Курсова работа по Математически анализ 2020/2021

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ЗАДАЧИ ЗА САМОСТОЯТЕЛНА РАБОТА
за Първи курс Информатика

1. Задача: Намерете границите на числовите редици:

1.1 $\lim\limits_{n\to\infty} \frac{7n^2+n-3}{8n^2-n+1} = \lim\limits_{n\to\infty} \frac{n^2(7+\frac{1}{n}-\frac{3}{n^2})}{n^2(8-\frac{1}{n}+\frac{1}{n^2})} = \frac{7}{8}$

1.2 $\lim\limits_{n\to\infty} \frac{3-n^2+4n^4}{2+n+3n^2+2n^4} = \lim\limits_{n\to\infty} \frac{4n^4-n^2+3}{2n^4+3n^2+n+2} = \lim\limits_{n\to\infty} \frac{n^4(4-\frac{1}{n^2}+\frac{3}{n^4})}{n^4(2+\frac{3}{n^2}+\frac{1}{n^3}+\frac{2}{n^4})} =$
$\frac{4}{2} = 2$

1.3 $\lim\limits_{n\to\infty} \frac{n^3+8n-2}{2n^2+3n-1} = \lim\limits_{n\to\infty} \frac{n^3(1+\frac{8}{n^2}-\frac{2}{n^3})}{n^2(2+\frac{3}{n}+\frac{1}{n^2})} = \lim\limits_{n\to\infty} = \frac{n}{2} = \infty$

1.4 $\lim\limits_{n\to\infty} \frac{4n-3}{n^3+2} = \lim\limits_{n\to\infty} \frac{n(4-\frac{3}{n})}{n^3(1+\frac{2}{n^3})} = \lim\limits_{n\to\infty} \frac{4}{n^2} = +\infty$

1.5 $\lim\limits_{n\to\infty} \frac{(n+1)^6}{(n+1)^7-n^7}$

1.6 $\lim\limits_{n\to\infty} \frac{a_k n^k + a_{k-1}n^{k-1}+\ldots+a_0}{b_l n^l + b_{l-1}n^{l-1}+\ldots+b_0}, a_k \neq 0, b_l \neq 0$

1.7 $\lim\limits_{n\to\infty} \frac{5^n+4^{n+1}}{6^{n+2}+5^n} = \lim\limits_{n\to\infty} \frac{5^n+4^n.4^1}{6^n.6^2+5^n} = \lim\limits_{n\to\infty} \frac{5^n(1+(\frac{4}{5}^n.4))}{5^n((\frac{5}{6}^n.6^2)+1)} = \frac{1}{1} = 1$

1.8 $\lim\limits_{n\to\infty}\dfrac{3^n+4^{n+3}}{4^n+3^{n+2}}=\lim\limits_{n\to\infty}\dfrac{3^n+4^n.4^3}{4^n.3^n+3^2}=\lim\limits_{n\to\infty}\dfrac{3^n(1+(\frac{4}{3}^n.4^3))}{3^n((\frac{4}{3}^n)+1.3^2)}=\dfrac{1}{9}$

1.9 $\lim\limits_{n\to\infty}\left(\dfrac{n-7}{n+3}\right)^n=\lim\limits_{n\to\infty}\left(\dfrac{n(1-\frac{7}{n})}{n(1+\frac{3}{n})}\right)^n=\dfrac{\varepsilon^{-7}}{\varepsilon^3}=\varepsilon^{-10}=\dfrac{1}{\varepsilon^{10}}$

1.10 $\lim\limits_{n\to\infty}\left(\dfrac{n-4}{n+6}\right)^{n+1}=\lim\limits_{n\to\infty}\left(\dfrac{n(1-\frac{4}{n})}{n(1+\frac{6}{n})}\right)^{n+1}=\dfrac{\varepsilon^{-4}}{\varepsilon^6}=\varepsilon^{-10}=\dfrac{1}{\varepsilon^{10}}$

1.11 $\lim\limits_{n\to\infty}\left(\dfrac{n^2-5n+6}{n^2-6n+5}\right)^n=\lim\limits_{n\to\infty}\left(\dfrac{(n-3).(n-2)}{(n-5).(n-1)}\right)^n=\lim\limits_{n\to\infty}\left(\dfrac{n(1-\frac{3}{n})}{n(1-\frac{5}{n})}\right)^n.\lim\limits_{n\to\infty}\left(\dfrac{n(1-\frac{2}{n})}{n(1-\frac{1}{n})}\right)^n=$
$\dfrac{\varepsilon^{-3}.\varepsilon^{-2}}{\varepsilon^{-5}.\varepsilon^{-1}}=\dfrac{\varepsilon^{-5}}{\varepsilon^{-6}}=\varepsilon^1=\varepsilon$

1.12 $\lim\limits_{n\to\infty}\left(\dfrac{n^2+2n+3}{2n^2-n+5}\right)^n=$

1.13 $\lim\limits_{n\to\infty}\left(\dfrac{n^2-7n+12}{n^2+5n+4}\right)^{\frac{n}{2}}=\lim\limits_{n\to\infty}\left(\dfrac{(n-4).(n-3)}{(n+4).(n+1)}\right)^{\frac{n}{2}}=\lim\limits_{n\to\infty}\left(\dfrac{n(1-\frac{4}{n})}{n(1+\frac{4}{n})}\right)^{\frac{n}{2}}.\lim\limits_{n\to\infty}\left(\dfrac{n(1-\frac{3}{n})}{n(1+\frac{1}{n})}\right)^{\frac{n}{2}}=$
$\lim\limits_{n\to\infty}\left(\dfrac{(1-\frac{4}{n})^n}{(1+\frac{4}{n})^n}\right)^{\frac{1}{2}}.\lim\limits_{n\to\infty}\left(\dfrac{(1-\frac{3}{n})^n}{(1+\frac{1}{n})^n}\right)^{\frac{1}{2}}=\dfrac{\varepsilon^{-4}.\varepsilon^{-3}}{\varepsilon^4.\varepsilon^1}=\dfrac{\varepsilon^{-7}}{\varepsilon^5}=\varepsilon^{-12}=\dfrac{1}{\varepsilon^{12}}$

1.14 Докажете, че $\lim\limits_{n\to\infty}\dfrac{2^n}{n^n}=0$

1.15 $\lim\limits_{n\to\infty}c_n=?$, където $c_n=\frac{1}{\sqrt{n^2+1}}+\frac{1}{\sqrt{n^2+2}}+\ldots+\frac{1}{\sqrt{n^2+n}}$

2. Задача: Да се намерят границите на функциите:

2.1 $\lim\limits_{x\to3}\dfrac{x^2-9}{x^2-4x+3}=\lim\limits_{x\to3}\dfrac{(x-3).(x+3)}{(x-3).(x-1)}=\lim\limits_{x\to3}\dfrac{x+3}{x-1}=\lim\limits_{x\to3}\dfrac{3+3}{3-1}=\dfrac{6}{2}=3$

2.2 $\lim\limits_{x\to3}\dfrac{x^3-27}{2x-3}$

2.3 $\lim\limits_{x\to2}\dfrac{x^4-5x^2+4}{x^2-4}=\lim\limits_{x\to3}\dfrac{(x-2).(x+2).(x-1).(x+1)}{(x-2).(x+2)}=(x+1).(x-1)=3.1=$
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2.4 $\lim_{x\to 1}\frac{x^3-2x-1}{x^5-2x-1}=\lim_{x\to 1}\frac{(x+1).(x^2-x-1)}{(x+1).(x^4-x^3+x^2-x-1)}=\lim_{x\to 1}\frac{x^2-x-1}{x^4-x^3+x^2-x-1}=$
$\lim_{x\to 1}\frac{1^2-1-1}{1^4-1^3+1^2-1-1}=\lim_{x\to 1}\frac{-1}{-1}=1$

2.5 $\lim_{x\to\pm\infty}\frac{3x^2+2x+1}{x^2-x+5}=$

2.6 $\lim_{x\to\pm\infty}\frac{3x^4-2x^2+x-4}{x^2+3x-7}$

2.7 $\lim_{x\to\pm\infty}\frac{3-7x+x^2}{4-8x+x^2-x^3}$

2.8 $\lim_{x\to\pm\infty}\frac{3^x+7^x}{7^{x+2}-5^x}=\lim_{x\to\pm\infty}\frac{3^x+7^x}{7^x.7^2-5^x}=\lim_{x\to\pm\infty}\frac{7^x((\frac{3}{7})^x+1)}{7^x(1.7^2-(\frac{5}{7})^x)}=\frac{1}{49}$

