 78 71 80 101 124 126 119 101 107 114 131 119] [63 65 75 88 89 71 62 81 120 138 135 105 81 98 110 118] [87 65 71 87 106 95 60 45 76 130 126 107 92 94 105 112] [118 97 82 86 117 123 116 66 41 51 95 93 89 95 102 107] [164 146 112 80 82 120 124 104 76 48 45 66 88 101 102 109] [157 170 157 120 93 86 114 132 112 97 69 55 70 82 99 94] [130 128 134 161 139 108 106 118 121 134 114 87 65 53 59 86] [128 112 96 117 150 144 120 115 104 107 102 93 87 81 72 79] [123 107 96 86 83 112 153 149 122 109 104 75 80 107 112 99] [122 121 102 80 82 86 94 117 145 148 153 102 58 78 92 107] [122 164 148 103 71 56 78 83 93 103 119 139 102 61 69 84]]
--

What the computer sees

An image is just a big grid of numbers between [0, 255]:

e.g. 800 x 600 x 3
(3 channels RGB)

Challenges: Viewpoint variation



[[105 112 108 111 104 99 106 99 96 103 112 119 184 97 93 87]
[91 98 102 106 104 79 98 107 99 105 123 130 118 105 94 85]
[76 85 90 93 102 105 106 100 106 108 115 121 126 105 96 86]
[99 106 103 108 106 102 127 100 99 106 108 115 121 126 105 96]
[104 91 61 64 60 61 88 85 101 107 109 98 75 84 94 95]
[114 108 85 55 55 69 64 54 64 87 112 129 98 74 84 91]
[132 137 147 139 65 61 88 65 52 54 74 84 182 93 85 82]
[120 125 131 133 133 133 133 133 133 133 133 133 133 133 133]
[125 133 140 137 119 121 137 94 65 79 89 85 54 64 72 98]
[127 125 131 147 133 127 126 131 111 99 89 75 61 64 72 84]
[115 114 100 123 100 148 121 118 112 109 180 92 74 65 72 78]
[89 93 98 97 100 147 131 118 113 114 113 108 186 95 77 88]
[63 77 88 81 77 79 102 123 113 111 114 113 108 186 95 77 88]
[62 76 87 82 81 77 102 123 113 111 114 113 108 186 95 77 88]
[93 89 75 88 89 71 62 91 128 138 135 105 82 98 118 118]
[87 65 71 87 106 95 69 45 76 138 126 107 92 94 105 112]
[118 57 82 86 117 123 116 66 41 51 95 93 89 95 102 107]
[104 104 112 107 109 82 128 123 104 78 51 45 60 88 105 100]
[131 129 137 139 139 139 139 139 139 139 139 139 139 139 97]
[136 128 134 161 139 188 189 116 121 134 114 87 69 53 69 86]
[128 112 96 117 158 144 128 115 104 107 182 93 87 81 72 79]
[123 107 96 86 83 112 153 149 122 109 184 75 80 107 112 99]
[122 121 107 88 82 86 94 117 145 148 153 182 58 78 92 107]
[122 164 148 183 71 56 78 83 93 103 119 139 182 51 65 84]]

All pixels change when the camera moves!

Challenges: Illumination



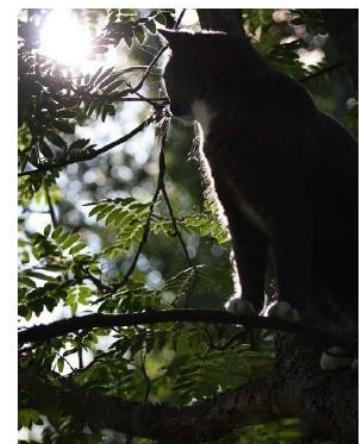
[This image is CC0 1.0 public domain](#)



[This image is CC0 1.0 public domain](#)

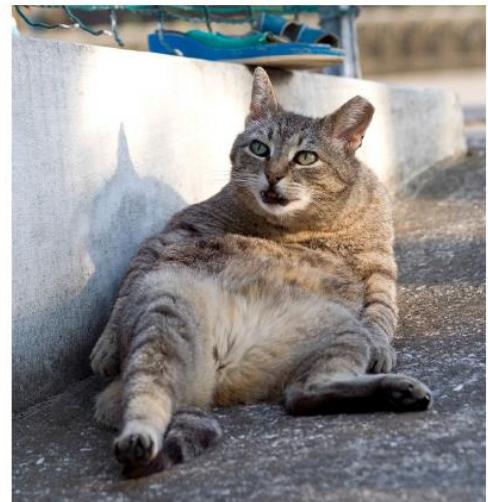


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Challenges: Deformation



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This image by [Umberto Salvagnin](#)
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This image by [sare bear](#) is
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This image by [Tom Thai](#) is
licensed under [CC-BY 2.0](#)

Challenges: Occlusion



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[This image by jonsson is licensed under CC-BY 2.0](#)

Challenges: Background Clutter



[This image](#) is CC0 1.0 public domain



[This image](#) is CC0 1.0 public domain

Challenges: Intraclass variation



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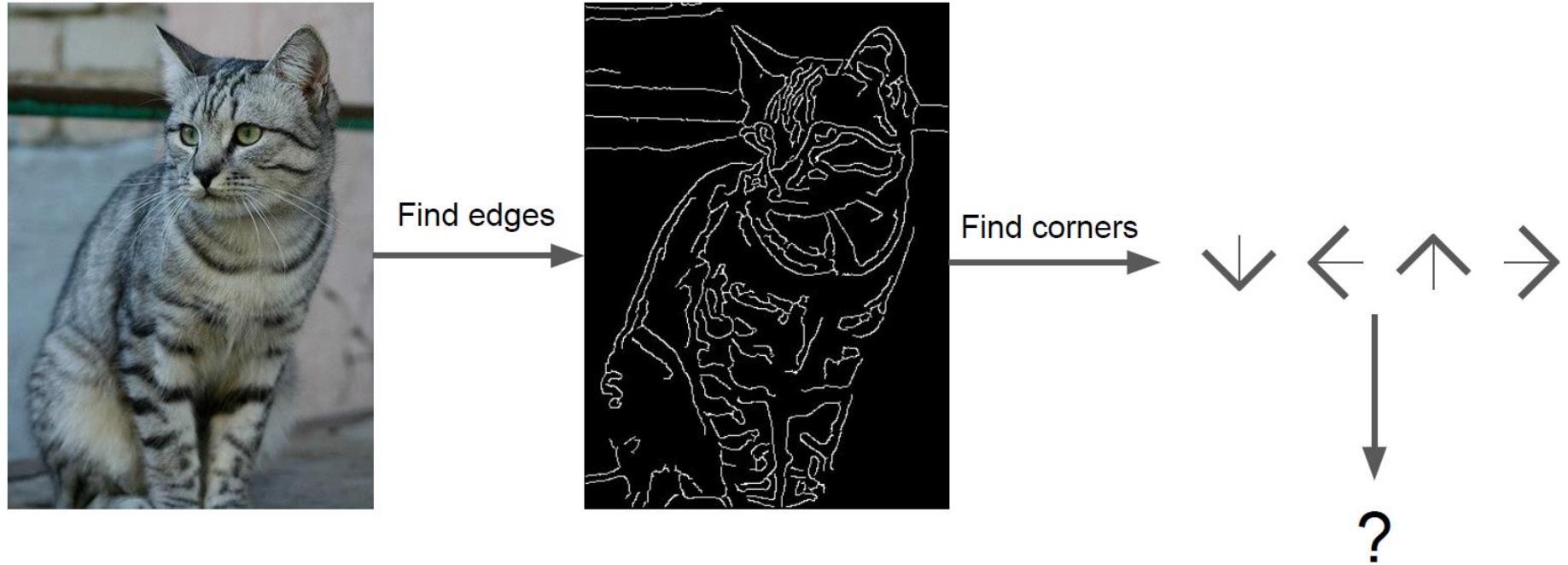
An image classifier

```
def classify_image(image):  
    # Some magic here?  
    return class_label
```

Unlike e.g. sorting a list of numbers,

no obvious way to hard-code the algorithm for
recognizing a cat, or other classes.

Attempts have been made



Machine Learning: Data-Driven Approach

1. Collect a dataset of images and labels
2. Use Machine Learning to train a classifier
3. Evaluate the classifier on new images

Example training set

```
def train(images, labels):  
    # Machine learning!  
    return model
```

```
def predict(model, test_images):  
    # Use model to predict labels  
    return test_labels
```

airplane



automobile



bird



cat



deer



First classifier: Nearest Neighbor

```
def train(images, labels):  
    # Machine learning!  
    return model
```

→ Memorize all
data and labels

```
def predict(model, test_images):  
    # Use model to predict labels  
    return test_labels
```

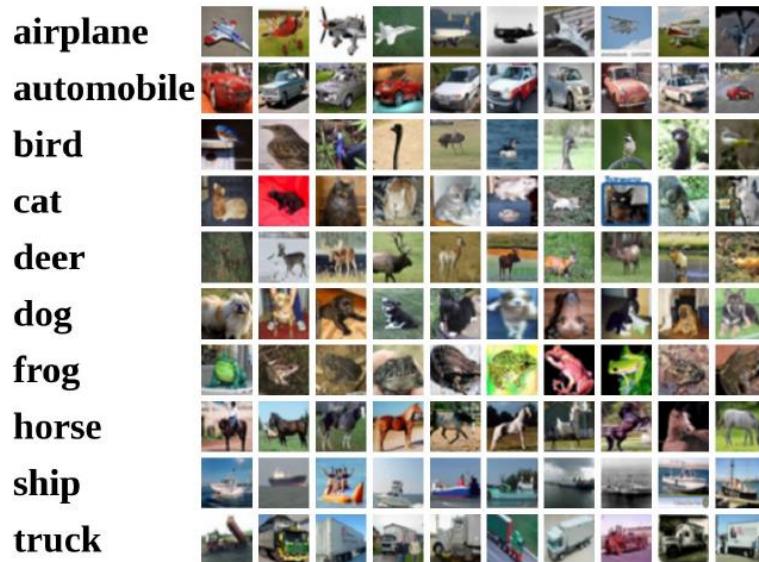
→ Predict the label
of the most similar
training image

Example Dataset: CIFAR10

10 classes

50,000 training images

10,000 testing images

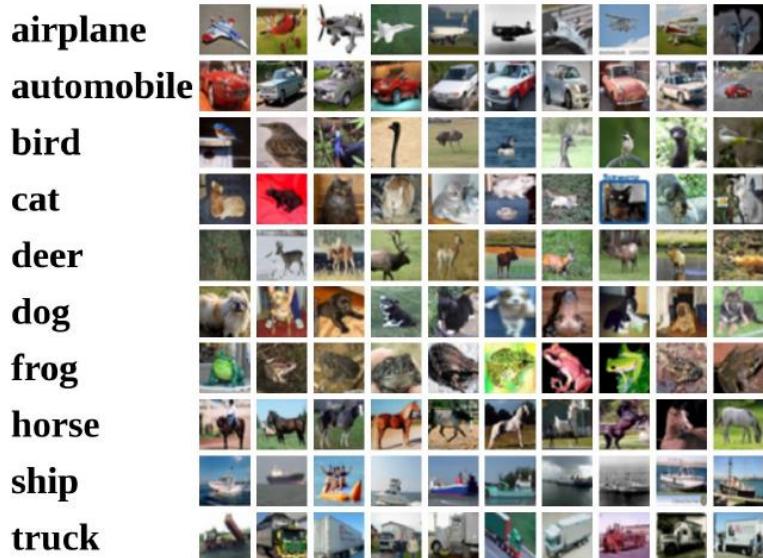


Example Dataset: CIFAR10

10 classes

50,000 training images

10,000 testing images



Test images and nearest neighbors



Distance Metric to compare images

L1 distance:

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$

test image

56	32	10	18
90	23	128	133
24	26	178	200
2	0	255	220

training image

10	20	24	17
8	10	89	100
12	16	178	170
4	32	233	112

-

pixel-wise absolute value differences

46	12	14	1
82	13	39	33
12	10	0	30
2	32	22	108

=

add → 456

```
import numpy as np

class NearestNeighbor:
    def __init__(self):
        pass

    def train(self, X, y):
        """ X is N x D where each row is an example. Y is 1-dimension of size N """
        # the nearest neighbor classifier simply remembers all the training data
        self.Xtr = X
        self.ytr = y

    def predict(self, X):
        """ X is N x D where each row is an example we wish to predict label for """
        num_test = X.shape[0]
        # lets make sure that the output type matches the input type
        Ypred = np.zeros(num_test, dtype = self.ytr.dtype)

        # loop over all test rows
        for i in xrange(num_test):
            # find the nearest training image to the i'th test image
            # using the L1 distance (sum of absolute value differences)
            distances = np.sum(np.abs(self.Xtr - X[i,:]), axis = 1)
            min_index = np.argmin(distances) # get the index with smallest distance
            Ypred[i] = self.ytr[min_index] # predict the label of the nearest example

        return Ypred
```

