Векторни и матрични норми

Вектори

In[ ]:= `a = {3, 7, -69, 17}`

Out[ ]= {3, 7, -69, 17}

In[ ]:= `n = Length[a]`

Out[ ]= 4

In[ ]:= `Norm[a, ∞]`

Out[ ]= 69

Първа норма

In[ ]:= `Max[Abs[a]]`

Out[ ]= 69

Втора норма

In[ ]:= $\sum_{i=1}^{n} \text{Abs}[a[\![i]\!]]$

Out[ ]= 96

In[ ]:= `Norm[a, 1]`

Out[ ]= 96

Трета норма

In[ ]:= $\sqrt{\sum_{i=1}^{n} \text{Abs}[a[\![i]\!]]^2}$

Out[ ]= $2\sqrt{1277}$

```
In[•]:= % // N
Out[•]=
71.4703
```

```
In[•]:= Norm[a]
Out[•]=
```
$2\sqrt{1277}$

Матрици

In[•]:=
$$A = \begin{pmatrix} 12 & 7 & 11 \\ 9 & 2 & 22 \\ 0.5 & 0.6 & -20.5 \end{pmatrix}$$

```
Out[•]=
{{12, 7, 11}, {9, 2, 22}, {0.5, 0.6, -20.5}}
```

```
In[•]:= Norm[A]
Out[•]=
33.975
```

```
In[•]:= n = Length[A]
Out[•]=
3
```

Първа норма

In[•]:=
$$\text{Table}\left[\sum_{j=1}^{n} \text{Abs}[A[[i, j]]], \{i, n\}\right]$$

```
Out[•]=
{30, 33, 21.6}
```

In[•]:=
$$\text{Max}\left[\text{Table}\left[\sum_{j=1}^{n} \text{Abs}[A[[i, j]]], \{i, n\}\right]\right]$$

```
Out[•]=
33
```

Втора норма

In[•]:=
$$\text{Table}\left[\sum_{i=1}^{n} \text{Abs}[A[[i, j]]], \{j, n\}\right]$$

```
Out[•]=
{21.5, 9.6, 53.5}
```

In[∘]:=
```
Max[Table[∑_{i=1}^{n} Abs[A[[i, j]]], {j, n}]]
```
Out[∘]=
```
53.5
```

Трета норма

In[∘]:=
$$\sqrt{\left(\sum_{i=1}^{n}\sum_{j=1}^{n} A[\![i, j]\!]^2\right)}$$

Out[∘]=
```
36.109
```

Метод на Якоби (простата итерация) за решаване на СЛАУ

In[∘]:=
$$A = \begin{pmatrix} 20 & 0.63 & 3.22 \\ 4.20 & -30 & 1.11 \\ 2.7 & 8.7 & 45.7 \end{pmatrix}; \; b = \{44, 308, 32.8\};$$

In[∘]:=
```
Print["За сравнение, точното решение е ", LinearSolve[A, b]]
```
```
За сравнение, точното решение е {2.11294, -9.87933, 2.47364}
```

Конструиране на метода - получаваме матрицата **B** и вектора **c**

In[∘]:=
```
(*Инициализация наматрицата B и вектора c*)
```
In[∘]:=
```
n = Length[A];
```
In[∘]:=
```
c = Table[0, n];
```
In[∘]:=
```
B = Table[0, {i, n}, {j, n}]
```
Out[∘]=
```
{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}
```
In[∘]:=
```
B // MatrixForm
```
Out[∘]//MatrixForm=
$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

In[ ]:=
```
For[i = 1, i ≤ n, i++,
  B[[i]] = -A[[i]]/A[[i, i]];
  B[[i, i]] = 0;
  c[[i]] = b[[i]]/A[[i, i]]
]
```

In[ ]:=
```
Print["Итерационният процес е x^(k+1) = ", B // MatrixForm, ". x^(k) + ", c // MatrixForm]
```

