Числено диференциране

Задача:

	$i=0$	$i=1$	$i=2$	$i=3$	$i=4$	
x	0.1	0.2	0.3	0.4	0.5	-> h = 0.1
y	-4	?(-1)	1	11	20	
y'	35	25	60	95	85	
y''	X	-100	800	-100	X	

Изп. формули (на листа): 3,4,5,6

$y_1 = ?$

$y_0' = \frac{-3y_0 + 4y_1 - y_2}{2h}$ |.2h

$y_1 = \frac{1}{4}(2h^* y_0' + 3y_0 + y_2)$

$y_1 = \frac{1}{4}(2^*0.1^*35 + 3^*(-4) + 1) = -1$

$y_1' = \frac{y_2 - y_0}{2h} = \frac{1-(-4)}{2*0.1} = \frac{5}{0.2} = 25$

$y_2' = \frac{y_3 - y_1}{2h} = \frac{11-(-1)}{2*0.1} = \frac{12}{0.2} = 60$

$y_3' = \frac{y_4 - y_2}{2h} = \frac{20-1}{2*0.1} = \frac{19}{0.2} = 95$

$y_4' = \frac{y_2 - 4y_3 + 3y_4}{2h} = \frac{1-4*11+3*20}{2*0.1} = \frac{17}{0.2} = 95$

$y_1'' = \frac{y_0 - 2y_1 + y_2}{h^2} = \frac{-4+2+1}{0.01} = \frac{-1}{0.01} = -100$

$y_2'' = \frac{y_1 - 2y_2 + y_3}{h^2} = \frac{-1-2+11}{0.01} = \frac{8}{0.01} = 800$

$y_3'' = \frac{y_2 - 2y_3 + y_4}{h^2} = \frac{1-22+20}{0.01} = \frac{-1}{0.01} = -100$

Задача:

$x_i = 4 + i^*0.1, i = \overline{0, 10}$

f(x) = 2sin(x+3)

$y_i = f(x_i), i = \overline{0, 10}$

In[1]:=
```
xt = Table[4 + i * 0.1, {i, 0, 10}];
```

In[2]:=
```
f[x_] := 2 Sin[x + 3]
yt = f[xt]
```

Out[3]=
```
{1.31397, 1.45794, 1.58734, 1.70087, 1.79742,
 1.876, 1.93584, 1.97634, 1.99709, 1.99788, 1.97872}
```

In[4]:=
```
h = 0.1
```

Out[4]=
```
0.1
```

In[5]:=
```
n = Length[xt]
```

Out[5]=
```
11
```

Формули с точност O(h) - първи порядък

Първа производна

In[6]:=
$$\texttt{yp1 = Table}\left[\frac{\texttt{yt[[i+1]] - yt[[i]]}}{\texttt{h}}\texttt{, \{i, 1, n - 1\}}\right]$$

Out[6]=
```
{1.43965, 1.29398, 1.13538, 0.96543, 0.785838,
 0.598394, 0.404971, 0.207502, 0.00795993, -0.191662}
```

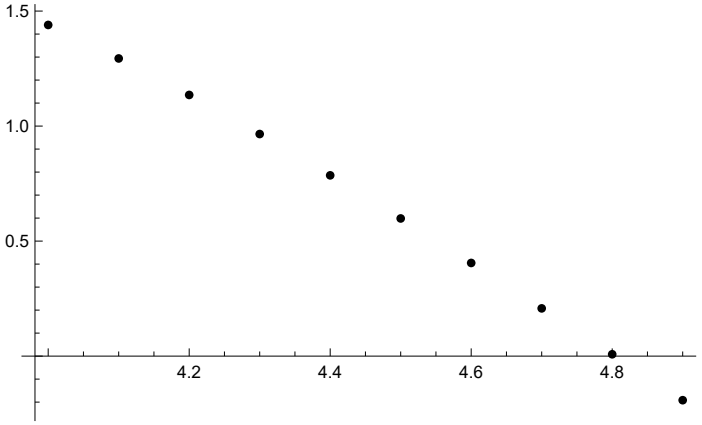
In[7]:=
$$\texttt{AppendTo}\left[\texttt{yp1, }\frac{\texttt{yt[[n]] - yt[[n-1]]}}{\texttt{h}}\right]$$