Nearest Neighbor classifier

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Nearest Neighbor classifier

Memorize training data

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```

Nearest Neighbor classifier

For each test image:
 Find closest train image
 Predict label of nearest image

```

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Nearest Neighbor classifier

Q: With N examples, how fast are training and prediction?

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Nearest Neighbor classifier

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and prediction?

A: Train $O(1)$,
predict $O(N)$

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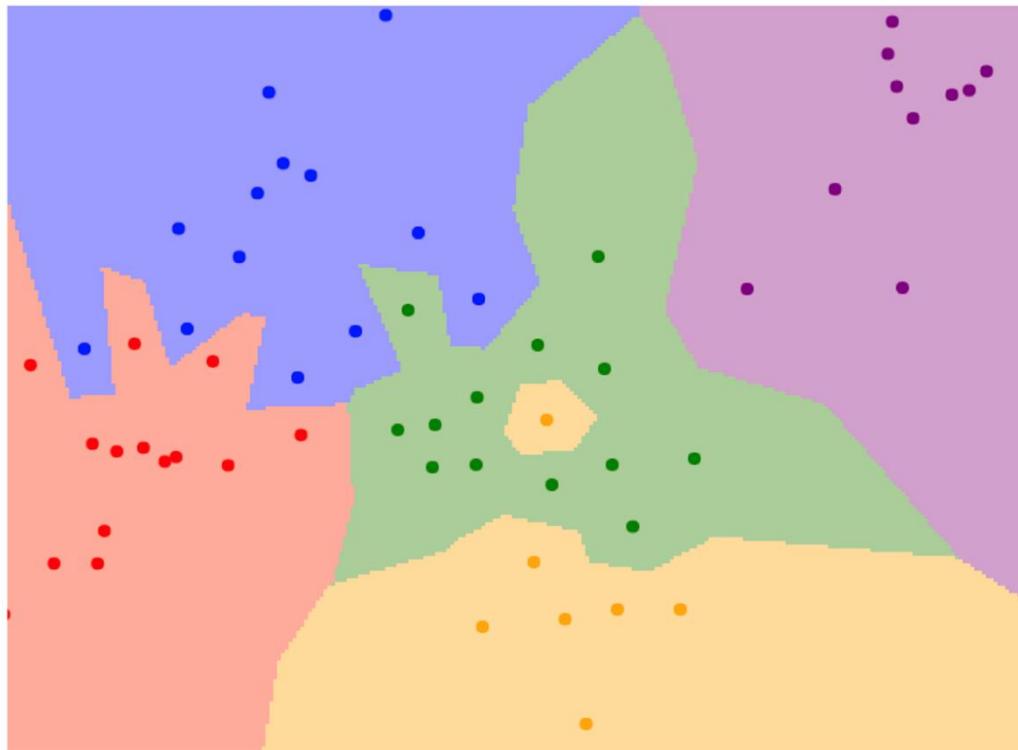
Nearest Neighbor classifier

Q: With N examples, how fast are training and prediction?

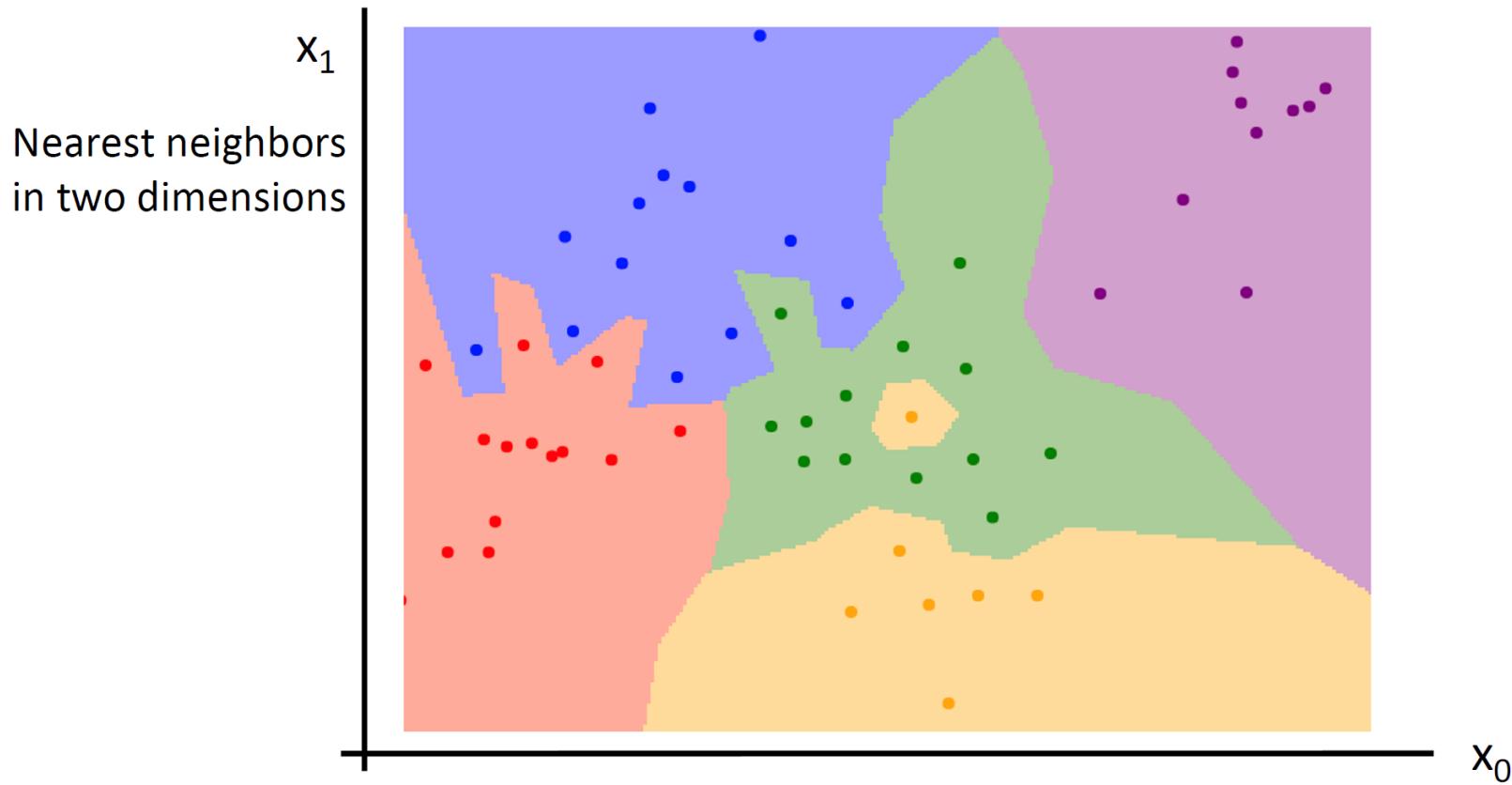
A: Train $O(1)$, predict $O(N)$

This is bad: we want classifiers that are **fast** at prediction; **slow** for training is ok

What does this look like?



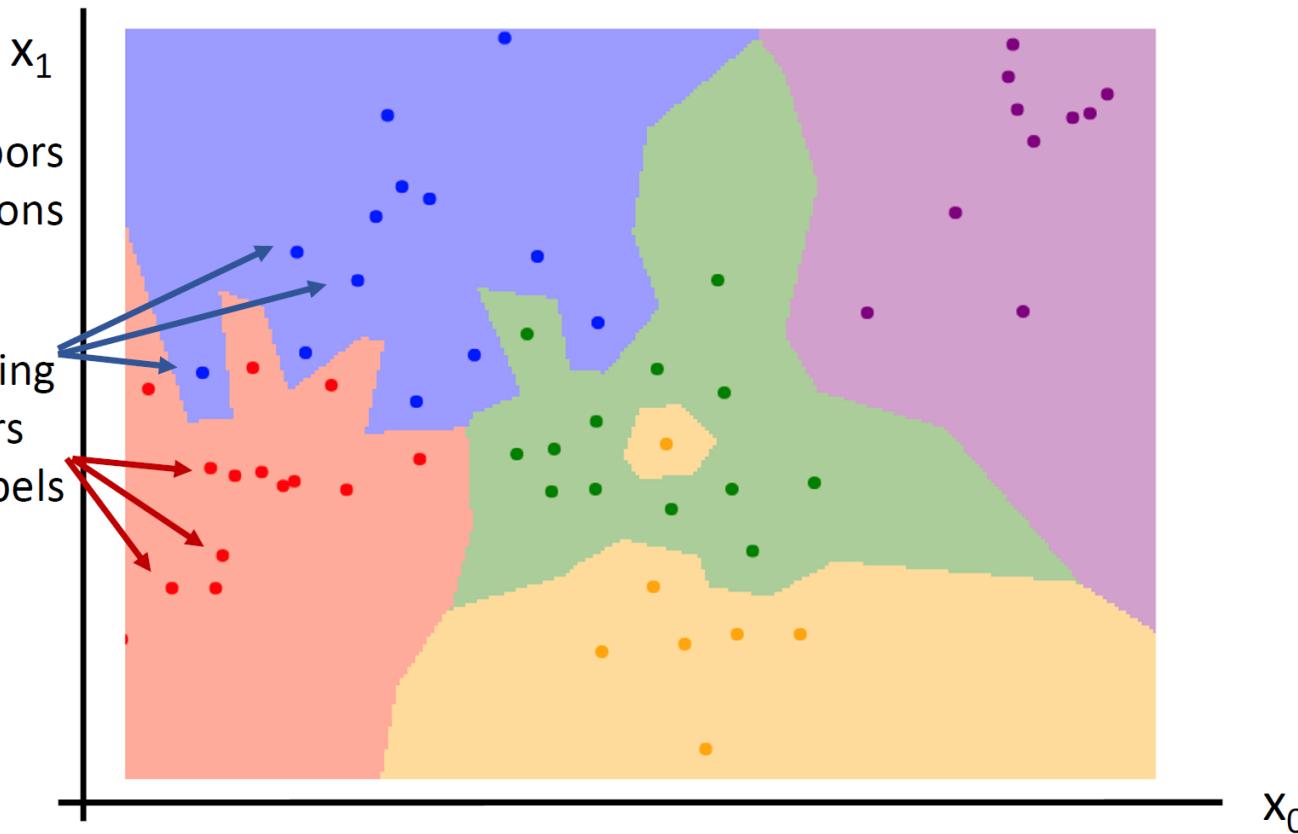
Nearest Neighbor Decision Boundaries



Nearest Neighbor Decision Boundaries

Nearest neighbors
in two dimensions

Points are training
examples; colors
give training labels

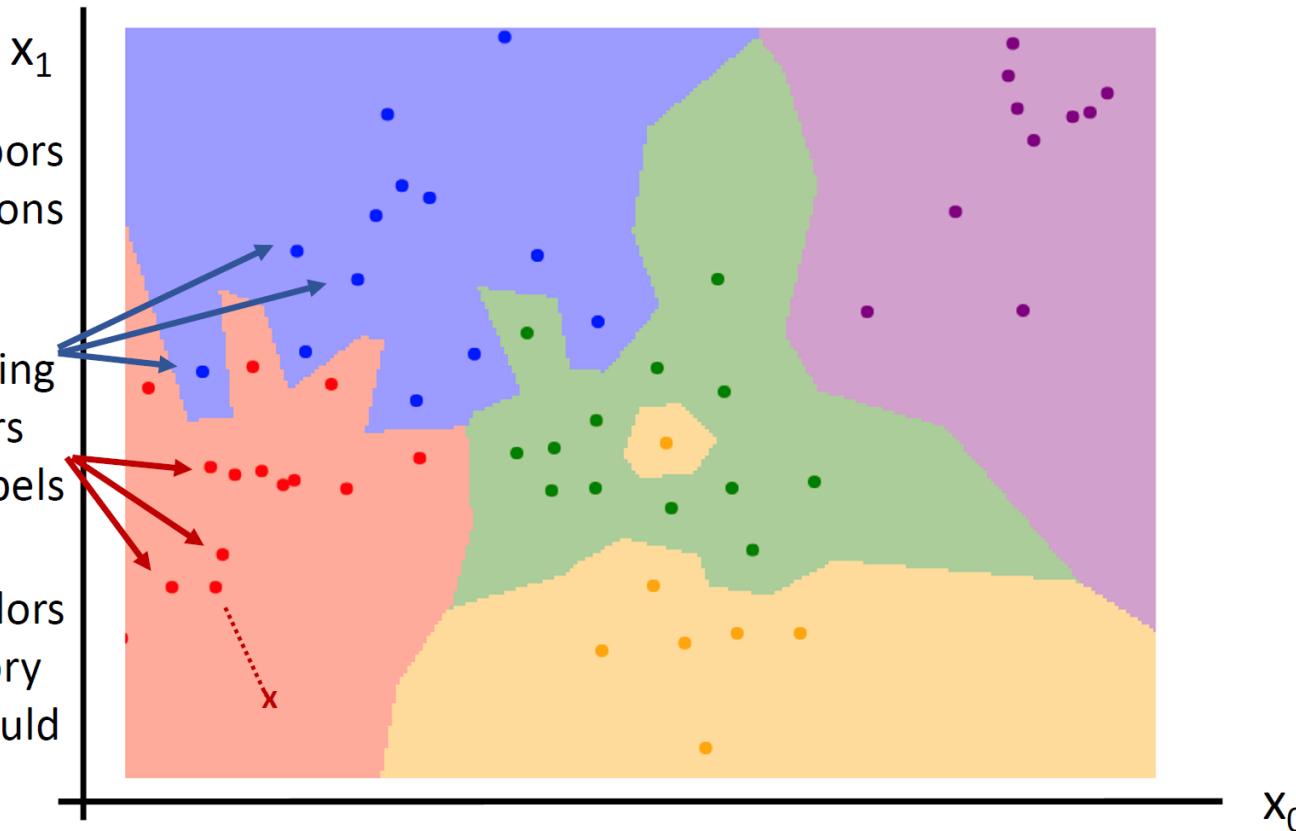


Nearest Neighbor Decision Boundaries

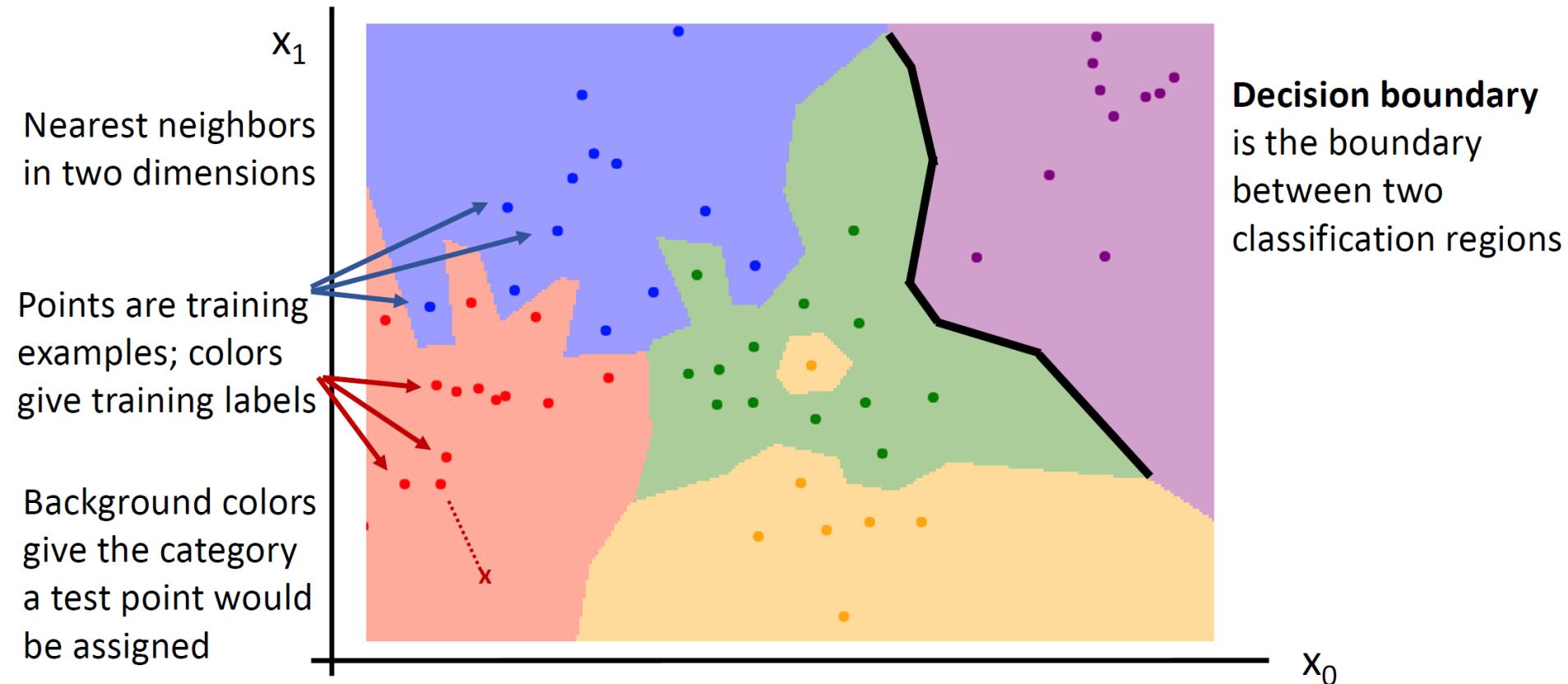
Nearest neighbors
in two dimensions

Points are training
examples; colors
give training labels

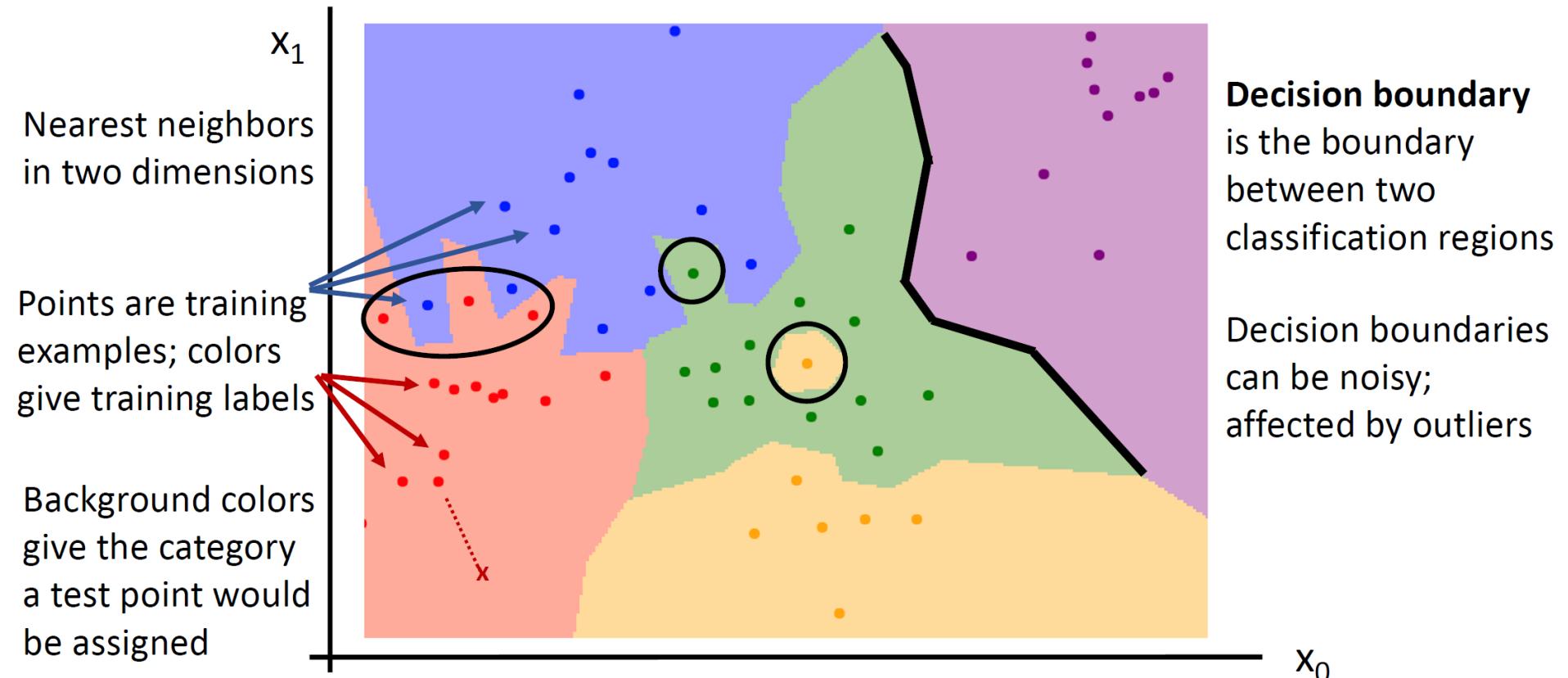
Background colors
give the category
a test point would
be assigned



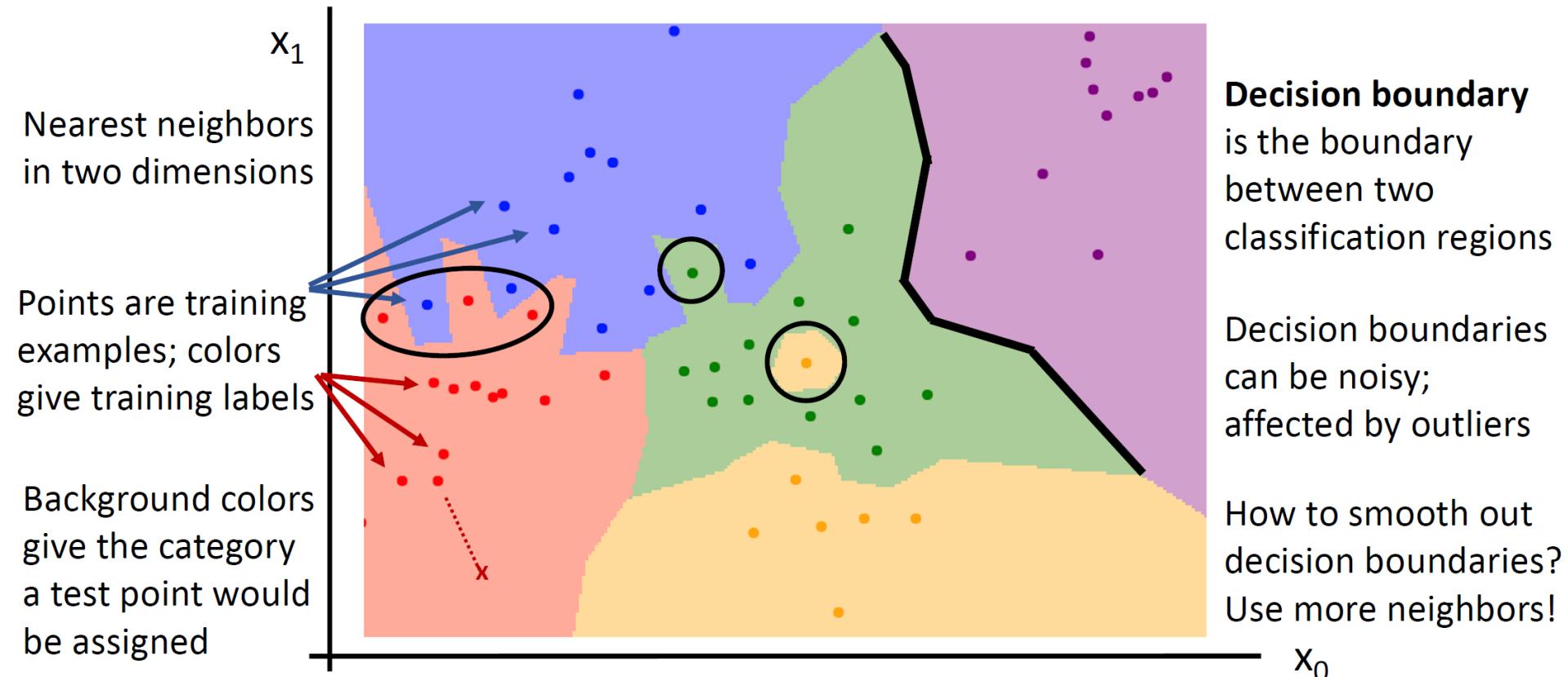
Nearest Neighbor Decision Boundaries



Nearest Neighbor Decision Boundaries



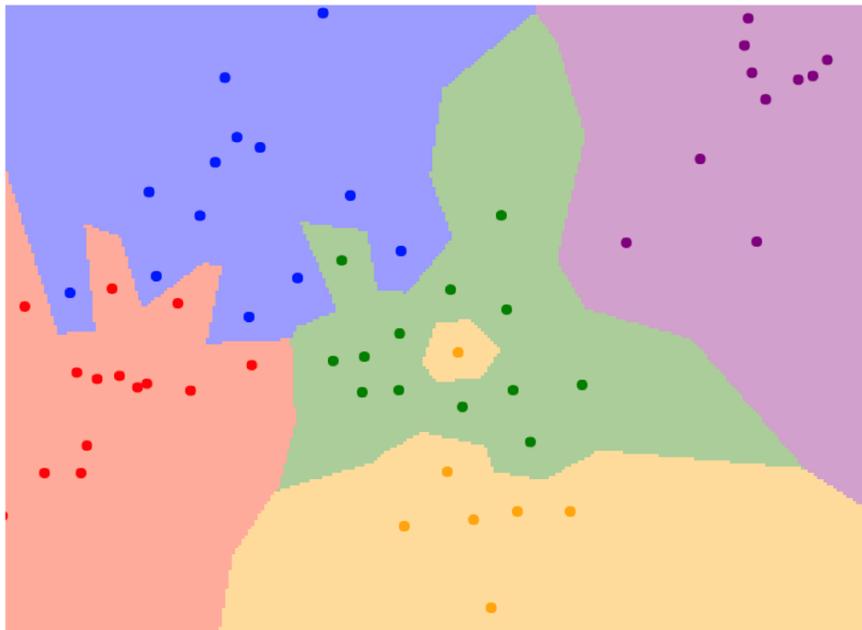
Nearest Neighbor Decision Boundaries



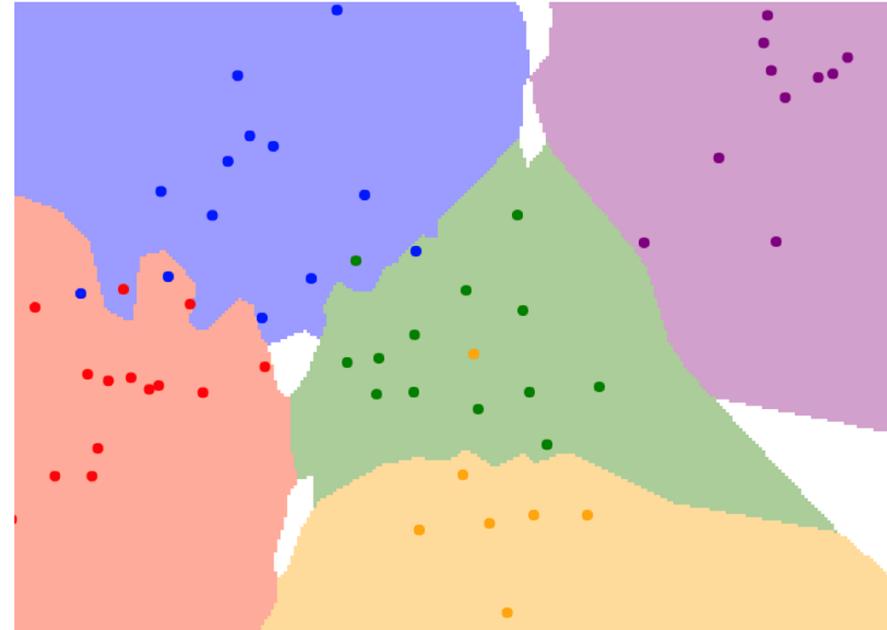
K-Nearest Neighbors

Instead of copying label from nearest neighbor,
take **majority vote** from K closest points

$K = 1$



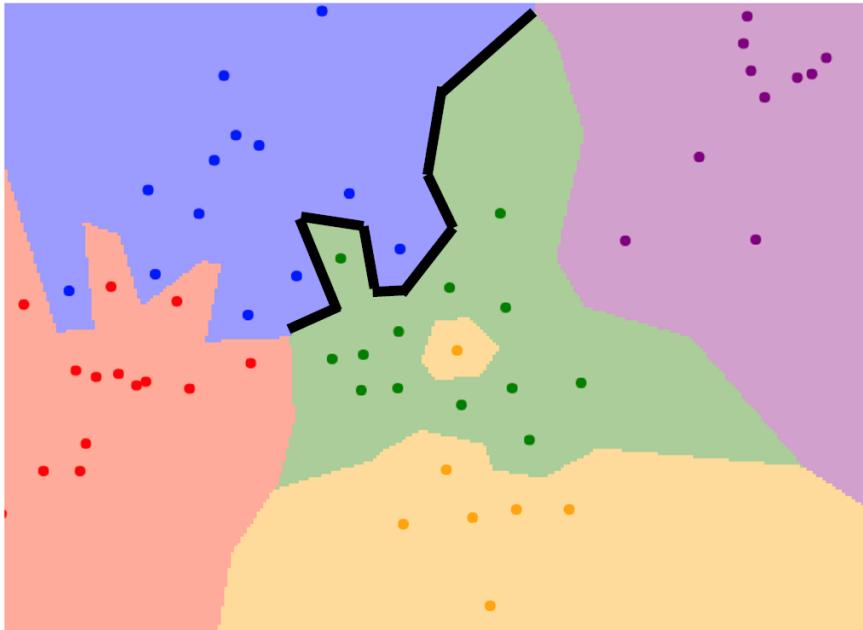
$K = 3$



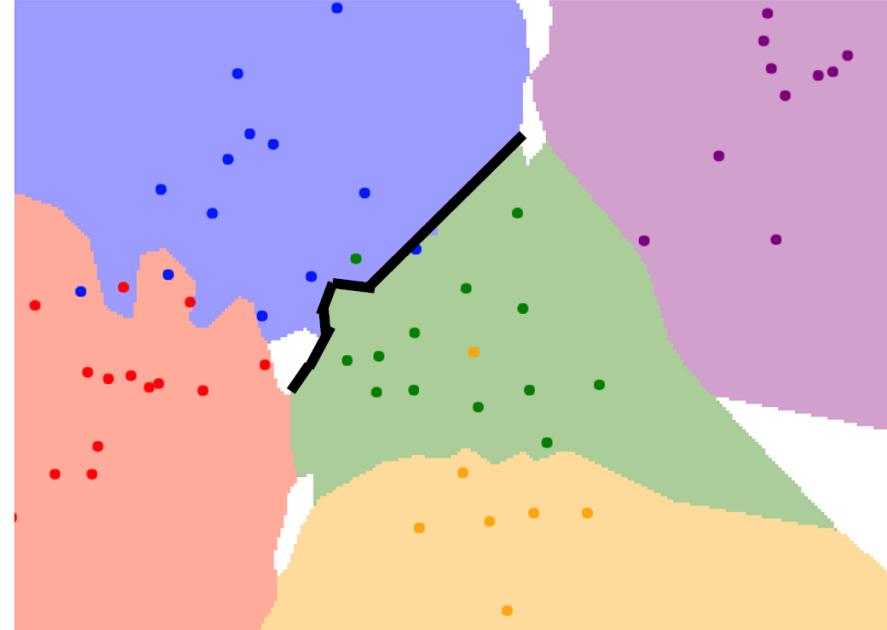
K-Nearest Neighbors

