

Метод за решаване на последователни приближения за решаване на СЛАУ

In[351]:=

$$A = \begin{pmatrix} 1.1 & 0.02 & -0.12 \\ 0.13 & 0.98 & -0.01 \\ 0 & 0.01 & 1.2 \end{pmatrix}; \quad b = \{6.5, 12, -21.2\};$$

In[352]:=

```
Print["За сравнение точното решение е ", LinearSolve[A, b]]
```

За сравнение точното решение е {3.76104, 11.5647, -17.763}

Построяване на метода

In[353]:=

```
n = Length[A];
```

In[354]:=

```
IM = IdentityMatrix[n];
```

In[355]:=

```
B = IM - A;
```

In[356]:=

```
c = b;
```

In[357]:=

```
Print["Итерационният процес е  $x^{(k+1)} =$ ", B // MatrixForm, " $. x^{(k)} +$ ", c // MatrixForm]
```

$$\text{Итерационният процес е } x^{(k+1)} = \begin{pmatrix} -0.1 & -0.02 & 0.12 \\ -0.13 & 0.02 & 0.01 \\ 0 & -0.01 & -0.2 \end{pmatrix} \cdot x^{(k)} + \begin{pmatrix} 6.5 \\ 12 \\ -21.2 \end{pmatrix}$$

Пояснения във Wolfram

In[358]:=

E

Out[358]=

e

```
In[359]:= IdentityMatrix[3] // MatrixForm
```

```
Out[359]//MatrixForm=
```

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Проверка за сходимост $\|B\| < 1$

първа норма

```
In[360]:= Max[Table[Sum[Abs[B[[i, j]]], {j, n}], {i, n}]]
```

```
Out[360]=
```

0.24

втора норма

```
In[361]:= Max[Table[Sum[Abs[B[[i, j]]], {j, n}], {i, n}]]
```

```
Out[361]=
```

0.33

трета норма

```
In[362]:=
```

$$\sqrt{\sum_{i=1}^n \sum_{j=1}^n B[[i, j]]^2}$$

```
Out[362]=
```

0.28688

Избираме най-малката възможна норма, която в случая е **първа**.

Нормата на матрицата B е по-малка от 1, следователно процесът ще е сходящ при всеки избор на начално приближение.

Извършване на итерациите - окончателен код в една клетка

In[411]:=

```

A =  $\begin{pmatrix} 1.1 & 0.02 & -0.12 \\ 0.13 & 0.98 & -0.01 \\ 0 & 0.01 & 1.2 \end{pmatrix}$ ; b = {6.5, 12, -21.2};

n = Length[A];
IM = IdentityMatrix[n];
B = IM - A;
c = b;
Print["Итерационният процес е  $x^{(k+1)} =$ ", B // MatrixForm, " $. x^{(k)} +$ ", c // MatrixForm]
Print[]
x = {9, 12,  $\frac{1}{2}$ }; (*изборът на начално приближение е произволен*)

(*изчисляваме нормите според избора на норма,
който сме направили по време на проверка на условието на сходимост*)
normB = Max[Table[ $\sum_{j=1}^n \text{Abs}[B[[i, j]]]$ , {i, n}]];

Print["Нормата на B е ", normB]
normx0 = Max[Abs[x]];
normc = Max[Abs[c]];
For[k = 0, k ≤ 5, k++,

  Print["k = ", k, "  $x^{(k)} =$ ", x, "  $\epsilon_k =$ ", eps = normBk  $\left( \text{normx0} + \frac{\text{normc}}{1 - \text{normB}} \right)$ ];

  x = B.x + c
]
Print["За сравнение, точното решение е ", LinearSolve[A, b]]

```

$$\text{Итерационният процес е } x^{(k+1)} = \begin{pmatrix} -0.1 & -0.02 & 0.12 \\ -0.13 & 0.02 & 0.01 \\ 0 & -0.01 & -0.2 \end{pmatrix} \cdot x^{(k)} + \begin{pmatrix} 6.5 \\ 12 \\ -21.2 \end{pmatrix}$$

Нормата на B е 0.24

$$k = 0 \quad x^{(k)} = \left\{ 9, 12, \frac{1}{2} \right\} \quad \varepsilon_k = 39.8947$$

$$k = 1 \quad x^{(k)} = \{ 5.42, 11.075, -21.42 \} \quad \varepsilon_k = 9.57474$$

$$k = 2 \quad x^{(k)} = \{ 3.1661, 11.3027, -17.0268 \} \quad \varepsilon_k = 2.29794$$

$$k = 3 \quad x^{(k)} = \{ 3.91413, 11.6442, -17.9077 \} \quad \varepsilon_k = 0.551505$$

$$k = 4 \quad x^{(k)} = \{ 3.72678, 11.545, -17.7349 \} \quad \varepsilon_k = 0.132361$$

$$k = 5 \quad x^{(k)} = \{ 3.76823, 11.5691, -17.7685 \} \quad \varepsilon_k = 0.0317667$$

За сравнение, точното решение е $\{ 3.76104, 11.5647, -17.763 \}$

Определяне на брой итерации за достигане на предварително зададена точност 10^{-6}

In[425]:=

$$\frac{\text{Log}\left[\frac{10^{-6}}{\text{normx0} + \frac{\text{normc}}{1-\text{normB}}}\right]}{\text{Log}[\text{normB}]}$$

Out[425]=

12.2637

Извод: Необходими са 13 на брой итерации.

