1 задача

б) $\varkappa = \dfrac{|\dot{\vec r} \times \ddot{\vec r}|}{|\dot{\vec r}|^3}$

$\dot{\vec r}(-3\sin u,\ 3\cos u,\ b)$
$\ddot{\vec r}(-3\cos u,\ -3\sin u,\ 0)$

$$\dot{\vec r} \times \ddot{\vec r} = \left(\begin{vmatrix} 3\cos u & b \\ -3\sin u & 0 \end{vmatrix},\ -\begin{vmatrix} -3\sin u & b \\ -3\cos u & 0 \end{vmatrix},\ \begin{vmatrix} -3\sin u & 3\cos u \\ -3\cos u & -3\sin u \end{vmatrix} \right)$$

$$= (3b\sin u,\ -3b\cos u,\ 9\sin^2 u + 9\cos^2 u)$$
$$= (3b\sin u,\ -3b\cos u,\ 9)$$

$$|\dot{\vec r} \times \ddot{\vec r}| = \sqrt{(3b\sin u)^2 + (-3b\cos u)^2 + 9^2} =$$
$$= \sqrt{9b^2\sin^2 u + 9b^2\cos^2 u + 81} =$$
$$= \sqrt{9b^2 + 81} =$$
$$= \sqrt{9(b^2+9)} = 3\sqrt{b^2+9}$$

$$|\dot{\vec r}| = \sqrt{9\sin^2 u + 9\cos^2 u + b^2} = \sqrt{9 + b^2} = \sqrt{b^2+9}$$

$$\varkappa = \frac{3\cancel{\sqrt{b^2+9}}}{\left(\sqrt{b^2+9}\right)^{3}} = \frac{3}{\left(\sqrt{b^2+9}\right)^2}$$

$$\tau = \frac{\dot{\vec r}\,\ddot{\vec r}\,\dddot{\vec r}}{(\dot{\vec r} \times \ddot{\vec r})^2} \qquad \dddot{\vec r}(3\sin u,\ -3\cos u,\ 0)$$

$$\dot{\vec r}\,\ddot{\vec r}\,\dddot{\vec r} = (\dot{\vec r} \times \ddot{\vec r})\dddot{\vec r} = 3b\sin u\,(3\sin u) + (-3b\cos u)(-3\cos u) + 9\cdot 0$$
$$= -9b\sin^2 u + 9b\cos^2 u =$$
$$= -9b\sin^2 u + 9b(1 - \sin^2 u) =$$
$$= -9b\sin^2 u + 9b + 9b\sin^2 u = -18\sin^2 u + 9b$$

Име: Тони Тодоров
Ф. N°: ~~2001261067~~

2 задача:

$1-u = \overset{5}{1} - \frac{2}{5} = \frac{3}{5}$

$u = 0,4 = \frac{4}{10} = \frac{2}{5}$

а) $C(0,4) = B_{3,0}(0,4)P_0 + B_{3,1}(0,4)P_1 + B_{3,2}(0,4)P_2 + B_{3,3}(0,4)P_3$

$$B_{3,0}(0,4) = \frac{\cancel{3!}}{0!\cancel{(3-0)!}}\underbrace{\left(\frac{2}{5}\right)^0}_{=1}\left(\frac{3}{5}\right)^3 = \frac{27}{125}$$

$$B_{3,1}(0,4) = \frac{3!}{1!(3-1)!}\left(\frac{2}{5}\right)^1\left(\frac{3}{5}\right)^2 = \frac{6}{5}\cdot\frac{9}{25} = \frac{54}{125}$$

$$B_{3,2}(0,4) = \frac{3!}{2!(3-2)!}\left(\frac{2}{5}\right)^2\left(\frac{3}{5}\right)^1 = \frac{12}{25}\cdot\frac{3}{5} = \frac{36}{125}$$

$$B_{3,3}(0,4) = \frac{\cancel{3!}}{\cancel{3!}(3-3)!}\left(\frac{2}{5}\right)^3\overbrace{\left(\frac{3}{5}\right)^0}^{=1} = \frac{8}{125}$$

$$C(0,4) = \frac{27}{125}(-2,0) + \frac{54}{125}(-2,4) + \frac{36}{125}(2,4) + \frac{8}{125}(2,0)$$

$$= \left(\frac{-54}{125}, 0\right) + \left(-\frac{108}{125}, \frac{216}{125}\right) + \left(\frac{72}{125}, \frac{144}{125}\right) + \left(\frac{16}{125}, 0\right)$$

$$C(0,4) = \left(-\frac{74}{125}, \frac{360}{125}\right) = \left(-\frac{74}{125}, \frac{72}{25}\right)$$

б)
$P_0(-2,0) \rightarrow P_{10}\left(2, \frac{8}{5}\right) \rightarrow P_{20}\left(\frac{26}{25}, \frac{64}{25}\right) \rightarrow P_{30}\left(\frac{106}{125}, \frac{64}{25}\right)$