Using more neighbors helps smooth out rough decision boundaries

$K = 1$



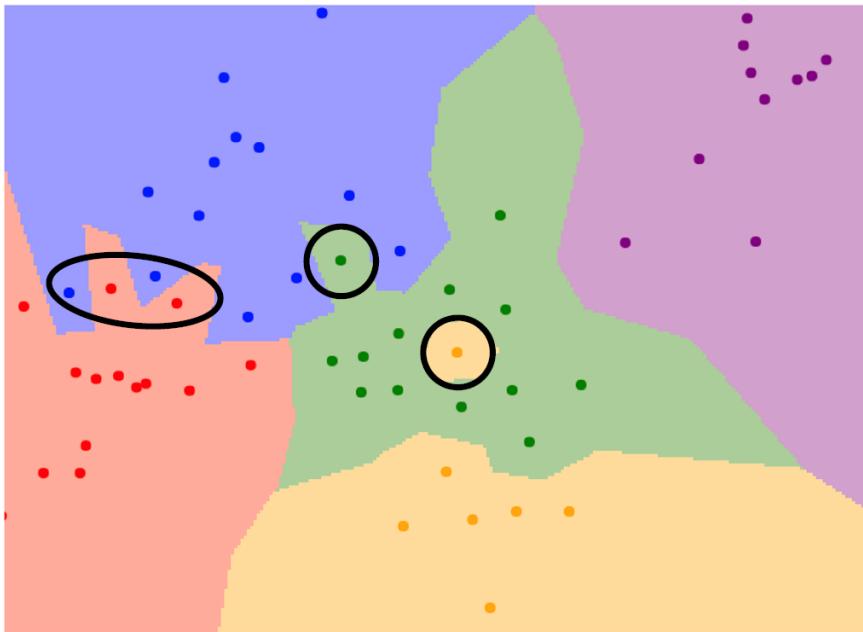
$K = 3$



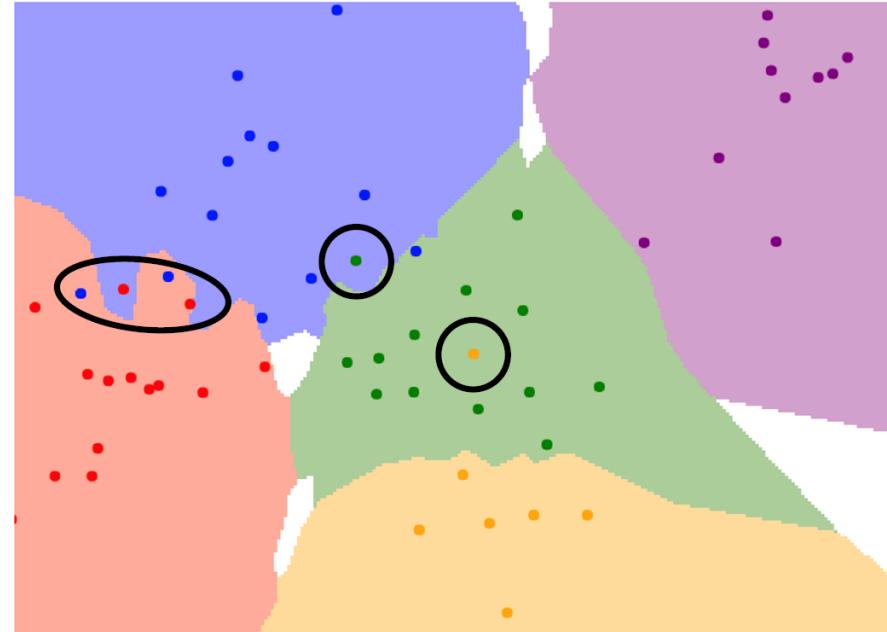
K-Nearest Neighbors

Using more neighbors helps reduce the effect of outliers

$K = 1$



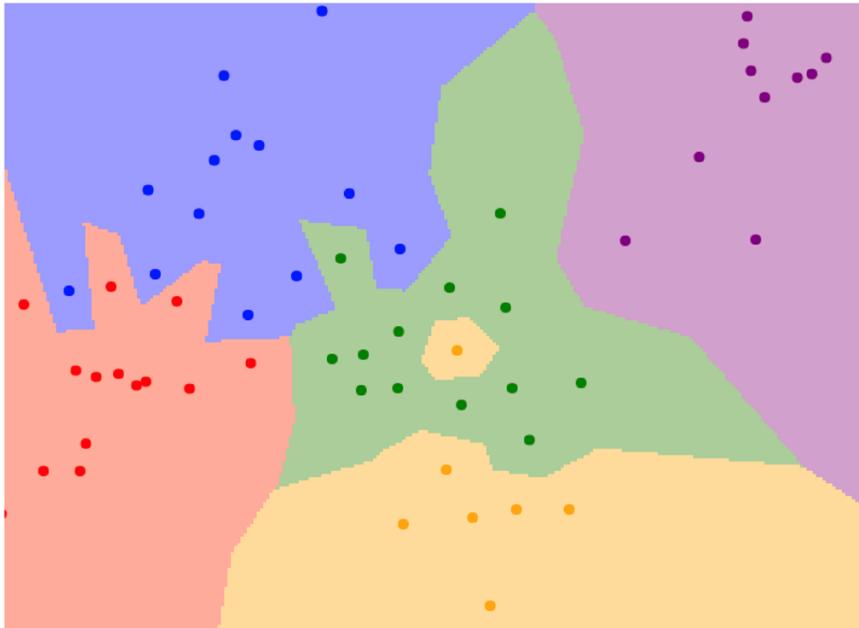
$K = 3$



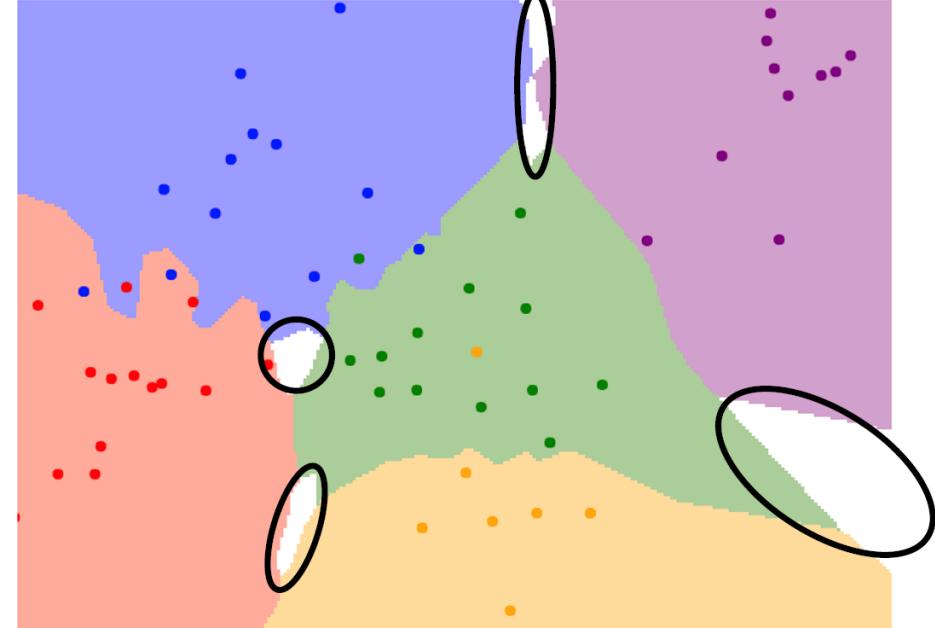
K-Nearest Neighbors

When $K > 1$ there can be ties between classes.
Need to break somehow!

$K = 1$

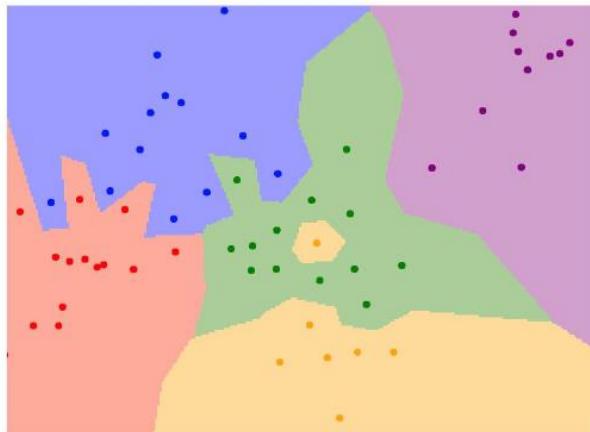


$K = 3$

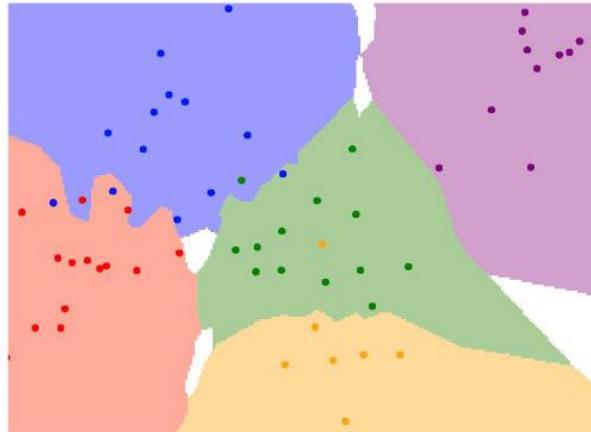


K-Nearest Neighbors

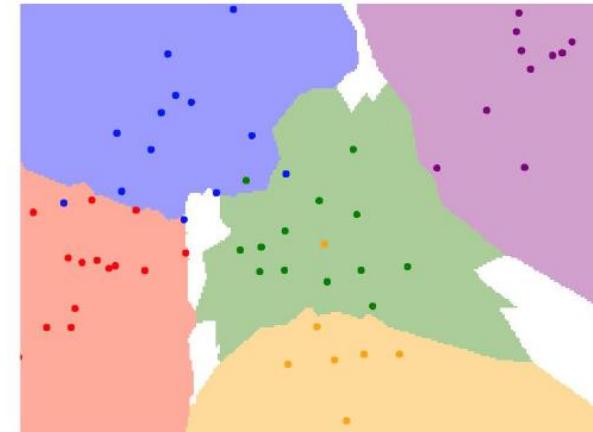
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$K = 1$

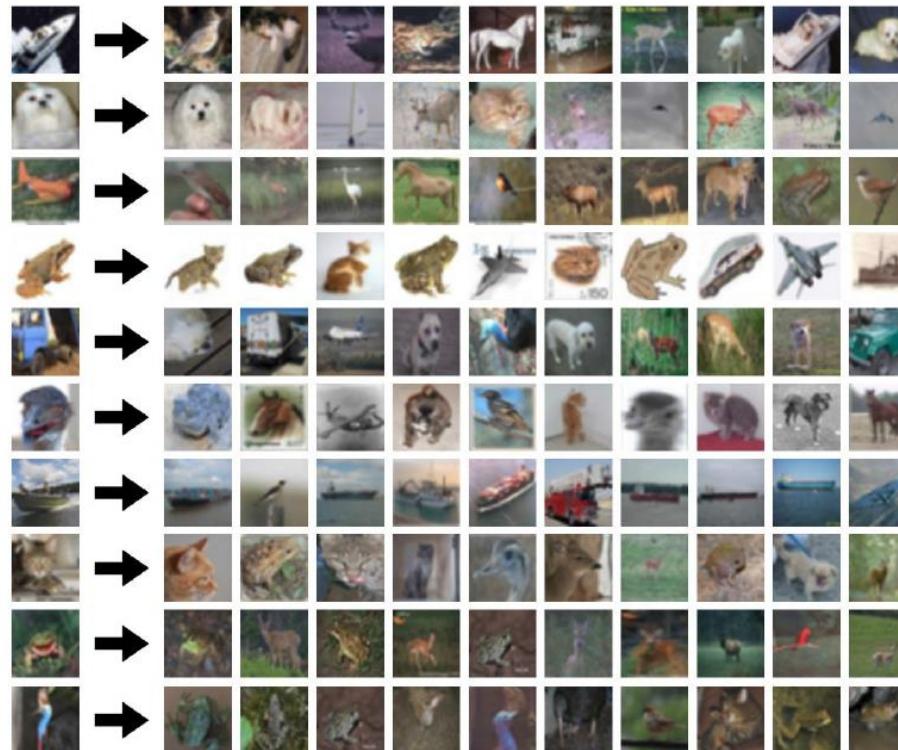


$K = 3$



$K = 5$

What does this look like?



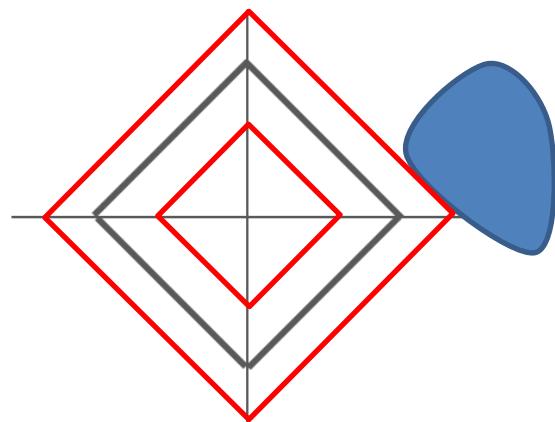
What does this look like?



K-Nearest Neighbors: Distance Metric

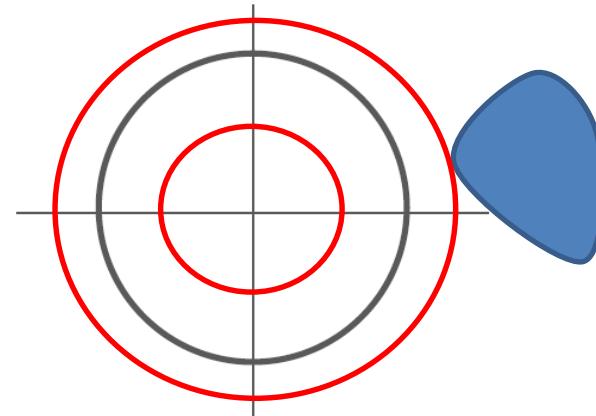
L1 (Manhattan) distance

$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$



L2 (Euclidean) distance

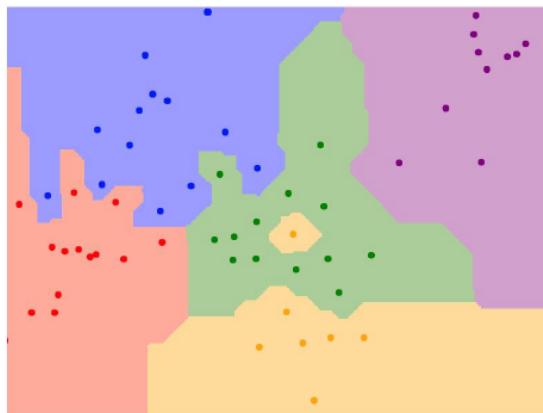
$$d_2(I_1, I_2) = \sqrt{\sum_p (I_1^p - I_2^p)^2}$$



K-Nearest Neighbors: Distance Metric

L1 (Manhattan) distance

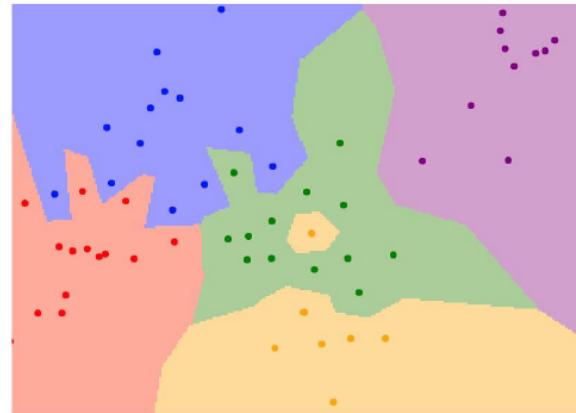
$$d_1(I_1, I_2) = \sum_p |I_1^p - I_2^p|$$



$K = 1$

L2 (Euclidean) distance

$$d_2(I_1, I_2) = \sqrt{\sum_p (I_1^p - I_2^p)^2}$$



$K = 1$

K-Nearest Neighbors: Distance Metric

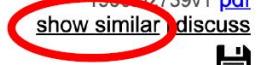
With the right choice of distance metric, we can apply K-Nearest Neighbor to any type of data!

K-Nearest Neighbors: Distance Metric

With the right choice of distance metric, we can apply K-Nearest Neighbor to any type of data!

Example:
Compare
research
papers using
tf-idf similarity

Mesh R-CNN
Georgia Gkioxari, Jitendra Malik, Justin Johnson
6/6/2019 cs.CV

1606.02739v1 [pdf](#) [show similar](#) [discuss](#) 



