U.S. Treasury Bond Yield Prediction Using Linear, SVR and LSTM Model

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Abstract

Decision-making process in fixed-income market could benefit from machine learning methods. Bond yield prediction is a field with relatively few investigations; therefore, we conduct this study to predict bond yields with linear models, support vector regression (SVR) and Long Short-Term Memory networks (LSTM). Our result showed that LASSO preforms the best.

1.Introduction

Fixed-income market is a significant component of the portfolios of governments, banks, private and public corporations. This market includes debt securities such as Treasury bonds and bonds issued by federal agencies, state and local municipalities, and private corporations. Issuance of Treasury bonds is often the primary way for a government to borrow funds from the public to cover its government deficit. According to the Securities Industry and Financial Markets Association (2020), U.S. Treasury Securities' trading volume is $16,673.3 billion in 2019, compared with $9,566.4 billion for U.S. corporate bond trading.

Despite the paramount importance of the Treasury bond in the fixed income assets, few studies have applied machine learning methods on Treasury bonds. Our study is inspired by the earlier work of Nunes, Gerding, McGroarty, & Niranjan (2019), which is the first comprehensive study on yield curve forecasting using artificial neural networks and multitasking learning techniques. We continued to consider a wide range of macroeconomic and financial time series data and to apply linear models. Additionally, we investigated SVR and LSTM models' predictive power in forecasting the U.S. Treasury bond yields.

Our research is focused on predicting U.S. Treasury bond yields for the following reasons. First, the U.S. government bond class is more liquid than other bond classes. Second, the size of the market is also considerably larger. Third and last, research on government bonds is of interest to entities such as central banks and asset management companies.

2. Data

In this section, we describe the targets and features selected for our research, and the methods applied for data preprocessing.

The time range of target data is from 2003-12-15 to 2019-01-01, a total of 3767 days. As we use 3 past values of features to predict 1 present value of targets, the time range of features is from 2003-12-12 to 2018-12-31.

2.1 Targets

Daily U.S. treasury yield curve rates with time to maturity of 3 months, 2 years, 5 years, 10 years and 30 years are selected as the five targets to be predicted, named 3M, 2Y, 5Y, 10Y ,30Y respectively for convenience. They were scrapped directly from the website of U.S. Department of the Treasury.

2.2 Original Features

The chosen original features are composed of macro-economic indicators, such as U.S. CPI and GDP, currency market data, such as USD/EUR, equity market data like S&P 500, and bond market data, consisting of corporate bond yields and government bond yields. The sources of feature data include FRED, Yahoo Finance and Quandl. The total number of original features is 112.

2.3 Generated Features

We generated several features from the original features.
13 percentage change data were generated based on the formula below.

$$P_t = \frac{X_t - X_{t-1}}{X_{t-1}}$$

84 generated technical indicators include 12 lagged U.S. treasury yield curve rates (1M, 2M, 3M, 6M, 1Y, 2Y, 3Y, 5Y, 7Y, 10Y, 20Y, 30Y), and 72 of their moving averages and moving average spreads. The formula of calculating a-day moving average of X at time t is presented below.

$$Ave_{a_t} = \frac{X_{t-a+1} + X_{t-a+2} + \ldots + X_t}{a}$$

The spread of a-day moving average and b-day moving average at time t is defined by

$$Spread_{ab_t} = Ave_{a_t} - Ave_{b_t}$$

Generated features also include 51 intra-market and inter-market bond spread data, derived from Japanese government bond yields, Euro area benchmark bond yields, and U.S. government bond yields. Intra-market bond spread is defined as the difference in yield between bonds with different time to maturity in the same market. Inter-markets bond spread is defined as the difference in yield between bonds with the same time to maturity but in different markets.
A detailed list of all original features and generated features is given in Appendix A.

2.4 Missing Data

Any data that are not available at the time when all the target data are available are considered as missing data. All missing data were dealt by linear interpolation, defined as follows. Given available data X_1 at time t_1 and X_2 at time t_2, any missing data X at time t (between t_1 and t_2) are calculated as

$$X = X_1 + (t - t_1)\frac{(X_2 - X_1)}{(t_2 - t_1)}$$

Features that still contain missing values after interpolation were dropped. The number of features left is 256.

2.5 Lagged Features Generation

As mentioned earlier, we use 3 past values of features to predict 1 present value of targets. Consequently, we generated a total of 768(256*3) lagged features, denoted by Xt-I (I ∈ [1,2,3], representing the number of days lagged).

2.6 Train-test split and Normalization

The data was divided into training set and test set using a 70%/30% split. Then all features were normalized based on the training set by the following procedure: Given the maximum value and minimum of value of feature X on the training set, every value of feature X is transformed as

$$X_{normalized} = \frac{X - X_{min}}{X_{max} - X_{min}}$$

2.7 Feature Selection

Another set of features were selected using LASSO regression on the normalized training set of 1-day lagged features. Features whose coefficients are 0 were discarded. A range of 0.0001 to 0.001 was considered for the regularization parameter, alpha. As shown in Figure 1, the average number of selected features goes below 20 when alpha becomes bigger than 0.0003. The speed of decline stabilizes after alpha surpasses 0.0004. As a result, 0.0005 was selected as alpha, where the average number of selected features is 13.6 (3M: 15, 2Y: 15, 5Y: 16, 10Y: 13, 30Y: 9). Selected features were then used to generate the other 2 lagged values following the procedure described in Section 2.5. After feature selection, we are only using around 5% of the original data. The list of variables name selected for each target is given in Appendix B.
Top 5 relevant features for each target sorted by the absolute value of their coefficients are listed in Table 1. For all targets, the last value of the target to predict

is always the most relevant feature. Apart from this dominant feature, additional relevant features tend to come from assets with the same or adjacent maturity type.

Figure 1.

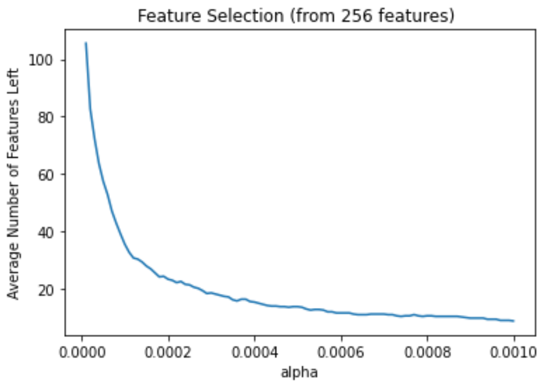


Table 1.

Top 5 relevant features for each target

Feature (1-day lagged)
3M
U.S. Treasury bond yield (3M)
U.S. Treasury bond yield (1M)
U.S.-Japan Govt bond yield Spread (1Y)
U.S. Treasury bond yield (6M)
U.S. 2Y-20Y Govt bond yield Spread
2Y
US Treasury bond yield (2Y)
5-day Moving Average (2Y)
U.S.-Japan Govt bond yield Spread (2Y)
U.S. Treasury bond yield (1Y)
U.S. Treasury bond yield (3Y)
5Y
U.S. Treasury bond yield (5Y)
5-day Moving Average (5Y)
U.S. Treasury bond yield (3Y)
U.S. Treasury bond yield (7Y)
U.S.-Japan Govt bond yield Spread (1Y)
10Y
U.S. Treasury bond yield (10Y)
U.S. Treasury bond yield (7Y)
CBOT U.S. Treasury Bond Futures price (-)
10-day Moving Average (20Y)
5-day Moving Average (20Y)
30Y
U.S. Treasury bond yield (30Y)
U.S. Treasury bond yield (20Y)
10-day Moving Average (30Y)
U.S. Purchasing Managers' Index
5-day Moving Average (30Y)

Note: (-) indicates negative coefficient

3. Methodology

3.1 Models

In this study, we compare the forecasting performance of linear models (OLS, Ridge and LASSO), support vector regression (SVR) and long short-term memory network (LSTM).

Due to the simplicity of linear regression, linear models are still popular nowadays. SVR is a type of support vector machine (SVM) which is able to model nonlinearity. LSTM is a type of recurrent neural network introducing memory cells, which can be used to deal with long-term dependencies (Chen et al., 2015).

For details of each model's algorithm, one could refer to Melkumova & Shatskikh (2017) for linear models, Drucker et al., (1996) for SVR, and Monner & Reggia (2012) for LSTM. In this section, we only discuss the hyperparameter tuning process.

3.1.1 Linear models

1) OLS Regression
   It is established as a baseline to set a benchmark for comparison between the rest of the models.
2) Ridge and LASSO
   The regularization parameter alpha:
   We repeatedly applied np.linspace(a,b,c) as possible values for alpha until the result is neither of the boundary points (a, b) and the value of (a-b)/c is equal to or smaller than 1.

3.1.2 Support Vector Regression

a) The regularization constant C:
   [0.01, 0.1, 1, 10, 100] was firstly used as possible values for C. After C is selected as c1, we repeatedly applied np.linspace(a,b,c) as possible values for C until the result is neither of the boundary points (a, b) and the value of (a-b)/c is equal to or smaller than 1.
b) Kernel function:
   Gaussian radial basis function (rbf) kernel
c) The Kernel parameter gamma: ['auto', 'scale']
d) Epsilon: [0.1, 0.3, 0.5]
e) Degree: [3, 8]

3.1.3 LSTM network

This model is composed of two LSTM layers and two dense layers.
The model written in python using Keras is shown in Figure 2.

Figure 2.

```
Inputs = Input (shape = (3, n_features))
x = LSTM(lstm1, activation='relu',
return_sequences=True)(Inputs)
x = LSTM(lstm2, activation='relu')(x)
x = Dense(dense1, activation="relu")(x)
Output = Dense(1,activation='relu'))(x)
```

Hyperparameters to be tuned include:

(a) Number of neurons for the first LSTM layer lstm1: [32, 64, 128, 256]
(b) Number of neurons for the second LSTM layer lstm2: [32, 64, 128, 256]
(c) Number of neurons for the first dense layer dense1: [32, 64, 128, 256]
(d) Epoch: [100, 300, 500]

3.2 Hyperparameter tuning

1) We applied grid search method for parameter tuning. Namely, we search exhaustively through a manually specified subset of the hyperparameter space of the targeted algorithm.
2) We applied walk-forward validation method on the training set. The set was split into three training-validation sets, as shown in Table 2. Models are trained on the training sets and tested on validation sets. Assessments were made based on model's average performance on the three validation sets.
3) We implemented mean squared error (MSE) for both hyperparameter tuning and cross-model comparison. MSE takes the form of:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(y_{actual,i} - y_{predicted,i}\right)^2$$

Model with the lowest MSE is considered as the best model.

Table 2.

split	Training sets	Validation sets	Test set
1	15/12/2003 - 4/8/2006 (661)	5/8/2006 - 26/3/2009 (661)	
2	15/12/2003 - 26/3/2009 (1322)	27/3/2009 -10/11/2011 (661)	8/7/2014 – 1/1/2019 (1123)
3	15/12/2003 – 10/11/2011 (1983)	11/11/2011 – 7/7/2014 (661)	

4. Results and Discussion

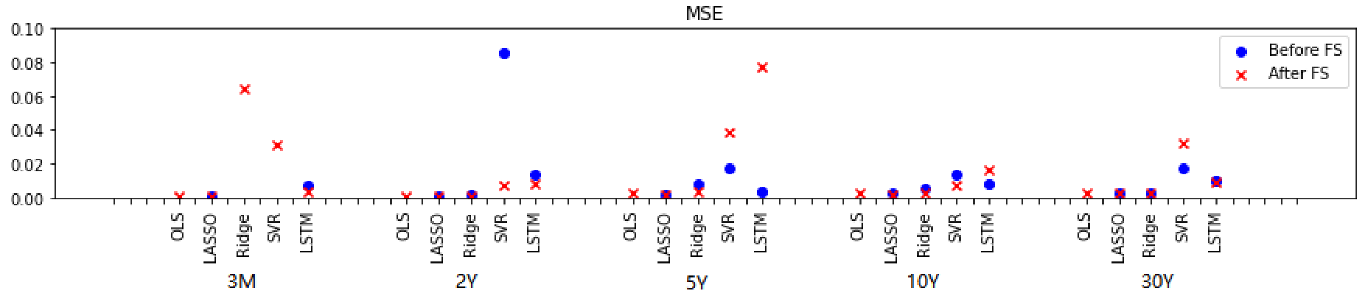
The best parameters selected for each model are listed in Table 3.

