Time Series Analysis: Recovering A Stochastic Signal

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1 Time Series Analysis

1.1 Recovering A Stochastic Signal

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This is a pretty basic example of how to filter and recover a random signal from a time series that that has a linear combination of confounding noise.

This notebook has done through several iterations as I learned more about Boostrapping, Jackknifing and KDE. My signal processing here is pretty solid, however you may want to take the statistical examples with a grain of salt. Ultimately, it's probably best to use a cross validation for any statistic that is important. I work on that in a separate notebook in the near future.

```
In [346]: #%install_ext https://raw.githubusercontent.com/rasbt/watermark/master/watermark
          %reload_ext watermark
          %watermark -p numpy,scipy,pandas,matplotlib
```

```
numpy 1.10.1
scipy 0.16.0
pandas 0.16.2
matplotlib 1.4.0
```

```
In [347]: import matplotlib.pyplot as plt
          import numpy as np
          import pandas as pd
          import scipy.signal as signal
          %matplotlib inline
```

```
In [348]: df = pd.read_csv("/home/daniel/git/Python2.7/DataScience/notebooks/TimeSeries/dat
```

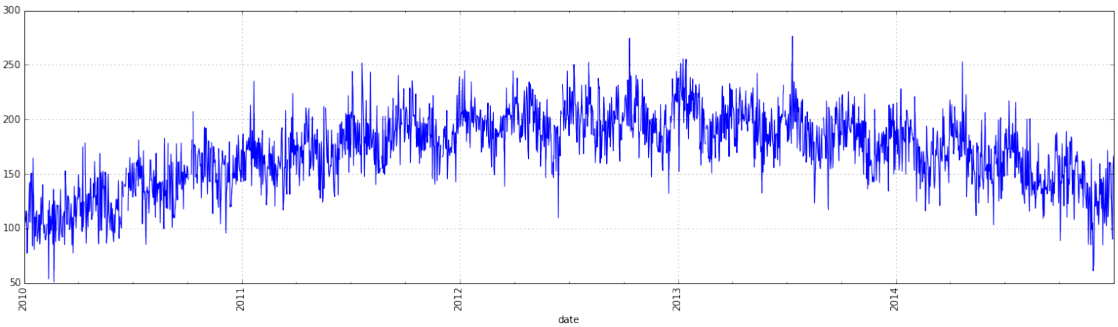
```
In [349]: df.head()
```

```
Out[349]:          date day.of.week  car.count  weather
          0  2010-01-01      friday       94.5     -0.1
          1  2010-01-02    saturday      108.4     -2.4
          2  2010-01-03      sunday      105.5     -0.5
          3  2010-01-04      monday      109.6     -2.1
          4  2010-01-05     tuesday      116.1      1.9
```

```
In [350]: # I like using Pandas b/c of the datetime features, resample or groupby
          df['date'] = pd.to_datetime(df['date'])
          df.set_index(df['date'], inplace=True)
```

1.2 Initial Plot

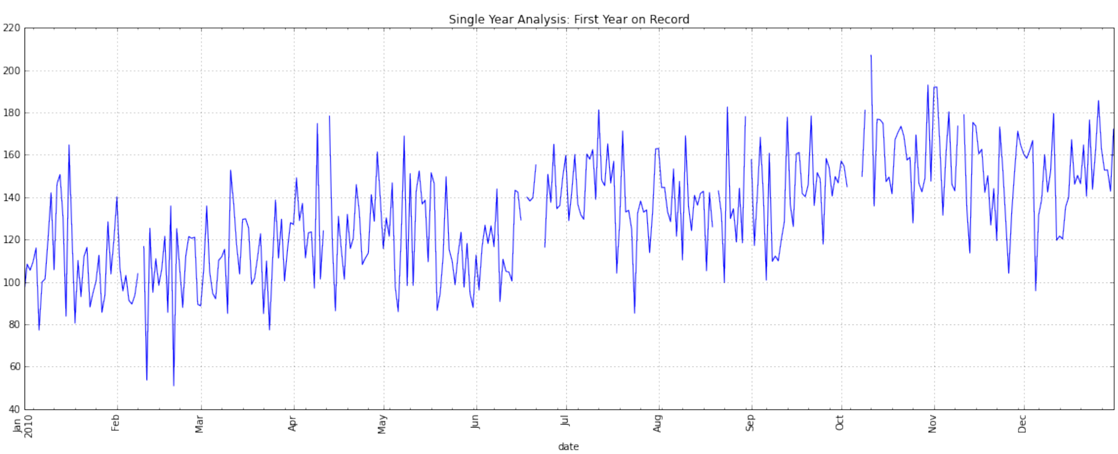
```
In [351]: df['car.count'].plot(rot=90, figsize=(20, 5), grid=True);
```



