导数列表

维基百科，自由的百科全书

以下的列表列出了许多函数的导数。f和g是可微函数，而别的皆为常数。用这些公式，可以求出任何初等函数的导数。

目录

一般求导法则

线性

$$\frac{\mathrm{d}Mf}{\mathrm{d}x} = M\frac{\mathrm{d}f}{\mathrm{d}x}$$

$$\frac{\mathrm{d}(f \pm g)}{\mathrm{d}x} = \frac{\mathrm{d}f}{\mathrm{d}x} \pm \frac{\mathrm{d}g}{\mathrm{d}x}$$

乘法定则

$$\frac{\mathrm{d}fg}{\mathrm{d}x} = \frac{\mathrm{d}f}{\mathrm{d}x}g + f\frac{\mathrm{d}g}{\mathrm{d}x}$$

倒数定则

$$\frac{\mathrm{d}\frac{1}{f}}{\mathrm{d}x} = \frac{-\frac{\mathrm{d}f}{\mathrm{d}x}}{f^2}$$

除法定则

$$\frac{\mathrm{d}\frac{f}{g}}{\mathrm{d}x} = \frac{\frac{\mathrm{d}f}{\mathrm{d}x}g - f\frac{\mathrm{d}g}{\mathrm{d}x}}{g^2} \qquad (g \neq 0)$$

复合函数求导法则

$$(f \circ g)' = (f' \circ g)g'$$

$$\frac{\mathrm{d}f(g(x))}{\mathrm{d}x} = \frac{\mathrm{d}f(g)}{\mathrm{d}g}\frac{\mathrm{d}g}{\mathrm{d}x}$$

反函数的导数

由於$g(f(x)) = x$，故$g(f(x))' = 1$，根據复合函数求导法则，则$(g' \circ f) \times f' = 1$

所以$f' = \dfrac{1}{g' \circ f}$

同理$g' = \dfrac{1}{f' \circ g}$

广义幂法则

$$(f^g)' = f^g \left(g' \ln f + \frac{g}{f} f' \right)$$

代数函数的导数

(n为任意实常数)

$$\frac{\mathrm{d}M}{\mathrm{d}x} = 0$$

$$\frac{\mathrm{d}x^n}{\mathrm{d}x} = nx^{n-1} \qquad x \neq 0$$

$$\frac{\mathrm{d}|x|}{\mathrm{d}x} = \frac{x}{|x|} = \operatorname{sgn} x \qquad x \neq 0$$

指数和对数函数的导数

$$\begin{aligned} \frac{\mathrm{d}e^x}{\mathrm{d}x} &= \lim_{\Delta x \to 0} \frac{e^x - e^{x-\Delta x}}{\Delta x} \\ &= e^x \lim_{\Delta x \to 0} \frac{1 - e^{-\Delta x}}{\Delta x} \\ &= e^x \end{aligned}$$

$$\begin{aligned} \frac{\mathrm{d}\ \alpha^x}{\mathrm{d}x} &= \frac{\mathrm{d}\ e^{x\ln\alpha}}{\mathrm{d}x} \\ &= \frac{\mathrm{d}e^{x\ln\alpha}}{\mathrm{d}\ x\ln\alpha} \cdot \frac{\mathrm{d}\ x\ln\alpha}{\mathrm{d}x} \\ &= e^{x\ln\alpha}\ln\alpha \\ &= \alpha^x\ln\alpha \end{aligned}$$

$$\frac{\mathrm{d}\ln|x|}{\mathrm{d}x} = \frac{1}{x}$$

$$\frac{\mathrm{d}\log_\alpha|x|}{\mathrm{d}x} = \frac{1}{\ln\alpha}\frac{\mathrm{d}\ln|x|}{\mathrm{d}x} = \frac{1}{x\ln\alpha}$$

$$\frac{dx^x}{dx} = x^x(1 + \ln x)$$

三角函数的导数

$$(\sin x)' = \cos x$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\cos x)' = -\sin x$$

$$(\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\tan x)' = \sec^2 x = \frac{1}{\cos^2 x}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\sec x)' = \sec x \tan x$$

$$(\operatorname{arcsec} x)' = \frac{1}{|x|\sqrt{x^2-1}}$$

$$(\csc x)' = -\csc x \cot x$$

$$(\operatorname{arccsc} x)' = -\frac{1}{|x|\sqrt{x^2-1}}$$

$$(\cot x)' = -\csc^2 x = -\frac{1}{\sin^2 x}$$

$$(\operatorname{arccot} x)' = -\frac{1}{1+x^2}$$

双曲函数的导数

$$(\sinh x)' = \cosh x = \frac{e^x + e^{-x}}{2}$$

$$(\operatorname{arsinh}\ x)' = \frac{1}{\sqrt{x^2+1}}$$

$$(\cosh x)' = \sinh x = \frac{e^x - e^{-x}}{2}$$

$$(\operatorname{arcosh}\ x)' = \frac{1}{\sqrt{x^2-1}} (x > 1)$$

$$(\tanh x)' = \operatorname{sech}^2\ x$$

$$(\operatorname{artanh}\ x)' = \frac{1}{1-x^2} (|x| < 1)$$

$$(\operatorname{sech}\ x)' = -\tanh x\ \operatorname{sech}\ x$$

$$(\operatorname{arsech}\ x)' = \frac{-1}{x\sqrt{1-x^2}}$$

$$(\operatorname{csch}\ x)' = -\ \coth\ x\ \operatorname{csch}\ x$$

$$(\operatorname{arcsch}\ x)' = -\frac{1}{|x|\sqrt{1+x^2}}$$

$$(\coth\ x)' = -\ \operatorname{csch}^2\ x$$

$$(\operatorname{arcoth}\ x)' = \frac{1}{1-x^2} (|x| > 1)$$

特殊函数的导数

伽玛函数

$$\frac{\mathrm{d}\Gamma(x)}{\mathrm{d}x} = \int_0^\infty e^{-t} t^{x-1} \ln t \mathrm{d}t$$