TALLINNA TEHNIKAÜLIKOOL
MEHAANIKAINSTITUUT

Pideva keskkonna mehaanika kodutöö nr. 1

Deformatsiooni analüüs

Variant 18

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Pideva keskkonna liikumine on kirjeldatud seadusega:

$$\begin{cases} x_1 = \frac{2}{3}X_1 + \frac{2}{3}X_2 \\ x_2 = -X_1 + X_2 \\ x_3 = X_3 \end{cases} \qquad (1)$$

1. Teisenduse (1) pöördteisendus ning siirdevektori komponendid Lagrange'i ja Euleri kirjelduses.

Pöördteisendus

$$\begin{cases} X_1 = \frac{3}{4}x_1 - \frac{1}{2}x_2 \\ X_2 = \frac{3}{4}x_1 + \frac{1}{2}x_2 \\ X_3 = x_3 \end{cases}$$

Siirdevektori komponendid Lagrange'i kirjelduses

$$\begin{cases} U_1 = -\frac{1}{3}X_1 + \frac{2}{3}X_2 \\ U_2 = -X_1 \\ U_3 = 0 \end{cases}$$

Siirdevektori komponendid Euleri kirjelduses

$$\begin{cases} u_1 = \frac{1}{4}x_1 + \frac{1}{2}x_2 \\ u_2 = -\frac{3}{4}x_1 + \frac{1}{2}x_2 \\ u_3 = 0 \end{cases}$$

2. Elementaarkuubi tippude siirded ja uued koordinaadid ning elementaarkuubi deformeerunud kuju.

'Tähis': Esialgne asukoht + Siire = Uus punkt

$$'0': \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$'A': \begin{pmatrix} dX \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -\frac{1}{3}dX \\ -dX \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{2}{3}dX \\ -dX \\ 0 \end{pmatrix}$$

$$'B': \begin{pmatrix} dX \\ dX \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{1}{3}dX \\ -dX \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{4}{3}dX \\ 0 \\ 0 \end{pmatrix}$$

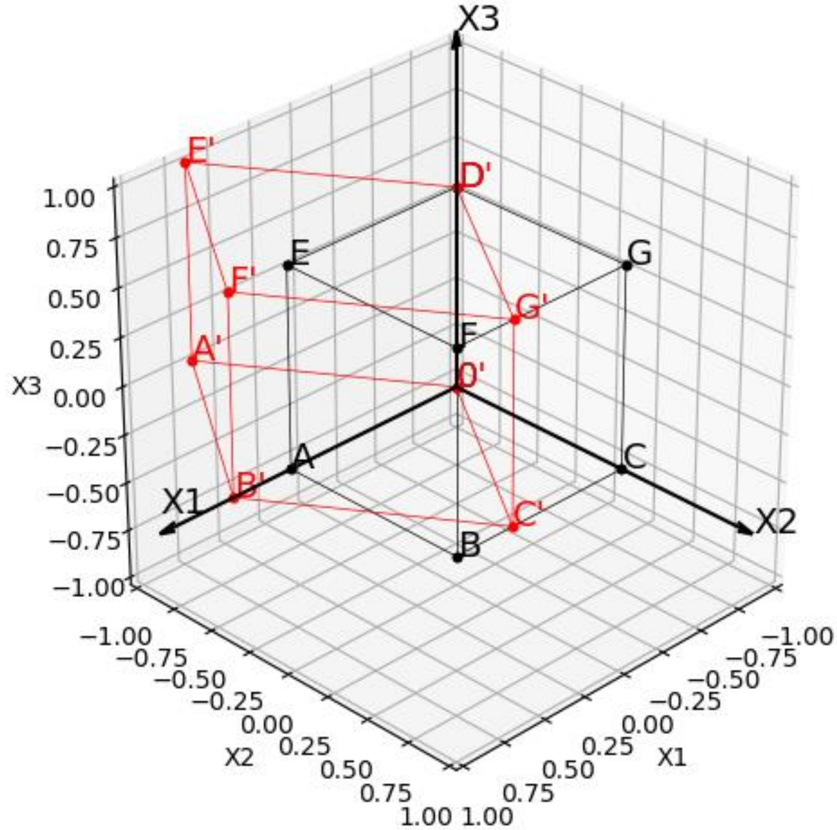
$$'C': \begin{pmatrix} 0 \\ dX \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{2}{3}dX \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{2}{3}dX \\ dX \\ 0 \end{pmatrix}$$

