Teaching Statement

Teaching Mathematics as the Art of Thinking

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Teaching, to me, is not the delivery of mathematics but the training of independent thinkers. My teaching philosophy rests on three principles: character before knowledge, motivation before instruction, and ideas before procedures. I want students not only to learn mathematics, but to experience what it feels like to think mathematically — to question, to generalize, to defend an idea, and to discover structure rather than memorize it.

1. Turning philosophy into classroom practice

(1) Motivation before instruction.

Every class I teach begins the same way: the first ten minutes are spent revisiting the previous lecture's key definitions and theorems, followed by one question that frames the purpose of the new material. Students should never feel that a theorem "drops from the sky"; they should feel the need for it before they see its statement. One student later wrote in a mid-semester survey: "Even though this class was required, it became surprisingly fun — I finally knew why we were learning each concept."

(2) Active feedback instead of passive listening.

To avoid the familiar dynamic where only the most confident students speak, I brought a set of green/red YES/NO paddles to class — the kind used in TV game shows. With them, every student can respond instantly to a conceptual question, and I can diagnose misunderstanding before it accumulates. This also built a sense of shared participation, as reflected by the comment: "The paddles are a great way to make the class engaging." Mid-semester surveys, done twice, helped me calibrate pace and clarity in real time rather than waiting for course evaluations at the end.

(3) Ideas before procedures.

In recitations and office hours, I intentionally stop at the moment when the core idea of a proof or computation becomes visible, and ask students to complete the algebra themselves. Instead of giving the solution, I ask: "Which definition is actually doing the work here?", or "If this were false, where would the contradiction appear?" I want students to see how a mathematician thinks, not just how a solution is written.

2. Evidence of effectiveness

The results of these practices were tangible. In Spring 2022, I served as the instructor of record for MATH 1365 (Introduction to Mathematical Reasoning), a full-semester course with 38 students and seven parallel sections. My section achieved the second-highest average score

among all sections. Before the final exam, I organized an all-day series of one-on-one review sessions that students could book in advance. Out of 38 students, 23 signed up — an engagement that revealed how much they valued interactive learning. The energy in the course did not end with the grade sheet: a year and a half later, one student contacted me asking for a master's program recommendation letter, saying he "learned more from this class than any other proof-based course."

These outcomes matter to me not because they are flattering, but because they show that students felt ownership of the material — the one thing that cannot be mandated by syllabus or grading policy.

3. Teaching beyond a single course: mentorship and graduate-level instruction

My teaching experience spans multiple levels and subject areas — from introductory undergraduate courses such as Calculus, Differential Equations, and Mathematical Reasoning, to advanced undergrad and graduate courses including Number theory, Matrix Methods for Data Analysis, Optimization and Complexity, Machine Learning, etc. I have been a TA for four graduate courses, including Analysis I, Optimization and Complexity, Applied Linear Algebra, and Machine Learning and Statistical Learning Theory. During one semester, the office hour for Applied Linear Algebra became so popular that I had to move it from my office to the department lounge — around half the class attended regularly, many staying until the end of the session. It also happened when I held recitation sessions in Calculus and Differential Equations.

Beyond formal courses, I have also mentored students through Northeastern's Directed Reading Program (DRP) on three semesters. Each time, I guided a small group of undergraduates in exploring topics in Algebraic Topology through a student-led reading format: participants took turns presenting sections of the text each week, while I facilitated discussion and helped them connect ideas across chapters. What impressed me most was their intrinsic motivation — the program carried no grades, credits, or external incentives, yet my students prepared and delivered outstanding final presentations to share their learning with peers. The presentations were totally optional to them. These moments reaffirmed for me that genuine curiosity, once nurtured, is the strongest engine of mathematical growth.

I also have long-term mentoring experience beyond formal courses. I tutored a Penn State junior in Analysis and Probability for a semester, and I currently work with three high-school students at international schools in Dubai, who have remained with me for over a year. These experiences strengthened my ability to adjust explanations across age, background, and mathematical maturity.

4. Reflection and continuous improvement

A few students noted on the survey that they wished for more detailed feedback on homework solutions, or earlier announcements. Instead of treating this as criticism, I see it as data. Since then, I have adopted a system where sample solutions come not only with "the right answer," but a short note about why common wrong answers fail — reinforcing the "ideas first" philosophy. I now schedule announcements to auto-release ahead of class, ensuring no one misses logistics.

5. Future teaching vision

Depending on departmental needs, I see two natural directions for my future teaching:

A research-oriented topology seminar that brings external speakers to campus and connects students with the broader applied topology community.

A project-based "Introduction to TDA" course in which students analyze real data sets, present final results, and learn mathematical storytelling in addition to computation.

Both formats integrate students into active mathematical culture — not just as receivers of knowledge, but as contributors.

6. Closing

I am deeply passionate about teaching because it is not a one-way transmission of knowledge, but a dialogue that continually inspires me. In every class, I find myself not only guiding students' thinking but also being challenged and enlightened by theirs. Teaching gives me the most immediate and concrete sense of accomplishment — seeing a student grasp an abstract idea, or witnessing how our interaction shapes both their understanding and my own character as an educator. It also refines how I communicate mathematics: explaining concepts to diverse audiences forces me to reflect on the clarity, value, and accessibility of my research itself. For me, teaching and research are not separate pursuits but two directions of the same inquiry — both rooted in curiosity, precision, and human connection.