За сравнение и проверка пускаме итерациите:

In[426]:=

```

A =  $\begin{pmatrix} 1.1 & 0.02 & -0.12 \\ 0.13 & 0.98 & -0.01 \\ 0 & 0.01 & 1.2 \end{pmatrix}$ ; b = {6.5, 12, -21.2};

n = Length[A];
IM = IdentityMatrix[n];
B = IM - A;
c = b;
Print["Итерационният процес е  $x^{(k+1)} =$ ", B // MatrixForm, " $. x^{(k)} +$ ", c // MatrixForm]
Print[]
x = {9, 12,  $\frac{1}{2}$ }; (*изборът на начално приближение е произволен*)
(*изчисляваме нормите според избора на норма,
който сме направили по време на проверка на условието на сходимост*)
normB = Max[Table[ $\sum_{j=1}^n \text{Abs}[B[[i, j]]]$ , {i, n}]];
Print["Нормата на B е ", normB]
normx0 = Max[Abs[x]];
normc = Max[Abs[c]];
For[k = 0, k ≤ 13, k++,
  Print["k = ", k, "  $x^{(k)} =$ ", x, "  $\varepsilon_k =$ ", eps = normBk  $\left( \text{normx0} + \frac{\text{normc}}{1 - \text{normB}} \right)$ ];
  x = B.x + c
]
Print["За сравнение, точното решение е ", LinearSolve[A, b]]

```

$$\text{Итерационният процес е } x^{(k+1)} = \begin{pmatrix} -0.1 & -0.02 & 0.12 \\ -0.13 & 0.02 & 0.01 \\ 0 & -0.01 & -0.2 \end{pmatrix} \cdot x^{(k)} + \begin{pmatrix} 6.5 \\ 12 \\ -21.2 \end{pmatrix}$$

Нормата на B е 0.24

$$k = 0 \quad x^{(k)} = \left\{ 9, 12, \frac{1}{2} \right\} \quad \varepsilon_k = 39.8947$$

$$k = 1 \quad x^{(k)} = \{ 5.42, 11.075, -21.42 \} \quad \varepsilon_k = 9.57474$$

$$k = 2 \quad x^{(k)} = \{ 3.1661, 11.3027, -17.0268 \} \quad \varepsilon_k = 2.29794$$

$$k = 3 \quad x^{(k)} = \{ 3.91413, 11.6442, -17.9077 \} \quad \varepsilon_k = 0.551505$$

$$k = 4 \quad x^{(k)} = \{ 3.72678, 11.545, -17.7349 \} \quad \varepsilon_k = 0.132361$$

$$k = 5 \quad x^{(k)} = \{ 3.76823, 11.5691, -17.7685 \} \quad \varepsilon_k = 0.0317667$$

$$k = 6 \quad x^{(k)} = \{ 3.75958, 11.5638, -17.762 \} \quad \varepsilon_k = 0.007624$$

$$k = 7 \quad x^{(k)} = \{ 3.76133, 11.5649, -17.7632 \} \quad \varepsilon_k = 0.00182976$$

$$k = 8 \quad x^{(k)} = \{ 3.76098, 11.5647, -17.763 \} \quad \varepsilon_k = 0.000439143$$

$$k = 9 \quad x^{(k)} = \{ 3.76105, 11.5647, -17.763 \} \quad \varepsilon_k = 0.000105394$$

$$k = 10 \quad x^{(k)} = \{ 3.76103, 11.5647, -17.763 \} \quad \varepsilon_k = 0.0000252946$$

$$k = 11 \quad x^{(k)} = \{ 3.76104, 11.5647, -17.763 \} \quad \varepsilon_k = 6.07071 \times 10^{-6}$$

$$k = 12 \quad x^{(k)} = \{ 3.76104, 11.5647, -17.763 \} \quad \varepsilon_k = 1.45697 \times 10^{-6}$$

$$k = 13 \quad x^{(k)} = \{ 3.76104, 11.5647, -17.763 \} \quad \varepsilon_k = 3.49673 \times 10^{-7}$$

