A Process-based Tide-induced Sediment Transport Capacity Formula

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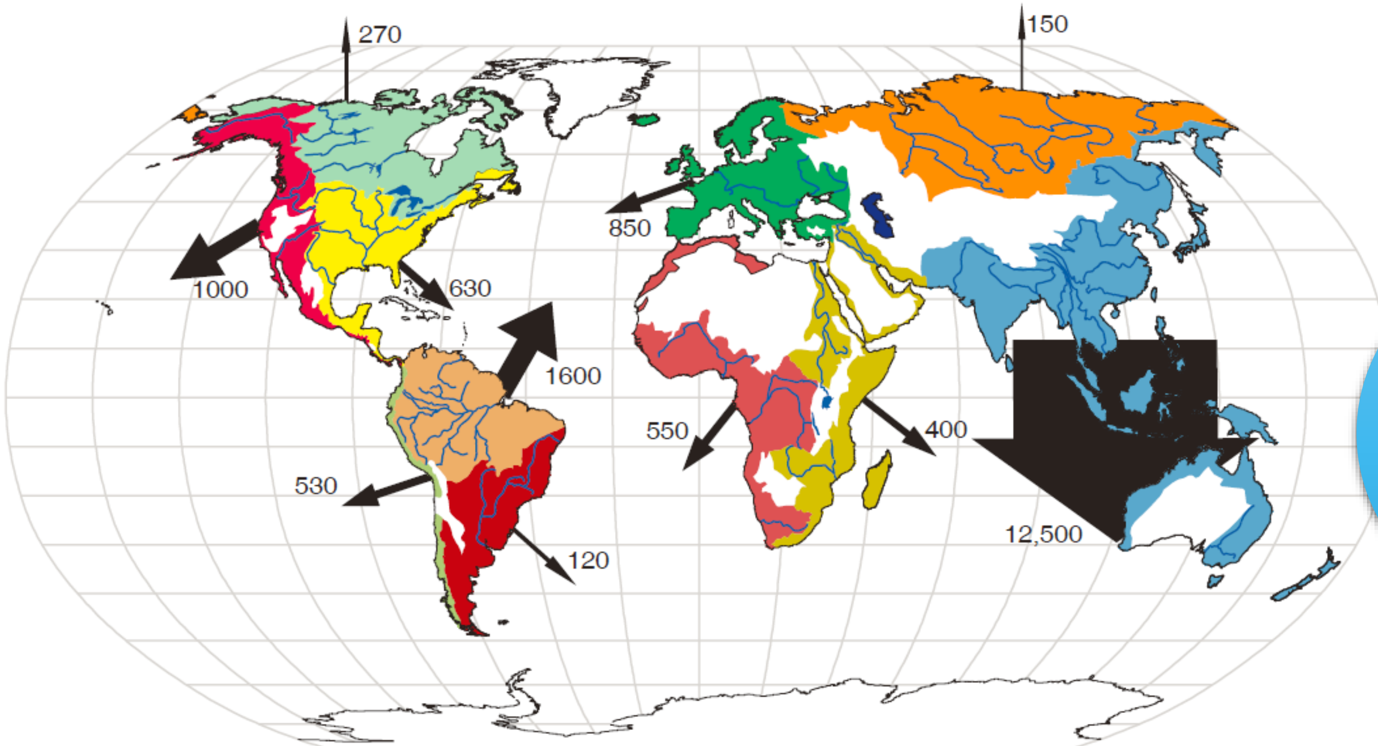
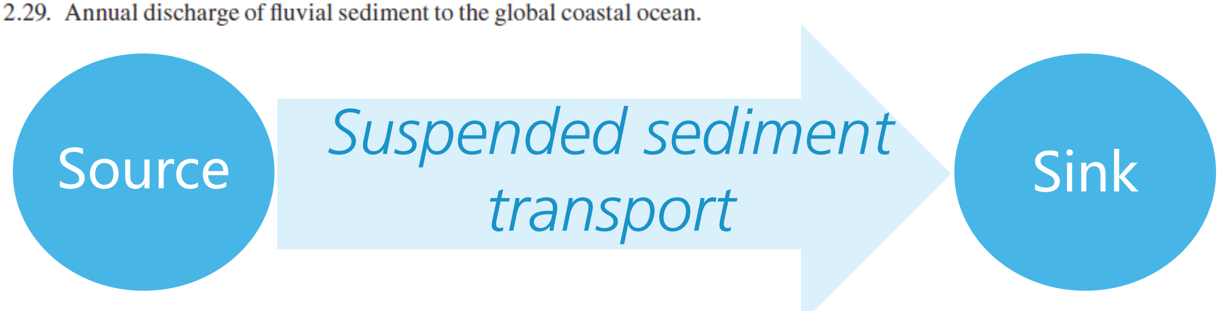
A crucial source-to-sink process



Milliman and Farnsworth, 2011 Total = 19 000 * 10^6 t/yr

Figure 2.29. Annual discharge of fluvial sediment to the global coastal ocean.



Transport capacity

$$S_* = \overline{c_z} = \frac{1}{h}\int_{z_a}^{h} c(z)dz$$

Mountains...

Coastal ocean, deep sea...

