Modeling concurrent systems

Def 2.1 (T.S.)

A transition system is a tuple $(S, Act, \rightarrow, I, AP, L)$ where

- S is a set of states
- Act is a set of actions
- $\rightarrow \subseteq S \times Act \times S$ transition relation
- $I \subseteq S$ set of initial states
- AP set of atomic propositions
- $L: S \longrightarrow 2^{AP}$ labeling function.

A T.S. is finite if S, Act and AP are finite

Notation $s \xrightarrow{\alpha} s'$ for $(s, \alpha, s') \in \rightarrow$

Note:

$L(s) \in 2^{AP}$ are the set of atomic properties satisfied at s.

We say a state s satisfies a formula φ if
$$L(s) \models \varphi.$$
and write $s \models \varphi$.

Ex 2.2

$S = \{pay, select, soda, beer\}$ $\qquad I = \{pay\}$

$Act = \{get\text{-}soda, get\text{-}beer, insert\text{-}coin, soda\_button, beer\_button\}$

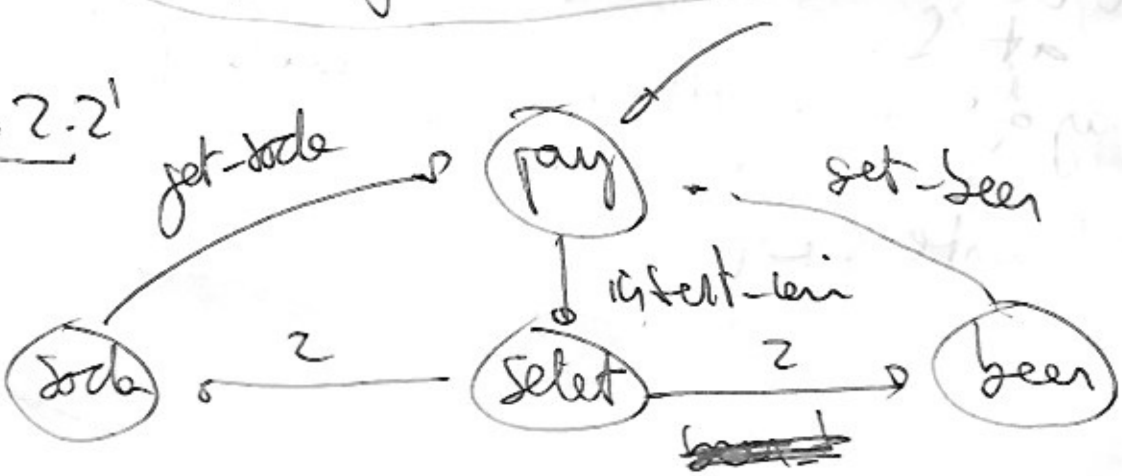
$AP = \{paid, drink\}$

$L(pay) = \emptyset \qquad L(soda) = L(beer) = \{paid, drink\}$

$L(select) = \{paid\}$

depends on the properties we want to study about the system.

Ex. 2.2'



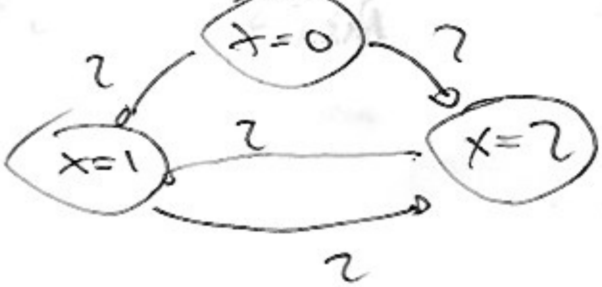
Same as above except

$$Act = \{\text{get-soda}, \text{get-beer}, \text{insert-coin}, \tau\}$$

special action called internal action; not observable

This machine chooses a soda or beer for you.

Ex.



$S = \{x=0, x=1, x=2\}$

$I = \{x=0\}$

$Act = \{\tau\}$

AP, L not shown.

