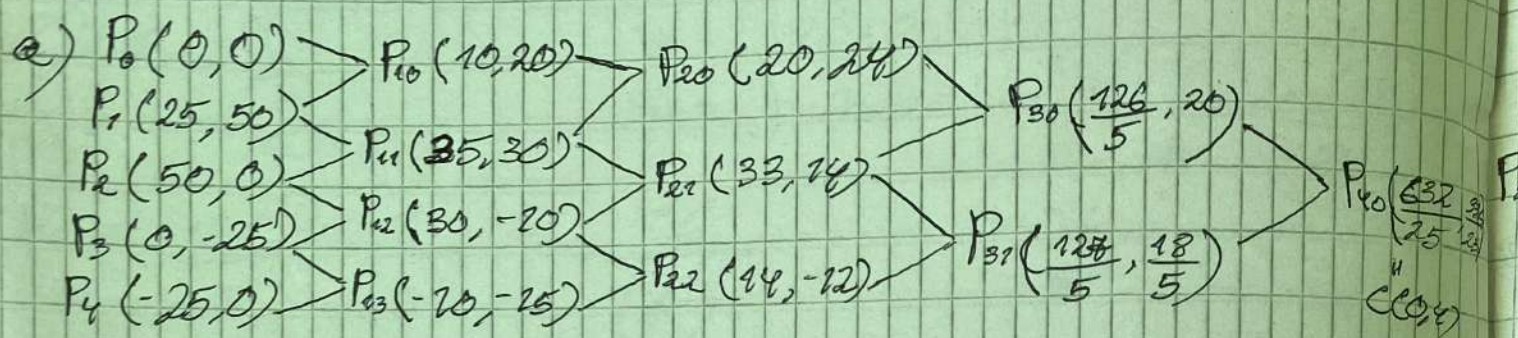


# 3agavea 1:



$$u = 0,4 = \frac{4}{10} = \frac{2}{5}$$

$$1-u = 1 - \frac{2}{5} = \frac{3}{5}$$

$$P_{10} = \frac{3}{5} P_0 + \frac{2}{5} P_1 = \frac{3}{5} (0,0) + \frac{2}{5} (25,50) = (10,20)$$

$$P_{11} = \frac{3}{5} P_1 + \frac{2}{5} P_2 = \frac{3}{5} (25,50) + \frac{2}{5} (50,0) = (15,30) + (20,0) = (35,30)$$

$$P_{12} = \frac{3}{5} P_2 + \frac{2}{5} P_3 = \frac{3}{5} (50,0) + \frac{2}{5} (0,-25) = (30,0) + (0,-10) = (30,-10)$$

$$P_{13} = \frac{3}{5} P_3 + \frac{2}{5} P_4 = \frac{3}{5} (0,-25) + \frac{2}{5} (-25,0) = (0,-25) + (-10,0) = (-10,-15)$$

$$P_{20} = \frac{3}{5} P_{10} + \frac{2}{5} P_{11} = \frac{3}{5} (10,20) + \frac{2}{5} (35,30) =$$

$$= (6,12) + (14,12) = (20,24)$$



$$P_{21} = \frac{3}{5} P_{11} + \frac{2}{5} P_{12} = \frac{3}{5} (35, 30) + \frac{2}{5} (30, -10) =$$

$$= (21, 18) + (12, -4) = (33, 14)$$

$$P_{22} = \frac{3}{5} P_{12} + \frac{2}{5} P_{13} = \frac{3}{5} (30, -20) + \frac{2}{5} (-10, -25) =$$

$$= (18, -6) + (-4, -6) = (14, -12)$$

$$P_{30} = \frac{3}{5} P_{20} + \frac{2}{5} P_{21} = \frac{3}{5} (20, 24) + \frac{2}{5} (33, 14) =$$

$$= \left( \frac{60}{5}, \frac{72}{5} \right) + \left( \frac{66}{5}, \frac{28}{5} \right) = \left( \frac{126}{5}, \frac{20}{5} \right)$$

$$P_{31} = \frac{3}{5} P_{21} + \frac{2}{5} P_{22} = \frac{3}{5} (33, 14) + \frac{2}{5} (14, -12) =$$

$$= \left( \frac{99}{5}, \frac{42}{5} \right) + \left( \frac{28}{5}, \frac{-24}{5} \right) = \left( \frac{127}{5}, \frac{18}{5} \right)$$

