```
clear
clc
close all

syms N_b Omega R c theta C_la C_d rho y dy
assume(N_b>0);
assume(Omega>0);
assume(R>0);
assume(c>0);
assume(theta>0);
assume(C_la>0);
assume(C_d>0);
assume(rho>0);
assume(y>0);
assume(dy>0);

r = y / R ;
dr = dy / R ;
```

Assumptions

```
syms lambda_c lambda_i mu beta betadot psi theta_0 sigma sigma_0
assume(lambda_c>0);
assume(lambda_i>0);
assume(mu>0);
assume(beta>0);
assume(betadot>0);
assume(psi>0);
assume(theta_0>0);
assume(sigma>0);
assume(sigma_0>0);

beta = 0;                                    % Assuming NO blade flapping
betadot = 0;                                 % Assuming NO blade dynamics
mu = 0;                                      % Assuming NO fordward flight
theta = theta_0;                             % Assuming constant pitch
sigma = sigma_0;                             % Assuming constant solidity
arg1 = ( sigma*C_la/16 - lambda_c/2);
arg2 = sigma*C_la*theta*r/8;
lambda_BEMT = sqrt( arg1^2 + arg2 ) - arg1;
lambda = lambda_BEMT;                        % Assuming BEMT inflow
% lambda = lambda_c + lambda_i;                % Assuming vetical fligth

lambda
```

lambda =

$$\frac{\lambda_c}{2}+\sqrt{\left(\frac{\lambda_c}{2}-\frac{C_{\mathrm{la}}\,\sigma_0}{16}\right)^2+\frac{C_{\mathrm{la}}\,\sigma_0\,\theta_0\,y}{8\,R}}-\frac{C_{\mathrm{la}}\,\sigma_0}{16}$$

```
U_T = (Omega*R) * ( r + mu*sin(psi) );
U_P = (Omega*R) * ( lambda + r*betadot/Omega + mu*beta*cos(psi) );
U_R = (Omega*R) * ( mu*cos(psi) );

phi = U_P / U_T;
alpha = theta - phi;                          % sectional angle of attack

q = 0.5*rho*U_T^2;                            % dynamic pressure
Cl = C_la*(alpha);                            % lift coefficient
dL = q*c*Cl*dy;                               % sectional lift
dD = q*c*C_d*dy;                              % sectional drag

dT = N_b * ( dL );                            % sectional thrust
dQ = N_b * ( phi*dL + dD ) * y;               % sectional torque
dP = dQ * Omega;                              % sectional power

% sectional thrust
dT = simplify(dT)
```

dT =

$$\frac{C_{\mathrm{la}}\, N_b\, \Omega^2\, c\, \mathrm{dy}\, \rho\, y^2 \left(\theta_0 - \frac{R\left(\frac{\lambda_c}{2} + \sqrt{\left(\frac{\lambda_c}{2} - \frac{C_{\mathrm{la}}\,\sigma_0}{16}\right)^2 + \frac{C_{\mathrm{la}}\,\sigma_0\,\theta_0\, y}{8\,R}} - \frac{C_{\mathrm{la}}\,\sigma_0}{16}\right)}{y}\right)}{2}$$

```
% sectional power
dP = simplify(dP)
```

dP =

$$N_b\,\Omega\, y \left(\frac{C_d\,\Omega^2\, c\, \mathrm{dy}\, \rho\, y^2}{2} + \frac{C_{\mathrm{la}}\,\Omega^2\, R\, c\, \mathrm{dy}\, \rho\, y \left(\theta_0 - \frac{R\,\sigma_1}{y}\right)\sigma_1}{2}\right)$$

where

$$\sigma_1 = \frac{\lambda_c}{2} + \sqrt{\left(\frac{\lambda_c}{2} - \frac{C_{\mathrm{la}}\,\sigma_0}{16}\right)^2 + \frac{C_{\mathrm{la}}\,\sigma_0\,\theta_0\, y}{8\,R}} - \frac{C_{\mathrm{la}}\,\sigma_0}{16}$$

```
% Integrate along the blade
T_psi = int(dT, y, 0, R)
```

