CVE 211

STRESS AND STRAIN

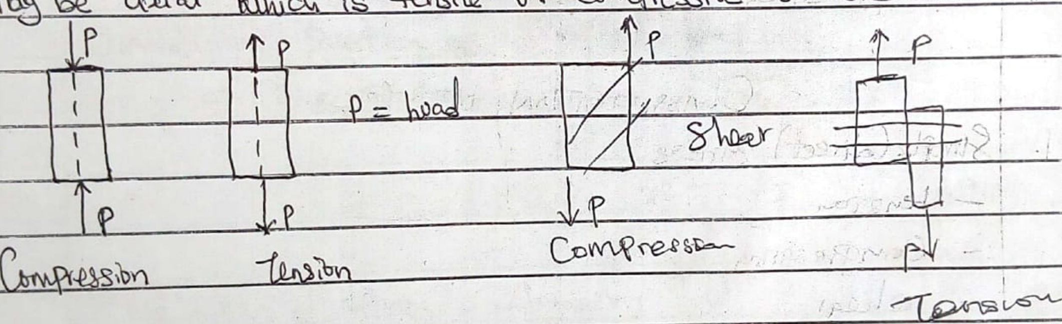
MATERIALS

Strength of materials also called Mechanics of material is a subject which deals with the behaviour of solid obj. subject to stresses and strains.

In Material science, the strength of a material is its ability to withstand an applied load without failure.

A load applied to a mechanical member will induce internal Fs within the member called stresses. When those Fs are expressed on a unit basis.

Deformation of a material is called strain when those deformation too are placed on a unit basis. The applied load may be axial which is tensile or compressive or shears.



The stresses and strains that developed within a mechanical member must be calculated in order to access the load capacity of that member.

Consideration for accessing the Load Capacity

This requires a complete description of the geometry of the member. It constrains, the loads applied to the member and the properties of the material of which the member is composed.

Once the state of stress and strain within the member is known, the strength (load carrying capacity) of that member is deformation (stiffness qualities and its stability (the ability to maintain its original configuration) can be calculated.

STRESS

When f is transmitted via a solid body, the body tends to undergo a Δ in shape. This tendency to deform is resisted by the unit cohesion of the body and the body is said be in a state of stress

This a stress may be described as a mobilized internal force which resist any tendency towards deformation.

By definition stress is F per unit Area

By formula stress = $\frac{\text{Force}}{\text{Area}}$ By symbol $\sigma = \frac{F}{A}$

TYPES OF STRESSES

1. Tensile stress
2. Compressive stress
3. Shearing stress

CLASSIFICATION OF STRESS

1. Simple (direct) stress
   - tension
   - Compression
   - shear
2. Indirect stress
   - Bending
   - Torsion

3. Combined stress — Any possible combination of eg 1 and 2 above.

UNIT OF STRESS

The concept of stress as F/A becomes more obvious when the unit of stress are considered. The basic unit will be the F unit divided by the area unit. Eg

Newton per sq. millimeter

Mega newton per sq. metre

Kilonewton per sq metre

1 Pa = $1\,N/m^2$

N/mm^2

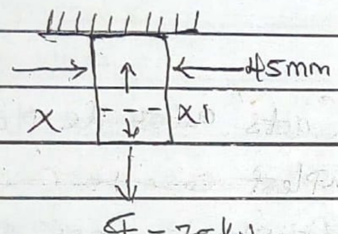
MPa = KN/mm^2

DIRECT AXIAL STRESS

Direct stress are those normal stress in which the F direction lies parallel to longitudinal axis of the member. In the case of an axial stress, the f is axially applied (i.e aligned along the longitudinal axis).

DAS may be tensile or compressive, depending on the sense of the force.

Eg Determine the tensile stress induce in the rod shown below due to an axial load of 75KN.



Considering section xx' ~~of the load~~ rod

Area of cross sectional plane $= \frac{\pi d^2}{4}$

$$= \frac{\pi \cdot 45^2}{4} = 1591 mm^2$$

Thus tensile stress $= \frac{\text{tension load}}{\text{Area of cross section}}$

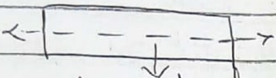
$$\sigma_t = \frac{75 \times 10^3}{1591} = 47.14 N/mm^2$$

C.V.E. 24/05/2017

* Strength of material is the study of the behaviour of structural and machine members, under the action of external loads, taken into account external forces and the resulting deformation.

LOAD: It's any engineering design which is build up of a number of member is in equilibrium under the action of external forces and the reaction at the Point of support.

* External forces which acts on each individual members is know as load.



Axial load. (acts accross the plane.)

Axially Loaded bar: the simplest case to consider is an initially stretch metal bar of constant Cross Section, Loaded at its ends by a Pair of Oppositely directed to Collinear forces. Colliding with the longditudinal axis of the bar & acting through the Centroid of each Cross Section.

For Certain equilibrium, the magnitute of the forces must be equal. ① If the forces are directed away from the bar, the bar is said to be in tension. P ← → P tension

②. If the forces are directed towards the bar, the bar is said to be in Compression. P → ← P Compression

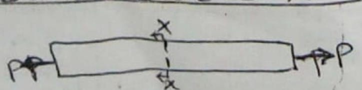
* Centroid is the point which is the Point Centre of a material.

STRESS

Under the action of these Pair of applied forces, internal resisting forceses are setup within the bar and their characteristic maybe studies by imagining a Plane to be cut through the bar any way along the ~~string~~ length of oriented perpendicular to the longitudal axis of the bar. Such a Plane is resigned as x – x

If for purposing of analysis the ~~Purpose~~ Portion of a bar to the right of these section is considered to be removed then the stress.

Across any Section such as x – x in Figure 3, the total



Force Carry must be equal to the load P.

The force transmitted across any section divided by the area of that section is Called the Intensity of Stress. It's usually Called Sigma (σ)

$$\sigma = \frac{Force}{Area} = \frac{N}{mm^2} \quad \therefore \sigma = N/mm^2.$$

TYPES OF STRESSES

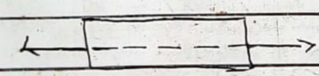
We have only two mainly stresses that exist namely;

1) Normal stresses
2) shearing stresses

* Normal stresses are stresses which acts normal (Perpendicular) to the stress surfaces under consideration.
* Shearing stresses are stresses which acts Parallel to the Stress surface

Normal stress may be either tensile or Compressive. Other stresses either as similar to those two stresses or Combination of the two basic stresses.

