Метод на най-малките квадрати (МНМК)

$P_n(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots.. + a_1 x + a_0$ (n + 1) на брой неизвестни

$\sqrt{\sum_{i=1}^{N} d_i^2}$ -> min -> $\Phi(a_0, a_1, \ldots, a_n) = \sum_{i=1}^{N} (y_i - (a_n x_i^n + \ldots.. + a_1 x_i + a_0))^2$

ДУ за min на $\Phi(a_0, a_1, \ldots, a_n)$:

$\frac{\partial \Phi}{\partial a_k} = 0$, k = 0,n (n + 1 на брой уравнения)

$\frac{\partial \Phi}{\partial a_k} = \frac{\partial}{\partial a_k} \{\sum_{i=1}^{N} [y_i - (a_n x_i^n + \ldots.. + a_1 x_i + a_0)]^2\} =$

$= \sum_{i=1}^{N} 2[y_i - (a_n x_i^n + \ldots.. + a_1 x_i + a_0)]\ x_i^k = 0 \ \ / : 2$

$[\sum_{i=1}^{N} y_i - \sum_{i=1}^{N} x_i^n . a_n - \ldots.. - \sum_{i=1}^{N} a_n - a_0].x_i^k = 0$

$\sum_{i=1}^{N} x_i^k a_0 + \sum_{i=1}^{N} x_i^{k+1} a_1 + \ldots.. + \sum_{i=1}^{N} x_i^{n+k} = \sum_{i=1}^{N} y_i x_i^k$ k = 0,n

СЛАУ:

k = 0: $\sum_{i=1}^{N} a_0 + \sum_{i=1}^{N} x_i a_1 + \ldots.. + \sum_{i=1}^{N} x_i^n a_n = \sum_{i=1}^{N} y_i$

k = 1: $\sum_{i=1}^{N} x_i . a_0 + \sum_{i=1}^{N} x_i^2 a_1 + \ldots.. + \sum_{i=1}^{N} x_i^{n+1} a_n = \sum_{i=1}^{N} y_i x_i$

k = n: $\sum_{i=1}^{N} x_i^n . a_0 + \sum_{i=1}^{N} x_i^{n+1} a_1 + \ldots.. + \sum_{i=1}^{N} x_i^{2\,n} a_n = \sum_{i=1}^{N} y_i x_i^n$

Пример:

x_i 0 1 2 3 4

y_i 1 2 1 0 4

P_1^*, P_2^* = ?

P_1^* = ? - линейна регресия

$P_1(x) = a_1 x + a_0$, a_0, a_1 = ?

$N.a_0 + \sum_{i=1}^{N} x_i . a_1 = \sum_{i=1}^{N} y_i$

$\sum_{i=1}^{N} x_i a_0 + \sum_{i=1}^{N} x_i^2 a_1 = \sum_{i=1}^{N} y_i x_i$

	i	x_i	y_i	x_i^2	$x_i y_i$
	1	0	1	0	0
	2	1	2	1	2
	3	2	1	4	2
	4	3	0	9	0
N =	5	4	4	16	16
$\sum$		10	8	30	20

$5a_0 + 10a_1 = 8$

$10a_0 + 30a_1 = 20$

=> $a_0^* = \frac{4}{5}$, $a_1^* = \frac{2}{5}$

=> $P_1^*(x) = \frac{2}{5}x + \frac{4}{5}$

P_2^* = ? - линейна регресия

$P_1(x) = a_2 x^2 + a_1 x + a_0$, a_0, a_1, a_2 = ?

$N.a_0 + \sum_{i=1}^{N} x_i.a_1 + \sum_{i=1}^{N} x_i^2.a_2 = \sum_{i=1}^{N} y_i$

$\sum_{i=1}^{N} x_i a_0 + \sum_{i=1}^{N} x_i^2 a_1 + \sum_{i=1}^{N} x_i^3.a_2 = \sum_{i=1}^{N} y_i x_i$

$\sum_{i=1}^{N} x_i^2 a_0 + \sum_{i=1}^{N} x_i^3 a_1 + \sum_{i=1}^{N} x_i^4.a_2 = \sum_{i=1}^{N} y_i x_i^2$

	i	x_i	y_i	x_i^2	$x_i y_i$	x_i^3	x_i^4	$y_i x_i^2$
	1	0	1	0	0	0	0	0
	2	1	2	1	2	1	1	2
	3	2	1	4	2	8	16	4
	4	3	0	9	0	27	81	0
N =	5	4	4	16	16	64	256	64
$\sum$		10	8	30	20	100	354	70

$5a_0 + 10a_1 + 30a_2 = 8$

$10a_0 + 30a_1 + 100a_2 = 20$

$30a_0 + 100a_2 + 354a_2 = 70$

In[∘]:=
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A = (5 10 30; 10 30 100; 30 100 354); b = {8, 20, 70};
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$$A = \begin{pmatrix} 5 & 10 & 30 \\ 10 & 30 & 100 \\ 30 & 100 & 354 \end{pmatrix};\ b = \{8, 20, 70\};$$

In[∘]:=
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LinearSolve[A, b]
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Out[∘]= $\left\{\frac{58}{35}, -\frac{46}{35}, \frac{3}{7}\right\}$

=> $a_0{}^* = \frac{58}{35}$, $a_1{}^* = -\frac{46}{35}$, $a_2{}^* = \frac{3}{7}$

=> $P_2{}^*(x) = \frac{3}{7} x^2 - \frac{46}{35} x + \frac{58}{35}$