Foundations for statistical inference - Confidence intervals

completion time: approx 8 hours

If you have access to data on an entire population, say the opinion of every adult in the United States on whether or not they think climate change is affecting their local community, it's straightforward to answer questions like, "What percent of US adults think climate change is affecting their local community?". Similarly, if you had demographic information on the population you could examine how, if at all, this opinion varies among young and old adults and adults with different leanings. If you have access to only a sample of the population, as is often the case, the task becomes more complicated. What is your best guess for this proportion if you only have data from a small sample of adults? This type of situation requires that you use your sample to make inference on what your population looks like.

Setting a seed: You will take random samples and build sampling distributions in this lab, which means you should set a seed on top of your lab. If this concept is new to you, review the lab on probability.

Getting Started

Load packages

In this lab, we will explore and visualize the data using the **tidyverse** suite of packages, and perform statistical inference using **infer**.

Let's load the packages.

```
library(tidyverse)
library(openintro)
library(infer)
library(tinytex)
```

The data

A 2019 Pew Research report states the following:

To keep our computation simple, we will assume a total population size of 100,000 (even though that's smaller than the population size of all US adults).

> Roughly six-in-ten U.S. adults (62%) say climate change is currently affecting their local community either a great deal or some, according to a new Pew Research Center survey.
>
> **Source:** Most Americans say climate change impacts their community, but effects vary by region

In this lab, you will assume this 62% is a true population proportion and learn about how sample proportions can vary from sample to sample by taking smaller samples from the population. We will first create our population assuming a population size of 100,000. This means 62,000 (62%) of the adult population think climate change impacts their community, and the remaining 38,000 does not think so.

```
us_adults <- tibble(
  climate_change_affects = c(rep("Yes", 62000), rep("No", 38000))
)
```

The name of the data frame is `us_adults` and the name of the variable that contains responses to the question *"Do you think climate change is affecting your local community?"* is `climate_change_affects`.

We can quickly visualize the distribution of these responses using a bar plot.

```
ggplot(us_adults, aes(x = climate_change_affects)) +
  geom_bar() +
  labs(
    x = "", y = "",
    title = "Do you think climate change is affecting your local community?"
  ) +
  coord_flip()
```

We can also obtain summary statistics to confirm we constructed the data frame correctly.

```
us_adults %>%
  count(climate_change_affects) %>%
  mutate(p = n /sum(n))
```

```
# A tibble: 2 x 3
  climate_change_affects     n     p
  <chr>                  <int> <dbl>
1 No                     38000  0.38
2 Yes                    62000  0.62
```

In this lab, you'll start with a simple random sample of size 60 from the population.

```
n <- 60
samp <- us_adults %>%
  sample_n(size = n)
```

1. What percent of the adults in your sample think climate change affects their local community? **Hint:** Just like we did with the population, we can calculate the proportion of those **in this sample** who think climate change affects their local community.

1. Insert your answer here

62.2%, not too far off!

```
>samp <- us_adults %>% sample_n(size = 60)
> table(samp)
climate_change_affects
 No Yes
 23  37
> 23/37
[1] 0.6216216
```

2. Would you expect another student's sample proportion to be identical to yours? Would you expect it to be similar? Why or why not?

2. Insert your answer here

I would not expect them to be the same unless we both set the same seed. In the context of sampling, a "seed" is a fixed initial value that is used to initialize the random number generator, ensuring that the same sequence of random selections is generated each time the code is run with the same seed, thus enabling reproducibility of random sampling results.

Confidence intervals

Return for a moment to the question that first motivated this lab: based on this sample, what can you infer about the population? With just one sample, the best estimate of the proportion of US adults who think climate change affects their local community would be the sample proportion, usually denoted as $\hat{p}$ (here we are calling it `p_hat`). That serves as a good **point estimate**, but it would be useful to also communicate how uncertain you are of that estimate. This uncertainty can be quantified using a **confidence interval**.