$$'D': \begin{pmatrix} 0 \\ 0 \\ dX \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ dX \end{pmatrix}$$

$$'E': \begin{pmatrix} dX \\ 0 \\ dX \end{pmatrix} + \begin{pmatrix} -\frac{1}{3}dX \\ -dX \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{2}{3}dX \\ -dX \\ dX \end{pmatrix}$$

$$'F': \begin{pmatrix} dX \\ dX \\ dX \end{pmatrix} + \begin{pmatrix} \frac{1}{3}dX \\ -dX \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{4}{3}dX \\ 0 \\ dX \end{pmatrix}$$

$$'G': \begin{pmatrix} 0 \\ dX \\ dX \end{pmatrix} + \begin{pmatrix} \frac{2}{3}dX \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{2}{3}dX \\ dX \\ dX \end{pmatrix}$$



Esialgne kuup (must) ja deformeerunud kuup (punane)

3. Punktist (0,0,0) väljuva elementaarkuubi diagonaali pikkuse ruut pärast deformatsiooni.

Deformatsioonigradient

$$x_{k,K} = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Valemi järgi $ds^2 = x_{k,K} x_{k,L} dX_K dX_L$ arvutamine

$$x_{k,K} \cdot x_{k,L} = \begin{pmatrix} \frac{13}{9} & -\frac{5}{9} & 0 \\ -\frac{5}{9} & \frac{13}{9} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$ds^2 = \frac{13}{9}(dX_1)^2 + \frac{13}{9}(dX_2)^2 + 1(dX_3)^2 + 2\left(-\frac{5}{9}\right) dX_1 dX_2 + 0 dX_1 dX_3 + 0 dX_2 dX_3 = \frac{25}{9} dX^2$$

4. Punktist (0,0,0) väljuvate elementaarkuubi servade (OA, OC, OD) ja diagonaalide (OE, OB, OG, OF) pikenemiskoefitsendid Λ ja pöördenurgad β.

Greeni deformatsioonitensor

$$C_{KL} = \begin{pmatrix} \frac{13}{9} & -\frac{5}{9} & 0 \\ -\frac{5}{9} & \frac{13}{9} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Servade pikenemiskoefitsendid

$\Lambda_{OA} = \sqrt{C_{11}} = \sqrt{1.44} \approx 1.2$ $\Lambda_{OC} = \sqrt{C_{22}} = \sqrt{1.44} \approx 1.2$ $\Lambda_{OD} = \sqrt{C_{33}} = \sqrt{1} = 1.0$

Diagonaalide pikenemiskoefitsendid ds^2 valemiga

$$\mathrm{N}_{OE} = (\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}})$$

$$\Lambda_{OE} = \sqrt{\frac{13}{9} \cdot (\frac{1}{\sqrt{2}})^2 + \frac{13}{9} \cdot 0^2 + 1 \cdot (\frac{1}{\sqrt{2}})^2 + 2 \cdot (-\frac{5}{9}) \cdot \frac{1}{\sqrt{2}} \cdot 0} \approx 1.11$$

$$\mathrm{N}_{OB} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$$

$$\Lambda_{OB} = \sqrt{\frac{13}{9} \cdot (\frac{1}{\sqrt{2}})^2 + \frac{13}{9} \cdot (\frac{1}{\sqrt{2}})^2 + 1 \cdot 0^2 + 2 \cdot (-\frac{5}{9}) \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}}} \approx 0.94$$

$$\mathrm{N}_{OG} = (0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$$

$$\Lambda_{OG} = \sqrt{\frac{13}{9} \cdot 0^2 + \frac{13}{9} \cdot (\frac{1}{\sqrt{2}})^2 + 1 \cdot (\frac{1}{\sqrt{2}})^2 + 2 \cdot (-\frac{5}{9}) \cdot 0 \cdot \frac{1}{\sqrt{2}}} \approx 1.11$$

$$\mathrm{N}_{OF} = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$$

$$\Lambda_{OF} = \sqrt{\frac{13}{9} \cdot (\frac{1}{\sqrt{3}})^2 + \frac{13}{9} \cdot (\frac{1}{\sqrt{3}})^2 + 1 \cdot (\frac{1}{\sqrt{3}})^2 + 2 \cdot (-\frac{5}{9}) \cdot \frac{1}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}} \approx 0.96$$