За сравнение, точното решение е $\{ 3.76104, 11.5647, -17.763 \}$

Пример за разходящ процес

In[377]:=

```

A =  $\begin{pmatrix} 11 & 0.02 & -0.12 \\ 0.13 & 98 & -0.01 \\ 0 & 0.01 & 12 \end{pmatrix}$ ; b = {6.5, 12, -21.2};

n = Length[A];
IM = IdentityMatrix[n];
B = IM - A;
c = b;
Print["Итерационният процес е  $x^{(k+1)} =$ ", B // MatrixForm, " $. x^{(k)} +$ ", c // MatrixForm]
Print[]
x = {9, 12,  $\frac{1}{2}$ }; (*изборът на начално приближение е произволен*)

(*изчисляваме нормите според избора на норма,
който сме направили по време на проверка на условието на сходимост*)

normB = Max[Table[ $\sum_{j=1}^n \text{Abs}[B[[i, j]]]$ , {i, n}]];

Print["Нормата на B е ", normB]
normx0 = Max[Abs[x]];
normc = Max[Abs[c]];
For[k = 0, k ≤ 5, k++,

  Print["k = ", k, "  $x^{(k)} =$ ", x, "  $\epsilon_k =$ ", eps = normBk  $\left( \text{normx0} + \frac{\text{normc}}{1 - \text{normB}} \right)$ ];

  x = B.x + c
]
Print["За сравнение, точното решение е ", LinearSolve[A, b]]

Итерационният процес е  $x^{(k+1)} = \begin{pmatrix} -10 & -0.02 & 0.12 \\ -0.13 & -97 & 0.01 \\ 0 & -0.01 & -11 \end{pmatrix} . x^{(k)} + \begin{pmatrix} 6.5 \\ 12 \\ -21.2 \end{pmatrix}$ 

```

Нормата на B е 97.14

$$k = 0 \quad x^{(k)} = \left\{ 9, 12, \frac{1}{2} \right\} \quad \epsilon_k = 11.7795$$

$$k = 1 \quad x^{(k)} = \{-83.68, -1153.17, -26.82\} \quad \epsilon_k = 1144.26$$

$$k = 2 \quad x^{(k)} = \{863.145, 111880., 285.352\} \quad \epsilon_k = 111153.$$

$$k = 3 \quad x^{(k)} = \{-10828.3, -1.08524 \times 10^7, -4278.86\} \quad \epsilon_k = 1.07974 \times 10^7$$

$$k = 4 \quad x^{(k)} = \{324824., 1.05269 \times 10^9, 155571.\} \quad \epsilon_k = 1.04886 \times 10^9$$

$$k = 5 \quad x^{(k)} = \{-2.42833 \times 10^7, -1.02111 \times 10^{11}, -1.22382 \times 10^7\} \quad \epsilon_k = 1.01887 \times 10^{11}$$

За сравнение, точното решение е {0.571414, 0.121511, -1.76677}

Модификация на метода при положително определена матрица A

Проверка на приложимостта на модификацията

```
In[391]:=
```

$$A = \begin{pmatrix} 11 & 0.02 & -0.12 \\ 0.13 & 98 & -0.01 \\ 0 & 0.01 & 12 \end{pmatrix};$$

```
In[392]:=
```

```
PositiveDefiniteMatrixQ[A]
```

```
Out[392]=
```

```
True
```

Определяне стойността на ρ

```
In[393]:=
```

```
Norm[A]
```

```
Out[393]=
```

```
98.0001
```

```
In[394]:=
```

```
ro = 200
```

```
Out[394]=
```

```
200
```


Итерациите

In[440]:=

```

A =  $\begin{pmatrix} 11 & 0.02 & -0.12 \\ 0.13 & 98 & -0.01 \\ 0 & 0.01 & 12 \end{pmatrix}$ ; b = {6.5, 12, -21.2};

n = Length[A];
IM = IdentityMatrix[n];
ro = 200;
B = IM -  $\frac{2}{ro}$  A;
c =  $\frac{2}{ro}$  b;
Print["Итерационният процес е  $x^{(k+1)} =$ ", B // MatrixForm, " $. x^{(k)} +$ ", c // MatrixForm]
Print[]
x = {9, 12,  $\frac{1}{2}$ }; (*изборът на начално приближение е произволен*)
(*изчисляваме нормите според избора на норма,
който сме направили по време на проверка на условието на сходимост*)
normB = Max[Table[ $\sum_{j=1}^n$  Abs[B[[i, j]]], {i, n}]];
Print["Нормата на B е ", normB]
normx0 = Max[Abs[x]];
normc = Max[Abs[c]];
For[k = 0, k <= 5, k++,
  Print["k = ", k, "  $x^{(k)} =$ ", x, "  $\epsilon_k =$ ", eps = normB^k  $\left( \text{normx0} + \frac{\text{normc}}{1 - \text{normB}} \right)$ ];
  x = B.x + c
]
Print["За сравнение, точното решение е ", LinearSolve[A, b]]