T_psi =

$$\frac{C_{\mathrm{la}}^{2}\,N_b\,\Omega^2\,R^3\,c\,\mathrm{dy}\,\rho\,\sigma_0}{64}+\frac{C_{\mathrm{la}}\,N_b\,\Omega^2\,R\,c\,\mathrm{dy}\,\rho\left(\frac{R^2\,\sigma_1^{3/2}\,\left(768\,\sigma_1-320\,\lambda_c^{2}-\sigma_3+80\,C_{\mathrm{la}}\,\lambda_c\,\sigma_0\right)}{\sigma_2}-\frac{R^2\left(\sigma_1+\frac{C_{\mathrm{la}}}{}\right.}{}\right.}{2}$$

where

$$\sigma_1=\left(\frac{\lambda_c}{2}-\frac{C_{\mathrm{la}}\,\sigma_0}{16}\right)^2$$

$$\sigma_2=30\,C_{\mathrm{la}}^{2}\,\sigma_0^{2}\,\theta_0^{2}$$

$$\sigma_3=5\,C_{\mathrm{la}}^{2}\,\sigma_0^{2}$$

```
% Integrate along the blade
P_psi = int(dP, y, 0, R)
```

```
P_psi =
```

$$\frac{N_b\,\Omega^3\,R^4\,c\,\mathrm{dy}\,\rho\,\left(573440\,\lambda_c^{4}\,\sigma_1^{3/2}-2752512\,\lambda_c^{2}\,\sigma_1^{5/2}-819200\,\lambda_c^{4}\,\sqrt{\sigma_2^{6}}+2752512\,\lambda_c^{2}\,\sqrt{\sigma_2^{10}}+3932160\right.}{}$$

where

$$\sigma_1=\sigma_2^{2}+\frac{C_{\mathrm{la}}\,\sigma_0\,\theta_0}{8}$$

$$\sigma_2=\frac{\lambda_c}{2}-\frac{C_{\mathrm{la}}\,\sigma_0}{16}$$

```
% Integrate along the azimuth
dy = 1;
T_psi = subs(T_psi);
P_psi = subs(P_psi);
T_tot= int(T_psi, psi, 0, 2*pi)
```

```
T_tot =
```

$$\frac{\pi\, C_{la}{}^2\, N_b\, \Omega^2\, R^3\, c\, \rho\, \sigma_0}{32} + \pi\, C_{la}\, N_b\, \Omega^2\, R\, c\, \rho \left(\frac{R^2\, \sigma_1{}^{3/2}\, \left(768\, \sigma_1 - 320\, \lambda_c{}^2 - \sigma_3 + 80\, C_{la}\, \lambda_c\, \sigma_0\right)}{\sigma_2} - \frac{R^2 \left(\sigma_1 + \frac{C_{la}\, \sigma_{\cdot}}{8}\right.}{} \right.$$

where

$$\sigma_1 = \left(\frac{\lambda_c}{2} - \frac{C_{la}\, \sigma_0}{16}\right)^2$$

$$\sigma_2 = 30\, C_{la}{}^2\, \sigma_0{}^2\, \theta_0{}^2$$

$$\sigma_3 = 5\, C_{la}{}^2\, \sigma_0{}^2$$

```
% Integrate along the azimuth
P_tot = int(P_psi, psi, 0, 2*pi)
```

```
P_tot =
```

$$\frac{\pi\, N_b\, \Omega^3\, R^4\, c\, \rho\, \left(573440\, \lambda_c{}^4\, \sigma_1{}^{3/2} - 2752512\, \lambda_c{}^2\, \sigma_1{}^{5/2} - 819200\, \lambda_c{}^4\, \sqrt{\sigma_2{}^6} + 2752512\, \lambda_c{}^2\, \sqrt{\sigma_2{}^{10}} + 3932160\, \sigma\right.}{}$$

where

$$\sigma_1 = \sigma_2{}^2 + \frac{C_{la}\, \sigma_0\, \theta_0}{8}$$

$$\sigma_2 = \frac{\lambda_c}{2} - \frac{C_{la}\, \sigma_0}{16}$$

```
% Normalize to a1
% a1 = subs( ( pi*N_b*rho*c*Omega^2*C_la*R^3 )/6 );
% a1 = subs( ( pi*N_b*rho*c*C_la*R^3 )/6 );
% T_tot = simplify( subs(T_tot / a1) )
% P_tot = simplify( subs(P_tot / a1) )
```

