# QC Architecture and Electronics (CESE4080) Homework 5

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**There are a lot of files in each exercise folder. Please refer to the files mentioned in the report. Not every file is used, some are just for testing out assumptions.**

## 1 Exercise A

Start from the example in the file HW5_example.m demonstrating a $\pi$ rotation around the X axis of Q1 for an ideal system, i.e. with a very high sampling rate and a large number of bits.

### 1.1 Question 1

First, we are going to evaluate the effect of quantization, i.e. a limited number of bits in the DAC. While keeping the maximum sample rate fsample=1000e9 (i.e. emulating a continuous-time signal), find the minimum number of bits in the DAC (N) for which a fidelity of 99.9% is achieved for a $\pi$-rotation of Q1 and the idle operation on Q2 and Q3.

**Answer:** With the script "HW5_A_1.m", and achieving figure 1 as a result. It is shown clearly that when it is a 4-bit DAC, the fidelity of all 3 qubits are above 99.9%.

This is caused by the quantization process, the more bits there are, the relatively smaller the harmonics (noise) would be compared to desired signal. As shown in figure 2.

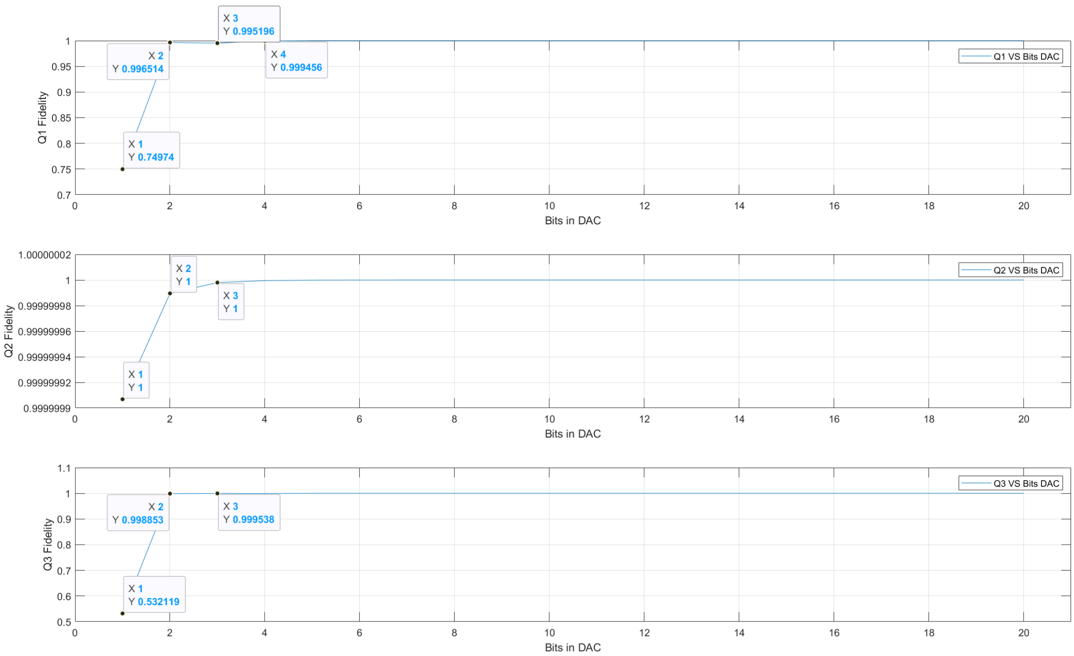



**Figure 1:** Qubits fidelity VS. Bits in DAC

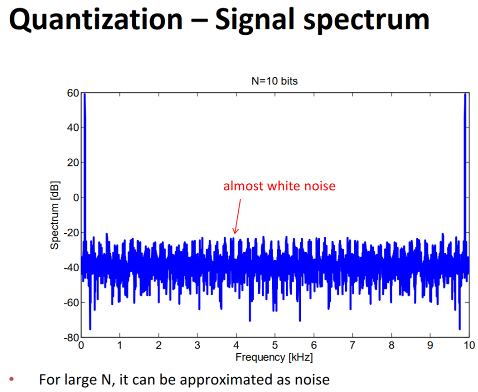



**Figure 2:** Quantization Harmonics

## 2 Exercise B

Set the number of bits of the DAC to 10 (N=10). We are going to evaluate the effect of a limited sample rate. Reduce the DAC sample rate (fsample) down to 10 GHz. If we keep the signal at the input of the DAC as in point A, we can observe a large degradation of the fidelity on Q1. Note: for this point, there are no requirements on the fidelity of Q2 and Q3.

### 2.1 Question 1

Change the phase of the signal at the input of the DAC (ph_in) and the gain G1 to bring back the fidelity of Q1 to above 99.9%.

**Answer:** After adjusting bits of DAC to 10, and the sample rate down to 10Ghz, the fidelity of Q1 is 89.9726%. We first adjust the input phase, and check its relation with fidelity of Q1. In figure 3, it is shown clearly that as the input phase gets shifted to the left (negative x-axis), the fidelity of Q1 first increases till a peak then decreases again. We set $ph\_in = 0.31$ static when tuning the G1, as when $ph\_in = 0.31$ delivers the maximum fidelity on Q1 of around 99.4%. This simulation is ran using the script "HW5_B_ph_in.m".

G1 is currently calculated as $G_1 = \frac{4}{sinc(\frac{fin(1)}{fsample})}$. As the denominator is a fixed value, the coefficient, in this case "4" is the only thing that could be adjusted. Hence we run a script which could visualize the relation between the coefficient and Q1 fideltiy. Running script "HW5_B_G1.m", the figure 4 is achieved. The pattern kind of follows the phase shift, the fidelity increases as G1 grows bigger until it reaches the peak, and then the fidelity decreases as G1 increases further. When $G_1 = 3.75$, the fidelity of Q1 exceeds 99.9%.

