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# Robust Adaptive Flocking Control of Nonlinear Multi-agent Systems

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***Abstract*— This paper studies robust adaptive flocking control of multi-agent systems with nonlinear dynamics. In this setting, the coupling weights, perturbed by asymmetric uncertain parameters, are dynamically updated while the network topology for velocity is fixed. By designing the coupling weights according to the distributed local information on the linked nodes, flock formation can be reached if the network is connected. It is found that flock formation can be achieved if the topology of the updated coupling weights forms a spanning tree. A simulation example is given to illustrate the theoretical analysis.**

**keywords**: Flokcking algorithm, multi-agent system, collective potential function, adaptive control, nonlinear dynamics.

## I. INTRODUCTION

Flocking behavior, one of the most typical collective behaviors in groups of autonomous mobile agents [2], [6], [8], [9], [10], [11], [12], [13], [19], has attracted much attention [3], [14], [15], [16], [17], [18] in recent years due to broad applications in biological systems, sensor networks [22], UAV (Unmanned Air Vehicles) formations, robotic teams, and so on. The main challenging problem in the study of flocking behavior is how this phenomenon emerges based only on local interactions among mobile individuals. It is quite interesting to see that a global flocking behavior can be realized under a distributed protocol where each individual can only share information with its neighboring agents.

In the literature, it has been shown that first-order consensus in a network with dynamically changing topologies can be reached if the union of the time-varying network topology contains a spanning tree frequently enough as the networked system evolves in time [8], [11]. However, this is not true in second-order consensus, which is the basis of flocking behavior since most flocking systems are modeled by second-order dynamics [14], [15], [16], [17], [18]. Some necessary and sufficient conditions for reaching second-order consensus in linear multi-agent systems with fixed directed topologies and time delays have been derived in [20] and it has been shown that the condition of containing a directed spanning tree is only necessary. In order to reach second-order consensus, an additional condition concerning the spectrum of the Laplacian matrix must be satisfied. The results were also extended to multi-agent systems with directed topologies and nonlinear dynamics in [21].

In the real world, the communication coupling weights are usually perturbed by some uncertainties through network information transmission. Even if the network is undirected, the uncertainties of perturbed coupling weights can make the network adjacency matrix asymmetric, which results in the difficult analysis of the flocking behavior. In order to show the robust flocking behavior, the coupling weights may be adjusted based on some distributed adaptive laws, which is the main motivation of this paper.

In [23], [24], some algorithms were designed to update coupling weights for reaching network synchronization by using both state and local network topology information, where theoretical analysis was not given. Recently, a general distributed adaptive strategy on the coupling weights was proposed and some theoretical conditions were also derived for reaching network synchronization in [4]. However, the conditions in [4] are not easy to apply and, in addition, it is ususally impossible to update all the coupling weights as proposed in [4], [23]. One contribution of the present paper is that a simple distributed adaptive strategy is applied on the coupling weights of the network and the derived conditions are very easy to apply. For example, if the corresponding edges and nodes with updated coupling weights form a spanning tree, then flock formation can be reached in the network, i.e., updating $N-1$ coupling weights is enough to achieve flocking. Another contribution is that the designed adaptive controllers can tolerate the asymmetric weight perturbations, which has not been investigated in the flocking control studies. Last but not least, flocking control with asymptotical nonlinear dynamics is also studied in this paper.

The rest of the paper is organized as follows. In Section 2, some preliminaries are briefly outlined. Distributed adaptive control strategies are then proposed in Section 3. In Section 4, a simulation example is given to verify the theoretical analysis. Conclusions are drawn in Section 5.

## II. PRELIMINARIES

Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be a weighted undirected graph of order $N$, with the set of nodes $\mathcal{V} = \{v_1, v_2, \cdots, v_N\}$, the set of undirected edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $A = (a_{ij})_{N\times N}$. As usual, assume there are no self-loops in $\mathcal{G}$. An edge $\mathcal{E}_{ij}$ in a weighted undirected network $\mathcal{G}$ is denoted by the unordered pair of nodes $(v_i, v_j)$. The weights $a_{ij} = a_{ji} > 0$ are positive if and only if there is an edge $(v_i, v_j)$ in $\mathcal{G}$; otherwise, $a_{ij} = a_{ji} = 0$. A path between nodes $v_i$ and $v_j$ is a sequence of edges, $(v_i, v_{i_1}), (v_{i_1}, v_{i_2}), \ldots, (v_{i_l}, v_j)$, in the network with distinct nodes $v_{i_k}$, $k = 1, 2, \ldots, l$. An undirected network $\mathcal{G}$ is *connected* if there is a path between any pair of distinct nodes in $\mathcal{G}$.