Analysis on a 0.25m blade

```
N_b = 2;
R = 1/4;                        % m
c = 0.1/4;                      % m
rho = 1.225;                    % kg/m3
C_la = 2*3.14;
theta_0 = 20*3.14/180;          % rad
C_d = C_la*theta_0 / 10;
sigma_0 = N_b*c*R/(pi*R^2);

T_tot = vpa(eval(subs(T_tot)), 2)
```

```
T_tot =
```

$$2.4\text{e-}3\,\Omega^2 - 4.7\text{e-}3\,\Omega^2\,\lambda_c - 0.3\,\Omega^2\,\left(0.11\,(\sigma_1 + 0.017)^{3/2}\,\left(32.0\,\lambda_c + 777.0\,\sigma_1 - 322.0\,\lambda_c{}^2 + 13.0\right) - 0.11\,\sigma_1{}^{3/}$$

where

$$\sigma_1 = (0.5\,\lambda_c - 0.025)^2$$

```
P_tot = vpa(eval(subs(P_tot)), 2)
```

```
P_tot =
```

$$2.3\text{e-}5\,\Omega^3\,\left(17.0\,\lambda_c + 577.0\,\lambda_c\,\sigma_2{}^{3/2} - 11.0\,\sigma_2{}^{3/2} + 6.9\text{e+}3\,\sigma_2{}^{5/2} + 3.9\text{e+}6\,\sigma_2{}^{7/2} - 455.0\,\lambda_c\,\sigma_1 - 8.6\text{e+}3\,\lambda_c{}^2\,\sigma_2{}^{:}$$

where

$$\sigma_1 = \sqrt{(0.5\,\lambda_c - 0.025)^6}$$

$$\sigma_2 = (0.5\,\lambda_c - 0.025)^2 + 0.017$$

$$\sigma_3 = \sqrt{(0.5\,\lambda_c - 0.025)^{10}}$$

BEMT inflow dependance on climb velocity

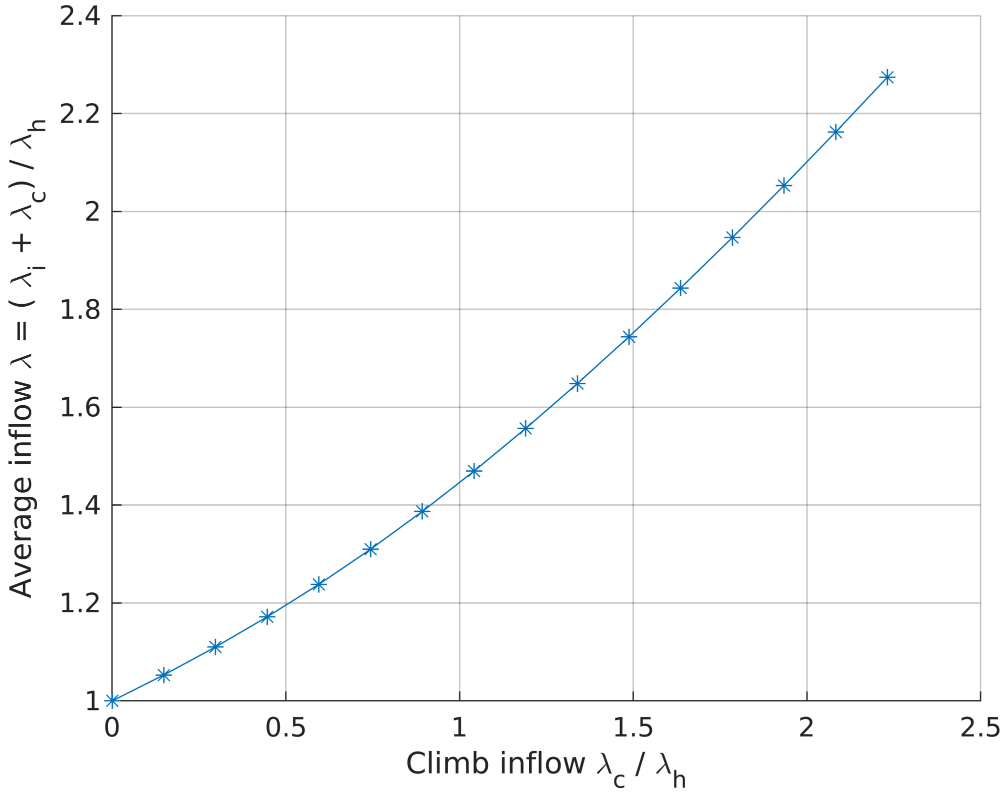
```
% Integrate along the blade
avg_lambda = int(lambda, y, 0, R) / R;
avg_lambda = vpa(eval(subs(avg_lambda)), 2)
```

avg_lambda = $0.5\,\lambda_c + 38.0\,\left((0.5\,\lambda_c - 0.025)^2 + 0.017\right)^{3/2} - 38.0\,\left((0.5\,\lambda_c - 0.025)^2\right)^{3/2} - 0.025$