Consider a straight bar of Uniform Cross Section subjected to a pair of Collinear forces acting in opposite direction and Coinsiding with the axis of the bar



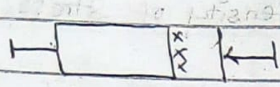
If the forces are directed away from the bar, then the bar tends to Increase in length under the action of applied forces and the stress developed in the bar are tensile forces.

Tensile stress are general denoted by the symbol sigma t [σ_t]

COMPRESSIVE STRESS

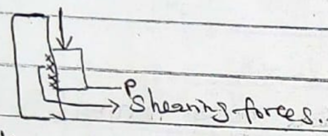
When a Pair of axial forces push on a member and shorten or Compressing, they are Called Compressive forces and they Could used axial Compressive stresses Internally on a Plane Perpendicular or normal

As in the case of tension, the stress area is normal or perpendicular to the direction of stress



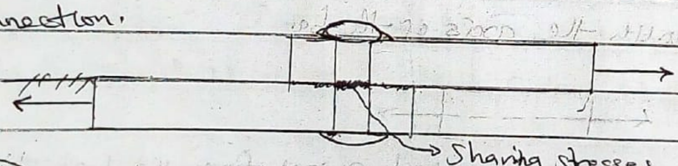
SHEARING STRESS.

These type of stress is differ from tensile & Compressive Stresses in that the stress length lies parallel to the direction of stress rather than perpendicular to it.

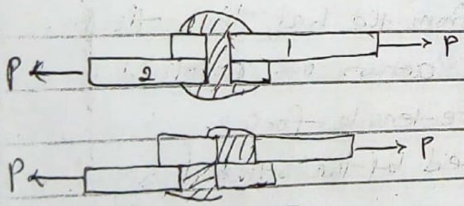


An illustration is shown in figure 1.5 (above) the vertical grained ~~bl~~ block of wood A is clamped unto a desting machine, A vertical force P applied to the upper surface of the projecting portion of the block will tend to sheer it off the main portion of the block along the sheer length. Such action is resisted by the shearing strength of the wood acting. The external forces developed in the shared length are all shearing stresses.

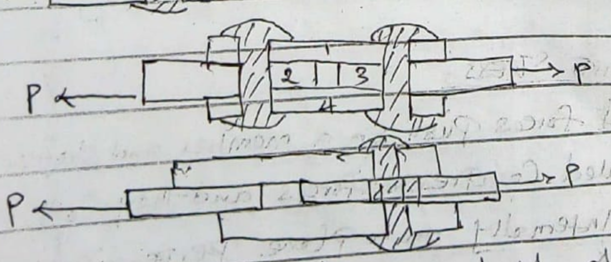
Two examples of shearing failure will be shown in the riveted connection.



fig[1.5]



single shear failure

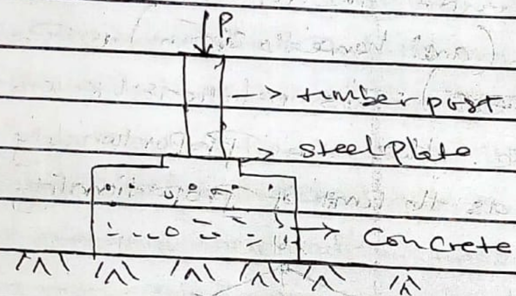


failure in double shear.

BEARING STRESS.

Compressing stress exerted on an external surface of a body as for example when one object pressing against another. It is reffered to as bearing stress.

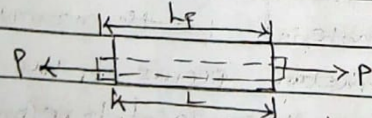
E.g let us assume that we have a timber post resting on a steel Pased plate supported on a concrete footing which inturn is ultimately supported by the soil the post base on the soil. The resulting compressive stresses at the external surfaces of contact between these bodies are bearing stresses. In their example just mentiones the stressed areas are plane surfaces all lying normal to the direction of stress.



STRAIN

Direct stresses produce a change in length in the direction of the strain. If a rod of length 'L' is on tension, and the elongation produced is 'D' the direct strain 'ε' is defined as;
Change in length per unit length.

i.e Strain $\varepsilon = \frac{\text{Elongation}}{\text{original length}}$



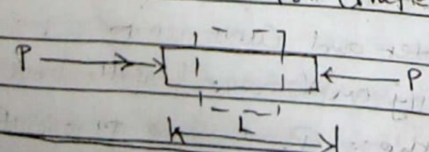
Where Lf is the final length; Elongation (D)

$L_f - L$

Strain $= \frac{D}{L}$

Tensile stresses increases the length whereas compressive stresses decreases the length. ∴ tensile strain caused by tensile stresses are normally considered positive while compressive strain caused by compressive stresses are negative.

for compression:



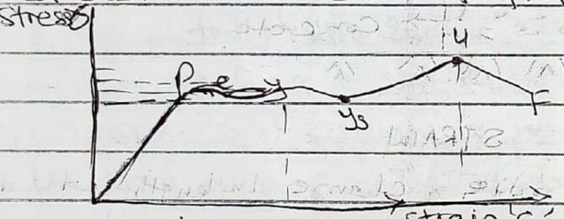
Strain is a ratio of length, hence it has no unit (it is dimensionless)

Strain $= \frac{\text{Increase in length (mm)}}{\text{Original length (mm)}}$

the test and remarks below.

Other engineering material show the same Phenomenon to a varing degree. The test is usually Carry out on a bar of uniform cross section, usually circular in a testing machine which indicates the tensile load been applied. For the very small strain involves in the early Part of the test, the longation is recorded by an extensome-ter or a strain gauage and later when the strain is the elongation

Consider a steel wire hee rigidly at the upper end and Carry a load at the lower end. The load is increase gradually, And at first, the elongation (and hence the strain) is Proportional to the load. (and hence the stress) This relationship was discovered in 1678 by Robert Hooke. this Proportionality hold up to the value of stress know as the limit of Proportionality (Point B)



Stress-Strain diagram.