Def 7.3 (Predecessors, successors)

Let $TS = (S, Act, \to, I, AP, L)$ be a T.S. For $s \in S$ and $\alpha \in Act$, we define

$Post(s, \alpha) \triangleq \{s' \in S \mid s \xrightarrow{\alpha} s'\}$ $\quad Post(s) \triangleq \bigcup_{\alpha \in Act} Post(s, \alpha)$

α-successors

$Pre(s, \alpha) \triangleq \{s' \in S \mid s' \xrightarrow{\alpha} s\}$ $\quad Pre(s) = \bigcup_{\alpha \in Act} Pre(s, \alpha)$

α-predecessors

Extended to $C \subseteq S$ pointwise.

Def 2.4 (Terminal State)

s in a T.S. is terminal iff $Post(s) = \phi$.

Def. 3.4 (Path fragment)

A finite path fragment $\hat{\pi}$ of TS is a sequence of states $s_0 s_1 s_2 \ldots s_n$ s.t. $s_i \in Post(s_{i-1})$ $\forall\ 0 < i \leq n$, $n \geq 0$.

An infinite path fragment π is a sequence of states $s_0 s_1 s_2 \ldots$ s.t. $s_i \in Post(s_{i-1})$ $\forall\ i > 0$.

Notation $\pi = s_0 s_1 \cdots$

$first(\pi) \triangleq s_0$

$\pi[j] \triangleq s_j \quad (j \geq 0)$

$\pi[..j] \triangleq s_0 s_1 \ldots s_j$

$\pi[j..] \triangleq s_j s_{j+1} \cdots$

Def 3.5 (Maximal and initial path fragment)

A maximal path fragment is either a finite path fragment that ends in a terminal state, or an infinite path fragment.

A path fragment is initial if $s_0 \in I$

Def 3.6 (Path)

A path in a T.S. is an initial, maximal path fragment

Notation

$Paths(s)$ set of maximal path fragments π with $start(\pi) = s$.

$Paths_{fin}(s)$ set of all finite path fragments $\hat{\pi}$ with $start(\hat{\pi}) = s$

Eg

also path

some path fragments

$\pi_1 =$ pay select soda pay select soda ...

$\pi_2 =$ select soda pay select beer ... not initial

$\hat{\pi} =$ pay select soda pay select soda

Def 3.8 (Trace)

The trace of an infinite path fragment $\pi = s_0 s_1 \ldots$ is

$$\text{Trace}(\pi) = L(s_0)\,L(s_1)\ldots$$

The trace of a finite path fragment $\hat{\pi} = s_0 \ldots s_n$ is

$$\text{Trace}(\hat{\pi}) = L(s_0)\ldots L(s_n)$$

Note: The trace of a path fragment is a word (possibly infinite) over the alphabet 2^{AP}.