**With $ph\_in = 0.31, G_1 = 3.75$, the fidelity of Q1 exceeds 99.9%, it is approximately 99.9171%. Note that the pattern mentioned above only applies to a single period, because the pattern is periodic.**

Running the script "HW5_Final.m", the final result with corresponding plots could be displayed directly. From figure 5, it is shown clearly that the rotation maintains a $\pi$ rotation.

### 2.2 Question 2

Explain why you had to change the phase.

**Answer:** This is due to the limitation of sampling of DAC, namely Jitter, as shown in figure 6. Due to the fact that we have reduced the sampling speed significantly compared to A, the effect of jitter is amplified, hence we compensate this issue by shifting the input signal to the left, in order to minimize the jitter and then contribute to a smaller quantization error. As shown in figure 7, it is a comparison of input signal with different shifts. It is shown clearly that the quantization process is better as the shifts gradually increase from 0 to 0.4 (Top left is the ideal signal, top middle is 0, top right is 0.1, botton left is 0.2, bottom middle is 0.3, bottom right is 0.4).

### 2.3 Question 3

Explain why you had to change the gain of G1. Hint: For a full understanding, observe what happens for fsample=10e9, fosc=12e9, f0 = [13e9, 14e9, 15e9].

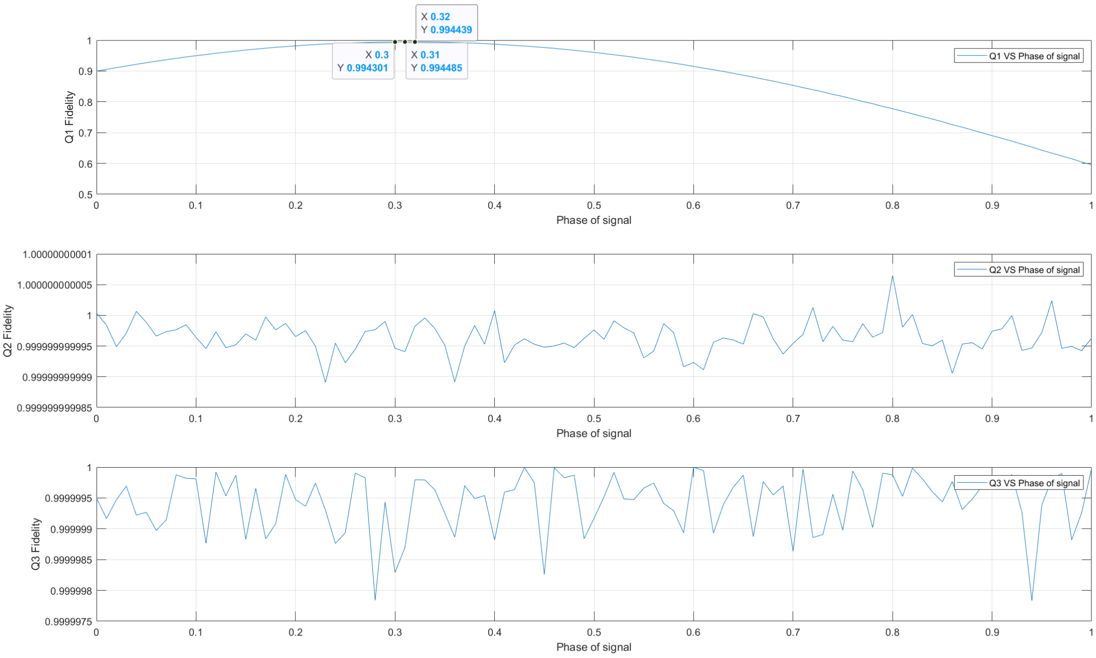



**Figure 3:** Fidelity VS. Phase shift of input signal

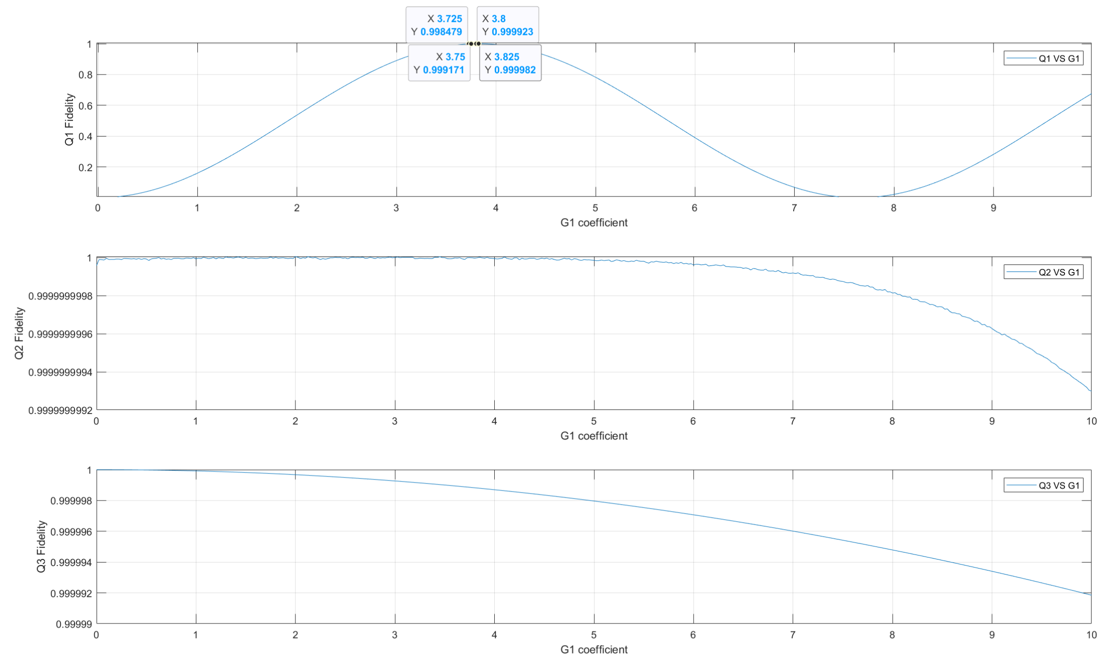



**Figure 4:** Fidelity VS. G1

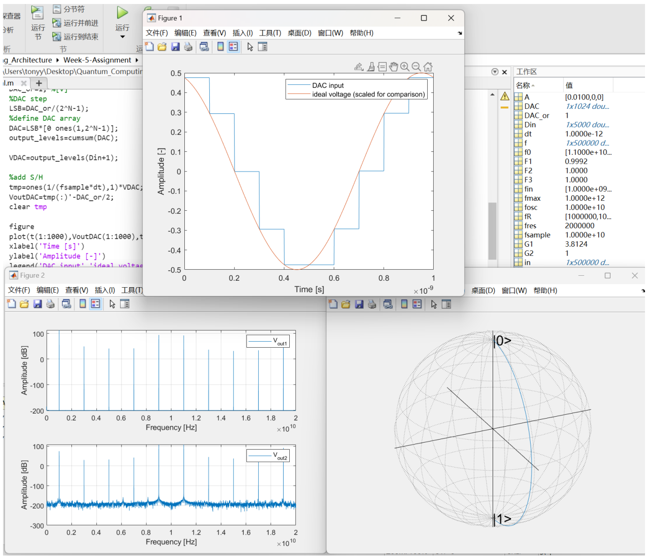


**Figure 5:** Final result

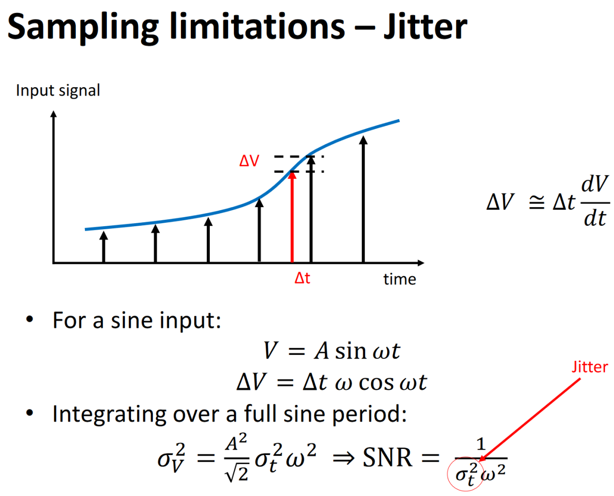



