Topic 6: Sequence Alignment II

ATALVALNIN ALVALLN

ATALVALNIN A--LVALL-N

ATALVAL-NIN
--ALVALLN--

Sequence Alignment

The sequence alignment problem:

- we have a pair of strings (sequences)
- we want to find the best way to "match" the two

Examples:

- leaf and please
- ATCCA and TCGA
- MPPDNE and PADQE

Sequence Alignment

Dot plots

	l	е	a	f
p				
p l				
е				
a				
S				
е				

	Т	С	G	Α
Α				
Т				
C				
C				
Α				

Sequence Dot Plots

	l	е	a	f
p				
p l	*			
е		*		
a			*	
S				
е		*		

	Т	С	G	Α
Α				*
Т	*			
C		*		
C		*		
Α				*

Sequence Dot Plots + windowing

	1	е	a	f
p				
l	*			
е		*		
a			*	
S				
е		*		

	Т	C	G	Α
Α				*
Т	*			
C		*		
C		*		
Α				*

Sequence Alignment

The sequence alignment problem:

- you have 2 strings (sequences)
- find the "best match" between the two

Ex: GCCAT and GAAT

GCCAT

+000

GAAT

A + means a match
A o means a mismatch

This alignment is probably not the best.

Sequence Alignment

Ex: GCCAT and GAAT

GCCAT +000 GAAT A + means a match
A o means a mismatch

GCCAT
oo++
GAAT

This alignment is better.

Can we do even better if we allow gaps?

Sequence Alignment with gaps

GCCAT

+-0++

G AAT

A + means a match

A o means a mismatch

A – is a gap _

GCCAT

+0-++

GA AT

GCCAT

+00-+

GAA T

GCCA T

+--+-+

G AAT

What we want is the best - an "optimal" - alignment

Sequence Alignment Algorithm

GCCAT

00++

GAAT

We want the *optimal alignment* of two strings.

Can we go through all the possible alignments and pick the best one?

If both sequences are N bases long, and gaps are not allowed, the best techniques take cN^2 time steps (c is a constant)

Optimal Alignment w/o gaps

Optimal alignment: If both sequences are N bases long, and gaps are not allowed, the best techniques take cN^2 time steps.

For large values of N, the "c" is not important and we say that the ungapped alignment has a time complexity of N².

Optimal Alignment with gaps

Optimal alignment: For sequences that are N bases long when gaps are allowed, the best techniques have a "time complexity" of N^{4N} .

As N becomes large, the time needed to find the best alignment becomes very large very quickly!

Finding the optimal sequence alignment problem is a hard problem to solve – so hard that we abandon our quest for the best and settle for something that is "good enough".

"Acceptable" Alignment with gaps

The optimal sequence alignment problem is hard: a "brute-force" search will take too long as sequence lengths increase.

Instead, we use methods that can give us reasonable – good enough - alignments but not necessarily the "optimal" one.

There are many such "heuristics" in computer science.

Traveling Salesman

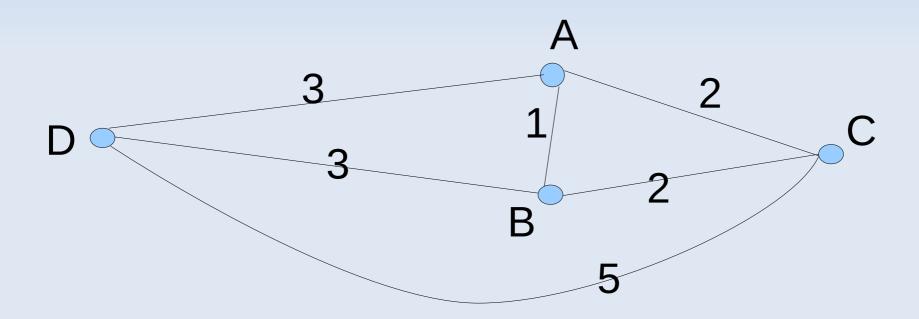
Another problem that is hard to solve is the Traveling Salesman problem:

A salesperson wants to visit a number of cities, A-D, returning to the starting city having been to each of the other cities only once; what route will cost him/her the least?

[&]quot;Cost" can be time, money, etc.

Traveling Salesman

Try a heuristic: start at a city, find the nearest city, and repeatedly visit the closest next city that we have not yet seen, ending back in the starting city



This heuristic is called a "Greedy" algorithm

Greedy Traveling Salesman

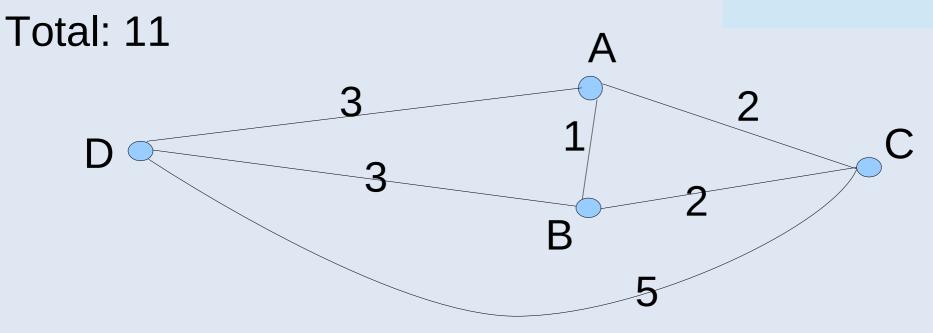
Start with at A, find the nearest city \rightarrow B; cost = 1