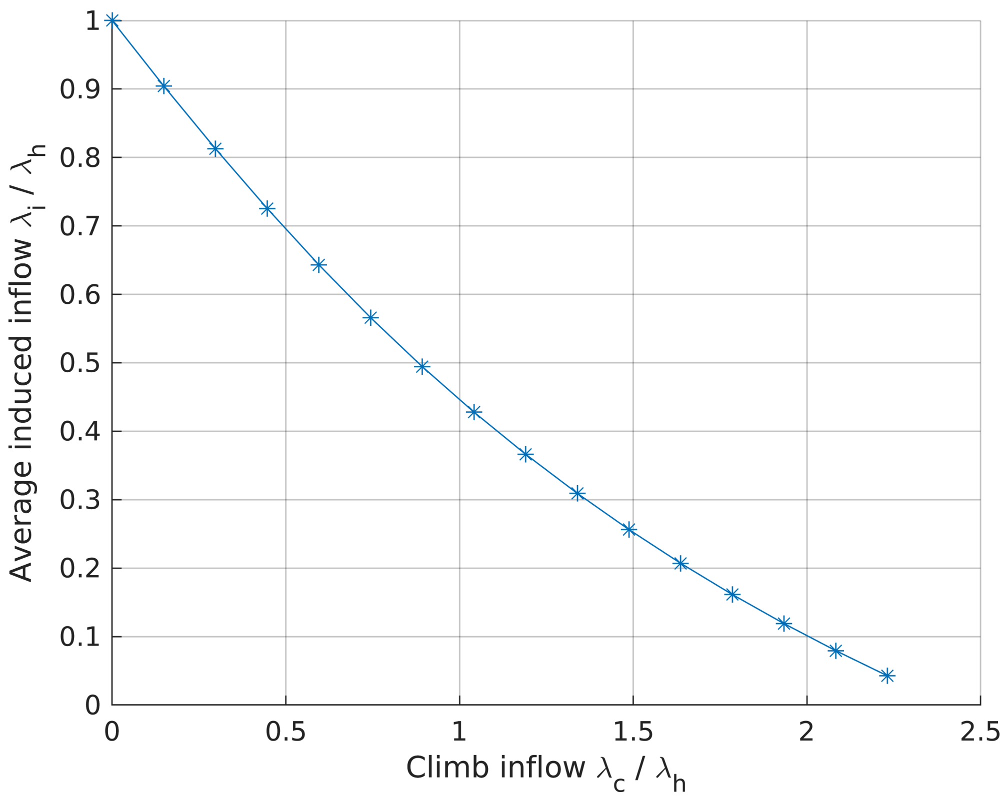
```
lambda_c_arr = 0:0.01:0.15;
ns = length(lambda_c_arr);

avg_lambda_arr = zeros(ns, 1);
avg_lambda_i_arr = zeros(ns, 1);
for i = 1:ns
    lc = lambda_c_arr(i); % / (Omega*R);
    avg_lambda_arr(i) = subs(avg_lambda, lambda_c, lc);
    avg_lambda_i_arr(i) = avg_lambda_arr(i) - lc;
end

lh = avg_lambda_arr(1)
```

```
lh = 0.0672
```

```
figure;
hold on;
grid on;
plot(lambda_c_arr./lh, avg_lambda_arr./lh, '-*')
% legend('isolation', 'interference')
xlabel('Climb inflow \lambda_c / \lambda_h')
ylabel('Average inflow \lambda = ( \lambda_i + \lambda_c) / \lambda_h')
```



