QC Architecture and Electronics (CESE4080) Homework 4

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1 Exercise A

Start from the example in the file HW4A_example.m demonstrating a pi rotation around the X axis of Q1 for an ideal system.

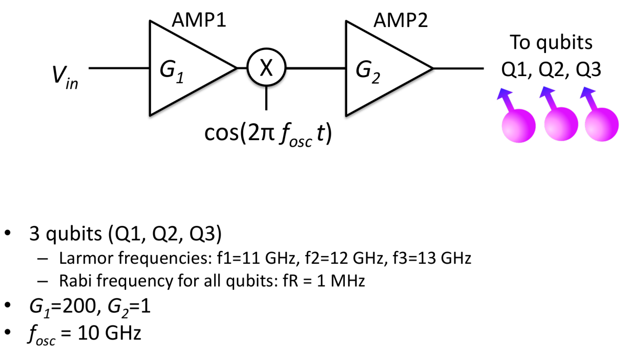

Figure 1: Question for Exercise 1

1.1 Question

The circuit is shown in figure 1. While keeping the same driving for Q1, determine the maximum 2nd and 3rd order distortion of AMP1 to maintain a fidelity above 99.9% in the idle operation, i.e. identity operation, on Q2 and Q3. Show how you derived your answer and explain.

Answer: First, as mentioned in the problem statement, this system is ideal. Hence we could assume in that $a_0 = 0$ (offset) in the equation $y = a_0 + a_1x + a_2x^2 + a_3x^3 + ...$ Now we introduce the 2nd and 3rd order distortion caused by non-linearity, hence we had 2nd and 3rd order on top of $V_{out1} = G_1 \times V_{in}$, leading to the final equation $V_{out1} = G_1 \times V_{in} + G_{1.2} \times V_{in}^2 + G_{1.3} \times V_{in}^3$. Note that the 2nd order distortion affects Q2, and the 3rd order distortion affects q3.

Strategy:

1. Find values of $G_{1.2}$ and $G_{1.3}$ when fidelity is above and below 99.9% or Q2 and Q3 respectively.
2. Create a "sweep" script which determines the values of $G_{1.2}$ and $G_{1.3}$ at a larger resolution.
3. Plot it in graph and find the precise value.
4. Deliver both the code of the script and the graphs plotted.
5. Deliver calculations of the maximum 2nd and 3rd order distortion.

Result: When above 99.9% $G_{1.2}$ and $G_{1.3}$ are 800 and 161000. When below 99.9% $G_{1.2}$ and $G_{1.3}$ are 830 and 161500.

Use the code in the file "Sweep_values.m" and the function in "spine_no_plot.m" to sweep among different values, and then plot the graphs. The graphs are shown in figure 2 and figure 3. Hence we pick $G_{1.2}$ and $G_{1.3}$ as 805.7 and 161162.

Using the equations $HD_2 = \frac{a_2}{2a_1}A$ and $HD_3 = \frac{a_3}{4a_1}A^2$, where $A = 0.001$, and $a_1 = 200$. $HD_2 = 0.0020145$ and $HD_3 = 0.0020145375$. These two values are nearly identical, the error is caused by not resolution not large enough, hence we consider them both as 0.0020145.