Итерационният процес е $x^{(k+1)} = \begin{pmatrix} 0 & -0.0315 & -0.161 \\ 0.14 & 0 & 0.037 \\ -0.059081 & -0.190372 & 0 \end{pmatrix} . x^{(k)} + \begin{pmatrix} \frac{11}{5} \\ -\frac{154}{15} \\ 0.717724 \end{pmatrix}$

Проверка условието на сходимост ||B|| < 1

Първа норма

In[ ]:=
```
Max[Table[Sum[Abs[B[[i, j]]], {j, 1, n}], {i, n}]]
```

Out[ ]=
0.249453

Втора норма

In[ ]:=
```
Max[Table[Sum[Abs[B[[i, j]]], {i, 1, n}], {j, n}]]
```

Out[ ]=
0.221872

Трета норма

In[ ]:=

$$\sqrt{\left(\sum_{i=1}^{n}\sum_{j=1}^{n} B[[i, j]]^2\right)}$$

Out[ ]=
0.295997

Избираме най-малката възможна норма, която в случая е втора.

Извършваме итерациите

```
In[•]:= x = {9, 12, 1/2}; (*изборът на начално приближение е произволен*)

In[•]:= For[k = 0, k ≤ 5, k++,
         Print["k = ", k, " x^(k) = ", x];
         x = B.x + c

        ]
```

k = 0 $x^{(k)}$ = $\{9, 12, \frac{1}{2}\}$

k = 1 $x^{(k)}$ = {1.7415, -8.98817, -2.09847}

k = 2 $x^{(k)}$ = {2.82098, -10.1005, 2.32593}

k = 3 $x^{(k)}$ = {2.14369, -9.78567, 2.47391}

k = 4 $x^{(k)}$ = {2.10995, -9.87502, 2.45399}

k = 5 $x^{(k)}$ = {2.11597, -9.88048, 2.47299}

Много далечно начално приближение:

```
In[•]:= x = {10^12, 12^13, -12 512 552 156 112 612 612}; (*изборът на начално приближение е произволен*)

In[•]:= For[k = 0, k ≤ 40, k++,
         Print["k = ", k, " x^(k) = ", x];
         x = B.x + c

        ]
```

k = 0 $x^{(k)}$ = {1 000 000 000 000, 106 993 205 379 072, -12 512 552 156 112 612 612}

k = 1 $x^{(k)}$ = $\{2.01452\times10^{18}, -4.62964\times10^{17}, -2.04276\times10^{13}\}$

k = 2 $x^{(k)}$ = $\{1.45867\times10^{16}, 2.82032\times10^{17}, -3.08842\times10^{16}\}$

k = 3 $x^{(k)}$ = $\{-3.91164\times10^{15}, 8.99418\times10^{14}, -5.45527\times10^{16}\}$

k = 4 $x^{(k)}$ = $\{8.75466\times10^{15}, -2.56608\times10^{15}, 5.98797\times10^{13}\}$

k = 5 $x^{(k)}$ = $\{7.11909\times10^{13}, 1.22787\times10^{15}, -2.87237\times10^{13}\}$

k = 6 $x^{(k)}$ = $\{-3.40533\times10^{13}, 8.90395\times10^{12}, -2.37958\times10^{14}\}$

k = 7 $x^{(k)}$ = $\{3.80307\times10^{13}, -1.35719\times10^{13}, 3.1684\times10^{11}\}$

k = 8 $x^{(k)}$ = $\{3.76504\times10^{11}, 5.33602\times10^{12}, 3.36818\times10^{11}\}$

k = 9 $x^{(k)}$ = $\{-2.22312\times10^{11}, 6.51728\times10^{10}, -1.03807\times10^{12}\}$

k = 10 $x^{(k)}$ = $\{1.65077\times10^{11}, -6.95325\times10^{10}, 7.27361\times10^{8}\}$

