Encoding Audio With DFT

EECS 16ML

Sampling

- Sampling a continuous time signal gives a discrete time signal that we can use for signal processing
- **Nyquist–Shannon sampling theorem**: a signal can be sampled and perfectly reconstructed from its samples if the waveform is sampled over twice as fast as it's highest frequency frequency
  - $f_s > 2 f_{max}$

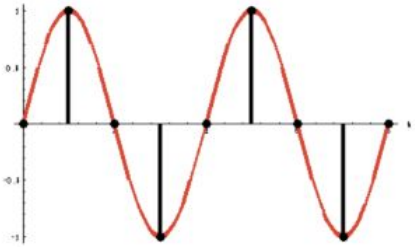


Figure 3: slower sampling rate (source: Berkeley microscopy)

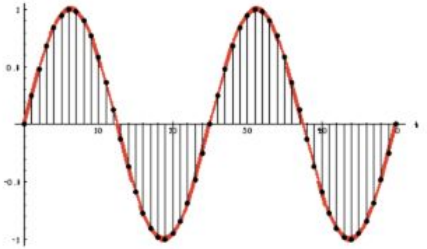


Figure 4: faster sampling rate (source: Berkeley microscopy)

Wave Fundamentals - Simple Waves

$\lambda = 2\pi$,

$f = 2\pi/\lambda = 1$

$A = 1$

- **Wavelength** (λ): The distance between 2 similar points on a periodic wave
  - Here we measure from peak to peak
- **Period** (T): wavelengths per cycle of unit circle
- **Frequency** (f) := $1/T = 2\pi/\lambda$
  - Wavelengths per cycle of unit circle
- **Amplitude** (A): vertical distance from center of wave to peak of wave
  - Tells us the strength of the wave

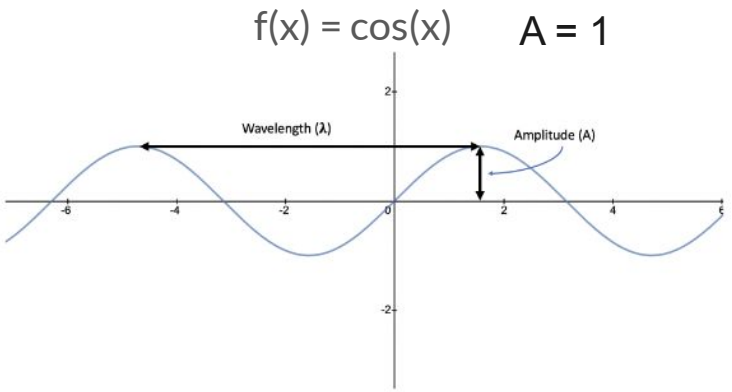


Figure 1: sine wave labeled with amplitude and wavelength

Wave Fundamentals - More Complex Waves

- Complex signal consists of cos(5x) and cos(2x)
- Each cosine contributes its own frequency to the signal
- Since the amplitudes of each cosine are the same, both cosines contribute an equal amount of their respective frequencies

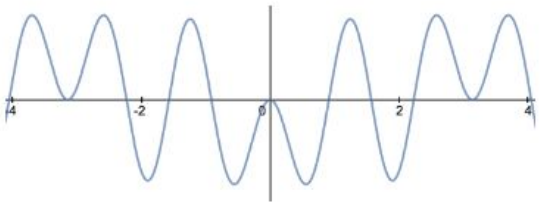


Figure 2: plot of $\cos(5x) + \cos(2x)$

As an exercise, calculate the wavelengths of cos(5x) and cos(2x)

Sound Waves

- A single tone is defined by its frequency
- All sound waves are a linear combination of tones with varying frequencies
- Human range of hearing from 20 Hz to 20,000 Hz