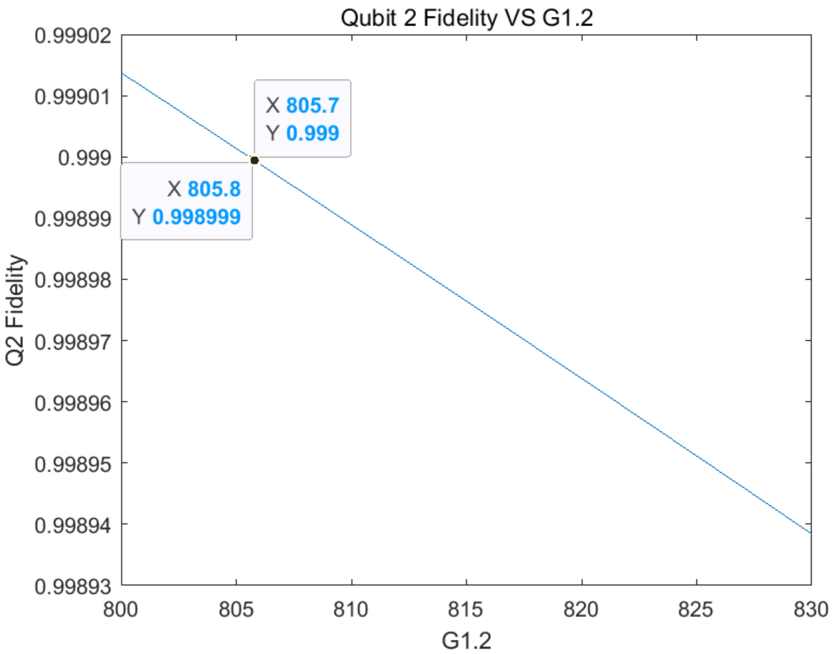

Figure 2: Q2 Fidelity VS G1.2

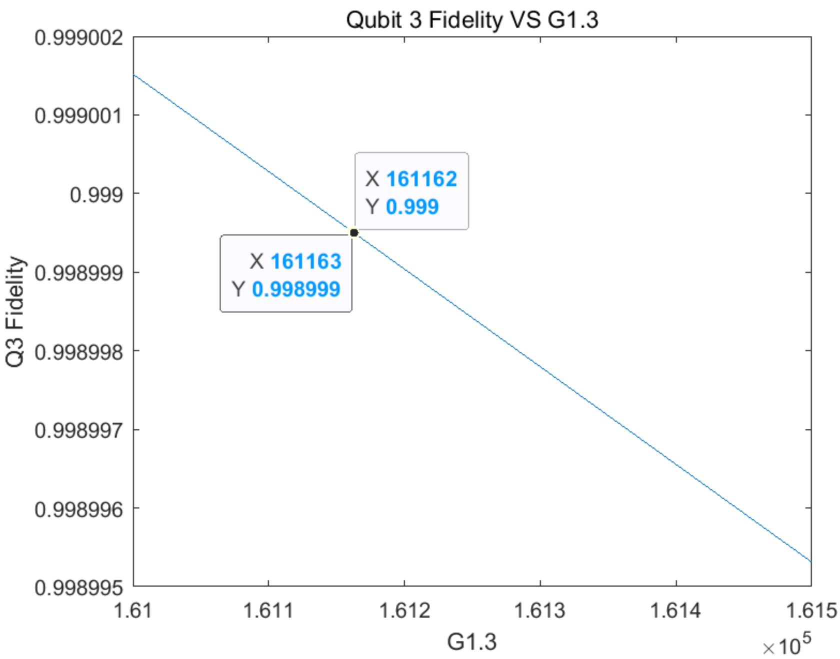

Figure 3: Q2 Fidelity VS G1.3

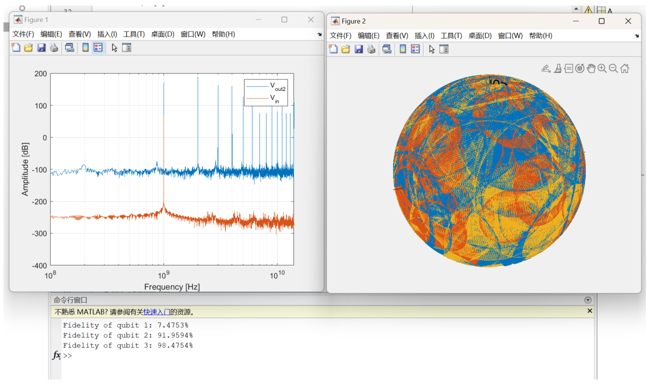

Figure 4: Only 2nd order distortion simulation result

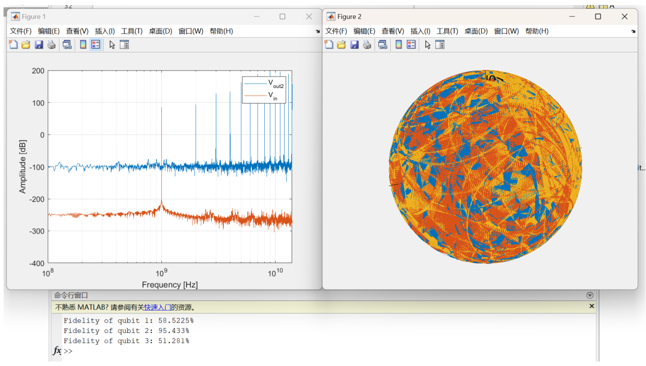

Figure 5: Only 3rd order distortion simulation result

1.2 Question

With the system found in A1 (including non-linearity in AMP1), can 2nd or 3rd order non-linearity in AMP2 affect the fidelity of Q2 and Q3? Explain and demonstrate by simulation.

Answer: Yes, it can. **We start by assigning random values to the coefficients, then we analyze the phenomenon with beautifully plotted graph. Note that the analyze part is more important.** We start by writing $V_{out1} = a_1 \times V_{in} + a_2 \times V_{in}^2 + a_3 \times V_{in}^3$. Now we mix the signal with another signal, we get $V_{mix} = V_{out1} * V_{add}$. We then use V_{mix} as the input of the second amplifier. With 2nd and 3rd distortion introduced, it could be easily observed that 2nd and 3rd order term still exists in the equation, and hence it still does have influence on Q2 and Q3.