k = 11 $x^{(k)}$ = $\{2.07317\times10^{9}, 2.31377\times10^{10}, 3.48413\times10^{9}\}$

k = 12 $x^{(k)}$ = $\{-1.28978\times10^{9}, 4.19156\times10^{8}, -4.52725\times10^{9}\}$

k = 13 $x^{(k)}$ = $\{7.15684\times10^{8}, -3.48078\times10^{8}, -3.59406\times10^{6}\}$

k = 14 $x^{(k)}$ = $\{1.15431\times10^{7}, 1.00063\times10^{8}, 2.3981\times10^{7}\}$

k = 15 $x^{(k)}$ = $\{-7.01291\times10^{6}, 2.50332\times10^{6}, -1.97311\times10^{7}\}$

k = 16 $x^{(k)}$ = $\{3.09786\times10^{6}, -1.71187\times10^{6}, -62231.6\}$

k = 17 $x^{(k)}$ = {63 945.4, 431 387., 142 868.}

k = 18 $x^{(k)}$ = {-36 588.3, 14 228.2, -85 901.3}

k = 19 $x^{(k)}$ = {13 384.1, -8310.97, -546.264}

k = 20 $x^{(k)}$ = {351.944, 1843.3, 792.148}

k = 21 $x^{(k)}$ = {-183.4, 68.315, -370.988}

k = 22 $x^{(k)}$ = {59.7771, -49.6692, -1.4521}

k = 23 $x^{(k)}$ = {3.99837, -1.9516, 6.64165}

k = 24 $x^{(k)}$ = {1.19217, -9.46115, 0.853027}

k = 25 $x^{(k)}$ = {2.36069, -10.0682, 2.44843}

k = 26 $x^{(k)}$ = {2.12295, -9.84558, 2.49496}

k = 27 $x^{(k)}$ = {2.10845, -9.87714, 2.46662}

k = 28 $x^{(k)}$ = {2.114, -9.88022, 2.47349}

k = 29 $x^{(k)}$ = {2.113, -9.87919, 2.47374}

k = 30 $x^{(k)}$ = {2.11292, -9.87932, 2.47361}

k = 31 $x^{(k)}$ = {2.11295, -9.87933, 2.47364}

k = 32 $x^{(k)}$ = {2.11294, -9.87933, 2.47364}

k = 33 $x^{(k)}$ = {2.11294, -9.87933, 2.47364}

k = 34 $x^{(k)}$ = {2.11294, -9.87933, 2.47364}

k = 35 $x^{(k)}$ = {2.11294, -9.87933, 2.47364}

k = 36 $x^{(k)}$ = {2.11294, -9.87933, 2.47364}

k = 37 $x^{(k)}$ = {2.11294, -9.87933, 2.47364}

k = 38 $x^{(k)}$ = {2.11294, -9.87933, 2.47364}

k = 39 $x^{(k)}$ = {2.11294, -9.87933, 2.47364}

k = 40 $x^{(k)}$ = {2.11294, -9.87933, 2.47364}

Добавяме оценка на грешката

In[◦]:= **x = $\{9, 12, \frac{1}{2}\}$; (*изборът на начално приближение е произволен*)**

In[◦]:= **(*Изчисляваме нормите според избора на норма,
който сме направили по време на проверка на условието на устойчивост*)**

In[◦]:= **normB = Max$\left[\text{Table}\left[\sum_{i=1}^{n}\text{Abs}[B[[i, j]]], \{j, n\}\right]\right]$;**

```
In[•]:= normx0 = Norm[x, 1];

In[•]:= normc = Norm[c, 1];

In[•]:= For[k = 0, k ≤ 5, k++,
          Print["k = ", k, " x^(k) = ", x, "εk = ", eps = normB^k (normx0 + normc/(1 - normB))];
          x = B.x + c
        ]
```