Table 3.

Model	Target	Best Parameters	Best Parameters (selected)
Ridge	3M	α: 979.05000	α: 31.53153
	2Y	α: 5.06000	α: 0.07000
	5Y	α: 6.30000	α: 0.14000
	10Y	α: 4.62000	α: 0.00006
	30Y	α: 5.69000	α: 0.02250
LASSO	3M	α: 0.00170	α: 0.00005
	2Y	α: 0.00050	α: 0.00021
	5Y	α: 0.00023	α: 0.00018
	10Y	α: 0.00021	α: 0.00013
	30Y	α: 0.00051	α: 0.00003
SVR	3M	C: 0.13143	C: 177.95918
	2Y	C: 12.00000	C: 1.80000
	5Y	C: 15.00000	C: 42.24490
	10Y	C: 11.00000	C: 5.00000
	30Y	C: 4.00000	C: 2.00000
For all models, gamma: 'auto', epsilon: 0.1, degree: 3			
LSTM	3M	[32,256,32,100]	[32, 32, 256,100]
	2Y	[32,64,256,100]	[32,128,128,500]
	5Y	[64,32, 32, 100]	[64, 64, 32, 100]
	10Y	[64,32, 32, 100]	[32, 32, 128,500]
	30Y	[128,128,64,100]	[64, 64, 128,500]
Results are presented as [lstm1, lstm2, dense1, epoch]			

Tuned models were tested on the test set. The outcomes are plotted in Figure 3 and listed in Table 4. The red color indicates the model has the lowest MSE among other models for each target.

Figure 3.



MSEs which are larger than 0.10 are not shown in this figure.

Table 4.

Comparison of model performance

MSE	3M	2Y	5Y	10Y	30Y	Average
OLS_MSE	160.50929	39.02990	13.91712	9.16283	16.30423	47.78467
OLS_MSE (FS)	0.00071	0.00122	0.00234	0.00274	0.00228	0.001858
Ridge_MSE	0.33166	0.00176	0.00818	0.00583	0.00228	0.06994
Ridge_MSE (FS)	0.06455	0.00099	0.00332	0.00272	0.00233	0.01478
LASSO_MSE	0.00043	0.00096	0.00181	0.00258	0.00227	0.00161
LASSO_MSE (FS)	0.00076	0.00098	0.00206	0.00210	0.00243	0.00167
SVR_MSE	0.16942	0.08587	0.01754	0.01362	0.01767	0.06082
SVR_MSE (FS)	0.03094	0.00728	0.03825	0.00756	0.03213	0.02323
LSTM_MSE	0.00720	0.01381	0.00395	0.00814	0.01035	0.00869
LSTM_MSE (FS)	0.00325	0.00866	0.07695	0.01649	0.00872	0.02281
Average_MSE	0.06766	0.01350	0.01716	0.00503	0.00869	

1. FS indicates the model was trained on selected features.

2. Average_MSE is calculated without OLS_MSE.

4.1 Results

1) All the models with the lowest MSE for each target are LASSO Regression. Moreover, on average, LASSO Regression performed better than other models both before and after feature selection.
2) On average, OLS, Ridge Regression and SVR performed better after feature selection, LSTM performed worse, while LASSO Regression gained similar results. It may be due to the fact that LASSO was the model used for feature selection.
3) On average, our models tend to perform better for bonds with longer time to maturity, especially for 10Y and 30Y bonds.

4.2 Discussion

4.2.1 Limitation on the LSTM model

Deep learning models like LSTM do not have a predetermined structure; thus, they potentially have a large amount of hyperparameters to tune. Due to our lack of practice in deep learning models and limited computational power, we only tuned a small number of parameters on one type of model structure.

It could lead to underperformance of LSTM model.

4.2.2 Suggestions

Further studies may investigate different models' performance after increasing the forecasting horizons or changing the time range of lagged features needed in the model as we only used lagged features up to 3 days.

5.Conclusion

Our study shows that LASSO performs consistently well in predicting U.S. Treasury bond yields compared with our other models. The good result even when it was trained on only around 5% of the original data suggests that LASSO could predicted U.S. Treasury bond yields well using only a small amount of technically indicators of the Treasury bond market.

The reason why LASSO provided a satisfying result could be the fact that shrinking and removing irrelevant features can reduce variance without a substantial increase of the bias, which is especially useful when we are using a large number of features. Our results above also revealed that LSTM is the second-best model in predicting U.S. bond yields on average. Considering the relatively rough hyperparameter tuning process we conducted on this model, it may still have potentials for better performance.

References

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Appendix A

Initial List of Features

Group	Ticker	Feature name	source
Original Features			
Government Bond Yields			
U.S.	1M	U.S. Treasury bond yield (1M)	U.S. Department of the Treasury
	3M	U.S. Treasury bond yield (3M)	U.S. Department of the Treasury
	6M	U.S. Treasury bond yield (6M)	U.S. Department of the Treasury
	1Y	U.S. Treasury bond yield (1Y)	U.S. Department of the Treasury
	2Y	U.S. Treasury bond yield (2Y)	U.S. Department of the Treasury
	3Y	U.S. Treasury bond yield (3Y)	U.S. Department of the Treasury
	5Y	U.S. Treasury bond yield (5Y)	U.S. Department of the Treasury
	7Y	U.S. Treasury bond yield (7Y)	U.S. Department of the Treasury
	10Y	U.S. Treasury bond yield (10Y)	U.S. Department of the Treasury
	20Y	U.S. Treasury bond yield (20Y)	U.S. Department of the Treasury
	30Y	U.S. Treasury bond yield (30Y)	U.S. Department of the Treasury
Europe	ECB2Y	Euro area 2Y Govt Benchmark bond yield	Quandl
	ECB3Y	Euro area 3Y Govt Benchmark bond yield	Quandl
	ECB5Y	Euro area 5Y Govt Benchmark bond yield	Quandl
	ECB7Y	Euro area 7Y Govt Benchmark bond yield	Quandl
	ECB10Y	Euro area 10Y Govt Benchmark bond yield	Quandl
Japan	JGB1Y	Japan Govt bond yield (1Y)	Quandl
	JGB2Y	Japan Govt bond yield (2Y)	Quandl
	JGB3Y	Japan Govt bond yield (3Y)	Quandl
	JGB4Y	Japan Govt bond yield (4Y)	Quandl
	JGB5Y	Japan Govt bond yield (5Y)	Quandl
	JGB6Y	Japan Govt bond yield (6Y)	Quandl
	JGB7Y	Japan Govt bond yield (7Y)	Quandl
	JGB8Y	Japan Govt bond yield (8Y)	Quandl
	JGB9Y	Japan Govt bond yield (9Y)	Quandl
	JGB10Y	Japan Govt bond yield (10Y)	Quandl
	JGB15Y	Japan Govt bond yield (15Y)	Quandl
	JGB20Y	Japan Govt bond yield (20Y)	Quandl
	JGB25Y	Japan Govt bond yield (25Y)	Quandl
	JGB30Y	Japan Govt bond yield (30Y)	Quandl
	JGB35Y	Japan Govt bond yield (35Y)	Quandl
	JGB40Y	Japan Govt bond yield (40Y)	Quandl
U.K.	BOE5Y	British Govt bond yield (5Y)	Quandl
	BOE10Y	British Govt bond yield (10Y)	Quandl
	BOE20Y	British Govt bond yield (20Y)	Quandl
Coporate Bond Yield			
	DAAA	Moody's Seasoned Aaa Corporate Bond Yield	FRED
	BAA10Y	DAAA Relative to Yield on 10Y Treasury bond yield	FRED
Interest Rate			
U.S.	DFF	Effective Federal Funds Rate	FRED
	DPRIME	U.S. Commercial Bank Prime Loan Rate	FRED
Europe	ECBinterest	Ecb Interest Rates For Main Refinancing Operations	Quandl
U.K.	USD1MTD156N	1-Month London Interbank Offered Rate (LIBOR)	FRED
	USD3MTD156N	3-Month London Interbank Offered Rate (LIBOR)	FRED
Equities			
	^DJI	Dow Jones Industrial Average	Yahoo Finance
	^GSPC	S&P 500 Index	Yahoo Finance
	^IXIC	NASDAQ Composite	Yahoo Finance
	FX.F	Euro Stoxx 50	Stooq
	X.F	FTSE 100 - Euronext	Stooq
	ES.F	S&P 500 E-Mini - CME	Stooq
	^W5000	Wilshire 5000 Total Market Index	Yahoo Finance
	^RUT	Russell 2000	Yahoo Finance
	^RUA	Russell 3000	Yahoo Finance
	^HSI	HANG SENG INDEX	Yahoo Finance
Currencies			
	JPY=X	USD/JPY	Yahoo Finance
	GBP=X	USD/GBP	Yahoo Finance
	EUR=X	USD/EUR	Yahoo Finance
	CHF=X	USD/CHF	Yahoo Finance
	CNY=X	USD/CNY	Yahoo Finance
	DX-Y.NYB	US Dollar/USDX (US Dollar Index)	Yahoo Finance

