Tony Cui

# CS 230 - Week 4

1)

a)

| A | B | C | $A \to B$ | $B \land C$ | $(A \to B) \to (B \land C)$ | $(B \land C) \to (A \to B)$ |
|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T |
| F | T | T | T | T | T | T |
| T | F | T | F | F | T | T |
| F | F | T | T | F | F | T |
| T | T | F | T | F | F | T |
| F | T | F | T | F | F | T |
| T | F | F | F | F | T | T |
| F | F | F | T | F | F | T |

$B \land C$ logically implies $A \to B$ since the truth table we used shows that $(B \land C) \to (A \to B)$ is true for all truth value combinations of A, B, and C.

b) $(A \lor B) \to (A \leftrightarrow C)$

| A | B | C | $(A \lor B)$ | $(A \leftrightarrow C)$ | $(A \lor B) \to (A \leftrightarrow C)$ | CNF | DNF |
|---|---|---|---|---|---|---|---|
| T | T | T | T | T | T | T | T |
| F | T | T | T | F | F | F | F |
| T | F | T | T | T | T | T | T |
| F | F | T | F | F | T | T | T |
| T | T | F | T | F | F | F | F |
| F | T | F | T | T | T | T | T |
| T | F | F | T | F | F | F | F |
| F | F | F | F | T | T | T | T |

CNF: $(\sim A \lor B \lor C) \land (A \lor B \lor \sim C) \land (A \lor \sim B \lor \sim C)$

DNF: $(A \land B \land C) \lor (A \land \sim B \land C) \lor (\sim A \land \sim B \land C) \lor (\sim A \land B \land \sim C) \lor$

$(\sim A \land \sim B \land \sim C)$

2) a) $\forall a,b \in \mathbb{Q}, \exists c \in \mathbb{Q}, (a=b) \vee ((\min(a,b) < c) \wedge (c < \max(a,b)))$

Proof: We can prove that S is true using a proof by cases, for $a,b,c \in \mathbb{Q}$.

Case 1: If a and b are the same rational number, then we can use the first part of S : $(a=b)$, since we know that $a=b$ would be true. Consequently, since $(a=b)$ is on one side of the Or statement, then propositional logic states that we have proved S to be true.

Case 2: If a and b are different rational numbers, then we can use the second part of S to prove S true:

$$((\min(a,b) < c) \wedge (c < \max(a,b))$$

This statement alone can also be expressed as "a rational number c that is in between a and b," which we can represent with $c = \frac{a+b}{2}$ or $c = (a+b)\frac{1}{2}$, which will always guarantee that c is between a and b. We know c would still be a rational number since the sum of two rational numbers is a number that's at least rational and when we multiply a rational number by another rational number we get a ~~rati~~ number that is at least rational.

This means the second part of S must be true since c can always represent a rational number that is inbetween a and b, and since the second part of S is on one side of the Or statement, then propositional logic states that we have proved S to be true. $\square$

Thus, we can conclude that S is true using a proof by cases.

2b)

Simplify : $\sim (\forall a,b \in \mathbb{Z}, \exists c \in \mathbb{Z}, (a=b) \vee ((\min(a,b) < c) \wedge (c < \max(a,b)))$

Simplified T
Used De Morgan's Law

$\exists a,b \in \mathbb{Z}, \forall c \in \mathbb{Z}, \sim ((a=b) \vee ((\min(a,b) < c) \wedge (c < \max(a,b))))$

$\exists a,b \in \mathbb{Z}, \forall c \in \mathbb{Z}, (a \neq b) \wedge \sim ((\min(a,b) < c) \wedge (c < \max(a,b)))$

$\boxed{\exists a,b \in \mathbb{Z}, \forall c \in \mathbb{Z}, (a \neq b) \wedge ((\max(a,b) \leq c) \vee (c \leq \min(a,b)))}$

2c) Proof: We can prove T to be true through a proof of cases where $a, b, c \in \mathbb{Z}$ and $a = b+1$.

Case 1: $c > a$

This would mean T is true because $a \neq b$ is true and $\max(a,b) \leq c$ is true since c would be greater than the larger of a,b since we defined a to be larger than b and c is larger than a. By propositional logic we have proven both sides of the and statement true so T is true.

Case 2: $c < b$

This would mean T is true because $a \neq b$ and $c \leq \min(a,b)$ is true since c is less than the smaller of a,b since we defined a to be larger than b and c is smaller than b. By propositional logic, we have proven both sides of the and statement true so T is true.

Case 3: $b \leq c \leq a$

Since $a = b+1$ and $b \leq c \leq a$, then c must equal to a or b due to c being an integer, which would prove the propositional statement true since $a \neq b$ and c is equal to either the smaller of a,b, which is just b, or the larger of a,b, which is just a.

Thus, T is proven to be true in all cases. □

3) a) $\mathbb{N}$ e) $\mathbb{Q}$

b) $\mathbb{Z}$ f) $\mathbb{N}$

c) $\mathbb{Z}$ g) $\mathbb{R}$

d) $\mathbb{Q}$

4) a) Everybody loves someone else ; True because each person has at least one person to love.

b) There is somebody that loves everybody ; False because no one loves everyone else. A loves everyone but not themself so they don't count.

c) There is someone that loves everyone except themself ; True because A loves everyone except themself

d) If someone loves someone else, then that person doesn't love you back ; False because D loves themself and so themself could love D.

e) There are 2 people who are not the same person and who either both love each other or both don't love each other ; True because B doesn't love D and D doesn't love B

5)

a) $x > 0 \wedge y \leq 0$

b) $\forall \varepsilon \in \mathbb{R}^{+}, \exists M \in \mathbb{R}^{+} \left[ x > M \rightarrow \left( |f(x) - b| \leq \varepsilon \right) \right]$

c) $\sin(x) \leq 0 \rightarrow \sim(0 \leq x \leq \pi)$

d) $\forall \, Prime(p), \exists \, Prime(q) \, (q > p)$

e) $\forall n \in \mathbb{N} \left[ \exists a, b, c, d \in \mathbb{Z} \, (n = a^2 + b^2 + c^2 + d^2) \right]$