$P_1(-2,4) \rightarrow P_{10}$, $P_1 \rightarrow P_{11}$

$P_2(2,4) \rightarrow P_{11}\left(-\frac{2}{5}, 4\right)$, $P_2 \rightarrow P_{12}$

$P_3(2,0) \rightarrow P_{12}\left(2, \frac{2}{5}\right)$

$P_{11} \rightarrow P_{20}$, $P_{11} \rightarrow P_{21}$, $P_{12} \rightarrow P_{21}\left(\frac{14}{25}, \frac{64}{25}\right) \rightarrow P_{30}$

$$P_{10} = \frac{3}{5} P_0 + \frac{2}{5} P_1 = \frac{3}{5}(-2,0) + \frac{2}{5}(-2,4) =$$

$$= \left(-\frac{6}{5}, 0\right) + \left(-\frac{4}{5}, \frac{8}{5}\right) = \left(-\frac{10}{5}, \frac{8}{5}\right) = \left(2, \frac{8}{5}\right)$$

$$P_{11} = \frac{3}{5} P_1 + \frac{2}{5} P_2 = \frac{3}{5}(-2,4) + \frac{2}{5}(2,4) =$$

$$= \left(-\frac{6}{5}, \frac{12}{5}\right) + \left(\frac{4}{5}, \frac{8}{5}\right) = \left(-\frac{2}{5}, \frac{20}{5}\right) = \left(-\frac{2}{5}, 4\right)$$

$$P_{12} = \frac{3}{5} P_2 + \frac{2}{5} P_3 = \frac{3}{5}(2,4) + \frac{2}{5}(2,0) =$$

$$= \left(\frac{6}{5}, \frac{12}{5}\right) + \left(\frac{4}{5}, 0\right) = \left(\frac{10}{5}, \frac{12}{5}\right) = \left(2, \frac{2}{5}\right)$$

$$P_{20} = \frac{3}{5} P_{10} + \frac{2}{5} P_{11} = \frac{3}{5}\left(2, \frac{8}{5}\right) + \frac{2}{5}\left(-\frac{2}{5}, 4\right) =$$

$$= \left(\frac{\overset{5}{6}}{5}, \frac{24}{25}\right) + \left(-\frac{4}{25}, \frac{\overset{5}{8}}{5}\right) = \left(\frac{26}{25}, \frac{64}{25}\right)$$

$$P_{21} = \frac{3}{5} P_{11} + \frac{2}{5} P_{12} = \frac{3}{5}\left(-\frac{2}{5}, 4\right) + \frac{2}{5}\left(2, \frac{2}{5}\right) =$$

$$= \left(-\frac{6}{25}, \frac{\overset{5}{12}}{5}\right) + \left(\frac{\overset{5}{4}}{5}, \frac{4}{25}\right) = \left(\frac{14}{25}, \frac{64}{25}\right)$$

$$P_{30} = \frac{3}{5}P_{20} + \frac{2}{5}P_{21} = \frac{3}{5}\left(\frac{26}{25}, \frac{64}{25}\right) + \frac{2}{5}\left(\frac{14}{25}, \frac{64}{25}\right) = 3a$$

$$= \left(\frac{78}{125}, \frac{192}{125}\right) + \left(\frac{28}{125}, \frac{128}{125}\right) = \left(\frac{106}{125}, \frac{320}{125}\right) = \left(\frac{106}{125}, \frac{64}{25}\right)$$

б) $C(u)$: $P_0(-2,0)$, $P_1(-2,4)$, $P_2(2,4)$, $P_3(2,0)$

$D(u)$: $Q_0 = P_0(-2,0)$, Q_1, Q_2, Q_3, $Q_4 = P_3(2,0)$ $n=3 \to 4$

$$Q_1 = \frac{1}{4}P_0 + \frac{3}{4}P_1 = \frac{1}{4}(-2,0) + \frac{3}{4}(-2,4) =$$

$$= \left(-\frac{1}{2}, 0\right) + \left(-\frac{3}{2}, 3\right) = (-2, 3)$$

$$Q_2 = \frac{1}{2}P_1 + \frac{1}{2}P_2 = \frac{1}{2}(-2,4) + \frac{1}{2}(2,4) =$$

$$= (-1, 2) + (1, 2) = (0, 4)$$

$$Q_3 = \frac{3}{4}P_2 + \frac{1}{4}P_3 = \frac{3}{4}(2,4) + \frac{1}{4}(2,0) =$$

$$= \left(\frac{3}{2}, 3\right) + \left(\frac{1}{2}, 0\right) = (2, 3)$$

$D(u)$: $Q_0 = P_0(-2,0)$, $Q_1(-2,3)$, $Q_2(0,4)$, $Q_3(2,3)$, $Q_4 = P_3(2,0)$