Задача 2:

а) $P_0(0,0)$, $P_1(0,4)$, $P_2(-4,4)$, $P_3(0,-4)$, $P_4(0,0)$

$P_{10}(0,3)$, $P_{11}(-3,4)$, $P_{12}(-1,-2)$, $P_{13}(0,-1)$

$P_{20}\left(-\frac{9}{4}, \frac{15}{4}\right)$, $P_{21}\left(-\frac{3}{2}, -\frac{1}{2}\right)$, $P_{22}\left(-\frac{1}{4}, \frac{-5}{4}\right)$

$P_{30}\left(\frac{-27}{16}, \frac{9}{16}\right)$, $P_{31}\left(-\frac{9}{16}, \frac{-27}{16}\right)$

$P_{40}\left(\frac{-54}{64}, \frac{-4[illegible]}{64}\right)$

$$u = 0{,}75 = \frac{75}{100} = \frac{3}{4}$$

$$1-u = 1 - \frac{3}{4} = \frac{1}{4}$$

$$P_{10} = \frac{1}{4}P_0 + \frac{3}{4}P_1 = \frac{1}{4}(0,0) + \frac{3}{4}(0,4) = (0,3)$$

$$P_{11} = \frac{1}{4}P_1 + \frac{3}{4}P_2 = \frac{1}{4}(0,4) + \frac{3}{4}(-4,4) = (-3,4)$$

$$P_{12} = \frac{1}{4}P_2 + \frac{3}{4}P_3 = \frac{1}{4}(-4,4) + \frac{3}{4}(0,-4) = (-1,-2)$$

$$P_{13} = \frac{1}{4}P_3 + \frac{3}{4}P_4 = \frac{1}{4}(0,-4) + \frac{3}{4}(0,0) = (0,-1)$$

$$P_{20} = \frac{1}{4}P_{10} + \frac{3}{4}P_{11} = \frac{1}{4}(0,3) + \frac{3}{4}(-3,4) = \left(0,\frac{3}{4}\right) + \left(-\frac{9}{4}, 3\right)$$

$$= \left(-\frac{9}{4}, \frac{15}{4}\right)$$

$$P_{21} = \frac{1}{4} P_{11} + \frac{3}{4} P_{12} = \frac{1}{4}(-3, 4) + \frac{3}{4}(-1, -2) =$$

$$= \left(-\frac{3}{4}, 1\right) + \left(-\frac{3}{4}, \frac{-3}{2}\right) =$$

$$= \left(\frac{-6}{4}, \frac{-1}{2}\right) = \left(\frac{-3}{2}, \frac{-1}{2}\right)$$

$$P_{22} = \frac{1}{4} P_{12} + \frac{3}{4} P_{13} = \frac{1}{4}(-1, -2) + \frac{3}{4}(0, -1) =$$

$$= \left(-\frac{1}{4}, \frac{-1}{2}\right) + \left(0, \frac{-3}{4}\right) =$$

$$= \left(-\frac{1}{4}, \frac{-5}{4}\right)$$

$$P_{30} = \frac{1}{4} P_{20} + \frac{3}{4} P_{21} = \frac{1}{4}\left(-\frac{9}{4}, \frac{15}{4}\right) + \frac{3}{4}\left(\frac{-3}{2}, \frac{-1}{2}\right) =$$

$$= \left(-\frac{9}{16}, \frac{15}{16}\right) + \left(-\frac{9}{8}, \frac{-3}{8}\right) =$$

$$= \left(-\frac{27}{16}, \frac{9}{16}\right)$$

$$P_{31} = \frac{1}{4} P_{21} + \frac{3}{4} P_{22} = \frac{1}{4}\left(-\frac{3}{2}, \frac{-1}{2}\right) + \frac{3}{4}\left(-\frac{1}{4}, \frac{-5}{4}\right) =$$

$$= \left(-\frac{3}{8}, -\frac{1}{8}\right) + \left(-\frac{3}{16}, \frac{-15}{16}\right) =$$

$$= \left(-\frac{9}{16}, -\frac{17}{16}\right)$$

$$P_{40} = \frac{1}{4} P_{30} + \frac{3}{4} P_{31} = \frac{1}{4}\left(\frac{-27}{16}, \frac{9}{16}\right) + \frac{3}{4}\left(-\frac{9}{16}, \frac{-17}{16}\right) =$$