closest next city \rightarrow C; cost = 2

closest next one \rightarrow D; cost = 5

And back to A; cost = 3

"Greedy" method: Find the nearest neighb



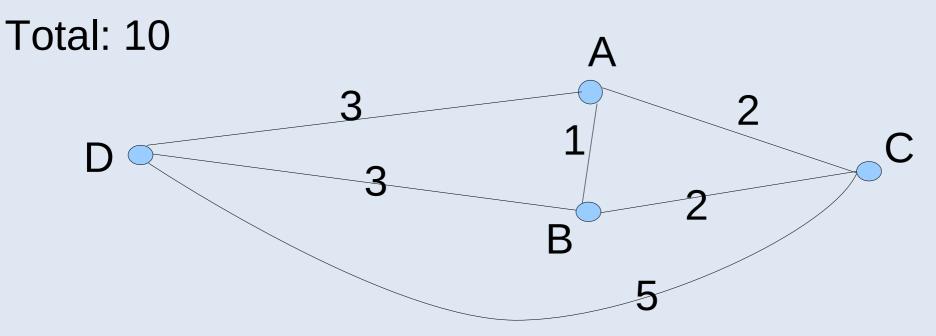
Optimal Solution Traveling Salesman

Start from A, go to D; cost = 3

next city \rightarrow B; cost = 3

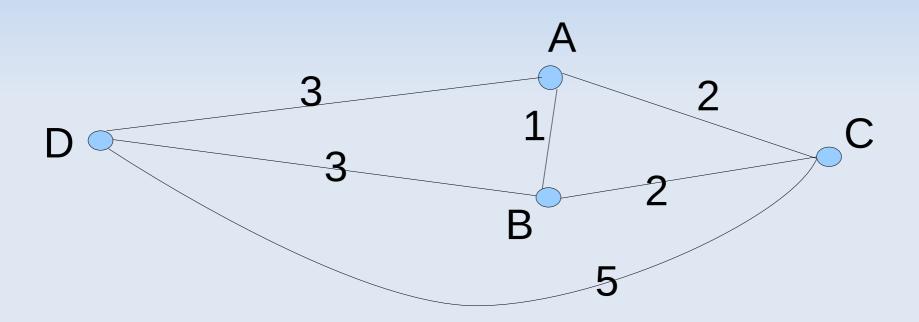
next one \rightarrow C; cost = 2

And back to A; cost = 2



What makes the Traveling Salesman Problem difficult

Knowing the optimal solution for N-1 cities (ABC), does not help with the optimal solution for N cities.



→ The greedy solution may not be the optimal one

"Acceptable" Alignment with gaps

We want a reasonably good alignment – may not be the optimal one

Dynamic Programming Strategy: Try to find an optimal solution by by finding optimal solutions to smaller subproblems.

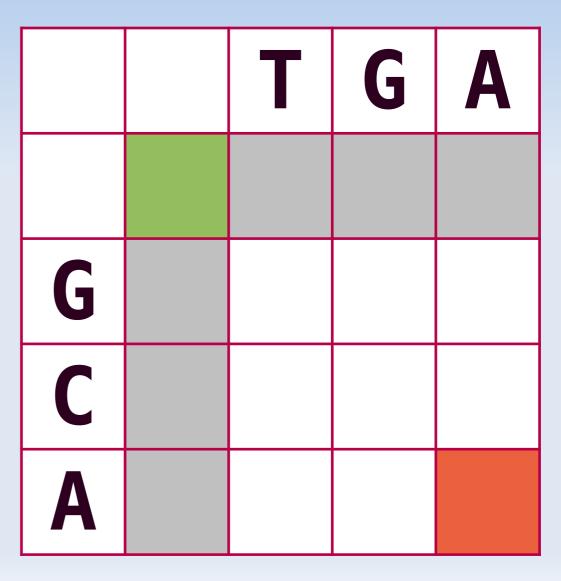
i.e. break a big problem into a bunch of smaller problems and "remember" the best previous solutions.

If a problem satisfies some properties, DP or *greedy* can find the optimal solution.

Dynamic Programming (DP) Dot Plot Approach

We will use a matrix like the one we used for dot plots:

TGA GCA

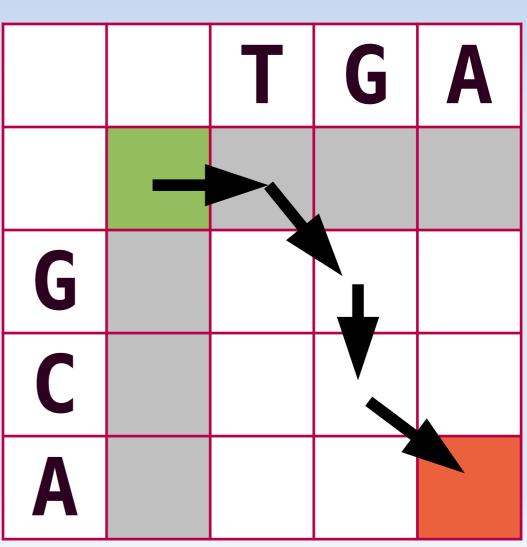


We will end up with a **path** from the upper-left corner to the lower-right corner. This path represents an alignment:

TG_A

- + - +

GCA



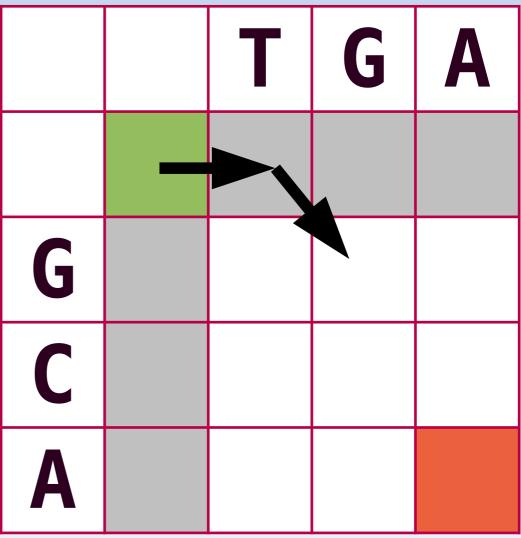
Path through the matrix: a horizontal step along a row is a gap (-) in the seq on the leftmost column

T

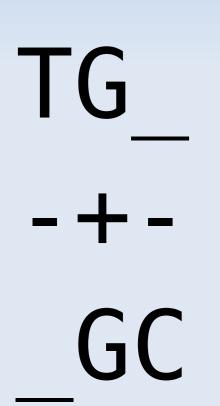
	T	G	A
G			
C			
A			

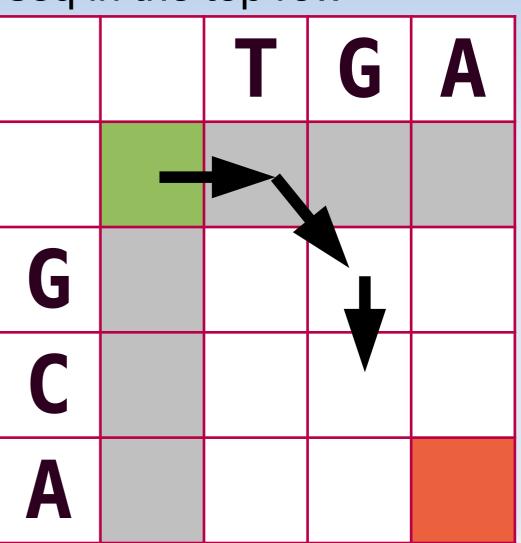
Path through the matrix: a diagonal step is an alignment (+) of one base in each sequence:

TG -+ G



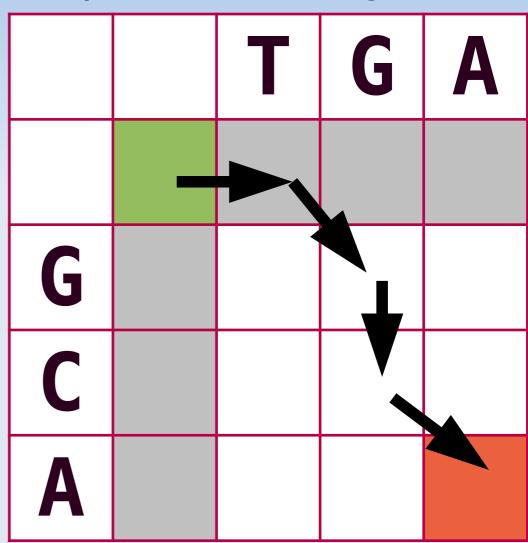
Path through the matrix: a vertical step along a column is a gap (-) in the seq in the top row





The path through the matrix represents an alignment

At each point in the path, we have to choose the best of 3 choices based on the path so far and the



nucleotides in the current row and column.

