A - Union of Grid Paths

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 400 points

Problem Statement

There is an $H \times W$ grid of cells. Let (h, w) denote the cell at the h-th row from the top and the w-th column from the left. Furthermore, you are given a string $S = S_1 \cdots S_{H+W-2}$ of length $H + W - 2$ consisting of `D`, `R`, and `?`.

Initially, all cells are painted white. You may perform the following operation, which consists of three steps, any number of times:

1. Choose a string $X = X_1 \cdots X_{H+W-2}$ of length $H + W - 2$ satisfying all of the following.
   - X consists of exactly $H - 1$ `D`s and $W - 1$ `R`s.
   - For each $1 \leq i \leq H + W - 2$, if S_i is `D` then X_i must also be `D`.
   - For each $1 \leq i \leq H + W - 2$, if S_i is `R` then X_i must also be `R`.
2. Stand on cell $(1, 1)$. Then for $i = 1, 2, \ldots$ in order, move one cell in the direction indicated by X_i: if X_i is `D`, move down one cell; if X_i is `R`, move right one cell. It can be shown that if X satisfies the conditions in step 1, the destination cell always exists within the grid.
3. For every cell you visited in step 2 (including the starting and ending cells), if the cell is currently white, paint it black.

Find the maximum possible number of cells that can be painted black in total.

There are T test cases; solve each one.

Constraints

- $1 \leq T \leq 2 \times 10^5$
- $2 \leq H, W \leq 2 \times 10^5$
- T, H, W are integers.
- S is a string of length $H + W - 2$ consisting of `D`, `R`, and `?`.
- The number of `D`s in S is at most $H - 1$.
- The number of `R`s in S is at most $W - 1$.
- The sum of $H + W$ over all test cases is at most 4×10^5.

Input

The input is given from Standard Input in the following format:

$$T$$
$$\mathrm{case}_1$$
$$\vdots$$
$$\mathrm{case}_T$$

Each case is given in the following format:

$$H \quad W$$
$$S$$

Output

Print T lines. The i-th line should contain the maximum number of cells that can be painted black for the i-th test case.

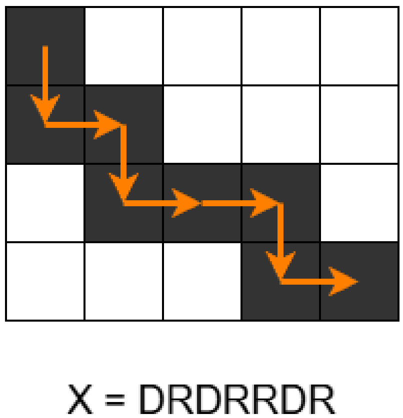
Sample Input 1

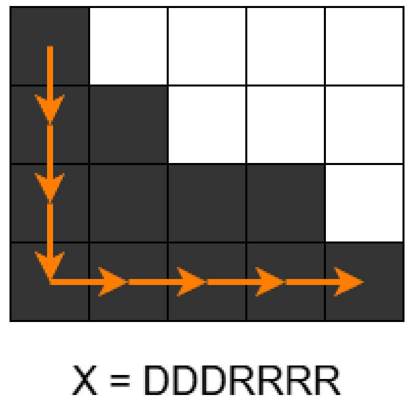
```
4
4 5
D?DRR?R
4 5
DDRRDRR
4 5
???????
2 2
DR
```

Sample Output 1

```
12
8
20
3
```

For the first test case, by choosing X as `DRDRRDR` in the first operation and `DDDRRRR` in the second operation, you can paint 12 cells black.





B - Greater Than Average

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 500 points

Problem Statement

Define the **score** of a non-empty integer sequence $x = (x_1, \ldots, x_n)$ as the number of elements of x that are strictly greater than the average of x.

That is, the score of x is the count of indices i satisfying $x_i > \dfrac{x_1 + \cdots + x_n}{n}$.

You are given a length-N integer sequence $A = (A_1, \ldots, A_N)$. Find the maximum score of a non-empty subsequence of A.