2.9 $\lim_{x\to-\infty}\frac{3^x+7^x}{7^{x+2}-5^x}=\lim_{x\to\pm\infty}\frac{3^x+7^x}{7^x.7^2-5^x}=\lim_{x\to\pm\infty}\frac{7^x((\frac{3}{7})^x+1)}{7^x(1.7^2-(\frac{5}{7})^x)}=-\frac{1}{49}$

2.10 $\lim_{x\to\pm\infty}\frac{2^x+4^{x+1}}{4^x+2^{x+1}}=\lim_{x\to\pm\infty}\frac{2^x+4^x.4^1}{4^x+2^x.2^1}=\lim_{x\to\pm\infty}\frac{4^x((\frac{2}{4})^x+1.4)}{4^x(1+(\frac{2}{4})^x.2)}=4$

2.11 $\lim_{x\to a}\frac{\sin x-\sin a}{x-a}=\lim_{x\to a}\frac{2sin(\frac{x-a}{2}).\cos(\frac{x+a}{2})}{x-a}=\lim_{x\to a}\frac{sin(\frac{x-a}{2})}{\frac{x-a}{2}}.\lim_{x\to a}\frac{x+a}{2}=\lim_{x\to a}1.\frac{\cos a+a}{2}=\lim_{x\to a}\frac{\cos 2a}{2}=\cos a$

2.12 $\lim_{x\to+\infty}(\sin\sqrt{x+1}-\sin\sqrt{x})$

3. Задача: Да се намерят границите на функциите чрез еквивалентни безкрайно малки функции:

3.1 $\lim_{x\to 0}\frac{\sin 6x}{x}=\lim_{x\to 0}6.\frac{\sin 6x}{6x}=6$

3.2 $\lim_{x\to 0}\frac{\sin^2 4x}{\ln(2x^2+1)}=$

3.3 $\lim_{x\to 1}\frac{x^2-4x+3}{\operatorname{arctg}(x^2+x-2)}=\lim_{x\to 1}\frac{(x-3).(x-1)}{\operatorname{arctg}(x+2).(x-1)}=\lim_{x\to 1}\frac{(x-3)}{\operatorname{arctg}(x+2)}=\lim_{x\to 1}\frac{(x-3)}{\frac{1}{1+x^2}(x+2)}=$

$$\lim_{x\to 1}\frac{(1-3)}{\frac{1}{1+1^2}(1+2)}=\lim_{x\to 1}-\frac{2}{\frac{3}{2}}=\lim_{x\to 1}-2.\frac{2}{3}=-\frac{4}{3}$$

3.4 $\lim\limits_{x\to 0}\dfrac{\ln(1+\tan x)}{e^{\arcsin x}-1}$

3.5 $\lim\limits_{x\to 0}\dfrac{e^{3x}-1}{\operatorname{arctg} 2x}$

3.6 $\lim\limits_{x\to 0}\dfrac{\ln(\cos x)}{\tan x^2}$

3.7 $\lim\limits_{x\to 0}(2x^3+1)^{\cot^3 2x}$

3.8 $\lim\limits_{x\to 0}\left(\dfrac{1+\tan x}{1+\sin x}\right)^{\frac{1}{\sin^3 x}}$

3.9 $\lim\limits_{x\to 0}\dfrac{\tan x.\arcsin^2 x}{\sin x.(1-\cos 4x)}$

4. Задача: Намерете производните на функциите:

4.1 $y=6x^2-7x+1$
$y'=6.2x-7$
$y'=12x-7$

4.2 $y=4x^3+5x-\frac{1}{x^2}+1$
$y=4x^3+5x-x^{-2}+1=$
$y'=4.3x^2+5-2x^{-2-1}$
$y'=12x^2+5-2x^{-3}$