$$= \left(-\frac{27}{64}, \frac{9}{64}\right) + \left(-\frac{27}{64}, \frac{-51}{64}\right) =$$

$$= \left(-\frac{54}{64}, -\frac{42}{64}\right)$$

б) $C_1(u)$: $u \in [0, 0{,}75]$: $P_0(0,0)$, $P_{10}(0,3)$, $P_{20}\left(-\frac{9}{4}, \frac{15}{4}\right)$,

$P_{30}\left(-\frac{27}{16}, \frac{9}{16}\right)$, $P_{40}\left(-\frac{54}{64}, \frac{-42}{64}\right)$

$C_2(u)$: $u \in [0{,}75, 1]$: $P_{40}\left(-\frac{54}{64}, -\frac{42}{64}\right)$, $P_{31}\left(-\frac{9}{16}, -\frac{17}{16}\right)$

$P_{22}\left(-\frac{1}{4}, \frac{-5}{4}\right)$, $P_{13}(0,-1)$, $P_4(0,0)$

в) $P_0(0,0)$, $P_1(0,4)$, $P_2(-4,4)$, $P_3(0,-4)$, $P_4(0,0) \Rightarrow n=4$

$\Rightarrow C(0) = C(1) = P_0 = P_4$

1\. $\dot{C}(1) \overset{?}{=} \dot{C}(0)$

$\dot{C}(0) = 4[(0,4)-(0,0)] = 4(0,4) = (0,16)$

$\dot{C}(1) = 4[(0,0)-(0,-4)] = 4(0,4) = (0,16)$

$\Rightarrow \dot{C}(1) = \dot{C}(0) \Rightarrow \exists\ C^1$-непр.

$\dot{C}(1) \uparrow\uparrow \dot{C}(0) \Rightarrow \exists\ G^1$-непр.

2. $\ddot{c}(1) \overset{?}{=} \ddot{c}(0)$

$\ddot{c}(0) = 12[(-4,4) - 2(0,4) + (0,0)] =$
$= 12[(-4,4) + (0,-8)] = 12(-4,-4) = (-48,-48)$

$\ddot{c}(1) = 12[(0,0) - 2(0,-4) + (-4,4)] =$
$= 12[(0,8) + (-4,4)] = 12(-4,12) = (-48,144)$

$\Rightarrow \ddot{c}(1) \neq \ddot{c}(0) \Rightarrow \nexists\ C^2$-непр.

3. $\ddot{c}(1) - \ddot{c}(0) \uparrow\uparrow \dot{c}(1) = \dot{c}(0)$

$\ddot{c}(1) - \ddot{c}(0) = (-48,144) - (-48,-48) =$
$= (0,192)$

$\ddot{c}(1) - \ddot{c}(0) \overset{?}{=} \lambda \dot{c}(1)$
$(0,192) \neq (0,16) \Rightarrow \nexists\ G^2$-непр.

4. $\varkappa_{c(1)} \overset{?}{=} \varkappa_{c(0)}$

$\varkappa_{c(0)} = \dfrac{|\dot{c}(0) \times \ddot{c}(0)|}{|\dot{c}(0)|^3} \overset{?}{=} \varkappa_{c(1)} = \dfrac{|\dot{c}(1) \times \ddot{c}(1)|}{|\dot{c}(1)|^3}$

$\dot{c}(0) = (0,16) \rightarrow (0,16,0)$
$\ddot{c}(0) = (-48,-48) \rightarrow (-48,-48,0)$

$\dot{c}(0) \times \ddot{c}(0) = \left(\begin{vmatrix} 16 & 0 \\ -48 & 0 \end{vmatrix}, -\begin{vmatrix} 0 & 0 \\ -48 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 16 \\ -48 & -48 \end{vmatrix} \right) =$