$$P_{40} = \frac{3}{5} P_{30} + \frac{2}{5} P_{31} = \frac{3}{5} \left( \frac{126}{5}, \frac{20}{5} \right) + \frac{2}{5} \left( \frac{127}{5}, \frac{18}{5} \right) =$$

$$= \left( \frac{378}{25}, \frac{60}{25} \right) + \left( \frac{254}{25}, \frac{36}{25} \right) = \left( \frac{632}{25}, \frac{96}{25} \right)$$

$$= \left( \frac{378}{25}, \frac{300}{25} \right) + \left( \frac{254}{25}, \frac{36}{25} \right) =$$

$$= \left( \frac{632}{25}, \frac{336}{25} \right)$$



$$C'(0,4) = 4[P_{31} - P_{30}] = 4\left[\left(\frac{128}{5}, \frac{18}{5}\right) - \left(\frac{126}{5}, \frac{20}{5}\right)\right] =$$

$$= 4\left[\left(\frac{1}{5}, -\frac{82}{5}\right)\right] = \left(\frac{4}{5}, -\frac{328}{5}\right)$$

$$C''(0,4) = 4.3[P_{22} - 2P_{21} + P_{20}] = 12[(14, -12) - 2(33, 14) + (20, 24)] =$$

$$= 12[(14, -12) - (66, 28) + (20, 24)] =$$

$$= 12(-32, -16) = (-384, -192)$$

$$\delta) P_0(0,0), P_1(25,50), P_2(50,0), P_3(0,-25), P_4(-25,0)$$

$$n = 4 \rightarrow 5$$

$$Q_0 = P_0 \Rightarrow Q_0(0,0)$$

$$Q_5 = P_4 \Rightarrow Q_5(-25,0)$$

$$Q_1 = \frac{1}{5} P_0 + \frac{4}{5} P_1 = \frac{1}{5}(0,0) + \frac{4}{5}(25,50) = (20,40)$$

$$Q_2 = \frac{2}{5} P_1 + \frac{3}{5} P_2 = \frac{2}{5}(25,50) + \frac{3}{5}(50,0) = (10,20) + (30,0) = (40,20)$$

$$Q_3 = \frac{3}{5} P_2 + \frac{2}{5} P_3 = \frac{3}{5}(50,0) + \frac{2}{5}(0,-25) = (30,0) + (0,-10) = (30,-10)$$

$$Q_4 = \frac{4}{5} P_3 + \frac{1}{5} P_4 = \frac{4}{5}(0,-25) + \frac{1}{5}(-25,0) = (0,-20) + (-5,0) = (-5,-20)$$

$$\Rightarrow Q_0(0,0), Q_1(20,40), Q_2(40,20), Q_3(30,-10), Q_4(-5,-20), Q_5(-25,0)$$



$$b) C^*(u) = C(u) + B_{n,j}(u) \cdot \vec{v}$$

$$P_2(50, 0) \rightarrow P_2^*(25, 25)$$

$$\vec{v} = P_2^* - P_2 = (25, 25) - (50, 0) = (-25, 25)$$

$$B_{4,2}(0, 4) = \frac{4!}{2!(4-2)!} \left(\frac{2}{5}\right)^2 \left(\frac{3}{5}\right)^{4-2} =$$

$$= \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1 \cdot 2 \cdot 1} \left(\frac{4}{25}\right) \left(\frac{9}{25}\right) = 6 \cdot \frac{36}{625} = \frac{216}{625}$$

$$\text{or a) } C(0, 4) = \left(\frac{632}{25}, \frac{336}{25}\right)$$

$$\Rightarrow C^*(0, 4) = \left(\frac{632}{25}, \frac{336}{25}\right) + \frac{216}{625} (-25, 25) =$$