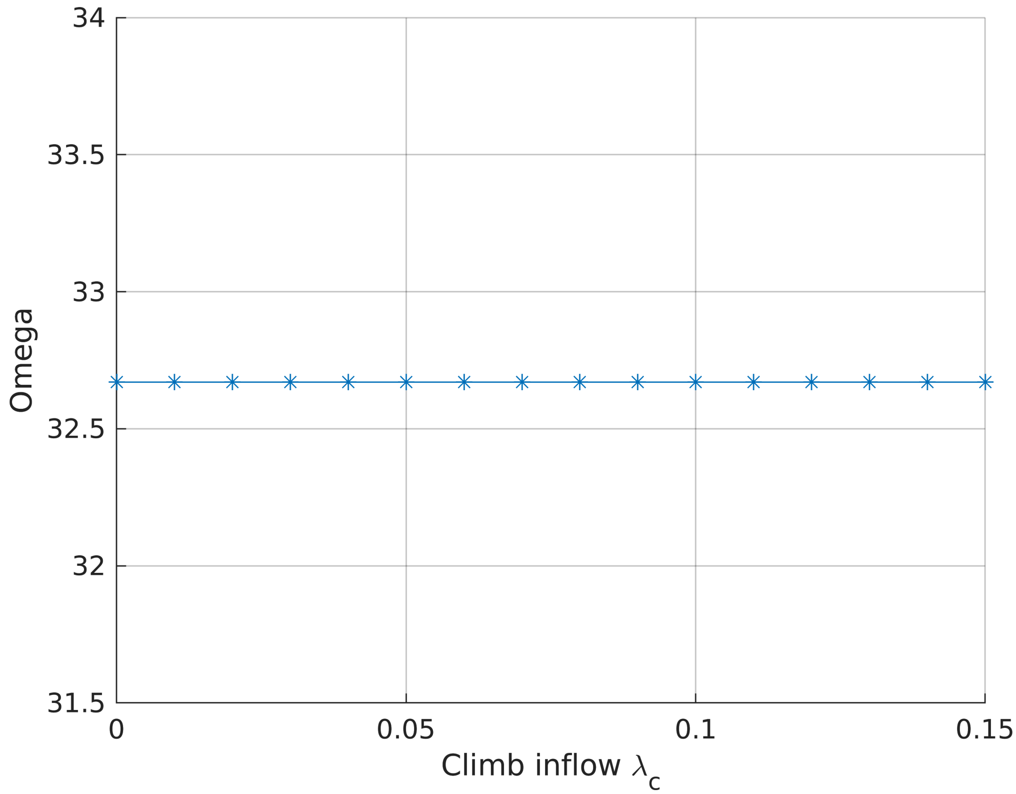

```
figure;
hold on;
grid on;
plot(lambda_c_arr./lh, avg_lambda_i_arr./lh, '-*')
% legend('isolation', 'interference')
xlabel('Climb inflow \lambda_c / \lambda_h')
ylabel('Average induced inflow \lambda_i / \lambda_h')
```



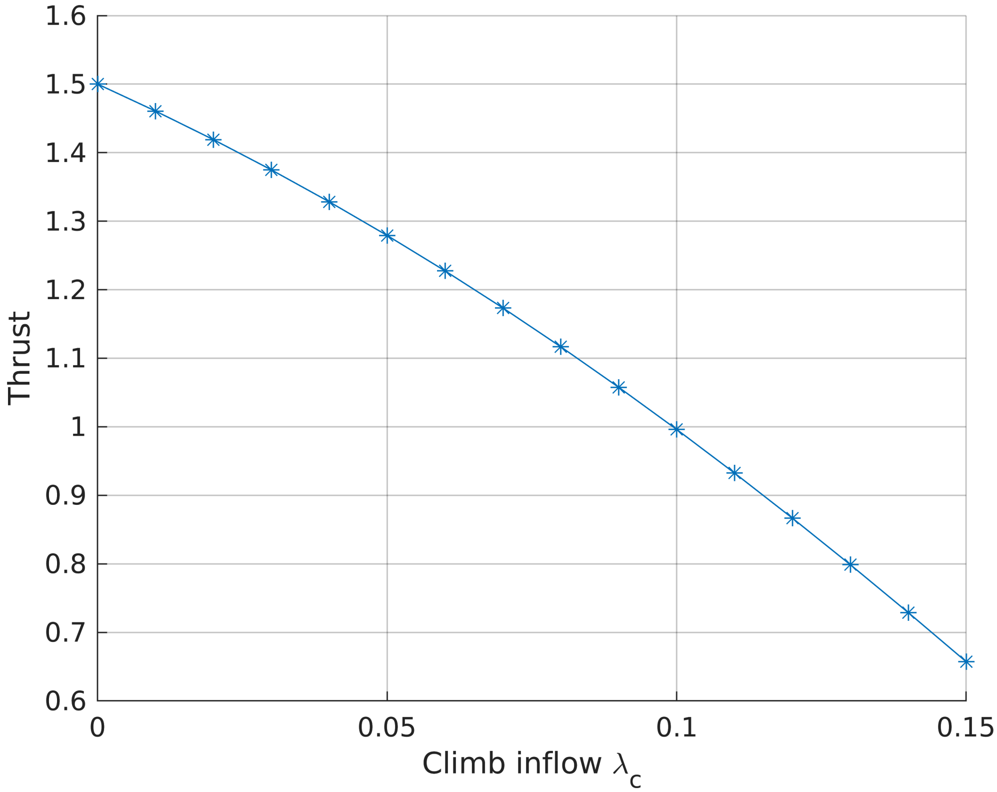
Thrust dependance on climb velocity (at fixed RPM)

```
lambda_c_arr = 0:0.01:0.15;
ns = length(lambda_c_arr);

Omega_arr = zeros(ns, 1);
T_arr = zeros(ns, 1);
P_arr = zeros(ns, 1);
for i = 1:ns
    lc = lambda_c_arr(i);
    T_i = subs(T_tot, lambda_c, lc);
    P_i = subs(P_tot, lambda_c, lc);

    Omega_0 = 32.6702;    % Hz
    solx = Omega_0;

    Omega_arr(i) = solx;
    T_arr(i) = subs(T_i, Omega, solx);
    P_arr(i) = subs(P_i, Omega, solx);
end

figure;
hold on;
grid on;
plot(lambda_c_arr, Omega_arr, '-*')
% legend('isolation', 'interference')
xlabel('Climb inflow \lambda_c')
```

