CS 600 Homework 12 Solutions

R-26.3 Minimizing costs

A web server company wants to minimize costs while maintaining a rack of standard servers that can handle at least 15,000 hits per minute. Additionally, the company wants to maintain at least 10 servers in their rack. We need to give a linear program to find the optimal server configuration and solve the program geometrically.

We know that the standard server costs 400 dollars, uses $300W$ of power, and takes up to 2 shelves. The cutting edge server costs 1600 dollars, uses $500W$ of power and takes up 1 shelf. The standard server can handle 1000 hits per minute and the cutting edge server can handle 2000 hits per minute. The company has $12,200W$ of power, and 44 shelves of server space.

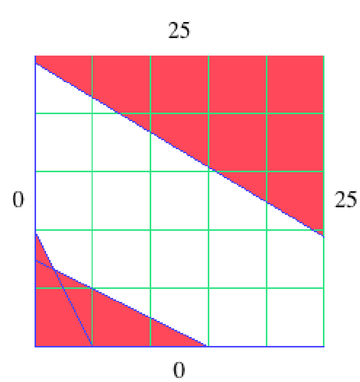
Our objective is to solve the following linear program,

$$\begin{array}{ll} \text{Minimize} & z = 400x_1 + 1600x_2 \\ \\ \text{subject to} & 1000x_1 + 2000x_2 \geq 15000 \\ & 2x_1 + x_2 \geq 10 \\ & 300x_1 + 500x_2 \leq 12200 \end{array}$$

Convering this into standard form we have,

$$\begin{array}{ll} \text{Maximize} & z = -400x_1 - 1600x_2 \\ \\ \text{subject to} & -1000x_1 - 2000x_2 \leq -15000 \\ & -2x_1 - x_2 \leq -10 \\ & 300x_1 + 500x_2 \leq 12200 \end{array}$$

Solving this using the simplex method, we get the following graph. The feasible region is shown in white.



R-26.7 Converting a linear program to standard form

We are given the following linear program and are required to convert it into standard form.

$$\begin{array}{ll} \text{Minimize} & z = 3y_1 + 2y_1 + y_3 \\ & \\ \text{subject to} & -3y_1 + y_2 + y_3 \geq 1 \\ & 2y_1 + y_2 - y_3 \geq 2 \\ & y_1, y_2, y_3 \geq 0 \end{array}$$

Recall that a linear program in standard form has the following form,

$$\begin{array}{ll} \text{Maximize} & z = \sum_{i \in V} c_i x_i \\ & \\ \text{subject to} & \sum_{j \in V} a_{ij} x_j \leq b_i \text{ for } i \in C \\ & x_i \geq 0 \text{ for } i \in V \end{array}$$

where V indexes over the set of variables and C indexers over the set of constraints. Using the rules from page 735 of the textbook, we can convert the given linear program into standard form,

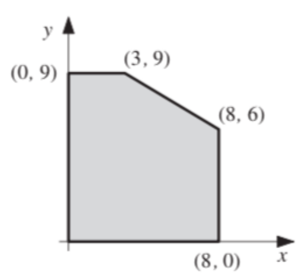
$$\begin{array}{ll} \text{Maximize} & z = -(3y_1 + 2y_1 + y_3) \\ & \\ \text{subject to} & -(-3y_1 + y_2 + y_3) \leq -1 \\ & -(2y_1 + y_2 - y_3) \leq -2 \\ & y_1, y_2, y_3 \geq 0 \end{array}$$

Simplifying, we get

$$\begin{array}{ll} \text{Maximize} & z = -3y_1 - 2y_1 - y_3 \\ & \\ \text{subject to} & 3y_1 - y_2 - y_3 \leq -1 \\ & y_3 - 2y_1 - y_2 \leq -2 \\ & y_1, y_2, y_3 \geq 0 \end{array}$$

R-26.10 Objective function with vertex as the optimal solution

We are given the following feasible region and have to provide an objective function that has the vertices $(3, 9)$ and $(8, 6)$ as the optimal solution.



There are many solutions to this problem, of which one solution is given below.

$$\begin{array}{ll} (3, 9) & \text{Maximize } z = 0.1x + y \\ (8, 6) & \text{Maximize } z = x + 0.1y \end{array}$$