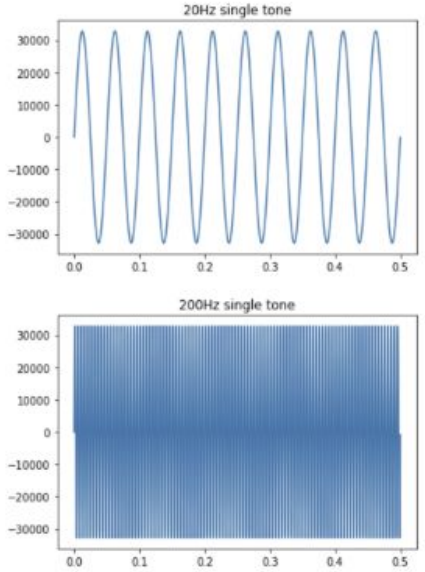


Figure 6: 20 Hz tone (top) and 200 Hz tone (bottom)

DFT of sound waves

- DFT tells us the intensity of different frequer a signal
- Since a tone only has a single frequency, the D tone is concentrated at a certain frequency
- Since sounds are a linear combination of frequencies, the DFT will show the spread of tones across the frequency spectrum

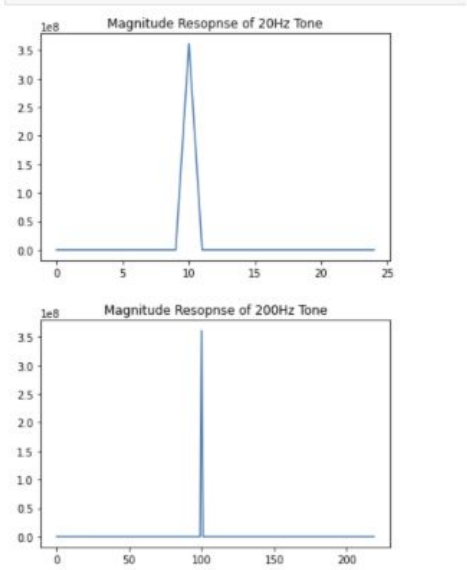


Figure 7: magnitude response of 20 Hz tone (top) and magnitude 200 Hz tone (bottom)

Frequency Spectrum

DFT:

- Transformation from time domain to frequency domain
- $$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-i\omega n}$$

STFT:

- Maps 1D signal to 2D spectrogram
- Gives us temporal information
-

$$X[n, \omega) = \sum_{m=-\infty}^{\infty} x[n+m] w[m] e^{-i\omega m}$$

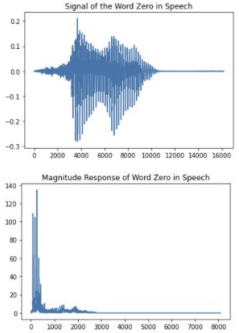


Figure 5: raw signal(top) and magnitude response (bottom) of the word "zero" in speech

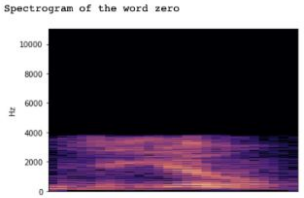
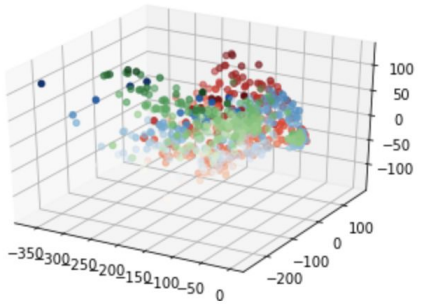


Figure 8: spectrogram of the word "zero"

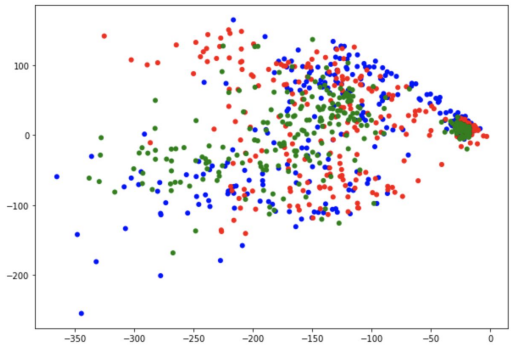
Raw Signal vs DFT vs STFT

Raw Time Varying Audio Signal

- Pretty bad clustering
- 3 words have don't appear distinguishable in the scatter plots