Material that show these character are said to obey hooke's law. Beyound the limit of proportionality, hooke's law S. to obey, although the material may be in elastic state, in the sense that if the load will remove the stress will also return to zero (0). Point e show the elastic limit of the material. i.e the maximum stress up to which no Permanent deformation occurs on the removal of the load. these Property of the material to recover its original position on removal of applied load is known as Ela-sticity.

N/B: That the stress strain Curve is linear, up to Point P, there after it becomes a little flatter and curve up to Point E. from O - E, the strain is applied fully on removal of the load. If the material is strain beyound these Point, Some plastic deform-ation will occur, i.e strain which is not recoverable if they load is remove, The Strain that does not disappear on removal of external load is known as Permanent ~~set~~. the next Important occurance is the yield Point (Y) at which the metal shows an appreciable strain, even without other Increase ~~including~~ in loading.

The stress Corresponding to the yield Point 'Y' is called the yield Point stress. At the yield stress, the material begin to flow. With mild steel Carefull testing has revealed a drop in the load immediately yielding commences so that there are two values the upper & the lower yield Point (Y & Ys). After yielding have occured, farther Straining Can only be taken by increasing the load, the stress-strain Curve continues to rise up to a Point U - the strain in the region Y to U is 100 times that of O - Y and it's Partly elastic but mainly Plastic, At Point U, the stress is Maximum and it's know as Ultimate or maximum tensile stress. beyound U, the steel bar elongest even with decreased to stress and finally fail at Point F. The ratio of the maximum load that the Specimen is Capable of sustaining to its original area of Cross Section is termed the maximum tensile stress of the material and its found by dividing the load at Point U, by the original Cross-sectional area of the material. At Point U, the ~~Statement~~ Specimen is breakly reduces in cross-sectionally area - ~~At~~ Point F, the Point of failure the reduces area is ~~the~~ released and this known as "necking". In elastic design the material strain or stress used will only be in the range O - P.

DEFINITION OF TERMS

i) Percentage Elongation: It's the total increase in guage length at fraction express as the Percentage of the original length.

ii) Contraction or Percentage reduction in area: It's the reduction in cross-sectional area at neckings, express as the percentage of the original area - this gives a greater measure of of the ductility of the ~~testie~~ tested material.

*Hooke's law state that for material loaded within its elastic limit strain is proportional to the stress producing it.

Materials, Mathematically $\frac{\text{Stress}}{\text{Strain}} = \text{Constant}$

These law is obeyed within certain limit of stress for most ferious alloy/Linear and can usually be assumed to apply within sufficient accuratecy to other engineering material such as timber, Concrete & non ferrous alloy.

MODULUS OF ELASTICITY (YOUNG'S MODULUS)

Within the limit of which hooke's law is ~~obeyed the~~ obeyed, the ratio of the direct stress to strain produced is called young's modulus or modulus of elasticity (E).

$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon}$. The unit of E is N/m^2 or kN/mm^2

Where $\sigma = \frac{F}{A}$

$\varepsilon = \frac{\Delta}{L}$

$E = \frac{F}{A} \times \frac{L}{\Delta} = \frac{FL}{A\Delta}$ $\qquad E = \frac{F}{A} \Big/ \frac{\Delta}{L}$

For a bar of uniform cross section of area A and length L.

* Where $\Delta \rightarrow$ Increase in length (Elongation)

$A \rightarrow$ (Cross-sectional area)

$L \rightarrow$ original length

P or $F \rightarrow$ Force

E is therefore a constant for any given material and is usually assumed to be the same for tension or compression since the unit of E is N/mm^2 or multiples of it.

CHANGE OF LENGTH

Consider a bar of length L of uniform cross-section area A subject to a pull of load P, let L = original length of the bar, Δ = change length caused by the applied load, A = Uniform cross sectional area

from Hooke

$E = \frac{\sigma}{\varepsilon}$

STATIC

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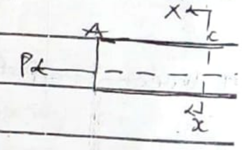

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$$E = \frac{\gamma}{\varepsilon}$$

STATIC EQUILIBRIUM: A structural system is said to be in a state of static equilibrium when the resultant of all forces and the resultant of all moments is equal to zero.

$$\sum_{i=1}^{n} f_i x = 0 \; ; \; \sum_{i=1}^{n} f_i y = 0.$$

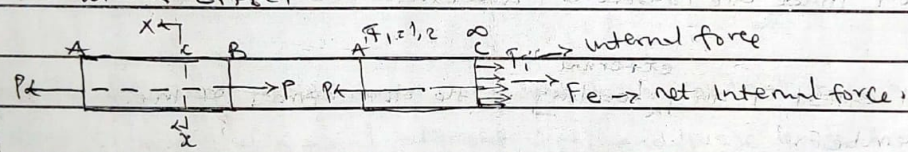
moment $\rightarrow M_e = \sum_{i=1}^{n} f_i q_i = 0$

where q_i is the perpendicular distance to the line of action of f_i from point O.

NB: moment M of all forces are taken about an axis through any point of convenience.

* Principle of Part (Cardinal) Equilibrium state that if a structural system is in equilibrium, any part of it must also be in equilibrium.

INTERNAL EFFECT



$$\sum_{i=1}^{\infty} f_i = f_e = P \quad - - - \; * $$

The internal moment, M_i, along the axis of the bar is zero since the external load P is along the axis of the bar.

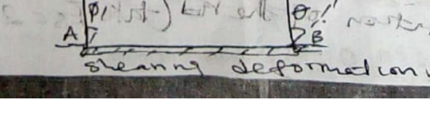
Equation * state that for any equilibrium to be maintained the resultant equal and opposite to the external load P. Instability leading to any fracture can occur when the body is not able to develop internal forces necessary to maintain equilibrium.

$$\sum_{i=1}^{\infty} f_i < P$$

When external forces are applied to a body, two internal effect are produced namely:

Internal forces which are developed to balanced the external loading.