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The Laplacian matrix $L = (L_{ij})_{N\times N}$ of graph $\mathcal{G}$ is defined as $L_{ij} = -a_{ij}$ for $i \neq j$, and $L_{ii} = k_i$, where $k_i = -\sum_{j=1,j\neq i}^{N} L_{ij}$ is the degree of node $v_i$, $i, j = 1, \dots, N$. Clearly, $\sum_{j=1}^{N} L_{ij} = 0$ for all $i = 1, 2, \dots, N$. The Laplacian matrix $L = (l_{ij})_{N\times N}$ of this graph $\mathcal{G}$ has the following properties:

*Lemma 1:* [5], [7]

(1) The Lapacian matrix $L$ in the undirected network $\mathcal{G}$ is semi-positive definite. It has a simple eigenvalues 0 and all the other eigenvalues are positive if and only if the network is connected. If the network is connected, the second smallest eigenvalue $\lambda_2(L)$ of the Laplacian matrix $L$ satisfies $\lambda_2(L) = \min\limits_{x^T\mathbf{1}_N=0, x\neq 0} \dfrac{x^T L x}{x^T x}$.

(2) For any $\eta = (\eta_1, \dots, \eta_N)^T \in R^N$, $\eta^T L \eta = \dfrac{1}{2}\sum\limits_{i=1}^{N}\sum\limits_{j=1}^{N} a_{ij}(\eta_i - \eta_j)^2$.

Consider a multi-agent system with nonlinear dynamics:

$$\begin{aligned} \dot{r}_i(t) &= v_i, \\ \dot{v}_i(t) &= f(v_i) + u_i, \end{aligned} \tag{1}$$

where $r_i \in R^n$ and $v_i \in R^n$ are the position and velocity vectors of agent $i$, $f(v_i)$ is the nonlinear dynamical term determining the asymptotic velocity states, and $u_i \in R^n$ is the control input, $i = 1, 2, \dots, N$.

In distributed flocking algorithms, the following control input is of particular interest [14], [15], [16], [17], [18]:

$$\begin{aligned} u_i(t) &= -\nabla_{r_i} V(r) \\ &\quad + \sum_{j\in\mathcal{N}_i} (a_{ij}(t) + \delta_{ij}(t))(v_j - v_i), \end{aligned} \tag{2}$$

where $\nabla_{r_i} V(r)$ is a gradient-based term of a collective potential function $V$, the second term is the velocity consensus term, $\delta_{ij} \neq \delta_{ji}$ is the asymmetric parameter perturbation, and $\mathcal{N}_i = \{j \in \mathcal{V} : \|a_{ij} > 0, j \neq i\}$ denotes the neighbors of agent $i$ in the velocity network.

For simplicity, assume that the velocity network topology is fixed and only the weights can evolve as time goes, which means that $a_{ij}$ is always positive if there is an edge between agents $i$ and $j$. Note that this assumption is reasonable since some agents can always communicate with each other about their velocity differences in a network. However, the position network involved in $V$ can be dynamic as commonly assumed in the literature [14], [15], [16], [17], [18].

As usual, the collective potential function $V(r)$, used for collision avoidance and cohesion in the group, is a nonnegative function and is related to the overall geometric shape of system (1). In [14], a smooth collective potential function was given and it was pointed out that the local minimum of $V(r)$ is an $\alpha$-lattice and vice versa.

Let $r = (r_1^T, r_2^T, \dots, r_N^T)^T$, $v = (v_1^T, v_2^T, \dots, v_N^T)^T$, $u = (u_1^T, u_2^T, \dots, u_N^T)^T$, $A\otimes B$ denote the Kroneker product [7] of matrices $A$ and $B$, $I_n$ be the $n$-dimensional identity matrix, $\mathbf{1}_N$ be the $N$-dimensional column vector with all entries being 1, and $\bar{r} = \dfrac{1}{N}\sum\limits_{j=1}^{N} x_j$ and $\bar{v} = \dfrac{1}{N}\sum\limits_{j=1}^{N} v_j$ be the center of the position and the velocity of the group, respectively.

