Time Series Analysis: Recovering A Stochastic Signal

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1 Time Series Analysis

1.1 Recovering A Stochastic Signal

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This is a pretty basic example of how to filter and recover a random signal from a time series that that has a linear combination of confounding noise.

```
In [1]: #%install_ext https://raw.githubusercontent.com/rasbt/watermark/master/watermark.py
        %reload_ext watermark
        %watermark -p numpy,scipy,pandas,matplotlib
```

```
numpy 1.10.1
scipy 0.16.0
pandas 0.16.2
matplotlib 1.4.0
```

```
In [2]: import matplotlib.pyplot as plt
        import numpy as np
        import pandas as pd
        import scipy.signal as signal
        %matplotlib inline
```

```
In [3]: df = pd.read_csv("/home/daniel/git/Python2.7/DataScience/notebooks/TimeSeries/data
```

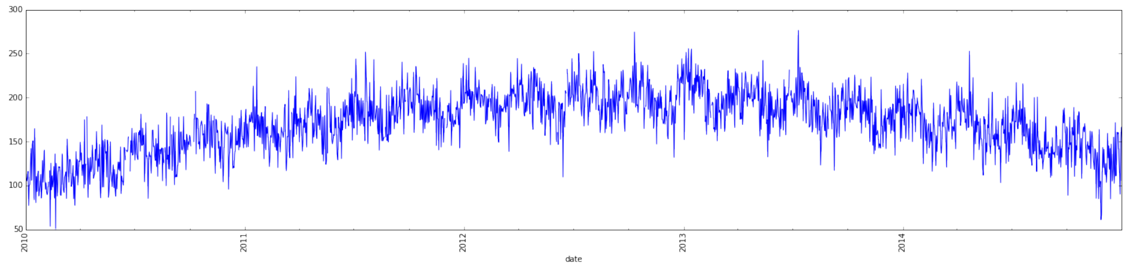
```
In [4]: df.head()
```

```
Out[4]:          date day.of.week  car.count  weather
        0  2010-01-01      friday       94.5     -0.1
        1  2010-01-02    saturday      108.4     -2.4
        2  2010-01-03      sunday      105.5     -0.5
        3  2010-01-04      monday      109.6     -2.1
        4  2010-01-05     tuesday      116.1      1.9
```

```
In [5]: # I like using Pandas b/c of the datetime features, resample or groupby
        df['date'] = pd.to_datetime(df['date'])
        df.set_index(df['date'], inplace=True)
```

1.2 Initial Plot

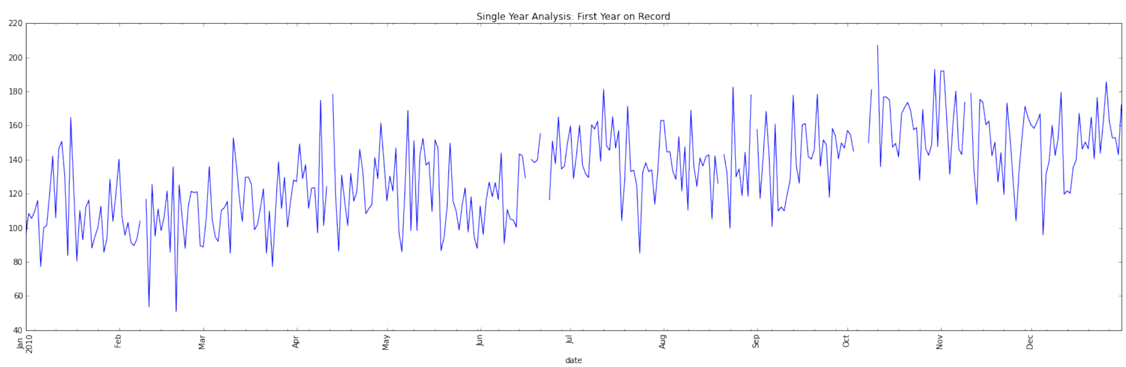
```
In [6]: df['car.count'].plot(rot=90, figsize=(25, 5));
```



I'd guess that we have a linear combination of a quadratic, sinusoid and random stochastic signal.

