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Задача 1:

$$\begin{vmatrix} x & 2x & 3 \\ 0 & x & 1 \\ 4x & -2 & 2 \end{vmatrix} = 0$$

$$2x^2 + 8x^2 - 12x^2 + 2x = 0$$
$$-2x^2 + 2x = 0 \quad :(-2)$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0 \quad \vee \quad x - 1 = 0$$
$$x = 1$$

Задача 2:

а) BC: $B(3, -3)$, $C(-4, 0)$

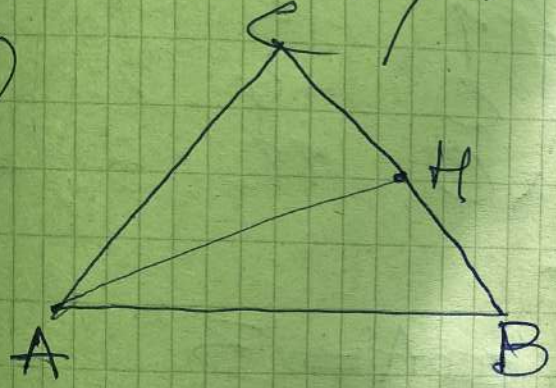
$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1}$$

$$\frac{x - 3}{-4 - 3} = \frac{y + 3}{0 - (-3)}$$

$$\frac{x-3}{-7} = \frac{y+3}{3}$$

$BC: 3x - 9 = -7y - 21$
$BC: 3x + 7y - 9 + 21 = 0$
$BC: 3x + 7y + 12 = 0$

б)



$$\text{на} \begin{cases} A(3,3) \\ \perp BC(-7,3) \end{cases}$$

на: $-7(x-3) + 3(y-3) = 0$
на: $-7x + 21 + 3y + 9 = 0$
на: $-7x + 3y + 11 = 0 \cdot (-1)$
на: $7x - 3y - 11 = 0$

в) $S_{ABC} = \frac{1}{2} \left| \begin{vmatrix} 3 & 3 & 1 \\ 3 & -3 & 1 \\ -4 & 0 & 1 \end{vmatrix} \right| =$

$= \frac{1}{2} |-9 - 12 - 12 + 31| = \frac{1}{2} |-24 + 31| = 12$ кв.ед.

Задача 3:

а)
$$A = \begin{pmatrix} 1 & -5 & 1 \\ 0 & -4 & 2 \\ 2 & 1 & -3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -5 & 1 \\ 0 & -4 & 2 \\ 2 & 1 & -3 \end{pmatrix} \cdot (-2) \sim \begin{pmatrix} 1 & -5 & 1 \\ 0 & -4 & 2 \\ 0 & 11 & -5 \end{pmatrix} \cdot 11/4$$

$$\sim \begin{pmatrix} 1 & -5 & 1 \\ 0 & -4 & 2 \\ 0 & 0 & 1/2 \end{pmatrix} \Rightarrow \operatorname{rg} A = 3$$

б) $AX = B \quad | \cdot A^{-1}$ отл.

$X = A^{-1}B$

$$\begin{pmatrix} 1 & -5 & 1 \\ 0 & -4 & 2 \\ 2 & 1 & -3 \end{pmatrix} X = \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

3×3 $\qquad$ 3×1

$|A| = 12 - 20 + 8 - 2 = -2 \neq 0$

$\Rightarrow A^{-1}$ есть.

$$A_{11} = (-1)^{1+1} \begin{vmatrix} -4 & 2 \\ 1 & -3 \end{vmatrix} = 10$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 2 \\ 2 & -3 \end{vmatrix} = -(0 - 4) = 4$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & -4 \\ 2 & 1 \end{vmatrix} = 8$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -5 & 1 \\ 1 & -3 \end{vmatrix} = -(15-1) = -14$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = -3-2 = -5$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -5 \\ 2 & 1 \end{vmatrix} = -(1+10) = -11$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -5 & 1 \\ -4 & 2 \end{vmatrix} = -10+4 = -6$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = -(2-0) = -2$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -5 \\ 0 & -4 \end{vmatrix} = -4-0 = -4$$

$$A^{-1} = \frac{1}{-2} \begin{pmatrix} 10 & -14 & -6 \\ 4 & -5 & -2 \\ 8 & -11 & -4 \end{pmatrix} =$$

$$A^{-1} = \begin{pmatrix} -5 & 7 & 3 \\ -2 & 5{,}2 & 1 \\ -4 & 1{,}12 & 2 \end{pmatrix}$$

$X = A^{-1} B$

$$X = \begin{pmatrix} -5 & 7 & 3 \\ -2 & 5{,}2 & 1 \\ -4 & 1{,}2 & 2 \end{pmatrix} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix}$$

$$X = \begin{pmatrix} -5 \cdot 0 + 7 \cdot (-1) + 3 \cdot 1 \\ -2 \cdot 0 + 5{,}2 \cdot (-1) + 1 \cdot 1 \\ -4 \cdot 0 + 1{,}2 \cdot (-1) + 2 \cdot 1 \end{pmatrix}$$

$$X = \begin{pmatrix} -4 \\ -3{,}2 \\ -7{,}2 \end{pmatrix}$$

Zadaca 4:

a) $g: \begin{cases} z A(1, 1, -2) \\ \perp 2: x - 2y + z - 9 = 0 \end{cases}$

$\vec{2}(1, -2, 1)$ u $d \Rightarrow \vec{2} \perp g$

$g: \begin{cases} z A(1, 1, -2) \\ \perp \vec{2}(1, -2, 1) \end{cases}$

$g: (x - 1) + 2(y - 1) - (z + 2)$

b) $g(A, 2) = \dfrac{|1 \cdot 1 - 2 \cdot 1 + 1 \cdot (-2) - 9|}{\sqrt{1^2 + (-2)^2 + 1^2}}$

$g(A, 2) = \dfrac{|-12|}{\sqrt{6}} = \dfrac{12}{\sqrt{6}} \cdot \dfrac{\sqrt{6}}{\sqrt{6}} = \dfrac{12\sqrt{6}}{6} = 2\sqrt{6}$