**Figure 6:** Jitter Effect

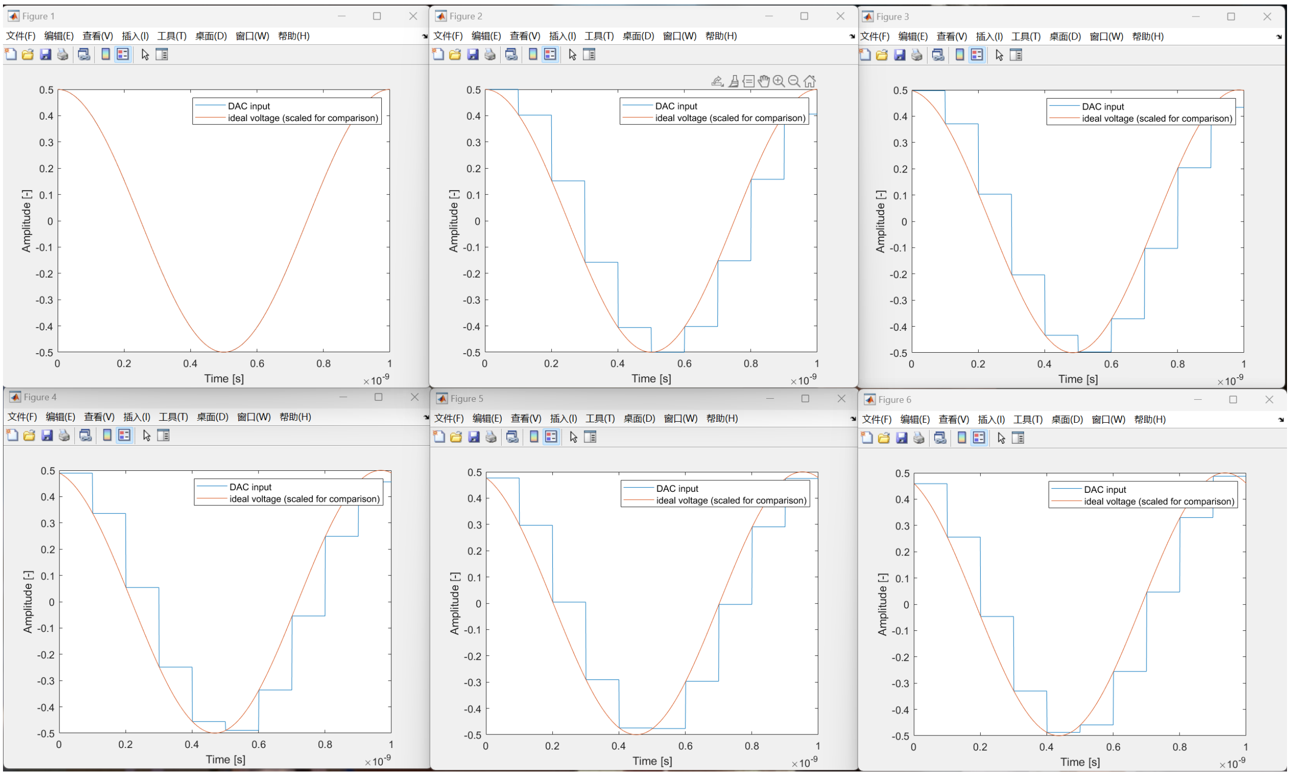



**Figure 7:** Quantization error comparison at sampling speed 10GHz

**Answer:** G1 is adjusted on top of the shift applied to the input signal. This is like a chain reaction, if one factor gets adjusted, the others would require some adjustment as well. First of all, G1 influences the amount of “power” applied to the rotaion. E.g. if the power is not strong enough, the rotation would be less than a $\pi$ rotation, while if the power is too strong, the rotation would exceed a $\pi$ rotation. Hence, this factor is decided after adjusting the phase shift of the input signal. As shown in figure 8, it shows clearly that as G1 increases, the power of rotation increases as well (from top left to bottom right, the coefficient of G1 is set to 1, 2, 3, 4 and 5 respectively).

In order to gain a full understanding, we remove the phase shift applied, and set fsample=10e9, fosc=12e9, f0 = [13e9, 14e9, 15e9] respectively. It is believed that by changing the settings would minimize the influence caused by harmonics and intermodulation. The result is shown in figure 9, it shows clearly that as G1 increases, the power of rotation increases as well (from top left to bottom right, the coefficient of G1 is set to 1, 2, 3, 4, 5, 6, 7 and 8 respectively), which follows the same pattern.

In a word, adjusting G1 is a compensate for the adjustment made for phase shift on the input signal, and the phase shift on the input signal is a compensate for the limitation of sampling at a relatively low speed.