1.2.1 Single Year Analysis: first year in the record

```
In [7]: # year = df['car.count'][0:365]  if you are in a rush
        year = df[df['date'] < pd.to_datetime('20110101')]['car.count']
        year.plot(rot=90, figsize=(25, 7), title="Single Year Analysis: First Year on Reco
```



It's not easy to see, but there are missing values in the series. We need to treat those.

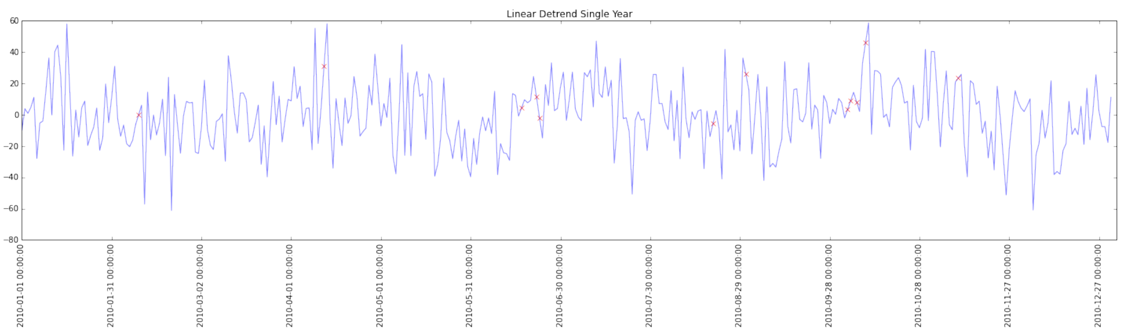
```
In [8]: #TODO: add to signal processing module

        def remove_nans(data, return_nan_index=False):
            nan_ind = np.nonzero(~np.isfinite(data))[0]
            good_data_ind = np.nonzero(np.isfinite(data))[0]
            good_data = data[good_data_ind]

            new_points = np.interp(nan_ind, good_data_ind, good_data)
            data[nan_ind] = new_points

            if return_nan_index:
                return data, nan_ind
            else:
                return data
```

```
In [9]: year, nan_ind = remove_nans(year.copy(), return_nan_index=True)
        year_linear_det = signal.detrend(year, axis=0, type='linear')
```

Single Year Linear Detrend Time Series

```
In [10]: plt.figure(figsize=(25, 5))
         plt.xticks(np.arange(year.shape[0])[0::30], year.index[0::30], rotation=90)
         plt.plot(year_linear_det, alpha=0.5)
         plt.title("Linear Detrend Single Year")
         plt.plot(nan_ind, year_linear_det[nan_ind], 'rx')
         plt.xlim(0, 366)
```

```
Out[10]: (0, 366)
```



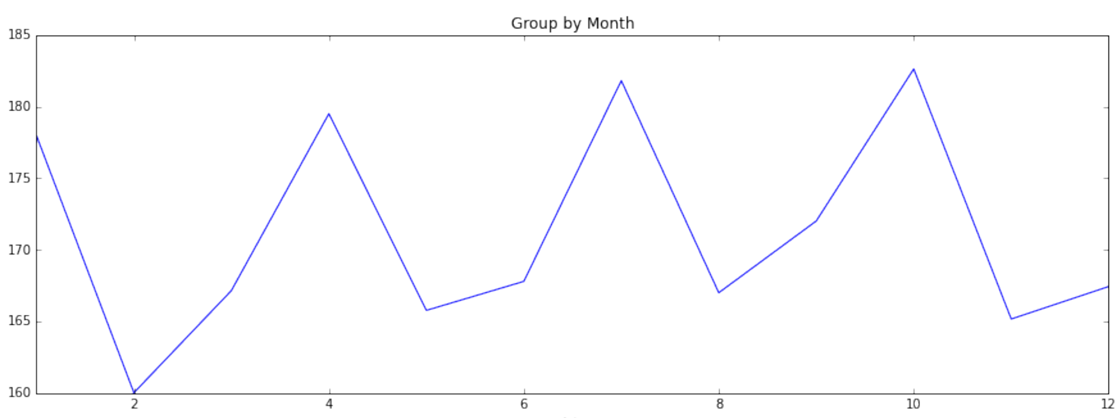
We see a ≈ 90 day period here.
Without more insight about the data we don't know if this is a nuisance or a feature we are looking for.
The NaN replacements look reasonable.