I'd guess that we have a linear combination of a quadratic, sinusoid and random stochastic signal.

1.2.1 Single Year Analysis: first year in the record

```
In [352]: # year = df['car.count'][0:365]  if you are in a rush
          year = df[df['date'] < pd.to_datetime('20110101')]['car.count']
          year.plot(rot=90, figsize=(20, 7), grid=True, title="Single Year Analysis: First
```



It's not easy to see, but there are missing values in the series. We need to treat those.

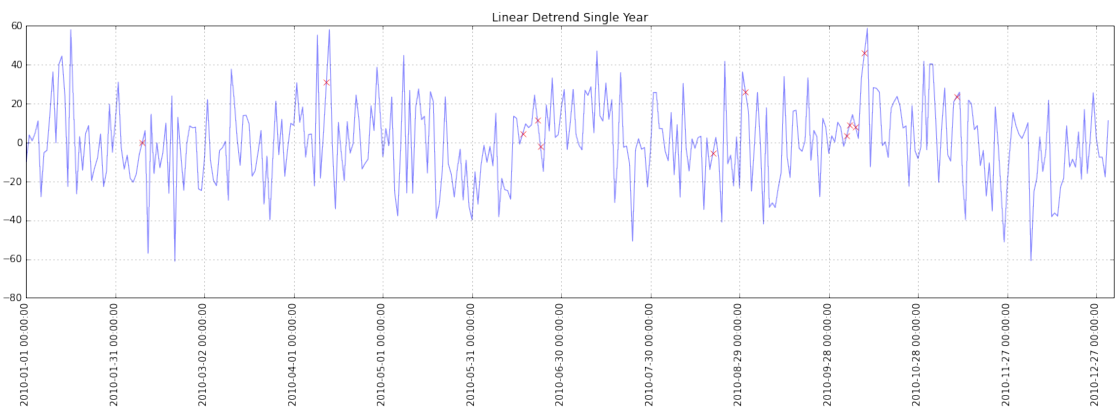
```
In [353]: #TODO: add to signal processing module

          def remove_nans(data, return_nan_index=False):
              nan_ind = np.nonzero(~np.isfinite(data))[0]
              good_data_ind = np.nonzero(np.isfinite(data))[0]
              good_data = data[good_data_ind]

              new_points = np.interp(nan_ind, good_data_ind, good_data)
              data[nan_ind] = new_points

              if return_nan_index:
```

```
                return data, nan_ind
            else:
                return data
```

```
In [354]: year, nan_ind = remove_nans(year.copy(), return_nan_index=True)
          year_linear_det = signal.detrend(year, axis=0, type='linear')
```

Single Year Linear Detrend Time Series

```
In [355]: plt.figure(figsize=(20, 5))
          plt.xticks(np.arange(year.shape[0])[0::30], year.index[0::30], rotation=90)
          plt.plot(year_linear_det, alpha=0.5)
          plt.title("Linear Detrend Single Year")
          plt.plot(nan_ind, year_linear_det[nan_ind], 'rx')
          plt.xlim(0, 366)
          plt.grid()
```



We see a ≈ 90 day period here.
Without more insight about the data we don't know if this is a nuisance or a feature we are looking for.
The NaN replacements look reasonable.
A group-by is sort of like a Fourier Transform where we choose just one frequency bin. There's the sinusodial period ≈ 90 days.

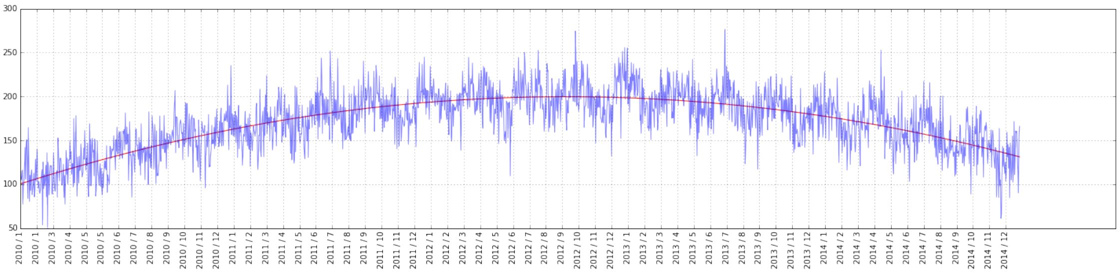
1.3 Removing Confounds

1.3.1 Quadratic Detrend Using PolyFit

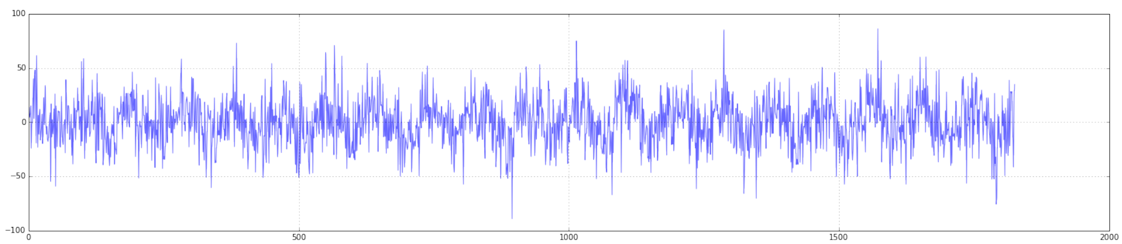
```
In [356]: poly =  np.polynomial.polynomial
          counts = remove_nans(df['car.count'].copy(), return_nan_index=False)

          t = np.arange(df.shape[0])
          coefs = poly.polyfit(t, counts, deg=2, full=False)
          fit_curve = poly.polyval(t, coefs)
```

```
In [357]: plt.figure(figsize=(25, 5))

          plt.plot(fit_curve, 'r')
          plt.plot(t, counts, alpha=0.5)
          plt.grid()
```

```
labels = df['date']
date_str = map(lambda x: str(x.year) + " / "  + str(x.month), labels)
plt.xticks(t[0::30], date_str[0::30], rotation="vertical");
```



```
In [358]: det_curve = counts - fit_curve
          plt.figure(figsize=(25, 5))
          plt.plot(det_curve, alpha=0.6);
          plt.grid()
```



1.4 Further Confound Removal

Lets suppose that the quadratic is a measurement error and that the ≈ 90 day sinusodial is a well understood or nuisance, then we'll examine the remainder of the signal.

```
In [359]: import sys
          sys.path.append("/home/daniel/git/Python2.7/MRI/Modules")
          import SignalProcessTools

          sigtools = SignalProcessTools.SignalProcessTools()
```

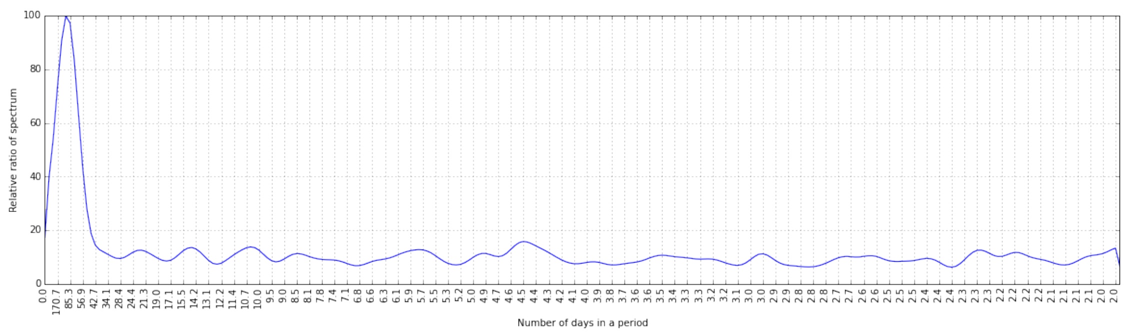
1.4.1 Frequency Domain Analysis Using FFT

I keep this method handy and it should be in my Sigtools Module. It's just as well that you can see inside the Welch call.

```
In [360]: def fft(data):
                  '''Plot FFT using Welch's method'''
                  f, y = signal.welch(data, fs=1.0, nperseg=128, noverlap=64, nfft=512, sca

                  interval = 3 # days
                  periods = np.round(1./f[0::interval], 1)
```

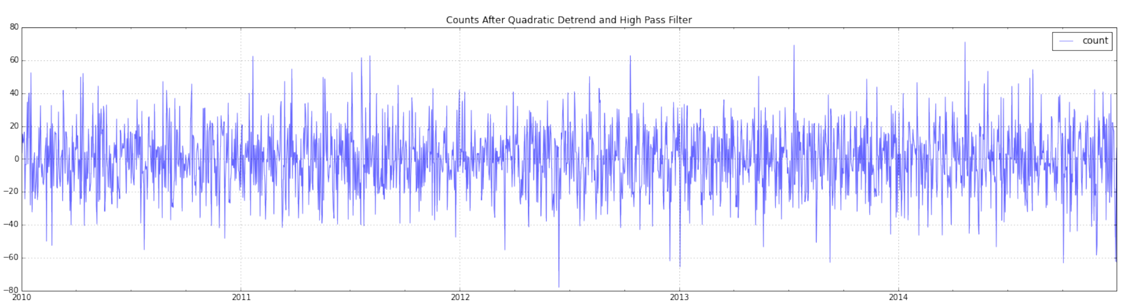
```
    # clean up frequency of 0 Hz
    periods[0] = 0 # avoid 1/ 0

    frqs = f[0::interval]
    plt.xticks(frqs, periods, rotation="vertical")

    plt.plot(f, y)
    #plt.semilogy(f, y)

    plt.grid(True)
    plt.ylabel("Relative ratio of spectrum")
    plt.xlabel("Number of days in a period")

    return f, y, frqs
```

In [361]:
```
plt.figure(figsize=(20, 5))
f, y, frqs = fft(det_curve)
```



In [362]:
```
frq = 1 / 56.9 # from FFT output above
out = sigtools.hi_pass_filter(det_curve, frq, 1.0, 3)

dff = pd.DataFrame({'count':out}, index=df.index)
dff.plot(title="Counts After Quadratic Detrend and High Pass Filter", grid=True,
```

Out[362]: <matplotlib.axes._subplots.AxesSubplot at 0x7f41f09c93d0>



2 'Group By' for Basic Analysis

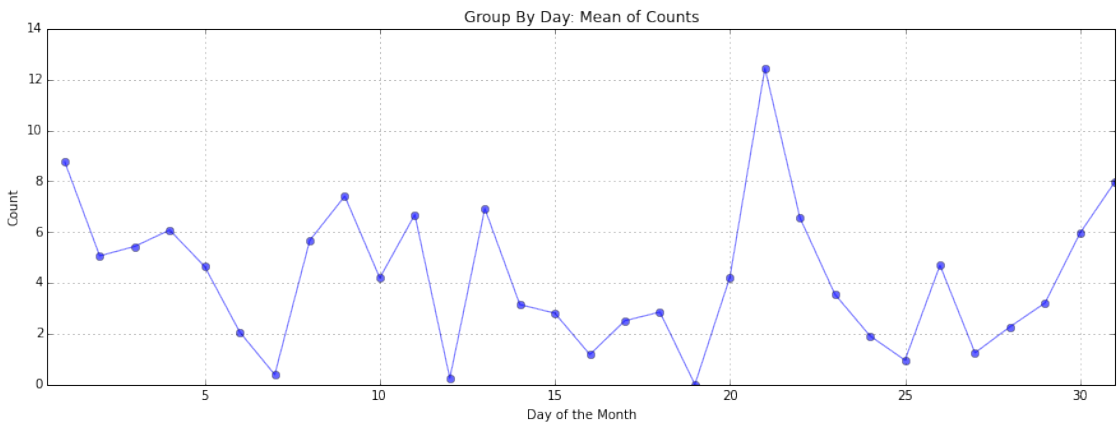
Grouping and aggregating is a simple and powerful way to gain insights into data sets. For large data, or even medium large, I use a SQL database. For small data like this, Pandas is perfect.

If we did this analysis before the detrending, then we'd see a periodic characteristic.

2.1 Tangent into statistical time series analysis

2.1.1 Group by Day: Global day trend averaging over the 5 years of 12 months = 60 data points

In [363]:
```
grp = dff.groupby(dff.index.map(lambda x:x.day)) #numerical day of the month, no
X = grp.mean()['count']
X -= X.min()

# decided to not use Pandas plot method b/c I wanted the -o style of lines
plt.figure(figsize=(15, 5))
plt.plot(X.index, X, '-o', alpha=0.6);

plt.title("Group By Day: Mean of Counts")
plt.xlabel("Day of the Month")
plt.ylabel("Count")

plt.xlim(0.5, 31)
plt.grid()
```



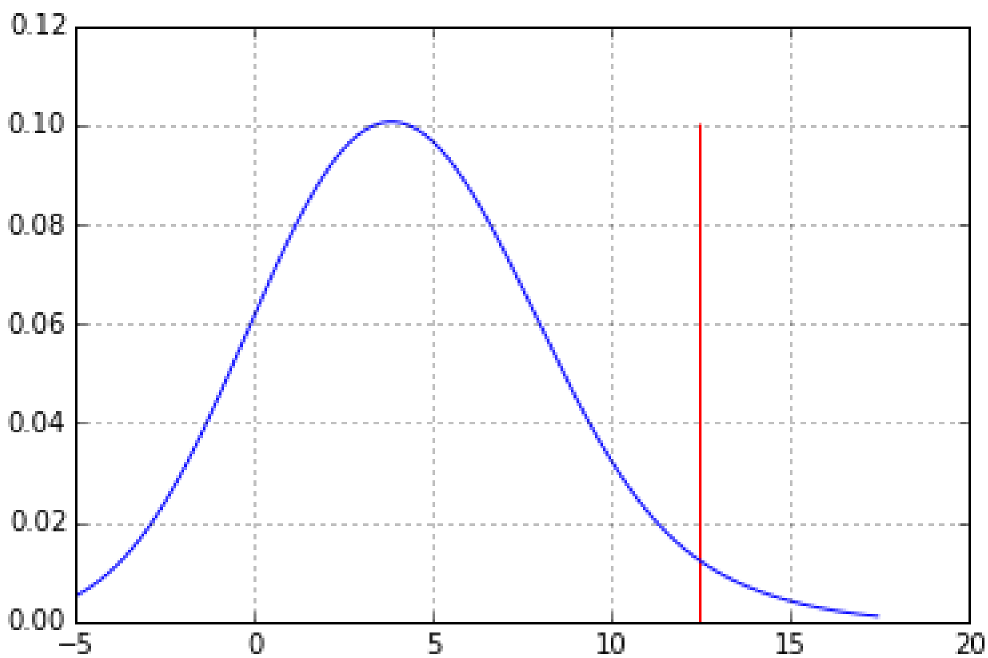
2.1.2 Significance

The density estimate below suggests that the max count on the 21st day is significant.

In [364]:
```
from scipy.stats import gaussian_kde

N = X.shape[0]
MIN = X.min() - 5
MAX = X.max() + 5
grid = np.linspace(MIN, MAX, 1000)

kde = gaussian_kde(X, bw_method=1.0)
```

```
out = kde.evaluate(grid)

plt.plot(grid, out)
plt.grid()

max_cnt_ind = X.argmax()
cnt_max = X[max_cnt_ind]

ht = out.max()
plt.vlines(cnt_max, 0, ht, 'r')

max_ind = out.argmax()
max_ = grid[max_ind]

print "Max count from density plot:     %.2f" % cnt_max
print "Mean count over days:             %.2f" % grid[max_ind]
print "Rememebr I added some extra points to make the graph pretty"
```

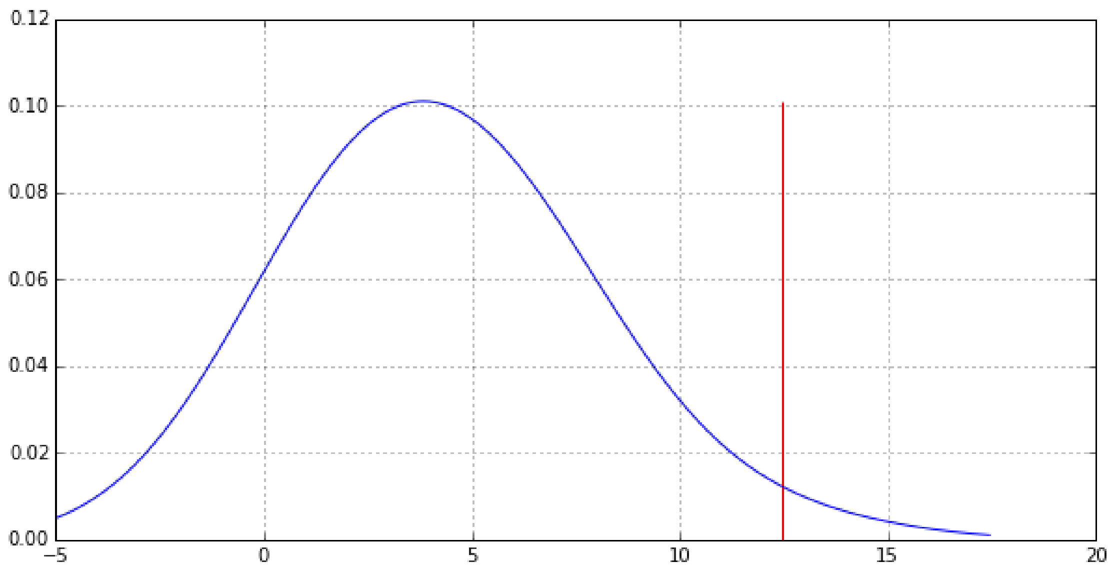
```
Max count from density plot:    12.44
Mean count over days:            3.83
Rememebr I added some extra points to make the graph pretty
```



2.2 Bootstrap for significance

```
In [365]: m = 2000
          cnt = []

          for i in range(m):
              sample = X.sample(n=N, replace=True)
```

```
        cnt.extend(sample)

    cnt = np.array(cnt)

    kde = gaussian_kde(cnt, bw_method=1.0)
    out = kde.evaluate(grid)

    plt.figure(figsize=(10, 5))
    plt.plot(grid, out)

    max_ind = cnt.argmax()
    cnt_max = cnt[max_ind]
    ht = out.max()

    plt.vlines(cnt_max, 0, ht, 'r')
    plt.grid()
```



3 Back to Complete Data Set

3.1 Kernel Density Estimate

http://docs.scipy.org/doc/scipy/reference/generated/scipy.stats.gaussian_kde.html#scipy.stats.gaussian_kde

Sometime it makes sense to fit the data to a distribution and rely upon the distribution for the parameters like a mean and variance.

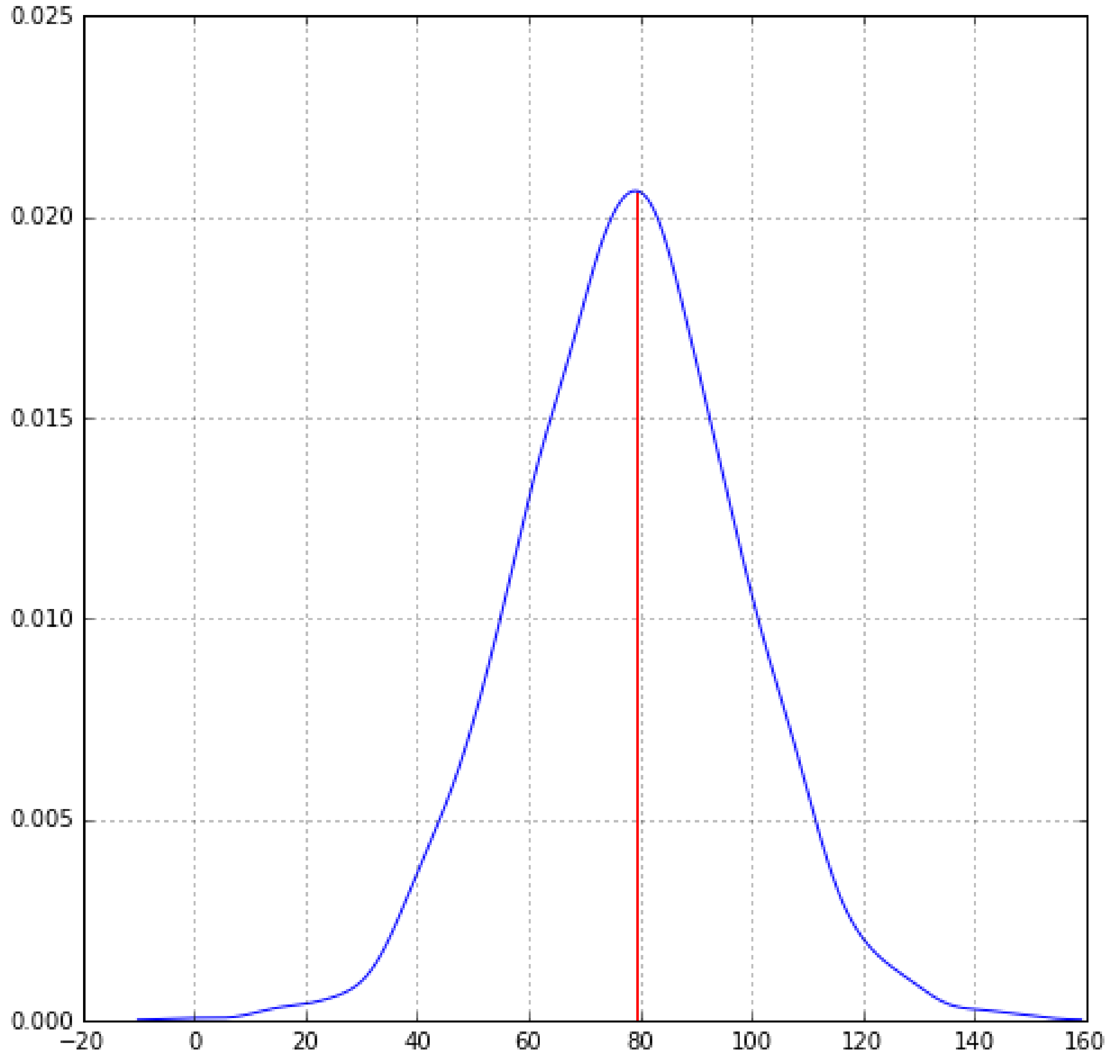
Let's assume that's a great thing to do here.

```
In [366]: # undo the centering that occures fromt he previous procesing
          count = dff['count'] - dff['count'].min()

          def comp_kde(data):
              N = data.shape[0]
              min_ = data.min() - 10
```

```
    max_ = data.max()+10
    grid = np.linspace(min_, max_, 1000)

    kde = gaussian_kde(data, bw_method=None)
    out = kde.evaluate(grid)

    return out, grid
```

```
In [367]: FIT, grid = comp_kde(count)
          plt.figure(figsize=(8, 8))
          plt.plot(grid, FIT)

          max_ind = FIT.argmax()
          MU_ = grid[max_ind]

          ht = FIT.max()
          plt.vlines(MU_, 0, ht, 'r')
          plt.grid()

          print "Mean of distribution: %.2f" % MU_
```

```
Mean of distribution: 79.18
```