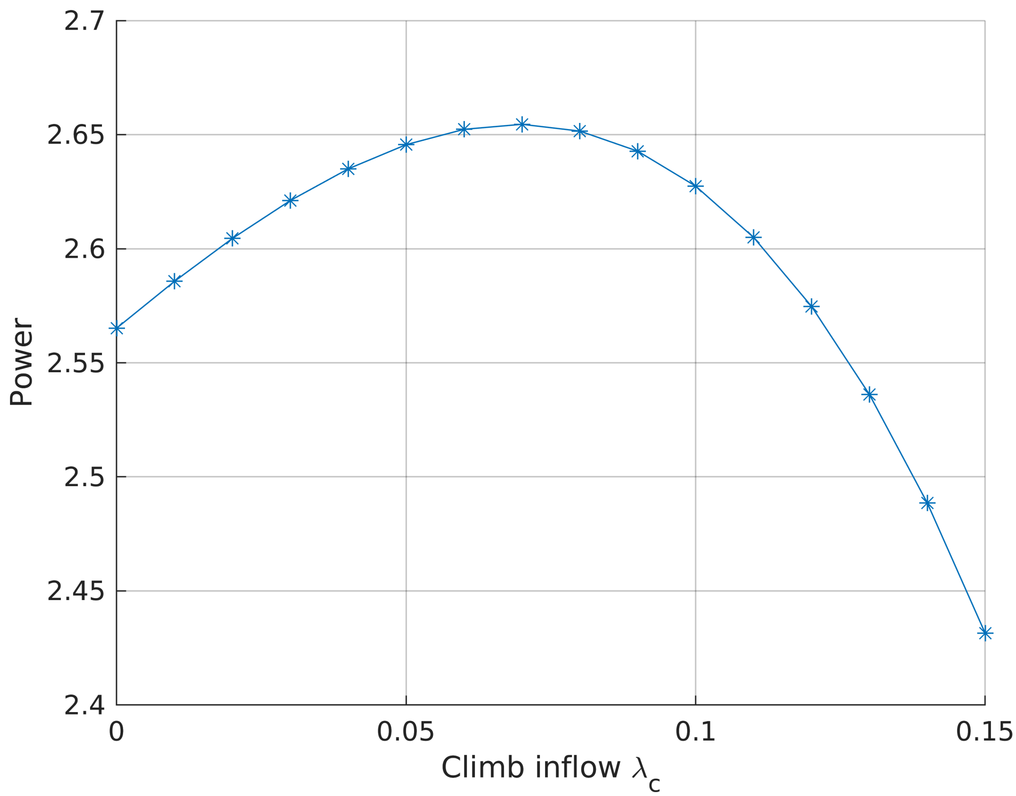
```
ylabel('Omega')
```




```
figure;
hold on;
grid on;
plot(lambda_c_arr, T_arr, '-*')
% legend('isolation', 'interference')
xlabel('Climb inflow \lambda_c')
ylabel('Thrust')
```