One way of calculating a confidence interval for a population proportion is based on the Central Limit Theorem, as $\hat{p} \pm z^\star SE_{\hat{p}}$ is, or more precisely, as $\hat{p} \pm z^\star \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

Another way is using simulation, or to be more specific, using **bootstrapping**. The term **bootstrapping** comes from the phrase "pulling oneself up by one's bootstraps", which is a metaphor for accomplishing an impossible task without any outside help. In this case the impossible task is estimating a population parameter (the unknown population proportion), and we'll accomplish it using data from only the given sample. Note that this notion of saying something about a population parameter using only information from an observed sample is the crux of statistical inference, it is not limited to bootstrapping.

In essence, bootstrapping assumes that there are more of observations in the populations like the ones in the observed sample. So we "reconstruct" the population by resampling from our sample, with replacement. The bootstrapping scheme is as follows:

- **Step 1.** Take a bootstrap sample - a random sample taken **with replacement** from the original sample, of the same size as the original sample.
- **Step 2.** Calculate the bootstrap statistic - a statistic such as mean, median, proportion, slope, etc. computed on the bootstrap samples.
- **Step 3.** Repeat steps (1) and (2) many times to create a bootstrap distribution - a distribution of bootstrap statistics.
- **Step 4.** Calculate the bounds of the XX% confidence interval as the middle XX% j knof the bootstrap distribution.

Instead of coding up each of these steps, we will construct confidence intervals using the **infer** package.

Below is an overview of the functions we will use to construct this confidence interval:

Function	Purpose
`specify`	Identify your variable of interest
`generate`	The number of samples you want to generate
`calculate`	The sample statistic you want to do inference with, or you can also think of this as the population parameter you want to do inference for
`get_ci`	Find the confidence interval

This code will find the 95 percent confidence interval for proportion of US adults who think climate change affects their local community.

```
samp %>%
  specify(response = climate_change_affects, success = "Yes") %>%
  generate(reps = 1000, type = "bootstrap") %>%
  calculate(stat = "prop") %>%
  get_ci(level = 0.95)
```

```
# A tibble: 1 x 2
  lower_ci upper_ci
     <dbl>    <dbl>
1    0.583    0.800
```

- In `specify` we specify the `response` variable and the level of that variable we are calling a `success`.
- In `generate` we provide the number of resamples we want from the population in the `reps` argument (this should be a reasonably large number) as well as the type of resampling we want to do, which is `"bootstrap"` in the case of constructing a confidence interval.
- Then, we `calculate` the sample statistic of interest for each of these resamples, which is `pro`portion.

Feel free to test out the rest of the arguments for these functions, since these commands will be used together to calculate confidence intervals and solve inference problems for the rest of the semester. But we will also walk you through more examples in future chapters.

To recap: even though we don't know what the full population looks like, we're 95% confident that the true proportion of US adults who think climate change affects their local community is between the two bounds reported as result of this pipeline.

Confidence levels

3. In the interpretation above, we used the phrase "95% confident". What does "95% confidence" mean?

3. Insert your answer here

In this context, confidence is a measure of estimate precision. It means that if we repeat the experiment, we are 95% confident our tested response will correctly fall within the boundaries of the confidence interval.

In this case, you have the rare luxury of knowing the true population proportion (62%) since you have data on the entire population.

4. Does your confidence interval capture the true population proportion of US adults who think climate change affects their local community? If you are working on this lab in a classroom, does your neighbor's interval capture this value?

4. Insert your answer here

My interval caught both my sample and the population. My confidence interval from my 1,000 repetition bootstrap was `lower_ci = .05 upper_ci=.75`*. My sample rate was* `0.6216216`*, and the population rate is 62%. If I was working with a group of people taking the same sample, given we've set up an interval that should be right 95% of the time, more than likely what they selected would fall within the interval as well.*

5. Each student should have gotten a slightly different confidence interval. What proportion of those intervals would you expect to capture the true population mean? Why?

5. Insert your answer here

I'd expect for mostly all the students to have confidence intervals that would capture the population mean. That's the purpose of the 95%, it means that 9.5 out of 10 times, you're going sample a mean that is within that band.