Out[7]=
```
{1.43965, 1.29398, 1.13538, 0.96543, 0.785838,
 0.598394, 0.404971, 0.207502, 0.00795993, -0.191662, -0.191662}
```

In[8]:=
```
pointsyp1 = Table[{xt[[i]], yp1[[i]]}, {i , 1, n - 1}]
gryp1 = ListPlot[pointsyp1, PlotStyle → Black]
```

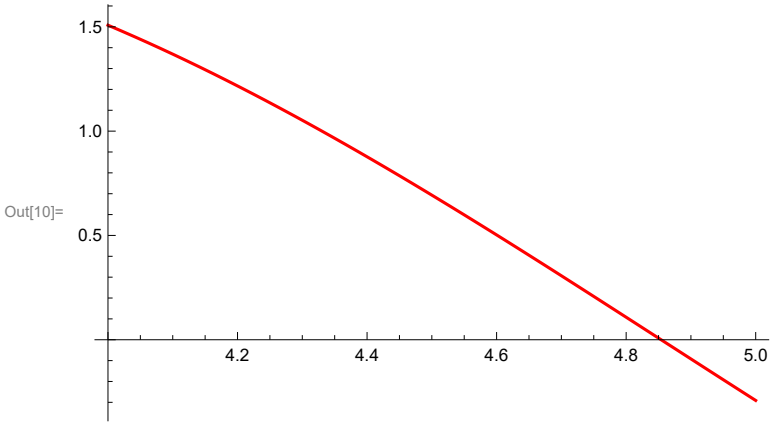
Out[8]=
```
{{4., 1.43965}, {4.1, 1.29398}, {4.2, 1.13538}, {4.3, 0.96543}, {4.4, 0.785838},
 {4.5, 0.598394}, {4.6, 0.404971}, {4.7, 0.207502}, {4.8, 0.00795993}, {4.9, -0.191662}}
```

Out[9]=



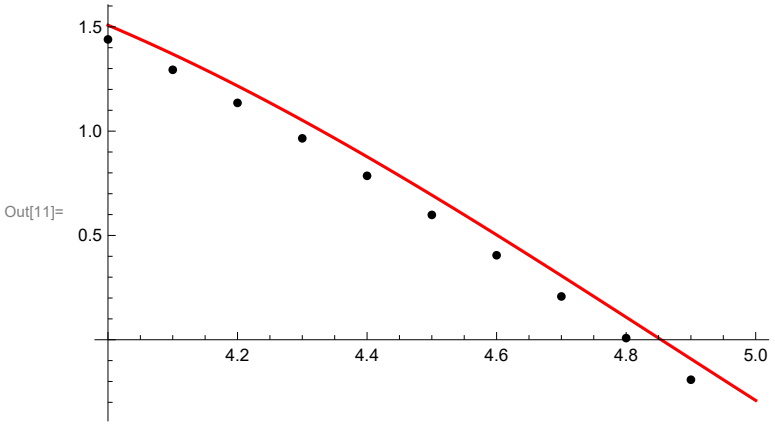
In[10]:=
```
grfyp = Plot[f'[x], {x, xt[[1]], xt[[n]]}, PlotStyle → Red]
```

Out[10]=



In[11]:=
```
Show[grfyp, gryp1]
```

Out[11]=



Формули с точност O(h^2) - втори порядък

Първа производна

Попълваме средните точки

In[12]:=
```
yp2 = Table[(yt[[i + 1]] - yt[[i - 1]])/(2 h), {i, 2, n - 1}]
```

Out[12]=
```
{1.36681, 1.21468, 1.0504, 0.875634, 0.692116, 0.501683, 0.306237, 0.107731, -0.091851}
```

Допълваме производната в десния край (последната)

In[13]:=
```
AppendTo[yp2, (yt[[n - 2]] - 4 yt[[n - 1]] + 3 yt[[n]])/(2 h)]
```

Out[13]=
```
{1.36681, 1.21468, 1.0504, 0.875634, 0.692116,
 0.501683, 0.306237, 0.107731, -0.091851, -0.291473}
```