Rapid advances in 2D perception have led to systems that accurately detect objects in real-world images. However, these systems make predictions in 2D, ignoring the 3D structure of the world. Concurrently, advances in 3D shape prediction have mostly focused on synthetic benchmarks and isolated objects. We unify advances in these two areas. We propose a system that detects objects in real-world images and produces a triangle mesh giving the full 3D shape of each detected object. Our system, called Mesh R-CNN, augments Mask R-CNN with a mesh prediction branch that outputs meshes with varying topological structure by first predicting coarse voxel representations which are converted to meshes and refined with a graph convolution network operating over the mesh's vertices and edges. We validate our mesh prediction branch on ShapeNet, where we outperform prior work on single-image shape prediction. We then deploy our full Mesh R-CNN system on Pix3D, where we jointly detect objects and predict their 3D shapes.

<http://www.arxiv-sanity.com/search?q=mesh+r-cnn>

Hyperparameters

What is the best value of k to use?

What is the best **distance** to use?

These are **hyperparameters**: choices about the algorithm that we set rather than learn

Hyperparameters

What is the best value of k to use?

What is the best **distance** to use?

These are **hyperparameters**: choices about the algorithm that we set rather than learn

Very problem-dependent.

Must try them all out and see what works best.

Setting Hyperparameters

Idea #1: Choose hyperparameters
that work best on the data

Your Dataset

Setting Hyperparameters

Idea #1: Choose hyperparameters
that work best on the data

BAD: K = 1 always works
perfectly on training data

Your Dataset

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the data

BAD: K = 1 always works perfectly on training data

Your Dataset

Idea #2: Split data into **train** and **test**, choose hyperparameters that work best on test data

train

test

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the data

BAD: K = 1 always works perfectly on training data

Your Dataset

Idea #2: Split data into **train** and **test**, choose hyperparameters that work best on test data

BAD: No idea how algorithm will perform on new data