There are T test cases; solve each one.

▶ Definition of subsequence

Constraints

- $1 \leq T \leq 2 \times 10^5$
- $2 \leq N \leq 2 \times 10^5$
- $1 \leq A_i \leq 10^9$
- All input values are integers.
- The sum of N over all test cases is at most 2×10^5.

Input

The input is given from Standard Input in the following format:

$$T$$
$$\mathrm{case}_1$$
$$\vdots$$
$$\mathrm{case}_T$$

Each case is given in the following format:

$$N$$
$$A_1 \quad \cdots \quad A_N$$

Output

Print T lines. The i-th line should contain the maximum score of a non-empty subsequence of A for the i-th test case.

Sample Input 1

```
3
5
2 6 5 7 5
5
10 10 10 10 10
5
1 10 100 1000 10000
```

Sample Output 1

```
3
0
1
```

For the first test case, below are examples of subsequences, their averages, and their scores.

- $x = (A_1) = (2)$: the average is 2, and the score is 0.
- $x = (A_3, A_5) = (5, 5)$: the average is 5, and the score is 0.
- $x = (A_1, A_3, A_4, A_5) = (2, 5, 7, 5)$: the average is $\frac{19}{4}$, and the score is 3.
- $x = (A_1, A_2, A_3, A_4, A_5) = (2, 6, 5, 7, 5)$: the average is 5, and the score is 2.

C - Removal of Multiples

Time Limit: 4 sec / Memory Limit: 1024 MiB

Score : 600 points

Problem Statement

Let S be the set of all positive integers.

Process Q queries in order. The i-th query gives you an integer A_i not less than 2 and a positive integer B_i, so perform the following two steps in order.

1. Remove from S all elements that are multiples of A_i.
2. Print the B_i-th smallest element of S. It can be shown that under the given constraints, S contains at least B_i elements at this point.

Constraints

- $1 \leq Q \leq 10^5$
- $2 \leq A_i \leq 10^9$
- $1 \leq B_i \leq 10^5$
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$
\begin{array}{l}
Q \\
A_1 \quad B_1 \\
\vdots \\
A_Q \quad B_Q
\end{array}
$$

Output

Print Q lines. The i-th line should contain the B_i-th smallest element of S after removing all multiples of A_i from S.

Sample Input 1

```
5
5 10
6 1
6 10
9 10
123456789 111
```

Sample Output 1

```
12
1
14
16
178
```

When the elements of S are listed in ascending order, the beginning of the list evolves as follows.

- Initially, $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, \ldots\}$.
- The first query removes multiples of 5, resulting in $S = \{1, 2, 3, 4, 6, 7, 8, 9, 11, 12, \ldots\}$.
- The second query removes multiples of 6, resulting in $S = \{1, 2, 3, 4, 7, 8, 9, 11, 13, 14, \ldots\}$.
- The third query removes multiples of 6, resulting in $S = \{1, 2, 3, 4, 7, 8, 9, 11, 13, 14, \ldots\}$.
- The fourth query removes multiples of 9, resulting in $S = \{1, 2, 3, 4, 7, 8, 11, 13, 14, 16, \ldots\}$.
- The fifth query removes multiples of 123456789, resulting in $S = \{1, 2, 3, 4, 7, 8, 11, 13, 14, 16, \ldots\}$.

D - Ancestor Relation

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 700 points

Problem Statement

You are given an $N \times N$ matrix $A = (A_{i,j})$ $(1 \leq i, j \leq N)$ whose entries are 0 or 1.

Find, modulo 998244353, the number of trees G on N vertices numbered 1 to N that satisfy the following condition.

- $A_{i,j} = 1$ if and only if at least one of the following holds:
  - When G is rooted at vertex 1, Vertex j is an ancestor of vertex i. That is, vertex j lies on the unique path in G between vertices 1 and i.
  - When G is rooted at vertex 1, Vertex i is an ancestor of vertex j. That is, vertex i lies on the unique path in G between vertices 1 and j.

Here, the endpoints of a path are considered to be on that path. Note that G being a tree guarantees uniqueness of the path between any two vertices.

There are T test cases; solve each one.

Constraints

- $1 \leq T \leq 10^5$
- $2 \leq N \leq 400$
- $A_{i,j} \in \{0, 1\}$ $(1 \leq i, j \leq N)$
- $A_{i,i} = 1$ $(1 \leq i \leq N)$
- $A_{i,j} = A_{j,i}$ $(1 \leq i, j \leq N)$
- The sum of N^2 over all test cases is at most 400^2.

Input

The input is given from Standard Input in the following format:

$$T$$
$$\mathrm{case}_1$$
$$\vdots$$
$$\mathrm{case}_T$$

Each case is given in the following format:

$$N$$
$$A_{1,1} \quad \cdots \quad A_{1,N}$$
$$\vdots$$
$$A_{N,1} \quad \cdots \quad A_{N,N}$$

Output

Print T lines. The i-th line should contain the number, modulo 998244353, of trees G satisfying the conditions for the i-th test case.

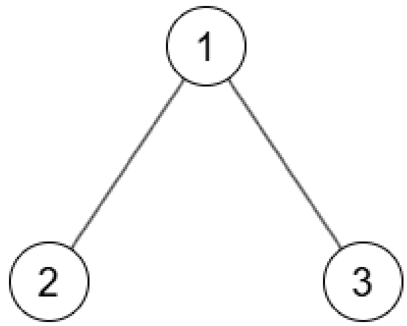
Sample Input 1

```
5
3
1 1 1
1 1 0
1 0 1
3
1 1 1
1 1 1
1 1 1
3
1 0 0
0 1 0
0 0 1
3
1 0 1
0 1 1
1 1 1
7
1 1 1 1 1 1 1
1 1 0 1 0 1 1
1 0 1 1 1 1 0
1 1 1 1 1 1 1
1 0 1 1 1 1 0
1 1 1 1 1 1 1
1 1 0 1 0 1 1
```

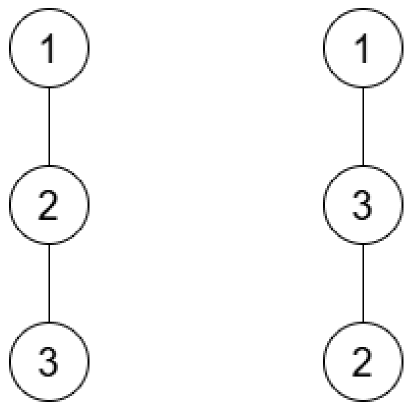
Sample Output 1

```
1
2
0
0
8
```

In the first test case, the following one tree G satisfies the condition:



In the second test case, the following two trees G satisfy the condition:



E - Four Square Tiles

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 800 points

Problem Statement

You are given positive integers N, H, and W, with $H, W \leq 3N - 1$.

Find the number, modulo 998244353, of ways to place four $N \times N$ square tiles on an $H \times W$ grid that satisfy all of the following conditions.

- Each tile exactly covers N^2 cells of the grid.
- No cell is covered by more than one tile.

Here, the tiles are indistinguishable.

There are T test cases; solve each one.

Constraints

- $1 \leq T \leq 2 \times 10^5$
- $1 \leq N, H, W \leq 10^9$
- $H, W \leq 3N - 1$
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$\begin{array}{l} T \\ \text{case}_1 \\ \vdots \\ \text{case}_T \end{array}$$

Each case is given in the following format:

$$N \quad H \quad W$$

Output

Print T lines. The i-th line should contain the number, modulo 998244353, of valid ways to place the tiles for the i-th test case.

Sample Input 1

```
4
2 4 5
2 5 5
1000 1000 1000
1000 2222 2025
```

Sample Output 1

```
9
79
0
262210557
```

For the first test case, there are 9 ways as illustrated in the following figure: