Задача 1:

а) $S: \vec{r}(u, uv, av^2)$

$\vec{r}(u, uv, av^2)$
$\vec{r_u}(1, v, 0)$
$\vec{r_v}(0, u, 2v)$

$g_{11} = \vec{r_u}^{\,2} = 1 + v^2$
$g_{12} = \vec{r_u} . \vec{r_v} = uv$
$g_{22} = \vec{r_v}^{\,2} = u^2 + 4v^2$

$\mathrm{I}(du, dv) = (1+v^2)du^2 + (2uv)du\,dv + (u^2+4v^2)dv^2$

$$\vec{N} = \frac{\vec{r_u} \times \vec{r_v}}{|\vec{r_u} \times \vec{r_v}|}$$

$$\vec{r_u} \times \vec{r_v} = \left(\begin{vmatrix} v & 0 \\ u & 2v \end{vmatrix}, -\begin{vmatrix} 1 & 0 \\ 0 & 2v \end{vmatrix}, \begin{vmatrix} 1 & v \\ 0 & u \end{vmatrix} \right)$$
$$= (2v^2, -2v, u)$$

$|\vec{r_u} \times \vec{r_v}| = \sqrt{(2v^2)^2 + (-2v)^2 + u^2} = \sqrt{4v^4 + 4v^2 + u^2}$

$$\Rightarrow N = \left(\frac{2v^2}{\sqrt{4v^4+4v^2+u^2}}, \frac{-2v}{\sqrt{4v^4+4v^2+u^2}}, \frac{u}{\sqrt{4v^4+4v^2+u^2}} \right)$$

Отбелязваме $\sqrt{4v^4+4v^2+u^2}$ е *

$\vec{r}_u(1, v, 0)$
$\vec{r}_v(0, u, 2v)$
$\vec{r}_{uu}(0, 0, 0)$
$\vec{r}_{uv}(0, 1, 0)$
$\vec{r}_{vv}(0, 0, 2)$

$h_{11} = \vec{N}\vec{r}_{uu} = 0$

$h_{12} = \vec{N}\vec{r}_{uv} = \dfrac{-2v}{*}$

$h_{22} = \vec{N}\vec{r}_{vv} = \dfrac{2u}{*}$

$\mathrm{II}(du, dv) = \left(-\dfrac{4v}{*}\right)du\,dv + \left(\dfrac{2u}{*}\right)dv^2$

б) $C: u = v$
$D: u = 2v - 1$

1)
$\left|\begin{array}{l} u = v \Rightarrow \boxed{u = 1} \\ u = 2v - 1 \Rightarrow v = 2v - 1 \Rightarrow -v = -1 \quad \boxed{v = 1} \end{array}\right.$

2)
Коеф. на първа осн. форма са:
$g_{11}^{p} = 1 + v^2 = 1 + 1^2 = 2$
$g_{12}^{p} = uv = 1$
$g_{22}^{p} = u^2 + 4v^2 = 1^2 + 4 \cdot 1^2 = 5$

3) $C: u = v \mid d$
$du = dv$
$\dfrac{du}{dv} = \dfrac{1}{1}$
$\Rightarrow (du, dv) = (1, 1)$

$D: u = 2v - 1 \mid \delta$
$\delta u = 2\delta v$
$\dfrac{\delta u}{\delta v} = \dfrac{2}{1}$
$(\delta u, \delta v) = (2, 1)$

4) $\cos \measuredangle(C,D) = \dfrac{g^P_{11} du\,\delta u + g^P_{12}(du\,\delta v + dv\,\delta u) + g^P_{22} dv\,\delta v}{\sqrt{g^P_{11} du^2 + 2g^P_{12} du\,dv + g^P_{22} dv^2} \cdot \sqrt{g^P_{11} \delta u^2 + 2g^P_{12} \delta u\,\delta v + g^P_{22} \delta v^2}}$

$$= \frac{2\cdot 1\cdot 2 + 1(1\cdot 1 + 1\cdot 2) + 5\cdot 1\cdot 1}{\sqrt{2\cdot 1^2 + 2\cdot 1\cdot 1\cdot 1 + 5\cdot 1^2} \cdot \sqrt{2\cdot 2^2 + 2\cdot 1\cdot 2\cdot 1 + 5\cdot 1^2}} =$$

$$= \frac{4 + 1 + 2 + 5}{\sqrt{2+2+5} \cdot \sqrt{8+4+5}} = \frac{12}{\sqrt{9}\cdot\sqrt{17}} = \frac{\cancel{12}}{\cancel{3}\sqrt{17}} = \frac{4}{\sqrt{17}}$$

b) $K = \dfrac{h}{g}$

$g = 4v^4 + 4v^2 + u^2$

$h = h_{11}h_{22} - h_{12}^2 = -\left(\dfrac{-2v}{\sqrt{4v^4 + 4v^2 + u^2}}\right)^2 = \dfrac{-4v^2}{4v^4 + 4v^2 + u^2}$

$$K = \frac{-4v^2}{4v^4 + 4v^2 + u^2} \cdot \frac{1}{4v^4 + 4v^2 + u^2} = \frac{-4v^2}{(4v^4 + 4v^2 + u^2)^2}$$

$$H = \frac{g_{11}h_{22} - 2g_{12}h_{12} + g_{22}h_{11}}{2g}, \quad h_{11} = 0$$

$$= \frac{(1+v^2)\underset{*}{(2u)} - (2uv)\underset{**}{(-2v)}}{2(4v^4 + 4v^2 + u^2)} =$$

$$\frac{2u + 2uv^2 - 4uv^2}{\sqrt{4v^4+4v^2+u^2}} \cdot \frac{1}{2(4v^4+4v^2+u^2)} =$$

$$= \frac{2u - 2uv^2}{2(4v^4+4v^2+u^2).(*)}$$

г) Проверяване броя на асимпт. линии, като приравним $\mathrm{II}(du, dv) = 0$

$\Rightarrow \mathrm{II}(du, dv) = h_{11} du^2 + 2h_{12} du dv + h_{22} dv^2 = 0$

$h = h_{11}h_{22} - h_{12}^2 = \frac{0}{*} \cdot \frac{2u}{*} - \left(\frac{-2v}{*}\right) = \frac{-2v}{*} < 0$

$\Rightarrow \exists$ 2 асимпт. линии

$\mathrm{II}(du, dv) = 0$

$\left(\frac{-4v}{*}\right) du dv + \left(\frac{2u}{*}\right) dv^2 = 0 \quad : (*)$

$-4v\, du dv + 2u\, dv^2 = 0 \quad : (-2)$

$2v\, du dv - u\, dv^2 = 0$

$dv(2v du - u dv) = 0$

$dv = 0 \quad \vee \quad 2v du - u dv = 0$

$\Rightarrow v = const \qquad 2v du = u dv$

$$\frac{du}{u} = \frac{dv}{2v} \quad : \int$$

$$\int \frac{du}{u} = \int \frac{dv}{2v}$$

$$\ln u = \frac{1}{2} \ln v + \ln c$$

Освобождаване се от ln:

$$\Rightarrow u = \frac{cv}{2} \quad \text{т. } B\,(u = v = 2)$$

$$2 = \frac{\cancel{2}c}{\cancel{2}} \Rightarrow \boxed{c = 2}$$