

# Formal Systems and their Applications: Implementing a dependently-typed calculus

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## 1 Base calculus (3.3.1)

$$\begin{array}{c}
\frac{x : t \in \Gamma}{\Gamma \vdash x \uparrow t} \quad (\text{T-VAR}) \\
\\
\frac{\Gamma \vdash t_1 \Downarrow \text{Set} \quad \Gamma, x : t_1 \vdash t_2 \uparrow t_3}{\Gamma \vdash \lambda x : t_1. t_2 \uparrow (x : t_1) \rightarrow t_3} \quad (\text{T-ABSANN}) \\
\\
\frac{\Gamma, x : t_1 \vdash t_2 \Downarrow t_3}{\Gamma \vdash \lambda x. t_2 \Downarrow (x : t_1) \rightarrow t_3} \quad (\text{T-ABS}) \\
\\
\frac{\Gamma \vdash t_1 \Downarrow \text{Set} \quad \Gamma, x : t_1 \vdash t_2 \Downarrow \text{Set}}{\Gamma \vdash (x : t_1) \rightarrow t_2 \uparrow \text{Set}} \quad (\text{T-PI}) \\
\\
\frac{\Gamma \vdash t_1 \uparrow t_3 \quad t_3 \rightarrow^* ((x : t_4) \rightarrow t_5) \quad \Gamma \vdash t_2 \Downarrow t_4}{\Gamma \vdash t_1 \ t_2 \uparrow t_5[x \mapsto t_2]} \quad (\text{T-APP}) \\
\\
\frac{}{\Gamma \vdash \text{Set} \uparrow \text{Set}} \quad (\text{T-SETINSET}) \\
\\
\frac{\Gamma \vdash A \uparrow \text{Set} \quad \Gamma \vdash t \uparrow A}{\Gamma \vdash t \Downarrow A} \quad (\text{T-SWITCH})
\end{array}$$

## 2 Sigma types (4.2)

$$\begin{array}{c}
\frac{\Gamma \vdash A \Downarrow \text{Set} \quad \Gamma(x : A) \vdash B \Downarrow \text{Set}}{\Gamma \vdash \Sigma[x : A] B \uparrow \text{Set}} \quad (\text{T-SIGMA}) \\
\\
\frac{\Gamma \vdash s \Downarrow A \quad \Gamma \vdash t \Downarrow [x \mapsto s] B}{\Gamma \vdash (s, t) \Downarrow \Sigma[x : A] B} \quad (\text{T-PAIR}) \\
\\
\frac{\Gamma \vdash t \Downarrow \Sigma[x : A] B}{\Gamma \vdash fst \ t \uparrow A} \quad (\text{T-FST}) \\
\\
\frac{\Gamma \vdash t \Downarrow \Sigma[x : A] B}{\Gamma \vdash snd \ t \uparrow [x \mapsto fst \ t] B} \quad (\text{T-SND})
\end{array}$$