```

Итерационният процес е $x^{(k+1)} = \begin{pmatrix} \frac{89}{100} & -0.0002 & 0.0012 \\ -0.0013 & \frac{1}{50} & 0.0001 \\ 0 & -0.0001 & \frac{22}{25} \end{pmatrix} \cdot x^{(k)} + \begin{pmatrix} 0.065 \\ \frac{3}{25} \\ -0.212 \end{pmatrix}$

Нормата на B е 0.8914

$$k = 0 \quad x^{(k)} = \left\{ 9, 12, \frac{1}{2} \right\} \quad \varepsilon_k = 13.9521$$

$$k = 1 \quad x^{(k)} = \{ 8.0732, 0.34835, 0.2268 \} \quad \varepsilon_k = 12.4369$$

$$k = 2 \quad x^{(k)} = \{ 7.25035, 0.116495, -0.0124508 \} \quad \varepsilon_k = 11.0863$$

$$k = 3 \quad x^{(k)} = \{ 6.51777, 0.112903, -0.222968 \} \quad \varepsilon_k = 9.8823$$

$$k = 4 \quad x^{(k)} = \{ 5.86553, 0.113763, -0.408223 \} \quad \varepsilon_k = 8.80908$$

$$k = 5 \quad x^{(k)} = \{ 5.28481, 0.114609, -0.571248 \} \quad \varepsilon_k = 7.85242$$

За сравнение, точното решение е $\{ 0.571414, 0.121511, -1.76677 \}$

In[455]:=

$$\frac{\text{Log}\left[\frac{10^{-6}}{\text{norm}x0 + \frac{\text{norm}c}{1-\text{norm}B}}\right]}{\text{Log}[\text{norm}B]}$$

Out[455]=

143.101

In[456]:=

```

A =  $\begin{pmatrix} 11 & 0.02 & -0.12 \\ 0.13 & 98 & -0.01 \\ 0 & 0.01 & 12 \end{pmatrix}$ ; b = {6.5, 12, -21.2};

n = Length[A];
IM = IdentityMatrix[n];
ro = 200;
B = IM -  $\frac{2}{ro}$  A;
c =  $\frac{2}{ro}$  b;
Print["Итерационният процес е  $x^{(k+1)} =$ ", B // MatrixForm, " $. x^{(k)} +$ ", c // MatrixForm]
Print[]
x = {9, 12,  $\frac{1}{2}$ }; (*изборът на начално приближение е произволен*)
(*изчисляваме нормите според избора на норма,
който сме направили по време на проверка на условието на сходимост*)
normB = Max[Table[ $\sum_{j=1}^n$  Abs[B[[i, j]]], {i, n}]];
Print["Нормата на B е ", normB]
normx0 = Max[Abs[x]];
normc = Max[Abs[c]];
For[k = 0, k ≤ 144, k++,
  Print["k = ", k, "  $x^{(k)} =$ ", x, "  $\varepsilon_k =$ ", eps = normB^k  $\left( \text{normx0} + \frac{\text{normc}}{1 - \text{normB}} \right)$ ];
  x = B.x + c
]
Print["За сравнение, точното решение е ", LinearSolve[A, b]]

```

$$\text{Итерационният процес е } x^{(k+1)} = \begin{pmatrix} \frac{89}{100} & -0.0002 & 0.0012 \\ -0.0013 & \frac{1}{50} & 0.0001 \\ 0 & -0.0001 & \frac{22}{25} \end{pmatrix} \cdot x^{(k)} + \begin{pmatrix} 0.065 \\ \frac{3}{25} \\ -0.212 \end{pmatrix}$$

Нормата на B е 0.8914

$$k = 0 \quad x^{(k)} = \left\{ 9, 12, \frac{1}{2} \right\} \quad \varepsilon_k = 13.9521$$

$$k = 1 \quad x^{(k)} = \{ 8.0732, 0.34835, 0.2268 \} \quad \varepsilon_k = 12.4369$$

$$k = 2 \quad x^{(k)} = \{ 7.25035, 0.116495, -0.0124508 \} \quad \varepsilon_k = 11.0863$$

$$k = 3 \quad x^{(k)} = \{ 6.51777, 0.112903, -0.222968 \} \quad \varepsilon_k = 9.8823$$

$$k = 4 \quad x^{(k)} = \{ 5.86553, 0.113763, -0.408223 \} \quad \varepsilon_k = 8.80908$$

$$k = 5 \quad x^{(k)} = \{ 5.28481, 0.114609, -0.571248 \} \quad \varepsilon_k = 7.85242$$