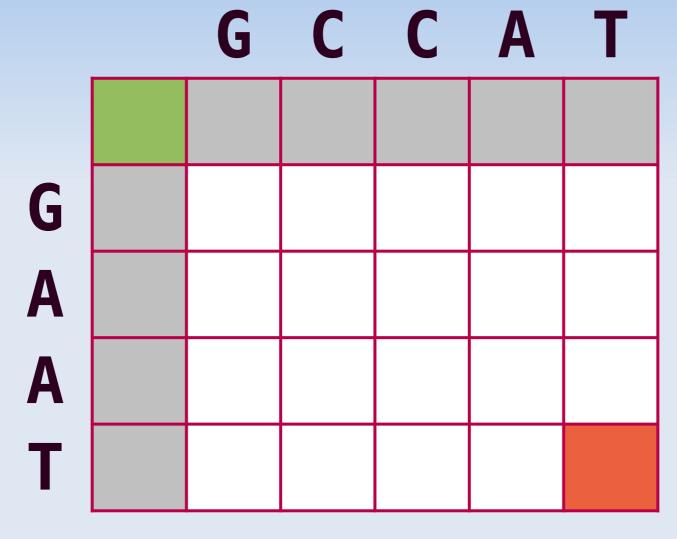
Sequence Alignment Problem

Suppose we have the sequences

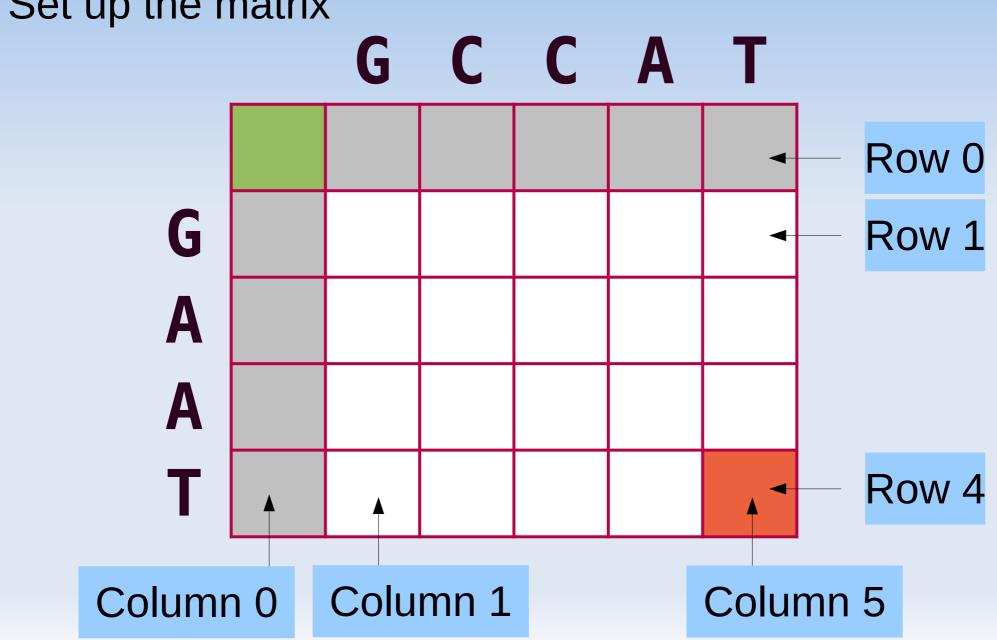
GCCAT GAAT

We want to find a "good" alignment of these two sequences using a Dynamic Programming algorithm