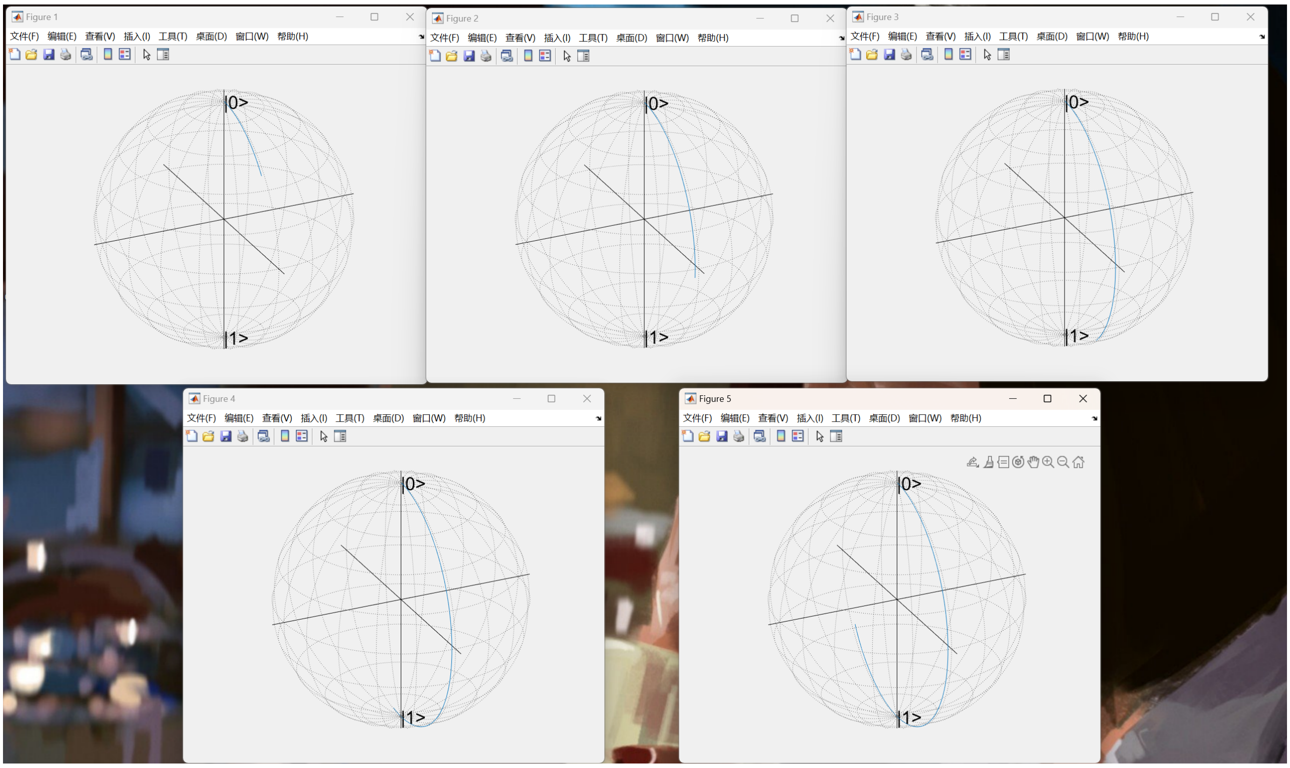



**Figure 8:** G1 coefficient comparison at sampling speed 10GHz

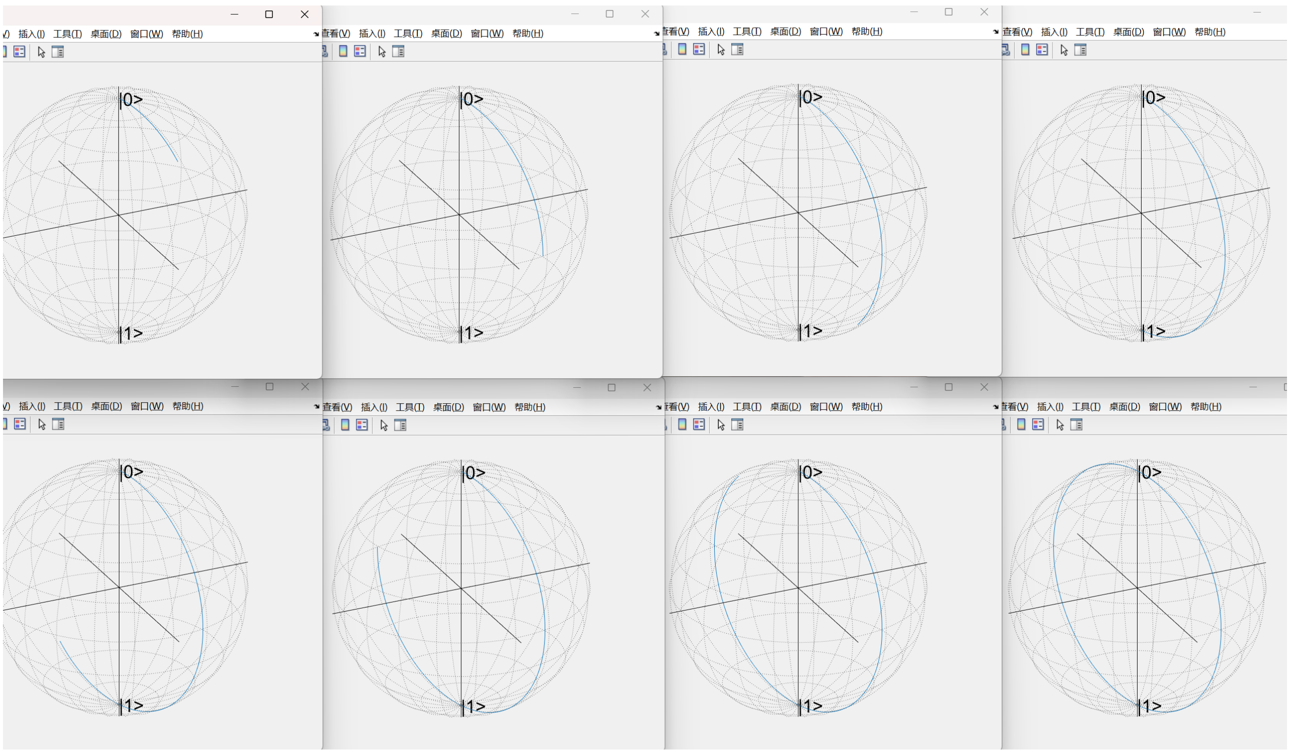



**Figure 9:** G1 coefficient comparison at sampling speed 10GHz with new settings

## 3 Exercise C

Set N=10 and set the other parameters to the initial conditions (fosc=10e9, fsample=1000e9, A=[10e-3 0 0]), i.e. drive a $\pi$ rotation around the X axis of Q1.

### 3.1 Question 1

Find at least 3 values for fsample (larger than the Nyquist rate) that cause a degradation of the fidelity on Q2 or Q3 with respect to the case in which fsample=1000e9. You may need to change the timestep dt.