& deformation within the body which manifest themselves as a change of shape and size.



shearing deformation.

Types of Load

1). Static Load :- they are loads which are applied at a very low rate and once applied, do not change significantly with time. e.g. the weight of the Concret deck, fillings, etc of a bridge which are carried by the supporting Pillars, structural engineers usually refer to such load as "dead" load to accentuate the fact that it does not change with time.

2). Dynamic Load have a finite rate of application and most importantly vary with time. e.g moving vechicle on a bridge, wind load (with very high speed) the effect of earthquake on structures, pile drivers, dynamic loads can be referred to as "live" Load. it Can Cause greater internal effect than static loads.

OTHER LOAD include;

Thermal Loads arises from temperature change and actually do Cause severe effect and must therefore not be over looked in engineering design.

* Inertia load are Caused by acceleration or retardation of a body.

* the net effect of any external loading is the development of internal effect in the member.

STRESS : Stress at a Point is defined as the intensity of internal force Per unit area.

$$\sigma = \frac{df}{dA}, \quad df = \sigma\, dA$$

$$P = \int_A df = \int_A \sigma\, dA$$

$\sigma = P/A$, where A is the cross sectional area at the Cut Plane.

PROBLEMS

1). A steel rod of 20mm diameter and 500cm is subjected to axial load of 3000kg.

i). Determine the intensity of stress in newton per millimeter square (N/mm^2).

ii). determine the strain Produced.

iii). Determine the elongation of the rod (take $E_s = 2.1 \times 10^6\, kg/cm^2$, $g = 10m/s^2$).

3000kg

Diameter =

length = 50

Pull, P = 3000kg

$E_s = 2.1 \times 10$

Area of steel

i). $P = \sigma A$

$\sigma = P/A$

ii) $\varepsilon = \frac{\sigma}{E}$

iii) $\varepsilon = \frac{D}{L}$

2). The follow

steel specime

unit of propo

was 0.048mm

ultimate maxi

fitted together

found to be

i) Young's m

ii) the stress

iii) the yield

iv) the strain

v) the ultimate

vi) the pore

vii) the pe

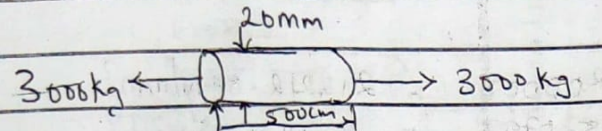
P = 80,000

d = mm

L =

N/B. ε = elongation.
P = Intensi Pull / forces acting on limit
γ = Intensity of stress / stress limit
A = Area of steel
E = Constant $2.1\times10^6 kg/cm^2$.

Sol.



Diameter = 20 mm

length = 500cm = 500×10 = 5000mm

Pull, P = 3000 kg = 3000×10 = 30,000N

$E = 2.1\times10^6 kg/cm^2 = 2.1\times10^6\times10N/100 = 2.1\times10^5 N/mm^2$

Area of steel $= \frac{\pi d^2}{4} = \frac{\pi\times20^2}{4} = \frac{3.142\times400}{4} = \frac{1256.8}{4} = 314.2\,mm^2$

i). $P = \gamma A$

$\gamma = P/A = \frac{30,000N}{314.2mm^2} = 95.48 N/mm^2$

ii) $\varepsilon = \frac{\gamma}{E} = \frac{95.48}{2.1\times10^5} = 4.546\times10^{-4}$

iii) $\varepsilon = \frac{\Delta}{L}$, $\Delta = \varepsilon L = 4.546\times10^{-4}\times5000mm$

$\Delta = 2.273mm$.

2). The following result were obtained in a tensile test on a mold steel specimen of original diameter 2 cm and guage length 4cm, at the unit of proportionality, the load was 80,000N and the extension was 0.048mm. The specimen yielded at a load of 85,000N and the ultimate maximum load with stud was 150,000N. when the two points were fixed together after been broken. the length between guage points was found to be 5.56cm and the diameter at the neck was 1.58cm, Calculate

i) Young's modulus
ii) the stress at the limit of proportionality
iii) the yield stress
iv) the strain at the limit of proportionality
v) the ultimate tensile stress
vi) the percentage elongation
vii) the Percentage Contraction

Solu.

P = 80,000N $\quad A = \frac{\pi d^2}{4} = \frac{3.142\times400}{4} = 314.15mm^2 = 314.2mm^2$

d = 20mm

L = 4cm = 40mm $\quad$ elongation $\Delta = 0.048mm$

i) Young's modulus, $E = \frac{PL}{\Delta A}$

$E = \frac{80{,}000 \times 40}{0.048 \times 314.15} = \frac{3200000}{15.0792} = 212212.84 N/mm^2$

$E = 2.12 \times 10^5 N/mm^2$

ii) Stress limit of Proportionality.

$\gamma = \frac{P}{A} = \frac{80{,}000}{314.2} = 254.61 N/mm^2$

$\gamma = 254.61 N/mm^2$

iii) Yield stress, $\gamma' = \frac{P_1}{A} = \frac{85{,}000}{314.2} = 270.52 N/mm^2$

iv). The strain at limit of Proportionality (L_p)

$\mathcal{E} = \frac{\Delta}{L} = \frac{0.048}{40} = 0.0012$

v). Ultimate tensile stress $\sigma_u = \frac{P_u}{A} = \frac{150{,}000 N}{314.2} = 477.40 N/mm^2$

vii) Percentage Contraction = $\frac{\text{decrease in area}}{\text{original area}} \times 100$

Original area = 314.2

Area of necking $= \frac{\pi d^2}{4} = \frac{\pi \times (15.8)^2}{4} = 196.09 mm^2$

d at necking = 1.58cm = 15.8mm

% Contraction $= \frac{314.2 - 196.09}{314.2} \times 100 = 37.58\% \simeq 38\%$

vi) Percentage elongation = $\frac{\text{Increase in length}}{\text{original length}} \times 100$

$= \frac{5.56 - 4}{4} \times 100 = \frac{1.56}{4} \times 100 = 39\%$

OR

Using diameter, $\frac{(\text{decrease in diameter})^2}{\text{original diameter}} \times 100 = \frac{2^2 - (1.58)^2}{2^2} \times 100$

% Contraction $= \frac{4 - 2.4964}{4} \times 100 = \frac{1.5036}{4} \times 100 = 37.5$

% Contraction = 38%

3). A 20cm long steel tube 150mm internal diameter and 1cm thick is surrounded closely by a brass tube of same length and thickness. The tubes carries an axial load of 15 tones. Estimate the load carried by each member. (take E for steel, $E_s = 2.1 \times 10^5 N/mm^2$, E for brass $E_b = 1 \times 10^5 N/mm^2$, $g = 10m/s^2$).