$k = 6 \quad x^{(k)} = \{4.76777, 0.115365, -0.71471\} \quad \varepsilon_k = 6.99964$
 $k = 7 \quad x^{(k)} = \{4.30743, 0.116038, -0.840956\} \quad \varepsilon_k = 6.23948$
 $k = 8 \quad x^{(k)} = \{3.89758, 0.116637, -0.952053\} \quad \varepsilon_k = 5.56187$
 $k = 9 \quad x^{(k)} = \{3.53268, 0.117171, -1.04982\} \quad \varepsilon_k = 4.95785$
 $k = 10 \quad x^{(k)} = \{3.20781, 0.117646, -1.13585\} \quad \varepsilon_k = 4.41943$
 $k = 11 \quad x^{(k)} = \{2.91856, 0.118069, -1.21156\} \quad \varepsilon_k = 3.93948$
 $k = 12 \quad x^{(k)} = \{2.66104, 0.118446, -1.27819\} \quad \varepsilon_k = 3.51165$
 $k = 13 \quad x^{(k)} = \{2.43177, 0.118782, -1.33682\} \quad \varepsilon_k = 3.13029$
 $k = 14 \quad x^{(k)} = \{2.22765, 0.119081, -1.38841\} \quad \varepsilon_k = 2.79034$
 $k = 15 \quad x^{(k)} = \{2.04592, 0.119347, -1.43381\} \quad \varepsilon_k = 2.48731$
 $k = 16 \quad x^{(k)} = \{1.88412, 0.119584, -1.47377\} \quad \varepsilon_k = 2.21719$
 $k = 17 \quad x^{(k)} = \{1.74008, 0.119795, -1.50893\} \quad \varepsilon_k = 1.9764$
 $k = 18 \quad x^{(k)} = \{1.61183, 0.119983, -1.53987\} \quad \varepsilon_k = 1.76176$
 $k = 19 \quad x^{(k)} = \{1.49766, 0.12015, -1.5671\} \quad \varepsilon_k = 1.57044$
 $k = 20 \quad x^{(k)} = \{1.39601, 0.120299, -1.59106\} \quad \varepsilon_k = 1.39989$
 $k = 21 \quad x^{(k)} = \{1.30552, 0.120432, -1.61214\} \quad \varepsilon_k = 1.24786$
 $k = 22 \quad x^{(k)} = \{1.22495, 0.12055, -1.6307\} \quad \varepsilon_k = 1.11234$
 $k = 23 \quad x^{(k)} = \{1.15323, 0.120655, -1.64702\} \quad \varepsilon_k = 0.991541$
 $k = 24 \quad x^{(k)} = \{1.08937, 0.120749, -1.66139\} \quad \varepsilon_k = 0.88386$
 $k = 25 \quad x^{(k)} = \{1.03252, 0.120833, -1.67404\} \quad \varepsilon_k = 0.787872$
 $k = 26 \quad x^{(k)} = \{0.981912, 0.120907, -1.68517\} \quad \varepsilon_k = 0.702309$
 $k = 27 \quad x^{(k)} = \{0.936855, 0.120973, -1.69496\} \quad \varepsilon_k = 0.626039$
 $k = 28 \quad x^{(k)} = \{0.896743, 0.121032, -1.70358\} \quad \varepsilon_k = 0.558051$
 $k = 29 \quad x^{(k)} = \{0.861033, 0.121085, -1.71116\} \quad \varepsilon_k = 0.497447$
 $k = 30 \quad x^{(k)} = \{0.829241, 0.121131, -1.71783\} \quad \varepsilon_k = 0.443424$
 $k = 31 \quad x^{(k)} = \{0.800939, 0.121173, -1.7237\} \quad \varepsilon_k = 0.395268$
 $k = 32 \quad x^{(k)} = \{0.775743, 0.12121, -1.72887\} \quad \varepsilon_k = 0.352342$
 $k = 33 \quad x^{(k)} = \{0.753313, 0.121243, -1.73342\} \quad \varepsilon_k = 0.314078$
 $k = 34 \quad x^{(k)} = \{0.733344, 0.121272, -1.73742\} \quad \varepsilon_k = 0.279969$
 $k = 35 \quad x^{(k)} = \{0.715567, 0.121298, -1.74094\} \quad \varepsilon_k = 0.249564$
 $k = 36 \quad x^{(k)} = \{0.699741, 0.121322, -1.74404\} \quad \varepsilon_k = 0.222461$
 $k = 37 \quad x^{(k)} = \{0.685652, 0.121342, -1.74677\} \quad \varepsilon_k = 0.198302$
 $k = 38 \quad x^{(k)} = \{0.67311, 0.121361, -1.74917\} \quad \varepsilon_k = 0.176767$
 $k = 39 \quad x^{(k)} = \{0.661945, 0.121377, -1.75128\} \quad \varepsilon_k = 0.15757$
 $k = 40 \quad x^{(k)} = \{0.652005, 0.121392, -1.75314\} \quad \varepsilon_k = 0.140458$
 $k = 41 \quad x^{(k)} = \{0.643157, 0.121405, -1.75477\} \quad \varepsilon_k = 0.125204$

$k = 42 \quad x^{(k)} = \{0.635279, 0.121417, -1.75621\} \quad \varepsilon_k = 0.111607$
 $k = 43 \quad x^{(k)} = \{0.628267, 0.121427, -1.75748\} \quad \varepsilon_k = 0.0994863$
 $k = 44 \quad x^{(k)} = \{0.622024, 0.121436, -1.75859\} \quad \varepsilon_k = 0.0886821$
 $k = 45 \quad x^{(k)} = \{0.616467, 0.121444, -1.75958\} \quad \varepsilon_k = 0.0790512$
 $k = 46 \quad x^{(k)} = \{0.61152, 0.121452, -1.76044\} \quad \varepsilon_k = 0.0704662$
 $k = 47 \quad x^{(k)} = \{0.607116, 0.121458, -1.7612\} \quad \varepsilon_k = 0.0628136$
 $k = 48 \quad x^{(k)} = \{0.603195, 0.121464, -1.76187\} \quad \varepsilon_k = 0.055992$
 $k = 49 \quad x^{(k)} = \{0.599705, 0.121469, -1.76245\} \quad \varepsilon_k = 0.0499113$
 $k = 50 \quad x^{(k)} = \{0.596599, 0.121474, -1.76297\} \quad \varepsilon_k = 0.0444909$
 $k = 51 \quad x^{(k)} = \{0.593833, 0.121478, -1.76343\} \quad \varepsilon_k = 0.0396592$
 $k = 52 \quad x^{(k)} = \{0.591371, 0.121481, -1.76383\} \quad \varepsilon_k = 0.0353522$
 $k = 53 \quad x^{(k)} = \{0.589179, 0.121484, -1.76418\} \quad \varepsilon_k = 0.031513$
 $k = 54 \quad x^{(k)} = \{0.587228, 0.121487, -1.76449\} \quad \varepsilon_k = 0.0280907$
 $k = 55 \quad x^{(k)} = \{0.585491, 0.12149, -1.76476\} \quad \varepsilon_k = 0.02504$
 $k = 56 \quad x^{(k)} = \{0.583945, 0.121492, -1.76501\} \quad \varepsilon_k = 0.0223207$
 $k = 57 \quad x^{(k)} = \{0.582569, 0.121494, -1.76522\} \quad \varepsilon_k = 0.0198967$
 $k = 58 \quad x^{(k)} = \{0.581344, 0.121496, -1.7654\} \quad \varepsilon_k = 0.0177359$
 $k = 59 \quad x^{(k)} = \{0.580253, 0.121498, -1.76557\} \quad \varepsilon_k = 0.0158098$
 $k = 60 \quad x^{(k)} = \{0.579282, 0.121499, -1.76571\} \quad \varepsilon_k = 0.0140928$
 $k = 61 \quad x^{(k)} = \{0.578418, 0.1215, -1.76584\} \quad \varepsilon_k = 0.0125623$
 $k = 62 \quad x^{(k)} = \{0.577649, 0.121501, -1.76595\} \quad \varepsilon_k = 0.0111981$
 $k = 63 \quad x^{(k)} = \{0.576964, 0.121502, -1.76605\} \quad \varepsilon_k = 0.00998196$
 $k = 64 \quad x^{(k)} = \{0.576354, 0.121503, -1.76613\} \quad \varepsilon_k = 0.00889792$
 $k = 65 \quad x^{(k)} = \{0.575812, 0.121504, -1.76621\} \quad \varepsilon_k = 0.0079316$
 $k = 66 \quad x^{(k)} = \{0.575329, 0.121505, -1.76628\} \quad \varepsilon_k = 0.00707023$
 $k = 67 \quad x^{(k)} = \{0.574899, 0.121506, -1.76634\} \quad \varepsilon_k = 0.00630241$
 $k = 68 \quad x^{(k)} = \{0.574516, 0.121506, -1.76639\} \quad \varepsilon_k = 0.00561796$
 $k = 69 \quad x^{(k)} = \{0.574175, 0.121507, -1.76643\} \quad \varepsilon_k = 0.00500785$
 $k = 70 \quad x^{(k)} = \{0.573872, 0.121507, -1.76647\} \quad \varepsilon_k = 0.004464$
 $k = 71 \quad x^{(k)} = \{0.573602, 0.121507, -1.76651\} \quad \varepsilon_k = 0.00397921$
 $k = 72 \quad x^{(k)} = \{0.573362, 0.121508, -1.76654\} \quad \varepsilon_k = 0.00354707$
 $k = 73 \quad x^{(k)} = \{0.573148, 0.121508, -1.76657\} \quad \varepsilon_k = 0.00316186$
 $k = 74 \quad x^{(k)} = \{0.572957, 0.121508, -1.76659\} \quad \varepsilon_k = 0.00281848$
 $k = 75 \quad x^{(k)} = \{0.572788, 0.121509, -1.76661\} \quad \varepsilon_k = 0.00251239$
 $k = 76 \quad x^{(k)} = \{0.572637, 0.121509, -1.76663\} \quad \varepsilon_k = 0.00223955$
 $k = 77 \quad x^{(k)} = \{0.572503, 0.121509, -1.76665\} \quad \varepsilon_k = 0.00199633$