Blackbox: Semi-empirical, semi-theoretical

$$S_* = \alpha \frac{\gamma \gamma_s}{\gamma_s - \gamma} \frac{U^3}{C^2 h w_s} \longrightarrow \boldsymbol{S_* = k_1 \frac{U^3}{g h w_s}}$$

(Dou, 1995)

$$S_* = 0.045 \frac{\gamma \gamma_s}{\gamma_s - \gamma} \frac{(|V_1| + |V_2|)^2}{g d_1} F^{\frac{1}{F}} \longrightarrow \boldsymbol{S_* = k_2 \frac{U^2}{g h}}$$

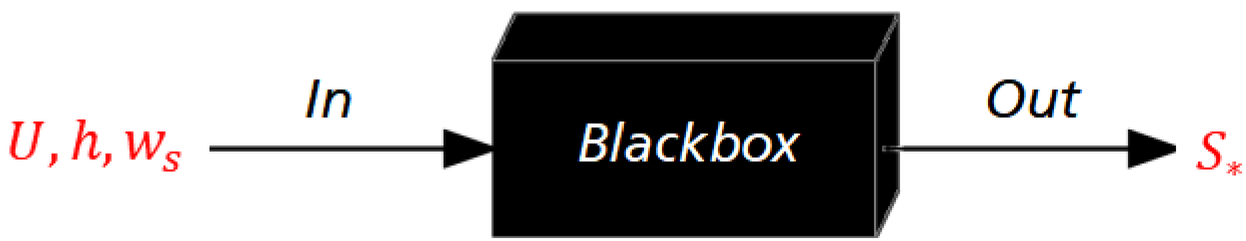
$$S_* = f(U, h, w_s)$$

(Liu, 2012)

$$S_* = K_0 \left(0.1 + 90 \frac{w_s}{U}\right) \frac{U^3}{g h w_s}$$

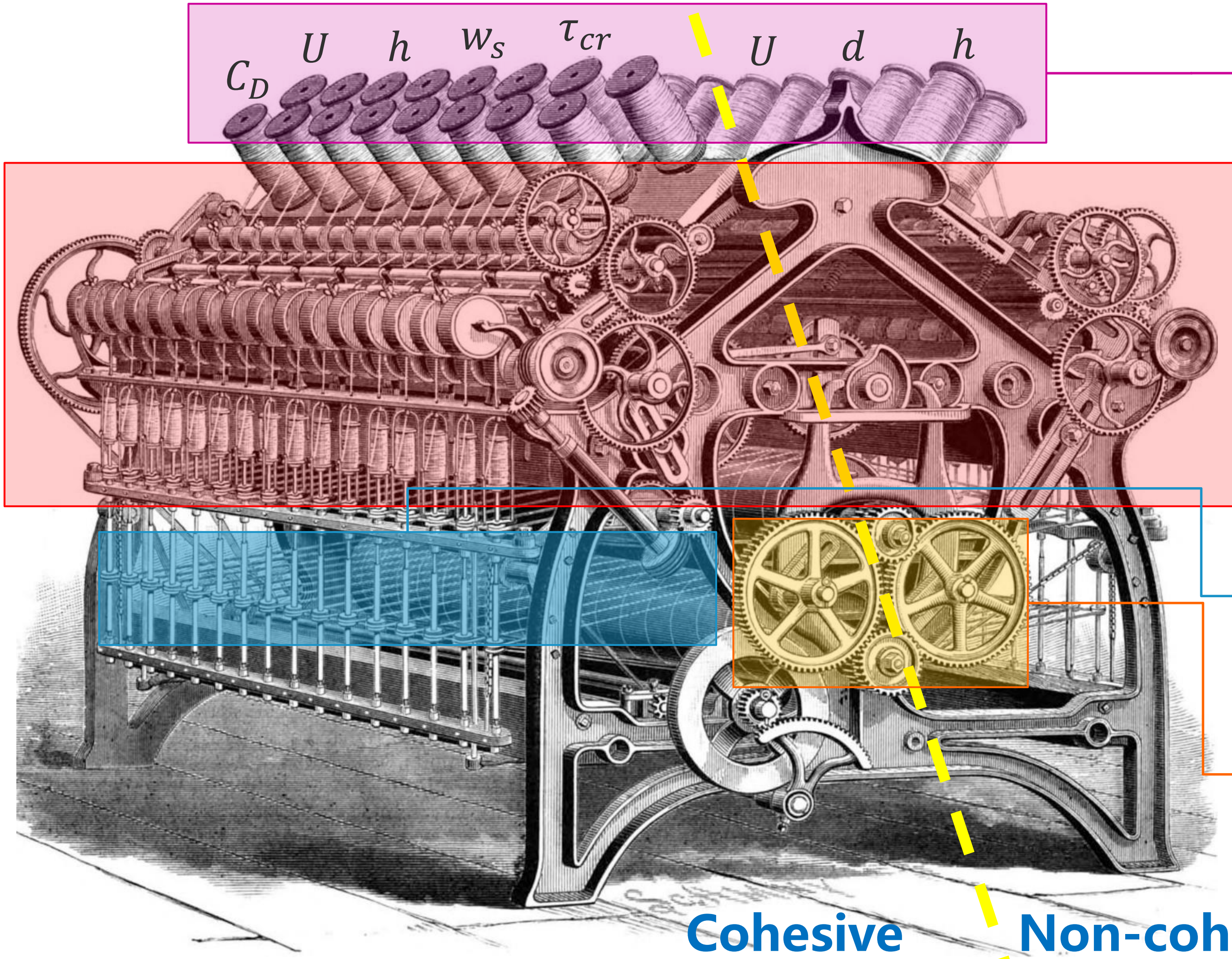
(Li, 1988)

How to open???



Energy conservation;
Dimensional analysis

Spinning machine x Sediment dynamics



Variables
(*Threads*)

Formulae describing sediment dynamic processes
(*Small gears*)

Calculated S_*
(*Separated threads*)

Regression
(*Large gears*)

Formula
(*Cloth products*)

Cohesive sediment

Non-cohesive sediment

Scientific American, 1882

Spinning machine: Basic assumptions

- Steady-uniform flow: $\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} = 0$
- Logarithmic horizontal velocity profile: $u = \frac{u_*}{\kappa} ln(\frac{z}{z_0})$
- Rouse (1937) profile of SSC: $\begin{cases} c_z = c_a \left(\frac{z}{z_a}\frac{h-z_a}{h-z}\right)^{-b} \\ b = \frac{w_s}{\kappa u_*} \end{cases}$

(SSC = Suspended Sediment Concentration)

Find c_a, z_a, h, w_s, and u_* to solve the SSC profile!