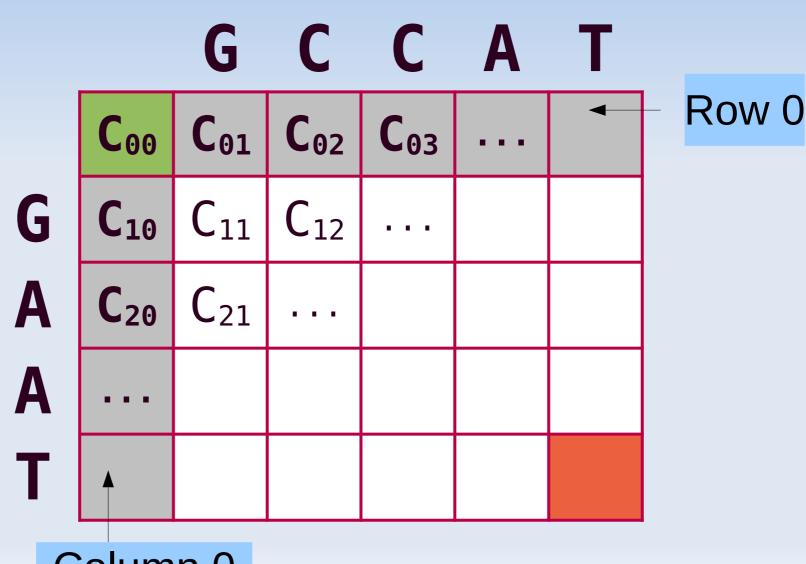
Set up the matrix



Set up the matrix

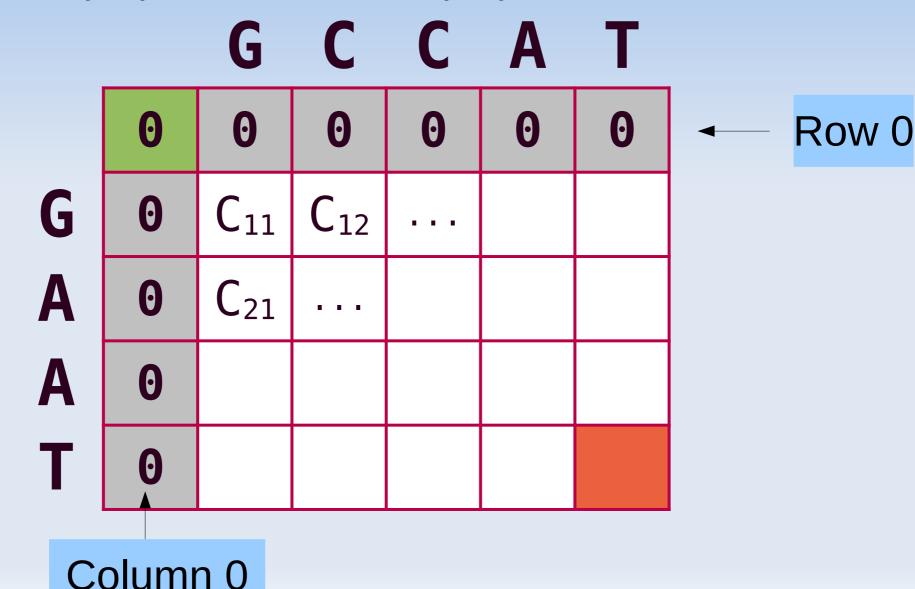


Set up the matrix

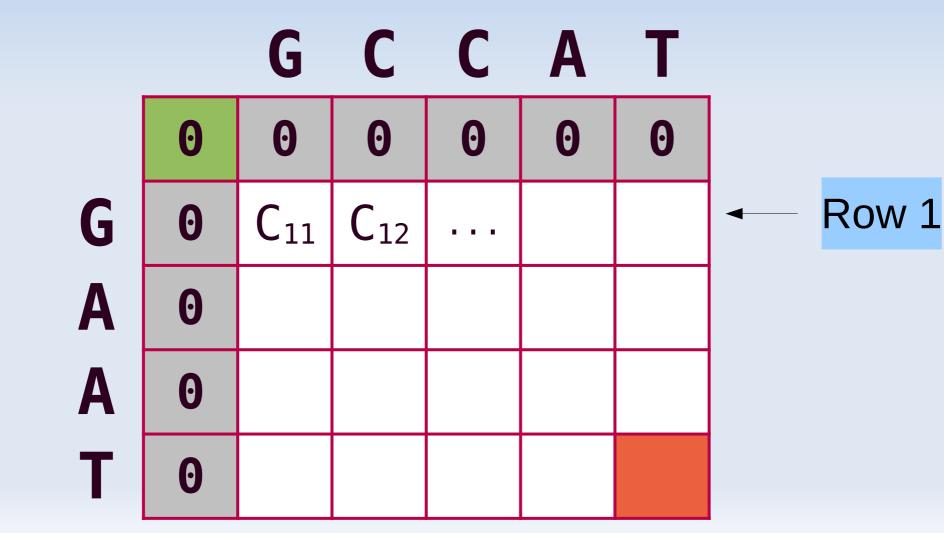


Column 0

Set row 0 (C_{0j}) and column 0 (C_{i0}) elements to 0



Set row 1 (C_{1j}) elements from left to right



DP Path algorithm Fill rule

Fill the matrix based on the cells to the left, above and diagonally up and left

		G	C	C	A	Т
	0	0	0	0	0	0
G	0					
A	0		C _{i-1,j-1}	C _{i-1,j}		
A	0		$C_{i,j-1}$	C_{ij}		
Т	0					

DP Path algorithm Fill rule

Fill the matrix based on the cells to the left, above and

diagonally up-and-left:

C_{ij} set to be the max of three possibilities:

- 1. C_{(i-1)j} 1
- $2. C_{i(i-1)} 1$

Diagonal Vertical gap + match score: score

Horizontal gap score

1 - match

0 -mismatch

Cell C_{ij} set to max of the 3. Remember source of max.

3. $C_{(i-1)(j-1)}$ + 1 for a match in C_{ij} or 0 for a mismatch in C_{ij}

DP Path algorithm Fill rule

Example:

	4	5
Т	3	4 4 2 ?

4 5 3 5 4 2 ?

Gap scores

Vertical

 Horizonta 	
-------------------------------	--

$$3 - 1$$

$$5 - 1$$

$$3 - 1$$

$$5 - 1$$

$$4 + 1$$

Set row 1 (C_{1j}) elements from left to right:

to the max of C_{0j} - 1, C_{1j-1} - 1, or C_{0j-1} + (1 if C_{1j} = match)

For C₁₁, the

3 values are

$$C_{01}$$
- 1 = -1

$$C_{10}$$
- 1 = -1

$$C_{00} + 1 = 1$$

A

A

T

	G			A	
0	0	0	0	0	0
0	C ₁₁	C ₁₂			
0					
0					
0					

For C_{12} , what are the 3 values?

$$C_{02}$$
- 1 = ?

$$C_{11}$$
- 1 = ?

$$C_{01}$$
+?=?

$$Max = ?$$

0	0	0	0	0
0	1	C ₁₂		
0				
0				
0				

G C C A T

U	
A	\
A	\
T	

Finish the row from left to right:

		G	C	C	A	Т
	0	0	0	0	0	0
G	0	1	0	0	0	0
Α	0					
Α	0					
T	0					

DP Path algorithm Step 4

Similar step for row 2 from left to right:

		G	C	C	A	Т
	0	0	0	0	0	0
G	0	1	0	0	0	0
Α	0	0	1	0	1	0
Α	0					
T	0					

DP Path algorithm Step 5

Row 3 from left to right:

		G	C	C	A	Т
	0	0	0	0	0	0
G	0	1	0	0	0	0
Α	0	0	1	0	1	0
Α	0	0	0	1	1	1
T	0					

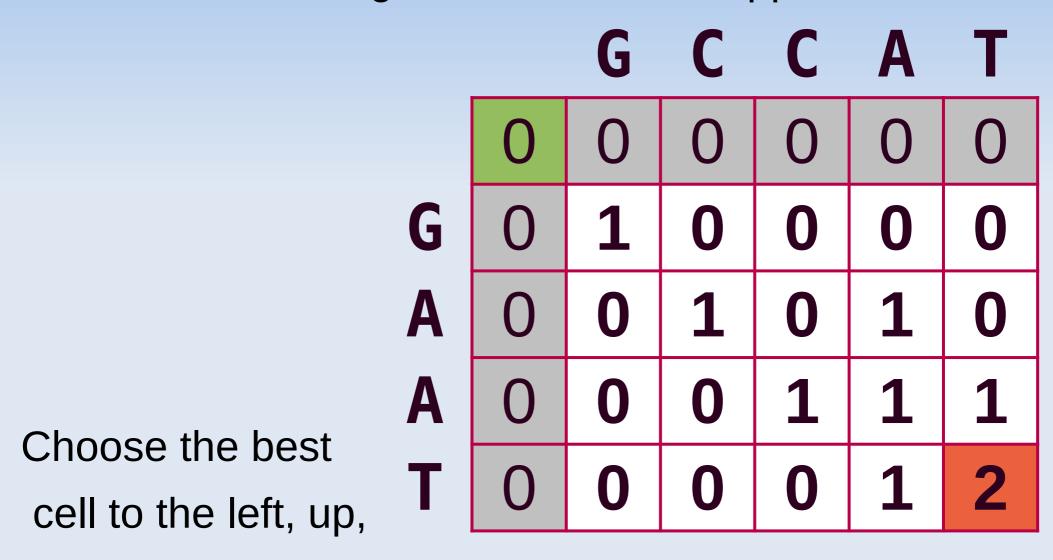
DP Path algorithm Step 6

Row 4 from left to right:

