Метод на най-малките квадрати (МНМК)

Генериране на данни

$x_t = 5 - t(0.13),\ \ t = -3, 15$

f(x) = 2cos(2x-2)

$y = f(x_t)$

Линейна регресия

$P_1(x) = a_1 x + a_0$

$a_1, a_0 = ?$

Квадратична регресия

$P_2(x) = a_0 + a_1 x + a_2 x^2$

$a_0, a_1, a_2 = ?$

Генериране на данни

In[621]:=

```
xt = Table[5 + t * 0.13, {t, -3, 15}]
```

Out[621]=

```
{4.61, 4.74, 4.87, 5., 5.13, 5.26, 5.39, 5.52, 5.65,
 5.78, 5.91, 6.04, 6.17, 6.3, 6.43, 6.56, 6.69, 6.82, 6.95}
```

In[622]:=

```
f[x_] := 2 Cos[2 x - 2]
yt = f[xt]
```

Out[623]=

```
{1.18471, 0.730649, 0.22747, -0.291, -0.789909, -1.23572,
 -1.59847, -1.85376, -1.98445, -1.98174, -1.84582, -1.58583,
 -1.21923, -0.770676, -0.270319, 0.24821, 0.750054, 1.20148, 1.57214}
```

In[624]:=

```
P = Length[xt]
```

Out[624]=

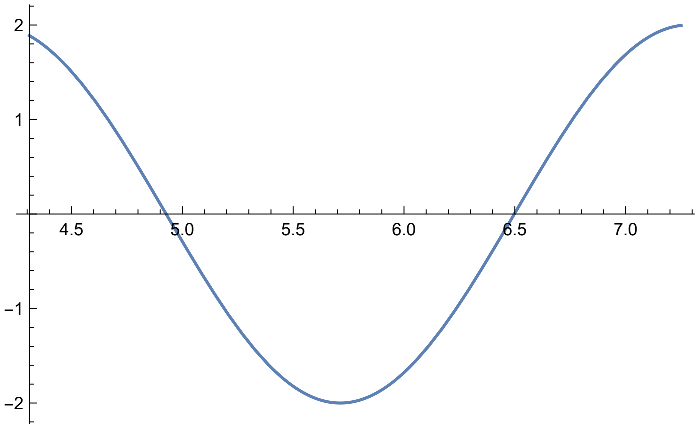
```
19
```

Визуализация

In[625]:=

```
grf = Plot[f[x], {x, xt⟦1⟧ - 0.3, xt⟦P⟧ + 0.3}]
```

Out[625]=



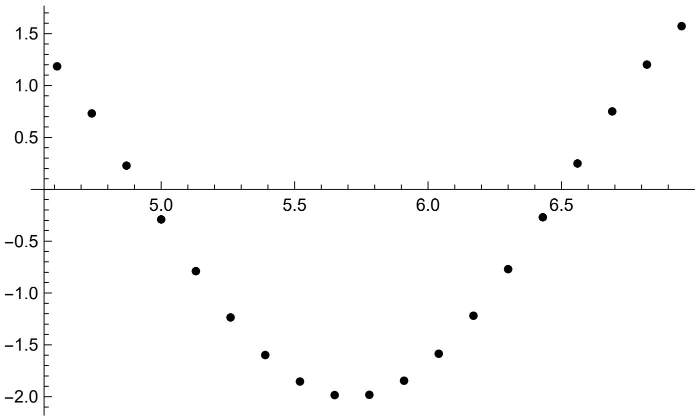
In[626]:=

```
points = Table[{xt⟦i⟧, yt⟦i⟧}, {i, 1, P}];
```