Spinning machine: Cohesive sediment

d ≤ 62.5 μm. No bedform formation.

- **Erosion rate**: $E = M\left(\frac{\tau_0}{\tau_{cr}} - 1\right)$ (Partheniades, 1965)

- **Deposition rate**: $D = w_s c_b$ (Partheniades, 1965)

- Boundary condition: $c_b = c_a, z_0 = z_a$

E = D → c_b → Solution to the Rouse Profile

Variable /Parameter	Source	Value	Unit
w_s	Gibbs et al. (1971); Hill & McCave (2001)	$\left(\sqrt{2}\right)^{t}, t = -4, -3, \dots 2$	mm/s
h	Pinet (2016)	2.50, 3.75, 5.00, … 20.00	m
τ_{cr}	van Ledden et al. (2004)	0.10, 0.15, 0.20, 0.25, 0.30	Pa
U } τ_0	Yu et al. (2014)	0.5, 0.6, … 1.5	m/s
C_D } τ_0	Soulsby (1997)	$(1.50, 1.75, \dots 3.00) \times 10^{-3}$	
M	Amos et al. (1992)	$10^{-5}, 10^{-4}$	kg/(m²s)
z_a	van Rijn (2007)	0.05	m

Spinning machine: Non-cohesive sediment

62.5 μm < *d* ≤ 250 μm (van Rijn, 2007). Formation of bedform.

- Solving the Rouse Profile:

(1) **Bedform roughness**: $z_{0f} = \frac{\Delta^2}{\lambda}$ (Soulsby, 1997)

$$\Delta = 0.11h\left(\frac{d_{50}}{h}\right)^{0.3}(1 - e^{-0.5T_s})(25 - T_s),\ \lambda = 7.3h,\ T_s = \frac{\tau_{0s}}{\tau_{cr}} - 1$$

(van Rijn, 1984)

(2) **Reference height & reference concentration**:

$$z_a = \frac{\Delta}{2},\ c_a = \rho_s \frac{0.015 d T_s^{1.5}}{z_a D_*^{0.3}}$$

(van Rijn, 1984)

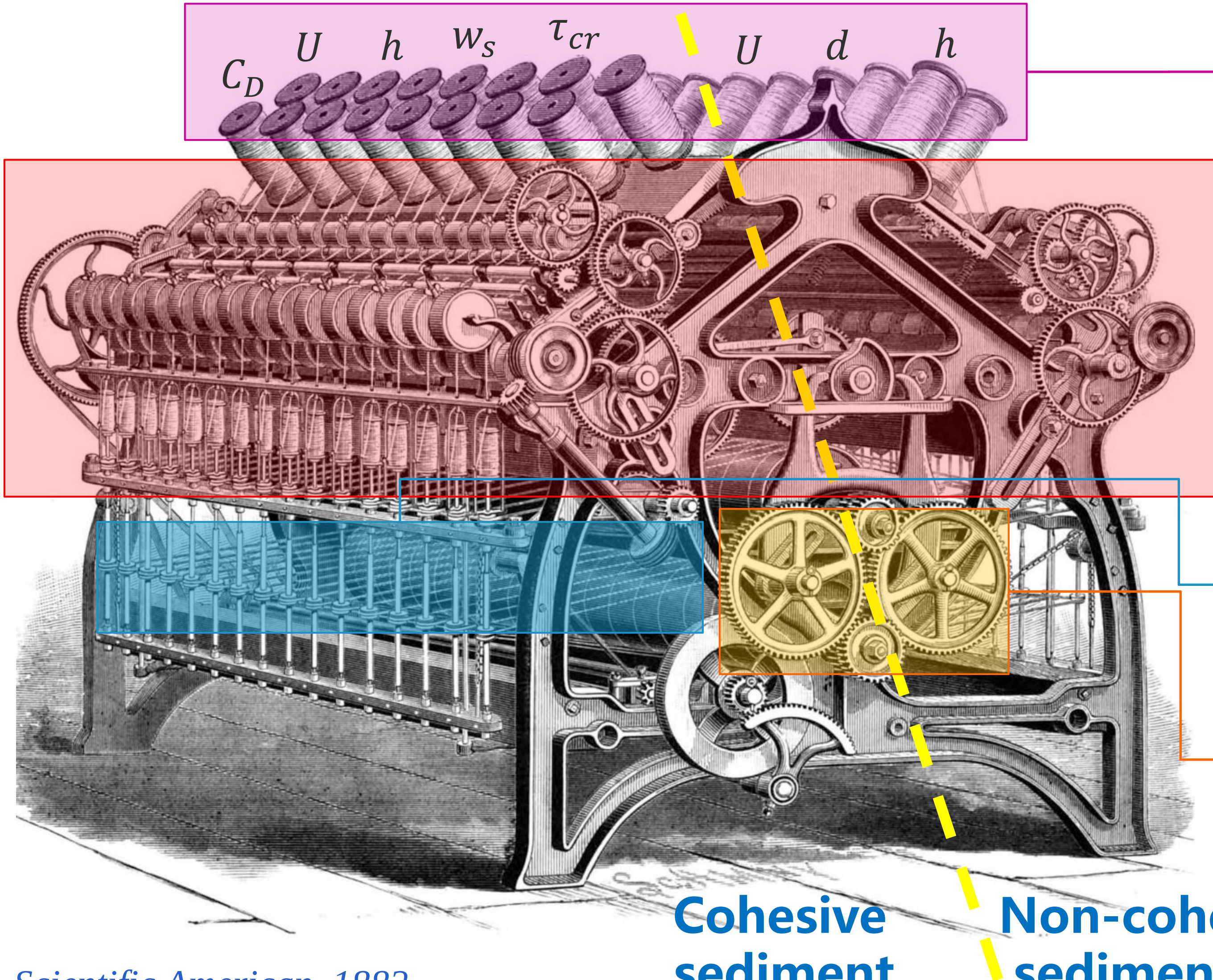
(3) **Settling velocity**:

$$w_s = \frac{\nu}{d}\left(\sqrt{10.36^2 + 1.049 D_*^3} - 10.36\right)$$

(Soulsby, 1997)