**Answer:** Currently we only care about rotation of qubit 1, hence the minimum value of fsample is 2GHz (Nyquist rate).

1. $fsample = 2e9, dt = 2e-12$: Figure 10 shows the result of the simulation. The fidelity of Q1, Q2 and Q3 are $\simeq 0\%$, 100% and 14.3662%. The fidelity of Q3 degraded hugely.

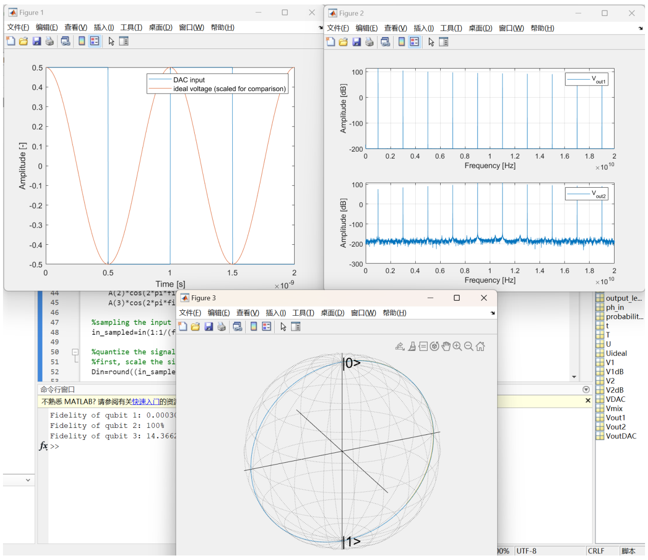



**Figure 10:** $fsample = 2e9, dt = 2e-12$

2. $fsample = 4e9, dt = 2e-12$: Figure 11 shows the result of the simulation. The fidelity of Q1, Q2 and Q3 are 49.8025%, 100% and 68.9468%. The fidelity of Q3 degraded hugely.