Commodities			
	^CRY	Commodity Research Bureau (CRB) Index	Yahoo Finance
	CL=F	Crude Oil Dec 20	Yahoo Finance
	GC=F	Gold Dec 20	Yahoo Finance
Forward Rates			
U.S.	THREEFF1	Fitted Instantaneous Forward Rate 1 Years Hence	FRED
	THREEFF2	Fitted Instantaneous Forward Rate 2 Years Hence	FRED
	THREEFF3	Fitted Instantaneous Forward Rate 3 Years Hence	FRED
	THREEFF5	Fitted Instantaneous Forward Rate 5 Years Hence	FRED
	THREEFF7	Fitted Instantaneous Forward Rate 7 Years Hence	FRED
	THREEFF10	Fitted Instantaneous Forward Rate 10 Years Hence	FRED
Futures			
U.S.	ZB=F	CBOT U.S. Treasury Bond Futures price	Yahoo Finance
	YM=F	Mini Dow Jones Indus.-$5 Dec 20	Yahoo Finance
Volatility			
U.S.	VIXCLS	CBOE Volatility Index: VIX	FRED
	EMVMONETARYPOL	Equity Market Volatility Tracker: Monetary Policy	FRED
	EMVEXRATES	Equity Market Volatility Tracker: Exchange Rates	FRED
	EMVMACROINFLATION	Equity Market Volatility Tracker: Inflation	FRED
	EMVGOVTSPEND	Equity Market Volatility Tracker: Government Deficts	FRED
	EMVMACROINTEREST	Equity Market Volatility Tracker: Interest Rates	FRED
	INFECTDISEMVTRACKD	Equity Market Volatility: Infectious Disease Tracker	FRED
Economic Indicators			
Global Indicators	EA19CLI	COMPOSITE LEADING INDICATOR: 19 EA COUNTRIES	OECD
	G-7CLI	COMPOSITE LEADING INDICATOR: G7	OECD
	USACLI	COMPOSITE LEADING INDICATOR: US	OECD
	OECDCLI	COMPOSITE LEADING INDICATOR: total	OECD
	CHNCLI	COMPOSITE LEADING INDICATOR: China	OECD
Govt Deficits	LUXdeficit	Government deficit /GDP	Trading Economics
	JPNdeficit	Government deficit /GDP	Trading Economics
	IRLdeficit	Government deficit /GDP	Trading Economics
	GBRdeficit	Government deficit /GDP	Trading Economics
	CHNdeficit	Government deficit /GDP	Trading Economics
	CHEdeficit	Government deficit /GDP	Trading Economics
	BRAdeficit	Government deficit /GDP	Trading Economics
	USAdeficit	Government deficit /GDP	Trading Economics
	HKdeficit	Government deficit /GDP	Trading Economics
U.S. Inflation	BOGZ1FL075035503Q	Commercial Real Estate Price Index	FRED
	GDPDEF	Gross Domestic Product: Implicit Price Deflator	FRED
	CPILFESL	Consumer Price Index for All Urban Consumers	FRED
	MICH	University of Michigan: Inflation Expectation	FRED
FED Balance Sheets	RELACBW027SBOG	Real Estate Loans: All Commercial Banks	FRED
	WALCL	FED Total Assets:Wednesday Level	FRED
	TOTBORR	Total Borrowings of Depository Institutions from FED	FRED
	WRESBAL	Reserve Balances with Federal Reserve Banks	FRED
	BOGZ1FL614090610Q	Finance Companies' Total Assets (Balance Sheet)	FRED
U.S. Employment	PAYEMS	All Employees, Total Nonfarm	FRED
	UNRATE	Unemployment Rate (percentage)	FRED
	ICSA	Initial Claims	FRED
U.S. GDP	GDPC1	Real Gross Domestic Product	FRED
	DSPIC96	Real Disposable Personal Income	FRED
	GPDIC1	Real Gross Private Domestic Investment	FRED
	GCEC1	Real Govt Consumption and Gross Investment	FRED
	ROWFDNQ027S	Foreign Direct Investment in U.S (Level)	FRED
	FBDIAEQ027S	U.S. Direct Investment Abroad (Level)	FRED
	ROWFDIQ027S	Foreign Direct Investment in U.S.(Current Cost)	FRED
	MEHOINUSA672N	Real Median Household Income in the U.S.	FRED
	PSAVERT	Personal Saving Rate	FRED
Others (U.S.)	M2	M2 Money Stock	FRED
	INDPRO	Industrial Production: Total Index	FRED
	FBTFASQ027S	Domestic Total Financial Assets (Level)	FRED
	TCU	Capacity Utilization: Total Index	FRED
	PMI	PMI Composite Index	FRED
Generated Features			
Government Bond Spreads			
Inter-markets	USJPBDSP1Y	U.S.-Japan Govt bond yield Spread (1Y)	
	USJPBDSP2Y	U.S.-Japan Govt bond yield Spread (2Y)	
	USJPBDSP3Y	U.S.-Japan Govt bond yield Spread (3Y)	
	USJPBDSP5Y	U.S.-Japan Govt bond yield Spread (5Y)	

	USJPBDSP7Y	U.S.-Japan Govt bond yield Spread (7Y)
	USJPBDSP10Y	U.S.-Japan Govt bond yield Spread (10Y)
	USJPBDSP20Y	U.S.-Japan Govt bond yield Spread (20Y)
	USJPBDSP30Y	U.S.-Japan Govt bond yield Spread (30Y)
	USEUBDSP2Y	U.S. -Euro Govt bond yield Spread (2Y)
	USEUBDSP3Y	U.S. -Euro Govt bond yield Spread (3Y)
	USEUBDSP5Y	U.S. -Euro Govt bond yield Spread (5Y)
	USEUBDSP7Y	U.S. -Euro Govt bond yield Spread (7Y)
	USEUBDSP10Y	U.S. -Euro Govt bond yield Spread (10Y)
Intra-markets	US1Y-2Y	U.S. Treasury bond yield Spread (1Y-2Y)
U.S.	US1Y-3Y	U.S. Treasury bond yield Spread (1Y-3Y)
	US1Y-5Y	U.S. Treasury bond yield Spread (1Y-5Y)
	US1Y-7Y	U.S. Treasury bond yield Spread (1Y-7Y)
	US1Y-10Y	U.S. Treasury bond yield Spread (1Y-10Y)
	US1Y-15Y	U.S. Treasury bond yield Spread (1Y-15Y)
	US1Y-20Y	U.S. Treasury bond yield Spread (1Y-20Y)
	US1Y-30Y	U.S. Treasury bond yield Spread (1Y-30Y)
	US2Y-3Y	U.S. Treasury bond yield Spread (2Y-3Y)
	US2Y-5Y	U.S. Treasury bond yield Spread (2Y-5Y)
	US2Y-7Y	U.S. Treasury bond yield Spread (2Y-7Y)
	US2Y-10Y	U.S. Treasury bond yield Spread (2Y-10Y)
	US2Y-20Y	U.S. Treasury bond yield Spread (2Y-20Y)
	US2Y-30Y	U.S. Treasury bond yield Spread (2Y-30Y)
	US3Y-5Y	U.S. Treasury bond yield Spread (3Y-5Y)
	US3Y-7Y	U.S. Treasury bond yield Spread (3Y-7Y)
	US3Y-10Y	U.S. Treasury bond yield Spread (3Y-10Y)
	US3Y-20Y	U.S. Treasury bond yield Spread (3Y-20Y)
	US3Y-30Y	U.S. Treasury bond yield Spread (3Y-30Y)
	US5Y-7Y	U.S. Treasury bond yield Spread (5Y-7Y)
	US5Y-10Y	U.S. Treasury bond yield Spread (5Y-10Y)
	US5Y-20Y	U.S. Treasury bond yield Spread (5Y-20Y)
	US5Y-30Y	U.S. Treasury bond yield Spread (5Y-30Y)
	US7Y-10Y	U.S. Treasury bond yield Spread (7Y-10Y)
	US7Y-20Y	U.S. Treasury bond yield Spread (7Y-20Y)
	US7Y-30Y	U.S. Treasury bond yield Spread (7Y-30Y)
	US10Y-20Y	U.S. Treasury bond yield Spread (10Y-20Y)
	US10Y-30Y	U.S. Treasury bond yield Spread (10Y-30Y)
	US20Y-30Y	U.S. Treasury bond yield Spread (20Y-30Y)
Europe	ECB2Y-ECB3Y	Euro area Govt Benchmark bond yield Spread (2Y-3Y)
	ECB2Y-ECB5Y	Euro area Govt Benchmark bond yield Spread (2Y-5Y)
	ECB2Y-ECB7Y	Euro area Govt Benchmark bond yield Spread (2Y-7Y)
	ECB2Y-ECB10Y	Euro area Govt Benchmark bond yield Spread (2Y-10Y)
	ECB5Y-ECB3Y	Euro area Govt Benchmark bond yield Spread (5Y-3Y)
	ECB5Y-ECB7Y	Euro area Govt Benchmark bond yield Spread (5Y-7Y)
	ECB5Y-ECB10Y	Euro area Govt Benchmark bond yield Spread (5Y-10Y)
	ECB7Y-ECB3Y	Euro area Govt Benchmark bond yield Spread (7Y-3Y)
	ECB7Y-ECB10Y	Euro area Govt Benchmark bond yield Spread (7Y-10Y)
	ECB10Y-ECB3Y	Euro area Govt Benchmark bond yield Spread (10Y-3Y)
Moving Averages		
1,2,3,5,7,10, 20,30Y (n)	n_SMA5	5-day Moving Average for variable n
	n_SMA10	10-day Moving Average for variable n
	n_SMA15	15-day Moving Average for variable n
	n_SMA20	20-day Moving Average for variable n
	n_SMA50	50-day Moving Average for variable n
	n_SMA200	200-day Moving Average for variable n
	n_SMA10_4	10-day MA minus 4-day MA for variable n
	n_SMA24_14	24-day MA minus 14-day MA for variable n
	n_SMA48_35	48-day MA minus 35-day MA for variable n
Percentage Change		
	BOGZ1FL075035503Q_ptc	Commercial Real Estate Price Index
	RELACBW027SBOG_ptc	Real Estate Loans: All Commercial Banks
	BOGZ1FL614090610Q_ptc	Finance Companies' Total Assets (Balance Sheet)
	FBDIAEQ027S_ptc	U.S. Direct Investment Abroad (Level)
	GDPC1_ptc	Real Gross Domestic Product
	DSPIC96_ptc	Real Disposable Personal Income
	GPDIC1_ptc	Real Gross Private Domestic Investment
	INDPRO_ptc	Industrial Production: Total Index
	FBTFASQ027S_ptc	Domestic Total Financial Assets (Level)
	ROWFDNQ027S_ptc	Foreign Direct Investment in U.S (Level)
	PAYEMS_ptc	All Employees, Total Nonfarm
	M2_ptc	M2 Money Stock

Appendix B

Part 1: Selected Features

3M: ['1Mt-1', '3Mt-1', '6Mt-1', '2Yt-1', 'EMVMONETARYPOLt-1', 'DFFt-1', 'EA19CLIt-1', 'CHNCLIt-1', 'DPRIMEt-1', 'USJPBDSP1Yt-1', 'US2Y-20Yt-1', 'US7Y-10Yt-1', '1Y_SMA5t-1', '2Y_SMA5t-1', '3Y_SMA24_14t-1']

2Y: ['1Yt-1', '2Yt-1', '3Yt-1', 'BOGZ1FL614090610Qt-1', 'EMVMONETARYPOLt-1', 'WALCLt-1', 'DFFt-1', 'JGB3Yt-1', 'JGB4Yt-1', 'MEHOINUSA672Nt-1', 'USJPBDSP2Yt-1', 'US3Y-10Yt-1', '2Y_SMA5t-1', '3Y_SMA5t-1', '2Y_SMA10t-1']

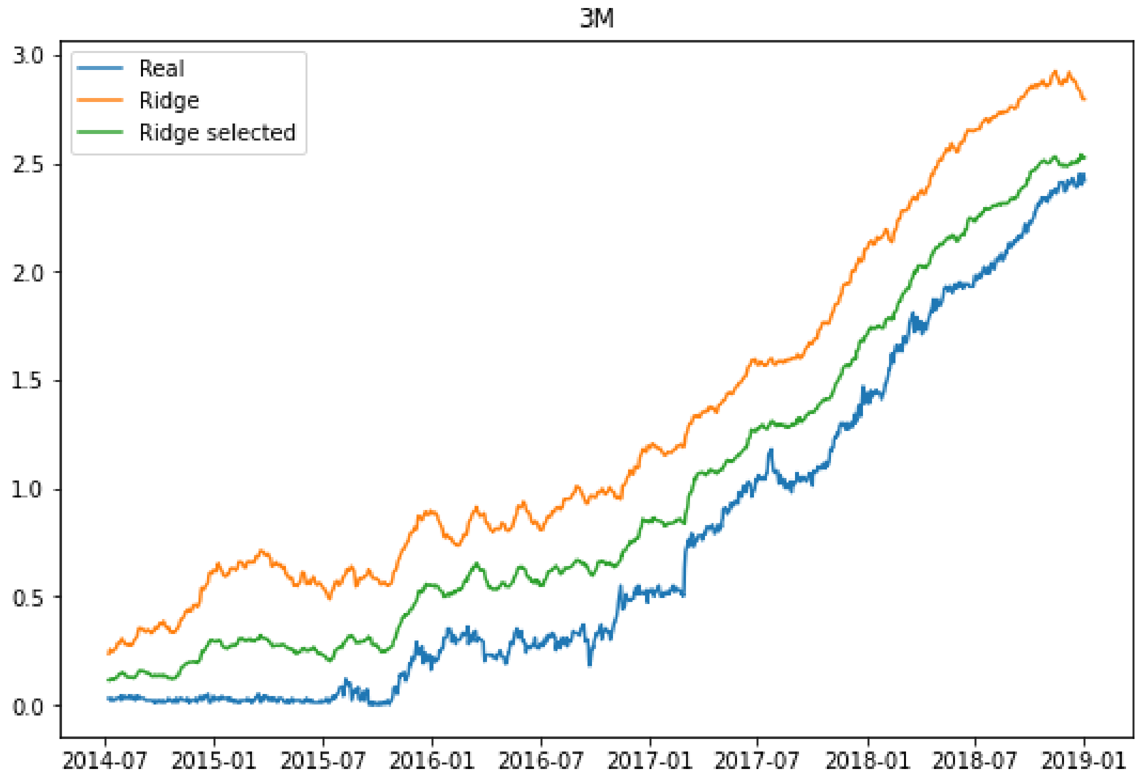
5Y: ['3Yt-1', '5Yt-1', '7Yt-1', 'EMVMONETARYPOLt-1', 'MICHt-1', 'GC=Ft-1', 'JPY=Xt-1', 'ZB=Ft-1', 'JGB7Yt-1', 'BOE5Yt-1', 'BRAdeficitt-1', 'USJPBDSP5Yt-1', 'USEUBDSP7Yt-1', 'USECB5Y-ECB7Yt-1', '5Y_SMA5t-1', '7Y_SMA10t-1']

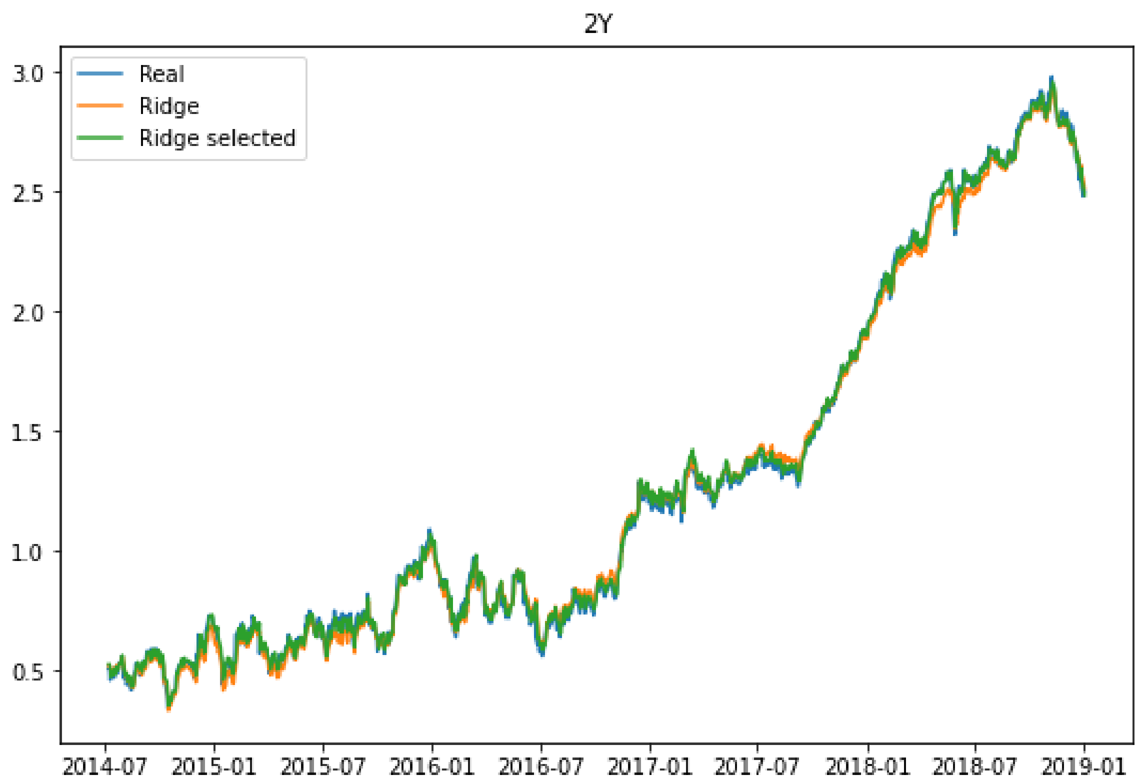
10Y: ['7Yt-1', '10Yt-1', 'EA19CLIt-1', 'DSWP30t-1', 'USD1MTD156Nt-1', 'JPY=Xt-1', 'ZB=Ft-1', 'BOE10Yt-1', 'USJPBDSP10Yt-1', 'USEUBDSP10Yt-1', '20Y_SMA5t-1', '10Y_SMA10t-1', '20Y_SMA10t-1']

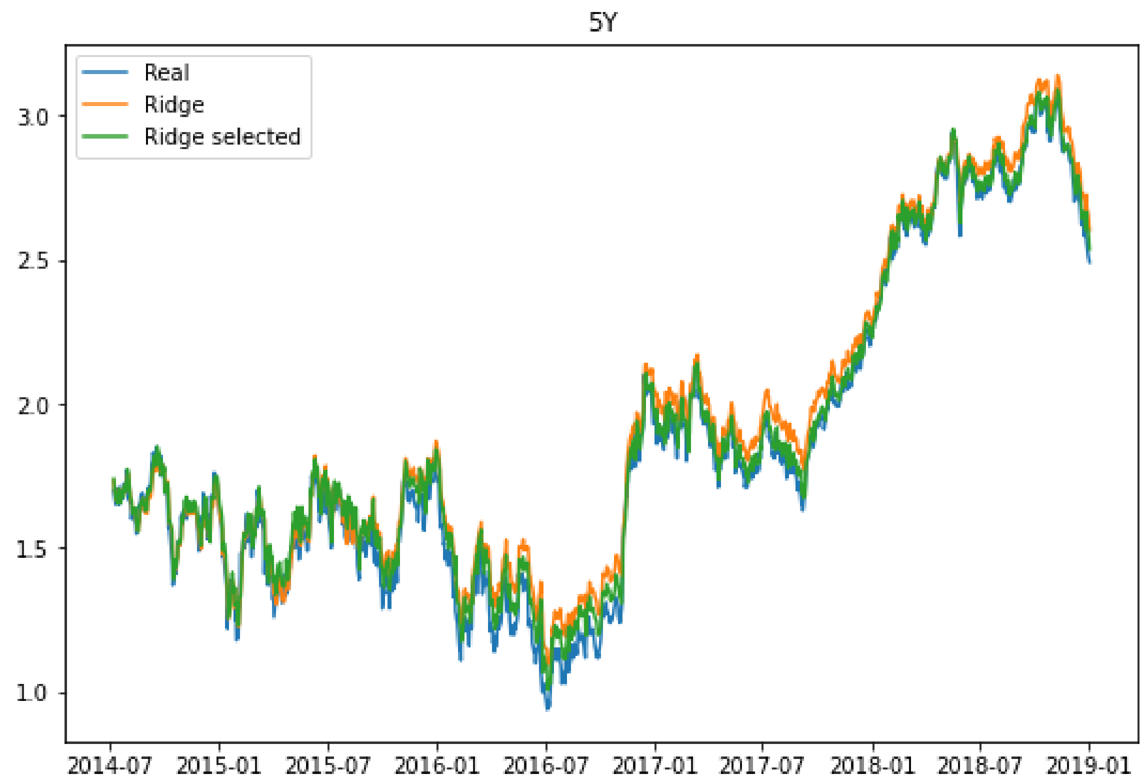
30Y: ['20Yt-1', '30Yt-1', 'WALCLt-1', 'DSWP30t-1', 'CNY=Xt-1', 'PMIt-1', 'BRAdeficitt-1', '30Y_SMA5t-1', '30Y_SMA10t-1']}

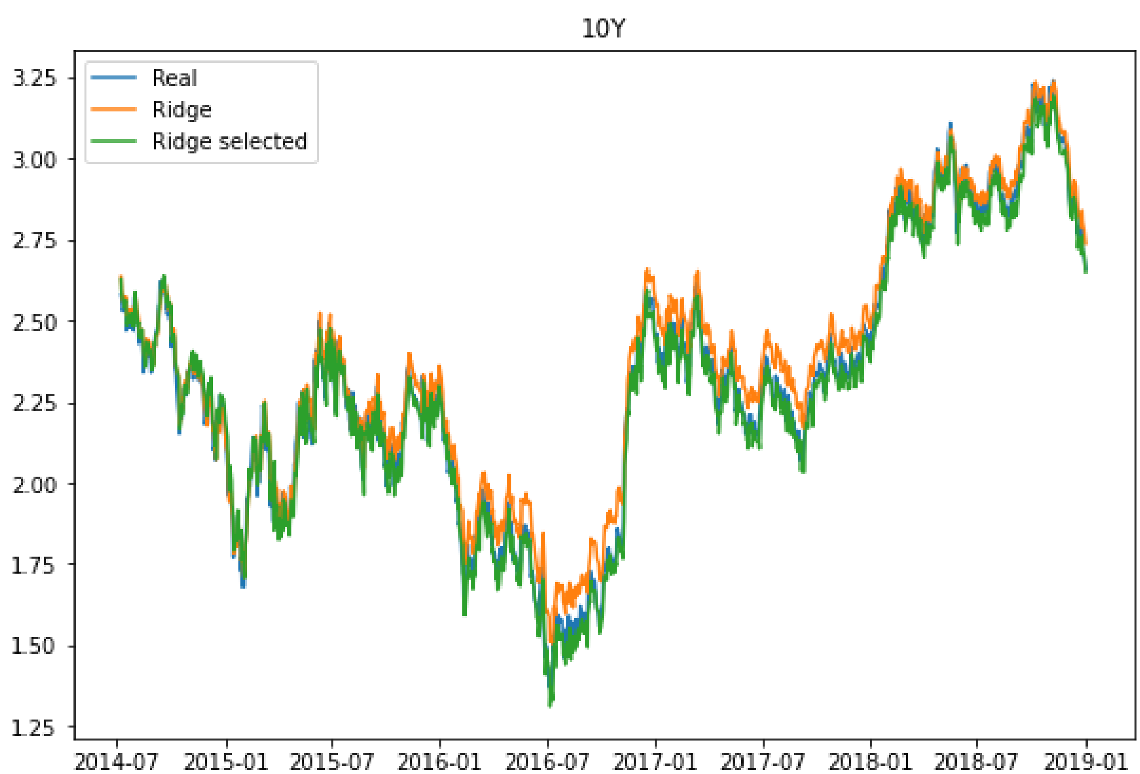
Part 2: Plots

Ridge Regression:






5Y
Real
Ridge
Ridge selected
3.0
2.5
2.0
1.5
1.0
2014-07
2015-01
2015-07
2016-01
2016-07
2017-01
2017-07
2018-01
2018-07
2019-01


