A - Median of Good Sequences

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 400 points

Problem Statement

You are given positive integers N and K.

An integer sequence of length NK where each integer from 1 to N appears exactly K times is called a **good** integer sequence.

Let S be the number of good integer sequences. Find the $\mathrm{floor}((S+1)/2)$-th good integer sequence in lexicographical order. Here, $\mathrm{floor}(x)$ represents the largest integer not exceeding x.

▶ What is lexicographical order for sequences?

Constraints

- $1 \leq N \leq 500$
- $1 \leq K \leq 500$
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$N \quad K$$

Output

Print the desired integer sequence, with elements separated by spaces.

Sample Input 1

```
2 2
```

Sample Output 1

```
1 2 2 1
```

There are six good integer sequences:

- $(1, 1, 2, 2)$
- $(1, 2, 1, 2)$
- $(1, 2, 2, 1)$
- $(2, 1, 1, 2)$
- $(2, 1, 2, 1)$
- $(2, 2, 1, 1)$

Therefore, the answer is the 3rd sequence in lexicographical order, $(1, 2, 2, 1)$.

Sample Input 2

```
1 5
```

Sample Output 2

```
1 1 1 1 1
```

Sample Input 3

```
6 1
```

Sample Output 3

```
3 6 5 4 2 1
```

Sample Input 4

```
3 3
```

Sample Output 4

```
2 2 2 1 3 3 3 1 1
```

B - Near Assignment

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 600 points

Problem Statement

You are given integer sequences of length N: $A = (A_1, A_2, \cdots, A_N)$ and $B = (B_1, B_2, \cdots, B_N)$, and an integer K.

You can perform the following operation zero or more times.

- Choose integers i and j ($1 \leq i, j \leq N$). Here, $|i - j| \leq K$ must hold. Then, change the value of A_i to A_j.

Determine whether it is possible to make A identical to B.

There are T test cases for each input.

Constraints

- $1 \leq T \leq 125000$
- $1 \leq K < N \leq 250000$
- $1 \leq A_i, B_i \leq N$
- The sum of N across all test cases in each input is at most 250000.
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$
\begin{array}{l}
T \\
case_1 \\
case_2 \\
\vdots \\
case_T
\end{array}
$$

Each test case is given in the following format:

$$
\begin{array}{l}
N \quad K \\
A_1 \quad A_2 \quad \cdots \quad A_N \\
B_1 \quad B_2 \quad \cdots \quad B_N
\end{array}
$$

Output

For each test case, print `Yes` if it is possible to make A identical to B, and `No` otherwise.

Sample Input 1

```
4
3 1
1 1 2
1 2 2
5 4
2 4 5 1 3
2 1 3 2 2
13 1
3 1 3 3 5 3 3 4 2 2 2 5 1
5 3 3 3 4 2 2 2 2 5 5 1 3
20 14
10 6 6 19 13 16 15 15 2 10 2 16 9 12 2 6 13 5 5 9
5 9 6 2 10 19 16 15 13 12 10 2 9 6 5 16 19 12 15 13
```

Sample Output 1

```
Yes
Yes
No
Yes
```

Consider the first test case. If we operate with $i = 2$ and $j = 3$, the value of A_2 will be changed to $A_3 = 2$, resulting in $A = (1, 2, 2)$.

C - Not Argmax

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 600 points

Problem Statement

Find the number, modulo 998244353, of permutations $P = (P_1, P_2, \cdots, P_N)$ of $(1, 2, \cdots, N)$ that satisfy all of the following M conditions.

- The i-th condition: The maximum among $P_{L_i}, P_{L_i+1}, \cdots, P_{R_i}$ is **not** P_{X_i}. Here, L_i, R_i, and X_i are integers given in the input.

Constraints

- $1 \leq N \leq 500$
- $1 \leq M \leq 10^5$
- $1 \leq L_i \leq X_i \leq R_i \leq N$
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$
\begin{array}{l}
N \quad M \\
L_1 \quad R_1 \quad X_1 \\
L_2 \quad R_2 \quad X_2 \\
\vdots \\
L_M \quad R_M \quad X_M
\end{array}
$$

Output

Print the answer.