1.3 Group By for Basic Analysis

Grouping the data points into bins and taking the mean, is very simialr to a Fourier Transform.
Pandas makes this easy and there's no reason not to. Especially if the data is related to business trends.

1.3.1 Group by Month: Global monthly trend averaging over the 5 samples of each month

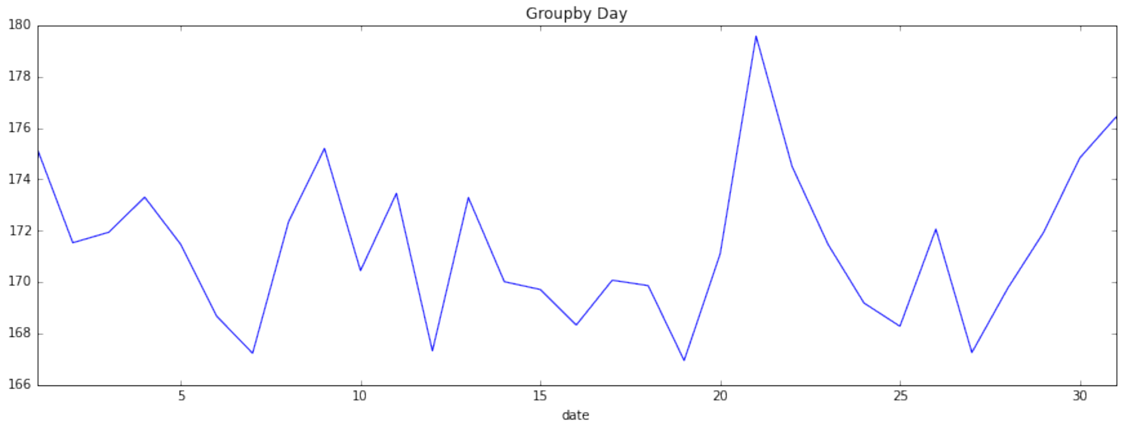
```
In [11]: grp = df.groupby(df.date.map(lambda x:x.month))
         grp.mean()['car.count'].plot(figsize=(15,5), title="Group by Month");
```



A group-by is sort of like a Fourier Transform where we choose just one frequency bin. There's the sinusodial period ≈ 90 days.

