Интерполационен полином на Лаграж

Стъпки за решаване:

1. Генерираме данни (съставяне на таблицата)
$x_i = 7 + i * (0.17)$, i = -7,7
$y_i = f(x_i)$
f(x) = 3sin(x-7)
x 5.81 5.98 ....
y -2.785 -2.556 ....

2. f(p) ≈ ?
а) p = 6.18 - интерполация
б) p = 5.78 - екстраполация
в) p = 30 - екстраполация
$|R_n(x)| <= \frac{M_{n+1}}{(n+1)!}|(x-x_0)....(x-x_n)|$
$L_n(x) = \sum .... \prod$

Линейна интерполация $L_1(x) = ?$, $L_1(p) = ?$ (n = 1)
x 6.15 6.32
y -2.2538 -1.886

Интерполационни условия:
$L_n(x_i) = y_i$

$L_1(x) = -2.2538 \frac{x-6.32}{6.15-6.32} - 1.886 \frac{x-6.15}{6.32-6.15}$

Генериране на данни

In[82]:=
```
xt = Table[7 + i * 0.17, {i, -7, 7}]
```

Out[82]=
```
{5.81, 5.98, 6.15, 6.32, 6.49, 6.66, 6.83, 7., 7.17, 7.34, 7.51, 7.68, 7.85, 8.02, 8.19}
```

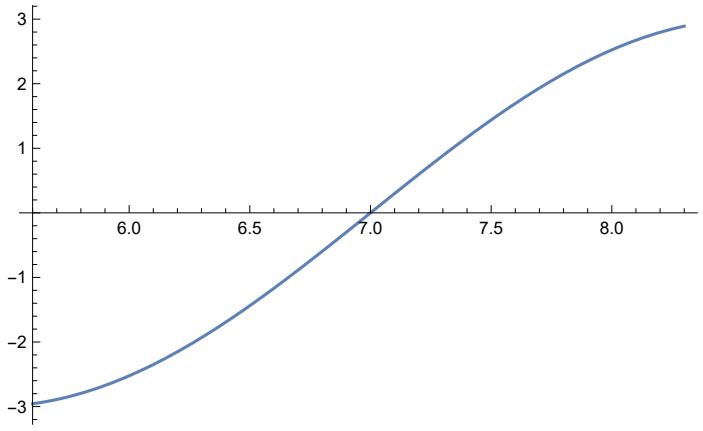
In[83]:=
```
f[x_] := 3 Sin[x - 7]
```

In[84]:=
```
yt = f[xt]
```

Out[84]=
```
{-2.78511, -2.55632, -2.25384, -1.88638, -1.46453, -1.00046,
 -0.507547, 0., 0.507547, 1.00046, 1.46453, 1.88638, 2.25384, 2.55632, 2.78511}
```

In[85]:=
```
grf = Plot[f[x], {x, 5.6, 8.3}]
```

Out[85]=



In[86]:=
```
n = Length[xt]
```

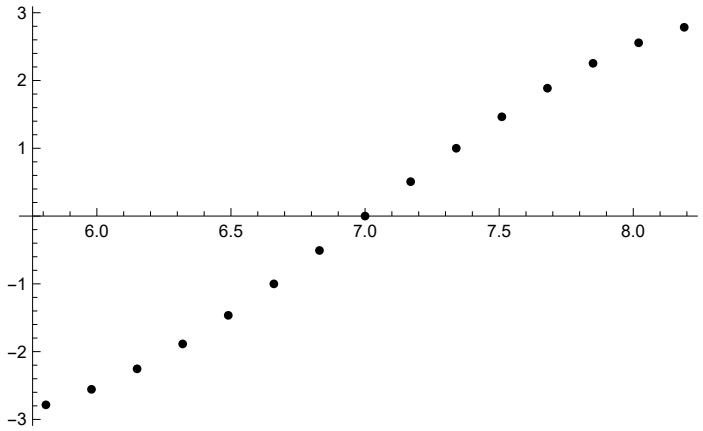
Out[86]=
```
15
```

In[87]:=
```
points = Table[{xt[[i]], yt[[i]]}, {i, 1, n}]
```

Out[87]=
```
{{5.81, -2.78511}, {5.98, -2.55632}, {6.15, -2.25384}, {6.32, -1.88638}, {6.49, -1.46453},
 {6.66, -1.00046}, {6.83, -0.507547}, {7., 0.}, {7.17, 0.507547}, {7.34, 1.00046},
 {7.51, 1.46453}, {7.68, 1.88638}, {7.85, 2.25384}, {8.02, 2.55632}, {8.19, 2.78511}}
```

