Encoding Audio with DFT Quiz Solutions

Anthony Shara, Adi Ganapathi, Dohyun Cheon, Larry Yan, Richard Shuai

December 2, 2020

1 Sampling Theorem

How fast must the sampling frequency be to perfectly construct each of the following periodic signals in discrete time? Also state the wavelength.

1.1

$f(x) = \cos(10x)$

Recall: $f_s > 2f_{max}$
$f_s > 10 \cdot 2 = 20$

$f_s = \frac{2\pi}{\lambda}$
$\lambda = \frac{2\pi}{f}$

$\lambda = \frac{2\pi}{20} = \frac{\pi}{10}$

1.2

$f(x) = \cos^2(18x)$

min $f_s = 18 \cdot 2 = 36$

$\lambda = \frac{2\pi}{36} = \frac{\pi}{18}$

1.3

$f(x) = \sin^2(22x + 9)$

min $f_s = 22 \cdot 2 = 44$

$\lambda = \frac{2\pi}{44} = \frac{\pi}{22}$

1.4

$f(x) = \sin(10x)$

$\min\ f_s = 10 \cdot 2 = 20$

$\lambda = \frac{2\pi}{20} = \frac{\pi}{10}$

2 Deriving Euler's Equations

Prove that $\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$, $\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$

(1) $e^{ix} = \cos(x) + i\sin(x)$
(2) $e^{-ix} = \cos(-x) + i\sin(-x) = \cos(x) - i\sin(x)$

2.1

using equations (1) and (2):

$e^{ix} + e^{-ix} = 2\cos(x)$
$\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$

2.2

using equations (1) and (2):

$e^{ix} - e^{-ix} = 2i\sin(x)$
$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$

3 DFT

Sketch the Magnitude response of the following equations:
a) $f(x) = \cos(x)$ b) $f(x) = \cos(12x)$ d) $f(x) = e^{i3}\cos(12x)$

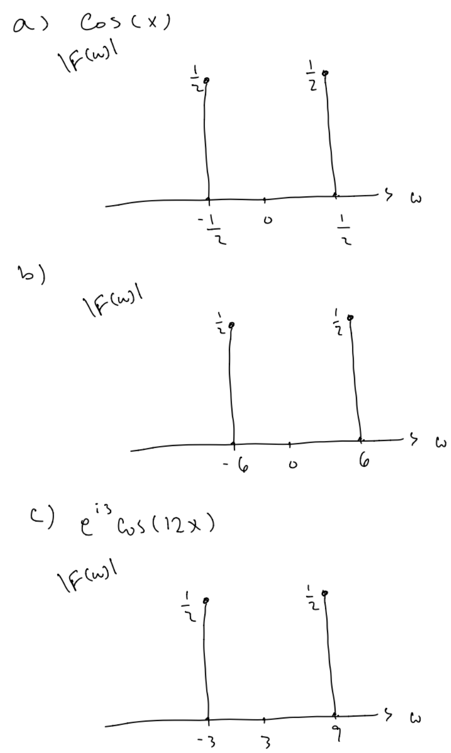

Figure 1: Problem 3 Solution

4 More DFT

What function $x(n)$ gives us the following magnitude response $F(\omega)$?

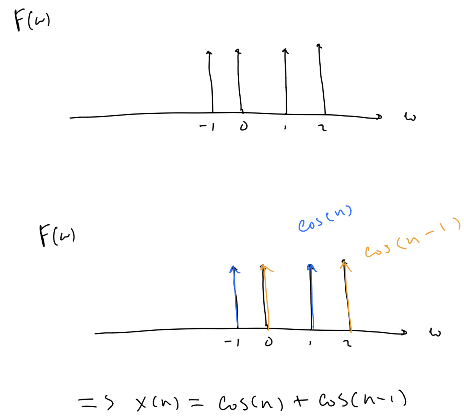

Figure 2: Problem 4 Solution

5 STFT

Sketch the STFT of $x(t), 0 \leq t < 15$ where $x(t)$ is defined as the following peicewise:

$$x(t) = \begin{cases} 2\cos(2t) & 0 \leq t < 5 \\ 2\cos(6t) & 5 \leq t < 10 \\ 2\cos(12t+1) & 10 \leq t < 15 \end{cases}$$

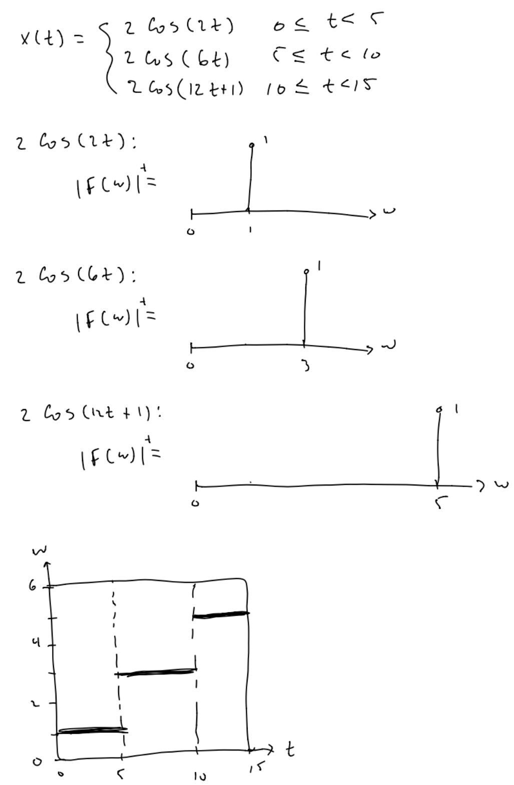

Figure 3: Problem 5 Solution

6 Encoding Audio with DFT Part 1

How can we use DFT to learn about hidden features in an audio signal?

By using STFT, we can take the DFT of a signal over small chunks of time. This reveals how frequencies in a signal change over time.

7 Encoding Audio with DFT Part 2

Describe a possible classification algorithm that incorporates Fourier features of the signal.

One possible algorithm is first perform DFT on the training data and then perform PCA to reduce dimensionality. Finally train a classification model using the dimensionally reduced training data encoded with DFT.

Another possible possible algorithm is to first perform STFT on each sample of the training data and then flatten each spectrogram into a 1D array. Then perform PCA to reduce dimensionally and finally train a classification model using this new processed dataset.

Any solution that involves training a classification model on a dataset with some form of DFT encoding is sufficient

8 Symmetry of Real Valued Continuous Signals in the Frequency Domain

If we have a real valued continuous time signal, it's magnitude spectrum should be symmetric about what axis?

symmetric about $\omega = 0$

9 Magnitude Spectrum of a Discrete Time Periodic Functions

If a discrete time signal is both real valued and periodic, where will there be symmetry in the magnitude spectrum of the signal?

There will be symmetry $\omega = \frac{k\dot{f}_s}{2}, k \in Z$

10 Decibels and the Magnitude Spectrum

What is one Advantage of using decibels to measure the magnitude spectrum of a signal?

> By using STFT, we can take the DFT of a signal over small chunks of time. This reveals how frequencies in a signal change over time.