1.3.2 Group by Day: Global day trend averaging over the 5 samples of each day

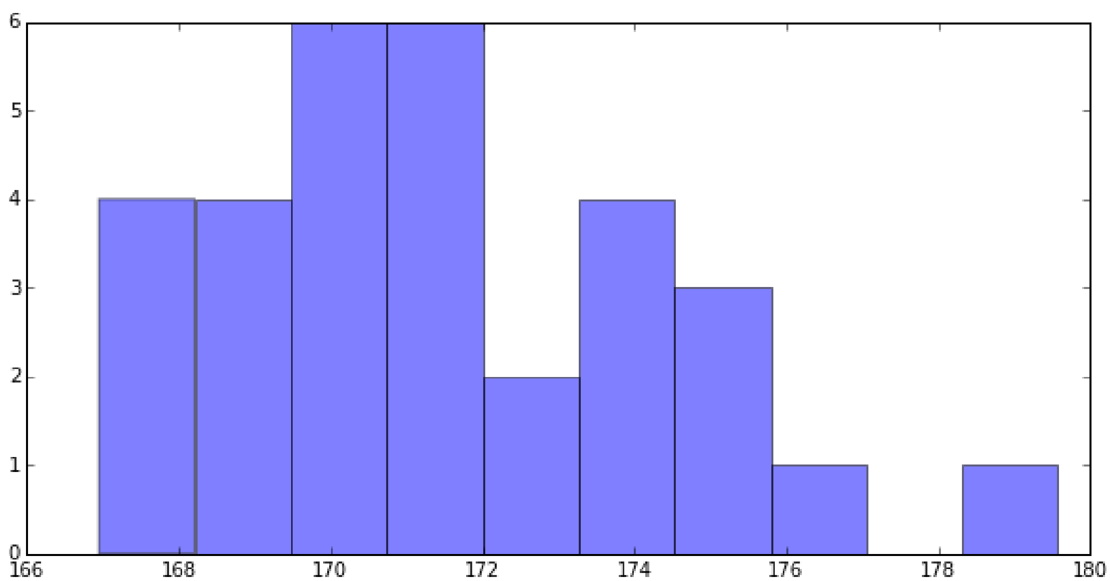
```
In [12]: grp = df.groupby(df.date.map(lambda x:x.day))
         grp.mean()['car.count'].plot(figsize=(15,5), title="Groupby Day");
```



It would appear as though the 21st day of each month saw greater count. However, we should check to see if it is statiscally significant.

The histogram below suggests that the 21st day of each month tends to have a higher count.

```
In [120]: data = np.array(grp.mean()['car.count'])

          plt.figure(figsize=(10, 5))
          plt.hist(data, bins=10, alpha=0.5);
```

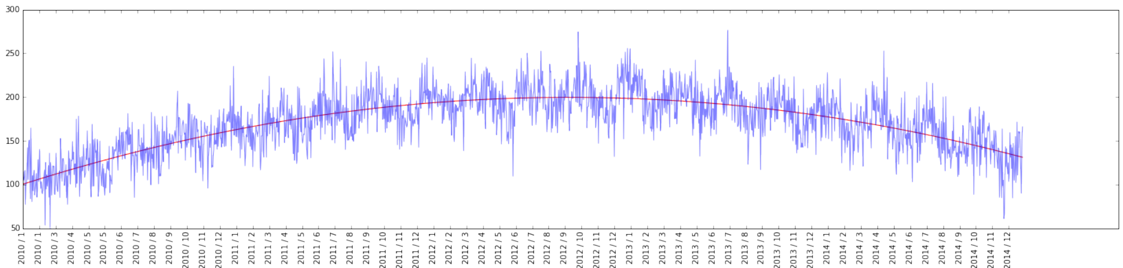


1.4 Removing Confounds

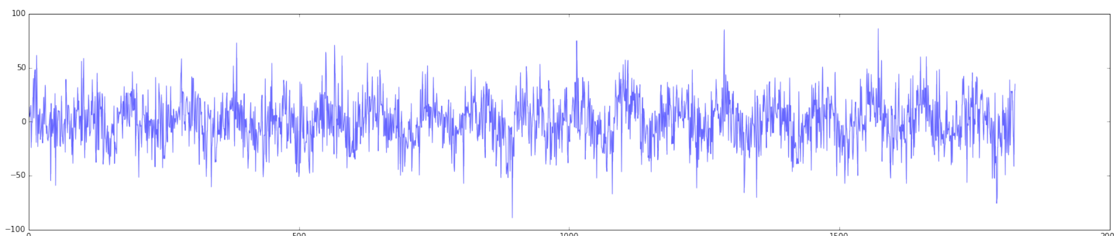
1.4.1 Quadratic Detrend Using PolyFit

```
In [13]: poly =  np.polynomial.polynomial
         counts = remove_nans(df['car.count'].copy(), return_nan_index=False)

         t = np.arange(df.shape[0])
         coefs = poly.polyfit(t, counts, deg=2, full=False)
         fit_curve = poly.polyval(t, coefs)
```

```
In [14]: plt.figure(figsize=(25, 5))

         plt.plot(fit_curve, 'r')
         plt.plot(t, counts, alpha=0.5)

         labels = df['date']
         date_str = map(lambda x: str(x.year) + " / "  + str(x.month), labels)
         plt.xticks(t[0::30], date_str[0::30], rotation="vertical");
```




```
In [15]: det_curve = counts - fit_curve
         plt.figure(figsize=(25, 5))
         plt.plot(det_curve, alpha=0.6);
```



1.5 Further Confound Removal

Lets suppose that the quadratic is a measurement error and that the ≈ 90 day sinusodial is a well understood or nuisance, then we'll examine the remainder of the signal.

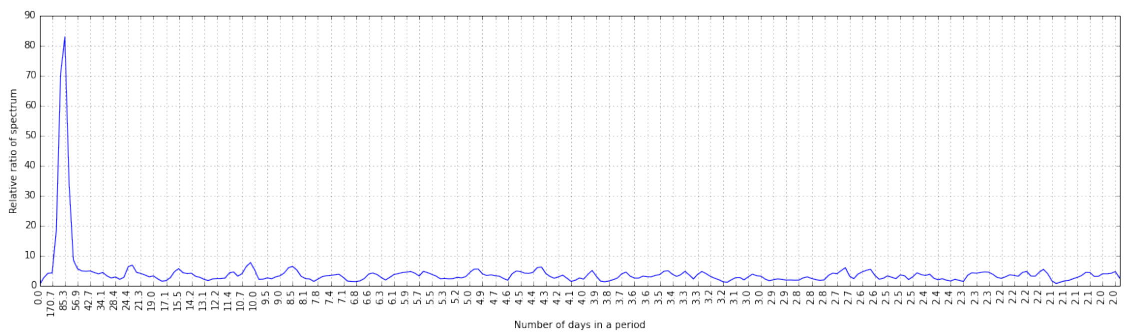
```
In [16]: import sys
         sys.path.append("/home/daniel/git/Python2.7/MRI/Modules")
         import SignalProcessTools

         sigtools = SignalProcessTools.SignalProcessTools()
```

1.5.1 Frequency Domain Analysis Using FFT

I keep this method handy and it should be in my Sigtools Module. It's just as well that you can see inside the Welch call.

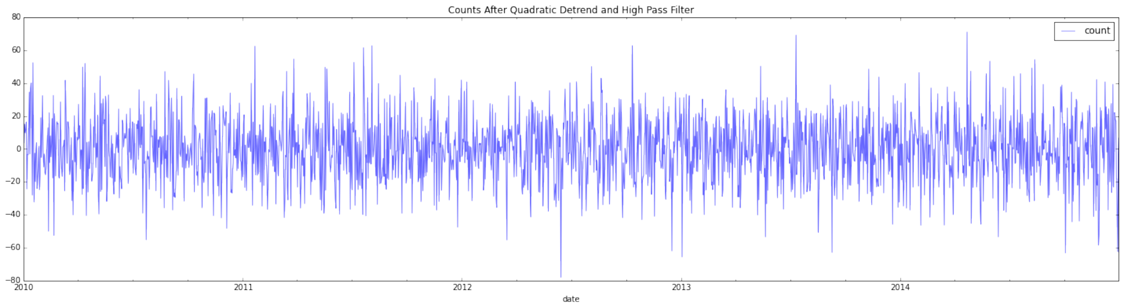
```
In [17]: def fft(data):
             '''Plot FFT using Welch's method, daily resolution '''
             f, y = signal.welch(data, fs=1.0, nperseg=365, noverlap=None, nfft=512, sc

             interval = 3 # days
             periods = np.round(1./f[0::interval], 1)
             # clean up frequency of 0 Hz
             periods[0] = 0

             frqs = f[0::interval]
             plt.xticks(frqs, periods, rotation="vertical")

             plt.plot(f, y)

             plt.grid(True)
             #plt.title("Welch FFT: Counts")
             plt.ylabel("Relative ratio of spectrum")
             plt.xlabel("Number of days in a period")

             return f, y, frqs
```

```
In [18]: plt.figure(figsize=(20, 5))
         f, y, frqs = fft(det_curve)
```




```
In [19]: frq = 1 / 56.9 # from FFT output above
         out = sigtools.hi_pass_filter(det_curve, frq, 1.0, 3)

         dff = pd.DataFrame({'count':out}, index=df.index)
         dff.plot(title="Counts After Quadratic Detrend and High Pass Filter", figsize=(25,
```

```
Out[19]: <matplotlib.axes._subplots.AxesSubplot at 0x7f226c540210>
```