Outer diamete
Inner ✓
Outer diameter
Inner ✓
Load P = 15
Area of stee
Area of bras

$\Delta = \frac{PL}{AE}$

but the elongati
same. Δb
hence, $\frac{P_s L}{A_s E_s}$

$\frac{A_b E_b}{A_s E_s} = \frac{P}{P}$

$P_b = 0.535$
$P_b + P_s = 15$
$P_b = 150{,}000$
$150{,}000 - P$
$150{,}000 = 0$
$150{,}000 = 1$
$P_s = \frac{150,}{1.5}$
$P_b = 150{,}00$

4). A steel
forces sh
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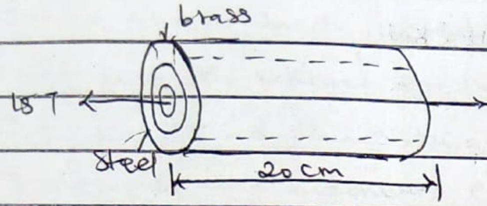
Sln.



Thickness of both side.

Outer diameter of steel = 150 mm + (10 + 10) mm = 170 mm

Inner " " " = 150 mm — same thickness as steel

Outer diameter of brass = 170 + (10 + 10) mm = 190 mm

Inner " " " = 170

Load $P = 15T = 15000\ kg \times 10 = 150,000\ N$

Area of steel $= \frac{\pi d^2}{4} - \frac{\pi d^2}{4} = \frac{\pi \times 170^2}{4} - \frac{\pi \times 150^2}{4} = 22,700.95 - 17,673.75$

$= 5,027.2\ mm^2$

Area of brass $= \frac{\pi d^2}{4}(\text{brass}) - \frac{\pi d^2}{4}(\text{inner brass})$

$= \frac{\pi \times 190^2}{4} - \frac{\pi \times 170^2}{4} = 28,356.55 - 22,700.95$

$= 5655.6\ mm^2$

$\Delta = \frac{PL}{AE}$

But the elongation in the brass will be the same as that of steel because the length are the same. $\Delta_b = \Delta$

hence, $\frac{P_s L}{A_s E_s} = \frac{P_b L}{A_b E_b}$, (The length are the same on both material.)

$\frac{A_b E_b}{A_s E_s} = \frac{P_b}{P_s} = \frac{5655.6 \times 1 \times 10^5}{5027.2 \times 2.1 \times 10^5} = 0.5357$

$P_b = 0.5357 P_s$ —— (1)

$P_b + P_s = 150,000$ —— (2) from eqn (2)

$P_b = 150,000 - P_s$ —— (3)

$150,000 - P_s = 0.5357 P_s$

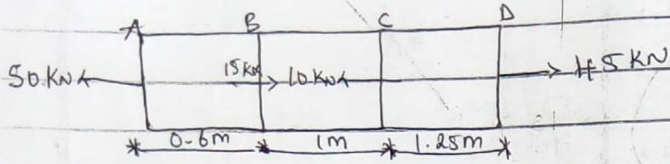
$150,000 = 0.5357 P_s + P_s$

$150,000 = 1.5357 P_s$

$P_s = \frac{150,000}{1.5357} = 97,675.3N$ from eqn (3)

$P_b = 150,000 - P_s = 150,000 - 97,675.3N = 52,324.67N$

4). A steel bar of cross-sectional area $500\ mm^2$ is acted upon by the forces shown in the figure below, determine the total elongation of the bar taking $E = 2.0 \times 10^5\ N/mm^2$.



Soln

for the system to be in equilibrium, $\Sigma F_{Left} = \Sigma F_{Right}$

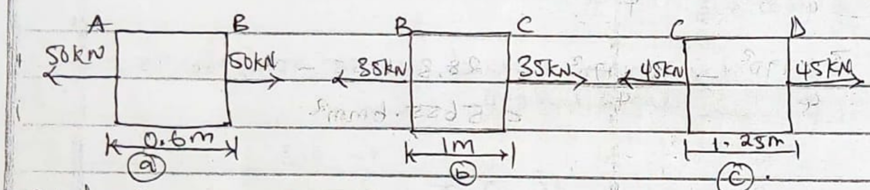
$50+10 = 15+45$

$60\,kN = 60\,kN$

Equilibrium Satisfied.

Applying the principles of cardinal equilibrium, which state that if a structural system is in equilibrium, any of its part (or part of it). Must also be in equilibrium.

Using free body diagram.



Total elongation $\Delta_{total} = \Delta_a + \Delta_b + \Delta_c$

$$\Delta = \frac{PL}{AE}$$

for "a" $\Delta_a = \frac{P_a L_a}{AE} = \frac{50\times10^3\times0.6\times10^3}{2.1\times10^5\times500\,mm^2}$. $\Delta_a = 0.286\,mm$.

for "b" $\Delta_b = \frac{P_b L_b}{AE} = \frac{35\times1\times10^6}{2.1\times10^5\times500\,mm^2}$. $\Delta_b = 0.333\,mm$.

for "c" $\Delta_c = \frac{P_c L_c}{AE} = \frac{45\times1.25\times10^6}{2.1\times10^5\times500}$, $\Delta_c = 0.5357\,mm \simeq 0.536\,mm$

$\Delta_{total} = (0.286+0.333+0.536)\,mm = 1.155\,mm$

PRINCIPLE OF SUPER POSITION

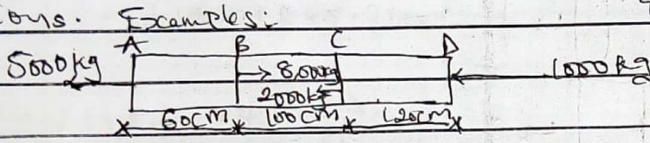
For an elastic body, acted upon by several forces it is possible to determine the internal stresses and strain due to each force independently and then obtained the resultant effect by algebraic sumation of the individual effect

* FREE BODY DIAGRAM

Deformation of individual section can be easily be verify with the help of free body diagram, In case of an elastic body subjected to a number of direct forces at different section along the length of the body. total deformation of the body will be equal to the algebraic sum of the deformation of the individual sections. Examples:

$E_b = 1.05 \times 10^6$ kg/cm^2

$A = 10cm^2$.