Sample Input 1

```
3 2
1 3 2
1 2 1
```

Sample Output 1

```
1
```

Only one permutation, $P = (1, 2, 3)$, satisfies the conditions.

Sample Input 2

```
5 1
1 1 1
```

Sample Output 2

```
0
```

Sample Input 3

```
10 5
3 8 4
3 10 4
1 7 2
1 8 3
3 8 7
```

Sample Output 3

```
1598400
```

Sample Input 4

```
15 17
2 11 9
2 15 13
1 14 2
5 11 5
3 15 11
1 6 2
4 15 12
3 11 6
9 13 10
2 14 6
10 15 11
1 8 6
6 14 8
2 10 2
6 12 6
3 14 12
2 6 2
```

Sample Output 4

```
921467228
```

D - Keep Perfectly Matched

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 800 points

Problem Statement

There is a tree with N vertices numbered from 1 to N. The i-th edge connects vertices A_i and B_i. Here, N is even, and furthermore, this tree has a perfect matching. Specifically, for each i ($1 \leq i \leq N/2$), it is guaranteed that $A_i = i \times 2 - 1$ and $B_i = i \times 2$.

You will perform the following operation $N/2$ times:

- Choose two leaves (vertices with degree exactly 1) and remove them from the tree. Here, the tree after removal must still have a perfect matching. In this problem, we consider a graph with zero vertices to be a tree as well.

For each operation, its score is defined as the distance between the two chosen vertices (the number of edges on the simple path connecting the two vertices).

Show one procedure that maximizes the total score. It can be proved that there always exists a procedure to complete $N/2$ operations under the constraints of this problem.

Constraints

- $2 \leq N \leq 250000$
- N is even.
- $1 \leq A_i < B_i \leq N$ ($1 \leq i \leq N-1$)
- $A_i = i \times 2 - 1, B_i = i \times 2$ ($1 \leq i \leq N/2$)
- The given graph is a tree.
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$
\begin{aligned}
&N \\
&A_1 \quad B_1 \\
&A_2 \quad B_2 \\
&\vdots \\
&A_{N-1} \quad B_{N-1}
\end{aligned}
$$

Output

Print a solution in the following format:

$$
\begin{aligned}
&X_1 \quad Y_1 \\
&X_2 \quad Y_2 \\
&\vdots \\
&X_{N/2} \quad Y_{N/2}
\end{aligned}
$$

Here, X_i and Y_i are the two vertices chosen in the i-th operation. If there are multiple solutions, you may print any of them.

Sample Input 1

```
4
1 2
3 4
2 3
```

Sample Output 1

```
4 1
2 3
```

The procedure in the sample output is as follows:

- 1st operation: Remove vertices 4 and 1. The remaining tree has vertices 2 and 3, and a perfect matching. The score of this operation is 3.
- 2nd operation: Remove vertices 2 and 3. The remaining tree has zero vertices and a perfect matching. The score of this operation is 1.
- The total score is $3 + 1 = 4$.

It is impossible to make the total score greater than 4, so this output solves this sample input.

Sample Input 2

```
8
1 2
3 4
5 6
7 8
2 3
1 5
1 7
```

Sample Output 2

```
4 8
7 6
5 3
2 1
```

Sample Input 3

```
14
1 2
3 4
5 6
7 8
9 10
11 12
13 14
2 8
4 11
5 12
7 13
11 14
9 13
```

Sample Output 3

```
1 6
5 2
8 12
3 7
10 4
11 9
13 14
```

Sample Input 4

```
20
1 2
3 4
5 6
7 8
9 10
11 12
13 14
15 16
17 18
19 20
8 10
16 18
16 19
5 9
10 17
2 13
7 14
3 7
3 12
```

Sample Output 4

```
6 1
2 15
20 13
14 19
16 4
11 18
17 12
3 5
9 7
8 10
```

E - Ascendant Descendant

Time Limit: 7 sec / Memory Limit: 1024 MiB

Score : 900 points

Problem Statement

There is a rooted tree with N vertices numbered from 1 to N. The root is vertex 1, and the parent of vertex i ($2 \leq i \leq N$) is vertex P_i ($P_i < i$).

There are also integer sequences of length M: $A = (A_1, A_2, \cdots, A_M)$ and $B = (B_1, B_2, \cdots, B_M)$, consisting of integers between 1 and N, inclusive.

