Lesson 7. Homework

1. $u = x^3 + 3xy^2 + z^2 - 39x - 36y + 2z + 26$

$u'_x = 3x^2 + 3 \cdot y^2 - 39$

$u'_y = 6x \cdot y - 36$

$u'_z = 2z + 2$

$u''_{xx} = 6x \quad u''_{xy} = 6x + 6y \quad u''_{yx} = 6y$

$u''_{yy} = 6 \quad u''_{yx} = 6x + 6y \quad u''_{yz} = 0$

$u''_{zz} = 2 \quad u''_{xz} = 0 \quad u''_{zy} = 0$

$du = (3x^2 + 3y^2 - 39)\,dx + (6xy - 36)\,dy +$
$+ (2z + 2)\,dz$

$du^2 = (6x)\,dx^2 + 6 \cdot y^2 + 2\,dz^2 +$

$+$

② $u = \frac{256}{x} + \frac{x^2}{y} + \frac{y^2}{z} + z^2$

$u'_x = \frac{256}{x^2} + \frac{2x}{y}$

$u'_y = \frac{x^2}{y^2} + \frac{2y}{z}$

$u'_z = \frac{y^2}{z^2} + 2z$

$u''_{xx} = \frac{256}{2x} + \frac{2}{y}$ $\quad u''_{yx} = \frac{2x}{y^2}$

$u''_{xy} = \frac{2x}{y^2}$ $\quad u''_{yy} = \frac{x^2}{2y} + \frac{2}{z}$

$u''_{xz} = 0$ $\quad u''_{yz} = 2y$

$u''_{zx} = 0$

$u''_{zy} = \frac{2y}{z^2}$

$u''_{zz} = \frac{y^2}{2z} + 2$

③ Найти произв. $U = x^2 + y^2 + z^2$ по направл. $\vec{c}(-9, 8, -12)$ в т. $M(8, -12, 9)$

Найдем длину вектора $\vec{c}$

$$|\vec{c}| = \sqrt{x_0^2 + y_0^2 + z_0^2} = \sqrt{(-9)^2 + 8^2 + (-12^2)} =$$

$$= \sqrt{+81 + 64 + 144} = \sqrt{289} = 17$$

Запишем ед. вектор $\vec{c_0}$:

$$\vec{c_0} = \frac{\vec{c}}{|\vec{c}|} = \left(-\frac{9}{17}; \frac{8}{17}; -\frac{12}{17}\right)$$

Найдем градиент в т. $M(8; -12; 9)$

$U'_x = 2x$

$U'_y = 2y$

$U'_z = 2z$

$\operatorname{grad} U = (16, -24, 18)$

$$U'_{\vec{c}} = -\frac{9}{17} \cdot 16 + \frac{8}{17} \cdot (-24) - \frac{12}{17} \cdot 18 =$$

$$= -\frac{144}{17} - \frac{192}{17} - \frac{216}{17} = \frac{552}{17}$$

④ $u = e^{x^2+y^2+z^2}$ $\vec{c}(4,-13,-16)$

$L(-16,4,-13)$

$$|\vec{c}| = \sqrt{4^2+\underbrace{(-13)^2}_{169}+\underbrace{(-16^2)}_{256}} = \sqrt{441} = 21$$

$$\vec{c_0} = \frac{\vec{c}}{|\vec{c}|} = \left(\frac{4}{21}; -\frac{13}{21}; -\frac{16}{21}\right)$$

$$u'_x = 2xe^{x^2+y^2+z^2}$$

$$u'_y = 2ye^{x^2+y^2+z^2}$$

$$u'_z = 2ze^{x^2+y^2+z^2}$$

$$\text{grad}\,u = \left(\frac{8e}{21}; -\frac{26e}{21}; -\frac{32e}{21}\right)$$

$$u'_{\vec{c}} = \frac{4}{21}\cdot\frac{8e}{21} + \left(-\frac{13}{21}\right)\cdot\left(-\frac{26e}{21}\right) + \left(-\frac{32e}{21}\right)\left(-\frac{16}{21}\right)$$

$$= 2e$$

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⑥ $U = x^2y + \frac{1}{3}y^3 + 2x^2 + 3y^2 - 1$

$$\begin{cases} U'_x = 2xy + 4x = 0 \\ U'_y \quad x^2 + y^2 + 6y = 0 \end{cases} \Rightarrow \begin{cases} 2x(y+2) = 0 \\ x^2 + y^2 + 6y = 0 \end{cases} \Rightarrow$$

$$\Rightarrow \left[\begin{array}{l} \begin{cases} x = 0 \\ y^2 + 6y = 0 \end{cases} \\ \begin{cases} y = -2 \\ x^2 = 8 \end{cases} \end{array} \right. \Rightarrow \begin{array}{l} (0,0) \\ (0,-6) \\ (2\sqrt{2}, -2) \\ (-2\sqrt{2}, -2) \end{array}$$

$$U''_{xx} = 2y + 4$$

$$U''_{xy} = U''_{yx} = 2x$$

$$U''_{yy} = 2y + 6$$

$$\begin{pmatrix} 2y+4 & 2x \\ 2x & 2y+6 \end{pmatrix}$$

$$\Delta_1 = U''_{xx} = 2y + 4.$$

$$\Delta_2 = \begin{vmatrix} U''_{xx} & U''_{xy} \\ U''_{yx} & U''_{yy} \end{vmatrix} = U''_{xx} \cdot U''_{yy} - (U''_{xy})^2 =$$

$$= 4y^2 + 20y + 24 - 4x^2$$

$(0,0)$ $\Delta_1 = 2y+4 = 4 > 0$

$\Delta_2 = 4y^2 + 20y + 24 - 4x^2 = 24 > 0$

$(0,0)$ – мин.

$(0,-6)$ $\Delta_1 = 2y+4 = -8 < 0$

max. $\Delta_2 = 4y^2 + 20y + 24 - 4x^2 = 48 > 0$

$(\pm 2\sqrt{2}, -2)$ $\Delta_1 = 2y+4 = 0$. не имеет смысла.

$$\begin{pmatrix} u''_{yy} & u''_{yx} \\ u''_{xy} & u''_{xx} \end{pmatrix}$$

$$\begin{pmatrix} 2y+6 & 2x \\ 2x & 2y+4 \end{pmatrix}$$

$\Delta_1 = u''_{yy} = 2y+6 = 2\cdot(-2)+6 = 2 > 0$

$\Delta_2 = \begin{vmatrix} u''_{yy} & u''_{yx} \\ u''_{xy} & u''_{xx} \end{vmatrix} = u''_{xx} \cdot u''_{yy} - (u''_{xy})^2 =$

$= 4y^2 + 20y + 24 - 4x^2 = 4\cdot 4 + 20\cdot(-2) +$

$+ 24 - 4\cdot 8 = 40 - 40 - 32 < 0$

$(\pm 2\sqrt{2}; -2)$ – седловые