train

test

Setting Hyperparameters

Idea #1: Choose hyperparameters that work best on the data

BAD: $K = 1$ always works perfectly on training data

Your Dataset

Idea #2: Split data into **train** and **test**, choose hyperparameters that work best on test data

BAD: No idea how algorithm will perform on new data

train

test

Idea #3: Split data into **train**, **val**, and **test**; choose hyperparameters on val and evaluate on test

Better!

train

validation

test

Setting Hyperparameters

Your Dataset

Idea #4: Cross-Validation: Split data into **folds**,
try each fold as validation and average the results

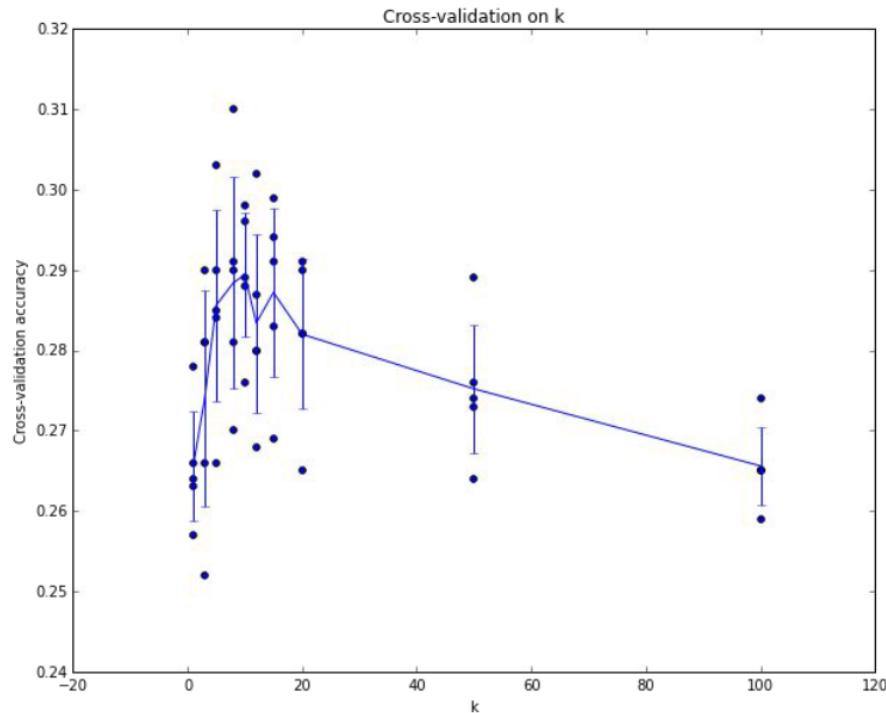
fold 1	fold 2	fold 3	fold 4	fold 5	test
--------	--------	--------	--------	--------	------

fold 1	fold 2	fold 3	fold 4	fold 5	test
--------	--------	--------	--------	--------	------

fold 1	fold 2	fold 3	fold 4	fold 5	test
--------	--------	--------	--------	--------	------

Useful for small datasets, but not used too frequently in deep learning

Setting Hyperparameters



Example of
5-fold cross-validation
for the value of k .

Each point: single
outcome.

The line goes
through the mean, bars
indicated standard
deviation

(Seems that $k \approx 7$ works best
for this data)

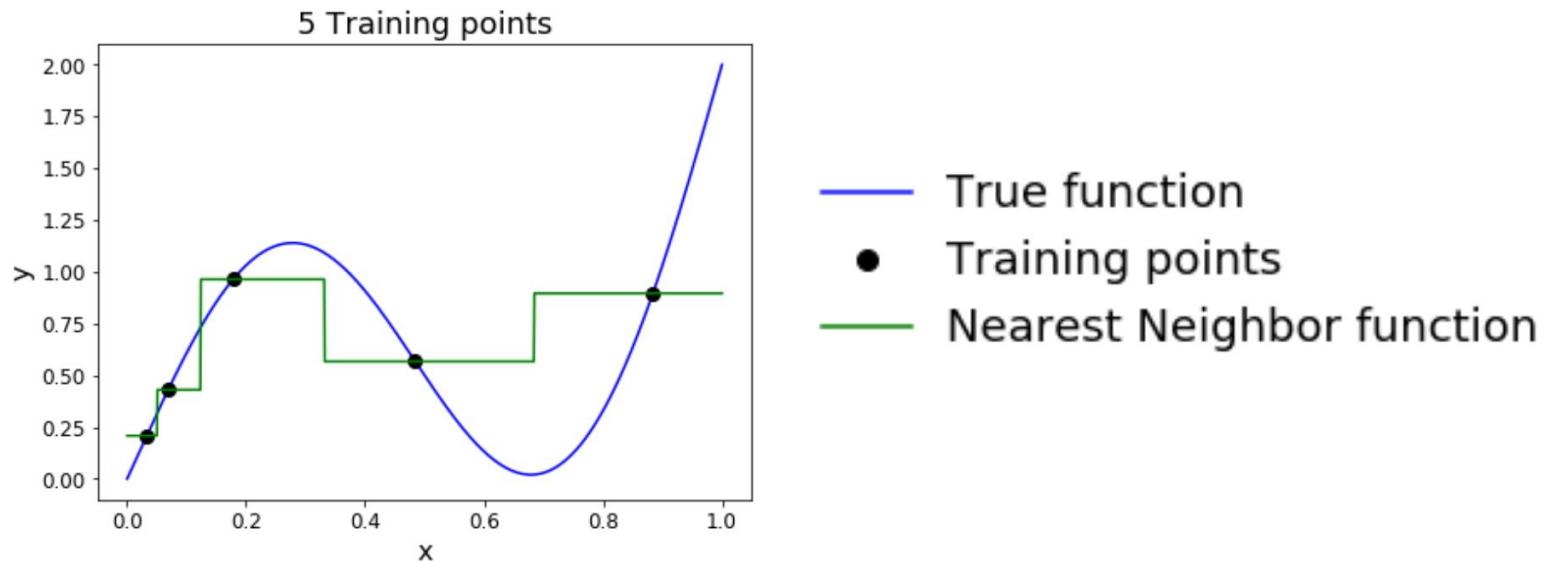
K-Nearest Neighbor: Universal Approximation

As the number of training samples goes to infinity, nearest neighbor can represent any^(*) function!

(*) Subject to many technical conditions. Only continuous functions on a compact domain; need to make assumptions about spacing of training points; etc.

K-Nearest Neighbor: Universal Approximation

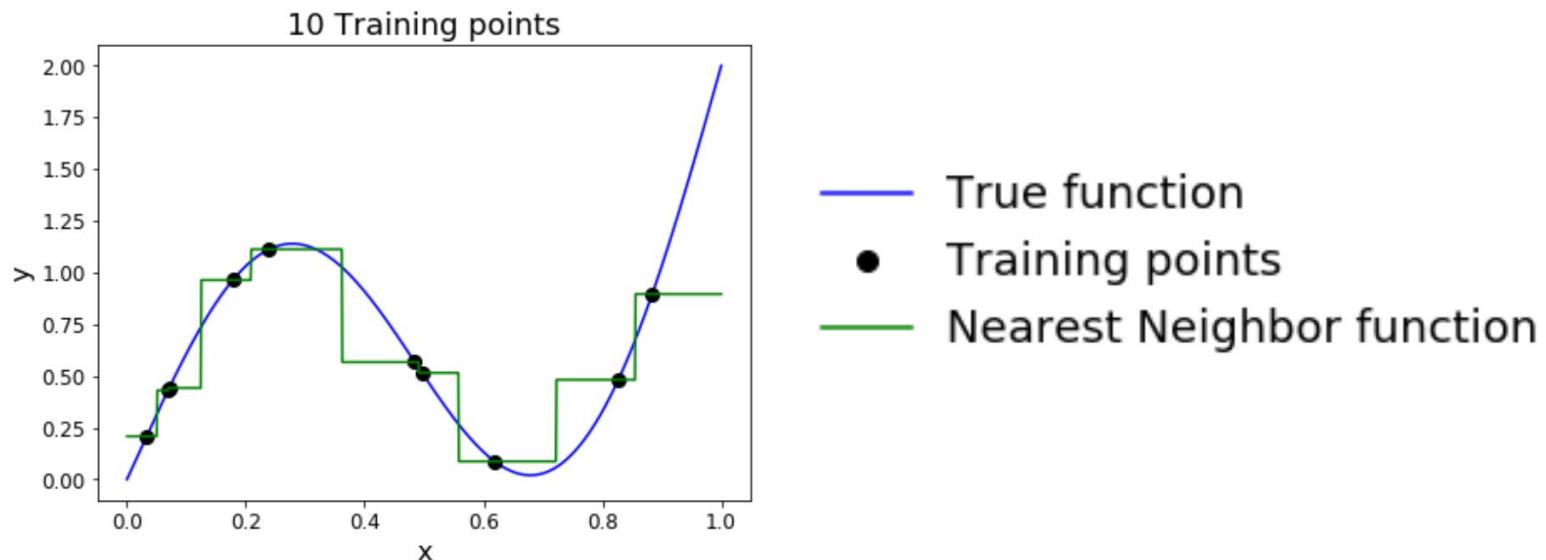
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K-Nearest Neighbor: Universal Approximation

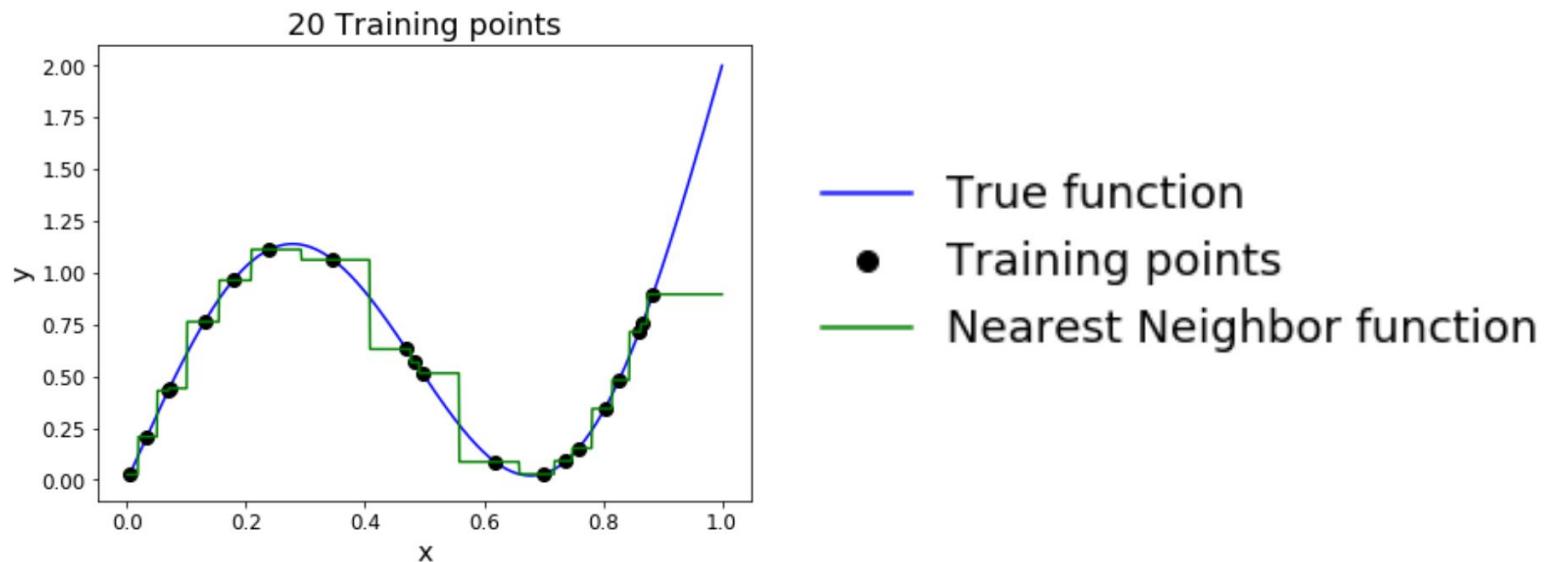
As the number of training samples goes to infinity, nearest neighbor can represent any^(*) function!



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K-Nearest Neighbor: Universal Approximation

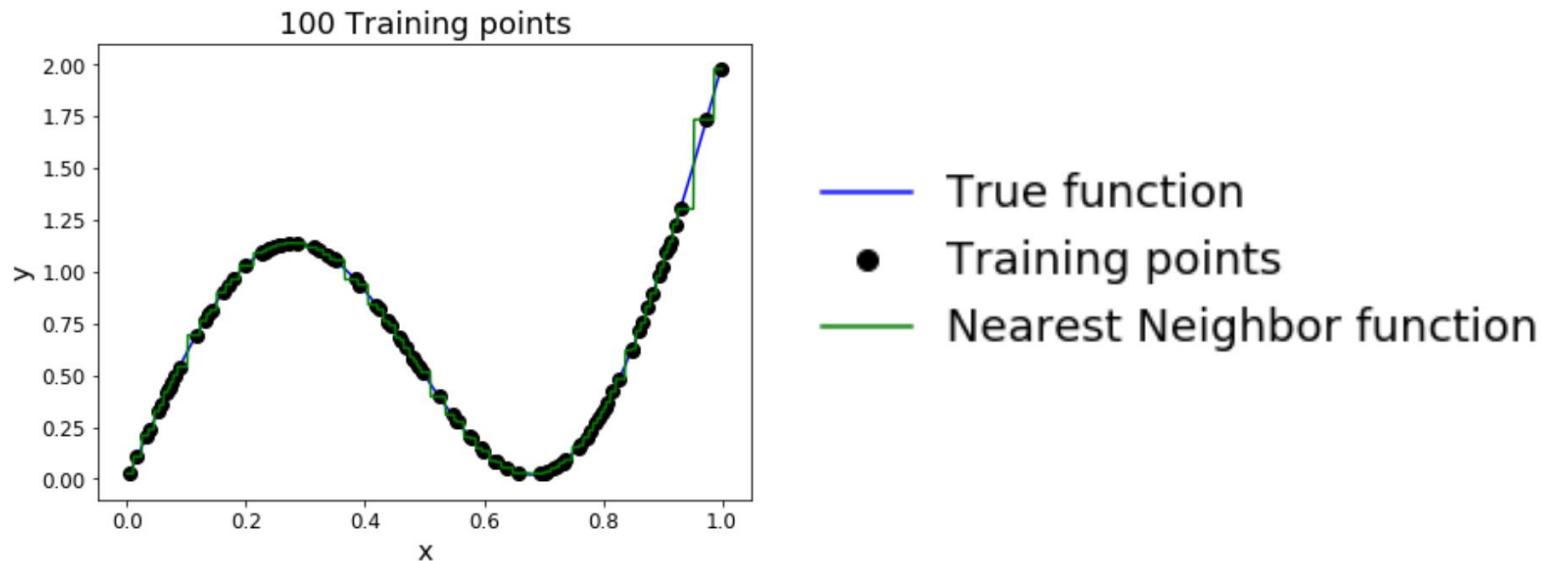