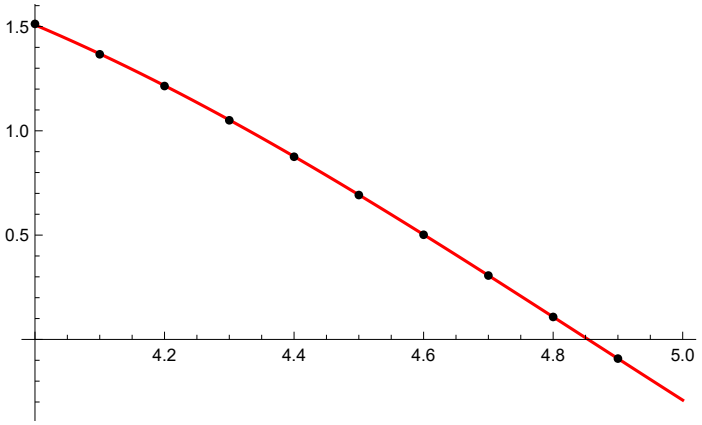
Допълваме производната в левия край (първата)

In[14]:=
$$\text{PrependTo}\left[\text{yp2},\ \frac{-3\,\text{yt}[\![1]\!] + 4\,\text{yt}[\![2]\!] - \text{yt}[\![3]\!]}{2\,h}\right]$$

Out[14]= {1.51249, 1.36681, 1.21468, 1.0504, 0.875634, 0.692116, 0.501683, 0.306237, 0.107731, −0.091851, −0.291473}

In[15]:=
```
pointsyp2 = Table[{xt[[i]], yp2[[i]]}, {i , 1, n - 1}] ;
grayp2 = ListPlot[pointsyp2, PlotStyle → Black];
grfyp = Plot[f'[x], {x, xt[[1]], xt[[n]]}, PlotStyle → Red];
Show[grfyp, gryp2]
```

Out[18]=



Втора производна

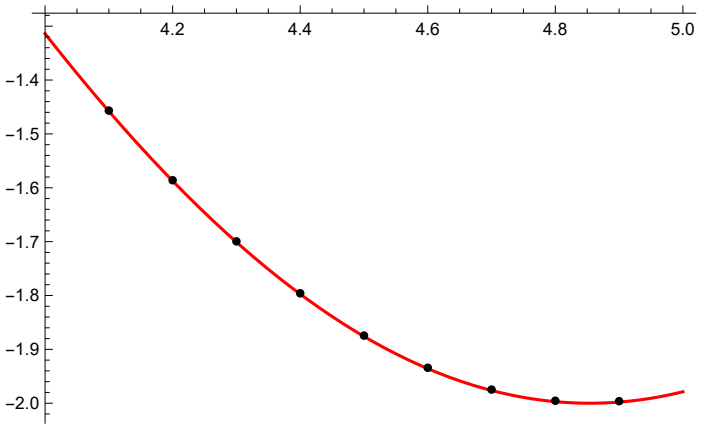
Попълваме средните точки

In[19]:=
$$\text{ypp2} = \text{Table}\left[\frac{\text{yt}[\![i+1]\!] - 2\,\text{yt}[\![i]\!] + \text{yt}[\![i-1]\!]}{h^2},\ \{i,\ 2,\ n-1\}\right]$$

Out[19]= {−1.45672, −1.58601, −1.69946, −1.79592, −1.87444, −1.93423, −1.97469, −1.99542, −1.99622}

In[20]:=
```
pointsypp2 = Table[{xt[[i + 1]], ypp2[[i]]}, {i , 1, n - 2}] ;
grypp2 = ListPlot[pointsypp2, PlotStyle → Black];
grfypp = Plot[f''[x], {x, xt[[1]], xt[[n]]}, PlotStyle → Red];
Show[grfypp, grypp2]
```