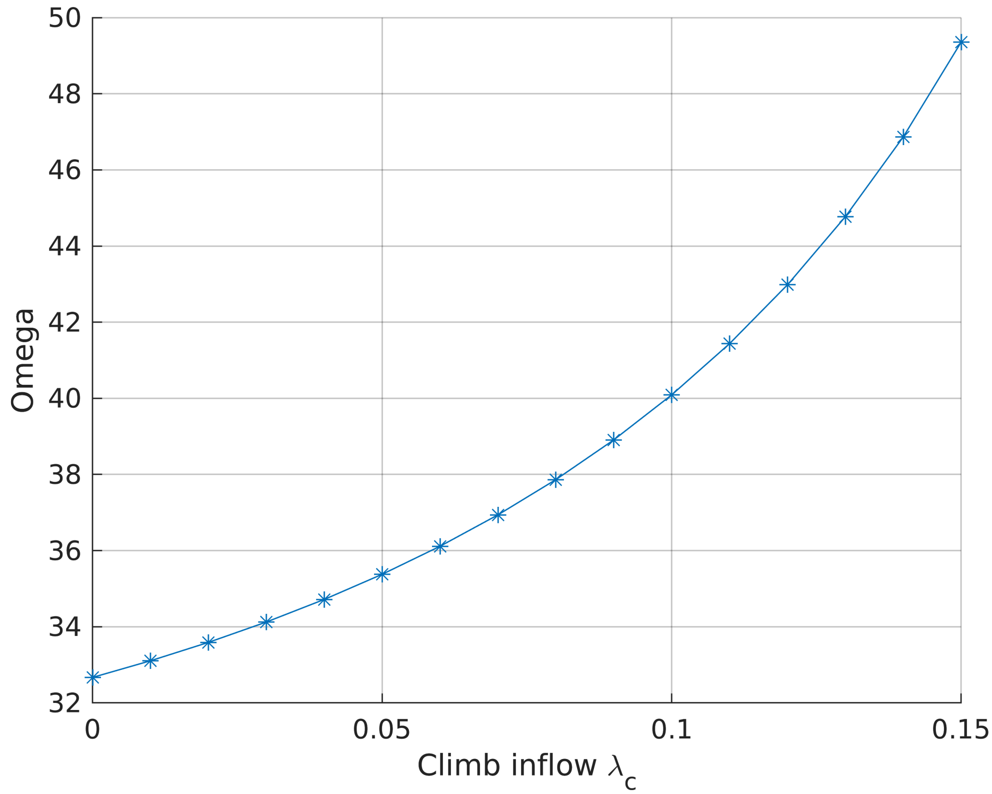

```
figure;
hold on;
grid on;
plot(lambda_c_arr, P_arr, '-*')
% legend('isolation', 'interference')
xlabel('Climb inflow \lambda_c')
ylabel('Power')
```



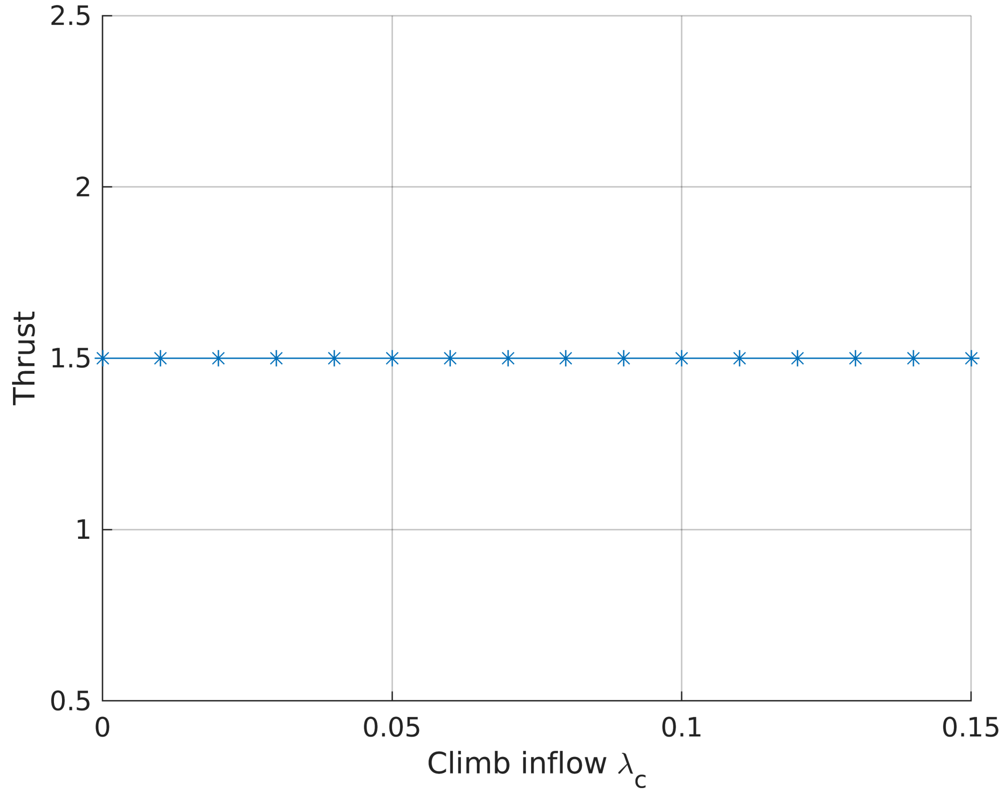
Power dependance on climb velocity (at fixed Thrust)

```
lambda_c_arr = 0:0.01:0.15;
ns = length(lambda_c_arr);

Omega_arr = zeros(ns, 1);
T_arr = zeros(ns, 1);
P_arr = zeros(ns, 1);
for i = 1:ns
    lc = lambda_c_arr(i);
    T_i = subs(T_tot, lambda_c, lc);
    P_i = subs(P_tot, lambda_c, lc);

    T_0 = 1.5;
    [solx, parameters, conditions] = solve(T_i == T_0, Omega, 'ReturnConditions', true)

    Omega_arr(i) = solx;
    T_arr(i) = subs(T_i, Omega, solx);
    P_arr(i) = subs(P_i, Omega, solx);
end

figure;
hold on;
grid on;
plot(lambda_c_arr, Omega_arr, '-*')
% legend('isolation', 'interference')
xlabel('Climb inflow \lambda_c')
```