Notation: If Π is a set of paths then,

$$\text{trace}(\Pi) = \{\text{trace}(\pi) \mid \pi \in \Pi\}$$

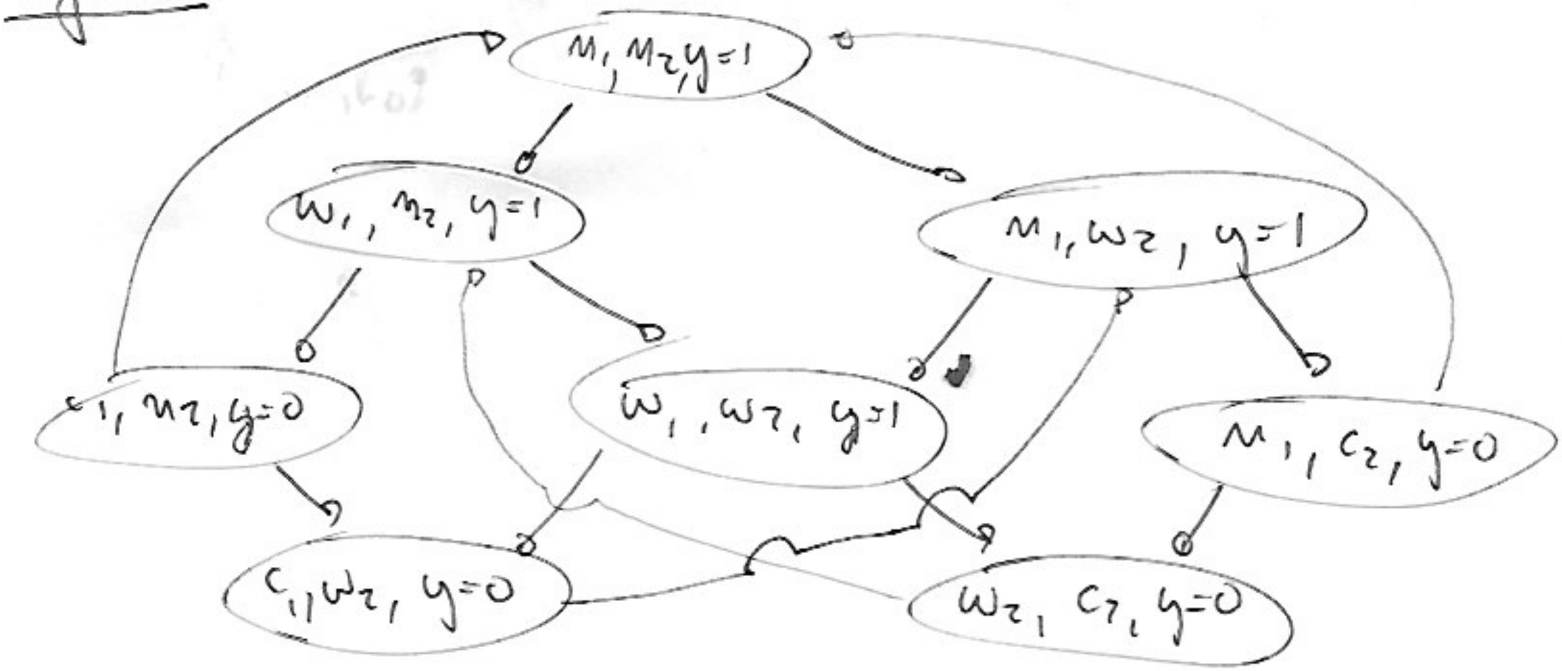
$$\text{Traces}(s) = \text{Trace}(\text{Paths}(s))$$

$$\text{Traces}(TS) = \bigcup_{s \in I} \text{Traces}(s)$$

$$\text{Traces}_{fin}(s) = \text{Trace}(\text{Paths}_{fin}(s))$$

$$\text{Traces}_{fin}(TS) = \bigcup_{s \in I} \text{Traces}_{fin}(s)$$

Eg. 3.9




```
while(true)
  n1: // noncrit
  w1: <await (y>0);
        y:=y-1>
  c1: // crit

while(true)
  n2: // noncrit
  w2: <await (y>0);
        y:=y-1>
  c2: // crit.
```

Assume $AP = \{crit1, crit2\}$

Consider the path:

$$\pi = (n_1, n_2, y=1) \to (w_1, n_2, y=1) \to (c_1, n_2, y=0) \to$$
$$\to (n_1, n_2, y=1) \to (n_1, w_2, y=1) \to (n_1, c_2, y=0) \to \ldots$$

$$Trace(\pi) = \emptyset\ \emptyset\ \{crit_1\}\ \emptyset\ \emptyset\ \{crit_2\}\ \emptyset\emptyset\ \{crit_1\}\ \emptyset\emptyset \ldots$$

Consider the path:

$$\hat{\pi} = (n_1, n_2, y=1) \to (w_1, n_2, y=1) \to (w_1, w_2, y=1) \to$$
$$(w_1, c_2, y=0) \to (w_1, n_2, y=1) \to (c_2, n_2, y=0)$$

$\mathrm{Trace}(\hat{\Pi}) = \phi\,\phi\,\phi\ \{\mathrm{cut}_2\}\ \phi\ \{\mathrm{cut}_1\}$