Lesson 6 Homework

① $\operatorname{arctg}\left(\frac{y}{x}\right) = \ln\sqrt{x^2+y^2}$

$\operatorname{arctg}\left(\frac{y}{x}\right) - \ln\sqrt{x^2+y^2} = 0$

$$\frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \frac{y' \cdot x - y \cdot x'}{x^2} - \frac{1}{\sqrt{x^2+y^2}} \cdot \frac{1}{2\sqrt{x^2+y^2}} \cdot 0$$

$$\frac{1}{\frac{x^2+y^2}{x^2}} \cdot \frac{y' \cdot x - y}{x^2} - \frac{1}{2(x^2+y^2)} = 0$$

$$\frac{1}{x^2+y^2} \cdot (y' \cdot x - y) - \frac{1}{2(x^2+y^2)} = 0$$

$$y' \cdot x - y - \frac{x^2+y^2}{2} = 0$$

$$2y' \cdot x - 2y - x^2 + y^2 = 0$$

$$2y' \cdot x = x^2 - y^2 + 2y$$

$$y' = \frac{x^2 - y^2 + 2y}{2x}$$

② $\begin{cases} y = \dfrac{t^2}{t-1} \\ x = \dfrac{t}{t^2-1} \end{cases}$ $\qquad y'_x = \dfrac{y'_t}{x'_t}$

$$y'_t = \frac{(t^2)' \cdot (t-1) - t^2 \cdot (t-1)'}{(t-1)^2} =$$

$$= \frac{2t(t-1) - t^2}{t^2 - 2t + 1} = \frac{2t^2 - 2t - t^2}{t^2 - 2t + 1} =$$

$$= \frac{t^2 - 2t + 1}{t^2 - 2t + 1 + 1} = \frac{1}{2}$$

$$x'_t = \frac{t' \cdot (t^2-1) - t \cdot (t^2-1)'}{(t^2-1)^2} =$$

$$= \frac{t^2 - 1 - t(2t)}{(t^2-1)^2} = \frac{t^2 - 2t^2 - 1}{(t^2-2)t^2 + 1} =$$

$$= \frac{-t^2 - 1}{t^4 - 2t^2 + 1} =$$

$$y'_x = \frac{t^4 - 2t^2 + 1}{-2t^2 - 2}$$

③ $y = (x^2+2)^5 \cdot (3x-x^3)^3$

$\ln y = \ln (x^2+2)^5 + \ln (3x-x^3)^3$

$\ln y = 5 \ln (x^2+2) + 3 \ln (3x-x^3)$

$\frac{1}{y} \cdot y' = 5 \frac{1}{x^2+2} \cdot 2x + 3 \frac{1}{3x-x^3} \cdot (3-3x^2)$

$y' = y \left(\frac{10x}{x^2+2} + \frac{9-9x^2}{3x-x^3} \right) =$

$(x^2+2)^5 \cdot (3x-x^3)^3 \cdot \left(\frac{10x}{x^2+2} + \frac{9-9x^2}{3x-x^3} \right)$

(4) $y = x^x$

$\ln y = \ln x^x$

$\ln y = x \ln x$

$\frac{1}{y} \cdot y' = x \cdot \frac{1}{x}$

$y' = y \cdot 1 = x^x$

(3) $$y = \frac{(2-x^2)^3 \cdot (x-1)^2}{(2x^3-3x)\cdot e^x}$$

$$\ln y = \ln(2-x^2)^3 + \ln(x-1)^2 - \ln(2x^3-3x) - \ln e^x$$

$$\ln y = 3\ln(2-x^2) + 2\ln(x-1) - \ln(2x^3-3x) - x\ln e$$

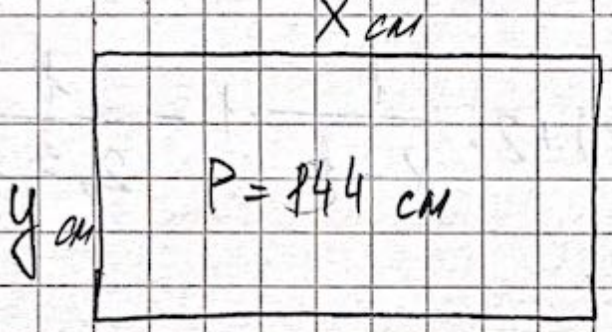
$$\frac{1}{y}\cdot y' = 3\cdot\frac{1}{2-x^2}\cdot(-2x) + 2\cdot\frac{1}{x-1}\cdot 1 - \frac{1}{2x^3-3x}\cdot$$

$$\cdot 6x^2 - 3 - x\cdot\frac{1}{e}$$

$$\frac{1}{y}\cdot y' = -\frac{6x}{2-x^2} + \frac{2}{x-1} - \frac{6x^2-3}{2x^3-3x} - \frac{x}{e}$$

$$y' = \frac{(2-x^2)^3\cdot(x-1)^2}{(2x^3-3x)\cdot e^x}\cdot\left(-\frac{6x}{2-x^2} + \frac{2}{x-1} - \frac{6x^2-3}{2x^3-3x} - \frac{x}{e}\right)$$

7. Найти длину x и ширину y прямоугольника при заданном периметре $P = 144$ см, при которых данный прям. имеет наибольш. площадь S.



$P = 2x + 2y = 2(x+y)$

$2x + 2y = 144$

$x + y = 72$

$y = 72 - x$

$x > 0 \quad y > 0$

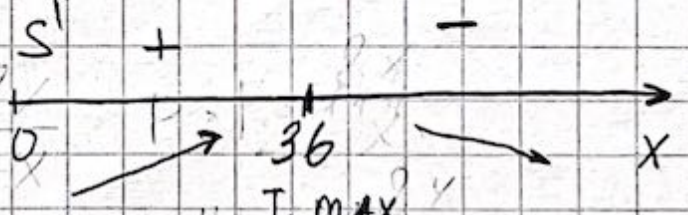
$S = x \cdot y \quad S = x \cdot (72 - x) \quad S = 72x - x^2$

$S' = 72 - 2x \quad S' = 0$

$72 - 2x = 0$

$-2x = -72$

$x = 36$



$y = 72 - x$

$y = 72 - 36$

$y = 36$

Сторона должна быть 36 см и 36 см чтобы при периметре 144 см S была наибольшей