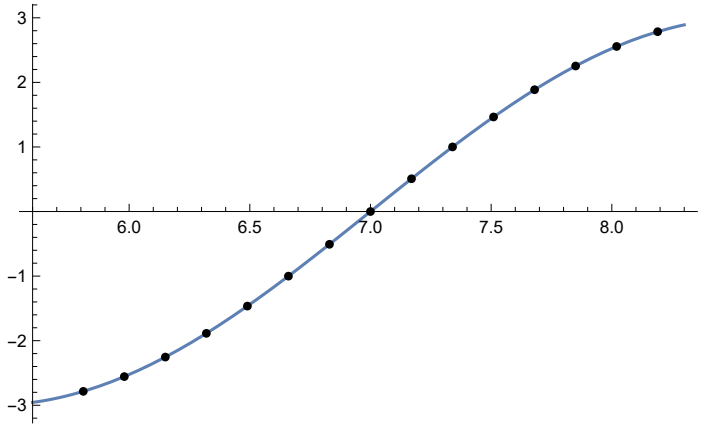
In[88]:= `grp = ListPlot[points, PlotStyle → Black]`

Out[88]=



In[89]:= `Show[grf, grp]`

Out[89]=



Линейна интерполация

In[90]:= $$L1[x\_] := -2.2538 * \frac{x - 6.32}{6.15 - 6.32} - 1.886 * \frac{x - 6.15}{6.32 - 6.15}$$

In[91]:= `Expand[L1[x]]`

Out[91]=

$$-15.5595 + 2.16353\, x$$

Проверка на интерполационните условия

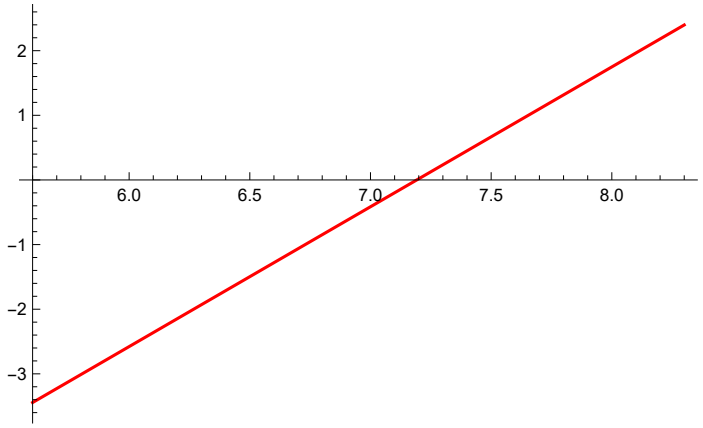
In[92]:=
```
L1[6.15]
L1[6.32]
```

Out[92]=
```
-2.2538
```

Out[93]=
```
-1.886
```

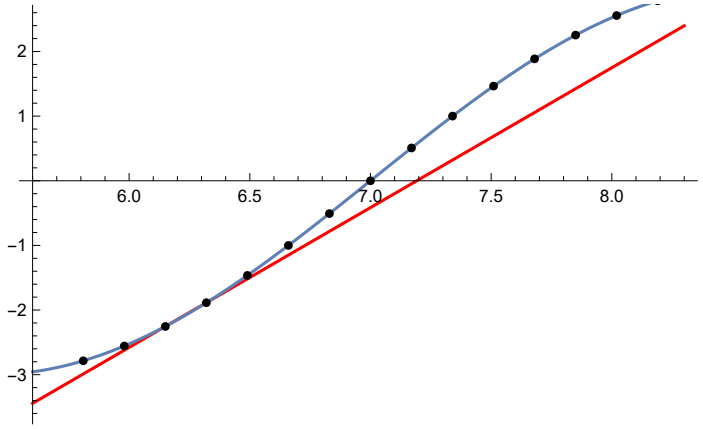
In[94]:=
```
grL1 = Plot[L1[x], {x, 5.6, 8.3}, PlotStyle → Red]
```

Out[94]=



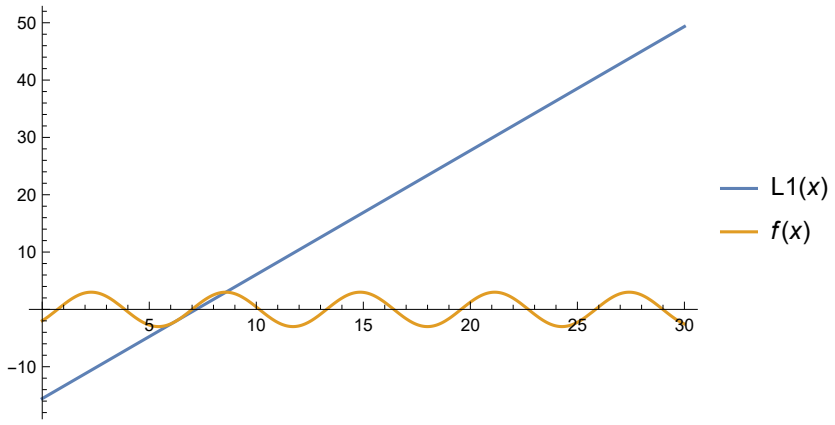
In[95]:=
```
Show[grL1, grf, grp]
```

Out[95]=



In[96]:=
```
grL1 = Plot[{L1[x], f[x]}, {x, 0, 30}, PlotLegends → "Expressions"]
```

Out[96]=



Пресмятане на приближена стойност

In[97]:=
```
L1[6.18]
```

Out[97]=

−2.18889

За сравнение с истинската стойност

In[98]:=
```
f[6.18]
```

Out[98]=

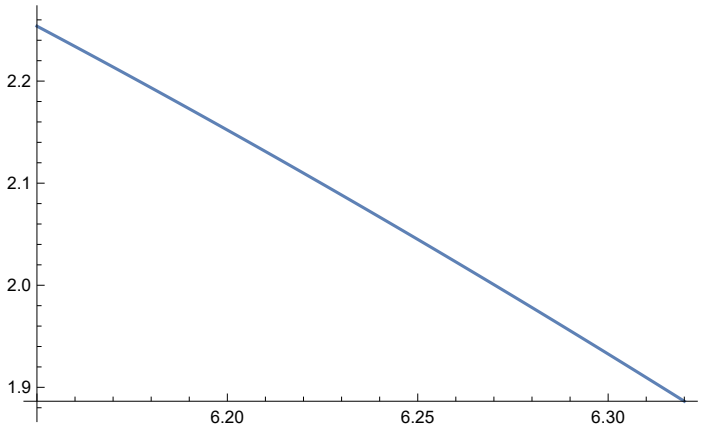
−2.19344

Оценка на грешката

Теоретична грешка

In[99]:=
```
Plot[Abs[f''[x]], {x, 6.15, 6.32}]
```

Out[99]=



In[100]:=
```
M2 = Abs[f''[6.15]]
```
Out[100]=
```
2.25384
```

In[101]:=
```
R1[x_] := M2/2! Abs[(x - 6.15) (x - 6.32)]
```

In[102]:=
```
R1[6.18]
```
Out[102]=
```
0.00473307
```

Истинска грешка

In[103]:=
```
Abs[L1 [6.18] - f[6.18]]
```
Out[103]=
```
0.00454337
```

Квадратична интерполация

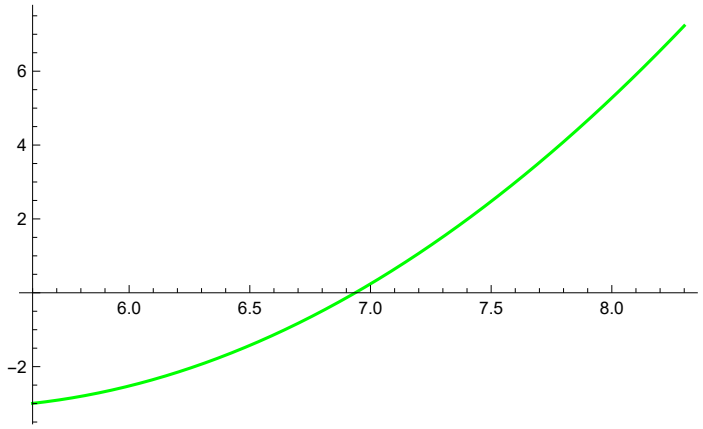
In[104]:=
```
L2[x_] := -2.556 * (x - 6.15) (x - 6.32)/((5.98 - 6.15) (5.98 - 6.32)) -
  2.2538 * (x - 5.98) (x - 6.32)/((6.15 - 5.98) (6.15 - 6.32)) - 1.886 * (x - 5.98) (x - 6.15)/((6.32 - 5.98) (6.32 - 6.15))
```

In[105]:=
```
Expand[L2[x]]
```
Out[105]=
$$28.5537 - 11.9893\, x + 1.13495\, x^2$$

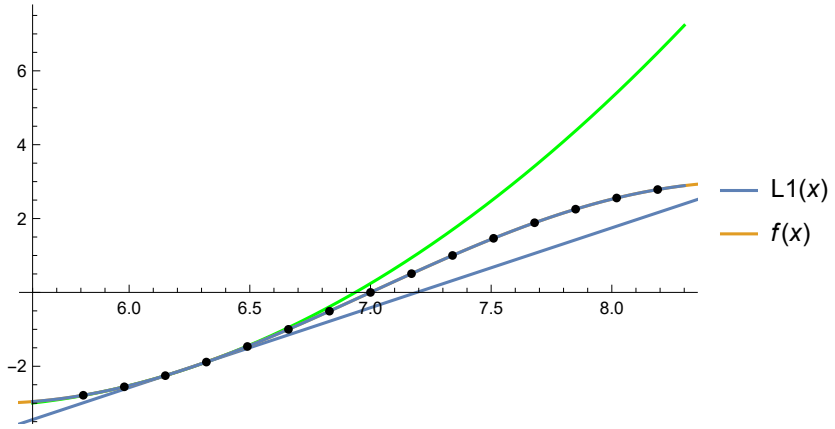
Проверка на интерполационните условия

In[106]:=
```
L2[5.98]
L2[6.15]
L2[6.32]
```
Out[106]=
```
-2.556
```
Out[107]=
```
-2.2538
```
Out[108]=
```
-1.886
```

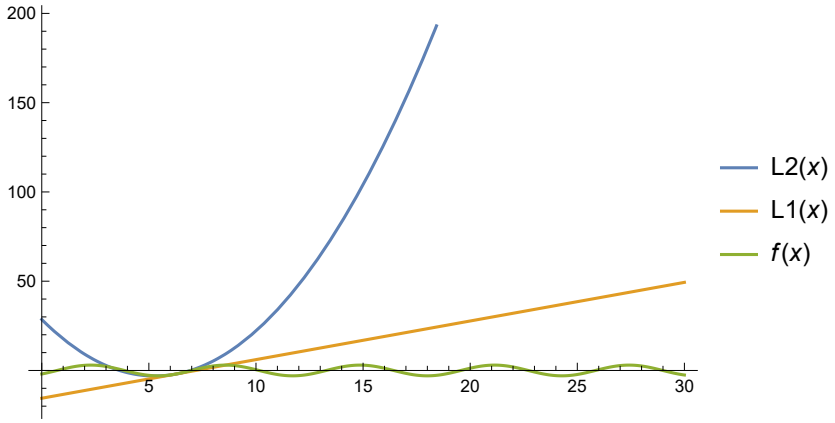
In[109]:=
```
grL2 = Plot[L2[x], {x, 5.6, 8.3}, PlotStyle → Green]
```
Out[109]=



In[110]:=
```
Show[grL2, grL1, grf, grp]
```
Out[110]=



In[112]:=
```
grL2 = Plot[{L2[x], L1[x], f[x]}, {x, 0, 30}, PlotLegends → "Expressions"]
```
Out[112]=



Пресмятане на приближена стойност

In[113]:=
```
L2[6.18]
```
Out[113]=
```
-2.19366
```

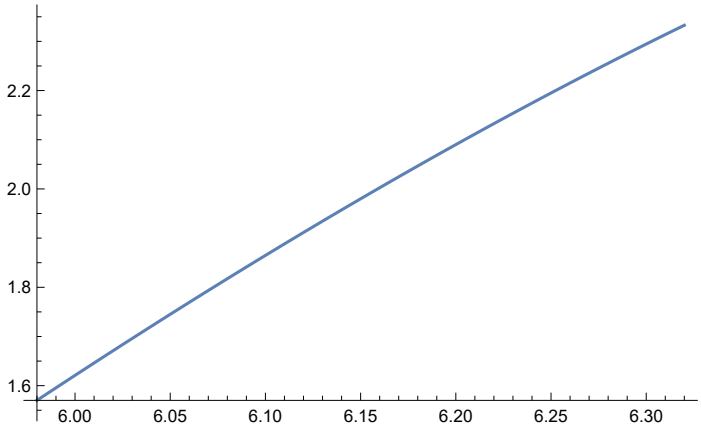
За сравнение с истинската стойност

In[121]:=
```
f[6.18]
```
Out[121]=
```
-2.19344
```

Оценка на грешката

Теоретична грешка

In[122]:=
```
Plot[Abs[f'''[x]], {x, 5.98, 6.32}]
```
Out[122]=



In[124]:=
```
M3 = Abs[f'''[6.32]]
```
Out[124]=
```
2.33272
```

In[125]:=
```
R2[x_] := M3/3! Abs[(x - 5.98) (x - 6.15) (x - 6.32)]
```

In[126]:=
```
R2[6.18]
```
Out[126]=
```
0.000326581
```

Истинска грешка

In[127]:=
```
Abs[L2 [6.18] - f[6.18]]
```
Out[127]=
```
0.00022341
```

Екстраполация

In[128]:=
```
L1[30]
```
Out[128]=
```
49.3464
```

In[129]:=
```
L2[30]
```
Out[129]=
```
690.329
```

In[131]:=
```
f[30.]
```
Out[131]=
```
-2.53866
```

In[132]:=
```
R1[30]
```
Out[132]=
```
636.449
```

In[133]:=
```
R2[30]
```
Out[133]=
```
5274.17
```