In[627]:=

```
grp = ListPlot[points, PlotStyle → Black]
```

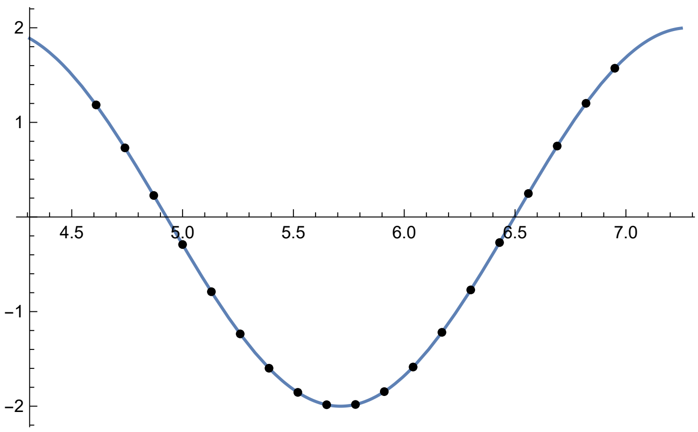
Out[627]=



In[628]:=

```
Show[grf, grp]
```

Out[628]=



Линейна регресия

Попълваме таблицата

In[629]:=

$$xt^2$$

Out[629]=

```
{21.2521, 22.4676, 23.7169, 25., 26.3169, 27.6676, 29.0521, 30.4704, 31.9225, 33.4084,
 34.9281, 36.4816, 38.0689, 39.69, 41.3449, 43.0336, 44.7561, 46.5124, 48.3025}
```

In[630]:=

```
yt * xt
```

Out[630]=

```
{5.46153, 3.46328, 1.10778, -1.455, -4.05223, -6.49989,
 -8.61573, -10.2328, -11.2121, -11.4545, -10.9088, -9.57838,
 -7.52264, -4.85526, -1.73815, 1.62826, 5.01786, 8.19409, 10.9264}
```

Намиране на сумите

In[631]:=

$$\sum_{i=1}^{P} xt[[i]]$$

Out[631]=

109.82

In[632]:=

$$\sum_{i=1}^{P} yt[[i]]$$

Out[632]=

-9.51221

In[633]:=

$$\sum_{i=1}^{P} xt[[i]]^2$$

Out[633]=

644.393

In[634]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]$$

Out[634]=

-52.3263

Решаваме системата

In[635]:=

$$A = \begin{pmatrix} 19 & 109.82 \\ 109.82 & 644.393 \end{pmatrix}; \; b = \{-9.512, -52.326\};$$

In[636]:=

```
LinearSolve[A, b]
```

Out[636]=

```
{-2.09264, 0.275433}
```

Съставяме полинома

In[637]:=

```
P1[x_] := -2.093 + 0.275 x
```

Таен коз (възможност за самопроверка)

In[638]:=

```
Fit[points, {1, x}, x]
```

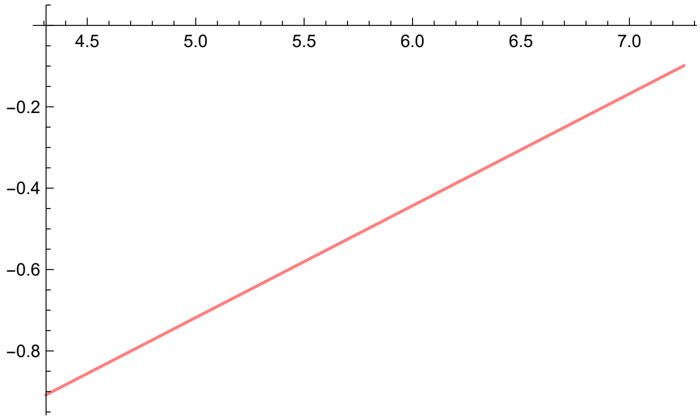
Out[638]=

```
-2.09324 + 0.275535 x
```

In[639]:=

```
grfP1 = Plot[P1[x], {x, xt[[1]] - 0.3, xt[[P]] + 0.3}, PlotStyle → Pink]
```