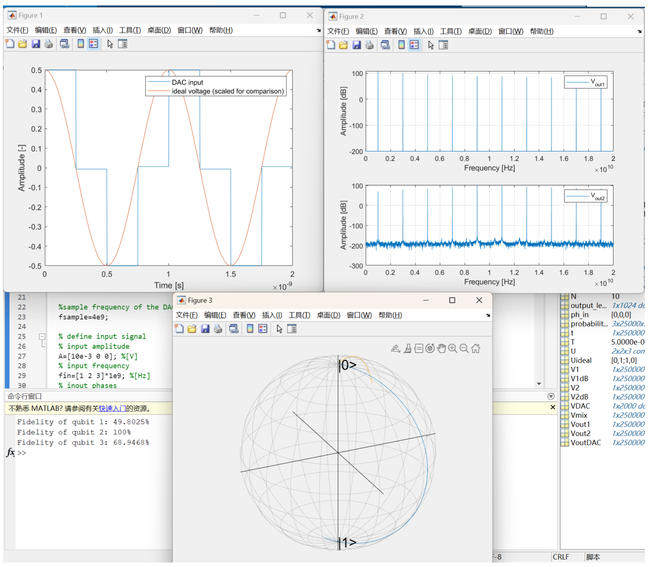



**Figure 11:** $fsample = 4e9, dt = 2e-12$

3. $fsample = 3e9, dt = 10e-12/3$: Figure 12 shows the result of the simulation. The fidelity of Q1, Q2 and Q3 are 25.8574%, 45.4465% and 100%. The fidelity of Q2 degraded hugely.

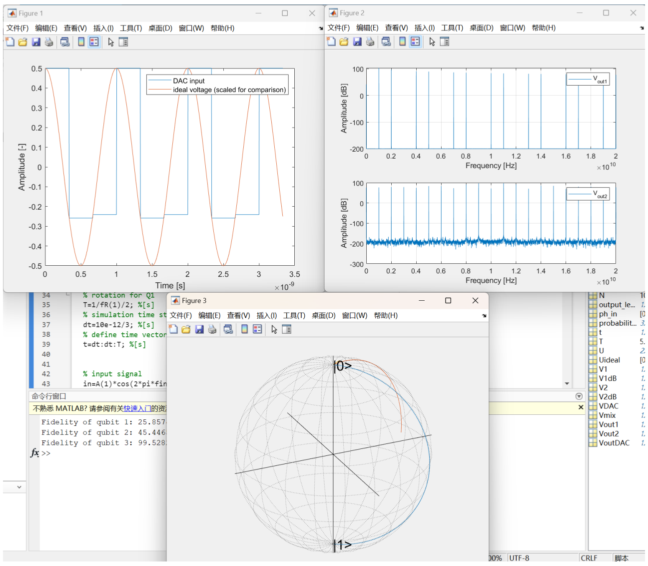
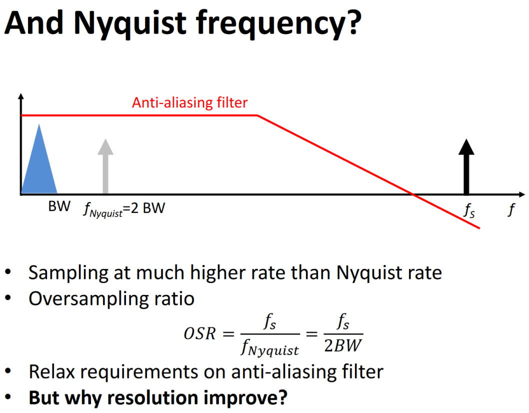
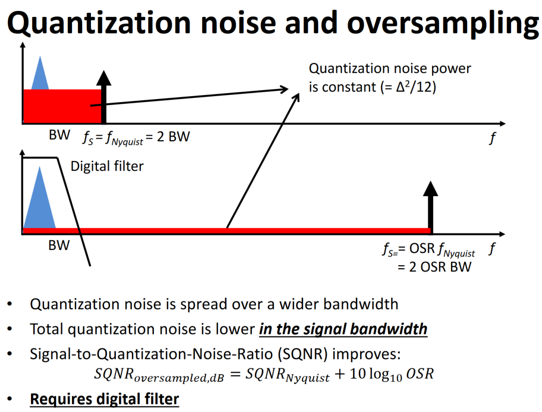


**Figure 12:** $fsample = 3e9, dt = 10e-12/3$




**(a)** Oversampling




**(b)** Digital Filter

**Figure 13:** Conditions & Relax requirements on fsample

## 3.2 Question 2

What are the conditions on fsample to avoid the degradation at point C.1? Which component can be added to the system to relax the requirements on fsample? Where should it be placed?

**Answer:** The conditions of fsample to avoid the degradation at point C.1 is to sample at a much higher rate than the Nyquist rate (in this case, much higer than 2GHz).The over sampling rate OSR is defined by $OSR = \frac{f_s}{f_{Nyquist}} = \frac{f_s}{2BW}$.

A digital filter, more specifically, an anti-aliasing filter could be added to the system to relax the requirements on fsample. It should be placed right after the sampling is done.

The principle is as shown in figure 13a and figure 13b.

# 4 Exercise D

## 4.1 Question 1

Set fsample=10e9, N=10. If mismatch is added to the DAC elements, what is the maximum variance in the mismatch of the DAC unit elements that keeps the fidelity of Q1, Q2 and Q3 above 99.9%?