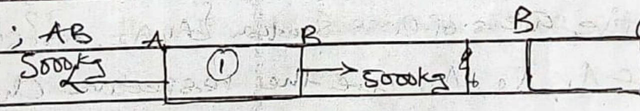
The bar AD showed is made up of brass with a cross-sectional area of $a = 10cm^2$, find the total change in length of the bar, take E for brass $(E_b) = 1.05 \times 10^6$ kg/cm^2

soln.

$\Sigma f_L = \Sigma f_R$

8000kg = 8000kg ∴ Bar is in equilibrium (L)

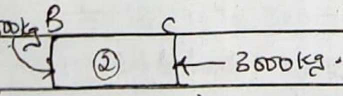
Free body diagram; AB



For Part AB it is subjected to tensile force of 5000kg at A the net force is 5000kg tensile. All forces to the right of B must balanced this 5000kg.

i.e 8000 − 2000 − 1000 = 5000kg

Similarly for section BC, All forces to the left of normal section Brace



(5000kg − 8000kg) = −3000kg C

the section BC results subjected to a compressive force of 3000kg.

C D

1000kg 3 1000kg 8000 − 5000 − 2000 = (1000) 1000kg 3 1000kg

$\Delta_1 = \frac{PL}{AE} = \frac{5000kg \times 60cm}{10 \times 1.05 \times 10^6 kg/cm^2} = \frac{300000}{10,500} = 0.2857 = 0.29cm$

$\Delta_2 = \frac{P_2 L}{AE} = \frac{3000 \times 100}{10 \times 1.05 \times 10^6} = \frac{300000}{10} = 0.2857$ $= -0.2857$ (ie Compression)

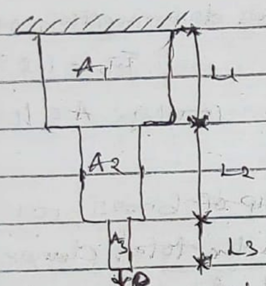
$\Delta L_3 = \frac{P_3 L}{EA} = \frac{1000 \times 120}{1.05 \times 1.0^6 \times 10\,cm^2} = \frac{120,000}{10,500} = 0.11428$ $= -0.114$ (Compressive)

Total deformation $= \Delta_1 + \Delta_2 + \Delta_3$

$= +0.0285 - 0.0285 - 0.0114\,cm = -0.0114\,cm$

$= -0.114\,mm.$

Elongation of bars of varying Cross-Section.



If a bar is made up of a number of Portion of different Cross-Sect-ion, then the total elongation of a bar is the sum of the elonga-tions of each Portion Constituting the bar length. Consider a bar Composed of three section of lengths $\{L_1, L_2, L_3\}$ in the diagram above have respective areas of Cross Section $\{A_1, A_2, A_3\}$ subjected to an axial Pull (P), If $\Delta_1, \Delta_2, \Delta_3$ are the respective Changes in length of the three sections then we have that $\Delta_1 = \frac{PL_1}{A_1 E}$, $\Delta_2 = \frac{PL_2}{A_2 E}$, $\Delta_3 = \frac{PL_3}{A_3 E}$; Where E = (modulus of the elasticity of the material) of which the bar is made). Now the change in Δ of the entire bar is

$\Delta = \Delta_1 + \Delta_2 + \Delta_3.$

$= \frac{PL_1}{A_1 E} + \frac{PL_2}{A_2 E} + \frac{PL_3}{A_3 E}$

$\frac{P}{E} = \left[\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right].$

Eg 5 the bar ABCD shown below is subjected to a tensile load of $18 \times 10^4 N$. If stress of the material is limited to $140 N/mm^2$ find the diameter of the Portion BC and it's length, if the total elongation of the bar is to be 0.024 cm take $E = 2 \times 10^5 N/mm^2$.

$18 \times 10^4 N$

Compr

$P = 18 \times 10^4 N$

$\sigma = 140 N/mm^2$

Required Cross

$A = \frac{18 \times 10^4}{140}$

$A = \frac{\pi d^2}{4} =$

$\pi d^2 = 1285$

$d^2 = \frac{1285}{3.}$

$d = \sqrt{1636.79}$

Cross-Section

If the length of

$400 - L_{BC}$

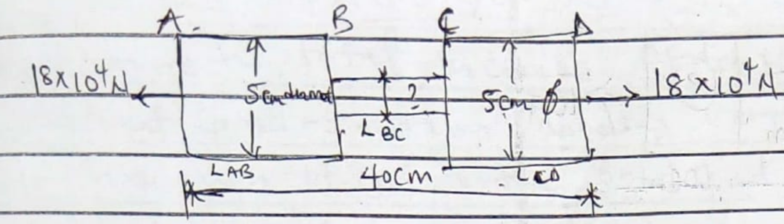
Elongation of

Elongation of Pa

Total elongatio

0.24 mm

What



Soln.

$P = 18\times10^4$N

$\sigma = 140$N/mm^2

Required Cross Section area of BC = $\sigma = \frac{P}{A}$ $\therefore A = \frac{P}{\sigma}$

$A = \frac{18\times10^4}{140} = 1285.71$mm^2.

$A = \frac{\pi d^2}{4} = 1285.71$mm^2

$\pi d^2 = 1285.71\times4$ mm^2

$d^2 = \frac{1285.7\times4}{3.142} = 1636.79$mm^2

$d = \sqrt{1636.79\text{mm}^2} = 40.46$mm.