*Assumption 1:* There exist a constant diagonal matrix $H = \mathrm{diag}(h_1, \dots, h_n)$ and a positive value $\varepsilon > 0$ such that

$$\begin{aligned} &(x-y)^T(f(x,t) - f(y,t)) - (x-y)^T H (x-y) \\ \leq\ & -\varepsilon (x-y)^T(x-y),\ \forall x, y \in R^n. \end{aligned} \tag{3}$$

*Assumption 2:* There exist positive constants $l_{ij}$ such that

$$|\delta_{ij}(t)| \leq l_{ij}, \forall t \geq 0, i \neq j; i, j = 1, \dots, N.$$

*Assumption 3:* The collective potential function $V$ satisfies

$$\begin{aligned} &\sum_{i=1}^{N} \nabla_{r_i} V(r) = 0, \\ &\nabla_{r_i} V(r) = \nabla_{r_i - \bar{r}} V(r - \mathbf{1}_N \otimes \bar{r}),\ i = 1, \dots, N. \end{aligned}$$

Note that Assumption 1 is very mild. For example, all linear and piecewise linear functions satisfy this condition. In addition, if $\partial f_i/\partial x_j$ $(i, j = 1, 2, \dots, n)$ are uniformly bounded, the above condition in Assumption 1 is satisfied, which includes many well-known systems.

*Lemma 2:* [1] If $V, W \in R^{N\times N}$ are two matrices satisfying $|V| = (|v_{ij}|_{n\times n}) \leq W = (w_{ij})_{n\times n}$, i.e., $|v_{ij}| \leq w_{ij}$, then $\|V\|_2 \leq \|W\|_2$.

*Definition 1:* The multi-agent system (1) is said to reach flocking formation if the velocities of all agents in the multi-agent system (1) reach consensus, i.e.,

$$\lim_{t\to\infty} \|v_i(t) - v_j(t)\| = 0,\ \forall\ i, j = 1, 2, \cdots, N, \tag{4}$$

and the positions of all agents reach the minimum of the potential function, i.e.,

$$\nabla_{r_i} V(r) = 0,\ \forall\ i = 1, 2, \cdots, N. \tag{5}$$

The objective of designing the control input here is to find some distributed adaptive control laws acting on $a_{ij}(t)$ under a fixed network topology such that the multi-agent system can reach flocking formation. Clearly, since $\sum_{j=1}^{N} L_{ij} = 0$, if a flocking formation can be reached, the asymptotic velocity state $s(t)$ of the system (1) is expected to be a possible trajectory of an isolated node satisfying

$$\dot{s}(t) = f(s(t)). \tag{6}$$

Here, $s(t)$ may be an isolated equilibrium point [14], a periodic orbit, or even a chaotic orbit [21].

## III. Robust Adaptive Control of Multi-agent Systems

In this section, some distributed adaptive laws on the weights $L_{ij}(t)$ or $a_{ij}(t)$ for $i \neq j$ are proposed. Let $\Delta = (\Delta_{ij})_{N\times N}$ be defined by $\Delta_{ij} = -\delta_{ij}$ for $i \neq j$, and $\Delta_{ii} = -\sum_{j=1,j\neq i}^{N} \Delta_{ij}$, $i,j = 1,\ldots,N$ . Since $\sum_{i=1}^{N} \nabla_{r_i} V(r) = 0$ and $\sum_{j=1}^{N} L_{ij} = 0$, one has

$$\begin{aligned}\dot{\bar{r}}(t) &= \bar{v}(t),\\ \dot{\bar{v}}(t) &= \frac{1}{N}\sum_{i=1}^{N} f(v_i(t)) - \frac{1}{N}\sum_{i=1}^{N}\sum_{j=1}^{N}\Delta_{ij}(t)v_j(t). \quad (7)\end{aligned}$$

By Assumption 3, subtracting (7) from (1) yields the following error dynamical network:

$$\begin{aligned}\dot{\widehat{r}}_i(t) &= \widehat{v}_i(t),\\ \dot{\widehat{v}}_i(t) &= f(v_i(t)) - \frac{1}{N}\sum_{j=1}^{N} f(v_j(t)) - \nabla_{\widehat{r}_i} V(\widehat{r})\\ &\quad -\sum_{j=1}^{N}\Big(L_{ij}(t) + (\Delta_{ij}(t) - \frac{1}{N}\sum_{k=1}^{N}\Delta_{kj}(t))\Big)\\ &\quad \times\widehat{v}_j(t),\ i = 1,2,\ldots,N, \qquad (8)\end{aligned}$$

where $\widehat{r}_i(t) = r_i - \bar{r}$, $\widehat{v}_i(t) = v_i - \bar{v}$, $\widehat{r} = r - \mathbf{1}_N \otimes \bar{r}$, and $\widehat{v} = v - \mathbf{1}_N \otimes \bar{v}$, $i = 1,\ldots,N$.

*Theorem 1:* Suppose that Assumptions 1-3 hold and the velocity network is connected. The multi-agent system (1) can reach flocking formation under the following distributed adaptive law:

$$\dot{L}_{ij}(t) = -\alpha_{ij}(v_i - v_j)^T(v_i - v_j), \qquad (9)$$

where $\alpha_{ij} = \alpha_{ji}$ are positive constants, $1 \leq i \neq j \leq N$.

**Proof.** Consider the Lyapunov functional candidate:

$$\begin{aligned}U(t) &= \frac{1}{2}\sum_{i=1}^{N}\widehat{v}_i^T(t)\widehat{v}_i(t) + V(\widehat{r})\\ &\quad +\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{1}{4\alpha_{ij}}(L_{ij}(t)+\sigma_{ij})^2, \quad (10)\end{aligned}$$

where $\sigma_{ij} = \sigma_{ji}$ are positive constants to be determined, $1 \leq i \neq j \leq N$.

From Lemma 1, the derivative of $U(t)$ along the trajectories of (8) gives

$$\begin{aligned}\dot{U} &= \sum_{i=1}^{N}\widehat{v}_i^T(t)\dot{\widehat{v}}_i(t) + \sum_{i=1}^{N}\nabla_{\widehat{r}_i}^T V(\widehat{r})\dot{\widehat{r}}_i\\ &\quad +\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\frac{1}{2\alpha_{ij}}(L_{ij}(t)+\sigma_{ij})\dot{L}_{ij}(t)\\ &= \sum_{i=1}^{N}\widehat{v}_i^T(t)\Big[f(v_i(t)) - \frac{1}{N}\sum_{j=1}^{N} f(v_j(t))\\ &\quad -\sum_{j=1}^{N}\Big(L_{ij}(t) + (\Delta_{ij}(t) - \frac{1}{N}\sum_{k=1}^{N}\Delta_{kj}(t))\Big)\widehat{v}_j(t)\Big]\\ &\quad -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}(L_{ij}(t)+\sigma_{ij})(v_i - v_j)^T(v_i - v_j)\\ &= \sum_{i=1}^{N}\widehat{v}_i^T(t)\Big[f(v_i(t)) - f(\bar{v}) + f(\bar{v})\\ &\quad -\frac{1}{N}\sum_{j=1}^{N} f(v_j(t))\\ &\quad -\sum_{j=1}^{N}\Big(\Delta_{ij}(t) - \frac{1}{N}\sum_{k=1}^{N}\Delta_{kj}(t)\Big)\widehat{v}_j(t)\Big]\\ &\quad -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\sigma_{ij}(v_i - v_j)^T(v_i - v_j). \qquad (11)\end{aligned}$$

Since $\sum_{i=1}^{N}\widehat{v}_i(t) = 0$, one has $\sum_{i=1}^{N}\widehat{v}_i^T(t)\Big[f(\bar{v}) - \frac{1}{N}\sum_{j=1}^{N} f(v_j(t))\Big] = 0$. From Assumption 1, it follows that

$$\begin{aligned}&\sum_{i=1}^{N}\widehat{v}_i^T(t)\left[f(v_i(t)) - f(\bar{v})\right]\\ &\leq -\varepsilon\sum_{i=1}^{N}\widehat{v}_i^T(t)\widehat{v}_i(t) + \sum_{i=1}^{N}\widehat{v}_i^T(t)H\widehat{v}_i(t). \qquad (12)\end{aligned}$$

Define the Laplacian matrix $\Sigma = (\widetilde{\sigma}_{ij})_{N\times N}$, where $\widetilde{\sigma}_{ij} = -\sigma_{ij}$, $i \neq j$; $\widetilde{\sigma}_{ii} = -\sum_{j=1,j\neq i}^{N}\widetilde{\sigma}_{ij}$. In view of Lemma 1, one obtains

$$\begin{aligned}&\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N}\sigma_{ij}(v_i - v_j)^T(v_i - v_j)\\ &= \sum_{i=1}^{N}\sum_{j=1}^{N}\widetilde{\sigma}_{ij}(t)\widehat{v}_i^T\widehat{v}_j. \qquad (13)\end{aligned}$$