10Y
Real
Ridge
Ridge selected
3.25
3.00
2.75
2.50
2.25
2.00
1.75
1.50
1.25
2014-07
2015-01
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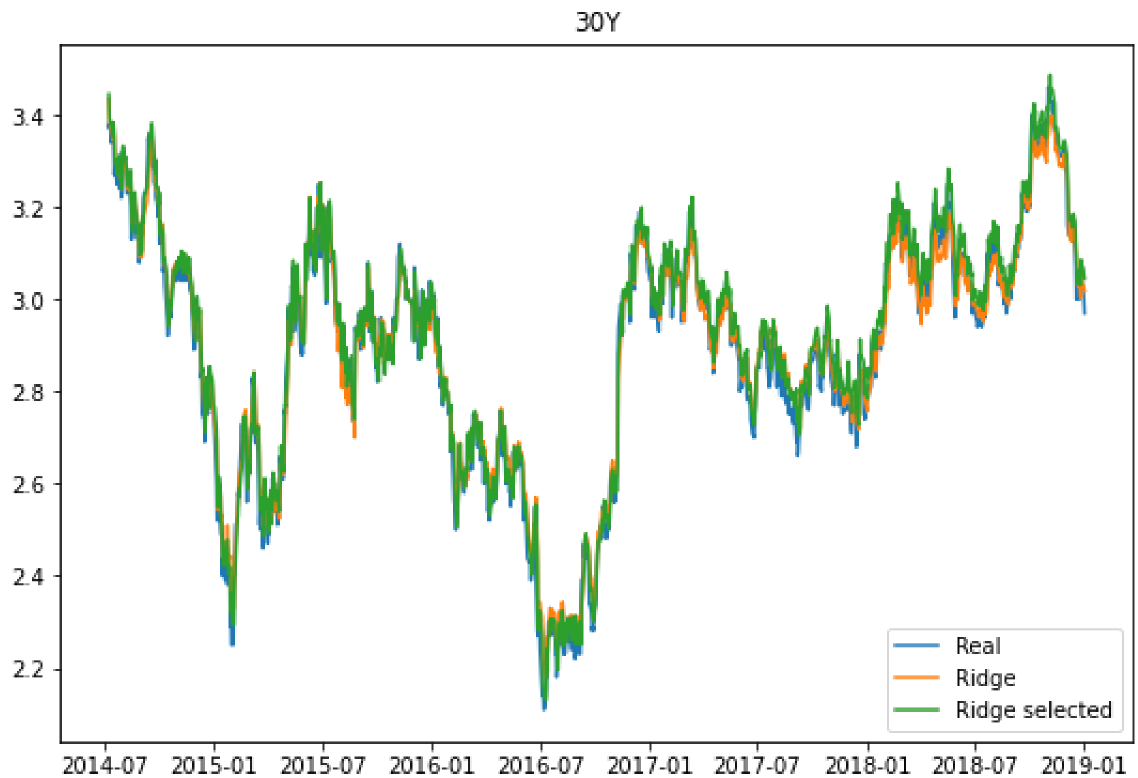
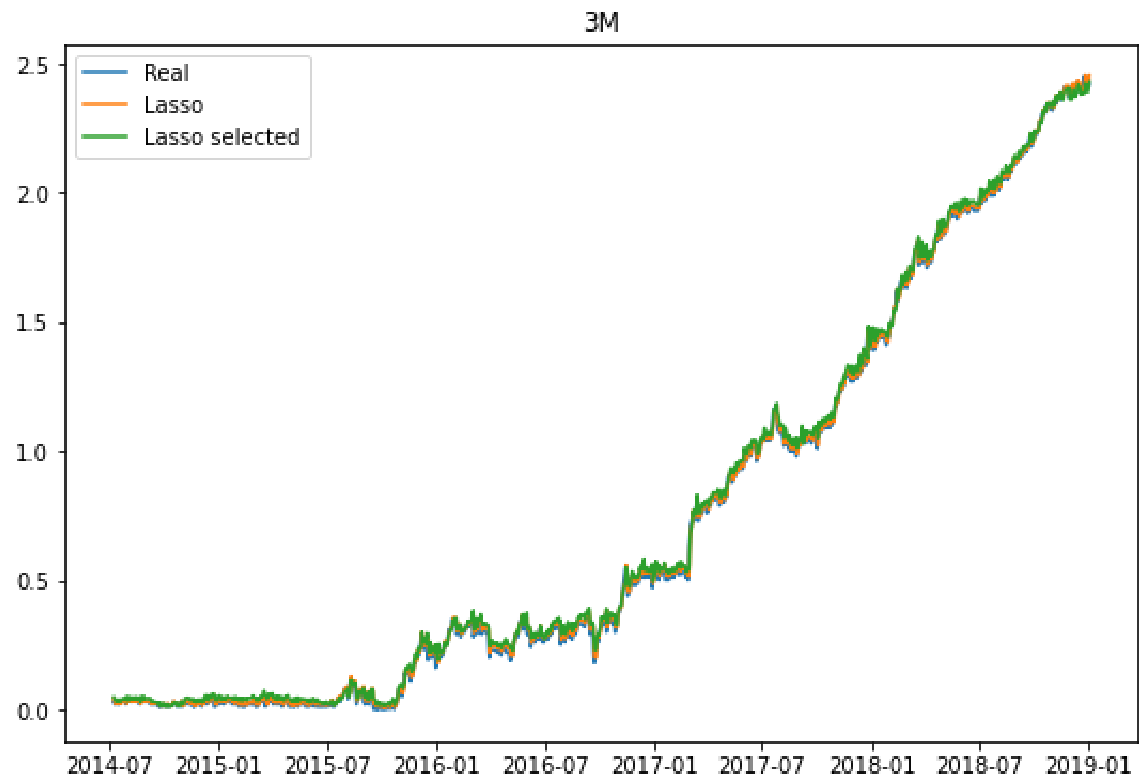
2018-01
2018-07
2019-01


30Y
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2014-07
2015-01
2015-07
2016-01
2016-07
2017-01
2017-07
2018-01
2018-07
2019-01
Real
Ridge
Ridge selected

LASSO:


3M
Real
Lasso
Lasso selected
2.5
2.0
1.5
1.0
0.5
0.0
2014-07
2015-01
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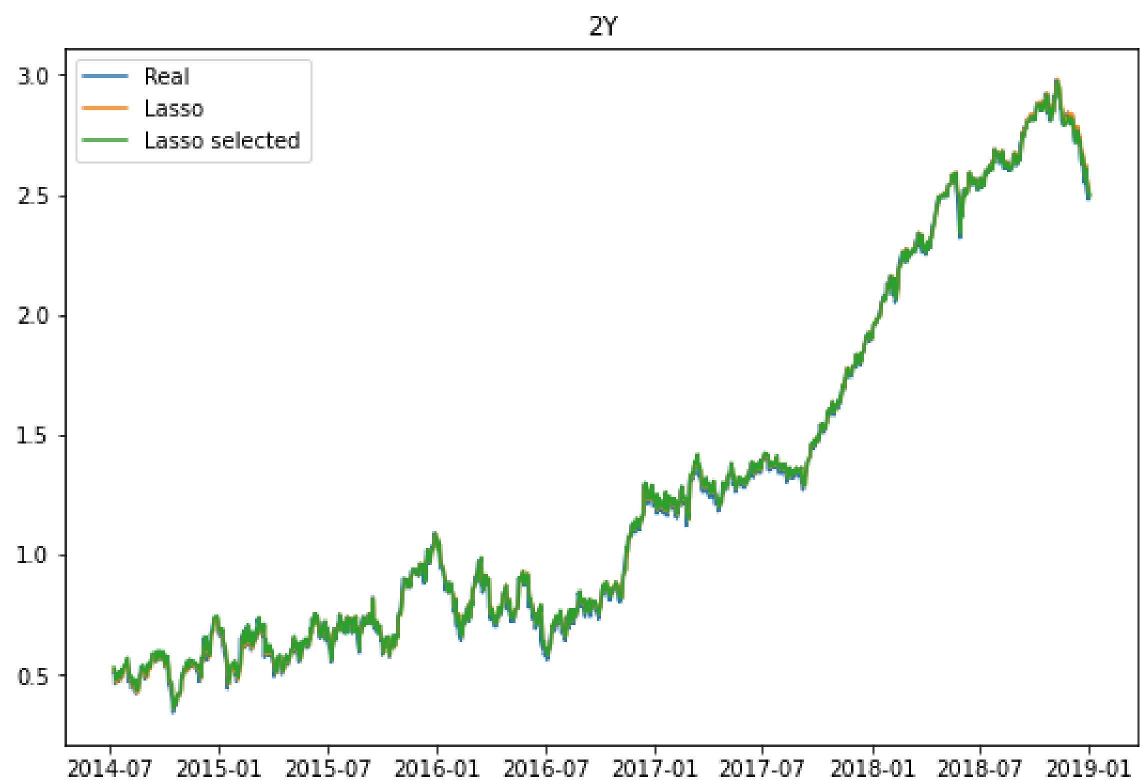
2018-01
2018-07
2019-01


2Y
Real
Lasso
Lasso selected
3.0
2.5
2.0
1.5
1.0
0.5
2014-07
2015-01
2015-07
2016-01
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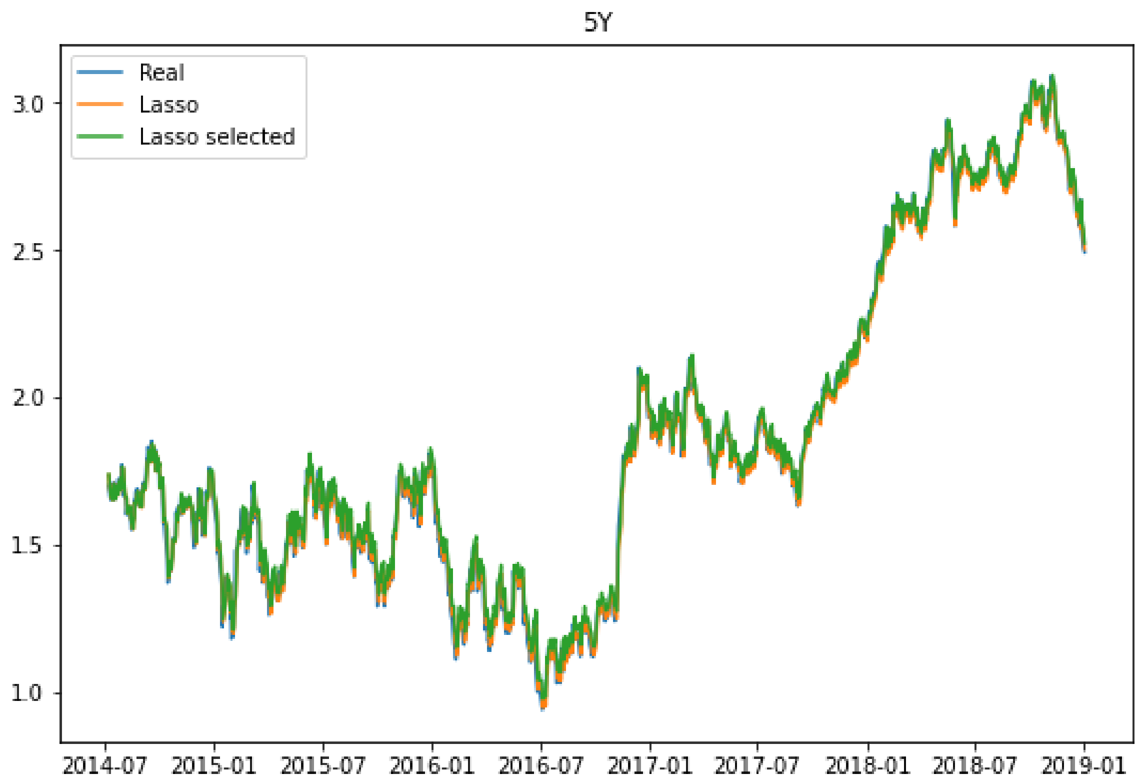
2018-01
2018-07
2019-01