Cross-Sectional area of AB or CD = $\frac{\pi\times50^2}{4} = \frac{3.142\times2500}{4} = \frac{7855}{4}$

$= 1963.75$mm^2

If the length of BC is L_{BC} then the Combined length of AB and CD is $400 - L_{BC}$

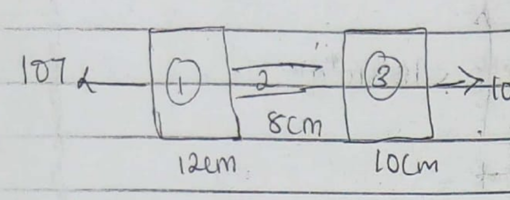
Elongation of Part AB+CD = $\frac{PL}{AE} = \frac{18\times10^4[400-L_{BC}]}{1963.75\times2\times10^5}$

Elongation of Part BC = $\frac{PL_{BC}}{AE} = \frac{18\times10^4\times L_{BC}}{AE} = \frac{18\times10^4\times L_{BC}}{1285.71\times2\times10^5}$

Total elongation = Elongation of AB + elongation CD + elongation of BC.

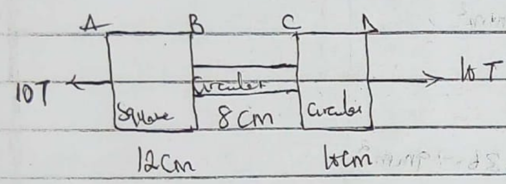
$0.24\text{mm} = \frac{18\times10^4\times(400-L_{BC})}{1963.5\times2\times10^5} + \frac{18\times10^4\times L_{BC}}{1285.71\times2\times10^5}$

Assignment (Tuesday).



A bar 30cm long is $5cm^2$ in section for 12cm of its lengths, 2.5cm diameter for 8cm, & 4cm diameter for the remaining length, If tensile force of 10T is applied to the bar, Calculate the max. & min. stresses produced in it and the total elongation of the bar. Assume uniform distribution of load over the cross section. take $E = 2 \times 10^3$ tonnes $/cm^2 = 2 \times 10^3 t/cm^2$.

Solution.



Given that:

Total length $= 30cm = 30 \times 10 = 300mm$

$E = 2 \times 10^3 T/cm^2 = \frac{2 \times 10^3 \times 1000kg \times 10ms^{-1}}{100mm^2} = 2 \times 10^5 N/mm^2$.

$P = 10T = 10 \times 1000kg \times 10ms^{-1} = 100,000N$.

At 8cm {80mm length} diameter $= 2.5cm = 25mm$

At 12cm, Area $= 5cm^2 = 500mm^2$, At 10cm, diameter $= 4cm = 40mm$.

Max. stress [occurs at the point of smallest cross section area when the load is uniformly distributed]

$\therefore$ Max. stress $= \frac{Load}{Area} = \frac{100,000N}{\frac{\pi}{4}(25)^2 mm^2} = \frac{100,000N}{490.87mm^2} = 203.7210 N/mm^2$.

$\therefore$ Max stress $\sigma = 203.72 N/mm^2$.

ii) min. stress $= \frac{Load/Force}{\text{Area of larger Cross-section}} = \frac{100,000N}{\frac{\pi (d^2)}{4}} = \frac{100,000N}{\pi (40)^2/4} = \frac{100,000N}{\pi \times 400mm^2} = 79.58N/mm^2$

$\sigma = 79.58N/mm^2$.

Total elongation of the bar $= \Delta_1 + \Delta_2 + \Delta_3 = \frac{PL_1}{A_1E} + \frac{PL_2}{A_2E} + \frac{PL_3}{A_3E} = \frac{P}{E}\left(\frac{L_1}{A_1} + \frac{L_2}{A_2} + \frac{L_3}{A_3}\right)$

$\frac{100,000N}{2 \times 10^5 N/mm^2}\left(\frac{120}{500} + \frac{80}{491.07mm^2} + \frac{100}{1257.14mm^2}\right) = \boxed{0.48244}$

$\Delta Total = 0.5(0.48244) = 0.2412282$

$= 0.24mm$

$= 0.024cm$

Elong

Determ

Constant

to its

figure below

the normal

is cause by the

elongation of

$dA = \frac{P}{A}$

density $= \frac{W}{V}$

To have the

the limit 0 & L

$\Delta = \int_0^L \frac{A \cdot \gamma}{A \cdot E}$

$\Delta = \int_0^L \frac{A \cdot \gamma}{A}$

$= \frac{A \cdot \gamma \cdot y^2}{A \cdot E \cdot 2}$

$= \frac{A \cdot \gamma L}{2AE}$

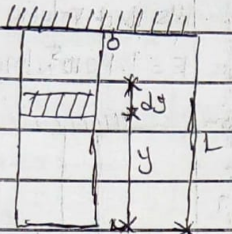
N/B: the Elongation of the bar is eq load of half its

Tuesday

own weight only

Elongation of a bar hanging and subjected to it's 21/11/17

Determine the total increase of length of a bar of Constant Cross-section handing vertically and subject to it's own weight as the only load as shown in the figure below. the bar is initially straight.



Soln.

Let the weight of the material be called W. the normal stress (tensile) over any horizontal cross-section is Cause by the weight (W) of the material below that section; the elongation of the element of fitness dy showed is given by

$d\Delta = \frac{P \cdot L}{A \cdot E} = \frac{A \cdot \gamma \cdot y \, dy}{A \cdot E}$ where A = Cross-sectional area of a bar
γ = it's specific weight (i.e weight/unit volume (density))

density $= \frac{W}{V}$ $\therefore W = d \times v$

To have the total elongation of the bar, we integrate the equation b/w the limit 0 & L

$\Delta = \int_0^L \frac{A \cdot \gamma \cdot y}{A \cdot E} dy$

$= \int_0^L \frac{\gamma}{E} = \int_0 y \, (dy)$

$\Delta = \int_0^L \frac{A \cdot \gamma \cdot y}{A \cdot E} dy$

$= \frac{A \cdot \gamma \cdot y^2}{A \cdot E \cdot 2} = \frac{A \cdot \gamma \cdot L^2}{2A \cdot E}$

$= \frac{(A \cdot \gamma L) L}{2AE} = \frac{WL}{2AE}$ **

N/B/ the Elongation Produced by the weight of the bar is equal to that Produced by a load of half it's weight applied at the end.