$k = 78 \quad x^{(k)} = \{0.572383, 0.121509, -1.76666\} \quad \varepsilon_k = 0.00177953$
 $k = 79 \quad x^{(k)} = \{0.572277, 0.121509, -1.76667\} \quad \varepsilon_k = 0.00158627$
 $k = 80 \quad x^{(k)} = \{0.572182, 0.12151, -1.76669\} \quad \varepsilon_k = 0.001414$
 $k = 81 \quad x^{(k)} = \{0.572098, 0.12151, -1.7667\} \quad \varepsilon_k = 0.00126044$
 $k = 82 \quad x^{(k)} = \{0.572022, 0.12151, -1.7667\} \quad \varepsilon_k = 0.00112356$
 $k = 83 \quad x^{(k)} = \{0.571956, 0.12151, -1.76671\} \quad \varepsilon_k = 0.00100154$
 $k = 84 \quad x^{(k)} = \{0.571896, 0.12151, -1.76672\} \quad \varepsilon_k = 0.000892773$
 $k = 85 \quad x^{(k)} = \{0.571843, 0.12151, -1.76672\} \quad \varepsilon_k = 0.000795818$
 $k = 86 \quad x^{(k)} = \{0.571796, 0.12151, -1.76673\} \quad \varepsilon_k = 0.000709392$
 $k = 87 \quad x^{(k)} = \{0.571754, 0.12151, -1.76673\} \quad \varepsilon_k = 0.000632352$
 $k = 88 \quad x^{(k)} = \{0.571717, 0.12151, -1.76674\} \quad \varepsilon_k = 0.000563679$
 $k = 89 \quad x^{(k)} = \{0.571684, 0.12151, -1.76674\} \quad \varepsilon_k = 0.000502463$
 $k = 90 \quad x^{(k)} = \{0.571654, 0.12151, -1.76675\} \quad \varepsilon_k = 0.000447896$
 $k = 91 \quad x^{(k)} = \{0.571628, 0.12151, -1.76675\} \quad \varepsilon_k = 0.000399254$
 $k = 92 \quad x^{(k)} = \{0.571604, 0.12151, -1.76675\} \quad \varepsilon_k = 0.000355895$
 $k = 93 \quad x^{(k)} = \{0.571583, 0.12151, -1.76675\} \quad \varepsilon_k = 0.000317245$
 $k = 94 \quad x^{(k)} = \{0.571565, 0.12151, -1.76675\} \quad \varepsilon_k = 0.000282792$
 $k = 95 \quad x^{(k)} = \{0.571548, 0.12151, -1.76676\} \quad \varepsilon_k = 0.000252081$
 $k = 96 \quad x^{(k)} = \{0.571534, 0.121511, -1.76676\} \quad \varepsilon_k = 0.000224705$
 $k = 97 \quad x^{(k)} = \{0.57152, 0.121511, -1.76676\} \quad \varepsilon_k = 0.000200302$
 $k = 98 \quad x^{(k)} = \{0.571509, 0.121511, -1.76676\} \quad \varepsilon_k = 0.000178549$
 $k = 99 \quad x^{(k)} = \{0.571498, 0.121511, -1.76676\} \quad \varepsilon_k = 0.000159159$
 $k = 100 \quad x^{(k)} = \{0.571489, 0.121511, -1.76676\} \quad \varepsilon_k = 0.000141874$
 $k = 101 \quad x^{(k)} = \{0.571481, 0.121511, -1.76676\} \quad \varepsilon_k = 0.000126467$
 $k = 102 \quad x^{(k)} = \{0.571474, 0.121511, -1.76676\} \quad \varepsilon_k = 0.000112732$
 $k = 103 \quad x^{(k)} = \{0.571467, 0.121511, -1.76676\} \quad \varepsilon_k = 0.00010049$
 $k = 104 \quad x^{(k)} = \{0.571461, 0.121511, -1.76676\} \quad \varepsilon_k = 0.0000895764$
 $k = 105 \quad x^{(k)} = \{0.571456, 0.121511, -1.76676\} \quad \varepsilon_k = 0.0000798484$
 $k = 106 \quad x^{(k)} = \{0.571452, 0.121511, -1.76676\} \quad \varepsilon_k = 0.0000711769$
 $k = 107 \quad x^{(k)} = \{0.571447, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000634471$
 $k = 108 \quad x^{(k)} = \{0.571444, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000565567$
 $k = 109 \quad x^{(k)} = \{0.571441, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000504147$
 $k = 110 \quad x^{(k)} = \{0.571438, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000449396$
 $k = 111 \quad x^{(k)} = \{0.571435, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000400592$
 $k = 112 \quad x^{(k)} = \{0.571433, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000357088$
 $k = 113 \quad x^{(k)} = \{0.571431, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000318308$

$k = 114 \quad x^{(k)} = \{0.571429, 0.121511, -1.76677\} \quad \varepsilon_k = 0.000028374$
 $k = 115 \quad x^{(k)} = \{0.571427, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000252925$
 $k = 116 \quad x^{(k)} = \{0.571426, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000225458$
 $k = 117 \quad x^{(k)} = \{0.571425, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000200973$
 $k = 118 \quad x^{(k)} = \{0.571424, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000179147$
 $k = 119 \quad x^{(k)} = \{0.571423, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000159692$
 $k = 120 \quad x^{(k)} = \{0.571422, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000142349$
 $k = 121 \quad x^{(k)} = \{0.571421, 0.121511, -1.76677\} \quad \varepsilon_k = 0.000012689$
 $k = 122 \quad x^{(k)} = \{0.57142, 0.121511, -1.76677\} \quad \varepsilon_k = 0.000011311$
 $k = 123 \quad x^{(k)} = \{0.571419, 0.121511, -1.76677\} \quad \varepsilon_k = 0.0000100826$
 $k = 124 \quad x^{(k)} = \{0.571419, 0.121511, -1.76677\} \quad \varepsilon_k = 8.98765 \times 10^{-6}$
 $k = 125 \quad x^{(k)} = \{0.571418, 0.121511, -1.76677\} \quad \varepsilon_k = 8.01159 \times 10^{-6}$
 $k = 126 \quad x^{(k)} = \{0.571418, 0.121511, -1.76677\} \quad \varepsilon_k = 7.14153 \times 10^{-6}$
 $k = 127 \quad x^{(k)} = \{0.571418, 0.121511, -1.76677\} \quad \varepsilon_k = 6.36596 \times 10^{-6}$
 $k = 128 \quad x^{(k)} = \{0.571417, 0.121511, -1.76677\} \quad \varepsilon_k = 5.67462 \times 10^{-6}$
 $k = 129 \quad x^{(k)} = \{0.571417, 0.121511, -1.76677\} \quad \varepsilon_k = 5.05836 \times 10^{-6}$
 $k = 130 \quad x^{(k)} = \{0.571417, 0.121511, -1.76677\} \quad \varepsilon_k = 4.50902 \times 10^{-6}$
 $k = 131 \quad x^{(k)} = \{0.571416, 0.121511, -1.76677\} \quad \varepsilon_k = 4.01934 \times 10^{-6}$
 $k = 132 \quad x^{(k)} = \{0.571416, 0.121511, -1.76677\} \quad \varepsilon_k = 3.58284 \times 10^{-6}$
 $k = 133 \quad x^{(k)} = \{0.571416, 0.121511, -1.76677\} \quad \varepsilon_k = 3.19374 \times 10^{-6}$
 $k = 134 \quad x^{(k)} = \{0.571416, 0.121511, -1.76677\} \quad \varepsilon_k = 2.8469 \times 10^{-6}$
 $k = 135 \quad x^{(k)} = \{0.571416, 0.121511, -1.76677\} \quad \varepsilon_k = 2.53773 \times 10^{-6}$
 $k = 136 \quad x^{(k)} = \{0.571415, 0.121511, -1.76677\} \quad \varepsilon_k = 2.26213 \times 10^{-6}$
 $k = 137 \quad x^{(k)} = \{0.571415, 0.121511, -1.76677\} \quad \varepsilon_k = 2.01646 \times 10^{-6}$
 $k = 138 \quad x^{(k)} = \{0.571415, 0.121511, -1.76677\} \quad \varepsilon_k = 1.79748 \times 10^{-6}$
 $k = 139 \quad x^{(k)} = \{0.571415, 0.121511, -1.76677\} \quad \varepsilon_k = 1.60227 \times 10^{-6}$
 $k = 140 \quad x^{(k)} = \{0.571415, 0.121511, -1.76677\} \quad \varepsilon_k = 1.42826 \times 10^{-6}$
 $k = 141 \quad x^{(k)} = \{0.571415, 0.121511, -1.76677\} \quad \varepsilon_k = 1.27315 \times 10^{-6}$
 $k = 142 \quad x^{(k)} = \{0.571415, 0.121511, -1.76677\} \quad \varepsilon_k = 1.13489 \times 10^{-6}$
 $k = 143 \quad x^{(k)} = \{0.571415, 0.121511, -1.76677\} \quad \varepsilon_k = 1.01164 \times 10^{-6}$
 $k = 144 \quad x^{(k)} = \{0.571415, 0.121511, -1.76677\} \quad \varepsilon_k = 9.01776 \times 10^{-7}$

За сравнение, точното решение е $\{0.571414, 0.121511, -1.76677\}$