4.3 $y=x^5-2\sqrt[4]{x^3}+\frac{3}{\sqrt{x}}+2=$
$y=x^5-2.x^{\frac{3}{4}}+3.x^{\frac{-1}{2}}+2$
$y'=5x^4-2.(\frac{3}{4})x^{\frac{3}{4}-1}+3.(\frac{1}{2})x^{\frac{-1}{2}-1}$
$y'=5x^4-\frac{3}{2}x^{\frac{-1}{4}}-\frac{3}{2}x^{\frac{-3}{2}}$

4.4 $y=\frac{3-x}{x^2}=$

$y' = \frac{(3-x)'(x^2)-(3-x)(x^2)'}{(x^2)^2}$
$y' = \frac{x^2-(3-x)(2x)}{(x^2)^2}$
$y' = \frac{x^2-6x-2x^2}{(x^2)^2}$
$y' = \frac{-x^2-6x}{(x^2)^2}$
$y' = \frac{x^2(-1-\frac{6}{x})}{(x^2)^2}$
$y' = \frac{-1-\frac{6}{x}}{x^2}$

4.5 $y = \sin x + \arcsin x + 2^x.x =$

4.6 $y = \cos(6x^2) - \cos^2 3x =$

4.7 $y = \frac{\mathrm{tg}\, x}{1+\cos x}$

4.8 $y = (3x^2 - x + 1)\ln(x+1)$

4.9$y = e^x(\tan x + \cot g3x)$

4.10$y = \frac{\sin 2x+1}{2x^2-1}$

4.11$y = \frac{\arctan(2x+1)}{1+3x}$

4.12$y = 5^{x^2} + e^{-x} + \ln(-x)$

4.13$y = \ln(\cos 4x + 1) + \arctan(\ln 2x)$

4.14$y = \sin^6(12x^3 + \tan(4x)) + 6x^2.\ln^3(\sin 2x)$

4.15$y = 2^{3x} + \frac{3x^2+\sqrt{x}+3}{\sin^4 3x} + (\arcsin 7x)^{5x}$

4.16$y = \frac{\arcsin 3x}{x^2} + 5\ln^3 x - e^{\sqrt{5x^2}} + (\mathrm{arctg}\, x)^{\cos 2x}$

5. Задача: Да се изчисли приближено стойността на:

5.1 $\sqrt{1620}$ с точност до $10^{-1} = \sqrt{1600+20} = \sqrt{1600} + \frac{1}{2\sqrt{1600}}.20 = 40 + \frac{1}{2.40}.20 = 40 + \frac{1}{80}.20 = 40 + \frac{1}{4} = \frac{161}{4} = 40,25$

5.2 $\sqrt[5]{250}$ с точност до 10^{-3}

5.3 $\sqrt[3]{1,02}$ с точност до $10^{-4} = \sqrt[3]{1+0,02} = 1 + \frac{0,02}{3.\sqrt[3]{1^2}} = 1 + \frac{0,02}{3} \approx 1,0067$

5.4 $\sin 85^\circ$ с точност до $10^{-3} \approx \sin(90^\circ - 5^\circ) \approx \sin\frac{\pi}{2} - \cos\frac{\pi}{2}.\frac{\pi}{36} \approx 1 - 0.\frac{\pi}{36} \approx 1$

5.5 $\cos 72^\circ$ с точност до $10^{-3} \approx \cos\frac{\pi}{4} - \sin\frac{\pi}{4}.\frac{3\pi}{20} \approx \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}.\frac{3\pi}{20}$

5.6 $\sin 1^\circ$ с точност до $10^{-6} \approx \sin(90^\circ - 89^\circ) \approx \sin\frac{\pi}{2} - \cos\frac{\pi}{2}.\frac{89\pi}{180} \approx 1 - 0.\frac{89\pi}{180} \approx 1$

5.7 e с точност до 10^{-7}

5.8 $\ln 3$ с точност до 10^{-3}

5.9 $\ln 11$ с точност до 10^{-4}

6. Задача: Намерете границите на функциите чрез теоремите на Лопитал:

6.1 $\lim\limits_{x\to 0}\dfrac{\cot 4x}{\cot 3x} = \lim\limits_{x\to 0}\dfrac{(\cot 4x)'}{(\cot 3x)'} = \lim\limits_{x\to 0}\dfrac{\frac{-4}{\sin^2 4x}}{\frac{-3}{sin^2 3x}} = \lim\limits_{x\to 0}\dfrac{4\sin^2 3x}{3sin^2 4x} = \lim\limits_{x\to 0}\dfrac{4}{3\sin x} = \lim\limits_{x\to 0}\dfrac{(4)'}{(3\sin x)'} = \dfrac{0}{3\cos x} = \dfrac{0}{3.1} = 0$

