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Problem Set 9

1. You have received the following two strings that were encoded using the Hadamard (16, 5) code:
2. { -1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1, 1, 1, -1, 1, -1 }
3. { 1, 1, -1, 1, -1, -1, -1, -1, -1, -1, 1, 1, 1, -1, 1, 1 }

Under the assumption that at most four errors occurred in the transmission of

each of these strings, do one of the following for each of (a) and (b):

1. Show that the string was transmitted without error, and determine

the integer it encoded;

1. Show that the string was transmitted with 1,2 or 3 errors, find the

corrected string, and determine the integer it encoded; or

1. Show that the string was transmitted with 4 errors, and so cannot

be unambiguously decoded.

1. We can find the decoded string by multiplying the encoded string vector m by the 16 x 16 generator matrix P. Then we can check the resulting decoded vector to check and correct errors:

m = { -1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1, 1, 1, -1, 1, -1 }

P =

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 |
| 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 |
| 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 |
| 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 |
| 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 |
| 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 |
| 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 |
| 1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 |
| 1 | 1 | -1 | -1 | 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 |
| 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 |
| 1 | 1 | 1 | 1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | -1 | 1 | 1 | 1 | 1 |
| 1 | -1 | 1 | -1 | -1 | 1 | -1 | 1 | -1 | 1 | -1 | 1 | 1 | -1 | 1 | -1 |
| 1 | 1 | -1 | -1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | -1 | -1 |
| 1 | -1 | -1 | 1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | -1 | 1 | -1 | -1 | 1 |

So we have that m x P = {0, 0, 0, 0, 4, -12, 4, 4, -4, -4, -4, -4, 0, 0, 0, 0}.

Since all values in the resulting decoded vector are from the range -4 to 4 except for the 6th column, this means that there are two errors in the encoded string. Since the outlier is negative, we can find the corrected string by negating the 6th row of the generator matrix:

Corrected string: {-1, 1, -1, 1, 1, -1, 1, -1, -1, 1, -1, 1, 1, -1, 1, -1}

Since the 6th value of the decoded vector was negative, this means that the value that was encoded was not 6, but instead 15 + 6 = 21.

1. For m = {1, 1, -1, 1, -1, -1, -1, -1, -1, -1, 1, 1, 1, -1, 1, 1}, we can multiply m x P again to get the decoded string:

m x P = {0, 0, -4, 4, 4, -4, 0, 0, -4, -4, 8, 0, 8, 0, 4, 4}

However, the decoded vector has all values in the range-8 to 8, and there are no outliers. This means that the encoded string must contain more than 3 errors, and cannot be unambiguously decoded.

1. This question concerns the Hamming (15,11) code described in class. (Recall that the first, second, fourth and eighth bits of the string x1 x2 x3 … are the parity bits, and that the parity check matrix is laid out in a manner similar to the parity check matrix of the Hamming (7,4) code.) You have received the string x = 100101011100000 which is the encoding of a message m using the Hamming (15, 11) code. Decode the string x, determine which – if any – of the bits is in error, and exhibit the message m encoded by x.

The parity bits for the Hamming (15, 11) code are bits 1, 2, 4, and 8. These parity bits must satisfy the equations:

x8 + x9 + x10 + x11 + x12 + x13 + x14 + x15 = 0 mod 2

x4 + x5 + x6 + x7 + x12 + x13 + x14 + x15 = 0 mod 2

x2 + x3 + x6 + x7 + x10 + x11 + x14 + x15 = 0 mod 2

x1 + x3 + x5 + x7 + x9 + x11 + x13 + x15 = 0 mod 2

When we check the equations above with the given bit string, we get:

1 + 1 + 1 + 0 + 0 + 0 + 0 + 0 = **1** mod 2

1 + 0 + 1 + 0 + 0 + 0 + 0 + 0 = **0** mod 1

0 + 0 + 1 + 0 + 1 + 0 + 0 + 0 = **0** mod 2

1 + 0 + 0 + 0 + 1 + 0 + 0 + 0 = **0** mod 2

According to the Hamming code, this means that bit 1000 in binary, or bit 8 in decimal is incorrect. We can flip this bit and rewrite the encoded message:

1 0 0 1 0 1 0 1 1 0 0 0 0 0

This means one of the check bits was incorrect. The original message is contained in bit positions 3, 5, 6, 7, 9, 10, 11, 12, 13, 14, 15, so the original message is:

0 0 1 0 1 1 0 0 0 0 0