For sanity I did a bit of my own testing with the code below. As long as the sample size reasonable, maybe over 10, the bands all seem to wrap around the population's 62% success rate.

```
calculate_ci <- function(n) {
  result <- us_adults %>%
    sample_n(size = n) %>%
    specify(response = climate_change_affects, success = "Yes") %>%
    generate(reps = 1000, type = "bootstrap") %>%
    calculate(stat = "prop") %>%
    get_ci(level = 0.95)
  result_str <- sprintf("Lower CI: %.3f, Upper CI: %.3f",
                        result$lower_ci, result$upper_ci)
  return(result_str)
}

sample_sizes <- c(2, 10, 60, 1000, 2000)
for (n in sample_sizes) {
  result <- calculate_ci(n)
  cat(sprintf("%5d: %s\n", n, result))
}
```

```
   2: Lower CI: 0.000, Upper CI: 1.000
  10: Lower CI: 0.600, Upper CI: 1.000
  60: Lower CI: 0.533, Upper CI: 0.767
1000: Lower CI: 0.593, Upper CI: 0.651
2000: Lower CI: 0.607, Upper CI: 0.648
```

In the next part of the lab, you will collect many samples to learn more about how sample proportions and confidence intervals constructed based on those samples vary from one sample to another.

- Obtain a random sample.
- Calculate the sample proportion, and use these to calculate and store the lower and upper bounds of the confidence intervals.

- Repeat these steps 50 times.

Doing this would require learning programming concepts like iteration so that you can automate repeating running the code you've developed so far many times to obtain many (50) confidence intervals. In order to keep the programming simpler, we are providing the interactive app below that basically does this for you and created a plot similar to Figure 5.6 on OpenIntro Statistics, 4th Edition (page 182).

---

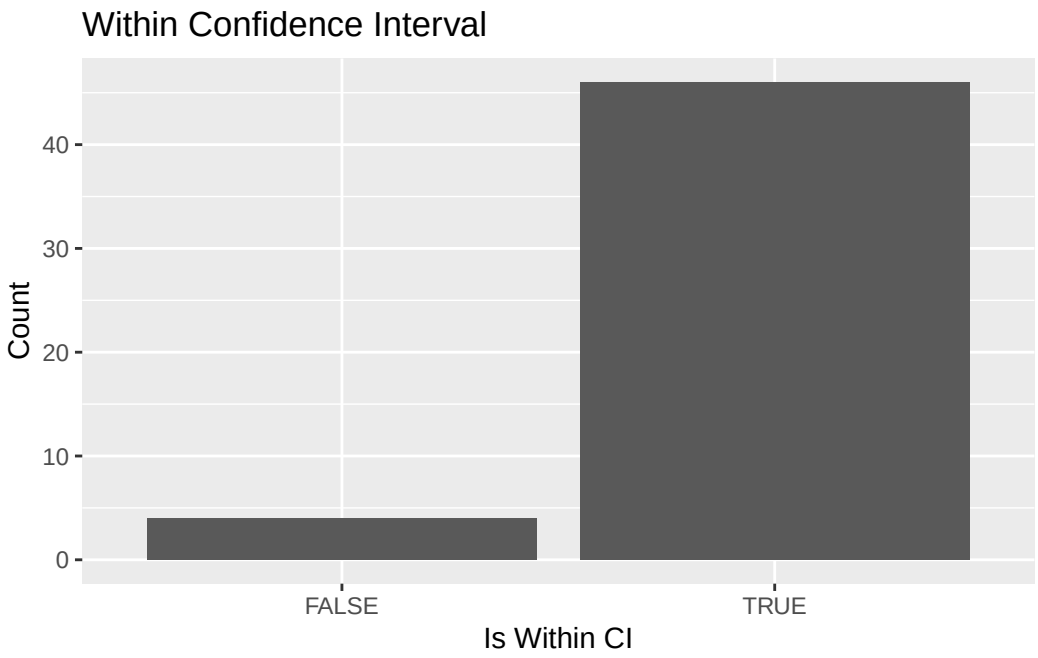
6. Given a sample size of 60, 1000 bootstrap samples for each interval, and 50 confidence intervals constructed (the default values for the above app), what proportion of your confidence intervals include the true population proportion? Is this proportion exactly equal to the confidence level? If not, explain why. Make sure to include your plot in your answer.