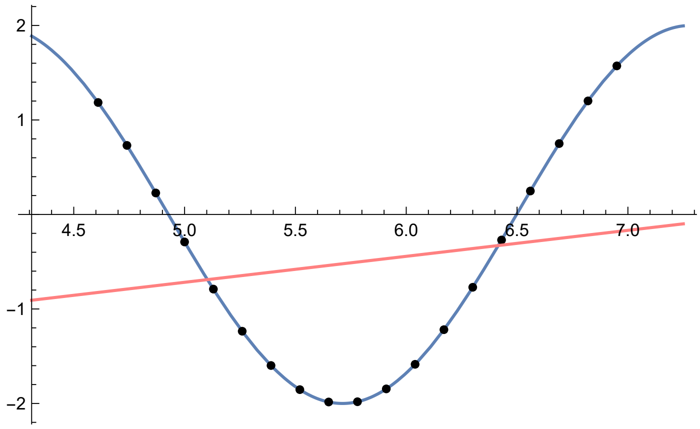
Out[639]=



In[640]:=

```
Show[grf, grp, grfP1]
```

Out[640]=



Намиране на приближена стойност (апроксимация)

In[641]:=

```
P1[4]
```

Out[641]=

```
-0.993
```

За сравнение истинската стойност

In[642]:=

```
f[4.]
```

Out[642]=

```
1.92034
```

Оценка на грешката

Теоретична грешка (средноквадратична)

In[643]:=

$$\sqrt{\sum_{i=1}^{P} (yt[[i]] - P1[xt[[i]]])^2}$$

Out[643]=

```
5.02027
```

Истинска грешка

In[644]:=

```
Abs[f[4.] - P1[4.]]
```

Out[644]=

```
2.91334
```

Квадратична регресия

Попълваме таблицата

In[645]:=

xt^2

Out[645]=

```
{21.2521, 22.4676, 23.7169, 25., 26.3169, 27.6676, 29.0521, 30.4704, 31.9225, 33.4084,
 34.9281, 36.4816, 38.0689, 39.69, 41.3449, 43.0336, 44.7561, 46.5124, 48.3025}
```

In[646]:=

```
yt * xt
```

Out[646]=

```
{5.46153, 3.46328, 1.10778, -1.455, -4.05223, -6.49989,
 -8.61573, -10.2328, -11.2121, -11.4545, -10.9088, -9.57838,
 -7.52264, -4.85526, -1.73815, 1.62826, 5.01786, 8.19409, 10.9264}
```

In[647]:=

xt^3

Out[647]=

```
{97.9722, 106.496, 115.501, 125., 135.006, 145.532, 156.591, 168.197, 180.362, 193.101,
 206.425, 220.349, 234.885, 250.047, 265.848, 282.3, 299.418, 317.215, 335.702}
```

In[648]:=

xt^4

Out[648]=

{451.652, 504.793, 562.491, 625., 692.579, 765.496, 844.025, 928.445, 1019.05, 1116.12, 1219.97, 1330.91, 1449.24, 1575.3, 1709.4, 1851.89, 2003.11, 2163.4, 2333.13}

In[649]:=

$yt * xt^2$

Out[649]=

{25.1777, 16.4159, 5.39488, -7.275, -20.788, -34.1894, -46.4388, -56.4849, -63.3486, -66.2069, -64.4711, -57.8534, -46.4147, -30.5881, -11.1763, 10.6814, 33.5695, 55.8837, 75.9383}

Намиране на сумите

In[650]:=

$$\sum_{i=1}^{P} xt[[i]]$$

Out[650]=

109.82

In[651]:=

$$\sum_{i=1}^{P} yt[[i]]$$

Out[651]=

-9.51221

In[652]:=

$$\sum_{i=1}^{P} xt[[i]]^2$$

Out[652]=

644.393

In[653]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]$$

Out[653]=

-52.3263

In[654]:=

$$\sum_{i=1}^{P} xt[[i]]^3$$

Out[654]=

3835.95

In[655]:=

$$\sum_{i=1}^{P} xt[[i]]^4$$

Out[655]=

23146.

In[656]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]^2$$

Out[656]=

-282.174

Решаваме системата

In[657]:=

$$A = \begin{pmatrix} 19 & 109.82 & 644.393 \\ 109.82 & 644.393 & 3835.95 \\ 644.393 & 3835.95 & 23146 \end{pmatrix}; \; b = \{-9.512, -52.326, -282.174\};$$

In[658]:=

```
LinearSolve[A, b]
```

Out[658]=

{80.6781, -28.8073, 2.51589}

Записваме в общ вид

In[659]:=

$$A = \begin{pmatrix} P & \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 \\ \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 \\ \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 \end{pmatrix};$$

$$b = \left\{\sum_{i=1}^{P} yt[[i]], \; \sum_{i=1}^{P} yt[[i]] * xt[[i]], \; \sum_{i=1}^{P} yt[[i]] * xt[[i]]^2\right\};$$

