

1. Show that the following statements are true:

a.  $\frac{n(n-1)}{2}$  is  $O(n^2)$   
 $\frac{n(n-1)}{2} = \frac{n^2-n}{2} = n^2$  constants are dropped and the leading exponent,  $n^2$  creates the asymptotic upper bound  $O(n^2)$ .

b.  $\max(n^3, 10n^2)$  is  $O(n^3)$  Constants don't matter for asymptotic analysis. The upper asymptote in this case is  $n^3$  so the "Oh" notation is  $O(n^3)$ .

c.  $\sum_{i=1}^n i^k$  is  $O(n^{k+1})$

Since k is a positive integer you will multiply all values of i by themselves k times and then you will do that n times. Ex:  $1^k + 2^k + \dots + n^k$ . Therefore  $n^k$ . Then you will sum those those n instance ok  $i^k$  for an O notation of  $n^{k+1}$

d. If  $p(x)$  is any  $k^{th}$  degree polynomial with a positive leading coefficient,  $p(n)$  is  $O(n^k)$

That is simply a polynomial with a highest exponent of k (i.e.  $p(n) = n^k$ ).

Therefore the O notation is  $n^k$

2. Which function grows faster?

a.  $n^{\log n}$ ;  $(\log n)^n$

$n^{\log(n)}$	$(\log(n))^n$	n
1	0	1
10	1	10
10000	1.26765E+30	100
1000000000	Number Overflow	1000
1E+16	Number Overflow	10000
1E+25	Number Overflow	100000

From the chart above,  $(\log n)^n$  grows faster.

b.  $\log n^k$ ;  $(\log n)^k$

For k=7

n	$\log(n^k)$	$(\log(n))^k$
1	0	0
10	7	1
100	14	128

1000	21	2187
10000	28	16384
100000	35	78125

From the chart above,  $(\log n)^k$  grows faster.

c.  $n^{\log \log \log n}$ ,  $(\log n)!$

$n^{\log(\log(\log(n)))}$	$(\log(n))!$	n
Error Log of negative number	1	1
Error Log of negative number	1	10
0.09061905829	2	100
0.1086141209	6	1000
0.1313890196	24	10000
0.1668371237	120	100000

From the chart above,  $(\log n)!$  grows faster.

d.  $n^n$ ;  $n!$

n	$n^n$	$n!$
1	1	1
10	10000000000	3628800
100	1E+200	9.33262E+157
1000	Number Overflow	Number Overflow
10000	Number Overflow	Number Overflow
100000	Number Overflow	Number Overflow

From the chart above,  $n^n$  grows faster.

3. If  $f_1(n)$  is  $O(g_1(n))$  and  $f_2(n)$  is  $O(g_2(n))$  where  $f_1$  and  $f_2$  are positive functions of  $n$ , show that the function  $f_1(n) + f_2(n)$  is  $O(\max(g_1(n), g_2(n)))$

This is true because, again, O notation is only concerned with the asymptotic upper bound of a function. So adding two function together, the max growth rate would

correspond to the function with the max O notation, each individual function's O notation doesn't affect the other.

4. Prove or disprove: Any positive  $n$  is  $O(\frac{n}{2})$ .

Multiplying a function by a constant only influences its growth rate by a constant amount, so linear functions still grow linearly, logarithmic functions still grow logarithmically, exponential functions still grow exponentially, etc. Since these categories aren't affected by constants, it doesn't matter that we drop the constants. Therefore any positive  $n$  will have an "Oh" notation of  $n$ .  $O(n/2)$  is the same as  $O(0.5*n)$  which is the same as  $0.5*O(n)$  or  $O(n)$ .

Not true

5. Prove or disprove:  $3^n$  is  $O(2^n)$

$3^n$  as a function simply climbs at a higher rate than  $2^n$ . In other words  $3^n$ 's asymptotic upper bound is greater than  $2^n$ 's. So the statement is false or disproven.