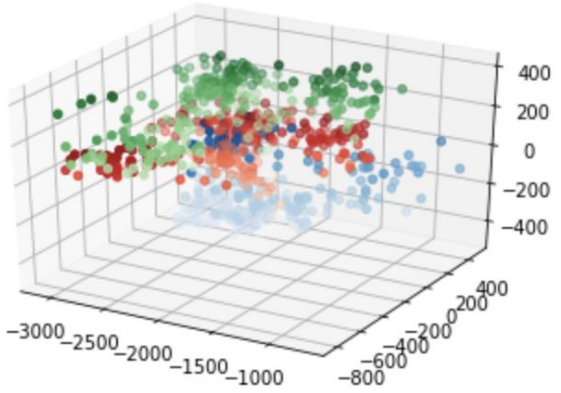


Plotting clusters of time domain signals for 3 different words

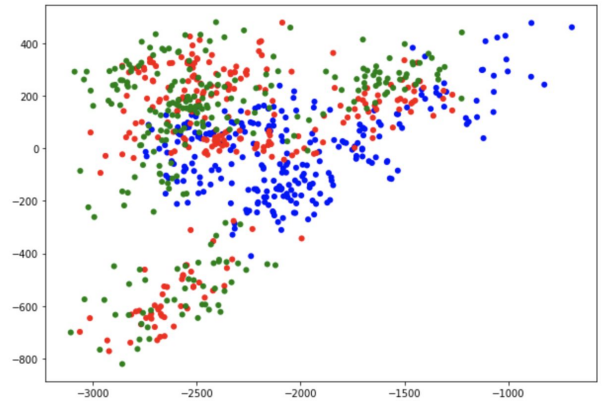


DFT of Audio Signal

- Much better than the raw signal
- 3 words seem fairly distinguishable, especially in 3d scatter plot

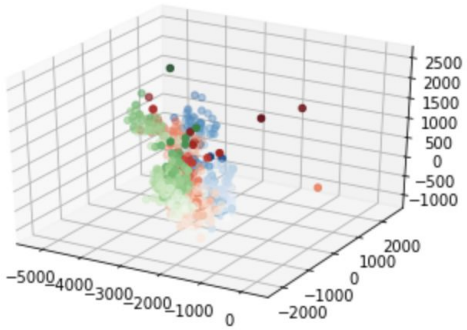


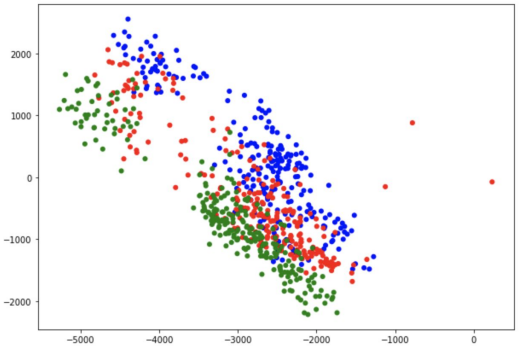
Plotting clusters of the DFT of signals for 3 different words. Here we use decibels to measure the magnitude spectrum to help us better differentiate between very small magnitudes



STFT of Audio Signal

- Very similar to DFT of the audio signal, but still performs a little better
- Both the 2D and 3D scatter plots are visually distinct





Plotting clusters of the STFT of signals for 3 different words

DFT and Convolutional Neural Networks

Convolutions in the Frequency Domain

- The frequency domain can also be useful for implementing convolutional layers: “FFT convolution”
- **Convolution theorem**: Pointwise multiplication in frequency domain is a convolution in the time domain
- FFT convolutions are more efficient when the kernel size is large relative to the input image

Using CNNs with STFT Spectrograms

- STFT spectrograms are very similar to 2D images
- Can use image classifying techniques such as CNNs
- The spectrogram's intensities at each time step and frequency can be transformed into a matrix input for the CNN, which can be trained to determine the presence or identity of target signal spectra

References

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