A - 01 Matrix Again

Time Limit: 4 sec / Memory Limit: 1024 MiB

Score: 400 points

Problem Statement

There is an $N \times N$ grid. Let (i, j) denote the cell at the i-th row from the top and the j-th column from the left.

You are to fill each cell with 0 or 1. Construct one method to fill the grid that satisfies all of the following conditions:

- The cells $(A_1, B_1), (A_2, B_2), \dots, (A_M, B_M)$ contain 1.
- The integers in the i-th row sum to M. $(1 \le i \le N)$
- The integers in the i-th column sum to M. $(1 \le i \le N)$

It can be proved that under the constraints of this problem, there is at least one method to fill the grid that satisfies the conditions.

Constraints

- $1 \le N \le 10^5$
- $1 \le M \le \min(N, 10)$
- $1 \le A_i, B_i \le N$
- $(A_i, B_i) \neq (A_j, B_j)$ if $i \neq j$.

Input

The input is given from Standard Input in the following format:

$$
\begin{array}{l}
N \quad M \\
A_1 \quad B_1 \\
A_2 \quad B_2 \\
\vdots \\
A_M \quad B_M
\end{array}
$$

Output

Let $(x_1, y_1), (x_2, y_2), \dots, (x_k, y_k)$ be the cells that contain 1, and print the following:

$$\begin{array}{ll} k & \\ x_1 & y_1 \\ x_2 & y_2 \\ \vdots & \\ x_k & y_k \end{array}$$

If multiple methods satisfy the conditions, any of them will be considered correct.

Sample Input 1

```
4 2
1 4
3 2
```

Sample Output 1

```
8
1 2
1 4
2 1
2 4
3 2
3 3
4 1
4 3
```

This output fills the grid as follows. All the conditions are satisfied, so this output is correct.

```
0101
1001
0110
1010
```

Sample Input 2

```
3 3
3 1
2 3
1 3
```

Sample Output 2

```
9
1 1
1 2
1 3
2 1
2 2
2 3
3 1
3 2
3 3
```

Sample Input 3

```
7 3
1 7
7 6
6 1
```

Sample Output 3

```
21
1 6
2 4
4 1
7 3
3 6
4 5
6 1
1 7
7 6
3 5
2 2
6 3
6 7
5 4
5 2
2 5
5 3
1 4
7 1
4 7
3 2
```

B - Simple Math 4

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 400 points

Problem Statement

Find the last digit of the remainder when 2^N is divided by $2^M - 2^K$.

You are given T test cases, each of which must be solved.

Constraints

- $1 \leq T \leq 2 \times 10^5$
- $1 \leq N \leq 10^{18}$
- $1 \leq K < M \leq 10^{18}$
- N, M, K are integers.

Input

The input is given from Standard Input in the following format, where case_i represents the i-th test case:

$$
\begin{aligned}
&T \\
&\mathrm{case}_1 \\
&\mathrm{case}_2 \\
&\vdots \\
&\mathrm{case}_T
\end{aligned}
$$

Each test case is given in the following format:

$$N \quad M \quad K$$

Output

Print the answer.

Sample Input 1

```
5
9 6 2
123 84 50
95 127 79
1000000007 998244353 924844033
473234053352300580 254411431220543632 62658522328486675
```

Sample Output 1

```
2
8
8
8
4
```

For the first test case, the remainder of 2^9 divided by $2^6 - 2^2$ is 32. Thus, the answer is the last digit of 32, which is 2.

C - Max Permutation

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score: 700 points

Problem Statement

Print the number, modulo 998244353, of permutations $P = (P_1, P_2, \ldots, P_N)$ of $(1, 2, \ldots, N)$ that satisfy all of the following conditions:

- $\max(P_{A_i}, P_{B_i}) = C_i \ (1 \leq i \leq M)$.

Constraints

- $2 \leq N \leq 2 \times 10^5$
- $1 \leq M \leq 2 \times 10^5$
- $1 \leq A_i < B_i \leq N$
- $1 \leq C_i \leq N$
- $(A_i, B_i) \neq (A_j, B_j)$ if $i \neq j$.

Input

The input is given from Standard Input in the following format:

```
N M
A_1 B_1 C_1
A_2 B_2 C_2
⋮
A_M B_M C_M
```

Output

Print the answer.

Sample Input 1

```
4 2
1 2 4
2 3 2
```

Sample Output 1

```
2
```

The two permutations P that satisfy the conditions are $(4, 1, 2, 3)$ and $(4, 2, 1, 3)$.

Sample Input 2

```
6 3
1 4 3
2 5 6
3 4 2
```

Sample Output 2

```
8
```

Sample Input 3

```
20 17
9 16 13
5 14 20
15 20 14
5 13 17
18 20 14
14 20 20
6 13 11
12 16 19
2 15 10
6 17 11
7 18 7
8 18 12
8 16 13
6 16 13
2 18 10
9 10 15
7 14 20
```

Sample Output 3

```
1209600
```

D - Swap Permutation

Time Limit: 4 sec / Memory Limit: 1024 MiB

Score: 700 points

Problem Statement

You are given a permutation $P = (P_1, P_2, \dots, P_N)$ of $(1, 2, \dots, N)$. You will perform the following operation M times:

- Choose a pair of integers (i, j) such that $1 \le i < j \le N$, and swap P_i and P_j.