Only 2nd order distortion simulation: $Vout2 = G_2 * Vmix + G_{2.2} * Vmix.^2$, we set $G_{2.2}$ to 10000. The fidelity of Q1, Q2 and Q3 are 7.5%, 91.9% and 98.48%, they obviously changed. Result shown in figure 4.

Only 3rd order distortion simulation: $Vout2 = G_2 * Vmix + G_{2.3} * Vmix.^3$, we set $G_{2.3}$ to 10000. The fidelity of Q1, Q2 and Q3 are 58.5%, 95.4% and 51.28%, they obviously changed. Result shown in figure 5.

Both 2nd and 3rd order distortion: $Vout2 = G_2 * Vmix + G_{2.2} * Vmix.^2 + G_{2.3} * Vmix.^3$, we set both values to 10000. The fidelity of Q1, Q2 and Q3 are 0.0002%, 70.05% and 82.1%, they obviously changed. Result shown in figure 6.

Analyze: With the script "Sweep_Cyclic.m", we could obtain four graphs. They each represent the effect of the coefficients of the 2nd and 3rd term on Qubit 2 and Qubit 3. The figures are figure 7, figure 8, figure 9 and figure 10. As shown clearly in these graphs, introducing the 2nd and the 3rd order distortion would have effect on the fidelity of Q2 and Q3. They also have "cross effects", e.g. the second order distortion affecting both Q2 and Q3, and the same case for the third order distortion. This is believed to be related to harmonics and intermodulation distortions, as shown in figure 11.