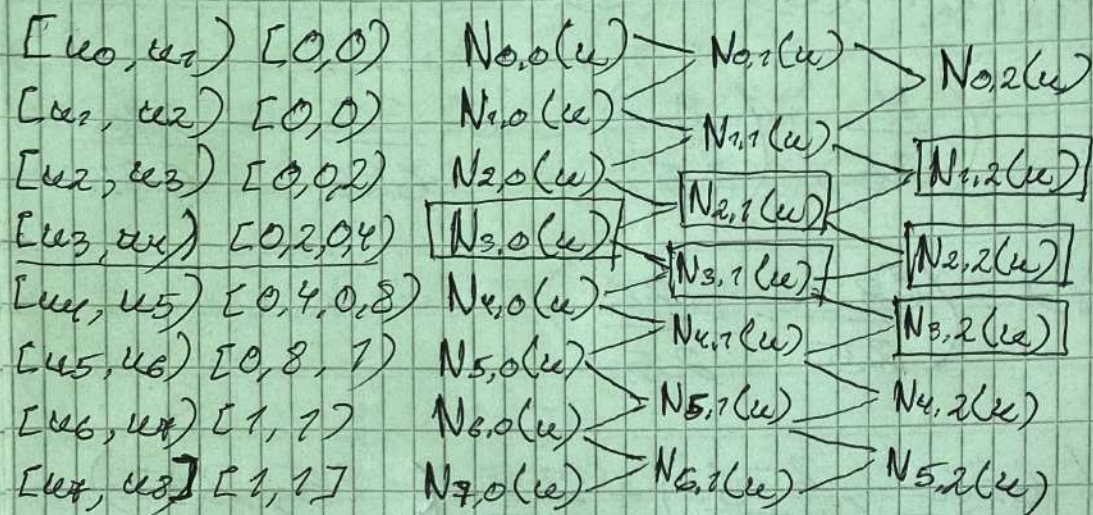
$$= \left(\frac{632}{25}, \frac{336}{25}\right) + \left(-\frac{216}{25}, \frac{216}{25}\right) = \left(\frac{416}{25}, \frac{552}{25}\right)$$



## Задача 2:

$$a) \quad u_0 = u_1 = u_2 \quad \left| \quad u_3 \quad \right| \quad u_4 \quad \left| \quad u_5 \quad \right| \quad u_6 = u_7 = u_8$$

$$\quad \quad \quad 0 \quad \quad \quad 0,2 \quad \left| \quad 0,4 \quad \right| \quad 0,8 \quad \quad \quad 1$$



$$u = 0,3 \in [0, 2, 0,4) \Rightarrow N_{3,0}(0,3) = 1$$

$$N_{2,1}(0,3) = \frac{u_4 - u}{u_4 - u_3} \quad N_{3,0}(0,3) = \frac{0,4 - 0,3}{0,4 - 0,2} \cdot 1 = \frac{0,1}{0,2} = \frac{1}{2}$$

$$N_{2,1}(0,3) + N_{3,1}(0,3) = 1$$

$$N_{3,1}(0,3) = 1 - N_{2,1}(0,3)$$

$$N_{3,1}(0,3) = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow N_{2,1}(0,3) = \frac{1}{2}, \quad N_{3,1}(0,3) = \frac{1}{2}$$

$$N_{1,2}(0,3) = \frac{u_4 - u}{u_4 - u_2} \quad N_{2,1}(0,3) = \frac{0,4 - 0,3}{0,4 - 0} \cdot \frac{1}{2} = \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{8}$$

$$N_{3,2}(0,3) = \frac{u - u_3}{u_5 - u_3} \quad N_{3,1}(0,3) = \frac{0,3 - 0,2}{0,8 - 0,2} \cdot \frac{1}{2} = \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$$



$$N_{1,2}(0,3) + N_{2,2}(0,3) + N_{3,2}(0,3) = 1$$

$$N_{2,2}(0,3) = 1 - N_{1,2}(0,3) - N_{3,2}(0,3)$$

$$N_{2,2}(0,3) = 1 - \frac{1^3}{8} - \frac{1^2}{12} = \frac{24 - 3 - 2}{24} = \frac{18}{24} = \frac{3}{4}$$

$$\delta) u_0 = u_1 = u_2 \mid u_3 \mid u_4 \mid u_5 \mid u_6 = u_7 = u_8$$

$$0 \mid 0,2 \mid 0,4 \mid 0,8 \mid 1$$

$$\text{I вызоване: } u = 0,3 \in [0,2, 0,4) = [u_3, u_4) \Rightarrow P_3, P_2, P_1$$

$$\begin{matrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{matrix} \begin{matrix} & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{matrix}$$