$= (0,0,768)$

$\Rightarrow |\dot{c}(0) \times \ddot{c}(0)| = 768$

$|\dot{c}(0)| = \sqrt{256}$

$\Rightarrow \kappa_{c(0)} = \dfrac{768}{(\sqrt{256})^3}$

$\dot{c}(1) = (0, 16) \to (0, 16, 0)$

$\ddot{c}(1) = (-48, 144) \to (-48, 144, 0)$

$$\dot{c}(1) \times \ddot{c}(1) = \left(\begin{vmatrix} 16 & 0 \\ 144 & 0 \end{vmatrix}, -\begin{vmatrix} 0 & 0 \\ -48 & 0 \end{vmatrix}, \begin{vmatrix} 0 & 16 \\ -48 & 144 \end{vmatrix} \right) =$$

$$= (0, 0, 768)$$

$\Rightarrow |\dot{c}(1) \times \ddot{c}(1)| = 768$

$|\dot{c}(1)| = \sqrt{256}$

$\Rightarrow \kappa_{c(1)} = \dfrac{768}{\sqrt{256}}$

$\Rightarrow \kappa_{c(0)} = \kappa_{c(1)} \to \exists\ \kappa$-непр.

г) $\dot{c}(0{,}75) = 4\,[P_{31} - P_{30}] = 4\left[\left(\dfrac{-9}{16}, \dfrac{-7}{16}\right) - \left(\dfrac{-27}{16}, \dfrac{9}{16}\right)\right]$

$= 4\left(\dfrac{18}{16}, \dfrac{-16}{16}\right) = 4\left(\dfrac{9}{8}, -1\right) = \left(\dfrac{9}{2}, -\dfrac{4}{2}\right)$

Задача 3:

I вмъкване:

$u_0=u_1=u_2$	u_3	u_4	u_5	$u_6=u_7=u_8$
0	0,5	0,6	0,9	1

$t=0,3 \in [0;0,5) \equiv [u_2, u_3) \to P_2, P_1, P_0$

$P_0, P_1 \to Q_1\left(-2, -\frac{4}{5}\right)$

$P_1, P_2 \to Q_2(-1, 0)$

$Q_1, Q_2 \to R_2\left(-\frac{7}{5}, -\frac{8}{25}\right) = C(0,3)$

$\equiv R_3$

P_0
P_1
P_2
P_3
P_4
P_5

От P_0 и $P_1 \Rightarrow Q_1$ и α_1

От P_1 и $P_2 \Rightarrow Q_2$ и α_2

$Q_1 = (1-\alpha_1)P_0 + \alpha_1 P_1$ $\qquad \alpha_1 = \frac{0,3-0}{0,5-0} = \frac{3}{5}$

$Q_1 = \frac{2}{5}P_0 + \frac{3}{5}P_1 =$

$= \frac{2}{5}(-2,-2) + \frac{3}{5}(-2,0) =$ $\qquad 1-\alpha_1 = 1-\frac{3}{5} = \frac{2}{5}$

$= \left(-\frac{4}{5}, -\frac{4}{5}\right) + \left(-\frac{6}{5}, 0\right) = \left(-2, -\frac{4}{5}\right)$

$Q_2 = (1-a_2)P_1 + a_2 P_2$ $\qquad a_2 = \frac{0,3-0}{0,6-0} = \frac{3}{6} = \frac{1}{2}$

$Q_2 = \frac{1}{2} P_1 + \frac{1}{2} P_2$

$= \frac{1}{2}(-2,0) + \frac{1}{2}(0,0) =$ $\qquad 1-a_2 = {}^{2}1 - \frac{1}{2} = \frac{1}{2}$

$= (-1,0)$

$v_0 = v_1 = v_2$	v_3	v_4	v_5	v_6	$v_7 = v_8 = v_9$
0	0,3	0,5	0,6	0,9	1

P'_0	P'_1	P'_2	P'_3	P'_4	P'_5	P'_6
P_0	Q_1	Q_2	P_2	P_3	P_4	P_5

II вмъкване:

$t = 0,3 \in [v_3, v_4) = [0,3,\ 0,5) \to P'_3, P'_2, P'_1$

От P'_1 и $P'_2 \Rightarrow R_2$ и $a_{2,2}$

От P'_2 и $P'_3 \Rightarrow R_3$ и $a_{3,2}$

$R_2 = (1-a_{2,2})P'_1 + a_{2,2}P'_2$ $\qquad a_{2,2} = \frac{0,3-0}{0,5-0} = \frac{3}{5}$

$R_2 = \frac{2}{5} Q_1 + \frac{3}{5} Q_2 =$ $\qquad 1-a_{2,2} = {}^{5}1 - \frac{3}{5} = \frac{2}{5}$

$= \frac{2}{5}\left(-2, -\frac{4}{5}\right) + \frac{3}{5}(-1,0) =$

$= \left(-\frac{4}{5}, -\frac{8}{25}\right) + \left(-\frac{3}{5}, 0\right) = \left(-\frac{7}{5}, -\frac{8}{25}\right)$

$R_3 = (1 - a_{3,2})P_2' + a_{3,2}P_3'$
$R_3 = (1-0)P_2' + 0P_3'$
$R_3 = P_2' = Q_2$

$a_{3,2} = \frac{0,3 - 0,3}{0,6 - 0,3} = 0$

$w_0 = w_1 = w_2$	w_3	w_4	w_5	w_6	w_7	$w_8 = w_9 = w_{10}$
0	$0,3$	$0,3$	$0,5$	$0,6$	$0,9$	1

P_0''	P_1''	P_2''	P_3''	P_4''	P_5''	P_6''	P_7''
P_0	Q_1	R_2	Q_2	P_2	P_3	P_4	P_5

$c(0,3) = R_2\left(-\frac{7}{5}, \frac{-8}{25}\right)$

Задача 1:

б) $\vec{r}(1+\sin u, 1+u, 1+\cos u)$

$\varkappa = \frac{|\dot{\vec{r}} \times \ddot{\vec{r}}|}{|\dot{\vec{r}}|^3}$

$\dot{\vec{r}}(\cos u, 1, -\sin u)$
$\ddot{\vec{r}}(-\sin u, 0, -\cos u)$

$\dot{\vec{r}} \times \ddot{\vec{r}} = \left(\begin{vmatrix} 1 & -\sin u \\ 0 & -\cos u \end{vmatrix}, -\begin{vmatrix} \cos u & -\sin u \\ -\sin u & -\cos u \end{vmatrix}, \begin{vmatrix} \cos u & 1 \\ -\sin u & 0 \end{vmatrix} \right)$

$= (-\cos u, -(-\cos^2 u - \sin^2 u), \sin u)$
$= (-\cos u, 1, \sin u)$

$$|\dot{\vec{r}} \times \ddot{\vec{r}}| = \sqrt{(-\cos u)^2 + (1)^2 + (\sin u)^2} = \sqrt{\cos^2 u + 1 + \sin^2 u} =$$

$$= \sqrt{\cos^2 u + \sin^2 u + 1} = \sqrt{1+1} = \sqrt{2}$$

$$|\dot{\vec{r}}| = \sqrt{(\cos u)^2 + (1)^2 + (-\sin u)^2} = \sqrt{\cos^2 u + \sin^2 u + 1} = \sqrt{1+1} = \sqrt{2}$$

$$\varkappa = \frac{\sqrt{2}}{(\sqrt{2})^3} = \frac{\sqrt{2}}{(\sqrt{2})^2 \sqrt{2}} = \frac{\cancel{\sqrt{2}}}{2\cancel{\sqrt{2}}} = \frac{1}{2}$$

$$\tau = \frac{\dot{\vec{r}}\,\ddot{\vec{r}}\,\dddot{\vec{r}}}{(\dot{\vec{r}} \times \ddot{\vec{r}})^2} \qquad \dddot{\vec{r}}(-\cos u, 0, \sin u)$$

$$\dot{\vec{r}}\,\ddot{\vec{r}}\,\dddot{\vec{r}} = (\dot{\vec{r}} \times \ddot{\vec{r}})\dddot{\vec{r}} = (-\cos u)\cdot(-\cos u) + 1 \cdot 0 + (\sin u)(\sin u)$$

$$= \cos^2 u + \sin^2 u = 1$$

$$(\dot{\vec{r}} \times \ddot{\vec{r}})^2 = \cos^2 u + 1 + \sin^2 u = 2$$

$$\tau = \frac{1}{2}$$