**Answer: Note that in the folder of Exercise D, you could find several scripts. I will first be explaining here the difference among different scripts, and why only one was chosen to be used in the end.**

Adding mismatch to the DAC component is done simply by introducing an offset error (generated with a certain variance/standard deviation) to the line that defines the input signal, namely VDAC (the adjustment to the code is displayed below). In this case, a mismatch with variance 0.01 is introduced to the DAC.

```
%DAC output range
DAC_or=1; %[V]
%DAC step
LSB=DAC_or/(2^N-1);
%define DAC array
DAC=LSB*[0 ones(1,2^N-1)];
output_levels=cumsum(DAC);

%define the variance of the mismatch and also calculate the standard
%deviation
var_mismatch = 0.001;
std_mismatch_dev = sqrt(var_mismatch);

% generate normally distributed mismatch errors for each element in output_levels
errors = normrnd(0, std_mismatch_dev, size(output_levels));

% add the errors to output_levels to simulate the mismatch error
noisy_levels = output_levels + errors;

% apply the noisy levels to the input signal
VDAC = noisy_levels(Din+1);
```

I used random numbers of normal distribution to inject the mismatch error because in practice, the mismatch between DAC elements is typically random and follows a statistical distribution. Therefore, by generating random numbers with a certain statistical distribution, we can simulate the effect of mismatch on the DAC output. Moreover, by adjusting the variance of the random numbers, the magnitude of the mismatch error can be controlled. In order to minimize the influence of randomness on fidelity, the result is averaged using the "movmean" function that Matlab provides.

As shown in the figure 14, the mismatch is successfully introduced with variance 0.01. The fidelity decreased from 99.9171% to 96.1347%. The result could be achieved by running the script "HW5_example2.m". Now we sweep among a wider range of value and see the relationship between mismatch and fidelity of qubits.

The result by running script "HW5_high_resoltuion_mismatch_2.m" (200 varainces between 0 and 1) is shown in figure 15.

The degradation of fidelity on all three qubits are fast, hence we change the range and sweep through narrower range. The new range is 0 to $\frac{LSB}{125}$ with 200 uniformed samples, the result is as shown in figure 16. For a more accurate and precise answer, we pick 2000 uniformed variances in the range 0 to $\frac{LSB}{125}$, the result is shown in figure 17.

Note that though the result has already been averaged in order to lower the impact of randomness, the mismatch errors are still generated randomly.

**The answer to this problem is, when the fidelity of Q1, Q2 and Q3 drops below 99.9% as the variance of the mismatch error exceeds 5.67244e-5, 2.47436ee-4 and 3.52082e-4. Hence the maximum value that keeps all three fidelity above 99.9% is 5.67244e-5. Moreover, the fidelity of qubits continuously decrease as the variance of mismatch error increases.**