5Y
Real
Lasso
Lasso selected
3.0
2.5
2.0
1.5
1.0
2014-07
2015-01
2015-07
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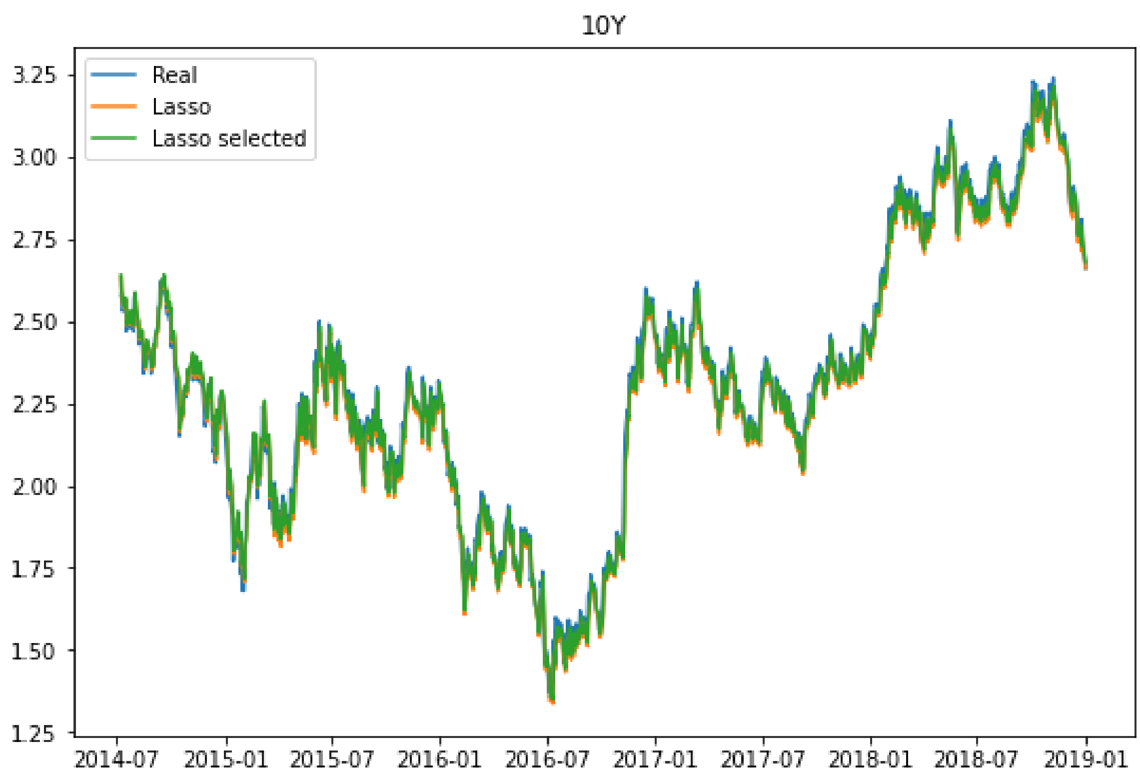
2018-01
2018-07
2019-01


10Y
Real
Lasso
Lasso selected
3.25
3.00
2.75
2.50
2.25
2.00
1.75
1.50
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2015-01
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2016-01
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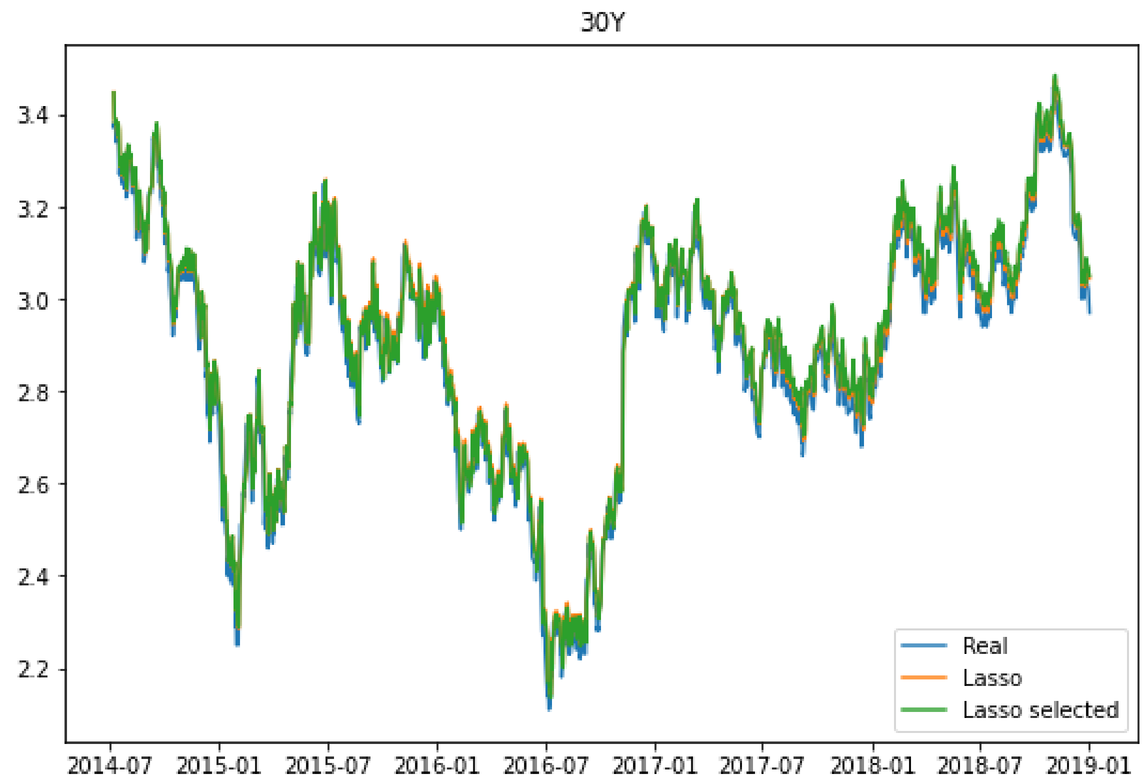
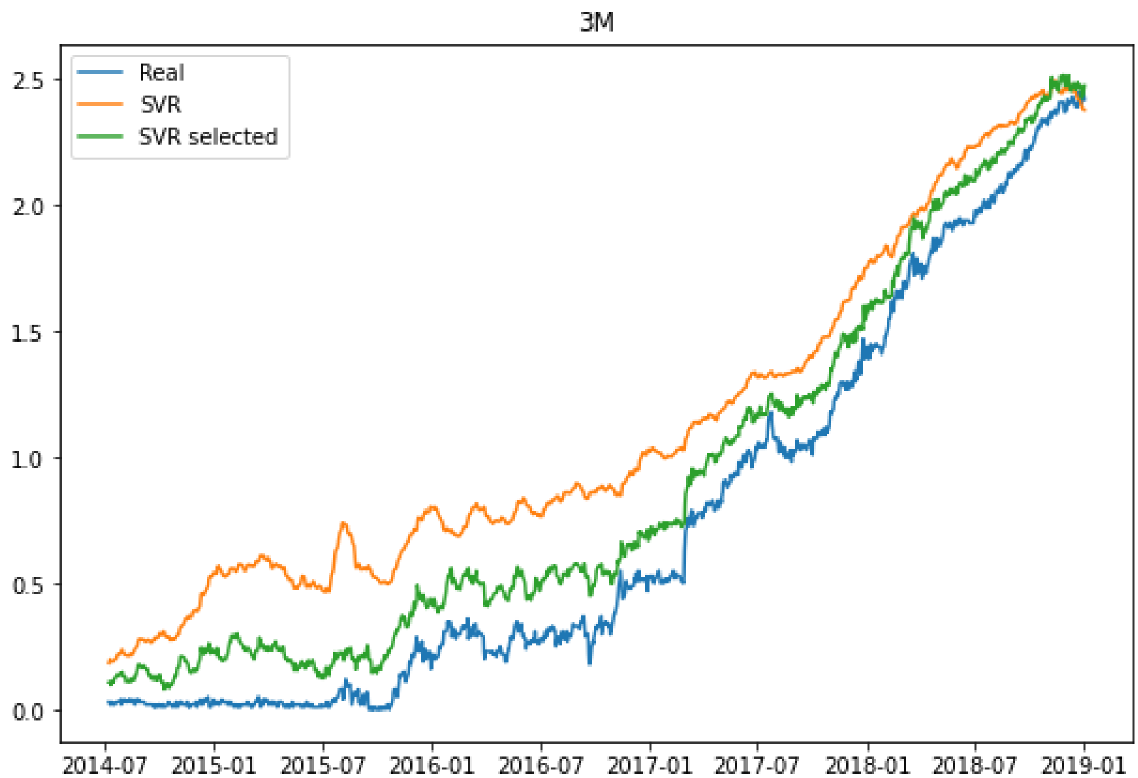
2018-01
2018-07
2019-01



30Y
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2014-07
2015-01
2015-07
2016-01
2016-07
2017-01
2017-07
2018-01
2018-07
2019-01
Real
Lasso
Lasso selected

SVR:


3M
Real
SVR
SVR selected
2.5
2.0
1.5
1.0
0.5
0.0
2014-07
2015-01
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2016-01
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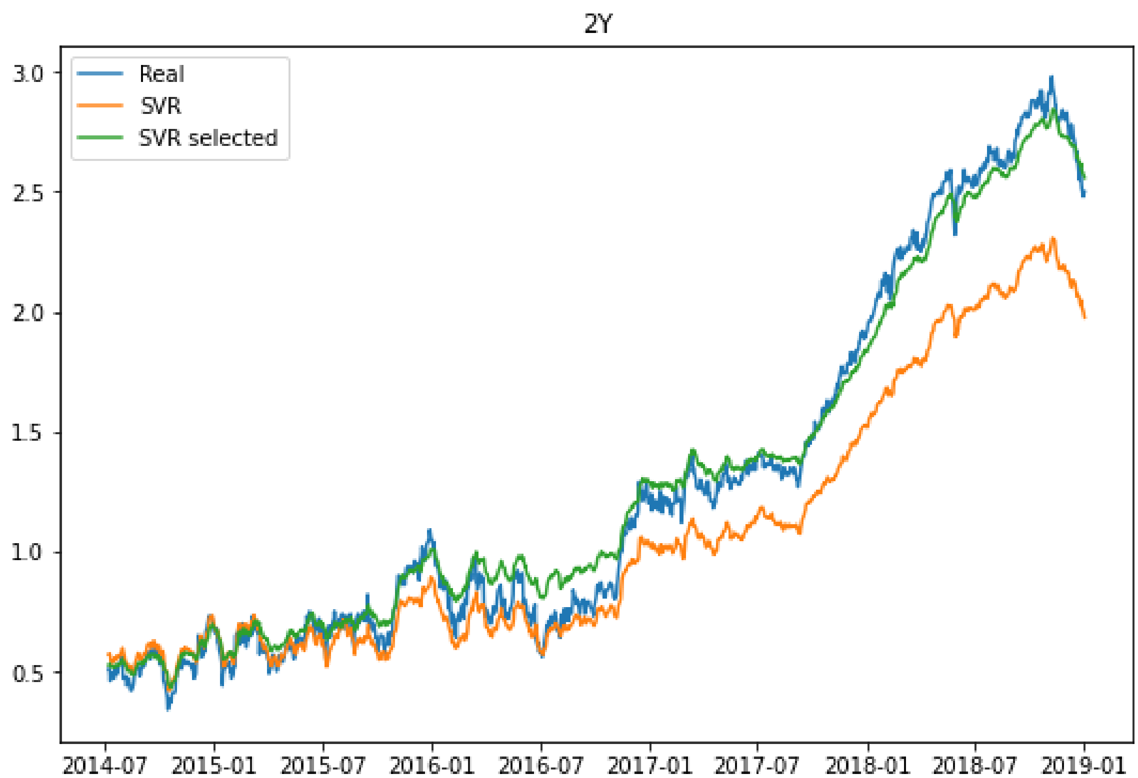
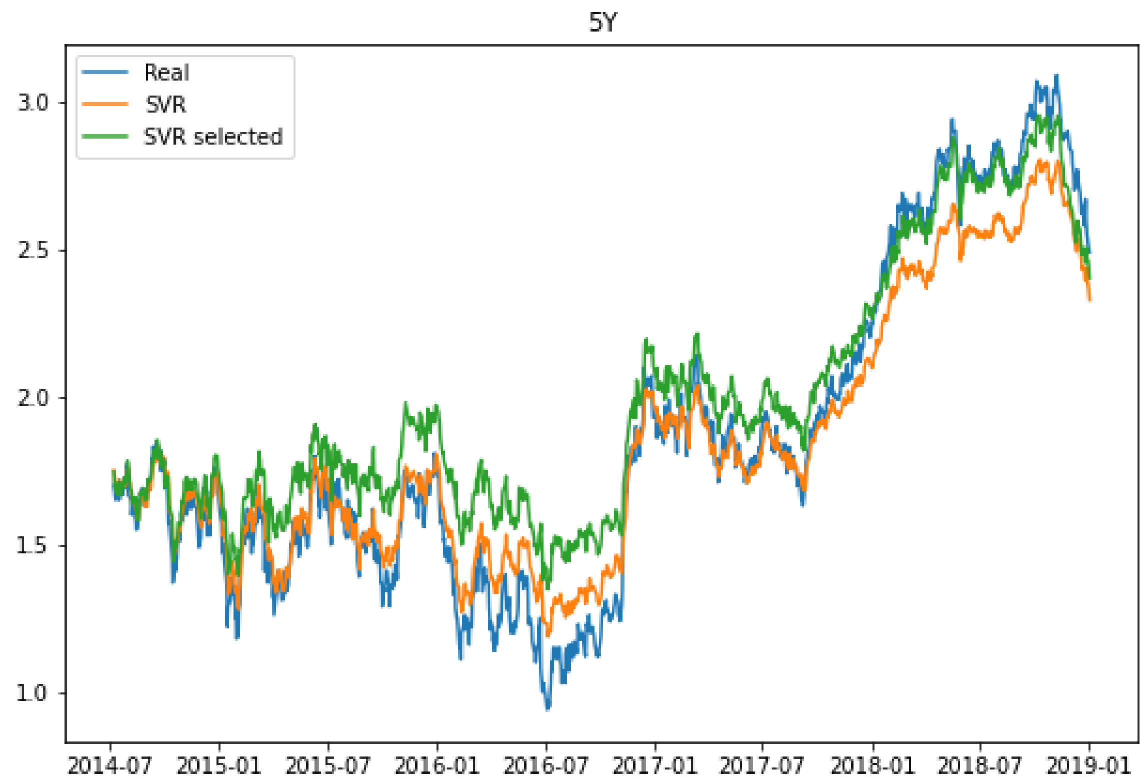
2017-01
2017-07
2018-01
2018-07
2019-01