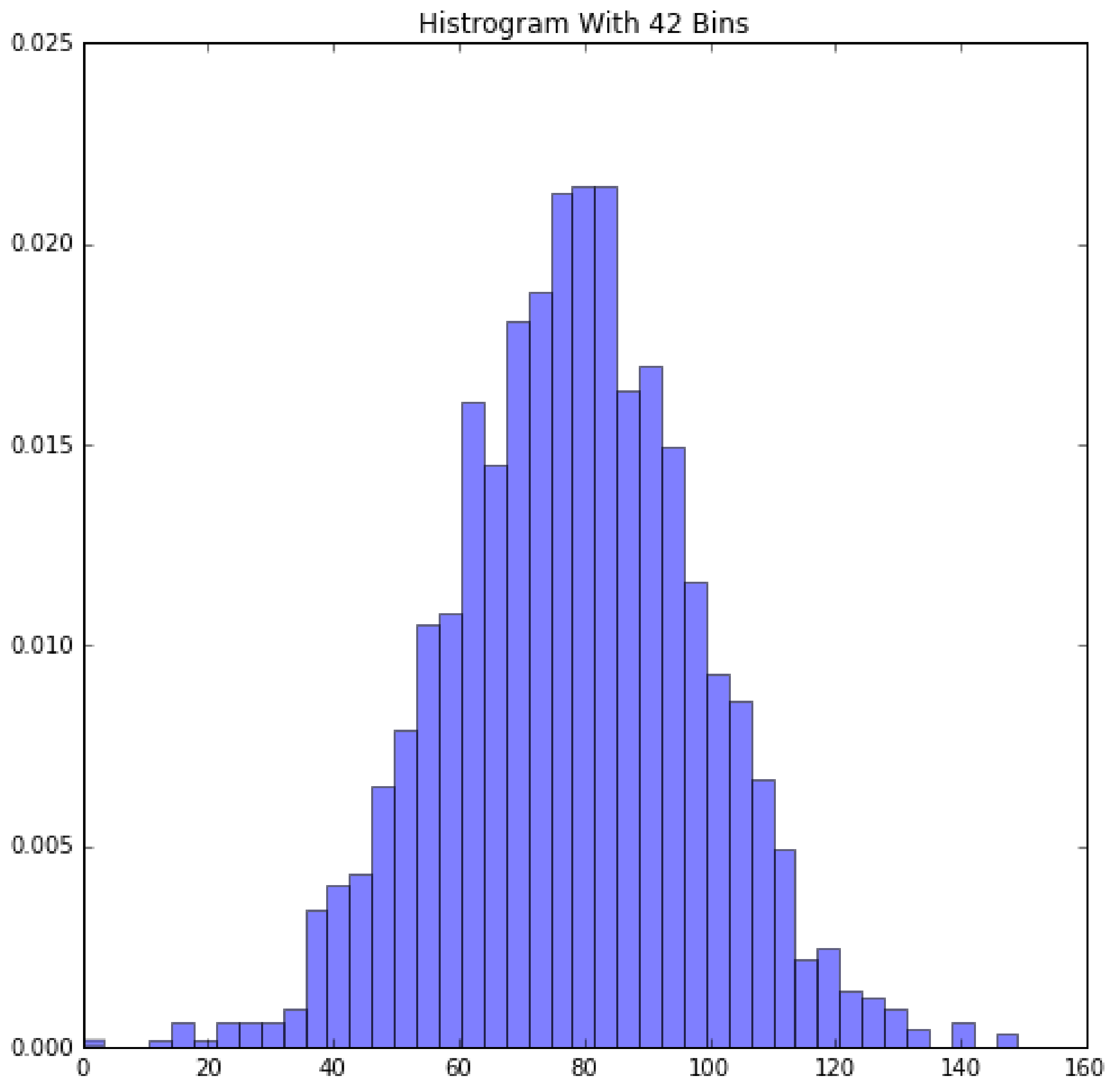


1.6 Fit A Distribution

Sometime it makes sense to fit the data to a distribution and rely upon the distribution for the parameters like a mean and variance.

Let's assume that's a great thing to do here.

```
In [20]: plt.figure(figsize=(8, 8))

         # undo the centering that occures fromt he previous procesing
         count = dff['count'] - dff['count'].min()

         nbins = np.floor(np.sqrt(count.shape[0]))
         y_counts, bin_x, patch = plt.hist(count, nbins, normed=True, alpha=0.5);
         string = "Histrogram With %s Bins" % str(int(nbins))
         plt.title(string)

Out[20]: <matplotlib.text.Text at 0x7f226c307510>
```



Perhaps a Gausian

```
In [21]: bin_x = bin_x[1:]# drop the first bin to match the array lengths
```

```
In [22]: from scipy.optimize import curve_fit

         def gauss(x, *p):
             A, mu, sig = p
             gau = A * np.exp(-(x-mu)**2 / (2 * sig)**2)
             return gau
```

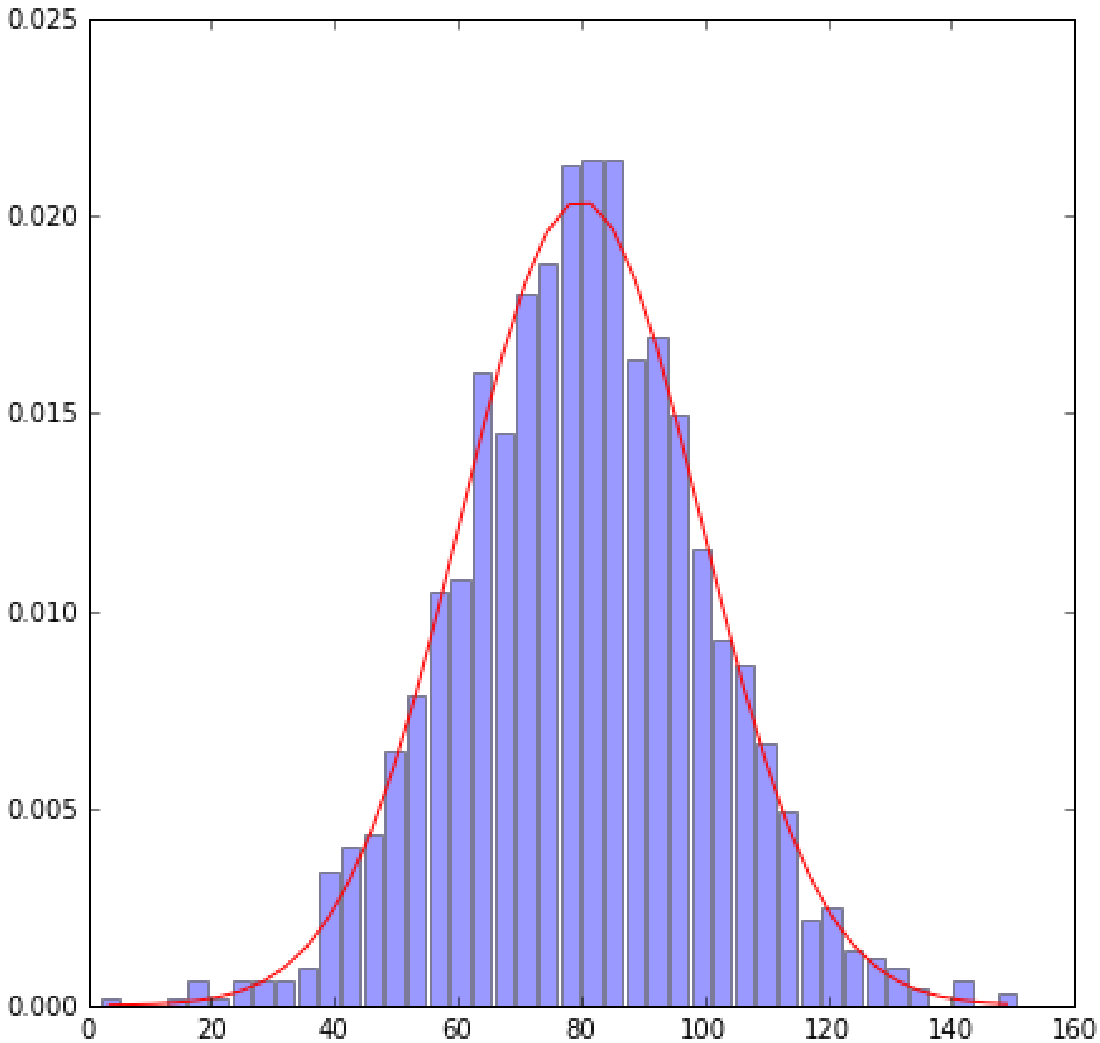
```
In [23]: coeff, var_matrix = curve_fit(gauss, bin_x, y_counts, p0=[0.001, 0.0001, 20.0])
         print "Amplitude:%f  mean:%f   std:%f" %(coeff[0], coeff[1], coeff[2])
```

```
Amplitude:0.020397  mean:79.954072   std:13.724083
```

```
In [24]: fit_gau = gauss(bin_x, *coeff)
         plt.figure(figsize=(7, 7))
```

```
        plt.plot(bin_x, fit_gau, 'r')
        plt.bar(bin_x, y_counts, alpha=0.4, width=3, align='center')
```

Out[24]: <Container object of 42 artists>



1.7 Jackknife Bias Estimation

Since the time dependant trends have been removed, the remaining records should be independent identically distributed records, or close to it.

We are putting a lot of weight ont he Guassian fit above. That mean and standard deviation are the main metrics that describe the data. To find out if our method of computing the mean and sigma are biased, we can use a resampling technique. The most simple is the Jackknife.

```
In [25]: ## Helper functions for the jackknife iterations

         def jk_gau_fit(sample):
```

```
            nbins = np.floor(np.sqrt(sample.shape[0]))
            y_counts, bin_x = np.histogram(sample, nbins, normed=True); # slightly differe
            bin_x = bin_x[1:]
            coeff, var_matrix = curve_fit(gauss, bin_x, y_counts, p0=[0.001, 0.0001, 20.0]

            return coeff[2] # amp, mean, sig

        def jk_params(data):
            sig = np.zeros_like(data)
            n = data.shape[0]
            for i in range(n):
                sample = np.delete(data, i)
                sig[i] = jk_gau_fit(sample)

            return sig
```

```
In [31]: count = np.array(count) # cast from dataframe/series into numpy array
         jksig = jk_params(count).mean()
```

1.7.1 Bias Calculation

```
In [45]: n = np.floor(np.sqrt(count.shape[0]))
         sig_bias = (n - 1) * (jksig - coeff[2])
```

```
In [60]: print "Original STD from Fit :   %.4f"    %coeff[2]
         print "Mean STD from Samples :   %.4f\n"  %jksig
         print "Bias:                    %07.4f" %(sig_bias)
```

```
Original STD from Fit :   13.7241
Mean STD from Samples :   13.3935

Bias:                    -13.5524
```

Update the standard deviation for the Guassian Fit.

```
In [66]: sig = coeff[2] - sig_bias

         print "sig:%.1f" %sig # rounding to three sig figs.
```

```
sig:27.3
```

The mean of the count taken to 3 sig figs.
$\mu_{count} = 80.0 \pm 27.3$