Out[23]=



Числено диференциране - повишаване на точността чрез сгъстяване на мрежата

In[24]:=
```
xt = Table[4 + i * 0.01, {i, 0, 100}];
```

In[25]:=
```
f[x_] := 2 Sin[x + 3]
yt = f[xt]
```

Out[26]=
```
{1.31397, 1.32899, 1.34386, 1.35861, 1.37322, 1.38769, 1.40202, 1.41622, 1.43027, 1.44417,
 1.45794, 1.47156, 1.48503, 1.49835, 1.51152, 1.52454, 1.53741, 1.55013, 1.56269,
 1.57509, 1.58734, 1.59942, 1.61135, 1.62312, 1.63472, 1.64616, 1.65744, 1.66855,
 1.67949, 1.69027, 1.70087, 1.71131, 1.72157, 1.73167, 1.74159, 1.75133, 1.7609,
 1.7703, 1.77952, 1.78856, 1.79742, 1.8061, 1.8146, 1.82292, 1.83105, 1.83901,
 1.84678, 1.85436, 1.86176, 1.86897, 1.876, 1.88284, 1.88949, 1.89595, 1.90222,
 1.9083, 1.9142, 1.91989, 1.9254, 1.93072, 1.93584, 1.94077, 1.9455, 1.95004, 1.95439,
 1.95854, 1.96249, 1.96625, 1.96981, 1.97317, 1.97634, 1.97931, 1.98208, 1.98465,
 1.98702, 1.9892, 1.99117, 1.99295, 1.99453, 1.99591, 1.99709, 1.99807, 1.99885,
 1.99942, 1.9998, 1.99998, 1.99996, 1.99974, 1.99932, 1.9987, 1.99788, 1.99686,
 1.99564, 1.99422, 1.99261, 1.99079, 1.98877, 1.98655, 1.98414, 1.98153, 1.97872}
```

In[27]:=
```
h = 0.01
```

Out[27]=
```
0.01
```

In[28]:=
```
n = Length[xt]
```

Out[28]=
```
101
```

Формули с точност O(h) - първи порядък

Първа производна

In[29]:=
```
yp1 = Table[(yt[[i + 1]] - yt[[i]])/h, {i, 1, n - 1}]
```

Out[29]=
```
{1.50121, 1.48792, 1.47448, 1.4609, 1.44716, 1.43329, 1.41927, 1.4051, 1.3908, 1.37636,
 1.36178, 1.34707, 1.33222, 1.31723, 1.30212, 1.28687, 1.2715, 1.256, 1.24037, 1.22462,
 1.20875, 1.19275, 1.17664, 1.16041, 1.14406, 1.1276, 1.11102, 1.09434, 1.07754,
 1.06064, 1.04363, 1.02652, 1.0093, 0.991988, 0.974572, 0.957059, 0.93945, 0.921747,
 0.903952, 0.886067, 0.868093, 0.850032, 0.831886, 0.813657, 0.795347, 0.776957,
 0.758489, 0.739946, 0.721329, 0.702639, 0.683879, 0.665051, 0.646156, 0.627197,
 0.608175, 0.589092, 0.56995, 0.550751, 0.531497, 0.51219, 0.492832, 0.473425, 0.45397,
 0.43447, 0.414926, 0.395341, 0.375716, 0.356054, 0.336356, 0.316624, 0.296861,
 0.277068, 0.257248, 0.237401, 0.217531, 0.197639, 0.177728, 0.157798, 0.137853,
 0.117894, 0.0979237, 0.0779432, 0.0579549, 0.0379608, 0.017963, -0.00203672,
 -0.0220362, -0.0420335, -0.0620265, -0.0820134, -0.101992, -0.121961, -0.141917,
 -0.161859, -0.181785, -0.201692, -0.22158, -0.241445, -0.261287, -0.281102}
```

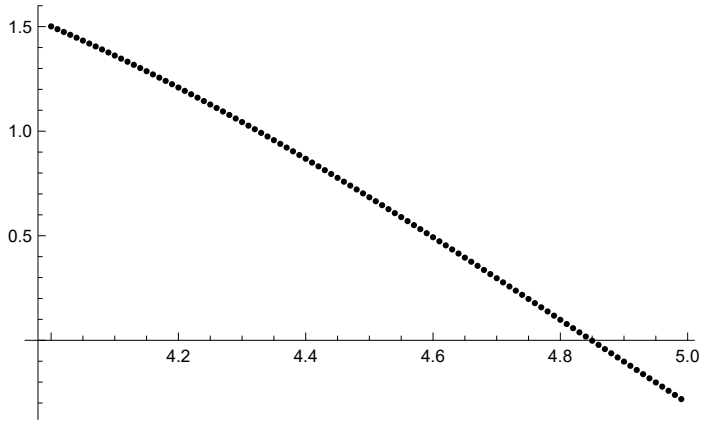
In[30]:=
```
AppendTo[yp1, (yt[[n]] - yt[[n - 1]])/h]
```

Out[30]=
```
{1.50121, 1.48792, 1.47448, 1.4609, 1.44716, 1.43329, 1.41927, 1.4051, 1.3908, 1.37636,
 1.36178, 1.34707, 1.33222, 1.31723, 1.30212, 1.28687, 1.2715, 1.256, 1.24037, 1.22462,
 1.20875, 1.19275, 1.17664, 1.16041, 1.14406, 1.1276, 1.11102, 1.09434, 1.07754, 1.06064,
 1.04363, 1.02652, 1.0093, 0.991988, 0.974572, 0.957059, 0.93945, 0.921747, 0.903952,
 0.886067, 0.868093, 0.850032, 0.831886, 0.813657, 0.795347, 0.776957, 0.758489,
 0.739946, 0.721329, 0.702639, 0.683879, 0.665051, 0.646156, 0.627197, 0.608175,
 0.589092, 0.56995, 0.550751, 0.531497, 0.51219, 0.492832, 0.473425, 0.45397, 0.43447,
 0.414926, 0.395341, 0.375716, 0.356054, 0.336356, 0.316624, 0.296861, 0.277068,
 0.257248, 0.237401, 0.217531, 0.197639, 0.177728, 0.157798, 0.137853, 0.117894,
 0.0979237, 0.0779432, 0.0579549, 0.0379608, 0.017963, -0.00203672, -0.0220362,
 -0.0420335, -0.0620265, -0.0820134, -0.101992, -0.121961, -0.141917, -0.161859,
 -0.181785, -0.201692, -0.22158, -0.241445, -0.261287, -0.281102, -0.281102}
```

In[31]:=
```
pointsyp1 = Table[{xt[[i]], yp1[[i]]}, {i , 1, n - 1}]
gryp1 = ListPlot[pointsyp1, PlotStyle → Black]
```

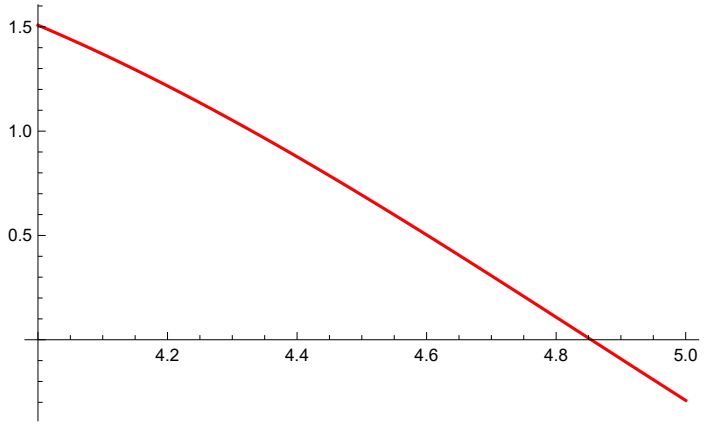
Out[31]=
```
{{4., 1.50121}, {4.01, 1.48792}, {4.02, 1.47448}, {4.03, 1.4609}, {4.04, 1.44716},
 {4.05, 1.43329}, {4.06, 1.41927}, {4.07, 1.4051}, {4.08, 1.3908}, {4.09, 1.37636},
 {4.1, 1.36178}, {4.11, 1.34707}, {4.12, 1.33222}, {4.13, 1.31723}, {4.14, 1.30212},
 {4.15, 1.28687}, {4.16, 1.2715}, {4.17, 1.256}, {4.18, 1.24037}, {4.19, 1.22462},
 {4.2, 1.20875}, {4.21, 1.19275}, {4.22, 1.17664}, {4.23, 1.16041}, {4.24, 1.14406},
 {4.25, 1.1276}, {4.26, 1.11102}, {4.27, 1.09434}, {4.28, 1.07754}, {4.29, 1.06064},
 {4.3, 1.04363}, {4.31, 1.02652}, {4.32, 1.0093}, {4.33, 0.991988}, {4.34, 0.974572},
 {4.35, 0.957059}, {4.36, 0.93945}, {4.37, 0.921747}, {4.38, 0.903952},
 {4.39, 0.886067}, {4.4, 0.868093}, {4.41, 0.850032}, {4.42, 0.831886},
 {4.43, 0.813657}, {4.44, 0.795347}, {4.45, 0.776957}, {4.46, 0.758489},
 {4.47, 0.739946}, {4.48, 0.721329}, {4.49, 0.702639}, {4.5, 0.683879},
 {4.51, 0.665051}, {4.52, 0.646156}, {4.53, 0.627197}, {4.54, 0.608175},
 {4.55, 0.589092}, {4.56, 0.56995}, {4.57, 0.550751}, {4.58, 0.531497},
 {4.59, 0.51219}, {4.6, 0.492832}, {4.61, 0.473425}, {4.62, 0.45397}, {4.63, 0.43447},
 {4.64, 0.414926}, {4.65, 0.395341}, {4.66, 0.375716}, {4.67, 0.356054},
 {4.68, 0.336356}, {4.69, 0.316624}, {4.7, 0.296861}, {4.71, 0.277068},
 {4.72, 0.257248}, {4.73, 0.237401}, {4.74, 0.217531}, {4.75, 0.197639},
 {4.76, 0.177728}, {4.77, 0.157798}, {4.78, 0.137853}, {4.79, 0.117894},
 {4.8, 0.0979237}, {4.81, 0.0779432}, {4.82, 0.0579549}, {4.83, 0.0379608},
 {4.84, 0.017963}, {4.85, -0.00203672}, {4.86, -0.0220362}, {4.87, -0.0420335},
 {4.88, -0.0620265}, {4.89, -0.0820134}, {4.9, -0.101992}, {4.91, -0.121961},
 {4.92, -0.141917}, {4.93, -0.161859}, {4.94, -0.181785}, {4.95, -0.201692},
 {4.96, -0.22158}, {4.97, -0.241445}, {4.98, -0.261287}, {4.99, -0.281102}}
```

Out[32]=



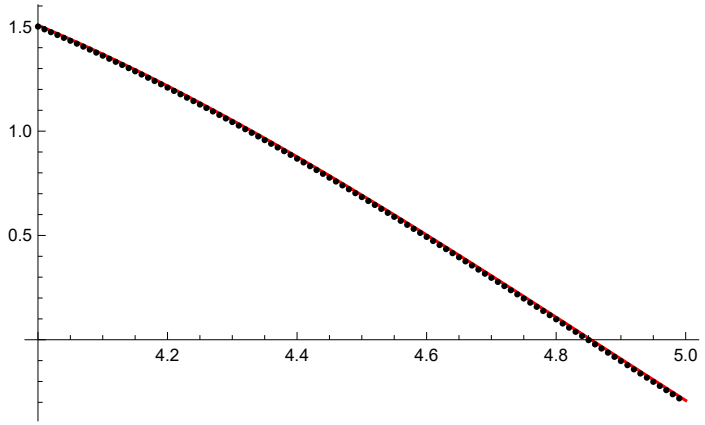
In[33]:=
```
grfyp = Plot[f'[x], {x, xt[[1]], xt[[n]]}, PlotStyle → Red]
```

Out[33]=



In[34]:=
```
Show[grfyp, gryp1]
```

Out[34]=



Формули с точност $O(h^2)$ - втори порядък

Първа производна

Попълваме средните точки

In[35]:=
```
yp2 = Table[(yt[[i + 1]] - yt[[i - 1]])/(2 h), {i, 2, n - 1}]
```

Out[35]=
```
{1.49456, 1.4812, 1.46769, 1.45403, 1.44022, 1.42628, 1.41219, 1.39795, 1.38358, 1.36907,
 1.35442, 1.33964, 1.32472, 1.30967, 1.29449, 1.27918, 1.26375, 1.24818, 1.23249,
 1.21668, 1.20075, 1.18469, 1.16852, 1.15223, 1.13583, 1.11931, 1.10268, 1.08594,
 1.06909, 1.05214, 1.03508, 1.01791, 1.00065, 0.98328, 0.965816, 0.948255, 0.930599,
 0.91285, 0.89501, 0.87708, 0.859063, 0.840959, 0.822772, 0.804502, 0.786152,
 0.767723, 0.749218, 0.730637, 0.711984, 0.693259, 0.674465, 0.655604, 0.636677,
 0.617686, 0.598633, 0.579521, 0.560351, 0.541124, 0.521844, 0.502511, 0.483128,
 0.463697, 0.44422, 0.424698, 0.405133, 0.385528, 0.365885, 0.346205, 0.32649,
 0.306743, 0.286965, 0.267158, 0.247324, 0.227466, 0.207585, 0.187684, 0.167763,
 0.147826, 0.127874, 0.107909, 0.0879334, 0.0679491, 0.0479579, 0.0279619, 0.00796311,
 -0.0120365, -0.0320348, -0.05203, -0.07202, -0.0920027, -0.111976, -0.131939,
 -0.151888, -0.171822, -0.191739, -0.211636, -0.231513, -0.251366, -0.271194}
```

Допълваме производната в десния край (последната)

In[36]:=
```
AppendTo[yp2, (yt[[n - 2]] - 4 yt[[n - 1]] + 3 yt[[n]])/(2 h)]
```

Out[36]=
```
{1.49456, 1.4812, 1.46769, 1.45403, 1.44022, 1.42628, 1.41219, 1.39795, 1.38358, 1.36907,
 1.35442, 1.33964, 1.32472, 1.30967, 1.29449, 1.27918, 1.26375, 1.24818, 1.23249,
 1.21668, 1.20075, 1.18469, 1.16852, 1.15223, 1.13583, 1.11931, 1.10268, 1.08594,
 1.06909, 1.05214, 1.03508, 1.01791, 1.00065, 0.98328, 0.965816, 0.948255, 0.930599,
 0.91285, 0.89501, 0.87708, 0.859063, 0.840959, 0.822772, 0.804502, 0.786152, 0.767723,
 0.749218, 0.730637, 0.711984, 0.693259, 0.674465, 0.655604, 0.636677, 0.617686,
 0.598633, 0.579521, 0.560351, 0.541124, 0.521844, 0.502511, 0.483128, 0.463697,
 0.44422, 0.424698, 0.405133, 0.385528, 0.365885, 0.346205, 0.32649, 0.306743, 0.286965,
 0.267158, 0.247324, 0.227466, 0.207585, 0.187684, 0.167763, 0.147826, 0.127874,
 0.107909, 0.0879334, 0.0679491, 0.0479579, 0.0279619, 0.00796311, -0.0120365,
 -0.0320348, -0.05203, -0.07202, -0.0920027, -0.111976, -0.131939, -0.151888,
 -0.171822, -0.191739, -0.211636, -0.231513, -0.251366, -0.271194, -0.291009}
```

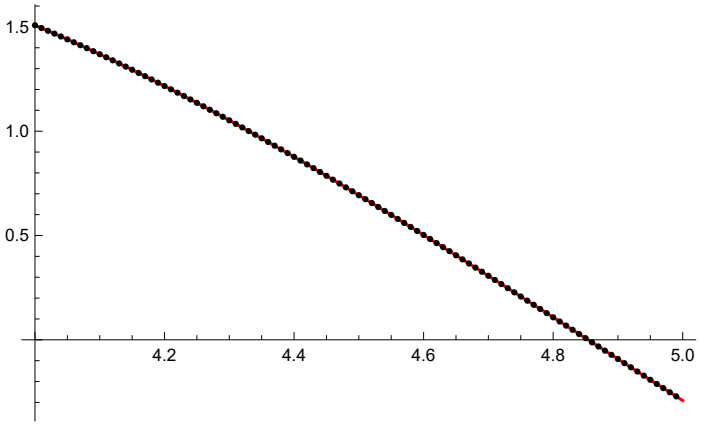
Допълваме производната в левия край (първата)

In[37]:=
```
PrependTo[yp2, (-3 yt[[1]] + 4 yt[[2]] - yt[[3]])/(2 h)]
```

Out[37]=
```
{1.50785, 1.49456, 1.4812, 1.46769, 1.45403, 1.44022, 1.42628, 1.41219,
 1.39795, 1.38358, 1.36907, 1.35442, 1.33964, 1.32472, 1.30967, 1.29449,
 1.27918, 1.26375, 1.24818, 1.23249, 1.21668, 1.20075, 1.18469, 1.16852,
 1.15223, 1.13583, 1.11931, 1.10268, 1.08594, 1.06909, 1.05214, 1.03508,
 1.01791, 1.00065, 0.98328, 0.965816, 0.948255, 0.930599, 0.91285, 0.89501,
 0.87708, 0.859063, 0.840959, 0.822772, 0.804502, 0.786152, 0.767723, 0.749218,
 0.730637, 0.711984, 0.693259, 0.674465, 0.655604, 0.636677, 0.617686, 0.598633,
 0.579521, 0.560351, 0.541124, 0.521844, 0.502511, 0.483128, 0.463697, 0.44422,
 0.424698, 0.405133, 0.385528, 0.365885, 0.346205, 0.32649, 0.306743, 0.286965,
 0.267158, 0.247324, 0.227466, 0.207585, 0.187684, 0.167763, 0.147826, 0.127874,
 0.107909, 0.0879334, 0.0679491, 0.0479579, 0.0279619, 0.00796311, -0.0120365,
 -0.0320348, -0.05203, -0.07202, -0.0920027, -0.111976, -0.131939, -0.151888,
 -0.171822, -0.191739, -0.211636, -0.231513, -0.251366, -0.271194, -0.291009}
```

In[38]:=
```
pointsyp2 = Table[{xt[[i]], yp2[[i]]}, {i , 1, n - 1}] ;
gryp2 = ListPlot[pointsyp2, PlotStyle → Black];
grfyp = Plot[f'[x], {x, xt[[1]], xt[[n]]}, PlotStyle → Red];
Show[grfyp, gryp2]
```

Out[41]=



Втора производна

Попълваме средните точки

In[42]:=
$$\text{ypp2 = Table}\left[\frac{\text{yt[[i + 1]] - 2 yt[[i]] + yt[[i - 1]]}}{\text{h}^2}, \{\text{i, 2, n - 1}\}\right]$$

Out[42]=
```
{-1.32897, -1.34385, -1.3586, -1.37321, -1.38768, -1.40201, -1.4162, -1.43025,
 -1.44416, -1.45793, -1.47154, -1.48501, -1.49834, -1.51151, -1.52453, -1.5374,
 -1.55011, -1.56267, -1.57508, -1.58732, -1.59941, -1.61134, -1.6231, -1.63471,
 -1.64615, -1.65742, -1.66853, -1.67948, -1.69025, -1.70086, -1.7113, -1.72156,
 -1.73165, -1.74157, -1.75132, -1.76089, -1.77028, -1.7795, -1.78854, -1.7974,
 -1.80608, -1.81458, -1.8229, -1.83104, -1.83899, -1.84676, -1.85435, -1.86174,
 -1.86896, -1.87598, -1.88282, -1.88947, -1.89594, -1.90221, -1.90829, -1.91418,
 -1.91988, -1.92539, -1.9307, -1.93582, -1.94075, -1.94549, -1.95003, -1.95437,
 -1.95852, -1.96247, -1.96623, -1.96979, -1.97315, -1.97632, -1.97929,
 -1.98206, -1.98463, -1.98701, -1.98918, -1.99116, -1.99293, -1.99451,
 -1.99589, -1.99707, -1.99805, -1.99883, -1.99941, -1.99979, -1.99997,
 -1.99995, -1.99973, -1.99931, -1.99869, -1.99787, -1.99685, -1.99563,
 -1.99421, -1.99259, -1.99077, -1.98875, -1.98654, -1.98412, -1.98151}
```

In[43]:=
```
pointsypp2 = Table[{xt[[i + 1]], ypp2[[i]]}, {i , 1, n – 2}] ;
grypp2 = ListPlot[pointsypp2, PlotStyle → Black];
grfypp = Plot[f''[x], {x, xt[[1]], xt[[n]]}, PlotStyle → Red];
Show[grfypp, grypp2]
```

Out[46]=