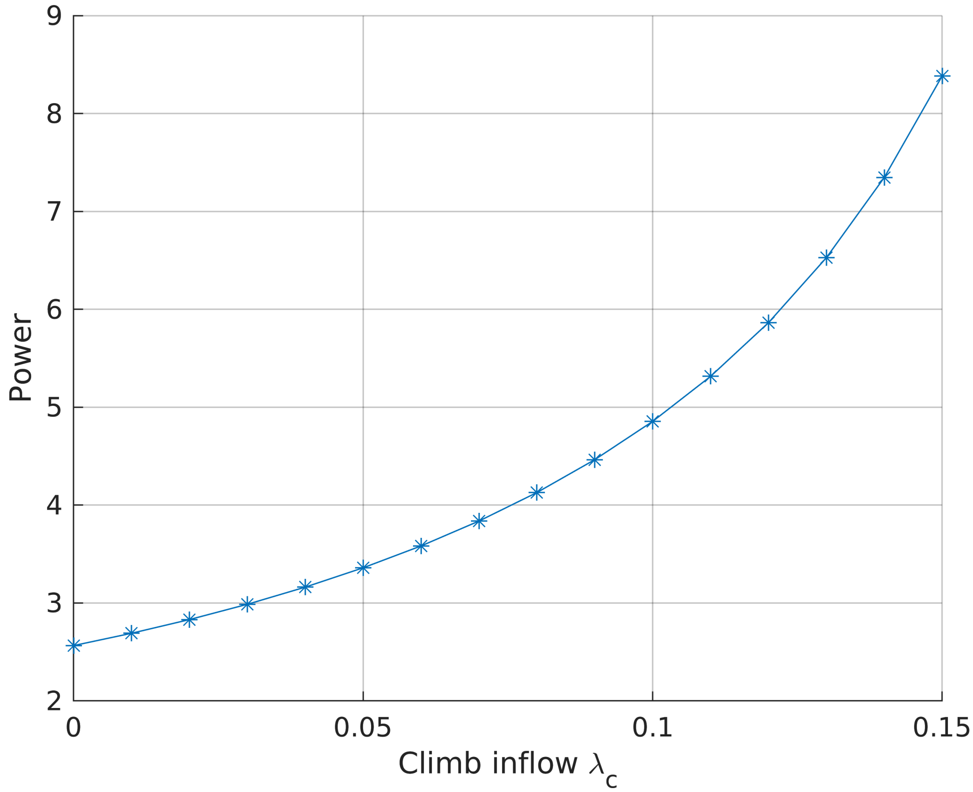
```
ylabel('Omega')
```




```
figure;
hold on;
grid on;
plot(lambda_c_arr, T_arr, '-*')
% legend('isolation', 'interference')
xlabel('Climb inflow \lambda_c')
ylabel('Thrust')
```




```
figure;
hold on;
grid on;
plot(lambda_c_arr, P_arr, '-*')
% legend('isolation', 'interference')
xlabel('Climb inflow \lambda_c')
ylabel('Power')
```


9
8
7
6
5
4
3
2
Power
0
0.05
0.1
0.15
Climb inflow λ_c