$k = 0\ x^{(k)} = \{9, 12, \frac{1}{2}\}\varepsilon_k = 38.4437$

$k = 1\ x^{(k)} = \{1.7415, -8.98817, -2.09847\}\varepsilon_k = 8.52959$

$k = 2\ x^{(k)} = \{2.82098, -10.1005, 2.32593\}\varepsilon_k = 1.89248$

$k = 3\ x^{(k)} = \{2.14369, -9.78567, 2.47391\}\varepsilon_k = 0.419888$

$k = 4\ x^{(k)} = \{2.10995, -9.87502, 2.45399\}\varepsilon_k = 0.0931613$

$k = 5\ x^{(k)} = \{2.11597, -9.88048, 2.47299\}\varepsilon_k = 0.0206699$

Окончателен код

In[•]:= $A = \begin{pmatrix} 20 & 0.63 & 3.22 \\ 4.20 & -30 & 1.11 \\ 2.7 & 8.7 & 45.7 \end{pmatrix}$; b = {44, 308, 32.8};

```
In[•]:= Print["За сравнение, точното решение е ", LinearSolve[A, b]]
```

За сравнение, точното решение е {2.11294, -9.87933, 2.47364}

```
In[•]:= (*Инициализация наматрицата B и вектора c*)

In[•]:= n = Length[A];

In[•]:= c = Table[0, n];

In[•]:= For[i = 1, i ≤ n, i++,
          B[[i]] = -A[[i]]/A[[i, i]];
          B[[i, i]] = 0;
          c[[i]] = b[[i]]/A[[i, i]]
        ]
```

```
In[∘]:= Print["Итерационният процес е x^(k+1) = ", B // MatrixForm, ". x^(k) + ", c // MatrixForm]
```

Итерационният процес е $x^{(k+1)} = \begin{pmatrix} 0 & -0.0315 & -0.161 \\ 0.14 & 0 & 0.037 \\ -0.059081 & -0.190372 & 0 \end{pmatrix} . x^{(k)} + \begin{pmatrix} \frac{11}{5} \\ -\frac{154}{15} \\ 0.717724 \end{pmatrix}$

```
In[∘]:= normB = Max[Table[Sum[Abs[B[[i, j]]], {i, 1, n}], {j, n}]];

In[∘]:= normx0 = Norm[x, 1];

In[∘]:= normc = Norm[c, 1];

In[∘]:= For[k = 0, k ≤ 5, k++,
         Print["k = ", k, " x^(k) = ", x, "ε_k = ", eps = normB^k (normx0 + normc/(1 - normB))];
         x = B.x + c

        ]
        Print["За сравнение точното решение е = ", LinearSolve[A, b]]
```

k = 0 $x^{(k)}$ = {2.11308, -9.87893, 2.47368}ε_k = 31.4094

k = 1 $x^{(k)}$ = {2.11292, -9.87931, 2.47355}ε_k = 6.96887

k = 2 $x^{(k)}$ = {2.11296, -9.87934, 2.47363}ε_k = 1.5462

k = 3 $x^{(k)}$ = {2.11294, -9.87933, 2.47364}ε_k = 0.343058

k = 4 $x^{(k)}$ = {2.11294, -9.87933, 2.47364}ε_k = 0.0761149

k = 5 $x^{(k)}$ = {2.11294, -9.87933, 2.47364}ε_k = 0.0168878

За сравнение точното решение е = {2.11294, -9.87933, 2.47364}

```
In[∘]:= Print["Итерационният процес е x^(k+1) = ", B // MatrixForm, ". x^(k) + ", c // MatrixForm]
```

Итерационният процес е $x^{(k+1)} = \begin{pmatrix} 0 & -0.0315 & -0.161 \\ 0.14 & 0 & 0.037 \\ -0.059081 & -0.190372 & 0 \end{pmatrix} . x^{(k)} + \begin{pmatrix} \frac{11}{5} \\ -\frac{154}{15} \\ 0.717724 \end{pmatrix}$