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week 12 - CS230

Q1] a) pairwise independent

A & B independent:

$Pr(A \cap B) = Pr(A) \cdot Pr(B)$

$\frac{1}{4} = \frac{1}{2} \cdot \frac{1}{2}$ ✓

A & C independent:

$Pr(A \cap C) = Pr(A) \cdot Pr(C)$ $\frac{6}{36}$

$\frac{3}{36} = \frac{1}{2} \cdot \frac{1}{6}$

$\frac{1}{12} = \frac{1}{12}$ ✓

B & C independent:

$Pr(B \cap C) = Pr(B) \cdot Pr(C)$

$\frac{3}{36} = \frac{1}{2} \cdot \frac{1}{6}$

$\frac{1}{12} = \frac{1}{12}$ ✓

so A, B, C are pairwise independent since every pair of two events are independent of each other

b) mutually independent:

By the definition of mutually independent events, events are only mutually independent if no subset of the events gives information about any of the other events. However we know this is not true with A, B, and C because having both A and B or $A \cap B$ greatly increases our odds of rolling a 7, since the first die is 1, 2, or 3 and the second die is 4, 5, or 6. Thus, $P(C) = \frac{1}{6} \neq P(C \mid A \cap B) = \frac{1}{3}$, so A, B, C are NOT mutually independent events.

c) Proof] We can prove the proposition true using a direct proof in which, if two events, A and B are disjoint and both have nonzero probability, then they are dependent. Using the independence formula of $P(A \cap B) = P(A)\,P(B)$, we can see that $P(A \cap B) = 0$, since the two events A and B are disjoint and thus have an empty intersection. However, since we know that $P(A)$ and $P(B)$ have nonzero probabilities, then we know that $P(A)\,P(B) > 0$. Thus we know that $P(A \cap B) \neq P(A)\,P(B)$ since $0 \neq P(A) \cdot P(B)$, where $P(A) \cdot P(B)$ are non zero, so since the independent formula does not hold, then the two events A and B must be dependent if A and B are disjoint and they both have nonzero probability. □

Q2] Distribution:

$\boxed{\text{One black in Jar 1, 49 black marbles and 50 white marbles in Jar 2}}$

$\frac{1}{2}$ chance you choose Jar 1 - auto win

$\frac{1}{2}$ chance you choose Jar 2 - $\frac{49}{99}$ chance to win

So in total you have a $\frac{1}{2} + \frac{1}{2}\left(\frac{49}{99}\right)$ chance to win → 74.74%

Q3) a) $Pr(A|B) = \frac{Pr(A \cap B)}{Pr(B)}$

$Pr(B) = \frac{100}{100{,}000} = \frac{1}{100}$ → people who ~~match~~ match description

$Pr(A \cap B) = \frac{99}{100{,}000}$ → People who match description + innocent

$\frac{\frac{99}{100{,}000}}{\frac{100}{100{,}000}} = \boxed{\frac{99}{100}}$

b)

$Pr(B)$ → Probability he is innocent: $\frac{99999}{100{,}000}$

$Pr(A|B) = \boxed{\frac{11}{11{,}111}}$

c) $1 - (0.999)^{1000} = \boxed{0.63}$

Q4) In a 64-team single-elimination, there are 63 ~~[illegible]~~ matches in the tournament, regardless of seeding and if there is a single winner. We can also observe that there are $\binom{64}{2}$ different possibilities for any given matchup. Thus, the probability that 2 given teams end up playing each other is $\frac{63}{\binom{64}{2}} = \boxed{\frac{1}{32}}$

Q5) We can observe that if the tally starts with a ballot B, then this will always draw a tie since A will end up winning and since A > B, then at one point the tally must be tied. We can also observe that if we flip our ballots such that all B votes become A and all A votes become B up to the tie, then we get a unique set of tallies that begin with A. Thus, we can observe that the # of tallies that start with B and have a tie and the # of tallies that start with A and have a tie, must be equal. We can represent this total probability of getting a tie in the tally at some point as 2 * the number of tallies that start with B or $\boxed{\frac{2b}{a+b}}$

Q6) a) $\boxed{119 \text{ number of trains}}$ $2 \times 60 - 1$

Assuming each train has uniform probability, or an equal chance of being picked and we only see #60, then our best guess would assume that #60 is the middle train car, and thus the total number of trains would be 59+1+59 = 119 # of trains. This guess is meant to be an unbiased estimator and be close to the average correct number of trains, given the single train number we observe.

We can make this assumption since we assume that the single number we observed will on average, represent the middle of the total number of train cars.

b) $\boxed{60 \text{ number of trains}}$

Assuming each train has uniform probability of being picked and we observe 5 different trains where our highest number picked, then our best guess would be 60, since this would aim to maximize the likelihood that our guess is correct. We know the number of cars can not be less than 60 and we have a sample of 5 observed cars with the largest number being 60 to back up our maximum likelihood estimate of 60. Additionally, there is a greater chance that there are 60 trains after observing 5 trains than observing 4, 3, 2, 1, or no trains.