In[660]:=

```
a = LinearSolve[A, b]
```

Out[660]=

{80.7295, -28.8245, 2.5173}

Съставяме полинома

In[661]:=

$$P2[x\_] := 80.678 - 28.807\,x + 2.516\,x^2$$

In[662]:=

$$P2[x\_] := a[[1]] + a[[2]]\,x + a[[3]]\,x^2$$

```
P2[x]
```

Out[663]=

$80.7295 - 28.8245\,x + 2.5173\,x^2$

Таен коз (възможност за самопроверка)

In[664]:=

$$\mathrm{Fit}\left[points, \{1, x, x^2\}, x\right]$$

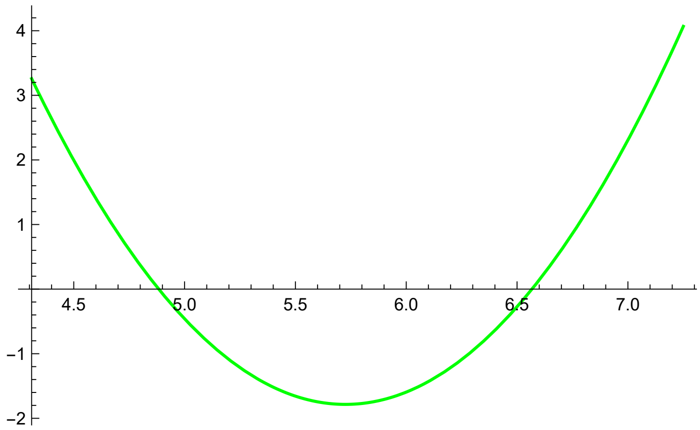
Out[664]=

$80.7295 - 28.8245\,x + 2.5173\,x^2$

In[665]:=

```
grfP2 = Plot[P2[x], {x, xt⟦1⟧ - 0.3, xt⟦P⟧ + 0.3}, PlotStyle → Green]
```

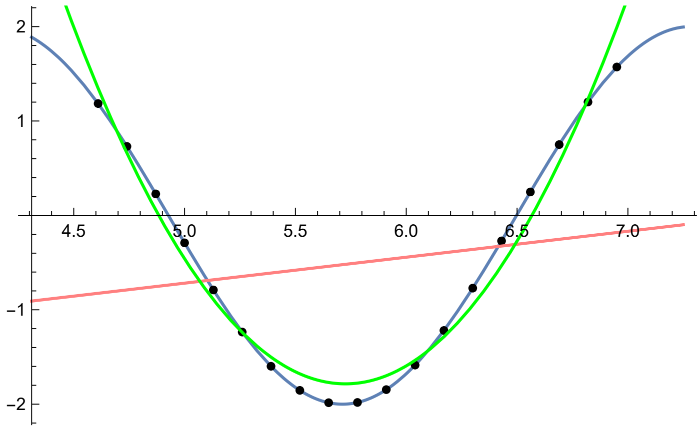
Out[665]=



In[666]:=

```
Show[grf, grp, grfP1, grfP2]
```

Out[666]=



Намиране на приближена стойност (апроксимация)

Стойност извън разглеждания интервал

In[667]:=

```
P2[4]
```

Out[667]=

5.70845

За сравнение истинската стойност

In[668]:=

```
f[4.]
```

Out[668]=

1.92034

Намиране на приближена стойност (апроксимация)

Стойност вътре в разглеждания интервал

In[669]:=

```
P2[5.3]
```

Out[669]=

```
-1.32918
```

За сравнение истинската стойност

In[670]:=

```
f[5.3]
```

Out[670]=

```
-1.35744
```

Оценка на грешката

Теоретична грешка (средноквадратична)

