Метод на най-малките квадрати (МНМК)

Задача: (**a** и **b** са съответно предпоследната и последната цифра от факултетния номер)

1. Да се състави таблицата (x_k, g(x_k)), където

$x_k =$ -b + k(0.1), k = $\overline{0, 10}$,   g(x) = $e^{\frac{(a+1)\,x}{10}}$

Търси се апроксимацията в точката s = -b + (0.17)a + 0.01. За тази цел:

2. Да се построи полином на ленейна регресия по получената таблица.
3. Да се построи полином на квадратична регресия по получената таблица.
4. Да се построи полином на кубична регресия по получената таблица.
5. Да се пресметне апроксимацията, използвайки всеки един от построените полиноми (общо 3).
6. Да се оцени грешката за всяка от получените апроксимации.
7. Да се направи сравнение между трите резултата.

Генериране на данни

In[1]:=
```
xt = Table[-7 + k * 0.1, {k, 0, 10}]
```

Out[1]=
```
{-7., -6.9, -6.8, -6.7, -6.6, -6.5, -6.4, -6.3, -6.2, -6.1, -6.}
```

In[2]:=
```
f[x_] := e^(7 x/10)
yt = f[xt]
```

Out[3]=
```
{0.00744658, 0.00798652, 0.00856561, 0.00918669, 0.0098528,
 0.0105672, 0.0113334, 0.0121552, 0.0130365, 0.0139818, 0.0149956}
```

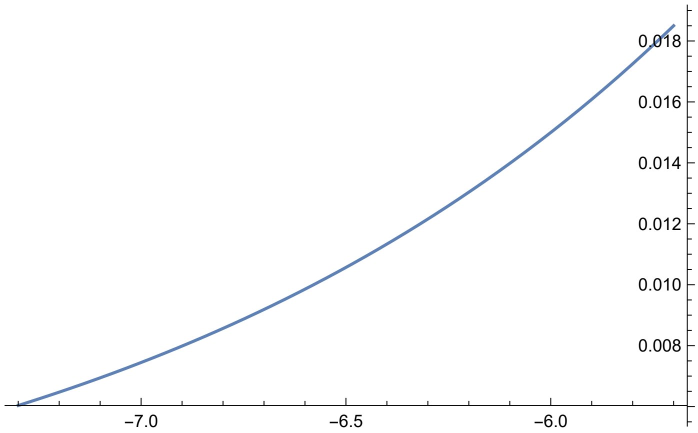
In[4]:=
```
P = Length[xt]
```

Out[4]=
```
11
```

Визуализация

In[5]:=
```
grf = Plot[f[x], {x, xt⟦1⟧ - 0.3, xt⟦P⟧ + 0.3}]
```

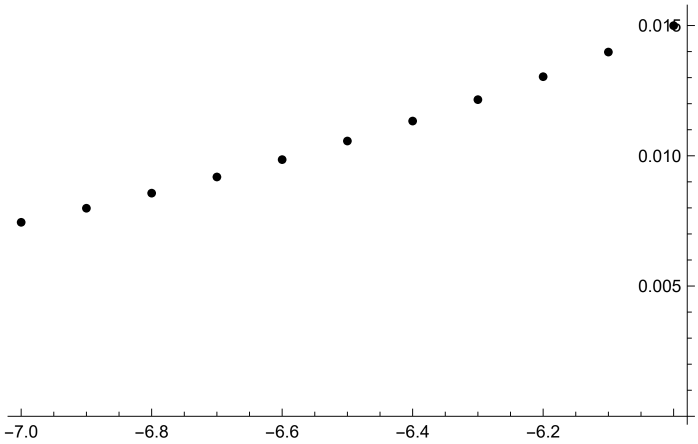
Out[5]=



In[6]:=
```
points = Table[{xt⟦i⟧, yt⟦i⟧}, {i, 1, P}];
```

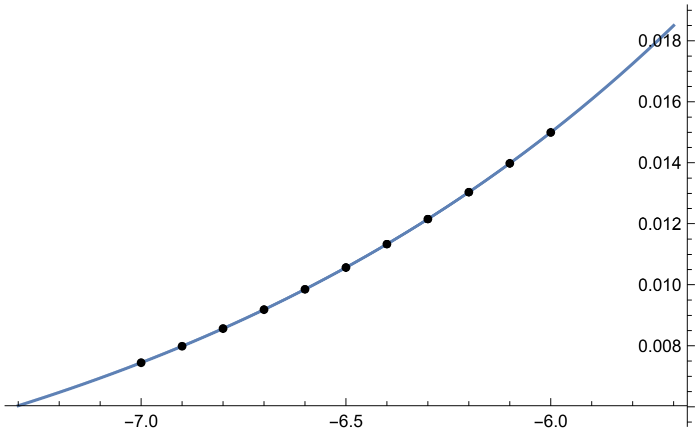
In[7]:=
```
grp = ListPlot[points, PlotStyle → Black]
```

Out[7]=



In[8]:=
```
Show[grf, grp]
```

Out[8]=



Линейна регресия

Попълваме таблицата

In[9]:= xt^2

Out[9]= {49., 47.61, 46.24, 44.89, 43.56, 42.25, 40.96, 39.69, 38.44, 37.21, 36.}

In[10]:= yt * xt

Out[10]= {-0.0521261, -0.055107, -0.0582461, -0.0615508, -0.0650285, -0.0686868, -0.0725338, -0.0765776, -0.0808265, -0.0852889, -0.0899735}

Намиране на сумите

In[11]:= $\sum_{i=1}^{P} xt[[i]]$

Out[11]= -71.5

In[12]:= $\sum_{i=1}^{P} yt[[i]]$

Out[12]= 0.119108

In[13]:= $\sum_{i=1}^{P} xt[[i]]^2$

Out[13]= 465.85

In[14]:= $\sum_{i=1}^{P} yt[[i]] * xt[[i]]$

Out[14]= -0.765946

Решаваме системата

In[15]:= $A = \begin{pmatrix} P & \sum_{i=1}^{P} xt[[i]] \\ \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 \end{pmatrix}; \; b = \left\{\sum_{i=1}^{P} yt[[i]], \; \sum_{i=1}^{P} yt[[i]] * xt[[i]]\right\};$

In[16]:= LinearSolve[A, b]

Out[16]= {0.0596113, 0.00750512}

Съставяме полинома

In[17]:= P1[x_] := 0.0596113 + 0.00750512 x

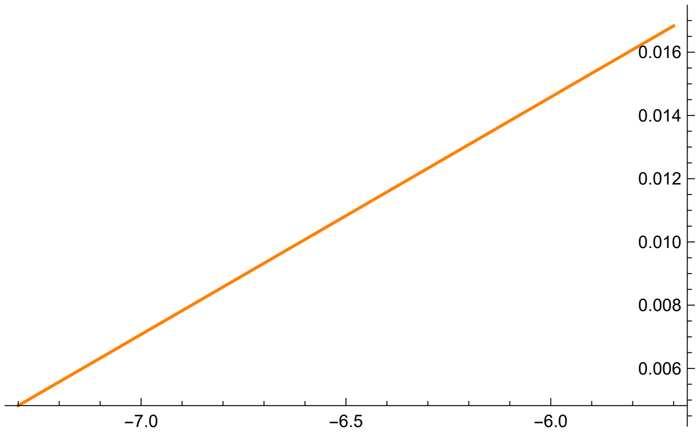
Таен коз (възможност за самопроверка)

In[18]:= Fit[points, {1, x}, x]

Out[18]= 0.0596113 + 0.00750512 x

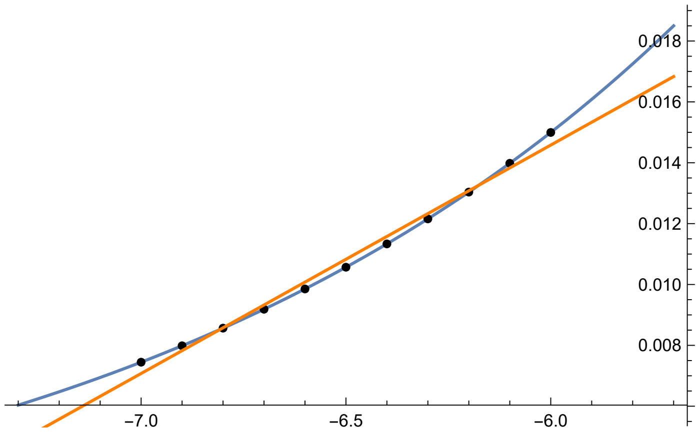
In[19]:=
```
grfP1 = Plot[P1[x], {x, xt[[1]] - 0.3, xt[[P]] + 0.3}, PlotStyle → Orange]
```

Out[19]=



In[20]:=
```
Show[grf, grp, grfP1]
```

Out[20]=



In[21]:=
```
P1[-5.97]
```

Out[21]= 0.0148057

За сравнение истинската стойност

In[22]:=
```
f[-5.97]
```

Out[22]= 0.0153138

Оценка на грешката

Теоретична грешка (средноквадратична)

In[23]:=

$$\sqrt{\sum_{i=1}^{P} (yt[[i]] - P1[xt[[i]]])^2}$$

Out[23]= 0.000767618

Истинска грешка

In[24]:=
```
Abs[f[-5.97] - P1[-5.97]]
```

Out[24]= 0.00050808

Квадратична регресия

Попълваме таблицата

In[25]:= xt^2

Out[25]= {49., 47.61, 46.24, 44.89, 43.56, 42.25, 40.96, 39.69, 38.44, 37.21, 36.}

In[26]:= yt * xt

Out[26]= {-0.0521261, -0.055107, -0.0582461, -0.0615508, -0.0650285, -0.0686868, -0.0725338, -0.0765776, -0.0808265, -0.0852889, -0.0899735}

In[27]:= xt^3

Out[27]= {-343., -328.509, -314.432, -300.763, -287.496, -274.625, -262.144, -250.047, -238.328, -226.981, -216.}

In[28]:= xt^4

Out[28]= {2401., 2266.71, 2138.14, 2015.11, 1897.47, 1785.06, 1677.72, 1575.3, 1477.63, 1384.58, 1296.}

In[29]:= $yt * xt^2$

Out[29]= {0.364883, 0.380238, 0.396074, 0.41239, 0.429188, 0.446464, 0.464217, 0.482439, 0.501124, 0.520262, 0.539841}

Намиране на сумите

In[30]:= $\sum_{i=1}^{P} xt[[i]]$

Out[30]= -71.5

In[31]:= $\sum_{i=1}^{P} yt[[i]]$

Out[31]= 0.119108

In[32]:= $\sum_{i=1}^{P} xt[[i]]^2$

Out[32]= 465.85

In[33]:= $\sum_{i=1}^{P} yt[[i]] * xt[[i]]$

Out[33]= -0.765946

In[34]:= $\sum_{i=1}^{P} xt[[i]]^3$

Out[34]= -3042.33

In[35]:= $\sum_{i=1}^{P} xt[[i]]^4$

Out[35]= 19 914.7

In[36]:= $\sum_{i=1}^{P} yt[[i]] * xt[[i]]^2$

Out[36]= 4.93712

Решаваме системата

In[37]:= $A = \begin{pmatrix} P & \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 \\ \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 \\ \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 \end{pmatrix};$

$b = \left\{\sum_{i=1}^{P} yt[[i]],\ \sum_{i=1}^{P} yt[[i]] * xt[[i]],\ \sum_{i=1}^{P} yt[[i]] * xt[[i]]^2\right\};$

In[38]:= `LinearSolve[A, b]`

Out[38]= {0.169855, 0.0415066, 0.0026155}

Таен коз (възможност за самопроверка)

In[39]:= `Fit[points, {1, x, x^2}, x]`

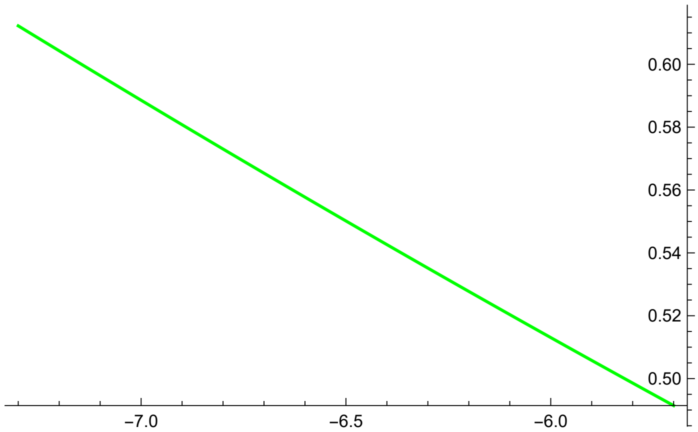
Out[39]= $0.169855 + 0.0415066\,x + 0.0026155\,x^2$

Съставяме полинома

In[40]:= `P2[x_] := 0.169855 - 0.0415066 x + 0.0026155 x^2`

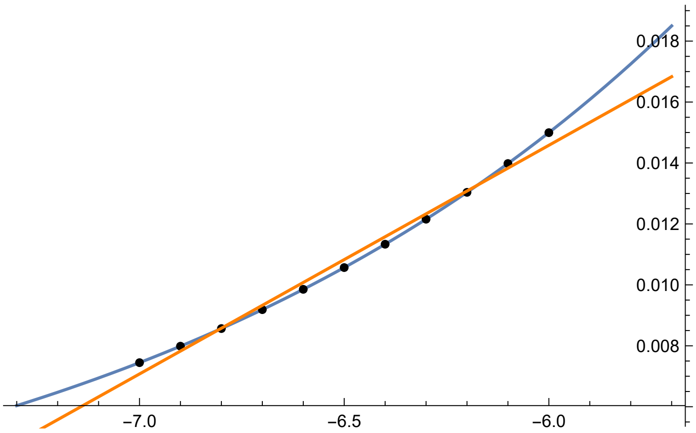
In[41]:= `grfP2 = Plot[P2[x], {x, xt[[1]] - 0.3, xt[[P]] + 0.3}, PlotStyle → Green]`

Out[41]=



In[42]:=
```
Show[grf, grp, grfP1, grfP2]
```

Out[42]=



In[43]:=
```
P2[-5.97]
```

Out[43]= 0.510868

За сравнение истинската стойност

In[44]:=
```
f[-5.97]
```

Out[44]= 0.0153138

Оценка на грешката

Теоретична грешка (средноквадратична)

In[45]:= $\sqrt{\sum_{i=1}^{p} (yt[[i]] - P2[xt[[i]]])^2}$

Out[45]= 1.79172

Истинска грешка

In[46]:=
```
Abs[f[-5.97] - P2[-5.97]]
```

Out[46]= 0.495554

Кубична регресия

Попълваме таблицата

In[47]:= xt^2

Out[47]= {49., 47.61, 46.24, 44.89, 43.56, 42.25, 40.96, 39.69, 38.44, 37.21, 36.}

In[48]:=
```
yt * xt
```

Out[48]= {-0.0521261, -0.055107, -0.0582461, -0.0615508, -0.0650285, -0.0686868, -0.0725338, -0.0765776, -0.0808265, -0.0852889, -0.0899735}

In[49]:= xt^3

Out[49]= {-343., -328.509, -314.432, -300.763, -287.496, -274.625, -262.144, -250.047, -238.328, -226.981, -216.}

In[50]:= xt^4

Out[50]= {2401., 2266.71, 2138.14, 2015.11, 1897.47, 1785.06, 1677.72, 1575.3, 1477.63, 1384.58, 1296.}

In[51]:= $yt * xt^2$

Out[51]= {0.364883, 0.380238, 0.396074, 0.41239, 0.429188, 0.446464, 0.464217, 0.482439, 0.501124, 0.520262, 0.539841}

In[52]:= $yt * xt^3$

Out[52]= {-2.55418, -2.62364, -2.6933, -2.76302, -2.83264, -2.90202, -2.97099, -3.03937, -3.10697, -3.1736, -3.23904}

In[53]:= xt^5

Out[53]= {-16 807., -15 640.3, -14 539.3, -13 501.3, -12 523.3, -11 602.9, -10 737.4, -9924.37, -9161.33, -8445.96, -7776.}

In[54]:= xt^6

Out[54]= {117 649., 107 918., 98 867.5, 90 458.4, 82 654., 75 418.9, 68 719.5, 62 523.5, 56 800.2, 51 520.4, 46 656.}

Намиране на сумите

In[55]:= $\sum_{i=1}^{P} xt[[i]]$

Out[55]= -71.5

In[56]:= $\sum_{i=1}^{P} yt[[i]]$

Out[56]= 0.119108

In[57]:= $\sum_{i=1}^{P} xt[[i]]^2$

Out[57]= 465.85

In[58]:= $\sum_{i=1}^{P} yt[[i]] * xt[[i]]$

Out[58]= -0.765946

In[59]:= $\sum_{i=1}^{P} xt[[i]]^3$

Out[59]= -3042.33

In[60]:= $\sum_{i=1}^{P} xt[[i]]^4$

Out[60]= 19 914.7

In[61]:= $\sum_{i=1}^{P} yt[[i]] * xt[[i]]^2$

Out[61]= 4.93712

In[62]:= $\sum_{i=1}^{P} xt[[i]]^5$

Out[62]= −130 659.

In[63]:= $\sum_{i=1}^{P} xt[[i]]^6$

Out[63]= 859 185.

In[64]:= $\sum_{i=1}^{P} yt[[i]] * xt[[i]]^3$

Out[64]= −31.8988

Решаваме системата

In[65]:= $A = \begin{pmatrix} P & \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 \\ \sum_{i=1}^{P} xt[[i]] & \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 \\ \sum_{i=1}^{P} xt[[i]]^2 & \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 & \sum_{i=1}^{P} xt[[i]]^5 \\ \sum_{i=1}^{P} xt[[i]]^3 & \sum_{i=1}^{P} xt[[i]]^4 & \sum_{i=1}^{P} xt[[i]]^5 & \sum_{i=1}^{P} xt[[i]]^6 \end{pmatrix};$

$b = \left\{\sum_{i=1}^{P} yt[[i]], \sum_{i=1}^{P} yt[[i]] * xt[[i]], \sum_{i=1}^{P} yt[[i]] * xt[[i]]^2, \sum_{i=1}^{P} yt[[i]] * xt[[i]]^3\right\};$

In[66]:= LinearSolve[A, b]

LinearSolve: Result for LinearSolve of badly conditioned matrix {{11., −71.5, 465.85, −3042.33}, {−71.5, 465.85, −3042.33, 19914.7}, {465.85, −3042.33, 19914.7, −130659.}, {−3042.33, 19914.7, −130659., 859185.}} may contain significant numerical errors.

Out[66]= {0.33634, 0.118563, 0.014487, 0.000608794}

Таен коз (възможност за самопроверка)

In[67]:= Fit[points, $\{1, x, x^2, x^3\}$, x]

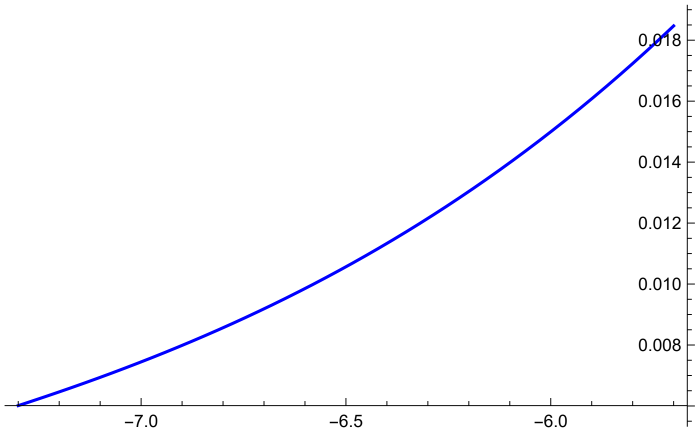
Out[67]= $0.33634 + 0.118563\,x + 0.014487\,x^2 + 0.000608793\,x^3$

Съставяме полинома

In[68]:= P3[x_] := $0.33634 + 0.118563\,x + 0.014487\,x^2 + 0.000608793\,x^3$

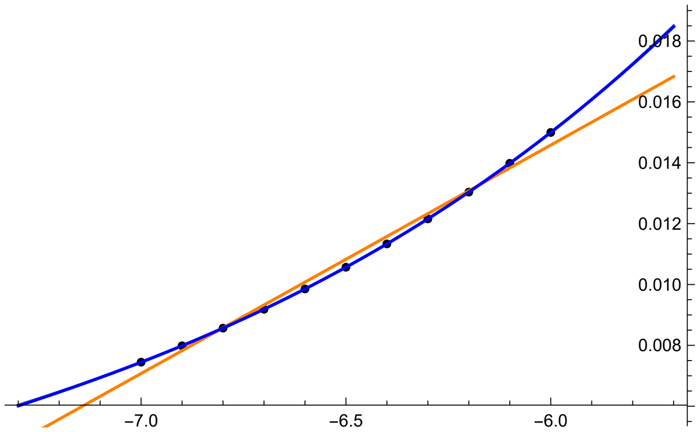
In[69]:=
```
grfP3 = Plot[P3[x], {x, xt⟦1⟧ - 0.3, xt⟦P⟧ + 0.3}, PlotStyle → Blue]
```

Out[69]=



In[70]:=
```
Show[grf, grp, grfP1, grfP2, grfP3]
```

Out[70]=



Намиране на приближена стойност (s = -b + (0.17)a + 0.01)

In[71]:=
```
s = -7 + (0.17 * 6) + 0.01
```

Out[71]= -5.97

In[72]:=
```
P3[-5.97]
```

Out[72]= 0.015312

За сравнение истинската стойност

In[73]:=
```
f[-5.97]
```

Out[73]= 0.0153138

Оценка на грешката

Теоретична грешка (средноквадратична)

In[74]:= $\sqrt{\sum_{i=1}^{P} (yt[[i]] - P3[xt[[i]]])^2}$

Out[74]= 2.17888×10^{-6}

Истинска грешка

In[75]:=
```
Abs[f[-5.97] - P3[-5.97]]
```

Out[75]= 1.85012×10^{-6}