6. Insert your answer here

Sure, let's do some quick coding first.

```
library(ggplot2)

# Let's build a function that creates a dataframe with a single record.
# that record will have three columns, sample_size, lower_ci and upper_ci.
# It's confiugred for bootstrap, 1,000 iterations and a CI of 95%.

calculate_ci_dataframe <- function(sample_size, ci_level) {
  results_df <- data.frame(sample_size = integer(0),
                           lower_ci = numeric(0), upper_ci = numeric(0),
                           ci_difference = numeric(0))
  result <- us_adults %>%
    sample_n(size = sample_size) %>%
    specify(response = climate_change_affects, success = "Yes") %>%
    generate(reps = 1000, type = "bootstrap") %>%
    calculate(stat = "prop") %>%
    get_ci(level = ci_level)

  ci_diff <- result$upper_ci - result$lower_ci

  results_df <- bind_rows(results_df,
                          data.frame(sample_size = sample_size,
                                     lower_ci = result$lower_ci,
                                     upper_ci = result$upper_ci,
                                     ci_difference = ci_diff))
```

```
  return(results_df)
}


# Now we run it 50 times, store it in a list.
results_list <- list()
for (i in 1:50) {
  result_df <- calculate_ci_dataframe(sample_size = 100, ci_level =  0.95)
  results_list[[i]] <- result_df
}

# now we'll convert it to a dataframe and then plot.
data95 <- do.call(rbind, results_list) %>%
  mutate(is_between = ifelse(0.62 >= lower_ci & 0.62 <= upper_ci, TRUE, FALSE))


ggplot(data95, aes(x = is_between)) +
  geom_bar() +
  labs(x = "Is Within CI", y = "Count", title = "Within Confidence Interval")
```




```
glimpse(data95)
```

```
Rows: 50
Columns: 5
$ sample_size   <dbl> 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 100, 1~
$ lower_ci      <dbl> 0.50000, 0.51000, 0.56000, 0.53975, 0.60000, 0.50000, 0.~
$ upper_ci      <dbl> 0.70000, 0.70000, 0.74000, 0.72000, 0.78000, 0.69000, 0.~
$ ci_difference <dbl> 0.20000, 0.19000, 0.18000, 0.18025, 0.18000, 0.19000, 0.~
$ is_between    <lgl> TRUE, TRUE, TRUE, TRUE, TRUE, TRUE, TRUE, TRUE, TRUE, TR~
```

I ran this a bunch of times. It looks to me like it matches the CI fairly closely, 2.5 out of 50 will be outside of the confidence interval. This was a fun exercise.

More Practice

7. Choose a different confidence level than 95%. Would you expect a confidence interval at this level to be wider or narrower than the confidence interval you calculated at the 95% confidence level? Explain your reasoning.

7. Insert your answer here

Interesting question! I modified the code above to support having a column that is the difference between the upper and lower band. We should be able to take the mean of that column and see the exact difference. I expect the 95 to have a lower mean average than the 99.

```r
results_list <- list()
for (i in 1:50) {
  result_df <- calculate_ci_dataframe(sample_size = 100, ci_level =  0.99)
  results_list[[i]] <- result_df
}

data99 <- do.call(rbind, results_list) %>%
  mutate(is_between = ifelse(0.62 >= lower_ci & 0.62 <= upper_ci, TRUE, FALSE))

cat(sprintf("Mean Differences between upper ad lower CI bands:  95CI:%s  99CI: %s",
  mean(data95$ci_difference), mean(data99$ci_difference)))
```

```
Mean Differences between upper ad lower CI bands:  95CI:0.18684  99CI: 0.241627
```

8. Using code from the **infer** package and data from the one sample you have (`samp`), find a confidence interval for the proportion of US Adults who think climate change is affecting their local community with a confidence level of your choosing (other than 95%) and interpret it.

8. Insert your answer here

Sure! We did this already above with 95% so let's do 68% and 99%. Of course we expect 99 to be wide and 68 to be narrow, but let's see. Please note that 68, 95, and 99 are also numbers associated with standard deviation – but that is not what a confidence interval is. These three numbers were picked out of personal curiosity.

After viewing the results below, as expected, 99% is much wider around our 62%.