Variable	Source	Value	Unit
h	Pinet (2016)	2.50, 3.75, 5.00, … 20.00	m
d* or *d_{50}	van Rijn (2007)	4.0, 3.8, … 2.2	φ
U	Yu et al. (2014)	0.5, 0.6, … 1.5	m

Spinning machine x Sediment dynamics



Scientific American, 1882

Variables
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（*Small gears*）

Calculated S_*
（*Separated threads*）

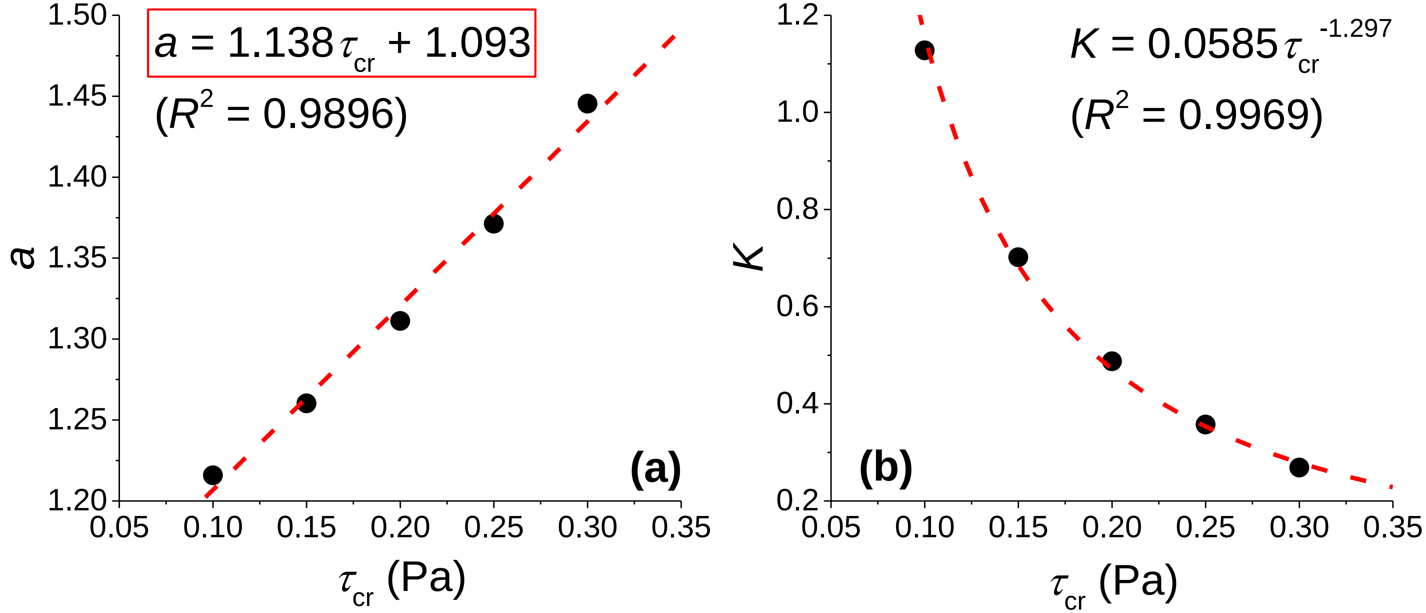
Regression
（*Large gears*）

Formula
（*Cloth products*）

Cohesive sediment: Role of τ_{cr}

$$\begin{cases} S_* = \alpha M K \rho^a C_D^a \dfrac{U^{\boxed{2a}}}{h^b w_s^c} \\ \alpha = 0.8709 \\ K = 0.0585\tau_{cr}^{-1.297} \\ \boxed{a = 1.138\tau_{cr} + 1.093} \\ b = 0.0517 \\ c = 1.25 \end{cases}$$

$2a \in (2.4, 2.9)$



Cohesive sediment: Verification

$$\begin{cases} S_* = \alpha M K \rho^a C_D^a \dfrac{U^{\boxed{2a}}}{h^b w_s^c} \\ \alpha = 0.8709 \\ K = 0.0585\tau_{cr}^{-1.297} \\ \boxed{a = 1.138\tau_{cr} + 1.093} \\ b = 0.0517 \\ c = 1.25 \end{cases}$$

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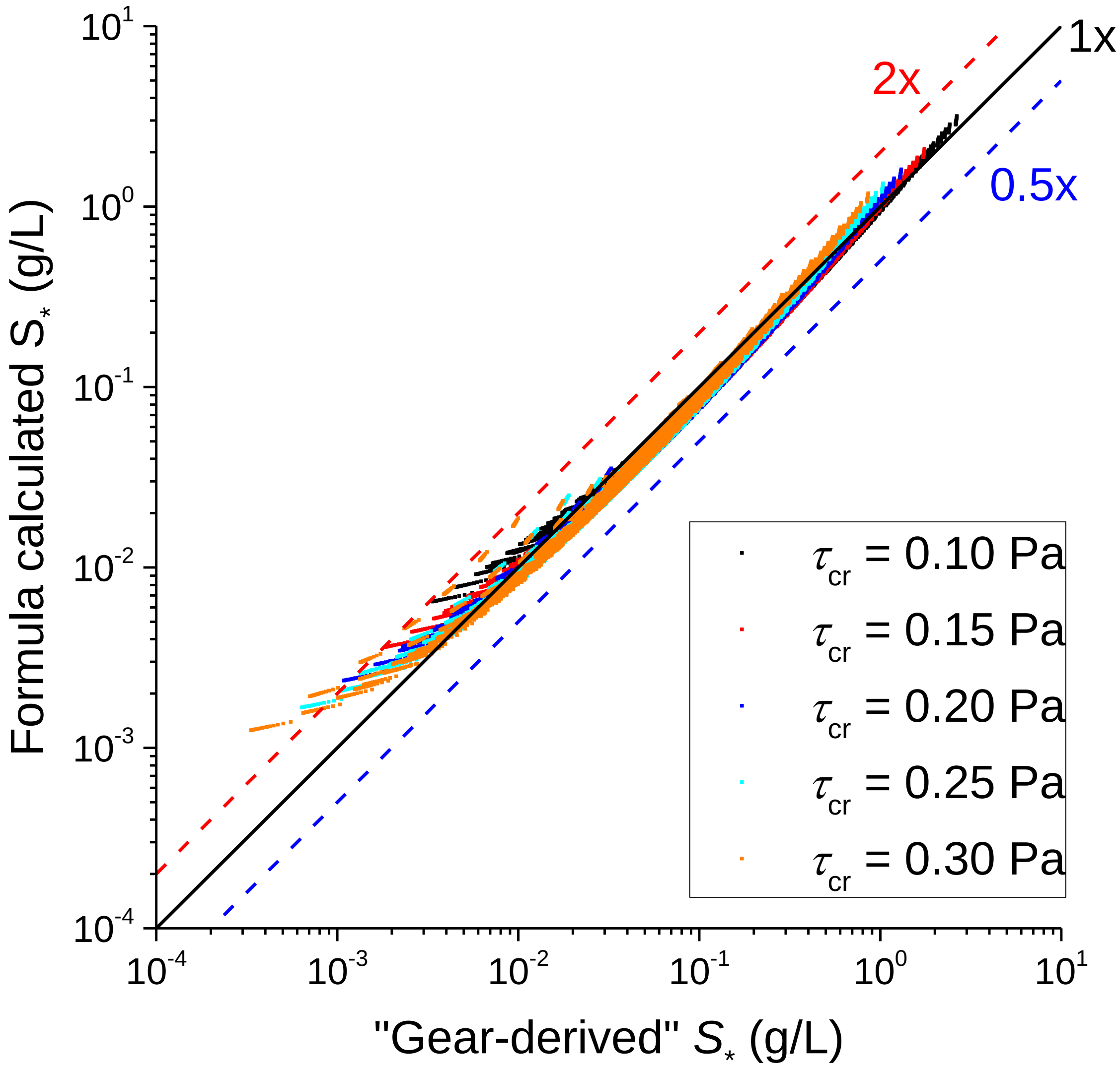
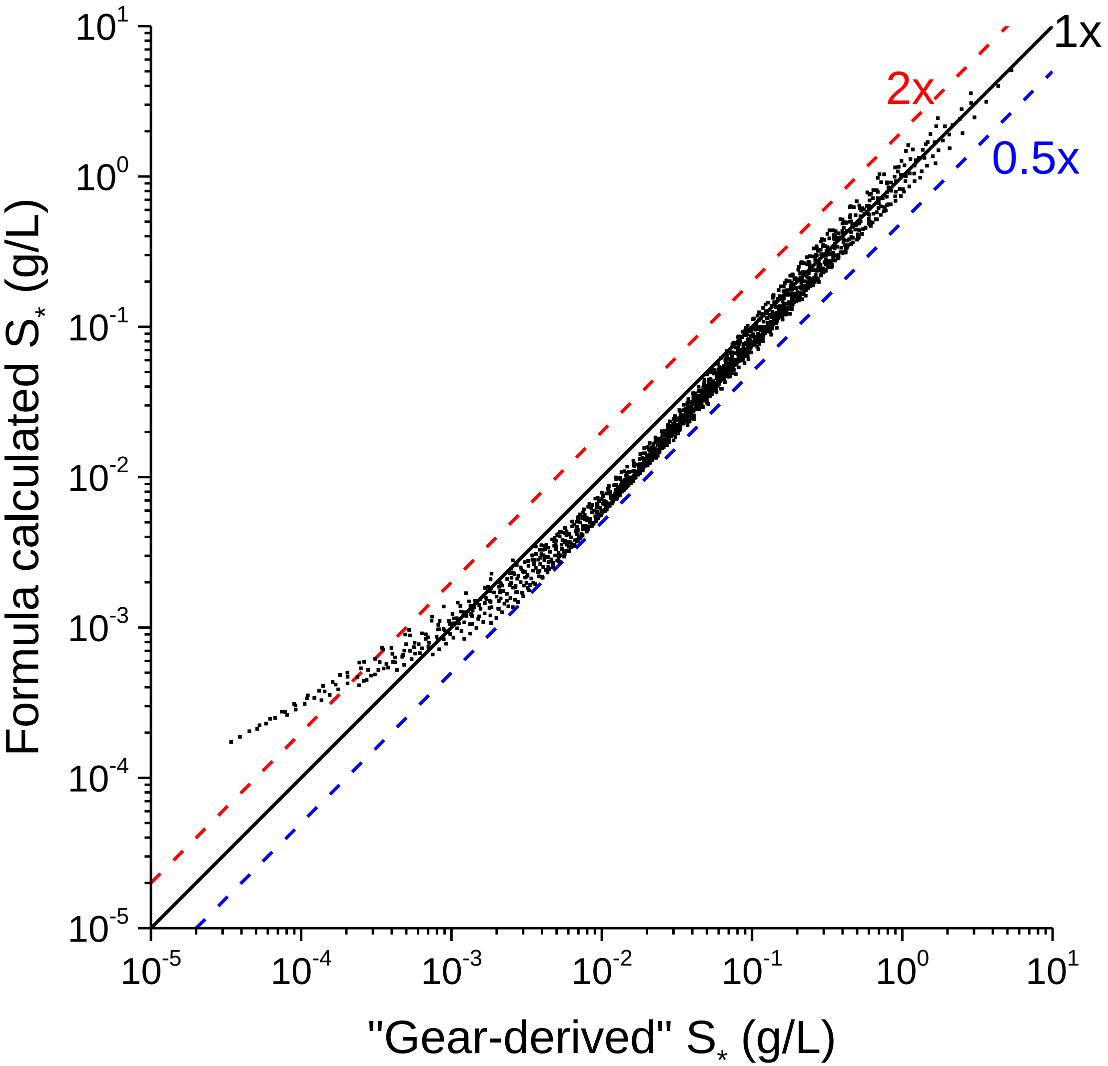