Pöördenurgad valemist $\cos\beta = \frac{\boldsymbol{v_1} \cdot \boldsymbol{v_2}}{|\boldsymbol{v_1}| \cdot |\boldsymbol{v_2}|}$

$\beta_{0A} = 56.31^o$ $\beta_{0C} = 33.7^o$ $\beta_{0E} = 41.1^o$

$\beta_{0B} = 45.0^o$ $\beta_{0D} = 0.0^o$ $\beta_{0F} = 36.1^o$

5. Algul X_1 ja X_2sihiliste servade vahelise täisnurga muutus deformatsioonil. Leida nurk $\Gamma_{(X_1,X_2)}$.

$\beta = \arccos\frac{\boldsymbol{OA'}\cdot\boldsymbol{OC'}}{|\boldsymbol{OA'}|\cdot|\boldsymbol{OC'}|} = 112.6^o$ $\quad\quad\quad$ $\Gamma_{(X_1,X_2)} = \frac{\pi}{2} - \beta = 90 - 112.6 = -22.6^0$

6. Deformatsioonitensorite maatriksid.

Deformatsioonigradiendid

$$x_{k,K} = \begin{pmatrix} \frac{2}{3} & \frac{2}{3} & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad\quad X_{K,k} = \begin{pmatrix} \frac{3}{4} & -\frac{1}{2} & 0 \\ \frac{3}{4} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Cauchy deformatsioonitensor c_{kl}

$$c_{kl} = X_{K,k} \cdot X_{K,l} = \begin{pmatrix} 1.125 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Greeni deformatsioonitensor C_{KL}

$$C_{KL} = x_{k,K} \cdot x_{k,L} = \begin{pmatrix} 1.444 & -0.556 & 0 \\ -0.556 & 1.444 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Lagrange'i deformatsioonitensor E_{KL}

$$E_{KL} = \frac{1}{2}(C_{KL} - \delta_{KL}) = \begin{pmatrix} 0.222 & -0.278 & 0 \\ -0.278 & 0.222 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Euleri deformatsioonitensor e_{kl}

$$e_{kl} = \frac{1}{2}(\delta_{kl} - c_{kl}) = \begin{pmatrix} -0.062 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

7. Tensorite C_{KL} ja c_{kl} peaväärtused C_α ja c_α ning peasuunad N_α ja n_α. Peasuunad joonisel.

Greeni peaväärtused valemist $(C_{KL} - C\delta_{KL})N_L = 0$. Reastatud $C_1 > C_2 > C_3$.

$C_1 = 2$ $\qquad$ $C_2 = 1$ $\qquad$ $C_3 = \frac{8}{9}$

Greeni peasuunad

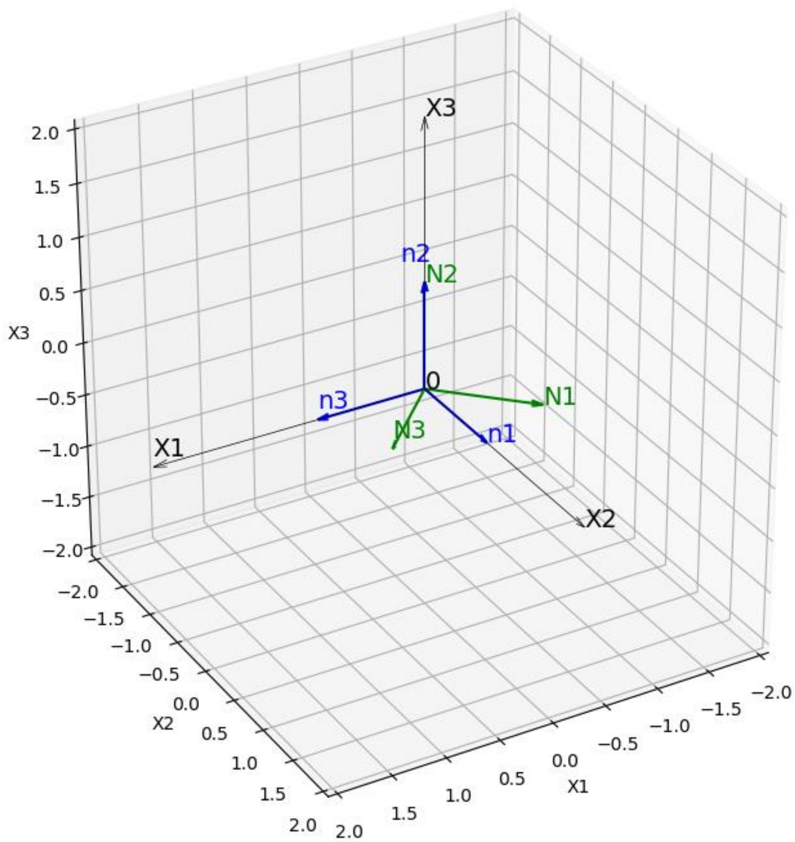
$N_1 = [-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0]$ $\qquad$ $N_2 = [0, 0, 1]$ $\qquad$ $N_3 = [\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0]$

Cauchy peaväärtused valemist $(c_{kl} - c\delta_{kl})n_L = 0$. Reastatud $c_1 < c_2 < c_3$.

$c_1 = 0.5$ $\qquad$ $c_2 = 1$ $\qquad$ $c_3 = 1.125$

Cauchy peasuunad

$n_1 = [0,1,0]$ $\qquad$ $n_2 = [0,0,1]$ $\qquad$ $n_3 = [1,0,0]$



Cauchy (sinised) ja Greeni (rohelised) peasuunad

8. Leida peasuundadele vastavad pikenemiskoefitsendid Λ_α ja suhtelised pikenemised E_α.

$\Lambda_1 = \sqrt{\mathrm{C}_1} = \sqrt{2} = 1.41$

$\Lambda_2 = \sqrt{\mathrm{C}_2} = \sqrt{1} = 1$

$\Lambda_3 = \sqrt{\mathrm{C}_3} = \sqrt{0.889} = 0.94$

$\mathrm{E}_1 = \sqrt{\mathrm{C}_1} - 1 = 0.41$

$\mathrm{E}_2 = \sqrt{\mathrm{C}_2} - 1 = 0$

$\mathrm{E}_3 = \sqrt{\mathrm{C}_3} - 1 = -0.06$

9. Pöördetensori R_{kK}maatriks.

$$R_{kK} = n_{k\alpha} \cdot N_{\alpha K} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

10. Materiaalse ja ruumilise deformatsiooniellipsoidi pooltelgede pikkused. Ellipsoidid joonisel.

Valemid $C_\alpha (dX_\alpha)^2 = k^2$, $a_\alpha^s = \frac{k}{\sqrt{C_\alpha}}$ (ruumiline) ja $a_\alpha^m = k\sqrt{C_\alpha}$ (materiaalne).

Greeni peaväärtused

$C_1 = 2$

$C_2 = 1$

$C_3 = \frac{8}{9}$

$$C_\alpha (dX_\alpha)^2 = 2(dX_1)^2 + 1(dX_2)^2 + \frac{8}{9}(dX_3)^2 = \frac{35}{9}(dX)^2 = k^2 \Rightarrow k = \sqrt{\frac{35}{9}} \approx 1.97$$

Ruumilise (Greeni suunal) deformatsiooniellipsoidi telgede pikkused

$a_1^s = \frac{k}{\sqrt{C_1}} \approx 1.39$

$a_2^s = \frac{k}{\sqrt{C_2}} \approx 1.97$

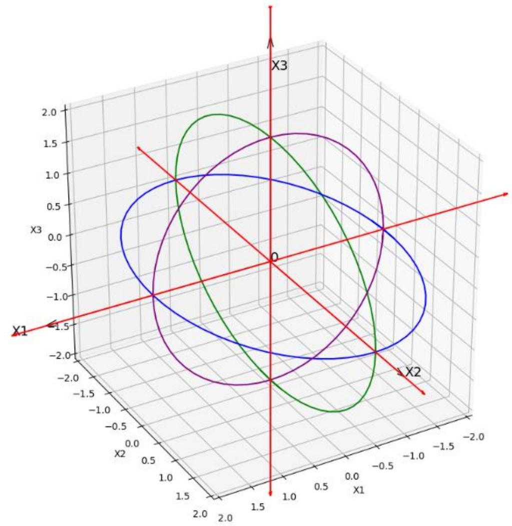
$a_3^s = \frac{k}{\sqrt{C_3}} \approx 2.09$

Materiaalse (Cauchy suunal) deformatsiooniellipsoidi telgede pikkused

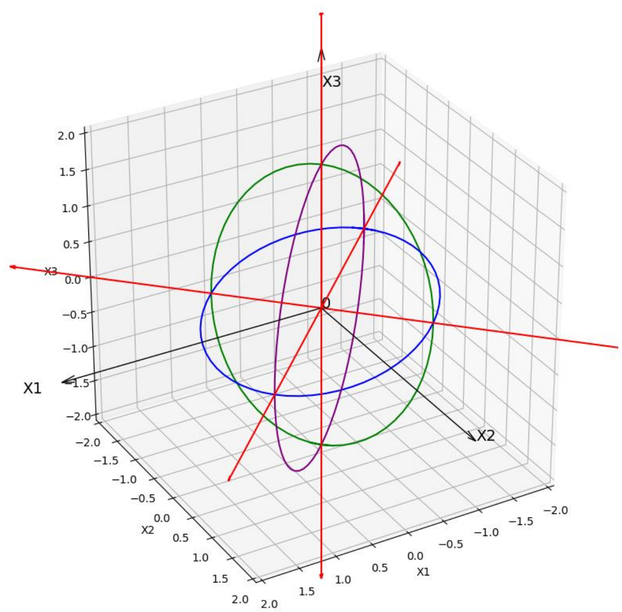
$a_1^m = k \cdot \sqrt{C_1} \approx 2.79$

$a_2^m = k \cdot \sqrt{C_2} \approx 1.97$

$a_3^m = k \cdot \sqrt{C_3} \approx 1.86$



Materiaalne deformatsiooniellipsoid (Cauchy peavektorid)



Ruumiline deformatsiooniellipsoid (Greeni peavektorid)

11. Tensorite C_{KL} ja c_{kl} invariandid.

Deformatsioonitensorite diagonaliseeritud maatriksid

$$C_{KL} = \begin{pmatrix} \frac{8}{9} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad c_{kl} = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{9}{8} \end{pmatrix}$$

Greeni deformatsioonitensori C_{KL} invariandid

$$I_C = C_1 + C_2 + C_3 = \frac{8}{9} + 1 + 2 = \frac{35}{9}$$

$$II_C = C_1C_2 + C_1C_3 + C_2C_3 = \frac{8}{9} \cdot 1 + \frac{8}{9} \cdot 2 + 1 \cdot 2 = \frac{14}{3}$$

$$III_C = C_1C_2C_3 = \frac{8}{9} \cdot 1 \cdot 2 = \frac{16}{9}$$

Cauchy deformatsioonitensori c_{kl} invariandid

$$I_c = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3} = 2 + 1 + \frac{8}{9} = \frac{35}{9}$$

$$II_c = \frac{1}{c_1}\frac{1}{c_2} + \frac{1}{c_1}\frac{1}{c_3} + \frac{1}{c_2}\frac{1}{c_3} = 2 \cdot 1 + 2 \cdot \frac{8}{9} + 1 \cdot \frac{8}{9} = \frac{14}{3}$$

$$III_c = \frac{1}{c_1}\frac{1}{c_2}\frac{1}{c_3} = 2 \cdot 1 \cdot \frac{8}{9} = \frac{16}{9}$$

12. Kas deformatsioon on isohooriline?

$$(\sqrt{III_c} = \sqrt{III_C}) \neq 1 \Rightarrow ei\ ole\ isohooriline$$