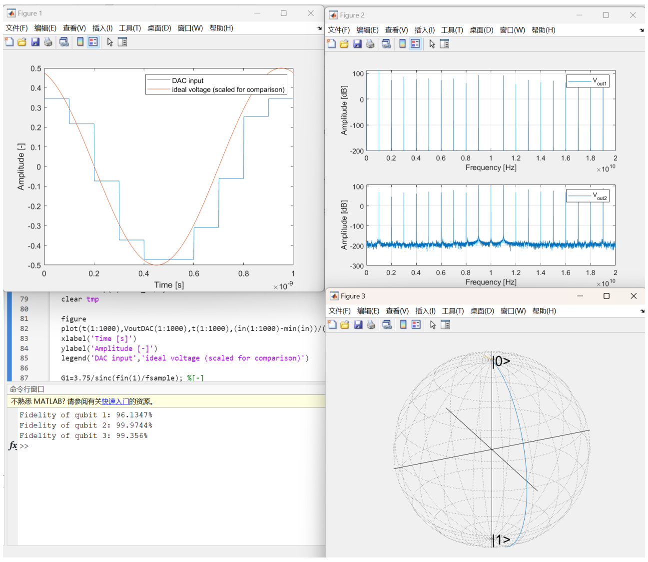



**Figure 14:** Result of the quantization after introducing a mismatch with variance 0.01

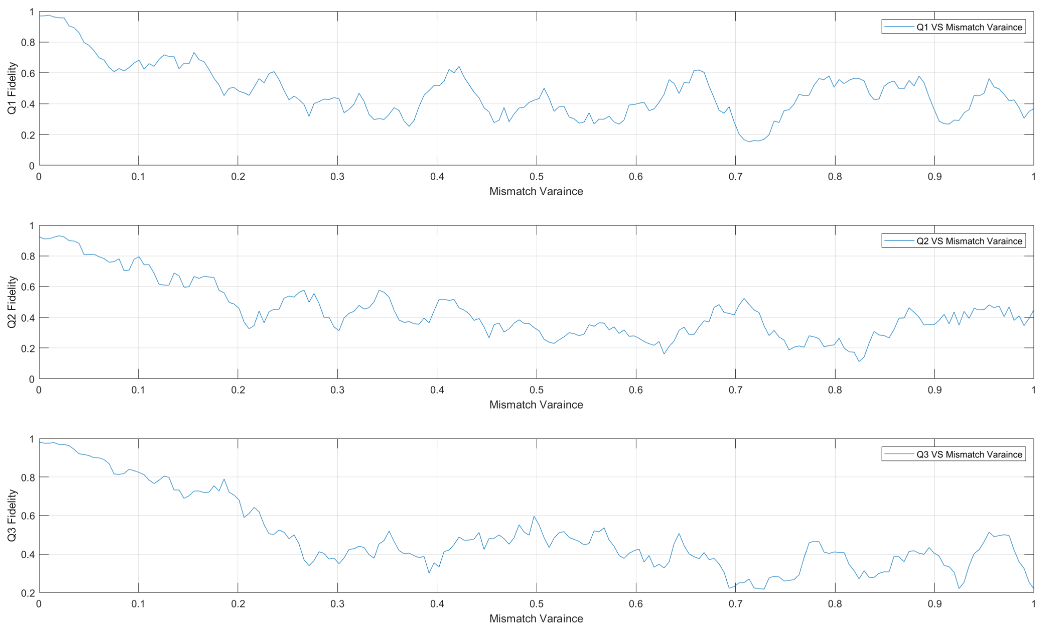



**Figure 15:** Variance(0 to 1) VS. Qubit fidelity

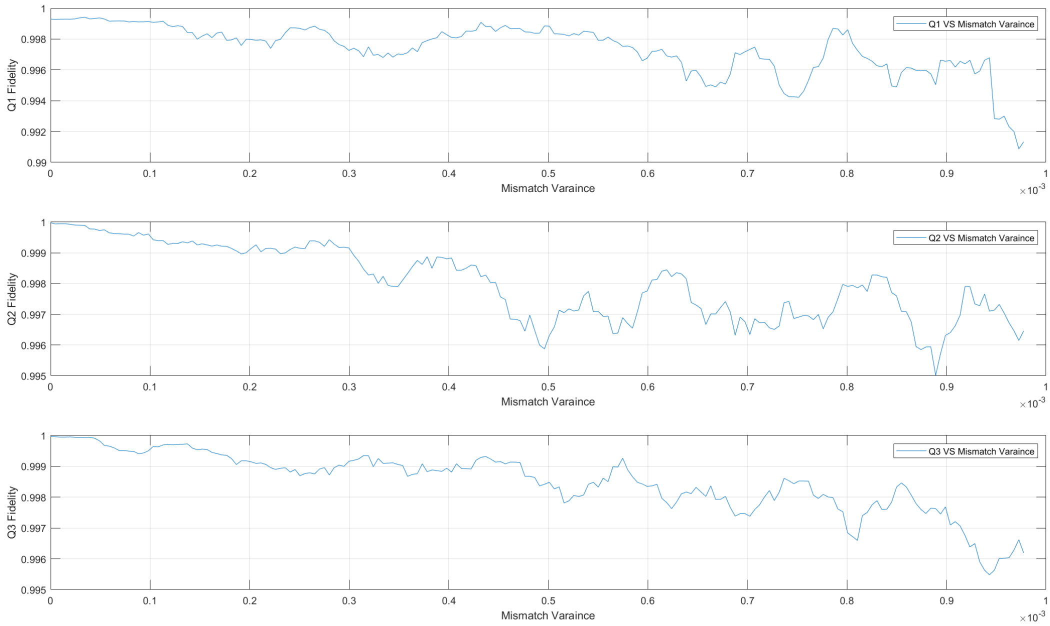



**Figure 16:** Variance(0 to $\frac{LSB}{125}$) VS. Qubit fidelity (Narrow Range)

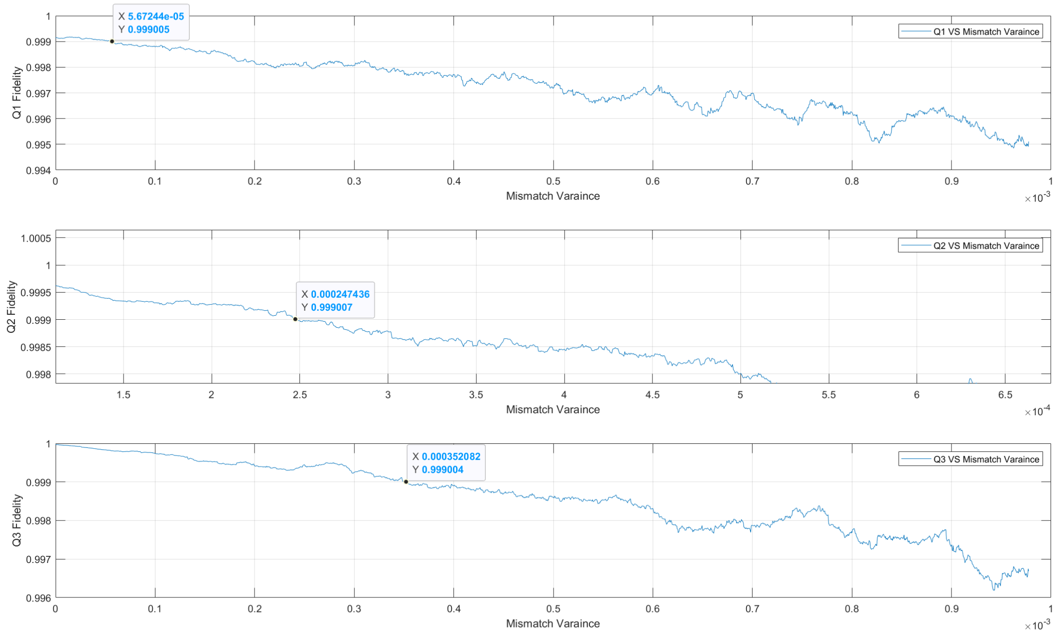



**Figure 17:** Variance(0 to $\frac{LSB}{125}$) VS. Qubit fidelity (High resolution)

**This homework is completed with the help of [1], [2] and [3].**

## References

[1] "How to calculate variance — calculator, analysis & examples," (Date last accessed 23-03-2023). [Online]. Available: https://www.scribbr.com/statistics/variance/

[2] J. van Dijk, "Impact of classical control electronics on qubit fidelity," Ph.D. dissertation, TUDelft, 2019.

[3] ——, "Supplemental online material for "the impact of classical control electronics on qubit fidelity"," Ph.D. dissertation, TUDelft, 2019.