3.2 Gaussian Fit

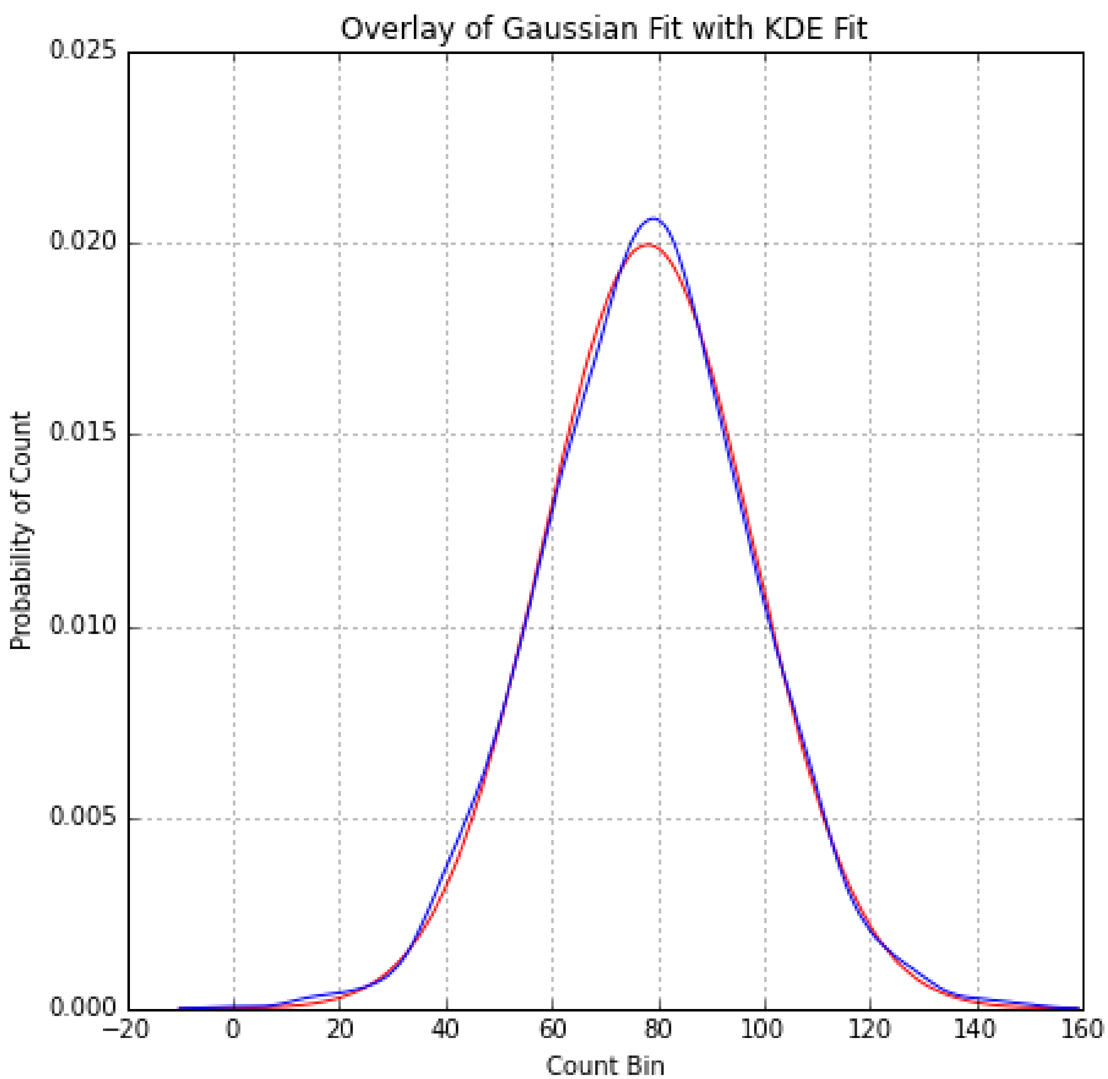
```
In [368]: from scipy.optimize import curve_fit

          def gauss(x, *p):
              A, mu, sig = p
              gau = A * np.exp(-(x-mu)**2 / (2 * sig)**2)
              return gau
```

```
In [369]: # fit the KDE to a Gaussian
          coeff, var_matrix = curve_fit(gauss, grid, FIT, p0=[0.02, 70.0, 14.0])
          print "Amplitude:%f  mean:%f   std:%f" %(coeff[0], coeff[1], coeff[2])
          fit_gau = gauss(grid, *coeff)

          plt.figure(figsize=(7, 7))
          plt.plot(grid, fit_gau, 'r');
          plt.plot(grid, FIT)
```

```
plt.grid()
plt.title("Overlay of Gaussian Fit with KDE Fit")
plt.xlabel("Count Bin")
plt.ylabel("Probability of Count")
```

```
Amplitude:0.019963  mean:78.152289   std:14.030888
```

Out[369]: <matplotlib.text.Text at 0x7f41f13d4950>



In [ ]: