A - mod M Game 2

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 600 points

Problem Statement

There are positive integers N and M, where $N < M$.
Alice and Bob will play a game. Each player has N cards with $1, 2, \ldots, N$ written on them, one for each number. Starting with Alice, the two players take turns repeatedly performing this action: choose one card from their hand and play it onto the table.
Immediately after a card is played onto the table, if the sum of the numbers on the cards that have been played so far is divisible by M, the player who just played the card loses, and the other player wins.
If both players play all their cards without satisfying the above condition, Alice wins.
Who will win, Alice or Bob, when both play optimally?

You are given T test cases. Solve each of them.

Constraints

- $1 \leq T \leq 10^5$
- $1 \leq N < M \leq 10^9$
- All input values are integers.

Input

The input is given from Standard Input in the following format. Here, case_i denotes the i-th test case.

$$
\begin{array}{l}
T \\
\mathrm{case}_1 \\
\mathrm{case}_2 \\
\vdots \\
\mathrm{case}_T
\end{array}
$$

Each test case is given in the following format:

$$N \quad M$$

Output

Print T lines. The i-th line should contain the answer for the i-th test case.

For each test case, if Alice wins when both play optimally, print `Alice`; if Bob wins, print `Bob`.

Sample Input 1

```
8
2 3
3 6
5 9
45 58
39 94
36 54
74 80
61 95
```

Sample Output 1

```
Alice
Alice
Bob
Bob
Alice
Bob
Bob
Alice
```

In the first test case, the game could proceed as follows.

- Initially, both Alice and Bob have two cards: the card with 1 and the card with 2.
- Alice plays the card with 1.
  - The sum of the numbers on the cards played so far is 1, which is not divisible by 3, so Alice does not lose.
- Bob plays the card with 1.
  - The sum is now 2, which is not divisible by 3, so Bob does not lose.
- Alice plays the card with 2.
  - The sum is now 4, which is not divisible by 3, so Alice does not lose.
- Bob plays the card with 2.
  - The sum is now 6, which is divisible by 3, so Bob loses and Alice wins.

In the first test case, no matter how Bob plays, Alice can win.

B - +1 and -1

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 600 points

Problem Statement

You are given an integer sequence $A = (A_1, A_2, \dots, A_N)$ of length N.
You can perform the following operation any number of times, possibly zero:

- Choose an integer pair (i, j) satisfying $1 \leq i < j \leq N$, and replace A_i with $A_i + 1$ and A_j with $A_j - 1$.

Determine whether it is possible to make A a non-decreasing sequence through the operations.

You are given T test cases. Solve each of them.

Constraints

- $1 \leq T \leq 2 \times 10^5$
- $2 \leq N \leq 2 \times 10^5$
- $0 \leq A_i \leq 10^9$
- The sum of N over all test cases is at most 2×10^5.
- All input values are integers.

Input

The input is given from Standard Input in the following format. Here, case_i denotes the i-th test case.

$$
\begin{array}{l}
T \\
\mathrm{case}_1 \\
\mathrm{case}_2 \\
\vdots \\
\mathrm{case}_T
\end{array}
$$

Each test case is given in the following format:

$$
\begin{array}{l}
N \\
A_1 \quad A_2 \quad \dots \quad A_N
\end{array}
$$

Output

Print T lines. The i-th line should contain the answer for the i-th test case.

For each test case, if it is possible to make A a non-decreasing sequence through the operations, print `Yes`; otherwise, print `No`.

Sample Input 1

```
3
3
1 7 5
2
9 0
10
607 495 419 894 610 636 465 331 925 724
```

Sample Output 1

```
Yes
No
Yes
```

In the first test case, you can make A into a non-decreasing sequence by performing the following operations:

- Choose $(i, j) = (1, 2)$. After the operation, A is $(2, 6, 5)$.
- Choose $(i, j) = (1, 2)$. After the operation, A is $(3, 5, 5)$.

In the second test case, you cannot make A into a non-decreasing sequence no matter how you perform the operations.

C - Sum of Three Integers

Time Limit: 3 sec / Memory Limit: 1024 MiB

Score : 600 points

Problem Statement

You are given an integer sequence $A = (A_1, A_2, \dots, A_N)$ and an integer X.
Print one triple of integers (i, j, k) satisfying all of the following conditions. If no such triple exists, report that fact.

- $1 \le i < j < k \le N$
- $A_i + A_j + A_k = X$

Constraints

- $3 \le N \le 10^6$
- $1 \le X \le 10^6$
- $1 \le A_i \le X$
- All input values are integers.

Input

The input is given from Standard Input in the following format:

$$
\begin{array}{l}
N \quad X \\
A_1 \quad A_2 \quad \dots \quad A_N
\end{array}
$$

Output

If there exists an integer triple (i, j, k) satisfying the conditions, print one in the following format. If there are multiple solutions, you may print any of them.

$$
i \quad j \quad k
$$

If no such triple exists, print `-1`.

Sample Input 1

```
5 16
1 8 5 10 13
```

Sample Output 1

```
1 3 4
```

The triple $(i, j, k) = (1, 3, 4)$ satisfies $1 \leq i < j < k \leq N$ and $A_i + A_j + A_k = 1 + 5 + 10 = 16 = X$.

Sample Input 2

```
5 20
1 8 5 10 13
```

Sample Output 2

```
-1
```

Sample Input 3

```
10 100000
73766 47718 74148 49218 76721 31902 21994 18880 29598 98917
```

Sample Output 3

```
4 6 8
```

D - Random Walk on Tree

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 800 points

Problem Statement

There is a tree with $N \times M + 1$ vertices numbered $0, 1, \dots, N \times M$. The i-th edge $(1 \leq i \leq N \times M)$ connects vertices i and $\max(i - N, 0)$.

Vertex 0 is painted. The other vertices are unpainted.

Takahashi is at vertex 0. As long as there exists an unpainted vertex, he performs the following operation:

- He chooses one of the vertices adjacent to his current vertex uniformly at random (all choices are independent) and moves to that vertex. Then, if the vertex he is on is unpainted, he paints it.

Find the expected number of times he performs the operation, modulo 998244353.

▶ What is the expected value modulo 998244353?

Constraints

- $1 \leq N \leq 2 \times 10^5$
- $1 \leq M \leq 2 \times 10^5$
- N and M are integers.

Input

The input is given from Standard Input in the following format:

$N \quad M$

Output

Print the expected number of times he performs the operation, modulo 998244353.

Sample Input 1

```
2 2
```

Sample Output 1

```
20
```

For example, Takahashi could behave as follows.

- Moves to vertex 1 and paints it. This action is chosen with probability $\frac{1}{2}$.
- Moves to vertex 0. This action is chosen with probability $\frac{1}{2}$.
- Moves to vertex 1. This action is chosen with probability $\frac{1}{2}$.
- Moves to vertex 3 and paints it. This action is chosen with probability $\frac{1}{2}$.
- Moves to vertex 1. This action is chosen with probability 1.
- Moves to vertex 0. This action is chosen with probability $\frac{1}{2}$.
- Moves to vertex 2 and paints it. This action is chosen with probability $\frac{1}{2}$.
- Moves to vertex 4 and paints it. This action is chosen with probability $\frac{1}{2}$.

He behaves in this way with probability $\frac{1}{128}$, in which case the number of operations is 8. The expected number of operations is 20.

Sample Input 2

```
123456 185185
```

Sample Output 2

```
69292914
```

E - Adjacent GCD

Time Limit: 2 sec / Memory Limit: 1024 MiB

Score : 800 points

Problem Statement

Define the **score** of a sequence of positive integers $B = (B_1, B_2, \dots, B_k)$ as $\displaystyle\sum_{i=1}^{k-1} \gcd(B_i, B_{i+1})$.

Given a sequence of positive integers $A = (A_1, A_2, \dots, A_N)$, solve the following problem for $m = 1, 2, \dots, N$.

- There are $2^m - 1$ non-empty subsequences of the sequence $(A_1, A_2, \dots, A_m)$. Find the sum of the scores of all those subsequences, modulo 998244353. Two subsequences are distinguished if they are taken from different positions in the sequence, even if they coincide as sequences.

Constraints

- $1 \leq N \leq 5 \times 10^5$
- $1 \leq A_i \leq 10^5$
- All input values are integers.

Input

The input is given from Standard Input in the following format:

N
$A_1 \quad A_2 \quad \dots \quad A_N$

Output

Print N lines. The i-th line should contain the answer for $m = i$.

Sample Input 1

```
3
9 6 4
```

Sample Output 1

```
0
3
11
```

Consider the case $m = 3$. Here are the non-empty subsequences of $(A_1, A_2, A_3) = (9, 6, 4)$ and their scores.

- (9): Score is 0.
- (6): Score is 0.
- (4): Score is 0.
- $(9, 6)$: Score is $\gcd(9, 6) = 3$.
- $(9, 4)$: Score is $\gcd(9, 4) = 1$.
- $(6, 4)$: Score is $\gcd(6, 4) = 2$.
- $(9, 6, 4)$: Score is $\gcd(9, 6) + \gcd(6, 4) = 3 + 2 = 5$.

Therefore, the answer for $m = 3$ is $0 + 0 + 0 + 3 + 1 + 2 + 5 = 11$.

Sample Input 2

```
5
3 8 12 6 9
```

Sample Output 2

```
0
1
13
57
155
```

Sample Input 3

```
10
47718 21994 74148 76721 98917 73766 29598 59035 69293 29127
```

Sample Output 3

```
0
2
14
35
97
372
866
1859
4273
43287
```