A - Good Permutation 2

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 400 points

Problem Statement

You are given a positive integer N and a sequence of M positive integers $A = (A_1, A_2, \dots, A_M)$.

Here, all elements of A are distinct integers between 1 and N, inclusive.

A permutation $P = (P_1, P_2, \dots, P_N)$ of $(1, 2, \dots, N)$ is called a **good permutation** when it satisfies the following condition for all integers i such that $1 \leq i \leq M$:

- No contiguous subsequence of P is a permutation of $(1, 2, \dots, A_i)$.

Determine whether a **good permutation** exists, and if it does, find the lexicographically smallest **good permutation**.

▶ What is lexicographical order?

Constraints

- $1 \leq M \leq N \leq 2 \times 10^5$
- $1 \leq A_i \leq N$
- All elements of A are distinct.
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$
\begin{array}{l}
N \quad M \\
A_1 \quad A_2 \quad \cdots \quad A_M
\end{array}
$$

Output

If a **good permutation** does not exist, print `-1`.

If it exists, print the lexicographically smallest **good permutation**, separated by spaces.

Sample Input 1

```
4 1
2
```

Sample Output 1

```
1 3 2 4
```

For example, $(4, 2, 1, 3)$ is not a **good permutation** because it contains $(2, 1)$ as a contiguous subsequence.

Other non-**good permutations** are $(1, 2, 3, 4)$ and $(3, 4, 2, 1)$.

Some **good permutations** are $(4, 1, 3, 2)$ and $(2, 3, 4, 1)$. Among these, the lexicographically smallest one is $(1, 3, 2, 4)$, so print it separated by spaces.

Sample Input 2

```
5 3
4 3 2
```

Sample Output 2

```
1 3 4 5 2
```

Examples of **good permutations** include $(3, 4, 1, 5, 2)$, $(2, 4, 5, 3, 1)$, and $(4, 1, 5, 2, 3)$.

Examples of non-**good permutations** include $(1, 2, 5, 3, 4)$, $(2, 3, 4, 1, 5)$, and $(5, 3, 1, 2, 4)$.

Sample Input 3

```
92 4
16 7 1 67
```

Sample Output 3

```
-1
```

If a **good permutation** does not exist, print `-1`.

Sample Input 4

```
43 2
43 2
```

Sample Output 4

```
-1
```

B - 1 + 6 = 7

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 600 points

Problem Statement

You are given positive integers A_1, A_2, A_3. Find the number, modulo 998244353, of tuples of positive integers (X_1, X_2, X_3) that satisfy all of the following conditions.

- X_1 is a positive integer with A_1 digits in decimal notation.
- X_2 is a positive integer with A_2 digits in decimal notation.
- X_3 is a positive integer with A_3 digits in decimal notation.
- $X_1 + X_2 = X_3$.

You are given T test cases per input file; solve each of them.

Constraints

- $1 \leq T \leq 10^5$
- $1 \leq A_i \leq 10^9$
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$\begin{array}{l} T \\ \mathrm{case}_1 \\ \mathrm{case}_2 \\ \vdots \\ \mathrm{case}_T \end{array}$$

Each case is given in the following format:

$$A_1 \quad A_2 \quad A_3$$

Output

Print T lines. The i-th line should contain the answer for case_i.

Sample Input 1

```
4
1 1 1
1 6 7
167 167 167
111 666 777
```

Sample Output 1

```
36
45
731780675
0
```

For the first case, tuples such as $(X_1, X_2, X_3) = (1, 6, 7), (2, 1, 3)$ satisfy the conditions.

On the other hand, tuples such as $(X_1, X_2, X_3) = (6, 7, 13), (3, 4, 5)$ do not.

There are 36 tuples (X_1, X_2, X_3) that satisfy the conditions, so print 36.

For the third case, remember to print the result modulo 998244353.

For the fourth case, there may be no tuples (X_1, X_2, X_3) that satisfy the conditions.

C - Sum of Abs 2

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 600 points

Problem Statement

You are given positive integers N and L, and a sequence of positive integers $A = (A_1, A_2, \dots, A_N)$ of length N.

For each $i = 1, 2, \dots, N$, answer the following question:

> Determine if there exists a sequence of L non-negative integers $B = (B_1, B_2, \dots, B_L)$ such that $\sum_{j=1}^{L-1} \sum_{k=j+1}^{L} |B_j - B_k| = A_i$. If it exists, find the minimum value of $\max(B)$ for such a sequence B.

Constraints

- $1 \leq N \leq 2 \times 10^5$
- $2 \leq L \leq 2 \times 10^5$
- $1 \leq A_i \leq 2 \times 10^5$
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$
\begin{array}{l}
N \quad L \\
A_1 \quad A_2 \quad \cdots \quad A_N
\end{array}
$$

Output

Print N lines. The k-th line should contain `-1` if no sequence B satisfies the condition for $i = k$; otherwise, it should contain the minimum value of $\max(B)$ for such a sequence B.

Sample Input 1

```
2 4
10 5
```

Sample Output 1

```
3
-1
```

For $A_1 = 10$, if we take $B = (1, 0, 2, 3)$, then $\sum_{j=1}^{L-1} \sum_{k=j+1}^{L} |B_j - B_k| = 10$, where $\max(B) = 3$. No non-negative integer sequence B satisfies the condition with $\max(B) < 3$, so print 3 in the first line.

For $A_2 = 5$, there is no non-negative integer sequence B that satisfies the condition, so print `-1` in the second line.

Sample Input 2

```
6 8
167 924 167167 167924 116677 154308
```

Sample Output 2

```
11
58
10448
10496
7293
9645
```

D - Delete Range Mex

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 700 points

Problem Statement

You are given a positive integer N and a sequence of M non-negative integers $A = (A_1, A_2, \ldots, A_M)$.

Here, all elements of A are distinct integers between 0 and $N-1$, inclusive.

Find the number, modulo 998244353, of permutations P of $(0, 1, \ldots, N-1)$ that satisfy the following condition.

- After initializing a sequence $B = (B_1, B_2, \ldots, B_N)$ to P, it is possible to make $B = A$ by repeating the following operation some number of times:
  - Choose l and r such that $1 \leq l \leq r \leq |B|$, and if $\mathrm{mex}(\{B_l, B_{l+1}, \ldots, B_r\})$ is contained in B, remove it from B.

▶ What is $\mathrm{mex}(X)$?

Constraints

- $1 \leq M \leq N \leq 500$
- $0 \leq A_i < N$
- All elements of A are distinct.
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$
\begin{array}{l}
N \quad M \\
A_1 \quad A_2 \quad \cdots \quad A_M
\end{array}
$$

Output

Print the answer.

Sample Input 1

```
4 2
1 3
```

Sample Output 1

```
8
```

After initializing $B = (2, 1, 0, 3)$, it is possible to make $B = A$ using the following steps:

- Choose $(l, r) = (2, 4)$, remove $\mathrm{mex}(\{1, 0, 3\}) = 2$ from B, making $B = (1, 0, 3)$.
- Choose $(l, r) = (3, 3)$, remove $\mathrm{mex}(\{3\}) = 0$ from B, making $B = (1, 3)$.

Thus, $P = (2, 1, 0, 3)$ satisfies the condition.

There are eight permutations P that satisfy the condition, including the above, so print 8.

Sample Input 2

```
4 4
0 3 2 1
```

Sample Output 2

```
1
```

Only $P = (0, 3, 2, 1)$ satisfies the condition.

Sample Input 3

```
16 7
9 2 4 0 1 6 7
```

Sample Output 3

```
3520
```

Sample Input 4

```
92 4
1 67 16 7
```

Sample Output 4

```
726870122
```

Find the count modulo 998244353.

E - Serval Survival

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 1000 points

Problem Statement

There are N servals on a bridge of length L.

The i-th serval is located at position A_i from the left end of the bridge.

Here, $0 < A_1 < A_2 < \cdots < A_N < L$ holds.

For each $i = 1, 2, \ldots, N$, answer the following question:

> The servals will perform the following three actions in order:
>
> - Action 1: The $N - 1$ servals other than the i-th serval face left or right.
> - Action 2: The i-th serval faces left or right.
> - Action 3: All servals start moving simultaneously. All servals move at a constant speed of exactly 1 unit distance per unit time. When a serval reaches the end of the bridge, it leaves the bridge. If two servals collide, they both reverse their direction and continue moving.
>
> The i-th serval is smart and loves this bridge, so when choosing a direction in Action 2, it will observe the directions of the other $N - 1$ servals and choose the direction that allows it to stay on the bridge the longer during Action 3. There are 2^{N-1} possible combinations of directions for the $N - 1$ servals in Action 1. Find the sum, modulo 998244353, over all these combinations, of the durations the i-th serval can stay on the bridge. It can be proved that the output value is an integer.

Constraints

- $1 \leq N \leq 10^5$
- $0 < A_1 < A_2 < \cdots < A_N < L \leq 10^9$
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$N \quad L$$
$$A_1 \quad A_2 \quad \cdots \quad A_N$$

Output

Print N lines. The k-th line should contain the answer for $i = k$.

Sample Input 1

```
2 167
9 24
```

Sample Output 1

```
182
301
```

For $i = 1$, it is always optimal to face right.

For $i = 2$, it is optimal to face the opposite direction from the first serval.

Sample Input 2

```
1 924
167
```

Sample Output 2

```
757
```

Sample Input 3

```
10 924924167
46001560 235529797 272749755 301863061 359726177 470023587 667800476 696193062 741860924 809211293
```

Sample Output 3

```
112048251
409175578
167800512
997730745
278651538
581491882
884751575
570877705
747965896
80750577
```

F - Long Sequence Inversion

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 1000 points

Problem Statement

You are given positive integers N, M, and K, and a sequence of M non-negative integers $A = (A_0, A_1, \ldots, A_{M-1})$. Here, $2^{N-1} \leq K < 2^N$ holds.

In the input, K is given as an N-digit number in binary notation, while the other integers are given in decimal notation.

Additionally, A is not given directly in the input. Instead, for each $i = 0, 1, \ldots, M-1$, you are given a sequence of L_i integers $X_i = (X_{i,0}, X_{i,1}, \ldots, X_{i,L_i-1})$ such that $A_i = \sum_{j=0}^{L_i-1} 2^{X_{i,j}}$. Here, $0 \leq X_{i,0} < X_{i,1} < \cdots < X_{i,L_i-1} < N$ holds.

Find the inversion number, modulo 998244353, of the sequence $B = (B_0, B_1, \ldots, B_{MK-1})$ defined as follows.

- For any integer a such that $0 \leq a < K$ and any integer b such that $0 \leq b < M$, the following holds:
  - B_{aM+b} is equal to the remainder when $\mathrm{popcount}(a \ \mathrm{AND}\ A_b)$ is divided by 2.

▶ What is AND?

▶ What is $\mathrm{popcount}$?

Constraints

- $1 \leq N \leq 2 \times 10^5$
- $1 \leq M \leq 2 \times 10^5$
- $2^{N-1} \leq K < 2^N$
- $0 \leq L_i \leq N$
- $\sum L_i \leq 2 \times 10^5$
- $0 \leq X_{i,0} < X_{i,1} < \cdots < X_{i,L_i-1} < N$
- All input values are integers.
- K is given in binary notation.
- All numbers except K are given in decimal notation.

Input

The input is given from Standard Input in the following format:

$$
\begin{array}{l}
N \quad M \\
K \\
L_0 \quad X_{0,0} \quad X_{0,1} \quad \cdots \quad X_{0,L_0-1} \\
L_1 \quad X_{1,0} \quad X_{1,1} \quad \cdots \quad X_{1,L_1-1} \\
\vdots \\
L_{M-1} \quad X_{M-1,0} \quad X_{M-1,1} \quad \cdots \quad X_{M-1,L_{M-1}-1}
\end{array}
$$

Output

Print the answer.

Sample Input 1

```
2 4
11
1 0
2 0 1
0
1 1
```

Sample Output 1

```
9
```

$A = (1, 3, 0, 2), B = (0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 1).$

Sample Input 2

```
3 3
101
2 1 2
2 0 1
1 0
```

Sample Output 2

```
23
```

$A=(6,3,1), B=(0,0,0,0,1,1,1,1,0,1,0,1,1,0,0)$.

Sample Input 3

```
16 7
1101010000100110
11 0 1 2 3 7 10 11 12 13 14 15
7 4 6 8 10 11 12 13
6 0 1 6 8 10 12
8 0 3 5 6 10 11 12 13
10 0 1 2 3 4 5 6 8 12 13
9 3 4 5 6 8 9 11 14 15
8 0 4 7 9 10 11 13 14
```

Sample Output 3

```
97754354
```

Sample Input 4

```
92 4
10101100101111111111011101111111101011001011111110011101111111011111111101001111000101110 11
23 1 2 5 13 14 20 28 32 34 39 52 56 59 60 62 64 67 69 71 78 84 87 91
20 15 17 22 28 36 40 43 47 52 53 57 67 72 77 78 81 87 89 90 91
23 7 8 9 10 11 13 16 19 22 23 30 33 42 49 51 52 58 64 71 73 76 79 83
22 1 13 19 26 27 28 29 35 39 40 41 46 55 60 62 64 67 74 79 82 89 90
```

Sample Output 4

```
291412708
```

Find the number modulo 998244353.