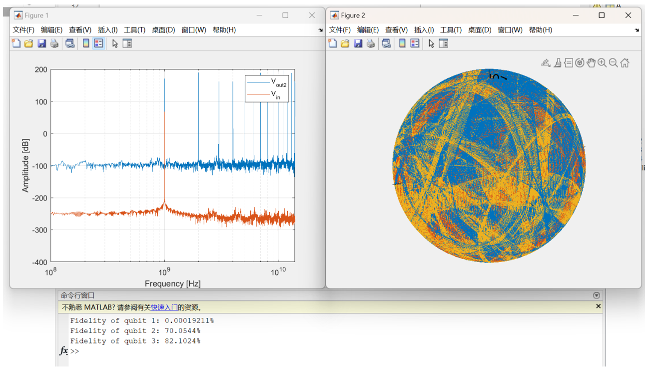

Figure 6: Both order distortion simulation result

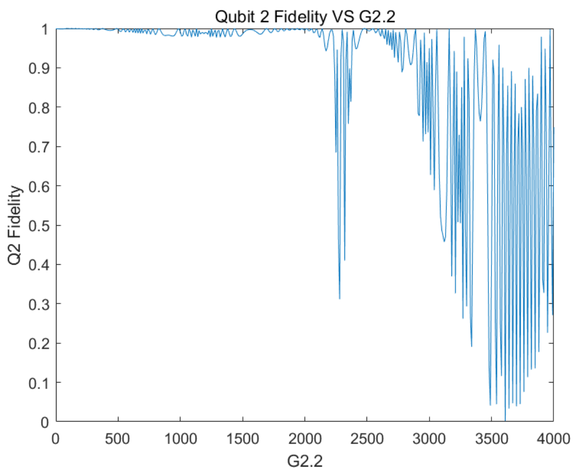

Figure 7: Q2 VS. G2.2

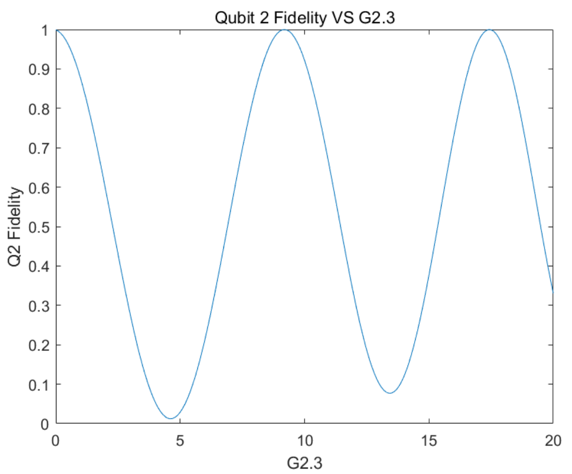

Figure 8: Q2 VS. G2.3

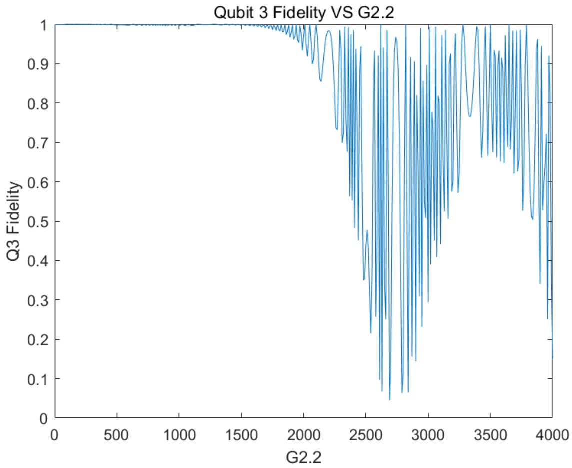

Figure 9: Q3 VS. G2.2

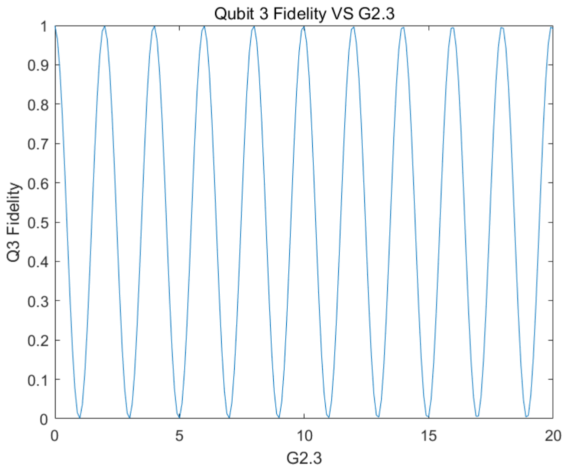

Figure 10: Q3 VS. G2.3

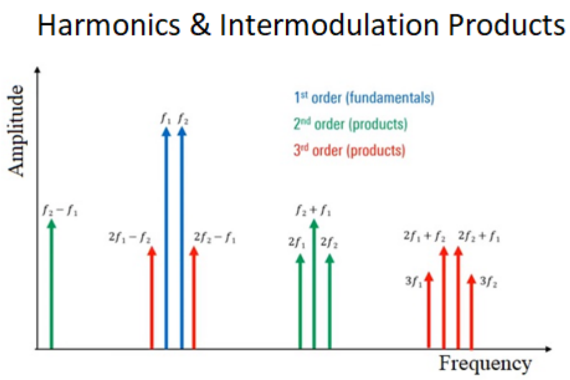

Figure 11: Effect of intermodulation and harmonics distortion

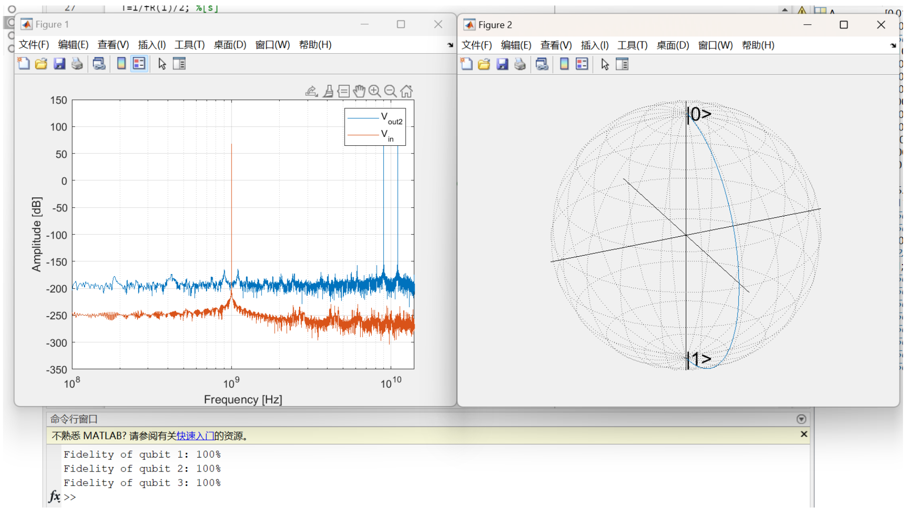

Figure 12: Without introducing non-linear distortions

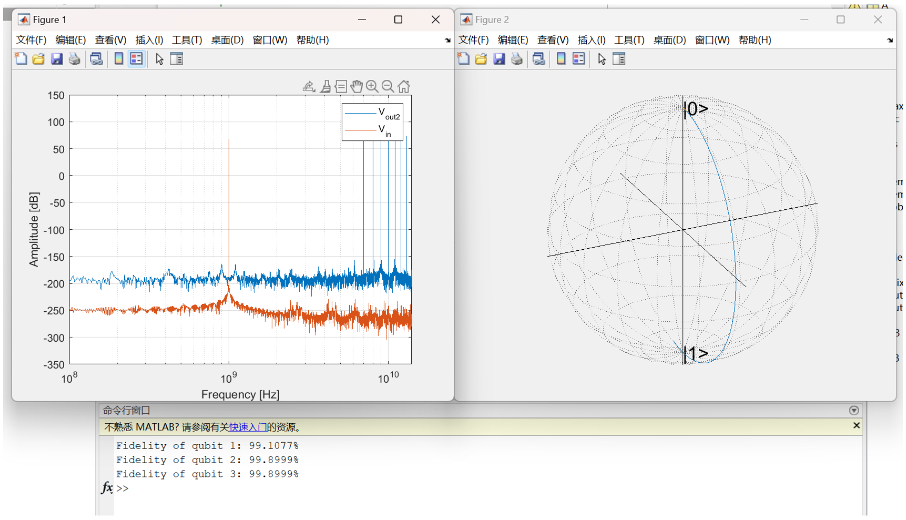

Figure 13: With introducing only non-linear distortions on Amp 1

1.3 Question

Change the parameters to only have distortion in AMP1. Do the introduced non-linearities affect the fidelity in the pi-rotation of Q1? If yes, how can you compensate for it? If not, why? Explain and demonstrate by simulation.

Answer: As shown in figure 12 and figure 13. The introducing of non-linear distortions does has effect on the fidelity in the pi-rotation of Q1. The way to compensate is via adjusting the value assigned to the input amplitude. When amplitude A = [9.48e-3 0 0], the result is shown in figure 14. The fidelity of the pi-rotation of Q1 is compensated. It is believed that with a very precise value around 9.48e-3, the fidelity could be back to 100%.

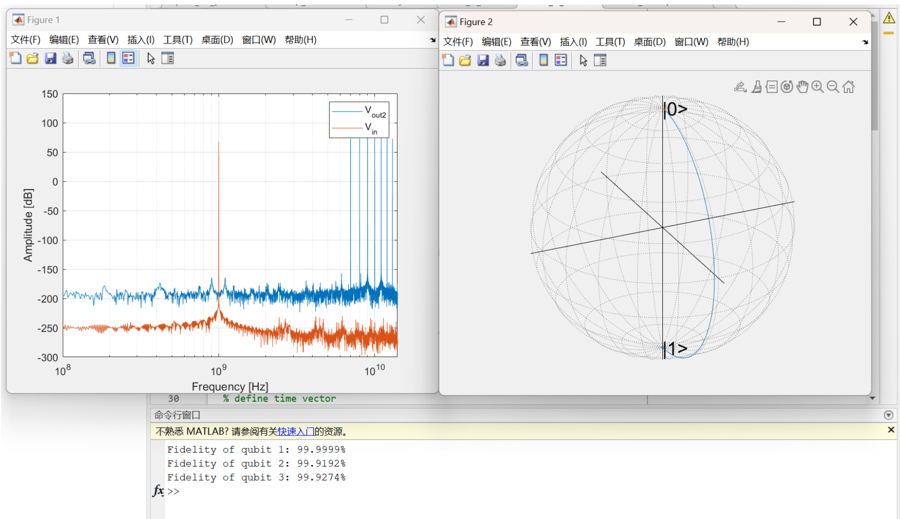

Figure 14: Result after adjusting the amplitude

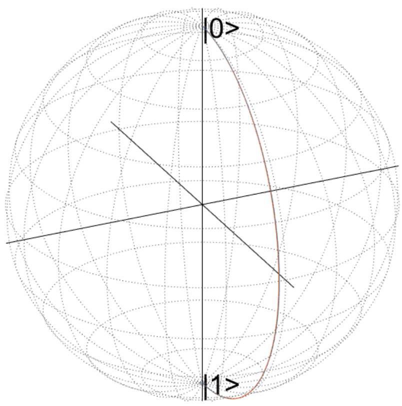

Figure 15: Both Q1 & Q2 rotating around the X-axis

2 Exercise B

Generate an input signal to drive a pi-rotation around the X axis for both Q1 and Q2.

Simply by changing A, into [10e-3 10e-3 0] does the job, the rotation graph is shown in figure 15.

2.1 Question

Find the nonlinear coefficients for AMP1 and AMP2 such that the fidelity of Q3 is above 99.9%. Divide the infidelity evenly over the two amplifiers.

Answer: We first define the meaning of "divide the infidelity evenly over two amplifiers", it is means that the first Amplifier should keep the fidelity above 99.95% while the second amplifier keeps it above 99.9%. We follow the same strategy as done in the previous exercise. Naming the coefficients of 2nd and 3rd order terms of Amp1 and Amp2 $G_{1.2}$, $G_{1.3}$, $G_{2.2}$ and $G_{2.3}$ respectively. The script used is a modified version of the same script used in A1, the name of the file is "Sweep_values.m", and it could be found in the corresponding folder of this exercise. When above 99.95% $G_{1.2}$ and $G_{1.3}$ are 280 and 28450. When below 99.95% $G_{1.2}$ and $G_{1.3}$ are 290 and 28500. As shown in figure 16 and figure 17. The values for $G_{1.2}$ and $G_{1.3}$ found are 284.9 and 28491.25. However, with two values combined, it does not give a fidelity of 99.95%, hence we lower both values and end up with 94.9 and 18891.25 respectively. It outputs a fidelity of 99.9503%.

We then set $G_{1.2}$ and $G_{1.3}$ static, and adjust $G_{2.2}$ and $G_{2.3}$ to keep the fidelity above 99.9%. We then take use of the script made of A2 to figure out how values of $G_{2.2}$ and $G_{2.3}$ would affect the

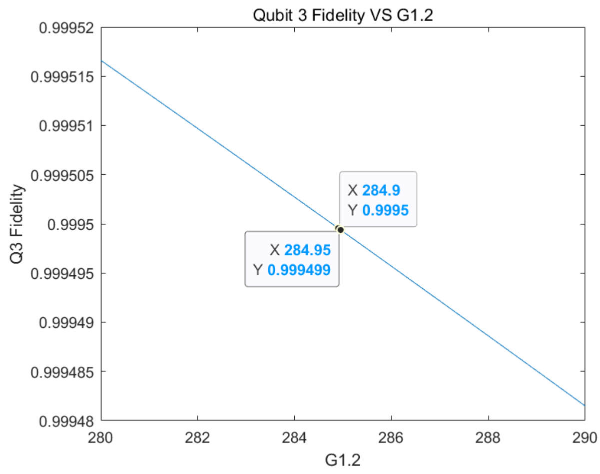

Figure 16: Q3 VS G1.2

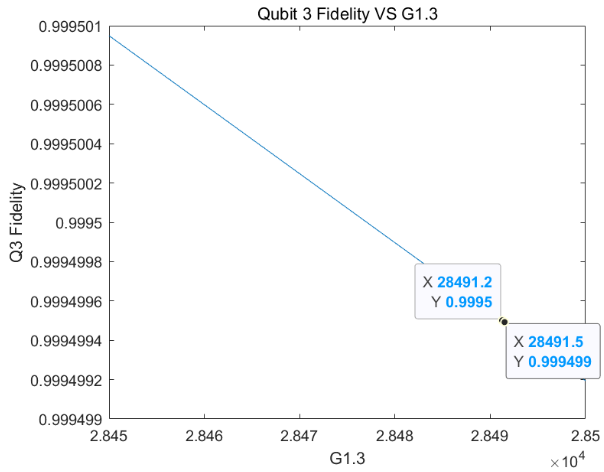

Figure 17: Q3 VS G1.3

fidelity of Q3. We obtain that with $G_{2.2} = 402.5$ and $G_{2.3} = 0$ would keep the fidelity above 99.9% respectively as in figure 18. However, when both values are applied, the fidelity is higher than 99.9%, hence we increase $G_{2.3}$ a bit.

To conclude, we set $G_{1.2} = 94.9$, $G_{1.3} = 18891.25$, $G_{2.2} = 402.5$ and $G_{2.3} = 0.26$. This results in a fidelity of 99.9993%. This file to simulate this directly is in the corresponding folder of this exercise with name "HW_4B_1.m".

2.2 Question

Do the constraints on the non-linearity of AMP2 change with respect to point A.2? How? (and why?) Explain and demonstrate by simulation.

Answer: No it does not change, the 2nd and 3rd order non-linearity in AMP2 affects fidelity of Q2 and Q3. With the script used for the last problem, we generate corresponding fidelity of Q2 and Q3 with different $G_{2.2}$ and $G_{2.3}$ values.

Figure 19 and figure 20 shows that Q3 is still affected by non-linearity in AMP2. Figure 21 and figure 22 shows that Q2 follows the same pattern.

To conclude, the constraints on the non-linearity of AMP2 does not change. Q2 and Q3 are still affected.

This is believed to be related to harmonics and intermodulation distortions, as shown in figure 11.

This homework is completed with the help of information found on Youtube, the "Qiskit" channel [1] and the explanation of intermodulation distortion found on [2].

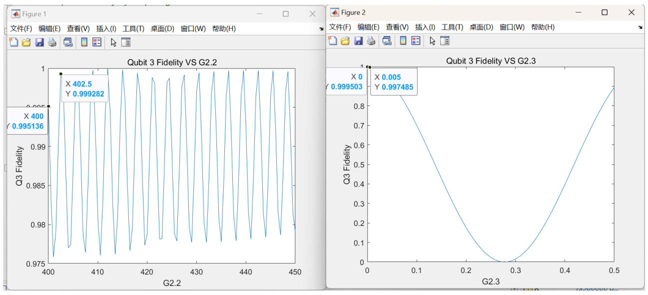

Figure 18: Q3 VS G2.2 & G2.3 (High resolution)

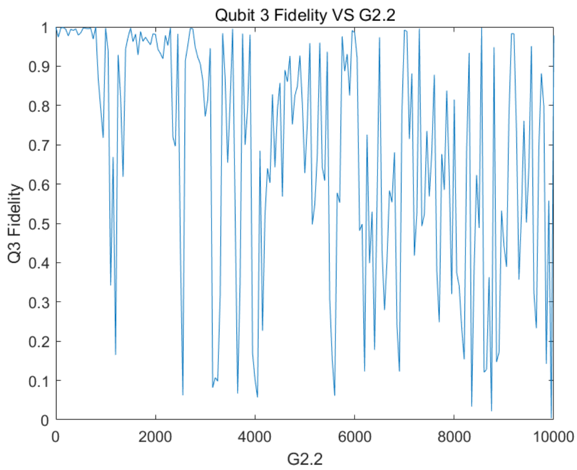

Figure 19: Q3 VS G2.2

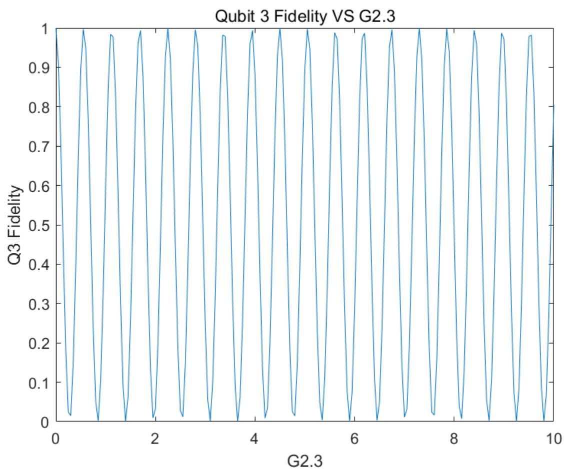

Figure 20: Q3 VS G2.3

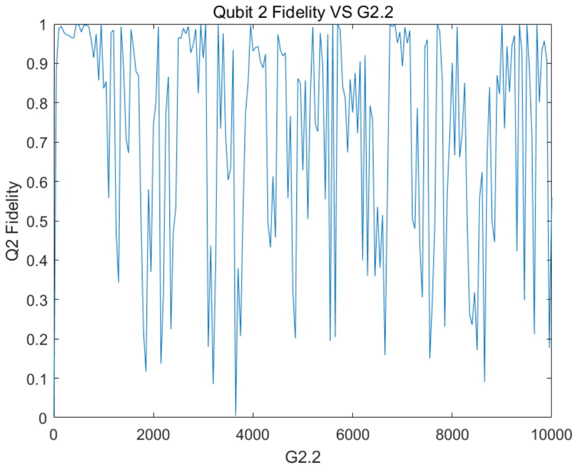

Figure 21: Q2 VS G2.2

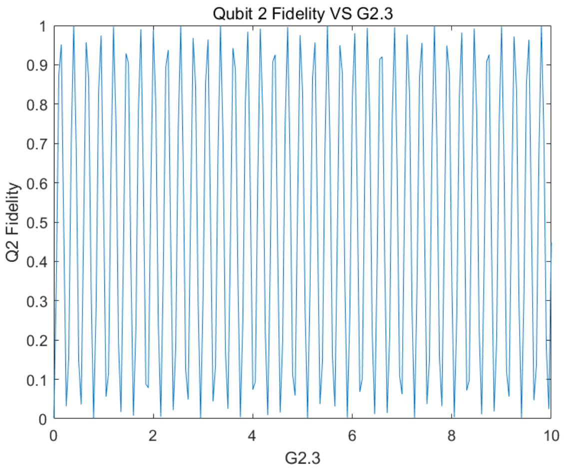

Figure 22: Q2 VS G2.3

References

[1] "Microwave amplifiers for quantum information processing," (Date last accessed 19-03-2023). [Online]. Available: https://www.youtube.com/watch?v=sVRWtSv0boc

[2] "What is intermodulation distortion?" (Date last accessed 19-03-2023). [Online]. Available: https://www.everythingrf.com/community/what-is-intermodulation-distortion