Non-cohesive sediment: Verification

$62.5\ \mu m < d \leq 200\ \mu m$

(van Rijn, 2007)

$$\begin{cases} S_* = \alpha K \dfrac{U^{\boxed{a}}}{h^b w_s^c} \\ \alpha = 0.8107 \\ K = 0.01107 \\ \boxed{a = 5.14} \\ b = 1.24 \\ c = 0.884 \end{cases}$$



Open the blackbox

$$S_* = f(U, h, w_s, \tau_{cr})$$

$$\begin{cases} S_* = \alpha M K \rho^a C_D^a \dfrac{U^{\boxed{2a}}}{h^b w_s^c} \\ \alpha = 0.8709 \\ K = 0.0585\tau_{cr}^{-1.297} \\ \boxed{a = 1.138\tau_{cr} + 1.093} \\ b = 0.0517 \\ c = 1.25 \end{cases}$$

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Cohesive

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Non-cohesive

Spinning machine: interlocking gears

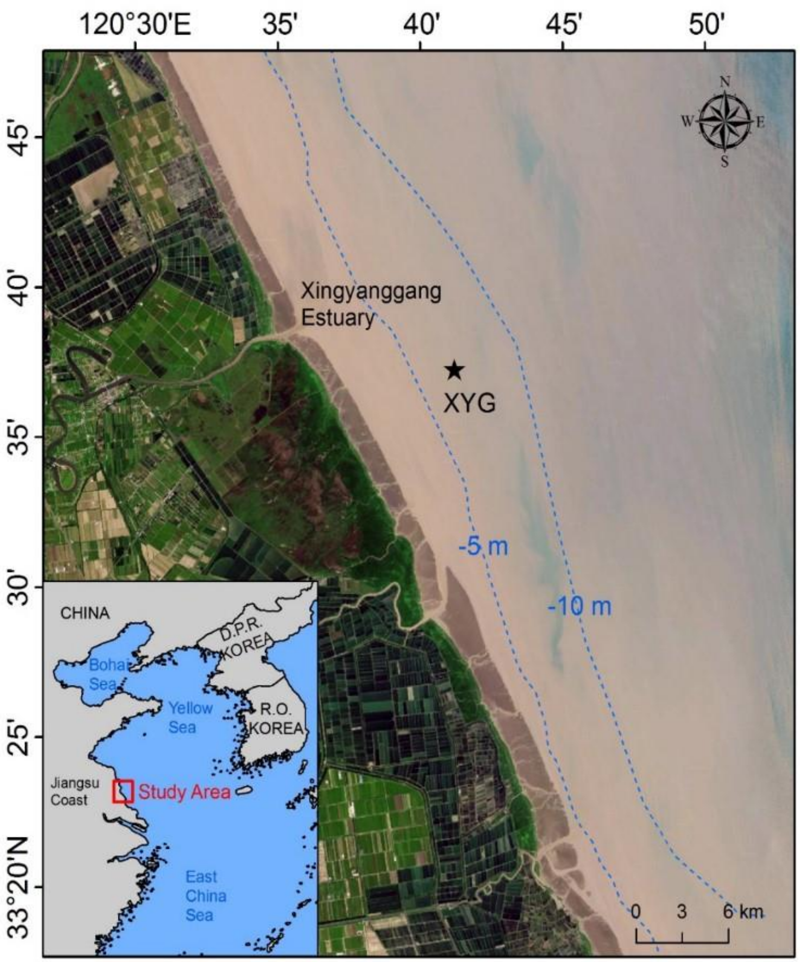
$$S_* = \alpha \frac{\gamma \gamma_s}{\gamma_s - \gamma} \frac{U^3}{C^2 h w_s} \xrightarrow[\text{(Dou, 1995)}]{} \boldsymbol{S_* = k_1 \frac{U^3}{g h w_s}}$$

$$S_* = 0.045 \frac{\gamma \gamma_s}{\gamma_s - \gamma} \frac{(|V_1| + |V_2|)^2}{g d_1} F^{\frac{1}{F}} \xrightarrow[\text{(Liu, 2012)}]{} \boldsymbol{S_* = k_2 \frac{U^2}{g h}}$$

$$S_* = K_0 \left(0.1 + 90 \frac{w_s}{U}\right) \frac{U^3}{g h w_s} \quad \text{(Li, 1988)}$$