As the number of training samples goes to infinity, nearest neighbor can represent any^(*) function!



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K-Nearest Neighbor: Universal Approximation

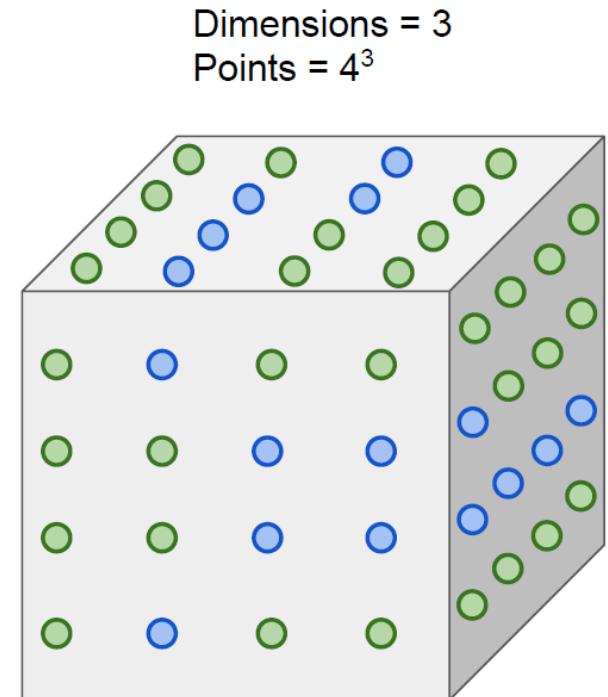
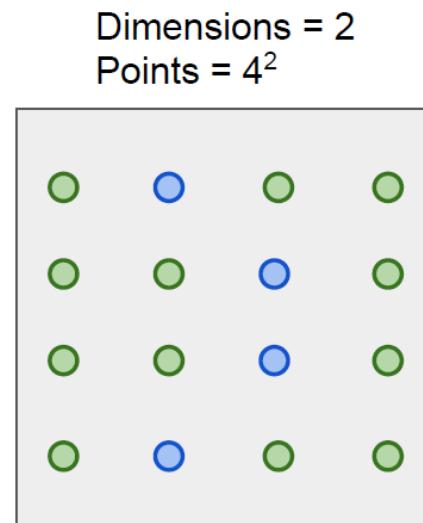
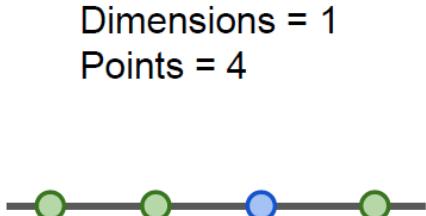
As the number of training samples goes to infinity, nearest neighbor can represent any^(*) function!



(*) Subject to many technical conditions. Only continuous functions on a compact domain; need to make assumptions about spacing of training points; etc.

k-Nearest Neighbor on images never used.

- Curse of dimensionality



Problem: Curse of Dimensionality

Curse of dimensionality: For uniform coverage of space, number of training points needed grows exponentially with dimension

Number of possible
32x32 binary images:

$$2^{32 \times 32} \approx 10^{308}$$

Number of elementary particles
in the visible universe: [\(source\)](#)

$$\approx 10^{97}$$

k-Nearest Neighbor on images **never used**.

- Very slow at test time
- Distance metrics on pixels are not informative

Original



Boxed



Shifted



Tinted



(all 3 images have same L2 distance to the one on the left)

Nearest Neighbor with ConvNet features works well!



Devlin et al, "Exploring Nearest Neighbor Approaches for Image Captioning", 2015

Nearest Neighbor with ConvNet features works well!

Example: Image Captioning with Nearest Neighbor



A bedroom with a bed and a couch.



A cat sitting in a bathroom sink.



A train is stopped at a train station.



A wooden bench in front of a building.

K-Nearest Neighbors: Summary

In **Image classification** we start with a **training set** of images and labels, and must predict labels on the **test set**

The **K-Nearest Neighbors** classifier predicts labels based on nearest training examples

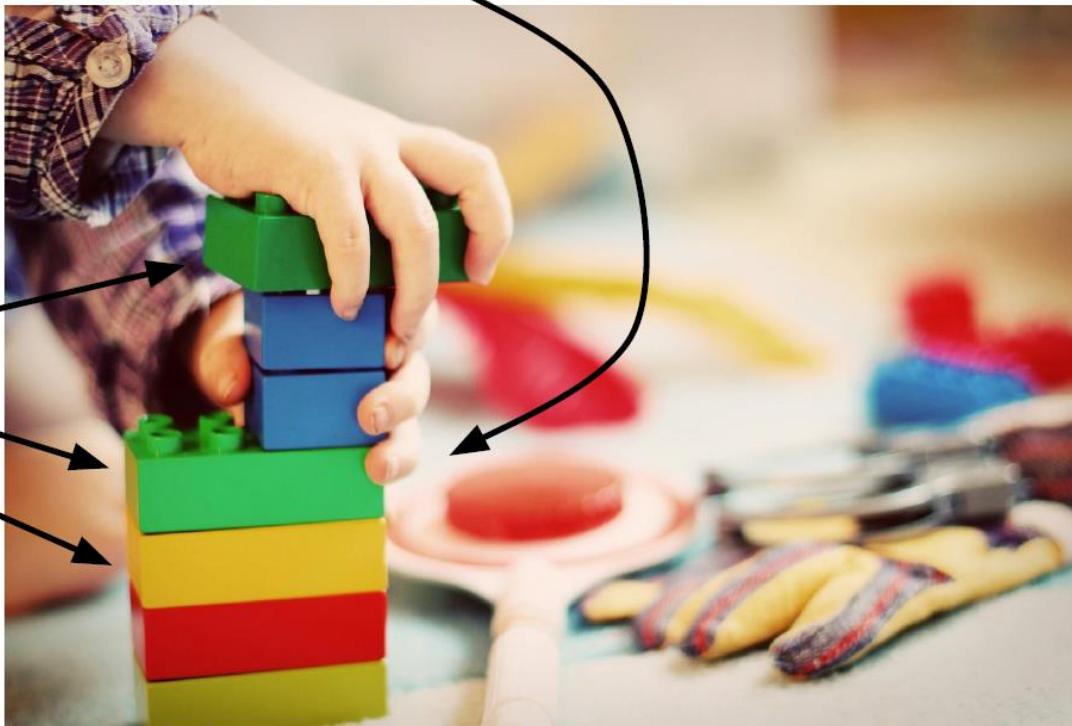
Distance metric and K are **hyperparameters**

Choose hyperparameters using the **validation set**; only run on the test set once at the very end!