6.2 $\lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\sin x - 1}{\cos 3x} = \lim\limits_{x\to\frac{\pi}{6}}\dfrac{(2\sin x)' - (1)'}{(\cos 3x)} = \lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\cos x}{-3\sin 3x} = \lim\limits_{x\to\frac{\pi}{6}}\dfrac{2\cos\frac{\pi}{6}}{-3\sin 3\frac{\pi}{6}} = \lim\limits_{x\to\frac{\pi}{6}}\dfrac{2.\frac{1}{2}}{-3.1} = -\dfrac{1}{3}$

6.3 $\lim_{x\to 0} \frac{\sqrt[7]{1+\sin x}-1}{\arcsin x}$

6.4 $\lim_{x\to +\infty} \frac{e^x}{x^3}$

6.5 $\lim_{x\to 2} \frac{\ln(x^2-3)}{x^2+3x-10}$

6.6 $\lim_{x\to 0} \left(\frac{\arcsin x}{x}\right)^{\frac{1}{x^2}}$

6.7 $\lim_{x\to 1} (x-1)^{\ln x}$

6.8 $\lim_{x\to \frac{\pi}{2}} (\cos x)^{\cot x}$

7. Задача: Докажете тъждествата и неравенствата:

8. Задача: Изследвайте функциите за монотонност и локални екстремуми:

8.1 $y = y^3 - 4x^2$

8.2 $y = \frac{x}{x^2+4}$

8.3 $y = \frac{x^3}{x^2-2}$

8.4 $y = xe^{-\frac{x^2}{2}}$

8.5 $y = \frac{e^x}{1+x}$

8.6 $y = x - 2\,\mathrm{arctg}\,x$

8.7 $y = \sqrt{e^{x^2}-1}$

9. Задача: Изследвайте за изпъкналост, вдлъбнатост и инфлексни точки функциите:

9.1 $y = 2e^x + 3x$

9.2 $y = \frac{x^3}{1-x^2}$

9.3 $y = \frac{\ln x}{x}$

9.4 $y = \sin x - \cos x$

10. Задача: Намерете най-голямата и най-малката стойност на функциите:

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11. Задача: Да се изследва функцията и да се построи графиката ѝ:

11.1 $y = \frac{x^2}{1-x}$

11.2$y = \frac{x^4}{x^3-1}$

11.3 $y = \frac{x^2}{x^2-2x+2}$

11.4 $y = \frac{x^3-4}{x^2}$

11.5 $y = \frac{x}{\ln x}$

11.6 $y = x^2.e^{\frac{1}{x}}$

11.7 $y = \ln(x^2-4)$

11.8 $y = 2\,\mathrm{arctg}\, x - x$

11.9 $y = (x-1)\sqrt{x}$

11.10 $y = \frac{x^2}{e^x}$

11.11 $y = \mathrm{arctg}\, x + \mathrm{arctg}\, \frac{1-x}{1+x}$

13. Задача: Да се пресметнат следните неопределени интеграли:

13.1 $\int (3x^2+2x+1)dx$

13.2 $\int (\cos x + 2\sqrt[5]{x} + \frac{1}{2x})dx$

13.3$\int (\frac{5}{\cos^2 3x} + \frac{3}{\sqrt{1-4x^2}})dx$

13.4 $\int \frac{x^4}{x^2-1}dx$

13.5 $\int \frac{\cos 2x}{\cos^2 x sin^2 x}dx$

13.6 $\int \sqrt{1-\cos 2x}dx$

13.7$\int \frac{\sqrt{1+x^2}-3\sqrt{1=x^2}}{\sqrt{1-x^4}}dx$

13.8 $\int \cos(5x-2)dx$

13.9 $\int \cos^2 x dx$

13.10 $\int (15x-18)^6 dx$

13.11 $\int \frac{dx}{\sin^2(x-3)}$

13.12 $\int \frac{\sin x}{\cos^4 x}dx$

13.13 $\int \frac{\cos^2 x}{\sin^4 x}dx$

13.14 $\int \cot x dx$

13.15 $\int \frac{1}{\cos x}dx$

13.16 $\int \frac{dx}{x\sqrt{4+\ln x}}$

13.17 $\int \frac{4x-\text{arctg}^3 5x}{1+25x^2}dx$

13.18 $\int \frac{dx}{x\ln^3 x}$

13.19 $\int \frac{\cos x}{1+3\cos^2 x}dx$

13.20 $\int \frac{\tan x+7}{\cos^2 x}dx$

13.21 $\int \frac{dx}{x\ln^3 x}$

13.22 $\int \frac{dx}{7+3x^2}$

13.23 $\int \frac{4x-\text{arctg}^3 5x}{1+25x^2}dx$

13.24 $\int \frac{\sin x-\cos x}{\sqrt[3]{\sin x+\cos x}}dx$

13.25 $\int \frac{dx}{x^2\tan(1/x)}$

14. Задача: Да се пресметнат следните неопределени интеграли:

14.1 $\int 3x\sin x dx$

14.2 $\int x^2\ln^2 x dx$

14.3 $\int (4x^2+8x-7)e^{2x}dx$

14.4 $\int \arcsin x dx$

14.5 $\int x^3 \operatorname{arctg} x dx$

14.6 $\int (2x+1)\sin 3x dx$

14.7 $\int x^2 \ln \frac{1-x}{1+x} dx$

14.8 $\int e^{3x} \cos 4x dx$

14.9 $\int \sqrt{9-x^2} dx$

14.10 $\int \frac{5x+4}{x^2+2x+5} dx$

14.11 $\int \frac{6-8x}{2x^2-3x+1} dx$

14.12 $\int \frac{x}{(x+1)(x-3)} dx$

14.13 $\int \frac{dx}{(x-2)(x-3)(x-4)}$

14.14 $\int \frac{dx}{x^3-1}$

14.15 $\int \frac{1}{(x-1)(x^2-x+3)} dx$

14.16 $\int \frac{x^2+1}{(x-1)(x+1)^2} dx$

14.17 $\int \frac{x}{x^3-1} dx$

14.18 $\int \frac{x^5+x^4-8}{x^3-4x} dx$

15. Задача: Да се пресметнат следните неопределени интеграли:

15.1 $\displaystyle\int \frac{\sin 2x}{1+\sin^2 x} dx$

15.2 $\displaystyle\int \frac{\cos^3 x}{\sin x - 3} dx$

15.3 $\displaystyle\int \frac{dx}{\sin^4 x + \cos^4 x}$

15.4 $\displaystyle\int \frac{x}{\sqrt{x+9}} dx$

15.5 $\displaystyle\int \frac{1}{x}\sqrt{\frac{1-x}{1+x}} dx$

15.6 $\displaystyle\int \frac{\sqrt{x^2-4}}{x^2} dx$

15.7 $\int \frac{dx}{x^2\sqrt[3]{(1+x^3)^5}}$

15.8 $\int \sqrt[3]{x}\sqrt[7]{1+\sqrt[3]{x^4}}dx$

16. Задача: Да се пресметнат следните определени интеграли:

16.1 $\int_1^{\sqrt{3}} \frac{1}{1+x^2}dx$

16.2 $\int_{-2}^{-1} \frac{1}{(5+2x)^4}dx$

16.3 $\int_{-1}^{0} \frac{dx}{\sqrt{4-3x}}$

16.4 $\int_0^1 \frac{\operatorname{arctg} x}{1+x^2}dx$

16.5 $\int_1^e \frac{\ln^4 x}{x}dx$

16.6 $\int_{-1}^{1} x(1-x)^{100}dx$

16.7 $\int_{-1}^{1} |x|dx$

16.8 $\int_0^1 x.\operatorname{arctg} dx$

16.9 $\int_0^{\frac{\pi}{2}} 2x\cos x dx$

16.10 $\int_{-\pi}^{\pi} x\sin x dx$

16.11 $\int_{-1}^{0} (2x+3)e^{-x}dx$

16.12 $\int_{-5}^{-1} \frac{dx}{x^2+6x+13}$