Lesson 8 Homework

(1) $U = 3 - 8x + 6y$

$x^2 + y^2 = 36$

$L(x, y, \lambda) = 3 - 8x + 6y + \lambda(x^2 + y^2 - 36)$

$$\begin{cases} L'_x = -8 + \lambda \cdot 2x = 0 \\ L'_y = 6 + \lambda \cdot 2y = 0 \\ L'_\lambda = x^2 + y^2 - 36 = 0 \end{cases}$$

$$\begin{cases} x = \dfrac{8}{2\lambda} \\ y = -\dfrac{6}{2\lambda} \\ \left(\dfrac{8}{2\lambda}\right)^2 + \left(-\dfrac{6}{2\lambda}\right)^2 = 36 \end{cases}$$

$\dfrac{64}{4\lambda^2} + \dfrac{36}{4\lambda^2} = 36$

$\dfrac{100}{4\lambda^2} = 36$

$144\lambda^2 = 100$

$\lambda^2 = \dfrac{100}{144} = \dfrac{25}{36}$

$$\begin{cases} x = \dfrac{8}{2\lambda} \\ y = -\dfrac{3}{2\lambda} \\ \lambda^2 = \dfrac{25}{36} \end{cases} \qquad \lambda = \pm \frac{5}{6}$$

2 точки $\left(8, -6, \frac{5}{6}\right)$ $\left(8, -6, -\frac{5}{6}\right)$

Матрица Гессе

$L''_{xx} = 2\lambda$

$L''_{yy} = 2\lambda$

$L''_{\lambda\lambda} = 0$

$L''_{xy} = L''_{yx} = 0$

$L''_{x\lambda} = L''_{\lambda x} = 2x$

$L''_{y\lambda} = L''_{\lambda y} = 2y$

$$\begin{pmatrix} L''_{\lambda\lambda} & L''_{\lambda x} & L''_{\lambda y} \\ L''_{x\lambda} & L''_{xx} & L''_{xy} \\ L''_{y\lambda} & L''_{yx} & L''_{yy} \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 2x & 2y \\ 2x & 2\lambda & 0 \\ 2y & 0 & 2\lambda \end{pmatrix}$$

$$\Delta = \begin{vmatrix} 0 & 2x & 2y \\ 2x & 2\lambda & 0 \\ 2y & 0 & 2\lambda \end{vmatrix} = \begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix} =$$

$$= 0 \begin{vmatrix} 2\lambda & 0 \\ 0 & 2\lambda \end{vmatrix} - 2x \begin{vmatrix} 2x & 0 \\ 2y & 2\lambda \end{vmatrix} +$$

$$+ 2y \cdot \begin{vmatrix} 2x & 2\lambda \\ 2y & 0 \end{vmatrix} = (-2x) \cdot 4x\lambda +$$

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$$2y\cdot(-4y\lambda) = -8\lambda(x^2+y^2)$$

$$\Delta = -8\lambda\cdot 36 = -288\lambda$$

Если $\lambda = \frac{5}{6}$, то $\Delta < 0$, $\Rightarrow \left(8, -6, \frac{5}{6}\right)$ точка min.

Если $\lambda = -\frac{5}{6}$, то $\Delta > 0 \Rightarrow \left(8, 6, -\frac{5}{6}\right)$ т. max.

(2) $u = 2x^2 + 12xy + 32y^2 + 15$

$$x^2 + 16y^2 = 64$$

$$L(x,y,\lambda) = 2x^2 + 12xy + 32y^2 + 15 + \lambda(x^2 + 16y^2 - 64)$$

$$\begin{cases} L'_x = 4x + 12y + \lambda\cdot 2x \\ L'_y = 12x + 64y + \lambda\cdot 32y \\ L'_\lambda = x^2 + 16y^2 - 64 = 0 \end{cases}$$

$x(4+2\lambda) = -12y$

$y(64+32\lambda) = -12x$

$$\begin{cases} x = -\dfrac{12y}{4+2\lambda} \\ y = -\dfrac{12x}{64+32\lambda} \\ \left(-\dfrac{12y}{4+2\lambda}\right)^2 + 16\cdot\left(-\dfrac{12x}{64+32\lambda}\right)^2 = 64 \end{cases}$$

$$\frac{36y^2}{l^2+4l+4} + \frac{2304x^2}{(32l+64)^2} = 64$$

$$\frac{9(x^2+16y^2)}{4(l+2)^2} = 64$$

$$\frac{36y^2}{l^2+4l+4} + \frac{9x^2}{4l^2+16l+16} = 64$$

$$\frac{36y^2}{\underset{(l+2)^2}{\underset{\shortparallel}{l^2+4l+4}}} + \frac{9x^2}{\underset{4(l+2)^2}{\underset{\shortparallel}{4(l^2+4l+4)}}} = 64$$

$$\frac{6y}{l+2} + \frac{3x}{2(l+2)} = 8$$

$$\frac{1}{l+2} \cdot \left(\frac{6y+3x}{2}\right) = 8$$

$$\frac{1}{l+2} \cdot (y+x) = 8$$

$$\frac{1}{l+2} = \frac{8}{y+x}$$

$$8l = \frac{y+x}{16}$$

$$l = \frac{y+x}{128}$$

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$$\begin{cases} x = -\dfrac{12y}{4+2\lambda} \\ y = -\dfrac{12x}{64+32\lambda} \\ \lambda = \dfrac{y+x}{128} \end{cases}$$

$$x = -\frac{12y}{4+2\left(\frac{y+x}{128}\right)} = -\frac{12y}{4+\frac{y+x}{64}} =$$

12