A is said to be **good** if and only if for each i, vertex A_i is an ancestor of vertex B_i or $A_i = B_i$. Initially, A is good.

Consider the following operation on A.

- Choose an integer i ($1 \leq i \leq M-1$) and swap the values of A_i and A_{i+1}. Here, A must remain good after the operation.

Find the number, modulo 998244353, of sequences that can result from performing this operation on A zero or more times.

Constraints

- $2 \leq N \leq 250000$
- $2 \leq M \leq 250000$
- $1 \leq P_i < i$
- $1 \leq A_i \leq B_i \leq N$
- Vertex A_i is an ancestor of vertex B_i or $A_i = B_i$.
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$
\begin{array}{l}
N \quad M \\
P_2 \quad P_3 \quad \cdots \quad P_N \\
A_1 \quad A_2 \quad \cdots \quad A_M \\
B_1 \quad B_2 \quad \cdots \quad B_M
\end{array}
$$

Output

Print the answer.

Sample Input 1

```
3 3
1 2
1 2 1
1 2 3
```

Sample Output 1

```
2
```

Consider choosing $i = 1$. The $A = (2, 1, 1)$ after the operation is not good, so this operation is invalid.

Consider choosing $i = 2$. The $A = (1, 1, 2)$ after the operation is good, so this operation is valid.

There are two sequences that can result from performing zero or more operations on A: $A = (1, 2, 1)$ and $(1, 1, 2)$.

Sample Input 2

```
4 3
1 1 1
2 3 4
2 3 4
```

Sample Output 2

```
1
```

Sample Input 3

```
8 13
1 2 2 3 4 4 3
5 3 2 5 4 6 2 8 2 6 7 4 7
5 5 8 5 6 6 5 8 3 6 7 4 7
```

Sample Output 3

```
8
```

Sample Input 4

```
30 27
1 2 1 1 5 1 7 1 5 10 1 12 12 13 15 16 12 18 19 18 21 21 23 13 18 18 27 27 13
1 18 1 5 11 12 1 1 1 12 1 12 1 15 1 1 21 1 12 10 2 8 3 1 1 30 12
14 27 30 5 11 17 1 18 24 27 29 27 19 15 28 5 21 21 29 11 2 8 3 4 10 30 22
```

Sample Output 4

```
60
```

F - Sum of Minimum Distance

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 1100 points

Problem Statement

You are given positive integers A, B, X, Y, and N. Here, the following is guaranteed:

- $A < B$
- $\gcd(A, B) = 1$
- $1 \leq N \leq A + B - 1$

For an integer n, define $f(n)$ as follows:

- You start with an integer $x = 0$. $f(n)$ is the minimum total cost to achieve $x = n$ by repeatedly performing the following operations.
    - Replace the value of x with $x + A$. The cost of this operation is X.
    - Replace the value of x with $x - A$. The cost of this operation is X.
    - Replace the value of x with $x + B$. The cost of this operation is Y.
    - Replace the value of x with $x - B$. The cost of this operation is Y.

It can be proved from the constraints on A and B that $f(n)$ is defined for any integer n.

Find the value of $\sum_{1 \leq n \leq N} f(n)$, modulo 998244353.

There are T test cases for each input.

Constraints

- $1 \leq T \leq 1000$
- $1 \leq A < B \leq 10^9$
- $\gcd(A, B) = 1$
- $1 \leq X, Y \leq 10^9$
- $1 \leq N \leq A + B - 1$
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$\begin{array}{l} T \\ case_1 \\ case_2 \\ \vdots \\ case_T \end{array}$$

Each test case is given in the following format:

$$A \quad B \quad X \quad Y \quad N$$

Output

Print the answer for each test case.

Sample Input 1

```
4
1 2 1 1 2
3 5 2 4 6
79 85 72 95 4
80980429 110892168 22712439 520643153 66132787
```

Sample Output 1

```
2
34
18111
785776602
```

In the first test case, $f(1) = 1$ and $f(2) = 1$.

In the second test case, $f(1) = 8$, $f(2) = 6$, $f(3) = 2$, $f(4) = 10$, $f(5) = 4$, and $f(6) = 4$.