Lesson 9 Homework

① $$\int \frac{2x+3}{(x-2)(x+5)}\,dx =$$ $\int \frac{dx}{x} = \ln|x| + C.$

$$=$$

$$\frac{2x+3}{(x-2)(x+5)} = \frac{A}{x-2} + \frac{B}{x+5}$$

$$2x+3 = A(x+5) + B(x-2)$$

$x=-5$: $-10+3 = 0 + B\cdot(-7) \Rightarrow 7 = -7B \Rightarrow B = -1$

$x = 2$: $7 = A\cdot 7 + 0 \Rightarrow A = 1$

$$\int \frac{1}{x+5}\,dx + \int \frac{1}{x-2}\,dx =$$

$$= \ln|x+5| + \ln|x-2| + C.$$

② $$\int e^{2x} \underbrace{\cos 3x}_{u}\,dx =$$

$u = \cos 3x \quad du = -3\sin 3x \cdot dx$

$dV = e^{2x}dx \quad V = \int e^{2x}dx = \frac{1}{2}\int e^{2x}d(2x) =$ ①

$\frac{1}{2}e^{2x}$

$$= \frac{1}{2}\cos 3x \cdot e^{2x} - \frac{1}{2}\int e^{2x}\cdot(-3)\cdot\sin 3x\,dx =$$

$$= \frac{1}{2}\cos 3x \cdot e^{2x} + \frac{3}{2}\int e^{2x}\cdot \sin 3x\,dx$$

$$\int e^{ax}\cos bx\,dx = \frac{(a\cdot\cos bx + b\sin bx)\cdot e^{ax}}{a^2+b^2} =$$

$a=2 \quad b=3$

$$= \frac{(2\cos 3x + 3\sin 3x)e^{2x}}{\underbrace{2^2+3^2}_{4+9=13}} =$$

$$= \frac{1}{13}e^{2x}(3\sin 3x + 2\cos 3x) + C.$$

3. $$\int_0^{\ln 2} xe^{-x}\,dx =$$

$u = x \Rightarrow du = dx$

$dV = e^{-x}dx \quad V = -e^{-x}$

$$= -e^{-x}x + \int e^{-x}dx = -e^{-x}(x+1)\Big|_0^{\ln 2} =$$

$$= -e^{-\ln 2}(\ln 2+1) + e^{-0}(0+1) = -\frac{1}{2}(\ln 2+1) +$$

$$\cancel{+}1 = 1+\frac{1}{2}(-1-\ln 2) = 1-\frac{1}{2}-\frac{1}{2}\cdot\ln 2.$$

4. $\int\limits_{2}^{+\infty} \frac{dx}{x^2+x-2} = \lim\limits_{b\to+\infty} \int\limits_{2}^{b} \frac{dx}{(x+2)(x-1)} =$

$= \lim\limits_{b\to+\infty} \frac{1}{3}\left(\ln|1-x| - \ln|x+2|\right) -$

$- \frac{1}{3}\left(\ln|1| - \ln|4|\right) = 0 - \frac{\ln 4}{3} =$

$= -\frac{\ln 4}{3}$