Linear Classification

Neural Network

Linear
classifiers



[This image](#) is CC0 1.0 public domain

Recall CIFAR10

airplane



automobile



bird



cat



deer



dog



frog



horse



ship



truck



50,000 training images
each image is **32x32x3**

10,000 test images.

Parametric Approach

Image



Array of **32x32x3** numbers
(3072 numbers total)

$$\xrightarrow{f(x, W)}$$

$$f(\mathbf{x}, \mathbf{W})$$

10 numbers giving
class scores



W

parameters
or weights

Parametric Approach: Linear Classifier

Image



$$f(x, W) = Wx$$

Array of **32x32x3** numbers
(3072 numbers total)

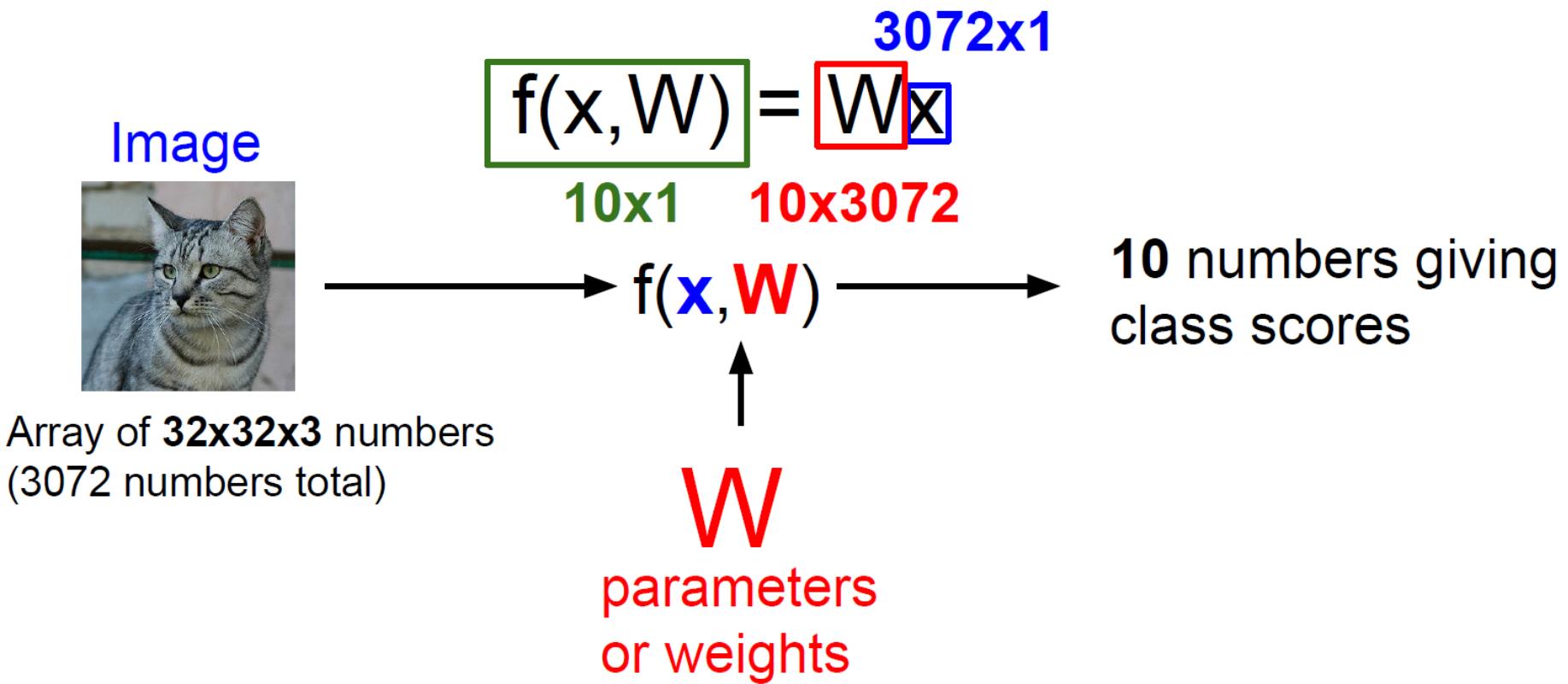
$$f(\mathbf{x}, \mathbf{W})$$

10 numbers giving
class scores

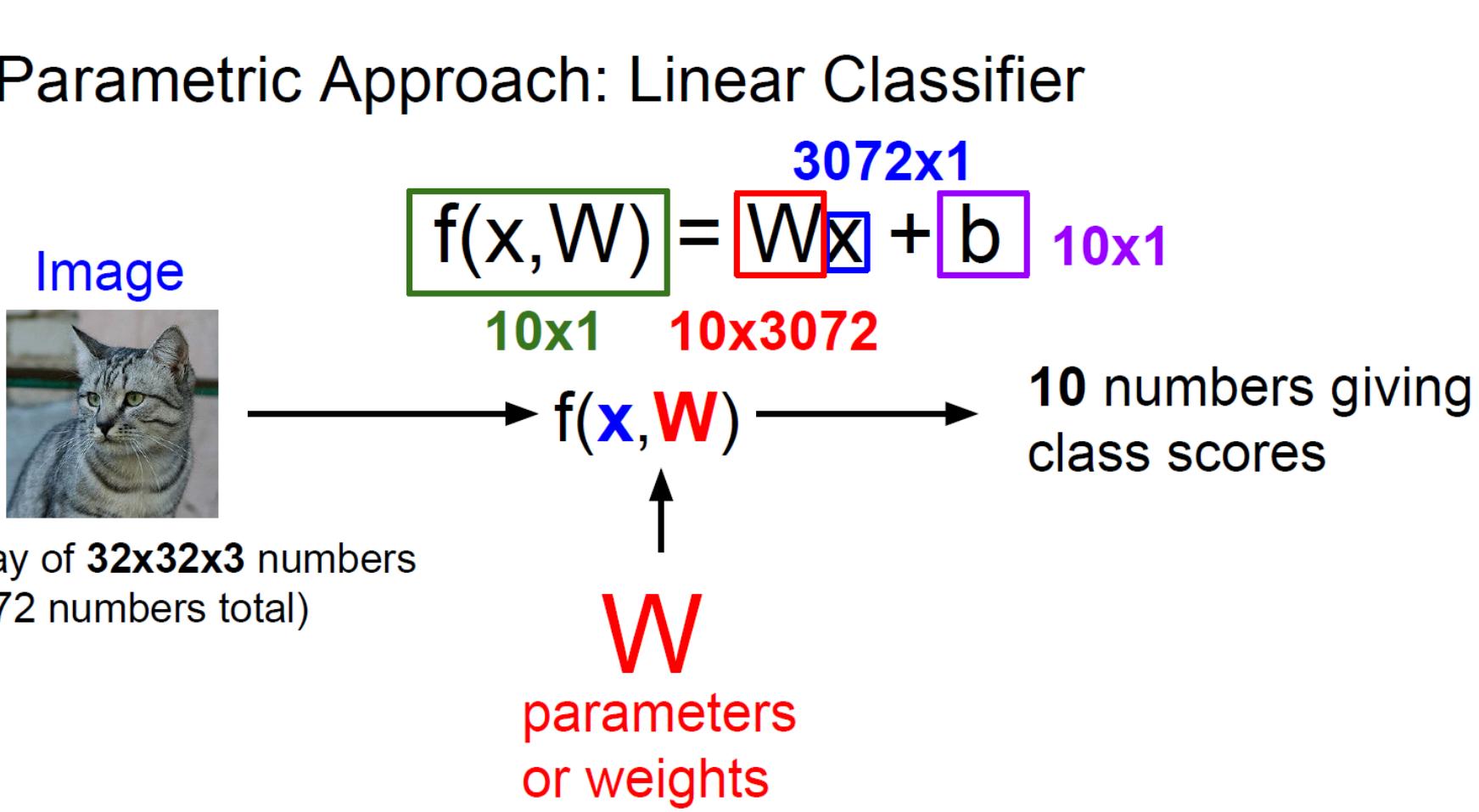


W
parameters
or weights

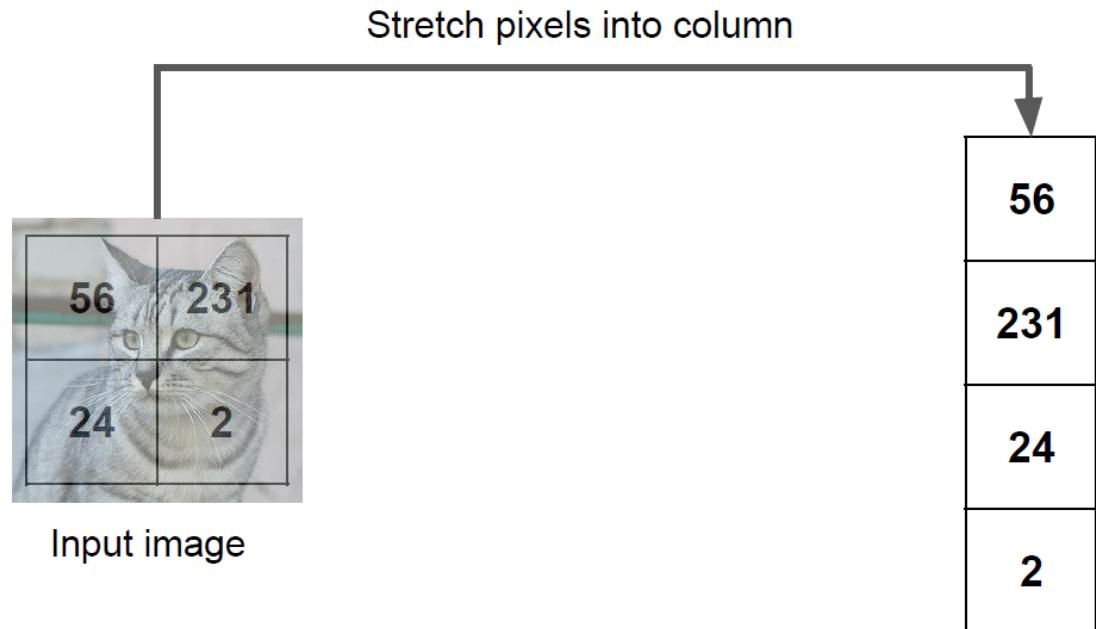
Parametric Approach: Linear Classifier



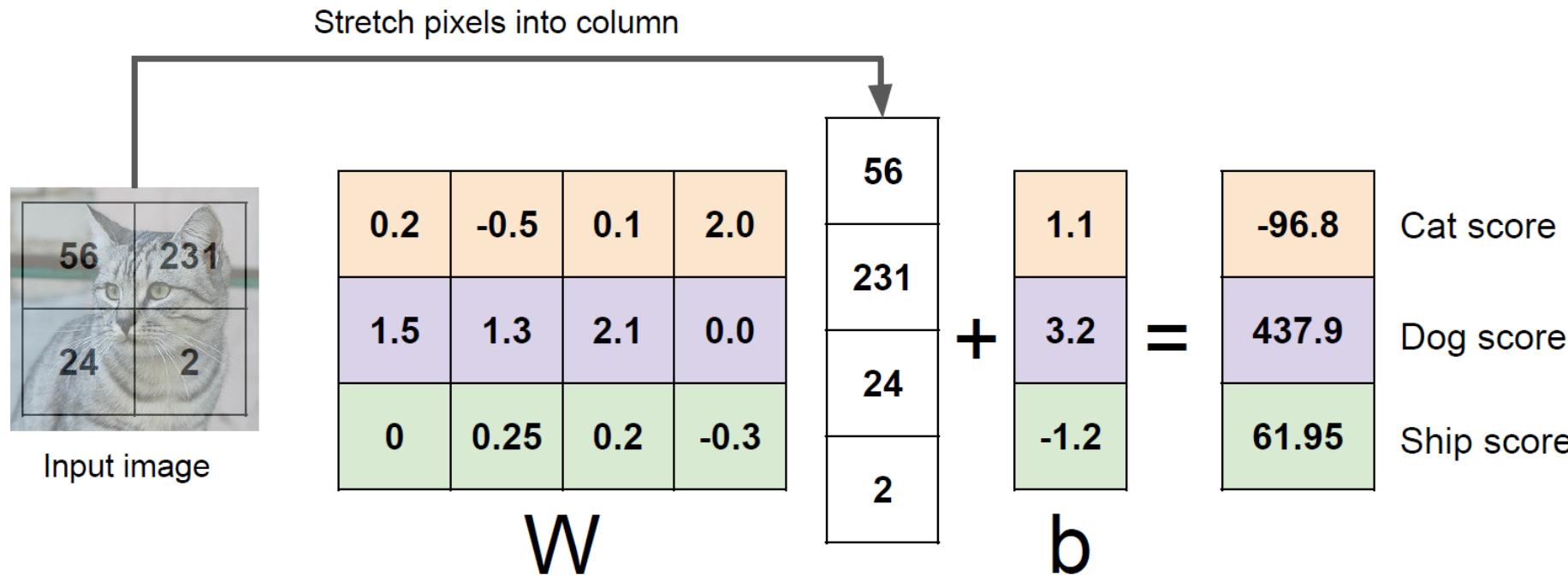
Parametric Approach: Linear Classifier



Example with an image with 4 pixels, and 3 classes (**cat/dog/ship**)



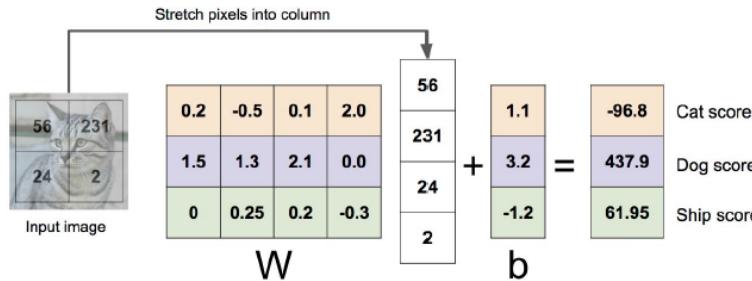
Example with an image with 4 pixels, and 3 classes (cat/dog/ship)



Example with an image with 4 pixels, and 3 classes (**cat/dog/ship**)

Algebraic Viewpoint

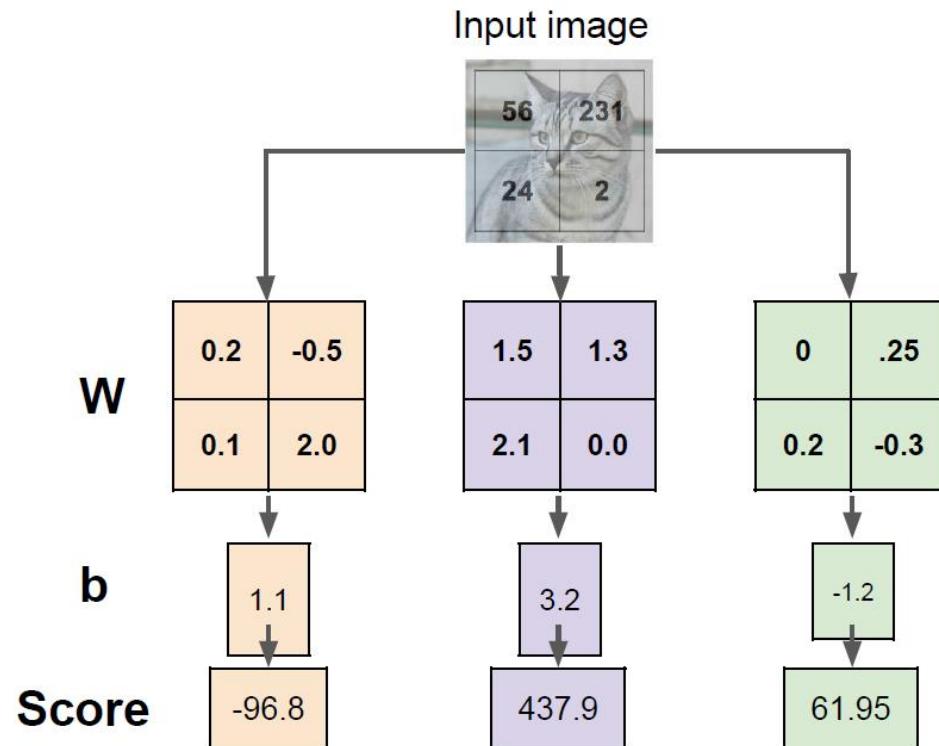
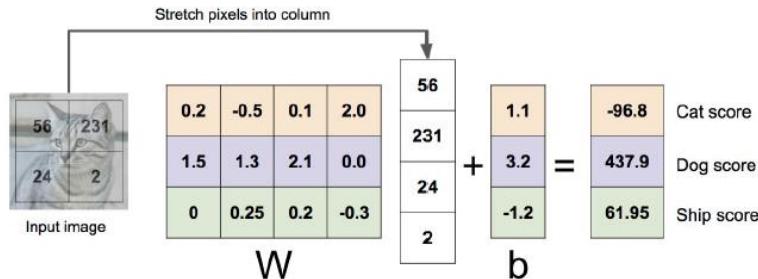
$$f(x, W) = Wx$$



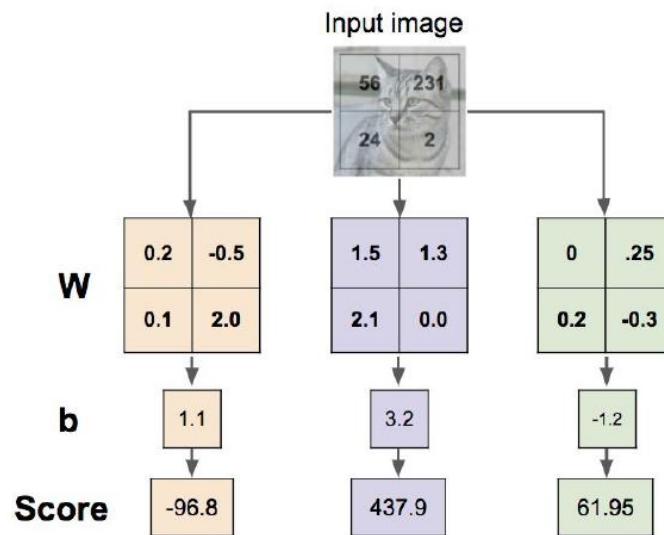
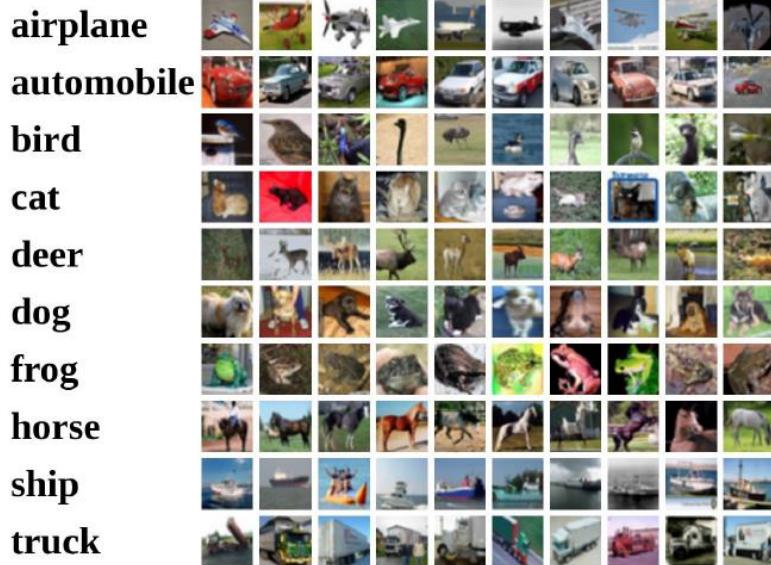
Example with an image with 4 pixels, and 3 classes (**cat/dog/ship**)

Algebraic Viewpoint

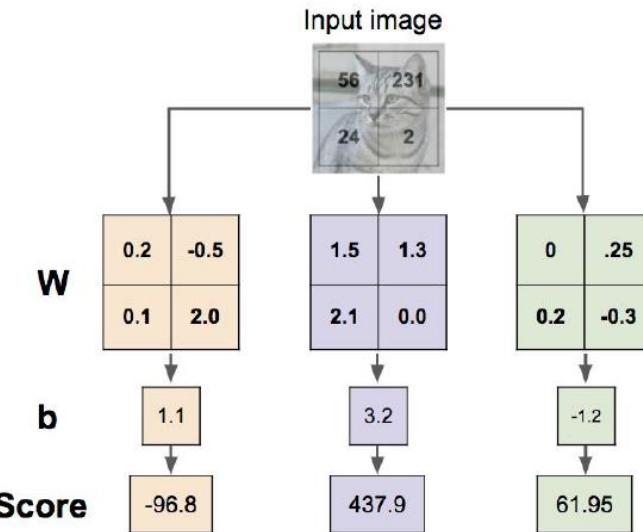
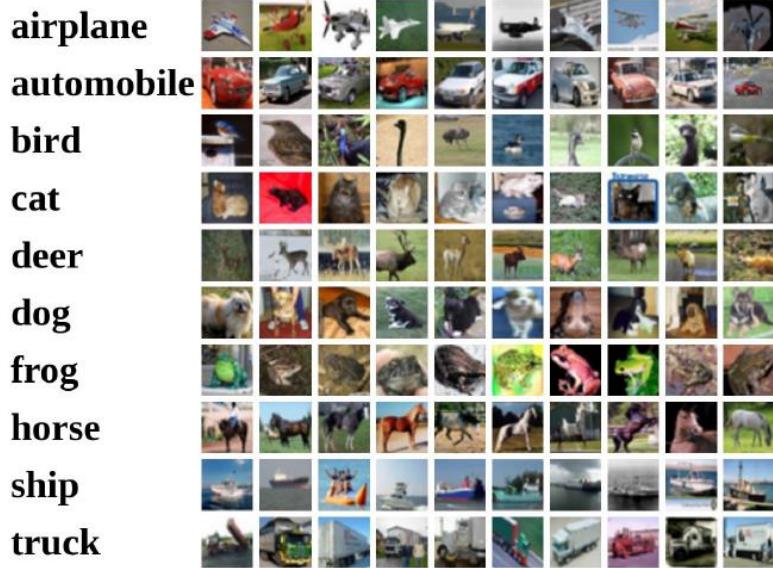
$$f(x, W) = Wx$$



Interpreting a Linear Classifier

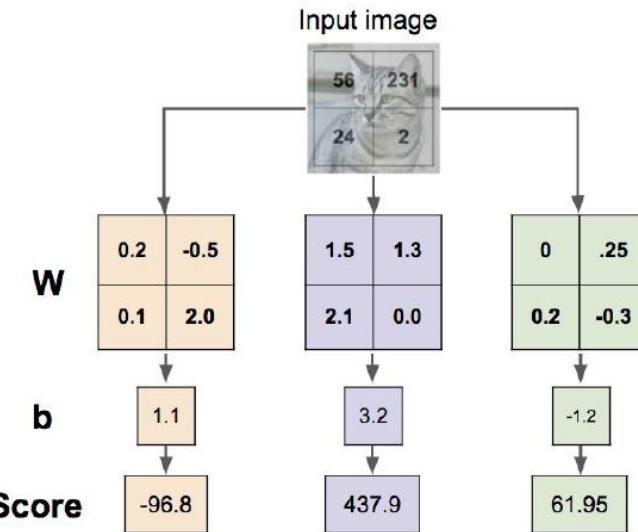


Interpreting a Linear Classifier: Visual Viewpoint



Interpreting a Linear Classifier: Visual Viewpoint

Linear classifier has one
“template” per category

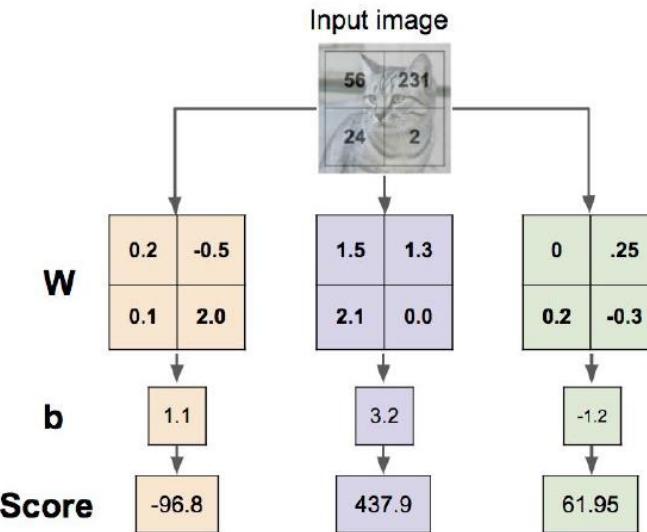


Interpreting a Linear Classifier: Visual Viewpoint

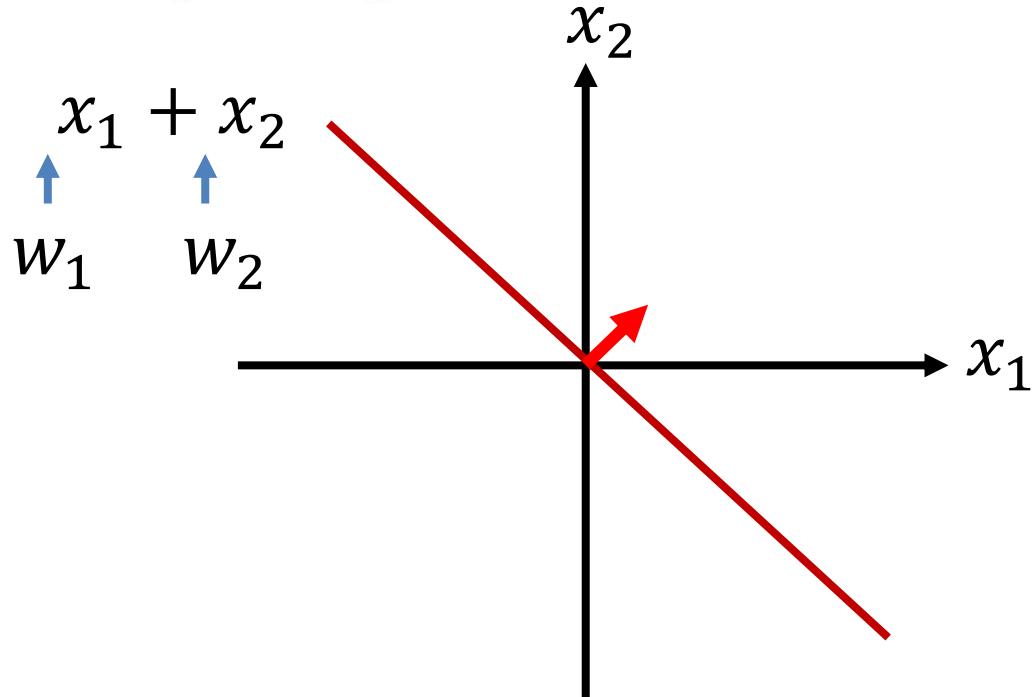
Linear classifier has one
“template” per category

A single template cannot capture
multiple modes of the data

e.g. horse template has 2 heads!

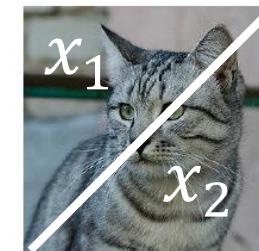


Interpreting a Linear Classifier: Geometric Viewpoint



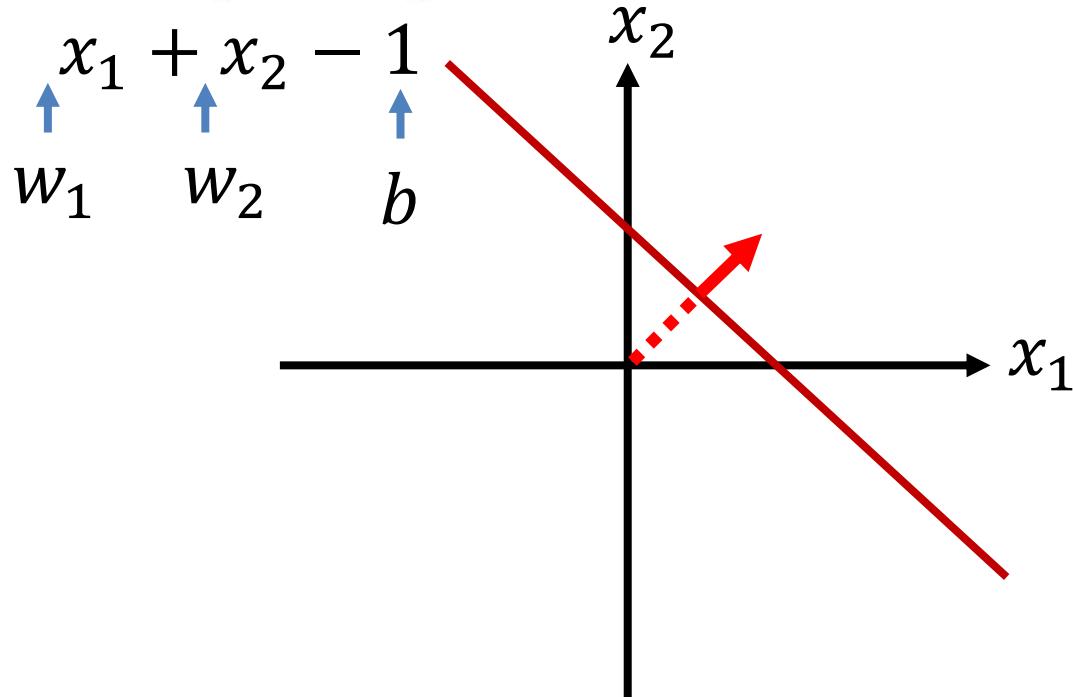
$$f(x, W) = Wx + b$$

$\begin{matrix} \text{---} \\ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{bmatrix}$



Array of **32x32x3** numbers
(3072 numbers total)

Interpreting a Linear Classifier: Geometric Viewpoint

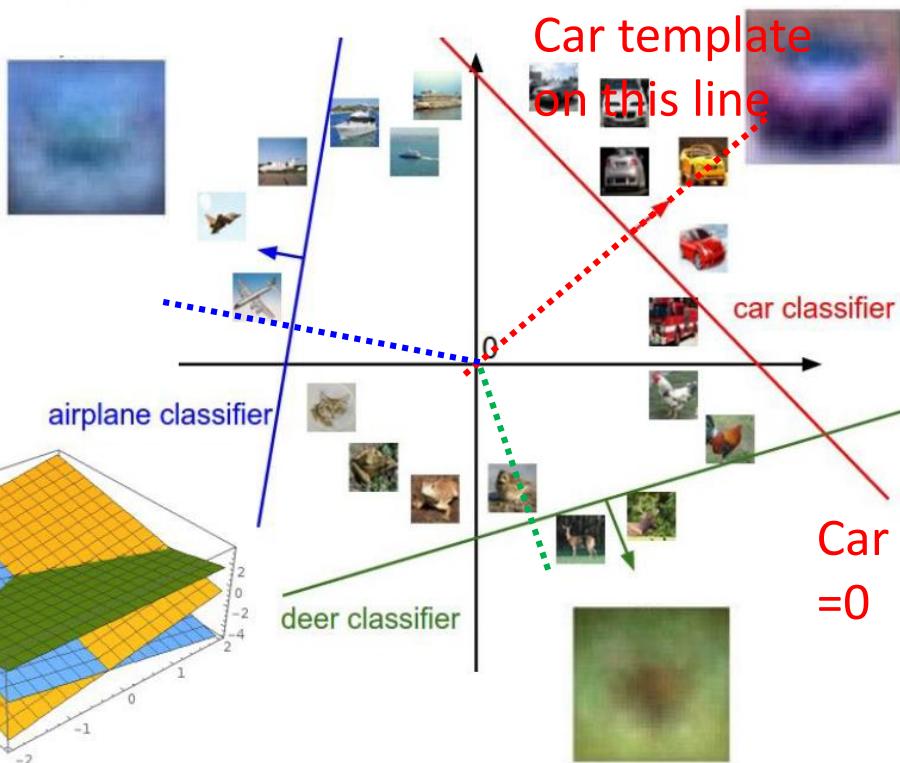


$$f(\mathbf{x}, \mathbf{W}) = \mathbf{W}\mathbf{x} + \mathbf{b}$$



Array of **32x32x3** numbers
(3072 numbers total)

Interpreting a Linear Classifier: Geometric Viewpoint



$$f(x, W) = Wx + b$$



Array of **32x32x3** numbers
(3072 numbers total)

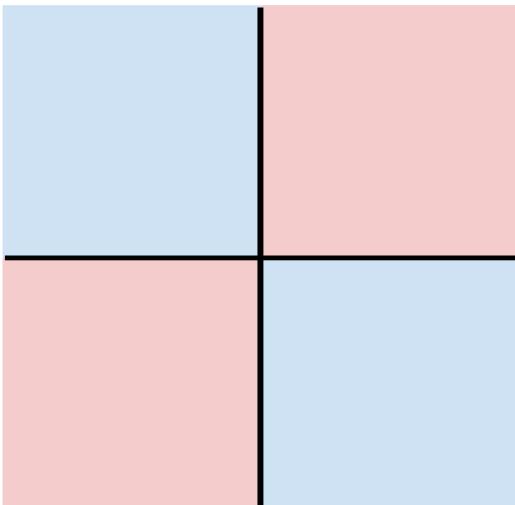
Hard cases for a linear classifier

Class 1:

First and third quadrants

Class 2:

Second and fourth quadrants

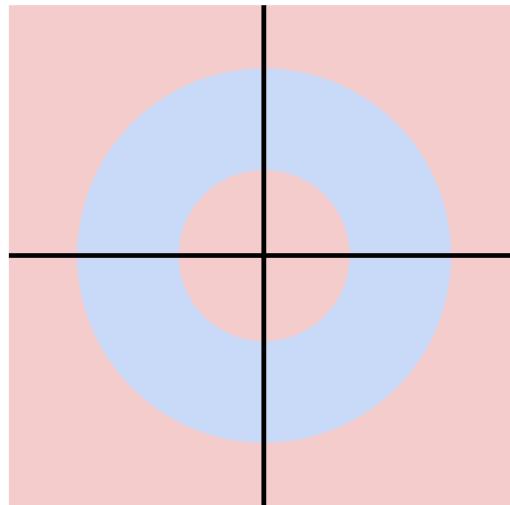


Class 1:

$1 \leq L_2 \text{ norm} \leq 2$

Class 2:

Everything else

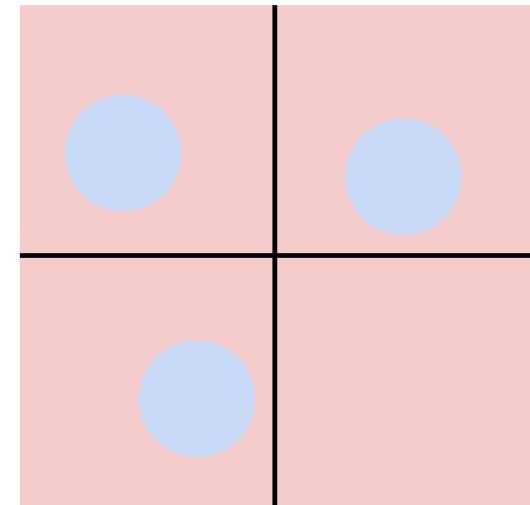


Class 1:

Three modes

Class 2:

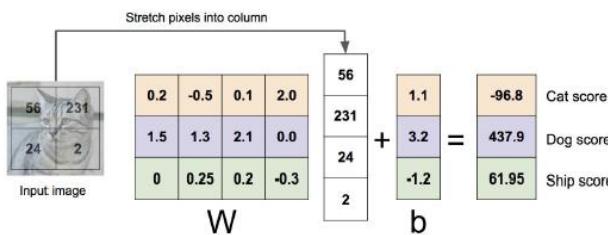
Everything else



Linear Classifier: Three Viewpoints

Algebraic Viewpoint

$$f(x, W) = Wx$$



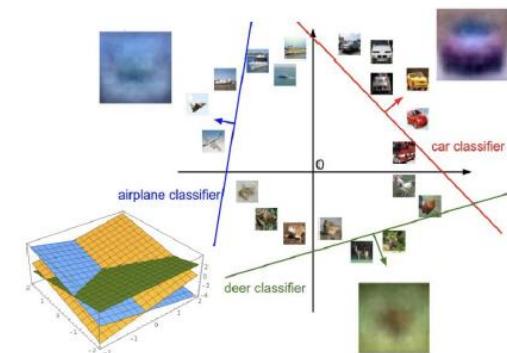
Visual Viewpoint

One template per class



Geometric Viewpoint

Hyperplanes cutting up space



So far: Defined a (linear) score function $f(x, W) = Wx + b$

Given a W , we can compute class scores for an image x .

How can we tell whether this W is good or bad?



airplane	-3.45	-0.51	3.42
automobile	-8.87	6.04	4.64
bird	0.09	5.31	2.65
cat	2.9	-4.22	5.1
deer	4.48	-4.19	2.64
dog	8.02	3.58	5.55
frog	3.78	4.49	-4.34
horse	1.06	-4.37	-1.5
ship	-0.36	-2.09	-4.79
truck	-0.72	-2.93	6.14

Cat image by Nikita is licensed under CC-BY 2.0

Car image is CC0 1.0 public domain

Frog image is in the public domain

$$f(x, W) = Wx + b$$

Coming up:

- Loss function
- Optimization
- ConvNets!

(quantifying what it means to have a “good” W)

(start with random W and find a W that minimizes the loss)

(tweak the functional form of f)