		G	C	C	Α	Т
	0	0	0	0	0	0
G	0	1	0	0	0	0
A	0	0	1	0	1	0
A	0	0	0	1	1	1
T	0	0	0	0	1	2

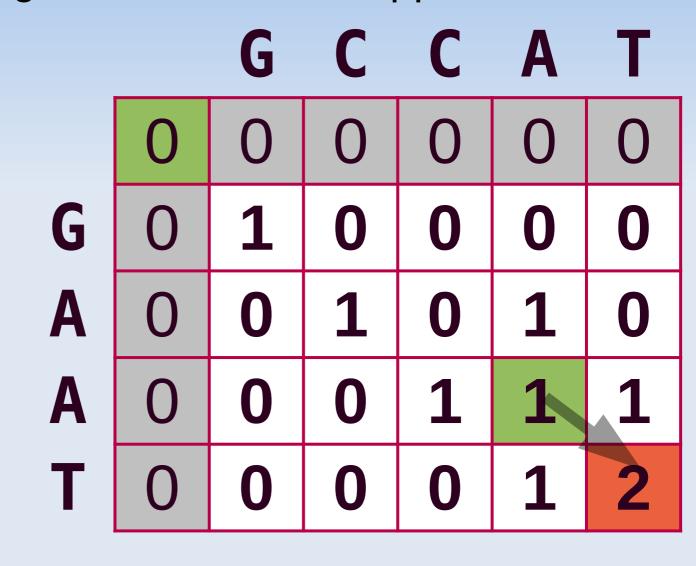
Done with the "Fill" phase

Work from lower-right corner back to upper-left



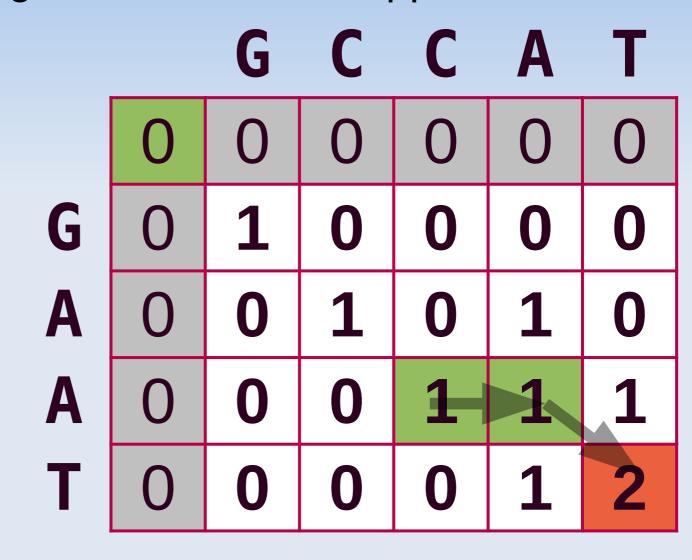
and diagonally; in this case all are $1 \rightarrow \text{try}$ all 3!

Work from lower-right corner back to upper-left



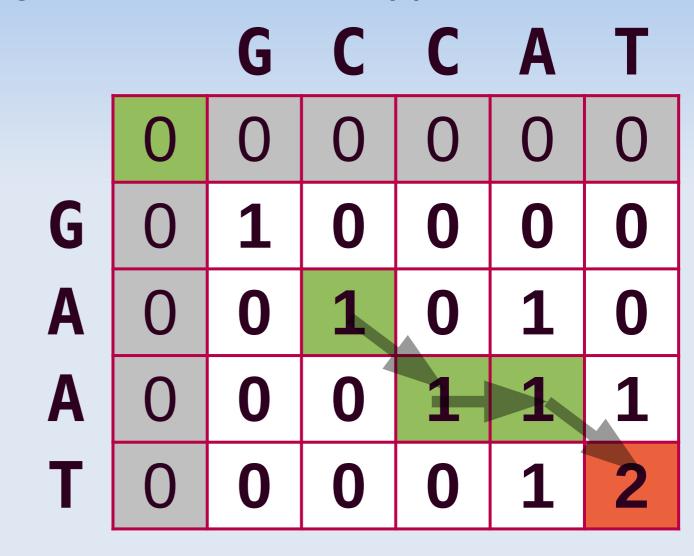
Turns out the diagonal one is best

Work from lower-right corner back to upper-left



Two possibilities turns out the left one is best

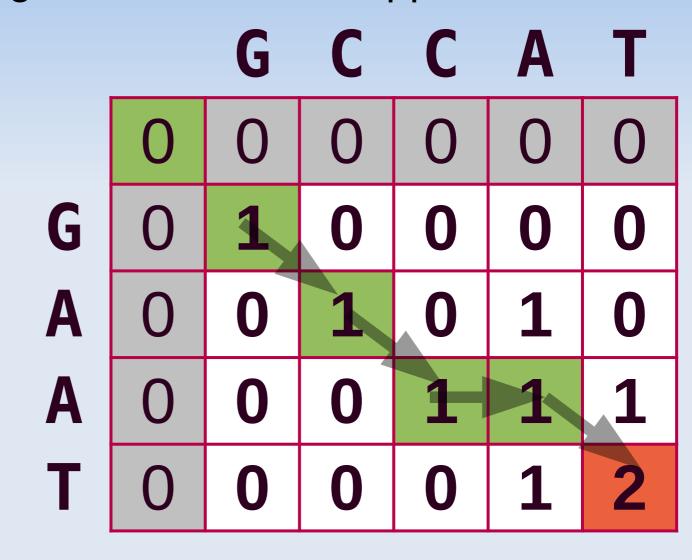
Work from lower-right corner back to upper-left



One best choice

- diagonal

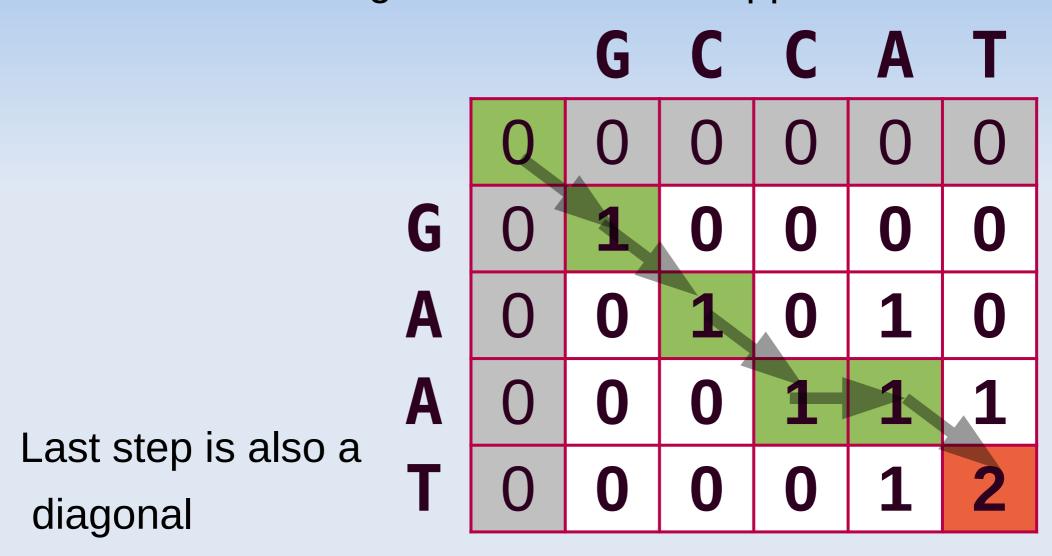
Work from lower-right corner back to upper-left



One best choice

- diagonal

Work from lower-right corner back to upper-left



DP Path algorithm - Align

Resulting path gives us this alignment:

			G	C	C	A	Т
GCCAT		0	0	0	0	0	0
+00-+	G	0	1	0	0	0	0
GAA T	A	0	0	1	0	1	0
<u> </u>	A	0	0	0	1	1	1
Probably not	T	0	0	0	0	1	2

optimal: one gap, two mismatches

DP Path alternative Step 2

Set row 0 (C_{0j}) and column 0 (C_{i0}) elements to penalize origination gaps

		G	C	C	Α	Τ
	0	-1	-2	-3	-4	-5
G	-1	C ₁₁	C ₁₂			
Α	-2	C ₂₁				
Α	-3					

Sequence Alignment Problem

The dynamic programming approach attempts to maximize the score of an alignment.

An alignment score depends on the scoring matrix and gap penalty – these can be combined into one if the gap penalty is a constant: A T C G - A

A 1 0 0 0 -1

T 0 1 0 0 -1

C 0 0 1 0 -1

G 0 0 0 1 -1

- -1 -1 -1 0

We can also use an "affine gap penalty": one penalty for starting a

gap and an additional penalty for continuing it.

Sequence Alignment - Alternative

We could also phrase the sequence alignment problem as the **minimum distance** between two sequences – the Edit Distance approach.

Not considered here but very interesting approach.

Sequence Alignment – Big Picture

We are trying to align sequences to see how closely related they may be: "sequence similarity".

Percent identity is a good measure of how close 2 sequences are. We may also want to measure the differences.

Similar sequences are usually related due to a common ancestor.

The main forces that changes sequences are: mutations, genetic drift, and natural selection

Homologous Sequences

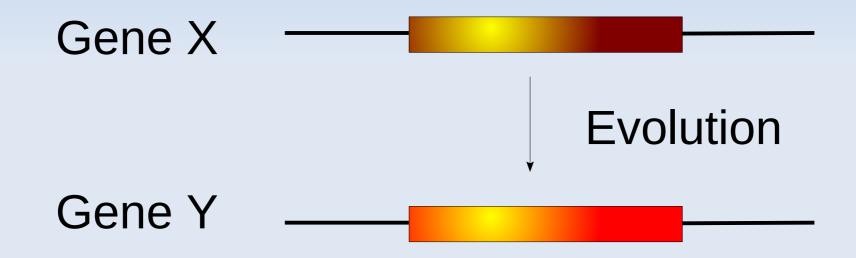
Sequences with significant similarity are said to be homologous

Homologs – two or more sequences possibly from different species and related by descent from a single common ancestral sequence.

Homologs can be orthologous or paralogous.

Orthologs

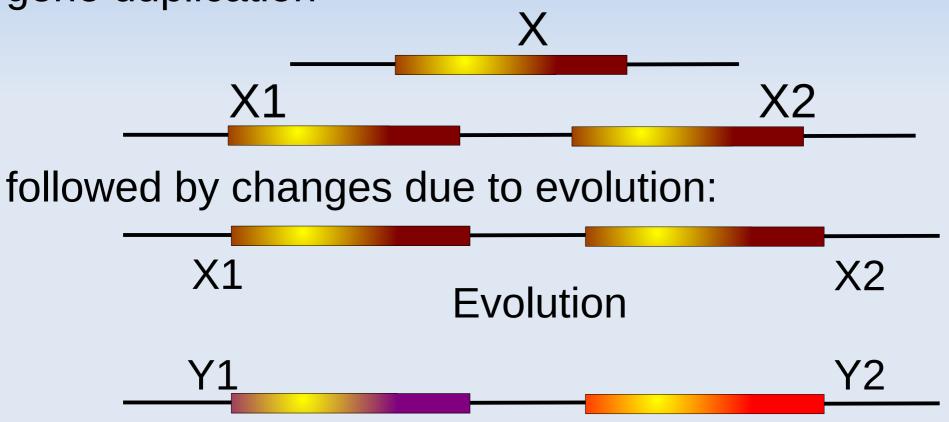
Homologous genes with the same function in two different species, evolved from a common ancestral gene by speciation



Likely to have similar protein sequence → similar structure → similar biological function

Paralogs

Homologous genes with similar function in two species, evolved from a common ancestral gene by gene duplication



E.g.: Apple MYB genes MYB10 and MYB110a appear to be paralogs that control fruit color. The Apple plant likely had a whole genome duplication event ~65 mya (Velasco 2010)

Mutations

Mutations in genomic DNA sequences occur when proteins responsible for copying DNA make mistakes. The three main kinds of mutations are:

Substitutions – one base is replaced by another Insertions – a base is inserted into a sequence Deletion – a base is deleted from a sequence

Substitution Mutations

Substitutions can occur in 3 ways. Eg:

GCCAT GCCAT GCCAT

GCGAT GCAAT GCTAT

Adenine and guanine are purines

Cytosine and thymine are pyramidines

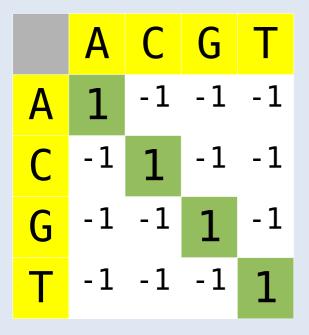
A **transition** substitution is a replacement of a purine by a purine or a pyramidine by a pyramidine

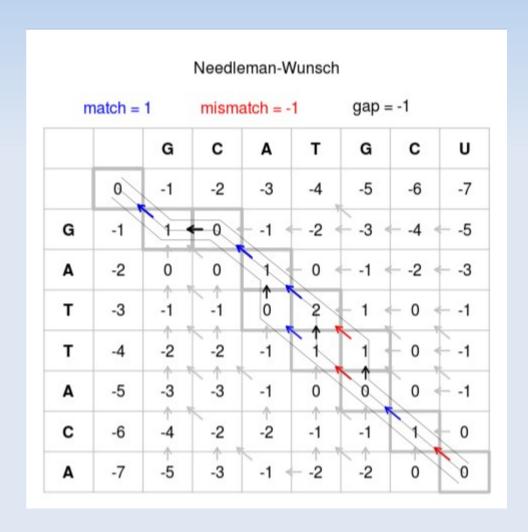
A **transversion** is a replacement of a purine by a pyramidine or a pyramidine by a purine.

Kinds of Alignments

Global alignment – Needleman-Wunsch algorithm

Scoring matrix:





Kinds of Alignments

Local alignment – **Smith-Waterman** algorithm, a variation of Needleman-Wunsch:

- Replace negative scores with zero
- Starts the traceback at the highest scoring cell and ends at a 0 → highest scoring local alignment

-	-	A	T	С	G	A	A
177	0	0	0	0	0	0	0
C	0	0	0	5 _	$\rightarrow 1$	0	0
A	0	5	→ 1	1 🔻	2	5	5
T	0	1 🔻	10	→ 6 -	→ 2	1 🔻	2
A	0	5	6	7 -	⇒ 3	7	6
С	0	1	2 🔻	11 -	→ 7 –	→ 3 🔻	4

Protein Alignments

Protein alignments are the same except that the scoring matrix is a 20 by 20 matrix and may also have weights based on physicochemical and biological properties of amino acids:

- Cys/Pro are important for structure and function
- Trp has a large side chain
- Lys/Arg are positively charged

Which residues can substitute for another without affecting protein function:

- Ile/Val are small and hydrophobic
- Ser/Thr are polar

BLAST

 A BLAST alignment is like the Smith-Waterman (SW) alignment

 SW is "too slow" → BLAST uses an algorithm that is 50X faster

 BLAST alignments are close to SW but may not be as good as SW

Outline of BLAST

- 1. Remove low-complexity areas of query
- 2. Divide query up into words and for each word
 - i. find good matches in database sequences
 - ii. create a search tree from high-scoring words
 - iii. extend matches to high-scoring segment pairs
- 3. List the best HSPs
 - Compute the E-values of HSPs
- 4. Combine HSPs into a longer alignment
- Show Smith-Waterman local alignments of query with database sequences

Remove low-complexity areas of query:

Replace repetitive sequences like "ATATATA" with things like "NNNNNN" or "XXXXXXXX"

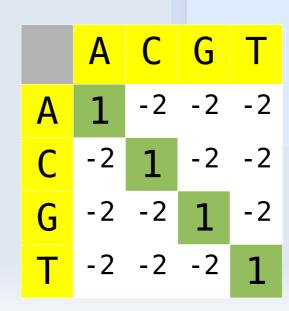
Code		Complement
А	Adenine	Т
С	Cytosine	G
G	Guanine	С
Т	Thymine	А
Υ	Pyrimidine (C,T)	R
R	Purine (A,G)	Υ
W	Weak (A,T)	W
S	Strong (G,C)	S
K	Keto (T,G)	M
M	Amino (A,C)	K
D	A, G, T	Н
V	A, C, G	В
Н	A, C, T	D
В	C, G, T	V
X/N	any	X/N
-	Gap	-

Divide query up into words

A "word" is a number of consecutive letters

Default nucleotide word size is 28 for megablast

Note: scoring matrix is



BLAST® Basic Local Alignment Search Tool Recent Results Saved Strategies Help CBI/ BLAST/ blastn suite/ Formatting Results - ARX4JHR9014 Edit and Resubmit Save Search Strategies ▶ Formatting options ▶ Download Nucleotide Sequence (596 letters) ARX4JHR9014 (Expires on 01-31 06:24 am) Query ID |cl|Query 16287 **Database Name Description** None Description Molecule type nucleic acid Program Query Length 596

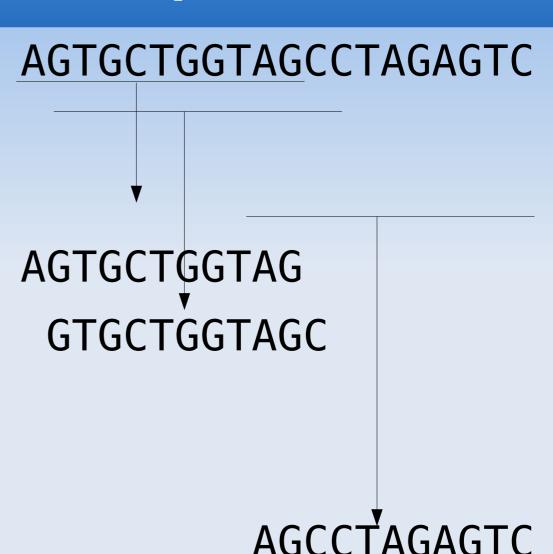
Other reports: ▼ Search Summary [Taxonomy reports] [Distance tree of results] [Genome view]

Si	earch Parameters
Program	blastn
Word size	28
Expect value	10
Hitlist size	100
Match/Mismatch scores	1,-2
Gapcosts	0,0
Low Complexity Filter	Yes
Filter string	L;R -d repeatmasker/repeat_9606;m;
Genetic Code	1

Divide query up into words

Default nucleotide word size can be 11 (blastn) to 28 (megablast)

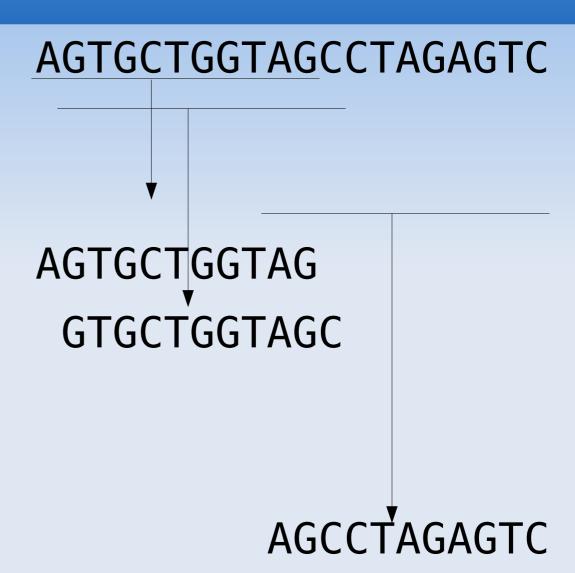
Protein words are smaller



There are 4¹¹ (> 4 million) possibile 11-nucleotide words

How many for a 28 nucleotide word?

Which is likely to have more exact matches in sequences in a database? A 11nt word or a 28nt word?



Find all common words between the query and each database sequence

Evaluate (use scoring matrix) word matches and keep those that exceed a threshold

Query: ACGAGATCAGGCACAGGA

Database: ACTAGATCAGTCACAGCA

For each good word match, the alignment is extended until the score drops (below 20 for nucleotides).

Remind you of somehing? Dot-plots with window and stringency?

Protein example of extending matches to Highscoring Segment Pairs

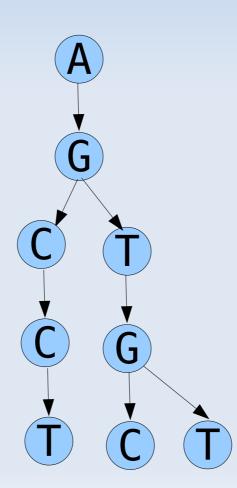
Optimal accumulated score =
$$7+7+2+6+1=23$$

Use an efficient search tree to list the high-scoring words that appear in a potential database sequence:

AGCCT

AGTGC

AGTGT



BLAST step 3: Expect (E) values

- E value meaning: measures randomness of relationship between sequences.
- e.g. E-value of 0.01 for an alignment means there is 1% chance that the two sequences are unrelated.
- < 1e-100 → identical sequences
- **1e-50 to 1e-100** → almost identical sequences
- **1e-10 to 1e-50** → closely related
- 1 to 1e-10 → possible but unlikely to be related
- **10 to 1** → unlikely to be related
- Default threshold for E value is 10 because >10 means alignments are no good

Karlin-Altschul eqn

Expectation value

$$E = kmNe^{-\lambda S}$$

measures randomness of relationship between 2 sequences; depends on sequence composition, length, scoring matrices (coming up)

E ~ number of HSPs purely by chance

k is a constant

m is the number of letters in query

N is the total number of letters in target database

%lambda is a normalizing constant

S is the score of the high-scoring segment pair

Scoring Matrices

An m × n matrix: the m rows are horizontal, the n columns are vertical.

Each element of a matrix is denoted by a variable with two subscripts.

e.g. $a_{2,1}$ is the element in the second row and first column.

Nucleotide Scoring Matrices

A way to rank "similarity" or "homology"
 e.g. nucleotide match/no match, w/o penalty

	Α	Т	С	G
Α	1	0	0	0
Т	0	1	0	0
С	0	0	1	0
G	0	0	0	1

Scoring Matrices

- Example: match/no match, different penalties for purines, pyramidines transitions/transversions
- Transition = purine (A ↔ G) pyramidine (C ↔ T)

	Α	Т	С	G
Α	5	-4	-4	-2
Т	-4	5	-2	-4
С	-4	-2	5	-4
G	-2	-4	-4	5

Proteins: PAM Matrices

"Point Accepted Mutation" or PAM scoring matrix.

- Margaret Dayhoff's group in the 1970s looked at mutations seen in proteins by natural selection:
 - 71 groups of related proteins
 - each group had 85% sequence identity
 - 1572 changes

Wanted to explain small changes in sequences

- PAM-1 ~ 1% divergence ~ 1 amino acid change per 100 residues = a PAM unit of time
- PAM-K predicts changes after K PAM time units

PAM Matrices

Matrix values, $M_{i,j}$ represents the probability of a J \rightarrow I substitution.

$$M_{ij} = \frac{m_j F_{ij}}{\sum_{i} F_{ij}}$$
 $M_{G,A} = \frac{2.8 \times 3}{4}$

The entries of the scoring matrix are the $M_{i,j}$ values divided by the frequency of occurrence - f_i - of residue i.

- e.g. $f_G = 10 \text{ GLY} / 63 \text{ residues} = 0.1587$
- $R_{GA} = \log(2.1/0.1587) = \log(12.760) = 1.106$
- Log-odds matrix
- Diagonal entries are $M_{ij} = 1 m_j$

Log odds matrix values:

- >0 if substitution is frequently seen
- <0 if infrequent

Compute PAM-K Matrices

Assume a Markov chain process:

changes at time T+1 are independent of the changes at time T

$$P(A \rightarrow B) = \sum_{X} P(A \rightarrow X) P(X \rightarrow B)$$

PAM-K = [PAM-K-1] [PAM-1] Matrix multiplication

Small K for closely related sequences

Large K for more distant sequences but Matrix multiplication magnifies errors

PAM-250 is common

Proteins: BLOSUM Matrices

- Used a set of related protein sequences to obtain "blocks".
 - ~2000 blocks from 500 families of related proteins → *more data than PAM*
- A block is the ungapped alignment of a highly conserved region of a family of proteins ~ functional protein "motif".
- BLOck SUBstitutions seen in blocks → BLOSUM matrices

Computing BLOSUM-K

- BLOSUM-K matrix created by weighting the degree of similarity between sequences.
 e.g. BLOSUM-62 is calculated from protein blocks: if two sequences are more than 62% identical, contribution of these sequences is weighted to sum to one.
- Contributions of multiple entries of closely related sequences is reduced.
- Larger numbers used to measure more recent divergence, default is BLOSUM-62

BLOSUM-62

Note these related residues:

M I L V are small hydrophobic residues

N D E Q are acidic and hydrophilic

HRK are basic

F Y W are aromatic

STPAG are small hydrophilic

C is a sulphydryl Lecture 11 has info on amino acids

Matrix values:

- >0 if substitution is frequently seen
- <0 if infrequent

PAM vs. BLOSUM

Approximate equivalences:
 Note: lower BLOSUM means more divergence

```
PAM-100 ~ BLOSUM-90
PAM-120 ~ BLOSUM-80
PAM-160 ~ BLOSUM-60
PAM-200 ~ BLOSUM-52
PAM-250 ~ BLOSUM-45
```

 BLOSUM-62 is a good default to use Both are available in BLAST

Local vs. Global Alignments

Local

Global

- Look for matching local regions
- Good for divergent sequences with some local similarities
- Sequence lengths can be very different
- Example: Smith-Waterman

- Align entire sequences
- Good for sequences related by homology
- Sequence lengths are similar

Example: Needleman-Wunsch