Possible tests around eta functions

I have realized that you do not give details about the accuracy of your way of generation of eta functions (item 1) and on the applications where these functions are used in your program (item 2).

Item 1.
To help you I send to you a fortran code for the double precision generation of $\eta_i(Z)$ for $i \in [-1, 50]$ and a set of values of $Z \in [-600.25, 16]$. I used subroutine GEBASEV from the CD attached to book [48] and another, more recent subroutine of mine, GEBASEV 2022. In the program ETA(I) and ETA 2022(I) are the values of $\eta_i(Z)$ produced by those subroutines for the indicated Z. The results for the two and their relative difference are listed in file fort.50. They indicate small deviations, affecting at most the last two figures, except when Z happens to be close to the zero of that $\eta_i(Z)$, a normal behavior.
Can you compare your results with these and tell to me the answer? You may use a similar table.

Item 2.
In sec. 3.2.3.1 (page 123) you give formulas of functions F^0, G^0, H^0 and J^0 for $i \neq j$ and $i = j$ but case $i \neq j$ does not mean automatically that the corresponding Z_i and Z_j are sufficiently far from each other for accepting the written formulas as suitable for a numerical evaluation. Take for example F^0,

$$F^0 := I^0_{-1,-1} = \frac{h}{\theta_i - \theta_j}(\theta_i \eta_0(Z_i)\eta_{-1}(Z_j) - \theta_j \eta_{-1}(Z_i)\eta_0(Z_j)), \tag{1}$$

which I express in terms of Z_i and Z_j

$$F^0 = \frac{h}{Z_i - Z_j}(Z_i \eta_0(Z_i)\eta_{-1}(Z_j) - Z_j \eta_{-1}(Z_i)\eta_0(Z_j)). \tag{2}$$

If Z_i and Z_j are close to each other the computation implies subtraction of like terms and this certainly reduces the accuracy. To cure this I should do as it follows.
Denote $\Delta Z = Z_i - Z_j$ and introduce a threshold T. If $|\Delta Z| > T$ then accept eqs.(1) or (2) but if $|\Delta Z| \leq T$ then some adaptation is needed. Rewrite (2) as

$$F^0 = \frac{hZ_j}{\Delta Z}[\eta_0(Z_j + \Delta Z)\eta_{-1}(Z_j) - \eta_{-1}(Z_j + \Delta Z)\eta_0(Z_j)] + h\eta_0(Z_i)\eta_{-1}(Z_j). \tag{3}$$

and use the Taylor series for $\eta_{-1}(Z_j + \Delta Z)$ and $\eta_0(Z_j + \Delta Z)$

$$\eta_n(Z_j + \Delta Z) = \eta_n(Z_j) + \sum_{m=1} \frac{\Delta Z^m}{m!2^m}\eta_{n+m}(Z_j),\ n = -1, 0$$

to obtain the acceptable expression

$$F^0 = \sum_{m=1}^{m_{max}} \frac{\Delta Z^{m-1}}{m!2^m}[\eta_m(Z_j)\eta_{-1}(Z_j) - \eta_{-1+m}(Z_j)\eta_0(Z_j)] + h\eta_0(Z_i)\eta_{-1}(Z_j),$$

when the upper limit m_{max} is conveniently fixed in terms of T. My guess is that values like $T = 1$ and $m_{max} = 10$ are sufficient for double precision computations but this should be checked. If $Z_i = Z_j$ you recover your expression for $i = j$.
The same can be done also for the other functions on p.123. I cannot figure out how big would be the influence of such corrections on your results but I should not exclude that sometimes this may be significant. Can you try this ? I only add that the expressions you give for G^0, H^0,J^0 and $i = j$ are of form 0/0 when $Z_i = 0$ and this is another matter of concern.