Automata on infinite words

NBA-0

Given a T.S. TS and an LTL formula φ, we shall be interested in determining whether

$$TS \models \varphi$$

for TS finite and without terminal states.

Some observations:

$$\begin{aligned} TS \models \varphi \text{ iff } & Traces(TS) \subseteq Words(\varphi) \\ \text{iff } & Traces(TS) \cap \left((2^{AP})^\omega \setminus Words(\varphi)\right) = \emptyset \\ \text{iff } & Traces(TS) \cap Words(\neg\varphi) = \emptyset \end{aligned}$$

We need a finite, computable description of this set

$\Rightarrow$ Büchi Automata

NBA (4.3.2)

NBA-1

Def (4.27)

A NBA A is a tuple $= (Q, \Sigma, \delta, Q_0, F)$ where

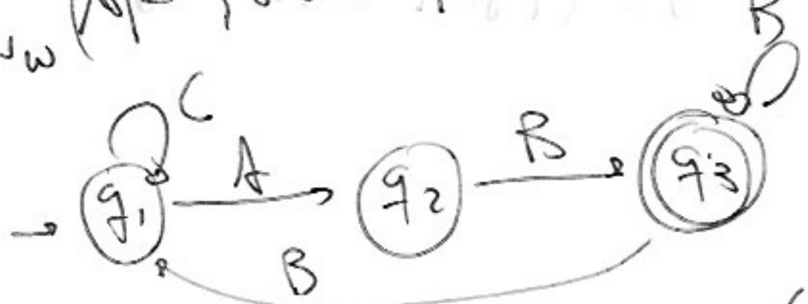
- Q finite set of states
- Σ alphabet
- $\delta : Q \times \Sigma \to 2^Q$
- $Q_0 \subseteq Q$
- $F \subseteq Q$

A run for $\sigma = A_0 A_1 A_2 \ldots \in \Sigma^\omega$ denotes an infinite sequence $q_0 q_1 \ldots$ of states of A s.t. $q_0 \in Q_0$ and $q_i \xrightarrow{A_i} q_{i+1}$, $i \geq 0$.

Run $q_0 q_1 \ldots$ is accepting if $q_i \in F$ for infinitely many i.

$$L_\omega(A) = \{\sigma \in \Sigma^\omega \mid \exists \text{ accepting run for } \sigma \text{ in } A\}$$

Eg.



C^ω has only one run $q_1 q_1 \ldots$ (q_1^ω)

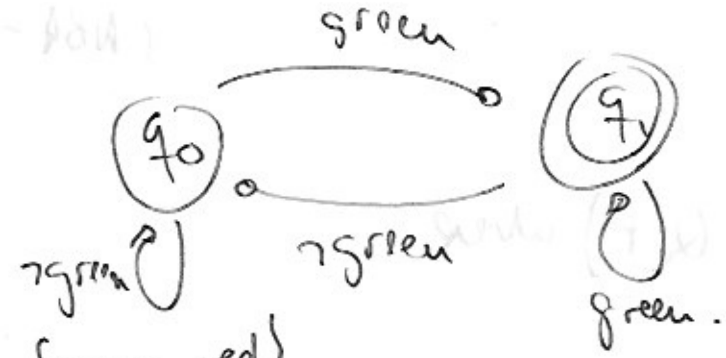
$q_1 q_2 q_3^\omega$ is a run for AB^ω

accepting: $(q_1 q_1 q_2 q_3)^\omega$ for $(CABB)^\omega$

not accepting: $(q_1 q_2 q_3)^4 q_1^\omega$ for $(ABB)^4 C^\omega$

$$L_\omega(A) = C^* AB(B + BC^* AB)^\omega$$

Eg



AP = {green, red}

Note: $\xrightarrow{\neg green}$ = $\xrightarrow{\{red\}}$ and $\xrightarrow{\{\}}$

$\mathcal{L}_\omega(A) = \{\sigma \in (2^{AP})^\omega \mid$ σ has infinitely many sets A_i with green $\in A_i\}$

"infinitely often green".

Eg $\sigma = \{green\}\{\}\{green\}\{\}\{green\}\{\}\ldots$

accepted by run $q_0 q_1 q_0 q_1 \ldots$

Same run obtained for

$\sigma' = (\{g,r\}\{\}\{g\}\{r\})^\omega$

<u>Thm</u> (5.31)

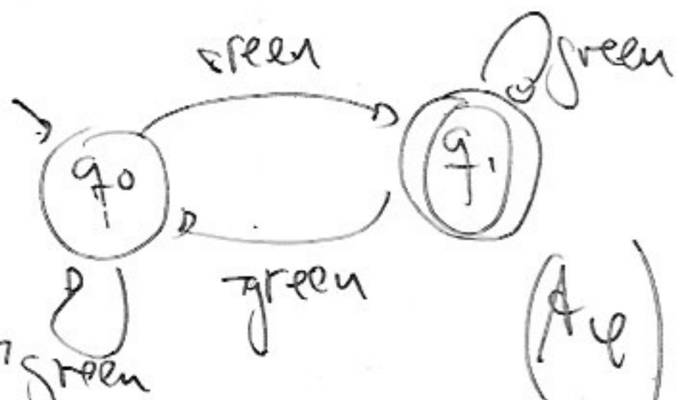
For any LTL formula φ (over AP) there exists an NBA A_φ s.t. $\mathrm{Words}(\varphi) = \mathcal{L}_\omega(A_\varphi)$

Here are some examples

<u>Eg.</u> (5.32)

$AP = \{green, red\}$

$\Box\Diamond\, green$ $\rightsquigarrow$

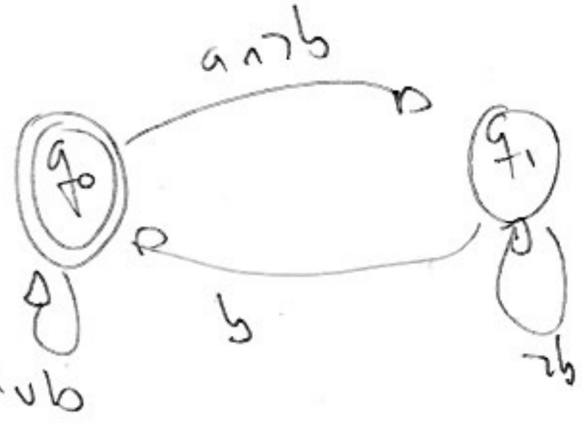


(φ)

$$\mathcal{L}_\omega(A_\varphi) = \mathrm{Words}(\varphi)$$

<u>Eg</u> $AP = \{a, b\}$

$\Box(a \to \Diamond b)$ $\rightsquigarrow$



Eg

$\{a\} \subseteq AP$

$\Diamond \Box a \rightsquigarrow$