In[671]:=

$$\sqrt{\sum_{i=1}^{p} (yt[[i]] - P2[xt[[i]]])^2}$$

Out[671]=

```
0.806493
```

Истинска грешка

In[672]:=

```
Abs[f[4.] - P2[4.]]
```

Out[672]=

```
3.78811
```

In[673]:=

```
Abs[f[5.3] - P2[5.3]]
```

Out[673]=

```
0.0282553
```

Кубична регресия

Попълваме таблицата

In[674]:=

xt^2

Out[674]=

```
{21.2521, 22.4676, 23.7169, 25., 26.3169, 27.6676, 29.0521, 30.4704, 31.9225, 33.4084,
 34.9281, 36.4816, 38.0689, 39.69, 41.3449, 43.0336, 44.7561, 46.5124, 48.3025}
```

In[675]:=

```
yt * xt
```

Out[675]=

```
{5.46153, 3.46328, 1.10778, -1.455, -4.05223, -6.49989,
 -8.61573, -10.2328, -11.2121, -11.4545, -10.9088, -9.57838,
 -7.52264, -4.85526, -1.73815, 1.62826, 5.01786, 8.19409, 10.9264}
```

In[676]:=

xt^3

Out[676]=

```
{97.9722, 106.496, 115.501, 125., 135.006, 145.532, 156.591, 168.197, 180.362, 193.101,
 206.425, 220.349, 234.885, 250.047, 265.848, 282.3, 299.418, 317.215, 335.702}
```

In[677]:=

xt^4

Out[677]=

```
{451.652, 504.793, 562.491, 625., 692.579, 765.496, 844.025, 928.445, 1019.05, 1116.12,
 1219.97, 1330.91, 1449.24, 1575.3, 1709.4, 1851.89, 2003.11, 2163.4, 2333.13}
```

In[678]:=

$yt * xt^2$

Out[678]=

```
{25.1777, 16.4159, 5.39488, -7.275, -20.788, -34.1894,
 -46.4388, -56.4849, -63.3486, -66.2069, -64.4711, -57.8534,
 -46.4147, -30.5881, -11.1763, 10.6814, 33.5695, 55.8837, 75.9383}
```

Допълваме необходимото

Намиране на сумите

In[679]:=

$$\sum_{i=1}^{P} xt[[i]]$$

Out[679]=

109.82

In[680]:=

$$\sum_{i=1}^{P} yt[[i]]$$

Out[680]=

-9.51221

In[681]:=

$$\sum_{i=1}^{P} xt[[i]]^2$$

Out[681]=

644.393

In[682]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]$$

Out[682]=

-52.3263

In[683]:=

$$\sum_{i=1}^{P} xt[[i]]^3$$

Out[683]=

3835.95

In[684]:=

$$\sum_{i=1}^{P} xt[[i]]^4$$

Out[684]=

23146.

In[685]:=

$$\sum_{i=1}^{P} yt[[i]] * xt[[i]]^2$$

Out[685]=

-282.174

Допълваме необходимото

Решаваме системата

In[686]:=

$$A = \begin{pmatrix} 19 & 109.82 & 644.393 \\ 109.82 & 644.393 & 3835.95 \\ 644.393 & 3835.95 & 23146 \end{pmatrix}; \ b = \{-9.512, -52.326, -282.174\};$$

In[687]:=

```
LinearSolve[A, b]
```

Out[687]=

{80.6781, -28.8073, 2.51589}

Записваме в общ вид

In[688]:=

$$A = \begin{pmatrix} P & \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 \\ \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 \\ \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 & \sum_{i=1}^{P} xt[[i]]^5 \\ \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 & \sum_{i=1}^{P} xt[[i]]^5 & \sum_{i=1}^{P} xt[[i]]^6 \end{pmatrix};$$

$$b = \left\{\sum_{i=1}^{P} yt[[i]], \sum_{i=1}^{P} yt[[i]] * xt[[i]], \sum_{i=1}^{P} yt[[i]] * xt[[i]]^2, \sum_{i=1}^{P} yt[[i]] * xt[[i]]^3\right\};$$

In[689]:=

```
a = LinearSolve[A, b]
```

LinearSolve: Result for LinearSolve of badly conditioned matrix

{{19., 109.82, 644.393, 3835.95}, {109.82, 644.393, 3835.95, 23146.}, {644.393, 3835.95, 23146., 141427.}, {3835.95, 23146., 141427., 874141.}} may contain significant numerical errors.

Out[689]=

{128.634, -54.1524, 6.93941, -0.255023}

Съставяме полинома

In[690]:=

```
P3[x_] := a[[1]] + a[[2]] x + a[[3]] x^2 + a[[4]] x^3
P3[x]
```

Out[691]=

$128.634 - 54.1524\,x + 6.93941\,x^2 - 0.255023\,x^3$

Таен коз (възможност за самопроверка)

In[692]:=

```
Fit[points, {1, x, x^2, x^3}, x]
```

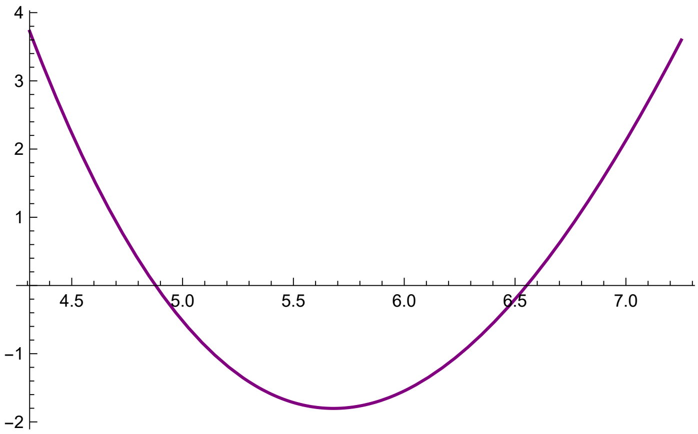
Out[692]=

$128.634 - 54.1524\,x + 6.93941\,x^2 - 0.255023\,x^3$

In[693]:=

```
grfP3 = Plot[P3[x], {x, xt[[1]] - 0.3, xt[[P]] + 0.3}, PlotStyle → Purple]
```

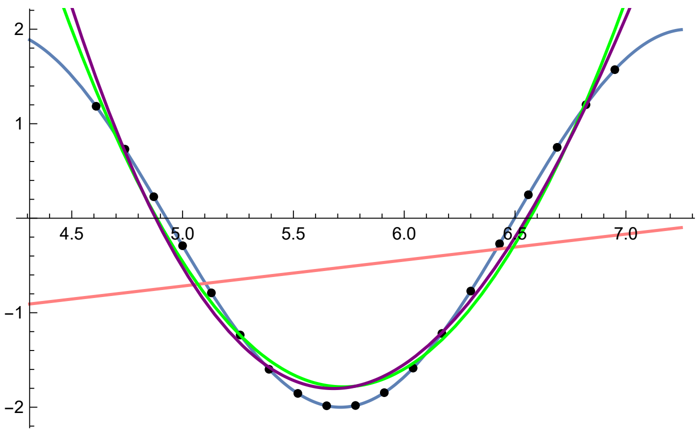
Out[693]=



In[694]:=

```
Show[grf, grp, grfP1, grfP2, grfP3]
```

Out[694]=



Намиране на приближена стойност (апроксимация)

Стойност извън разглеждания интервал

In[695]:=

```
P3[4]
```

Out[695]=

```
6.73399
```

За сравнение истинската стойност

In[696]:=

```
f[4.]
```

Out[696]=

```
1.92034
```

Намиране на приближена стойност (апроксимация)

Стойност вътре в разглеждания интервал

In[697]:=

```
P3[5.3]
```

Out[697]=

```
-1.41228
```

За сравнение истинската стойност

In[698]:=

```
f[5.3]
```

Out[698]=

```
-1.35744
```

Оценка на грешката

Теоретична грешка (средноквадратична)

In[699]:=

$$\sqrt{\sum_{i=1}^{p} (yt[[i]] - P3[xt[[i]]])^2}$$

Out[699]=

```
0.744355
```

Истинска грешка

In[700]:=

```
Abs[f[4.] - P3[4.]]
```

Out[700]=

```
4.81364
```

In[701]:=

```
Abs[f[5.3] - P3[5.3]]
```

Out[701]=

```
0.0548399
```