There are $\left(\frac{N(N-1)}{2}\right)^M$ possible sequences of operations. For each of them, consider the value $\sum_{i=1}^{N-1} |P_i - P_{i+1}|$ after all the operations. Find the sum, modulo 998244353, of all those values.

Constraints

- $2 \le N \le 2 \times 10^5$
- $1 \le M \le 2 \times 10^5$
- $(P_1, P_2, \dots, P_N)$ is a permutation of $(1, 2, \dots, N)$.

Input

The input is given from Standard Input in the following format:

$$
\begin{array}{l}
N \quad M \\
P_1 \quad P_2 \quad \dots \quad P_N
\end{array}
$$

Output

Print the answer.

Sample Input 1

```
3 1
1 3 2
```

Sample Output 1

```
8
```

There are three possible sequences of operations:

- Choose $(i, j) = (1, 2)$, making $P = (3, 1, 2)$.
- Choose $(i, j) = (1, 3)$, making $P = (2, 3, 1)$.
- Choose $(i, j) = (2, 3)$, making $P = (1, 2, 3)$.

The values of $\sum_{i=1}^{N-1} |P_i - P_{i+1}|$ for these cases are $3, 3, 2$, respectively. Thus, the answer is $3 + 3 + 2 = 8$.

Sample Input 2

```
2 5
2 1
```

Sample Output 2

```
1
```

Sample Input 3

```
5 2
3 5 1 4 2
```

Sample Output 3

```
833
```

Sample Input 4

```
20 24
14 1 20 6 11 3 19 2 7 10 9 18 13 12 17 8 15 5 4 16
```

Sample Output 4

```
203984325
```

E - Max Vector

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score: 800 points

Problem Statement

You are given two length-N sequences of positive integers: $X = (X_1, X_2, \ldots, X_N)$ and $Y = (Y_1, Y_2, \ldots, Y_N)$.

Additionally, you are given M length-N sequences of positive integers. The i-th sequence is $A_i = (A_{i,1}, A_{i,2}, \ldots, A_{i,N})$.

For each $i = 1, 2, \ldots, M$, you must perform one of the following operations. You can independently choose which operation to perform for each i.

- Replace X_j with $\max(X_j, A_{i,j})$ for all integers j such that $1 \leq j \leq N$.
- Replace Y_j with $\max(Y_j, A_{i,j})$ for all integers j such that $1 \leq j \leq N$.

Find the minimum possible value of $\sum_{j=1}^{N}(X_j + Y_j)$ after all operations.

Constraints

- $1 \leq N \leq 10$
- $1 \leq M \leq 500$
- $1 \leq X_j, Y_j, A_{i,j} \leq 500$

Input

The input is given from Standard Input in the following format:

$$
\begin{array}{l}
N \quad M \\
X_1 \quad X_2 \quad \ldots \quad X_N \\
Y_1 \quad Y_2 \quad \ldots \quad Y_N \\
A_{1,1} \quad A_{1,2} \quad \ldots \quad A_{1,N} \\
A_{2,1} \quad A_{2,2} \quad \ldots \quad A_{2,N} \\
\vdots \\
A_{M,1} \quad A_{M,2} \quad \ldots \quad A_{M,N}
\end{array}
$$

Output

Print the answer.

Sample Input 1

```
3 2
4 4 2
3 1 5
2 5 2
1 2 4
```

Sample Output 1

```
21
```

One optimal sequence of operations is as follows:

- Replace X_j with $\max(X_j, A_{1,j})$, making $X = (4, 5, 2)$.
- Replace Y_j with $\max(Y_j, A_{2,j})$, making $Y = (3, 2, 5)$.

This sequence of operations achieves $\sum_{j=1}^{N}(X_j + Y_j) = 21$.

Sample Input 2

```
3 5
4 13 10
14 9 4
4 6 4
13 18 16
8 13 5
7 18 17
20 20 14
```

Sample Output 2

```
84
```

Sample Input 3

```
5 12
330 68 248 387 491
295 366 376 262 192
280 121 17 168 455
288 179 210 378 490
150 275 165 264 287
66 331 207 282 367
303 215 456 214 18
227 326 103 443 427
395 57 107 350 227
318 231 146 2 116
57 325 124 383 260
147 319 23 177 445
254 198 32 85 56
68 177 356 41 471
```

Sample Output 3

```
3595
```

F - Colorful Star

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score: 1000 points

Problem Statement

There is a tree with $NM+1$ vertices numbered 0 to NM. The i-th edge ($1 \leq i \leq NM$) connects vertices i and $\max(i-N, 0)$.

Initially, vertex i is colored with color $i \bmod N$. You can perform the following operation zero or more times:

- Choose two vertices u and v connected by an edge. Repaint the color of vertex u with that of vertex v.

Find the number, modulo 998244353, of possible trees after the operations. Two trees are distinguished if and only if some vertex has different colors.

Constraints

- $1 \leq N, M \leq 2 \times 10^5$

Input

The input is given from Standard Input in the following format:

$$N \quad M$$

Output

Print the answer.

Sample Input 1

```
3 1
```

Sample Output 1

```
42
```

One possible sequence of operations is as follows. Including this one, there are 42 possible final trees.

Sample Input 2

```
4 2
```

Sample Output 2

```
219100
```

Sample Input 3

```
20 24
```

Sample Output 3

```
984288778
```

Sample Input 4

```
123456 112233
```

Sample Output 4

```
764098676
```