Задача 1:

a) $z = \dfrac{1+2i}{3-4i} - \dfrac{4-3i}{2-i}$

1) $\dfrac{1+2i}{3-4i} \cdot \dfrac{3+4i}{3+4i} = \dfrac{(1+2i)(3+4i)}{(3-4i)(3+4i)} =$

$= \dfrac{3+4i+6i+8i^2}{3^2+(4i)^2} = \dfrac{3+10i-8}{9+16} = \dfrac{-5+10i}{25} =$

$= \dfrac{\cancel{5}(-1+2i)}{\cancel{25}_5} = \dfrac{-1+2i}{5}$

2) $\dfrac{4-3i}{2-i} \cdot \dfrac{2+i}{2+i} = \dfrac{(4-3i)(2+i)}{(2-i)(2+i)} = \dfrac{8+4i-6i-3i^2}{2^2+(-i)^2} =$

$= \dfrac{8-2i+3}{4+1} = \dfrac{11-2i}{5}$

$\Rightarrow \dfrac{-1+2i-(11-2i)}{5} = \dfrac{-1+2i-11+2i}{5} = \dfrac{-12+4i}{5} =$

$= -\dfrac{12}{5} + \dfrac{4}{5}i$

в) $z = (1+i)^5$

Решаване чрез прилагане на Биномната теорема

$\Rightarrow 1^5 + 5\cdot 1^4(-i) + 10\cdot 1^3(-i)^2 + 10\cdot 1^2(-i)^3 + 5\cdot 1(-i)^4 + (-i)^5 =$

$= 1 - 5i + 10\cdot(-1)^2\cdot i^2 + 10\cdot(-1)^3\cdot(-(i^2\cdot i)) + 5\cdot(-1)^4(i^2)^2 - i^5 =$

$= 1 - 5i - 10 + 10i + 5 - (i^4\cdot i) =$

$= 1 - 5i - 10 + 10i + 5 - ((i^2)^2\cdot i) =$

$= 1 - 5i - 10 + 10i + 5 - i =$

$= -4 + 4i =$

$= 4(-1 + i)$

б) $z = (3+i)\overline{(2+i)}$

$z = (3+i)(2-i)$

$z = 6 - 3i + 2i - i^2 = 6 - i + 1 = 7 - i$

Задача 2:

a) $z = \sqrt{3} + i$

$tg\theta = \frac{b}{a} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3} = \frac{\pi}{6}$

$r = |z| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$

$\Rightarrow \begin{cases} \cos\theta = \frac{a}{|r|} = \frac{\sqrt{3}}{2} \\ \sin\theta = \frac{b}{|r|} = \frac{1}{2} \end{cases}$

$\varphi = \frac{\pi}{6} + 2k\pi$

$\Rightarrow z = 2\left(\cos\left(\frac{\pi}{6} + 2k\pi\right) + i\sin\left(\frac{\pi}{6} + 2k\pi\right)\right)$

б) $z = \dfrac{1}{(1-i)^2}$

$z_1 = 1$

$z_2 = (1-i)^2$

$z_1 = \sqrt{1^2+0^2} = 1 \quad tg\theta = \dfrac{b}{a} = \dfrac{0}{1} = 0$ $a \neq 0$ - не

$z_1 = \left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right) \quad cotg\theta = \dfrac{a}{b} = \dfrac{1}{0} = 0 \Rightarrow \dfrac{\pi}{2}$

$z_2 = \sqrt{1^2+(-1)^2} = \sqrt{2} \quad tg\theta = \dfrac{b}{a} = \dfrac{-1}{1} = -1 \Rightarrow \dfrac{3\pi}{4}$

$z_2 = \sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$

$$z = \frac{\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)}{\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)^2}$$

Прилагаме ф-лата на Моавър:

$$z = \frac{1}{(\sqrt{2})^2}\left(\frac{\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)}{\cos\left(\frac{6\pi}{4}\right) + i\sin\left(\frac{6\pi}{4}\right)}\right)$$

$$z = \frac{1}{2}\left(\cos\left(-\frac{5\pi}{4} + 2k\pi\right) + i\sin\left(-\frac{5\pi}{4} + 2k\pi\right)\right)$$

б) $z = \frac{(1+i)^3}{(1-i)^5}$

$z_1 = (1+i)^3$
$z_2 = (1-i)^5$

$z_1 = \sqrt{1^2+1^2} = \sqrt{2}$ $\quad tg\theta = \frac{b}{a} = 1 \Rightarrow \frac{\pi}{4}$

$z_1 = \sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$

$z_2 = \sqrt{1^2+(-1)^2} = \sqrt{2}$ $\quad tg\theta = \frac{b}{a} = -1 \Rightarrow \frac{3\pi}{4}$

$z_2 = \sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)$

$$z = \frac{\left(\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)\right)^3}{\left(\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right)\right)^5}$$

От ф-лата на Моавър

$$\Rightarrow z = \frac{\sqrt{2}^3}{\sqrt{2}^5}\left(\frac{\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)}{\cos\left(\frac{15\pi}{4}\right) + i\sin\left(\frac{15\pi}{4}\right)}\right)$$

$$z = \frac{1}{2}\left(\cos\left(-\frac{12\pi}{4}\right) + i\sin\left(-\frac{12\pi}{4}\right)\right)$$

$$z = \frac{1}{2}\left(\cos(-3\pi) + i\sin(-3\pi)\right)$$

$$z = \frac{1}{2}\left(\cos(-3\pi + 2k\pi) + i\sin(-3\pi + 2k\pi)\right)$$

Задача 3:

а) $z = (\sqrt{3} - i)^6$

Решаване чрез прилагане на биномната теорема

$$\Rightarrow (\sqrt{3})^6 - 6(\sqrt{3})^5 \cdot (-i) + 15(\sqrt{3})^4 \cdot (-i)^2 - 20 \cdot (\sqrt{3})^3 \cdot (-i)^3 + 15(\sqrt{3})^2 \cdot (-i)^4 -$$

$$- 6\sqrt{3} \cdot (-i)^5 + (-i)^6 =$$

$$= 3^{\frac{6}{2}} + 6 \cdot 3^{\frac{5}{2}} i - 15 \cdot 3^{\frac{4}{2}} - 20 \cdot 3^{\frac{3}{2}} i + 15 \cdot 3^{\frac{2}{2}} + 6\sqrt{3} i - 1 =$$

$$= 3^3 + 6 \cdot 3^{2+\frac{1}{2}} i - 15 \cdot 3^2 - 20 \cdot 3^{1+\frac{1}{2}} i + 15 \cdot 3 + 6\sqrt{3} i - 1 =$$

$$= 27 + 6 \cdot 3^2 \sqrt{3} i - 15 \cdot 9 - 20 \cdot 3\sqrt{3} i + 45 + 6\sqrt{3} i - 1 =$$

$$= 27 + \cancel{54\sqrt{3} i} - 135 - \cancel{60\sqrt{3} i} + 45 + \cancel{6\sqrt{3} i} - 1 =$$

$$= -136 + 72 = -64$$