2Y
Real
SVR
SVR selected
3.0
2.5
2.0
1.5
1.0
0.5
2014-07
2015-01
2015-07
2016-01
2016-07
2017-01
2017-07
2018-01
2018-07
2019-01


5Y
Real
SVR
SVR selected
3.0
2.5
2.0
1.5
1.0
2014-07
2015-01
2015-07
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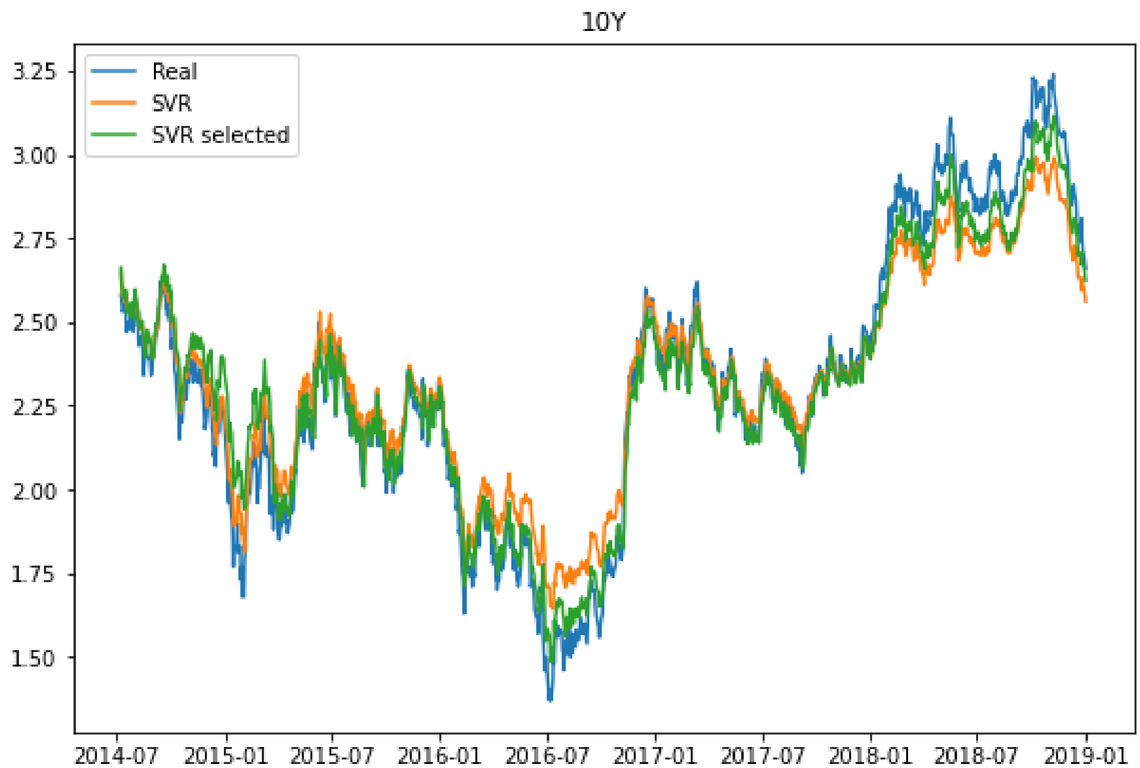
2018-01
2018-07
2019-01


10Y
Real
SVR
SVR selected
3.25
3.00
2.75
2.50
2.25
2.00
1.75
1.50
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2015-01
2015-07
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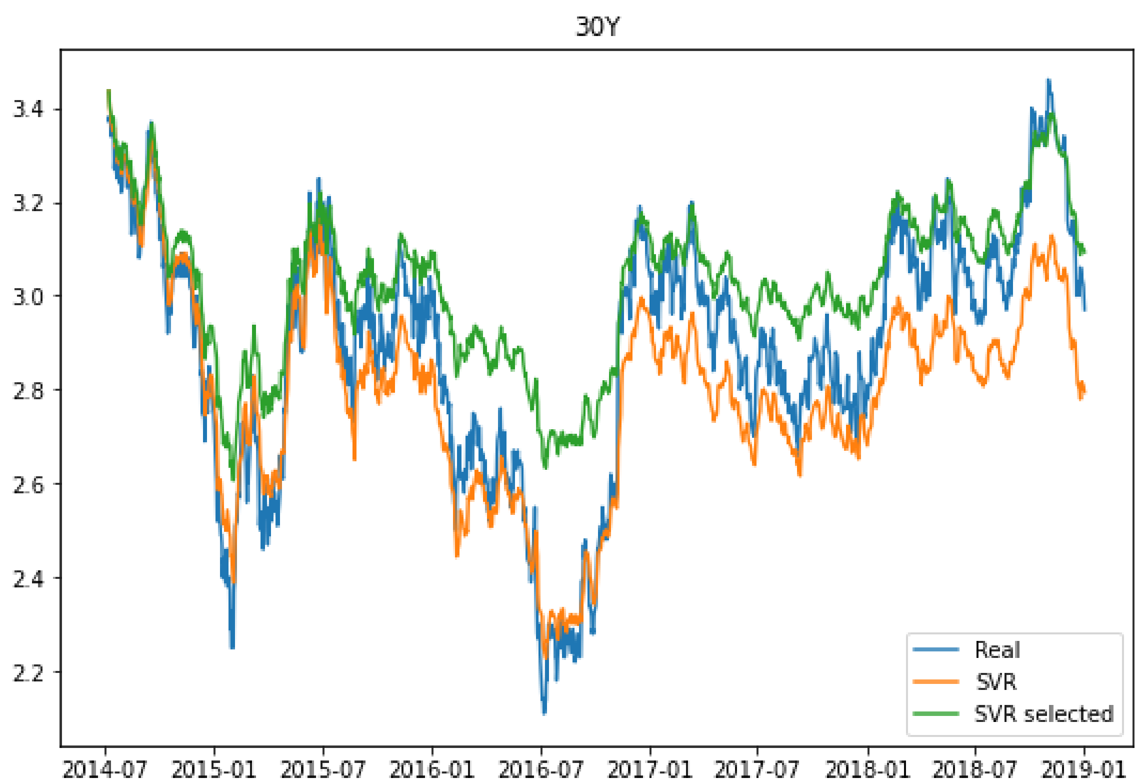
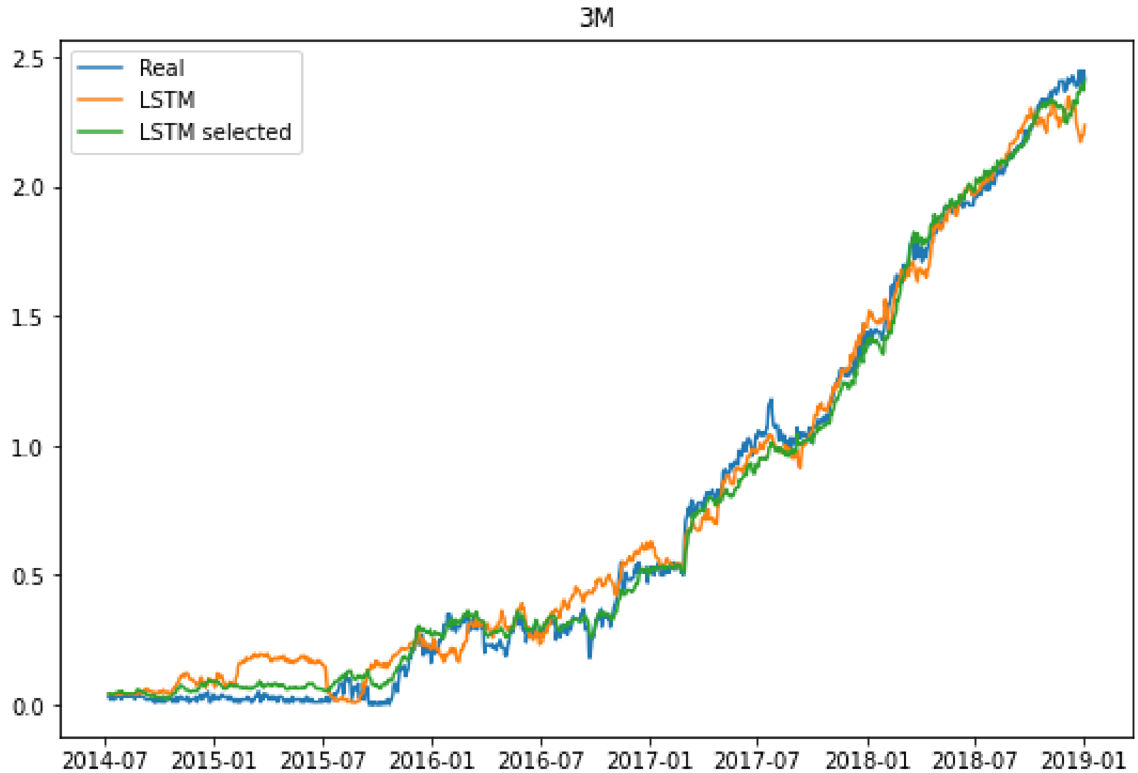
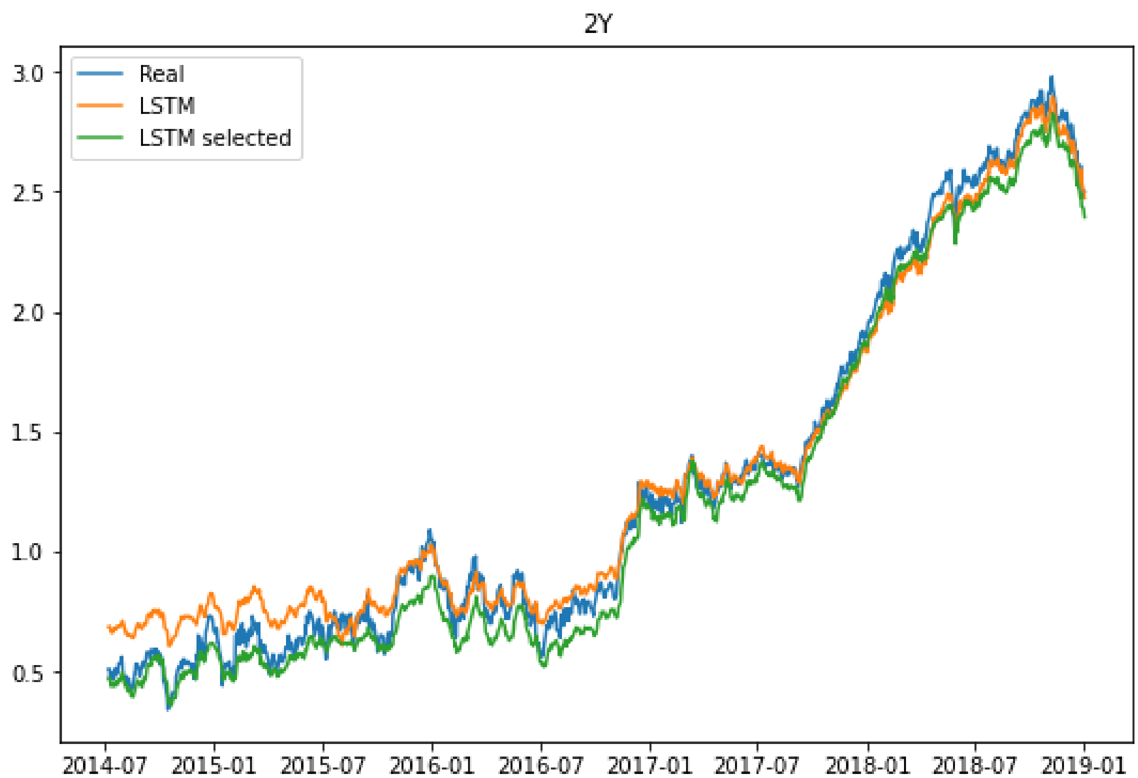
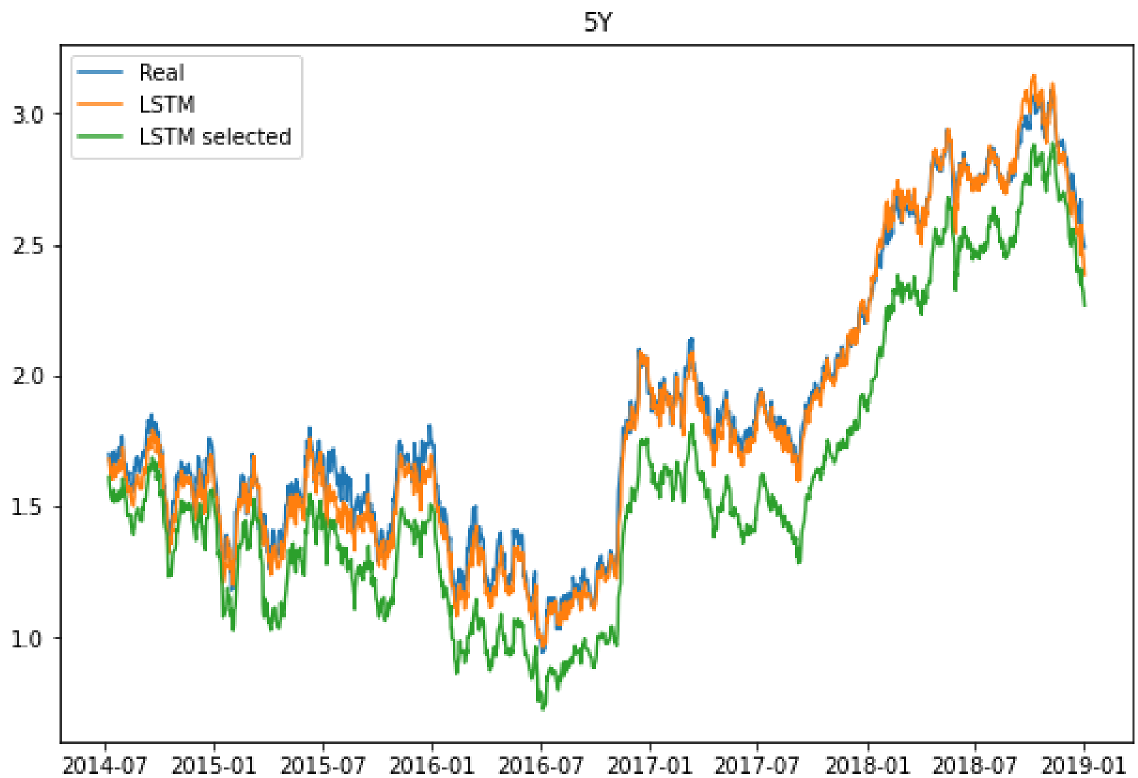
2018-01
2018-07
2019-01