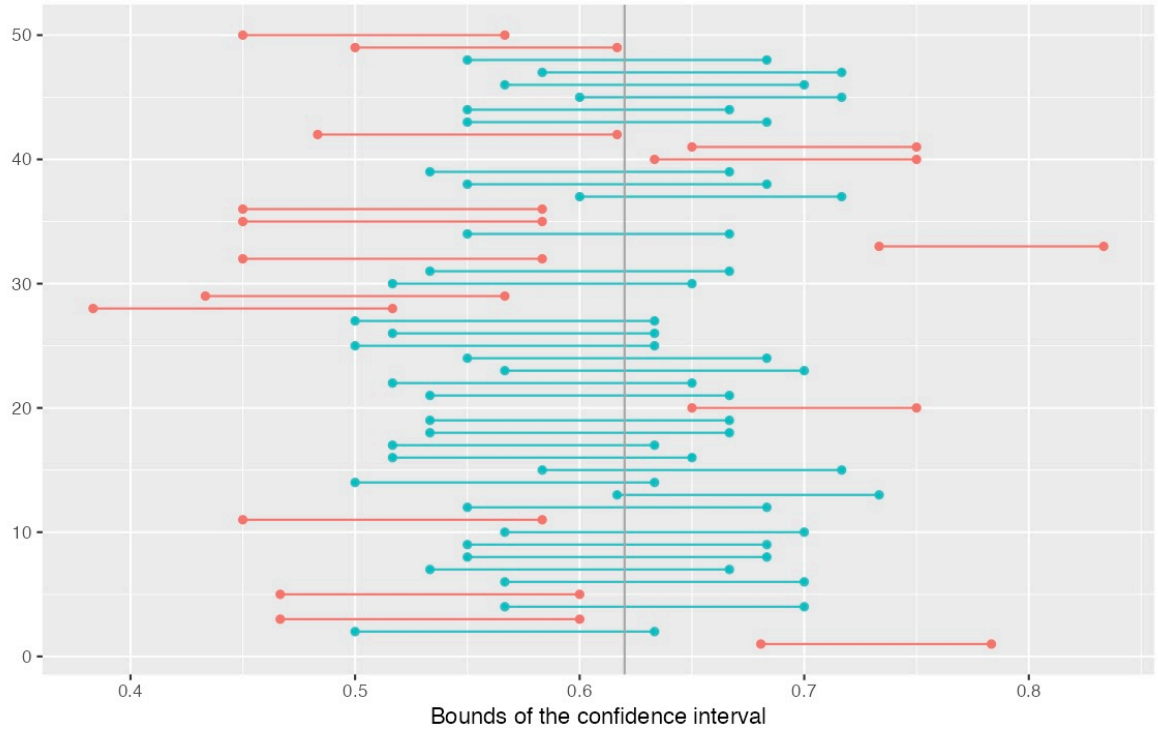
```
new_sample <- us_adults %>% sample_n(size = 60)

calculate_interval <- function (samp, interval) {
  return(samp %>%
    specify(response = climate_change_affects, success = "Yes") %>%
    generate(reps = 1000, type = "bootstrap") %>%
    calculate(stat = "prop") %>%
    get_ci(level =interval))
}

ninety_nine <- calculate_interval(new_sample, .99)
ninety_five <- calculate_interval(new_sample, .95)
sixty_eight <- calculate_interval(new_sample, .68)

print(sprintf("Bands: 99:[%s,%s], 95:[%s,%s] 68:[%s,%s]",
  round(ninety_nine$lower_ci, 2),
  round(ninety_nine$upper_ci, 2),
  round(ninety_five$lower_ci, 2),
  round(ninety_five$upper_ci, 2),
  round(sixty_eight$lower_ci, 2),
  round(sixty_eight$upper_ci, 2)
  ))
```

```
[1] "Bands: 99:[0.5,0.8], 95:[0.53,0.77] 68:[0.58,0.7]"
```

9. Using the app, calculate 50 confidence intervals at the confidence level you chose in the previous question, and plot all intervals on one plot, and calculate the proportion of intervals that include the true population proportion. How does this percentage compare to the confidence level selected for the intervals?

9. Insert your answer here

I did both 99 and 68. Let's do 68%. As expected, dropping the confidence requirement down to 68% opens up upper and lower bounds, and allows more errors to occur.

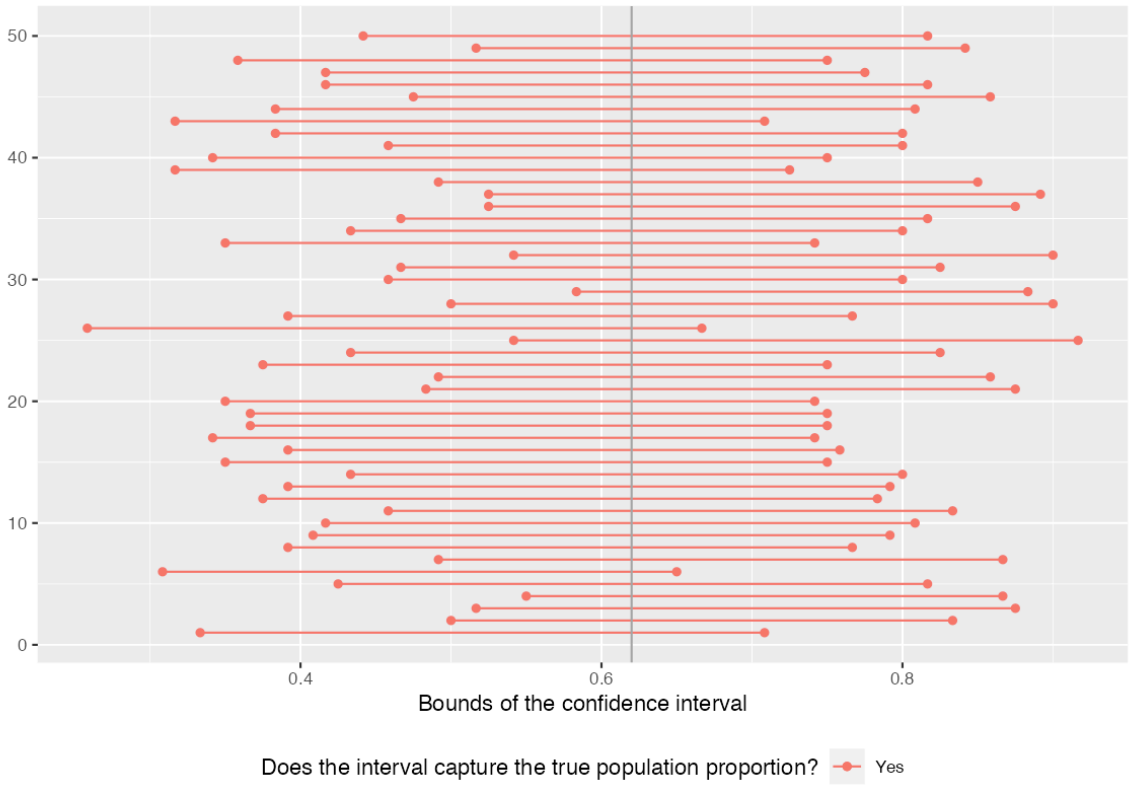


10. Lastly, try one more (different) confidence level. First, state how you expect the width of this interval to compare to previous ones you calculated. Then, calculate the bounds of the interval using the **infer** package and data from `samp` and interpret it. Finally, use the app to generate many intervals and calculate the proportion of intervals that are capture the true population proportion.

10. Insert your answer here

Let's try a confidence interval that is three nines. I expect the interval to be the widest yet. I also expect that when I put it in the app, there are not likely to be any out of bounds samples. Both proved to be true

```
> print(calculate_interval(next_new_sample, .999))
# A tibble: 1 × 2
  lower_ci upper_ci
     <dbl>    <dbl>
1    0.408      0.8 # <- check!! That's really wide!
```



11. Using the app, experiment with different sample sizes and comment on how the widths of intervals change as sample size changes (increases and decreases).

11. Insert your answer here

A larger sample size equals a tighter confidence band. At sample size of 20, the scale of the chart is from between .4 and .8, or a difference of .2. At at sample size 10,000, the difference is closer .03.

12. Finally, given a sample size (say, 60), how does the width of the interval change as you increase the number of bootstrap samples. **Hint:** Does changing the number of bootstap samples affect the standard error?

12. Insert your answer here

This feels like sort of a given question, but yes a difference in the number of repetitions will show up on standard error. A smaller value of reps will result in a greater standard error, and that means an estimate may have less precision compared to using a larger number of bootstrap samples.