Eg(1). In 1989, Jason, a research type summissible with remote tv monitoring capability and weight 35,200 N was lower to a depth of 646 m in an effort to send back to the attending surface vessle photographs of a sunken roman ship off shore from Italy. the summissible was lower at the end of a hollow steel Cable having an area of $452 \times 10^{-6} m^2$ and $e = 200$ GPa. the Central Core of a cable Contain the fibre optic system for transmitted of photographic images to the surface ship. determine the extension of the steel Cable. due to the small volume of the en the system bouyancy may be neglected & the effect of the fibre optic cable on the extension is also ne

D3.65429

(Jason was the system that took the first photography of the sinking titanic in 1986). Take the weight of steel per unit volume as 77kN/m³. ($\gamma = 78$ kN/m³)

Soln

the total cable extension is the sum of the extensions due to (a) the weight of Jason (b) the weight of the steel cable.

$A = 452 \times 10^{-6} m^2$, $L = 646$, $E = 200 \times 10^9$

a) Extension due to the weight of Jason

Extension $\Delta_J = \frac{PL}{A.E}$

$= \frac{35,200N \times 646m}{452 \times 10^{-6} \times 200 \times 10^9} = 0.252m$

B). Extension due to the weight of the cable. Δ_s

$\Delta_s = \frac{WL}{2AE}$

$W = V \times \gamma$ ~~$V = 77 \times 10^3$ volume $\times 646$~~

volume $= 452 \times 10^{-6} \times 646 =$

$W = 452 \times 10^{-6} \times 646 \times 77 \times 10^3$

$= 22,484N$

$\Delta_s = \frac{22484 \times 646m}{2 \times 452 \times 10^{-6} \times 200 \times 10^9}$

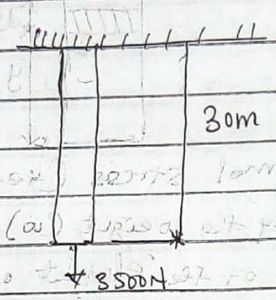
$= \frac{14524664}{2 \times 90400000} = 0.080m$

∴ Total elongation $\Delta = \Delta_J + \Delta_s$
$= 0.252m + 0.80m$
$= 0.332m$

180, 80, 0

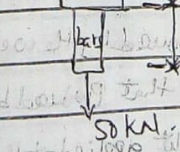
22,739,200 90,40

A figure below show a brass cable of area $400 \times 10^{-6} m^2$ suspending a load of 3500N. Calculate the total extension of the brass cable. If its unit volume $\gamma = 84kN/m^3$ and $E = 1.1 \times 10^5 N/mm^2$.



Assignment

Two prismatic bars are rigidly fastened together and support a vertical load of 50kN as shown. The upper bar is steel having a specific weight of 77kN/m³, length 1.2m and cross sectional area 0.012m². The lower bar is brass having a specific weight of 84kN/m³, length 0.8m and cross-sectional area 0.01m² for steel, $E = 2.1 \times 10^5 N/mm^2$ and brass $E = 1.1 \times 10^5 N/mm^2$. Determine the maximum stress in each material, and the total elongation of the prismatic bars.

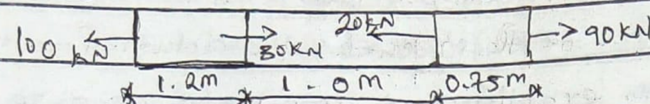


A bar of cross...
Total elongat...
100 kN

3) Using clear...
- Tensile s...
- Compressive...
- Shear

4). A Bar s...
diameter for ...
force of 8 Tone...
Stresses produ...
distribution

A bar of Cross-section $1000 mm^2$ is acted upon by the force shown determine the total elongation of the bar $E = 110 GN/m^2$.



3) Using clear diagram explain the following:
- Tensile stress
- Compressive Stress
- Shear stress.

4). A Bar 50cm long is 6cm square in section for 10cm of its length, 4cm diameter for 12cm and of 5cm diameter for the remaining length. If a tensile force of 8 Tones is applied to the bar ends, Calculate the maximum and minimum stresses produced in it and the total elongation of the bar. Assume uniform distribution of load over the x-section. Take $E = 2.1 \times 10^5 N/mm^2$.

FACTOR OF SAFETY

Stress is usually determined from a knowledge of the magnitude & position of application of the load, the dimensions of the member. and the properties of the material.. In actual practice none of this factors is known exactly and possible errors arise from various sources. moreover, as stress is below the proportionality limit, the material is perfectly elastic & beyond this limit, a part of the strain usually remains after unloading the member. In other to avoid permanent set, in structures, this used to adopt working stress (σ_w) well below the limit of proportionality. the usual practice is to adopt a working stress as factor of the ultimate stress (σ_{ult})

Definitions:

1). Ultimate stress (strength): is defined as the greatest unit stress, a material can withstand without rupture. In the practical design of structures & weariness, one would not of course for the above reasons of safety attempt to use the full strength of the material.

2). Working stress / allowable stress / permissible stress: This is that portion of the ultimate strength which may safely be used in designing or designs.

3). Factor of safety (n): This is defined as the ratio of the ultimate stress to the working stress and it is denoted by the symbol (n)

$$\text{Factor of Safety} = \frac{\text{Ultimate Stress}}{\text{Working Stress}}$$

The appropriate factor of safety for any given design depends upon a number of considerations which include the following.

a. the degree of safety required, ie what dangers to human life and property are involved.

b). the degree of economy desired.

c). Dependability of the material or homogenity of the material.

d). Permanency of design.

Lower factors of safety are after permitted in temporary designs.

e). Load Conditions: the type of load may be described as dead load ie static probably gravity), 'live load (such as vechicle crossing a bridges), 'fluctuating load' (e.g the alternating tension and compression in the connecting rod of a reciprocating engine), Impact load' or stock load

Each type
- Dead load
- live load
f. the exer
can be predi
g. Accessibil
h. Relative d
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Designers
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=> COMPOUN
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A tensile
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With both
Compressive
Both x an
E_y = Modu
A_y = Area
A_x = Cros
P_x = Lo
P_y = Lo