Задача 1: б)

$$(\underbrace{y^2-2x^3}_{M})dx+\underbrace{2xy}_{N}dy=0$$

$M'_y \overset{?}{=} N'_x$

$M'_y=(y^2-2x^3)'_y=(y^2)'_y-(2x^3)'_y=2y$

$N'_x=(2xy)'_x=2y(x)'_x=2y$

$\Rightarrow M'_y \equiv N'_x \Rightarrow$ точно диф. уравнение

$\exists F(x,y)$ $\left|\begin{array}{l} (1)\ F'_x=M=y^2-2x^3 \\ F'_y=N=2xy \end{array}\right.$

$F=\int 2xy\,dy+C(x)=2x\int y\,dy+C(x)=$

$=\cancel{2}x\dfrac{y^2}{\cancel{2}}+C(x)=xy^2+C(x)$

(3)

$F=xy^2+C(x)\rightarrow$ диференциране спрямо x

$F'_x=(xy^2)'_x+C'(x)$

$F'_x=y^2(x)'_x+C'(x)$

$F'_x=y^2+C'(x)$ (2)

От (1) и (2) $\Rightarrow \cancel{y^2}-2x^3=\cancel{y^2}+C'(x)$

$\Rightarrow C'(x)=-2x^3$

$C(x)=-2\int x^3dx+C$

$C(x)=-\cancel{2}\dfrac{x^4}{\cancel{4}}+C$

$C(x)=-\dfrac{x^4}{2}+C$

Заместваме в (3) и получаваме:

$$F = xy^2 + \left(-\frac{x^4}{2} + C\right)$$

$$F = xy^2 - \frac{x^4}{2} + C$$

Решението на диференциалното уравнение е:

$$xy^2 - \frac{x^4}{2} + C = C_1$$

$$xy^2 - \frac{x^4}{2} = \underbrace{C_1 - C}_{C_2}$$

$$\Rightarrow xy^2 - \frac{x^4}{2} = C_2$$

Задача 3:

$$y = y' + \frac{1}{2}(x - \ln y')$$

Полагаме: $y' = p$, $p = p(x)$, $p' = \frac{dp}{dx}$

$$y = p + \frac{1}{2}\cdot(x - \ln p)\,\Big|'_x \quad p \geq 0$$

$$y' = p' + \underbrace{\frac{1}{2}'}_{0}(x - \ln p) + \frac{1}{2}(x - \ln p)'$$

$$p = p' + \frac{1}{2}\left(1 - \frac{1}{p}p'\right)$$

$$p = \frac{dp}{dx} + \frac{1}{2}\left(1 - \frac{1}{p}\cdot\frac{dp}{dx}\right)$$

$$p = \frac{1}{2}\left(1 - \frac{1}{p}\right)\frac{dp}{dx} \cdot \Big| 2$$

$$2p = \left(1 - \frac{1}{p}\right)\frac{dp}{dx} \quad :\Big| 2p \cdot dx$$

$$dx = \frac{p-1}{p}\cdot\frac{1}{2p}\,dp$$

$$dx = \frac{p-1}{2p^2}\,dp$$

Интегриране:

$$\int dx = \int \frac{p-1}{2p^2}\,dp$$

$$x = \frac{1}{2}\left(\int \frac{p-1}{p^2}\,dp\right)$$

$$x = \frac{1}{2}\left(\int \frac{\cancel{p}}{p^{\cancel{2}}}\,dp - \int \frac{1}{p^2}\,dp\right)$$

$$x = \frac{1}{2}\left(\int \frac{1}{p}\,dp - \int \frac{1}{p^2}\,dp\right)$$

$$x = \frac{1}{2}\left(\ln|p| - \int p^{-2}\,dp\right)$$

$$x = \frac{1}{2}\left(\ln|p| - \left(\frac{p^{-1}}{-1}\right)\right)$$

$$\left|\begin{aligned} x &= \frac{1}{2}\left(\ln|p| + \frac{1}{p}\right) \\ y &= p + \frac{1}{2}\left(x - \ln p\right) \end{aligned}\right\} \text{общо решение в параметричен ви}$$

Задача 4:

$y - xy' + y'\ln y' = 0$

$y = xy' - y'\ln y'$ → уравн. на Клеро

Полагане: $y' = p$, $p = p(x)$, $p' = \frac{dp}{dx}$

$y = xp - p\ln p \,\Big|'_x$, $p > 0$

$y' = x'p + x\cdot p' - p'\ln p + p\cdot\ln p'$

$p = \cancel{p} + xp' - p'\ln p + \cancel{p}\cdot\frac{1}{\cancel{p}}\cdot p'$

$0 = xp' - p'\ln p + p'$

$0 = p'(x - \ln p + 1)$

$p' = 0$

$p = C$

$y = xC - C\ln C$

общо решение

$x - \ln p + 1 = 0$

$\begin{cases} x = \ln p - 1 \\ y = xp - p\ln p \end{cases}$

допълнително решение

Задача 5: б)

$y - 2xy' = (y')^4$

$y = 2xy' + (y')^4 \rightarrow$ уравнение на Лагранж

Полагаме: $y' = p$, $p = p(x)$, $p' = \frac{dp}{dx}$

$y = 2xp + p^4 \quad |'_x$

$y' = 2x'p + 2xp' + 4p^3p'$

$p = 2p + 2xp' + 4p^3p'$

$-p = p'(2x + 4p^3)$

$-p = \frac{dp}{dx}(2x + 4p^3) \cdot \frac{dx}{dp}$

$\frac{dx}{dp} = \frac{2x + 4p^3}{-p}$

$\frac{dx}{dp} = -\frac{2}{p}x - 4p^2$

$x = e^{\underbrace{-\int \frac{2}{p}dp}_{-2\ln|p|}}\left(C + -\int 4p^2 e^{\underbrace{\int \frac{2}{p}dp}_{2\ln|p|}} dp\right)$

$x = e^{-2}\left(C - 4\int p^2 \cdot e^{2\ln|p|} \cdot dp\right)$

$x = \ln|p|^{-2}\left(C - \frac{4p^3}{3} \cdot \ln|p|^2\right)$

$$\begin{cases} x = \frac{1}{\ln|p|^2}\left(C - \frac{4p^3}{3} \ln|p|^2\right) \\ y = 2xp + p^4 \end{cases}$$

} общо решение в параметричен вид