30Y
Real
SVR
SVR selected
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2014-07
2015-01
2015-07
2016-01
2016-07
2017-01
2017-07
2018-01
2018-07
2019-01

LSTM:


3M
Real
LSTM
LSTM selected
2.5
2.0
1.5
1.0
0.5
0.0
2014-07
2015-01
2015-07
2016-01
2016-07
2017-01
2017-07
2018-01
2018-07
2019-01


2Y
Real
LSTM
LSTM selected
3.0
2.5
2.0
1.5
1.0
0.5
2014-07
2015-01
2015-07
2016-01
2016-07
2017-01
2017-07
2018-01
2018-07
2019-01


5Y
Real
LSTM
LSTM selected
3.0
2.5
2.0
1.5
1.0
2014-07
2015-01
2015-07
2016-01
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2017-01
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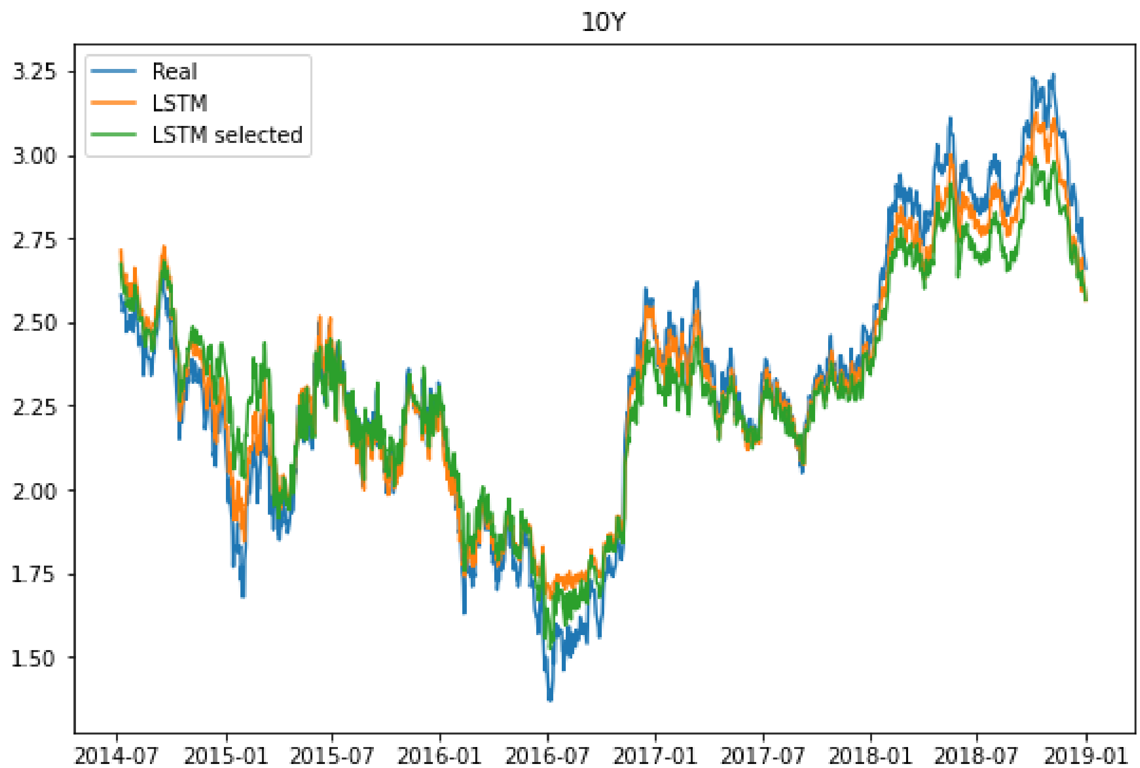
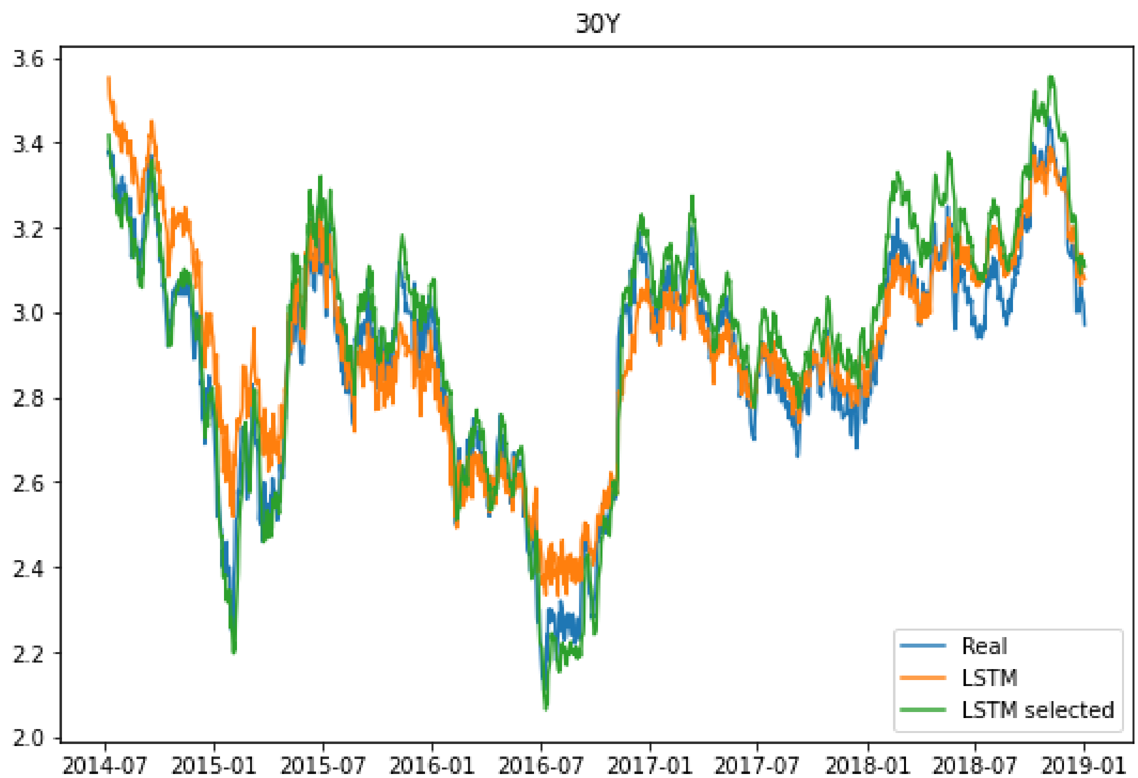
2018-01
2018-07
2019-01


10Y
Real
LSTM
LSTM selected
3.25
3.00
2.75
2.50
2.25
2.00
1.75
1.50
2014-07
2015-01
2015-07
2016-01
2016-07
2017-01
2017-07
2018-01
2018-07
2019-01


30Y
3.6
3.4
3.2
3.0
2.8
2.6
2.4
2.2
2.0
Real
LSTM
LSTM selected
2014-07
2015-01
2015-07
2016-01
2016-07
2017-01
2017-07
2018-01
2018-07
2019-01