Blackbox: semi-empirical, semi-theoretical

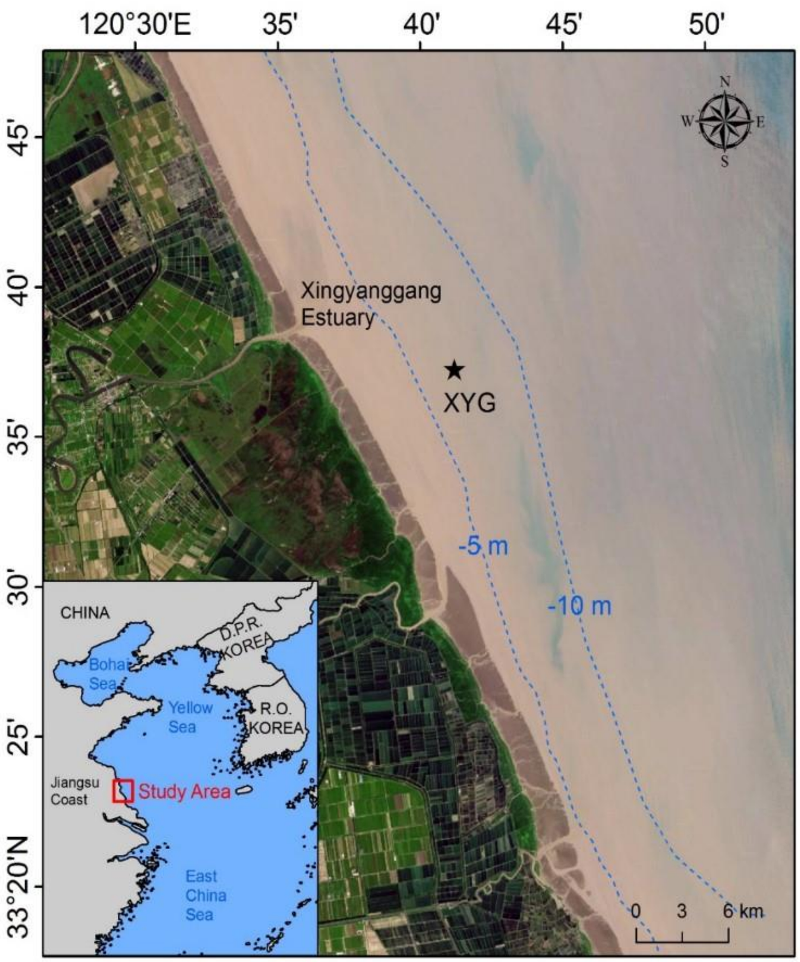
In-situ observation: General information



Water sampling:
3 depths, every hour,
in a 12.5-hour tidal cycle;