$$P_1 \rightarrow Q_2(0,1) \rightarrow R_3(-\frac{1}{6}, \frac{1}{2}) = C(0,3)$$

$$P_2 \rightarrow Q_3(-\frac{1}{3}, 0) = R_4$$

$$\text{от } P_1 \text{ и } P_2 \Rightarrow Q_2 \text{ и } Q_2$$

$$\text{от } P_2 \text{ и } P_3 \Rightarrow Q_3 \text{ и } Q_3$$

$$Q_2 = (1 - a_2)P_1 + a_2P_2$$

$$Q_2 = \frac{1}{2}P_1 + \frac{1}{2}P_2 =$$

$$a_2 = \frac{1 - u_2}{u_4 - u_2} = \frac{0,3 - 0,2}{0,4 - 0,2} = \frac{1}{2}$$

$$= \frac{1}{2}(0,2) + \frac{1}{2}(0,0) = (0,1)$$



$$Q_3 = (1 - a_3)P_2 + a_3P_3$$

$$Q_3 = \frac{5}{6}P_2 + \frac{1}{6}P_3$$

$$a_3 = \frac{t - u_3}{u_5 - u_3} = \frac{0,3 - 0,2}{0,8 - 0,2} = \frac{1}{6}$$

$$= \frac{5}{6}(0,0) + \frac{1}{6}(-2,0) = \left(-\frac{1}{3}, 0\right)$$

Сред I - во вярване:

$$v_0 = v_1 = v_2 \quad \begin{array}{c|c|c|c|c|c} u_3 & u_4 & u_5 & u_6 & u_7 & u_8 \\ \hline 0 & 0,2 & 0,3 & 0,4 & 0,8 & 1 \end{array} \quad v_7 = v_8 = v_9$$

$$\begin{array}{c|c|c|c|c|c|c} P_0' & P_1' & P_2' & P_3' & P_4' & P_5' & P_6' \\ \hline P_0 & P_1 & Q_2 & Q_3 & P_3 & P_4 & P_5 \end{array}$$

II вярване:  $u = 0,3 \in [0,3, 0,4) = [u_4, u_5) \Rightarrow P_4', P_3', P_2'$

От  $P_2'$  и  $P_3' \Rightarrow R_3$  и  $a_{3,2}$

От  $P_3'$  и  $P_4' \Rightarrow R_4$  и  $a_{4,2}$

$$R_3 = (1 - a_{3,2})P_2' + a_{3,2}P_3'$$

$$R_3 = \frac{1}{2}Q_2 + \frac{1}{2}Q_3$$

$$a_{3,2} = \frac{t - v_3}{v_5 - v_3} = \frac{0,3 - 0,2}{0,4 - 0,2} = \frac{1}{2}$$

$$= \frac{1}{2}(0,1) + \frac{1}{2}\left(-\frac{1}{3}, 0\right) = \left(0, \frac{1}{2}\right) + \left(-\frac{1}{6}, 0\right) = \left(-\frac{1}{6}, \frac{1}{2}\right)$$

$$R_4 = (1 - a_{4,2})P_3' + a_{4,2}P_4'$$

$$R_4 = P_3' = Q_3$$

$$a_{4,2} = \frac{t - v_4}{v_6 - v_4} = \frac{0,3 - 0,3}{0,8 - 0,3} = 0$$



след II - по вхождению:

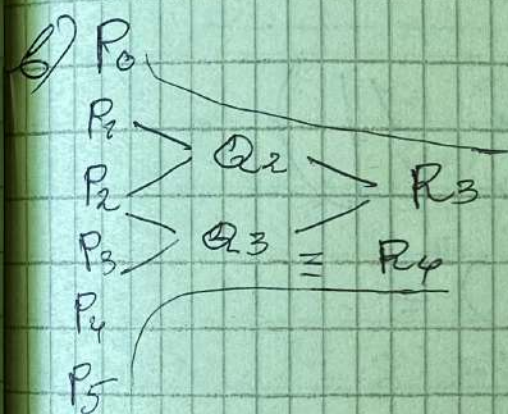
$$w_0 = w_1 = w_2 \mid w_3 \mid w_4 \mid w_5 \mid w_6 \mid w_7 \mid w_8 = w_9 = w_{10}$$

$$0 \mid 0,2 \mid 0,3 \mid 0,3 \mid 0,4 \mid 0,8 \mid 1$$

$$P_0'' \mid P_1'' \mid P_2'' \mid P_3'' \mid P_4'' \mid P_5'' \mid P_6'' \mid P_7''$$

$$P_0 \mid P_1 \mid Q_2 \mid R_3 \mid Q_3 \mid P_3 \mid P_4 \mid P_5$$

$$\Rightarrow C(0,3) = \left( -\frac{1}{6}, \frac{1}{2} \right)$$



$$C_1(u): u = \{0; 0; 0; 0,2; 0,3; 0,3; 0,3\}$$

$$C_2(u): u = \{0,3; 0,3; 0,3; 0,4; 0,8; 1; 1; 1\}$$



Bagaza 3:

