Assignment 6

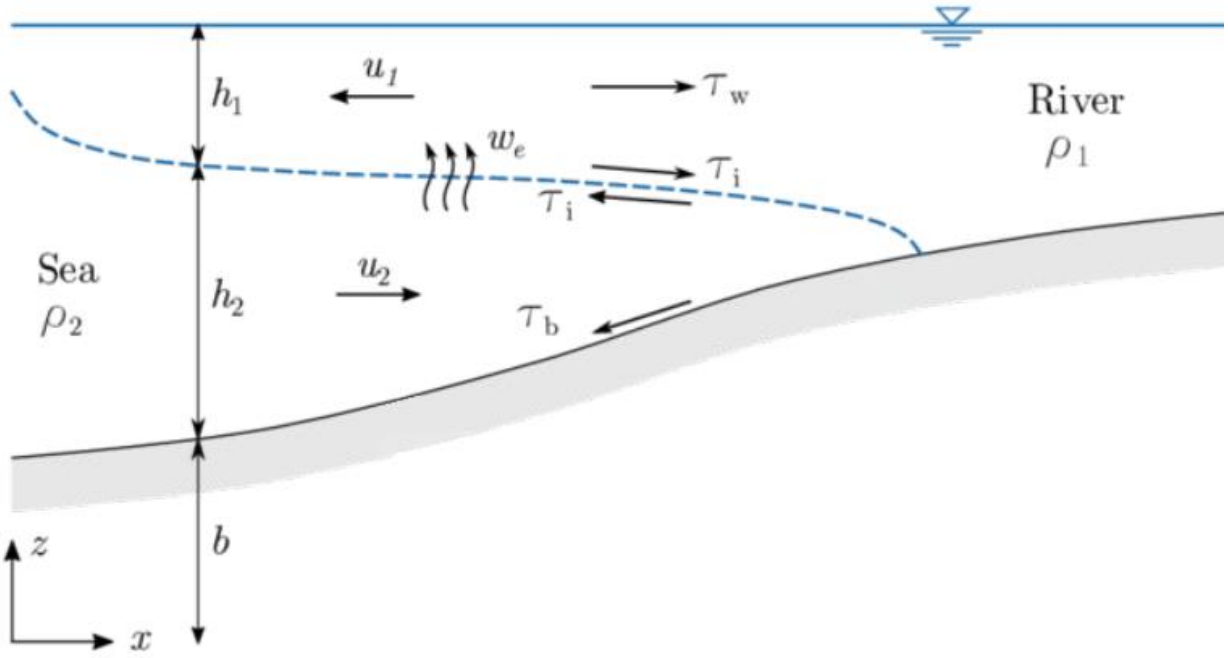

Image 1 Assignment visualization

Initial parameters:

$H = 10\,m$

$H_2 = \frac{1}{4} * H1\,m$

$H = H_1 + H_2 = \frac{5}{4} H_1$

$H_1 = 8\,m$

$H_2 = 2\,m$

$U_2 = 0.1\,\frac{m}{s}$

$\rho_2 = 1007\,\frac{kg}{m^3}$

$\rho_1 = 1001\,\frac{kg}{m^3}$

$\rho_0 = 1000\,\frac{kg}{m^3}$

$g = 9.81\,\frac{m}{s^2}$

Mixed state:

$$V_m = \frac{H_1V_1 + H_2V_2}{H} = \frac{8V_1 + 2 * 0.1}{10} = 0.8V_1 + 0.02\frac{m}{s}$$

$$\rho_m = \frac{H_1\rho_1 + H_2\rho_2}{H} = \frac{8 * 1001 + 2 * 1007}{10} = 1002\frac{kg}{m^3}$$

KE loss:

$$\int_0^{H_2} \frac{1}{2}\rho_0 U_2^2 dz + \int_{H_2}^{H} \frac{1}{2}\rho_0 U_1^2 dz - \int_0^{H} \frac{1}{2}\rho_0 U_m^2 dz = \frac{1}{2}\rho_0 U_2^2 + \frac{1}{2}\rho_0 U_1^2 (H - H_2) - \frac{1}{2}\rho_0 U_m^2 H =$$

$$= 10 + 4000U_1^2 - 5000\,(0.8U_1 + 0.02)^2 = 800U_1^2 - 160U_1 + 8$$

PE loss:

$$\int_0^{H} \rho_m g z dz - \int_0^{H_2} \rho_2 g z dz - \int_{H_2}^{H} \rho_1 g z dz = \rho_m g \frac{H^2}{2} - \rho_2 g \frac{H_2^2}{2} - \rho_2 g \left(\frac{H^2}{2} - \frac{H_2^2}{2}\right) =$$

$$= 1002 * 9.81 * \frac{100}{2} - 1007 * 9.81 * \frac{4}{2} - 1001 * 9.81 * \left(\frac{100}{2} - \frac{4}{2}\right) = 491\,481 - 19\,757 - 471\,351 = 373$$

If KE loss > PE gain, then mixing occurs:

$$800U_1^2 - 160U_1 + 8 > 373$$

After solving equation, results are:

$$U_{1,1} \geq -0.58\frac{m}{s}$$

$$U_{1,2} \geq 0.78\frac{m}{s}$$

Answer:

Fresh water flows toward the sea (minus sign) on top of salty water and must be at least U_1 = -0.58 m/s.

Salty sea water flows toward the land beneath fresh water at speed U_2 = 0.1 m/s.