From Assumption 2, one has

$$\begin{aligned}&\Big|\Delta_{ij}(t) - \frac{1}{N}\sum_{k=1}^{N}\Delta_{kj}(t)\Big|\\ &\leq \sum_{k=1,k\neq i}^{N} l_{ik} + \frac{1}{N}\sum_{k=1,k\neq j}^{N}(l_{kj} + l_{jk}). \qquad (14)\end{aligned}$$

Let $G = (g_{ij})_{N\times N}$ be defined by $g_{ij} = \sum_{k=1,k\neq i}^{N} l_{ik} + \frac{1}{N}\sum_{k=1,k\neq j}^{N}(l_{kj}+l_{jk})$. By combining (12)-(14) and by using Lemmas 1 and 2, one finally has

$$\begin{aligned}\dot{U} &\leq -\varepsilon\sum_{i=1}^{N}\widehat{v}_i^T(t)\widehat{v}_i(t) + \sum_{i=1}^{N}\widehat{v}_i^T(t)H\widehat{v}_i(t)\\ &\quad -\sum_{i=1}^{N}\sum_{j=1}^{N}\widetilde{\sigma}_{ij}(t)\widehat{v}_i^T\widehat{v}_j + \|G\|_2\widehat{v}^T(t)\widehat{v}(t)\\ &= \widehat{v}^T(t)[(-\varepsilon + \|G\|_2)I_{Nn} + (I_N\otimes H)\\ &\quad -(\Sigma\otimes I_n)]\widehat{v}(t)\\ &\leq \widehat{v}^T(t)[(-\varepsilon + \|G\|_2)I_{Nn} + (I_N\otimes H)\\ &\quad -\lambda_2(\Sigma)(I_N\otimes I_n)]\widehat{v}(t). \qquad (15)\end{aligned}$$

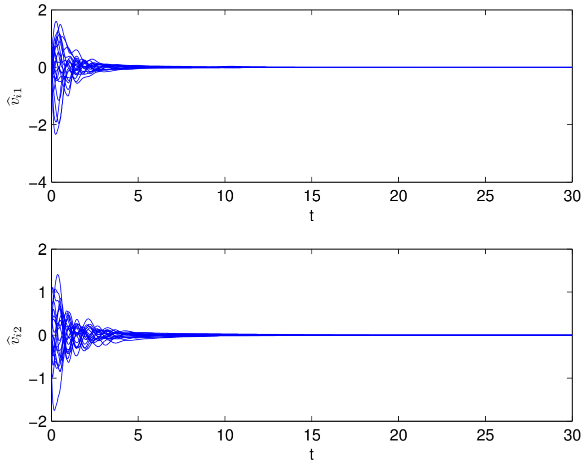


Fig. 1. Error velocities of all the agents, $i = 1, 2, \ldots, 20$.

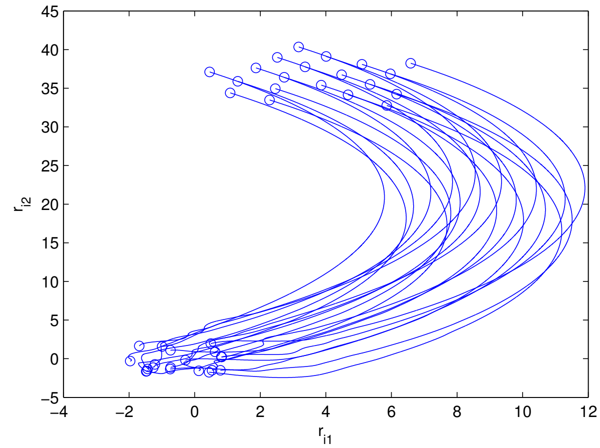


Fig. 2. Position states of all the agents, $i = 1, 2, \ldots, 20$.

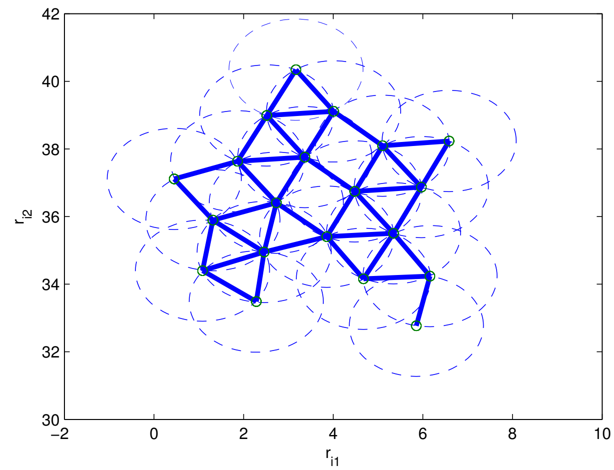


Fig. 3. Positions of all the agents at time $30s$ , $i = 1, 2, \ldots, 20$.

By choosing $\sigma_{ij}$ sufficiently large such that $\lambda_2(\Sigma) > \max_j(h_j) + \|G\|_2$, one has

$$\dot{U} \quad \leq \quad -\varepsilon \widehat{v}^T(t)\widehat{v}(t).$$

This completes the proof. $\square$

*Remark 1:* In Theorem 1, by designing a simple distributed adaptive law (9) on the coupling weights, flocking formation can be reached without satisfying any other conditions except the connectivity of the network. In order to let the coupling weights change very slowly, one can choose very small parameter $\alpha_{ij}$ if the communication ability between nodes $i$ and $j$ is limited.

*Remark 2:* Since only the connectivity of the network should be satisfied in Theorem 1, the multi-agent system (1) can reach flocking formation if the topology of the updated coupling weights forms a spanning tree. Therefore, the minimal number of the updated coupling weights under this scheme is $N-1$, which is much smaller than the number of updating all the coupling weights.

## IV. SIMULATION EXAMPLES

In this section, a simple simulation example is presented to verify the effectiveness and applicability of the theoretical analysis.

Consider the multi-agent system (1) and (2), where $N = 20$, the gradient-based term is the same as in [14], [15], $f(v_i) = \begin{pmatrix} 0 & -0.05 \\ 0.05 & 0 \end{pmatrix} v_i$, $a_{ij}(0) = 1$ if agents $i$ and $j$ are connected, and $\delta_{ij} \leq 1$.

In particular, the gradient-based term is given in the following form [14], [15]:

$$-\nabla_{r_i} V(r) \quad = \quad \sum_{j \in \mathcal{N}_i} \phi(\|r_j - r_i\|_\sigma) n_{ij},$$

where $\|r_j - r_i\|_\sigma = \frac{1}{\varepsilon}\left[\sqrt{1 + \varepsilon\|r_j - r_i\|^2} - 1\right]$, $n_{ij} = (r_j - r_i)/\sqrt{1 + \varepsilon\|r_j - r_i\|^2}$, $\phi(z) = \rho_h(z/r_s)\phi_1(z - d)$, $\phi_1(z) = \frac{1}{2}[(a + b)\sigma_1(z + c) + (a - b)]$, $\sigma_1(z) = z/\sqrt{1 + z^2}$, $c = |a - b|/\sqrt{4ab}$, and the bump function is defined to be

$$\rho_h(z) = \begin{cases} 1, & z \in [0, h), \\ \frac{1}{2}[1 + \cos(\pi \frac{z-h}{1-h})], & z \in [h, 1], \\ 0, & \text{otherwise}, \end{cases}$$

for $h \in (0, 1)$. Choose $a = 20$, $b = 50$, $h = 0.9$, $d = 0.8$, $\varepsilon = 1$, and $r_s = 1$. Then, it is easy to see that the potential function $V$ reaches its minimum at $\|r_j - r_i\|_\sigma = d$, i.e., $\|r_j - r_i\| = \sqrt{2.24}$.

In the simulation, the initial positions and velocities of the 20 agents are chosen randomly from $[-2, 2] \times [-2, 2]$. In this paper, $N - 1$ coupling strengths are updated where the corresponding edges and nodes with updated coupling weights form a spanning tree. It is easy to see that all the assumptions in Theorem 1 are satisfied, so flocking formation can be reached in the multi-agent system (1) under the distributed adaptive law (9). Specifically, all the agents move with the same velocity asymptotically as verified by Fig 1. The states of all agents are shown in Fig 2. It is easy to see that the positions of agents converge to the local minimum $\sqrt{2.24}$ of the potential function $V$ at time $30s$ as illustrated by Fig 3, where the distance between one agent and the other agents on the nearest dashed circle is $\sqrt{2.24}$.

## V. Conclusions

In this paper, robust adaptive flocking control of multi-agent systems with nonlinear dynamics has been studied. In this new setting, the coupling weights, perturbed by asymmetric parameters, are dynamical while the network topology for velocity is fixed. By designing the coupling weights according to the distributed local information of connected nodes, flocking formation can be reached if the network is connected. It is found that flock can even be achieved if the topology of the updated coupling weights forms a spanning tree.

Flocking behavior is one of the most studied collective behaviors in multi-agent systems. In the future, more complicated works, for example, networks with time-varying topologies for velocities, noise perturbations, uncertain nonlinear systems, etc., will be investigated.

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