COMS 4771 Machine Learning (Spring 2015) Problem Set #1

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Problem 1

Examples of blackboard and calligraphic letters: $\mathbb{R}^d \supset \mathbb{S}^{d-1}$, $\mathcal{C} \subset \mathcal{B}$. Examples of bold-faced letters (perhaps suitable for matrix and vectors):

$$L(\boldsymbol{x}, \boldsymbol{\lambda}) = f(\boldsymbol{x}) - \langle \boldsymbol{\lambda}, \boldsymbol{A}\boldsymbol{x} - \boldsymbol{b} \rangle. \tag{1}$$

Example of a custom-defined math operator:

$$\operatorname{var}(X) = \mathbb{E}X^2 - (\mathbb{E}X)^2.$$

Example of references: the Lagrangian is given in Eq. (1), and Theorem 1 is interesting. Example of adaptively-sized parentheses:

$$\left(\prod_{i=1}^n x_i \right)^{1/n} + \left(\prod_{i=1}^n y_i \right)^{1/n} \leq \left(\prod_{i=1}^n (x_i + y_i) \right)^{1/n}.$$

Example of aligned equations:

$$\begin{aligned} \Pr(X = 1 \,|\, Y = 1) &= \frac{\Pr(X = 1 \wedge Y = 1)}{\Pr(Y = 1)} \\ &= \frac{\Pr(Y = 1 \,|\, X = 1) \cdot \Pr(X = 1)}{\Pr(Y = 1)}. \end{aligned} \tag{2}$$

Example of a theorem:

Theorem 1 (Euclid)**.** *There are infinitely many primes.*

Euclid's proof. There is at least one prime, namely 2. Now pick any finite list of primes $p_1, p_2, \ldots, p_n$. It suffices to show that there is another prime not on the list. Let $p := \prod_{i=1}^n p_i + 1$, which is not any of the primes on the list. If p is prime, then we're done. So suppose instead that p is not prime. Then there is prime q which divides p. If q is one of the primes on the list, then it would divide $p - \prod_{i=1}^n p_i = 1$, which is impossible. Therefore q is not one of the n primes in the list, so we're done. $\square$

Problem 2

Problem 3

Problem 4

Problem 5