$$S: \vec{r}(u^2 - v^2, u + v, u - v)$$

$$\vec{r}(u^2 - v^2, u + v, u - v)$$

$$\vec{r}_u(2u, 1, 1)$$

$$\vec{r}_v(-2v, 1, -1)$$

$$g_{11} = \vec{r}_u \cdot \vec{r}_u = (2u)^2 + 1^2 + 1^2 = 4u^2 + 2$$

$$g_{12} = \vec{r}_u \cdot \vec{r}_v = 2u \cdot (-2v) + 1 \cdot 1 + 1 \cdot (-1) = -4uv$$

$$g_{22} = \vec{r}_v \cdot \vec{r}_v = (-2v)^2 + 1^2 + (-1)^2 = 4v^2 + 2$$



$$I(du, dv) = g_{11} du^2 + 2g_{12} du dv + g_{22} dv^2$$

$$I(du, dv) = (4u^2 + 2) du^2 + (8uv) du dv + (4v^2 + 2) dv^2$$

$$II(du, dv) = h_{11} du^2 + 2h_{12} du dv + h_{22} dv^2$$

$$h_{11} = \vec{N} \cdot \vec{r}_{uu}, \quad h_{12} = \vec{N} \cdot \vec{r}_{uv}, \quad h_{22} = \vec{N} \cdot \vec{r}_{vv}$$

$$\vec{N} = \frac{\vec{r}_u \times \vec{r}_v}{|\vec{r}_u \times \vec{r}_v|} = \frac{(-2, 2u+2v, 2u+2v)}{\sqrt{4+8u^2+8v^2}} = 0.341 \cdot C^*$$

$$\vec{r}_u(2u, 1, 1)$$

$$\vec{r}_v(-2v, 1, -2)$$

$$\vec{r}_u \times \vec{r}_v = \left( \begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix}, -\begin{vmatrix} 2u & 1 \\ -2v & -1 \end{vmatrix}, \begin{vmatrix} 2u & 1 \\ -2v & 1 \end{vmatrix} \right) =$$

$$= (-2, -(-2u+2v), 2u+2v) =$$

$$= (-2, 2u+2v, 2u+2v)$$

$$|\vec{r}_u \times \vec{r}_v| = \sqrt{(-2)^2 + (2u+2v)^2 + (2u+2v)^2} = \sqrt{4 + 4u^2 + 8uv + 4v^2 + 4u^2 + 8uv + 4v^2}$$

$$= \sqrt{4 + 8u^2 + 16uv + 8v^2} = \sqrt{4 + 8u^2 + 8v^2}$$

$$\vec{r}_{uu}(2, 0, 0)$$

$$\vec{r}_{uv}(0, 0, 0)$$

$$\vec{r}_{vv}(-2, 0, 0)$$

$$h_{11} = \frac{-4}{*}$$

$$h_{12} = 0, \quad h_{22} = \frac{-4}{*}$$



$$\Pi(2u, 2v) = \frac{-4}{\sqrt{4+8u^2+8v^2}} - \frac{4}{\sqrt{4+8u^2+8v^2}} 2v^2$$

$$8) \text{TPS: } \begin{cases} z + P = \vec{r}(u^2 - v^2, u+v, u-v) \\ \perp \vec{N} \parallel \vec{r}_u \times \vec{r}_v (-2, 2u-2v, 2u+2v) \end{cases}$$

$$\text{TPS: } -2(x - (u^2 - v^2)) + (2u - 2v)(y - (u+v)) - (2u+2v)(z - (u-v))$$

$$+ P(u=3, v=1)$$

$$\Rightarrow \text{TPS: } -2(x-8) + 3(y-2) - 8(z-2)$$

$$6) H = \frac{g_{11}h_{22} - 2g_{12}h_{12} + g_{22}h_{11}}{2g}$$

$$H = \frac{(4u^2+2) \cdot (-4)}{*} + \frac{(4v^2+2) \cdot (-4)}{*} \cdot \frac{1}{2*}$$

$$H = \frac{-16u^2 - 8 - 16v^2 - 8}{2(\sqrt{4+8u^2+8v^2})^3}$$

$$H = \frac{-16u^2 - 16v^2 - 16}{2(\sqrt{4+8u^2+8v^2})^3}$$

$$H = \frac{-16(u^2 + v^2 + 1)}{2(\sqrt{4+8u^2+8v^2})^3}$$

$$H = \frac{-8(u^2 + v^2 + 1)}{(\sqrt{4+8u^2+8v^2})^3}$$