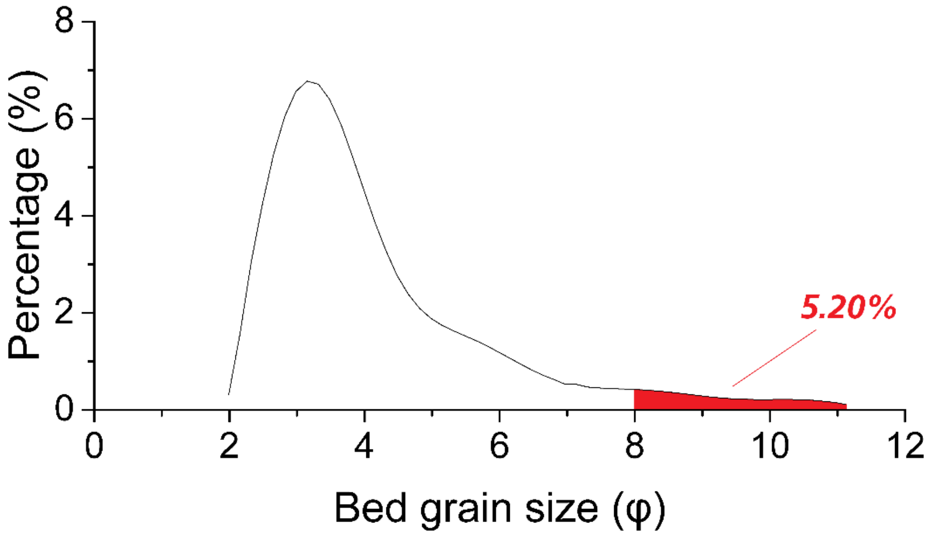
Flow measurement:
ADCP, ship-mounted

In-situ observation: Bed property



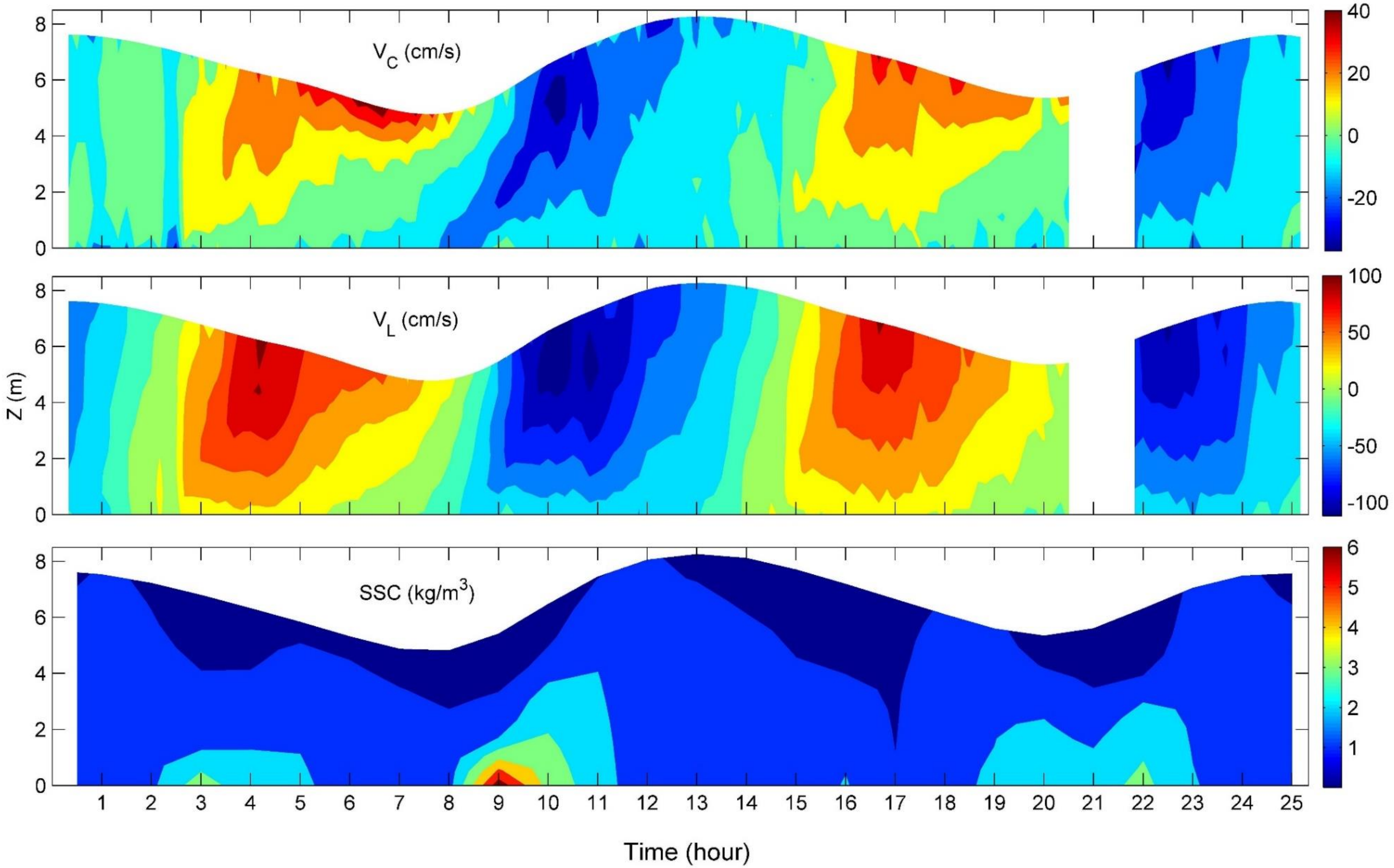
Cohesive bed

Cohesive fraction (<8 μm i.e. >7 φ) over 5% (van Rijn, 2007)



In-situ observation: Results

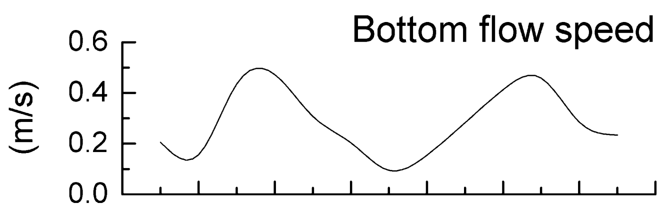
Observed flow & SSC



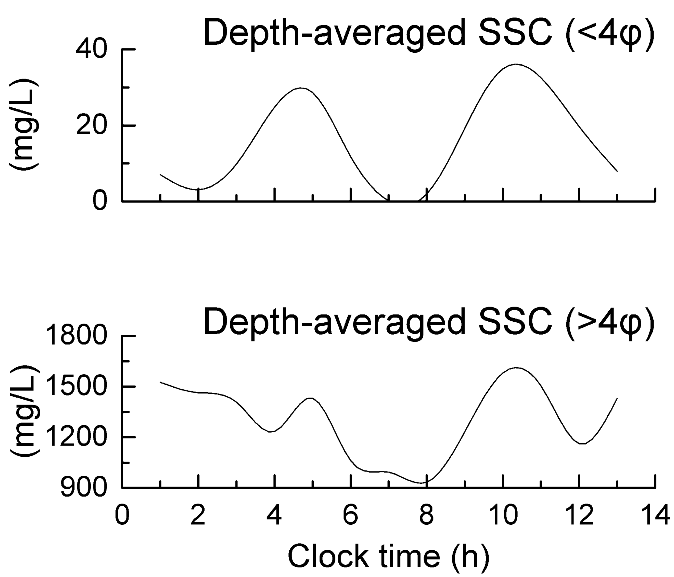
In-situ observation: Results

Observed flow & SSC

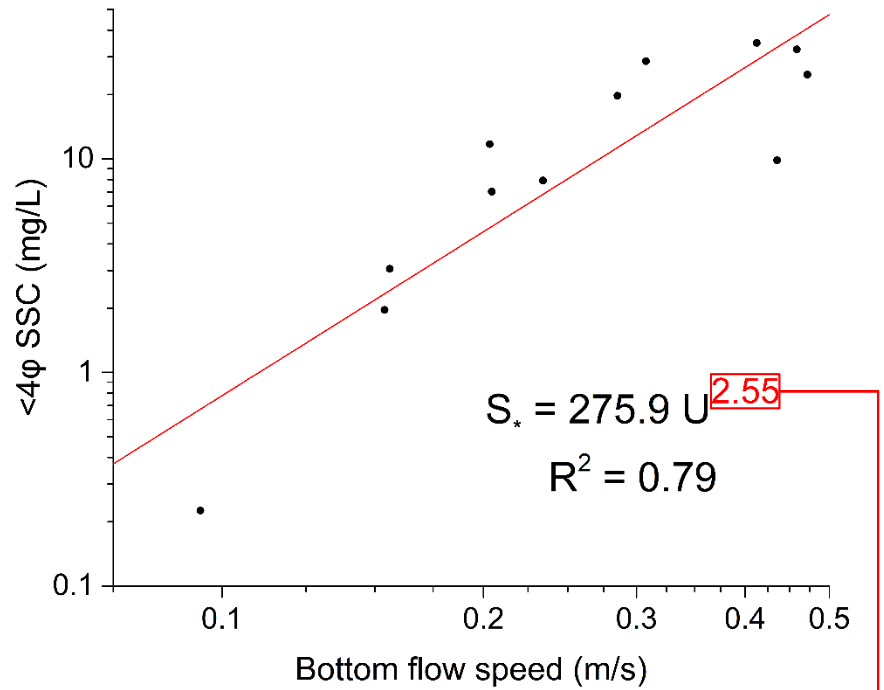
- Primary participant of vertical exchange
- Good indicator of local resuspension and settling (therefore local SSC)



Coarse: local erosion and deposition



Fine sediment: dominated by advection



Recall that $2a \in (2.4, 2.9)$!

Further work

- Non-cohesive bed: Request for flume or *in-situ* data

- Density (salinity & SSC) gradient: Dampen turbulence and vertical mixing, change the Rouse Profile

Conclusion

- A process-based tide-induced sediment transport capacity formula is derived from numerical experiment by separating cohesive and non-cohesive sediments.

- (Depth-averaged) Horizontal flow speed plays the dominant role in STC formulae; the power of flow speed increases linearly with τ_{cr} for the cohesive sediment.

- *In-situ* observation in middle Jiangsu coast has validated our derived formula.

Thank you !

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