Physica A 489 (2018) 47–55

Contents lists available at ScienceDirect

Physica A

journal homepage: www.elsevier.com/locate/physa

Robustness of weighted networks

Michele Bellingeri *, Davide Cassi

Dipartimento di Fisica, Università di Parma, via G.P. Usberti, 7/a, 43124 Parma, Italy

HIGHLIGHTS

- The efficiency of the attack strategies changed using binary or weighted measures.
- Removing nodes according to weighted rank produced the higher damage.
- Adopting binary or weighted measure changed the efficacy of the attack strategy.
- To find the best attack strategy, it is necessary to account the weight of the links.

ARTICLE INFO

Article history:
Received 27 April 2017
Received in revised form 11 July 2017
Available online 1 August 2017

Keywords:
Weighted networks
Network attack strategies
Weighted nodes rank
Network robustness
Links weight

ABSTRACT

Complex network response to node loss is a central question in different fields of network science because node failure can cause the fragmentation of the network, thus compromising the system functioning. Previous studies considered binary networks where the intensity (weight) of the links is not accounted for, i.e. a link is either present or absent. However, in real-world networks the weights of connections, and thus their importance for network functioning, can be widely different. Here, we analyzed the response of real-world and model networks to node loss accounting for link intensity and the weighted structure of the network. We used both classic binary node properties and network functioning measure, introduced a weighted rank for node importance (node strength), and used a measure for network functioning that accounts for the weight of the links (weighted efficiency).

We find that: (i) the efficiency of the attack strategies changed using binary or weighted network functioning measures, both for real-world or model networks; (ii) in some cases, removing nodes according to weighted rank produced the highest damage when functioning was measured by the weighted efficiency; (iii) adopting weighted measure for the network damage changed the efficacy of the attack strategy with respect the binary analyses.

Our results show that if the weighted structure of complex networks is not taken into account, this may produce misleading models to forecast the system response to node failure, i.e. consider binary links may not unveil the real damage induced in the system. Last, once weighted measures are introduced, in order to discover the best attack strategy, it is important to analyze the network response to node loss using nodes rank accounting the intensity of the links to the node.

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* Corresponding author.
E-mail address: michele.bellingeri@nemo.unipr.it (M. Bellingeri).

http://dx.doi.org/10.1016/j.physa.2017.07.020
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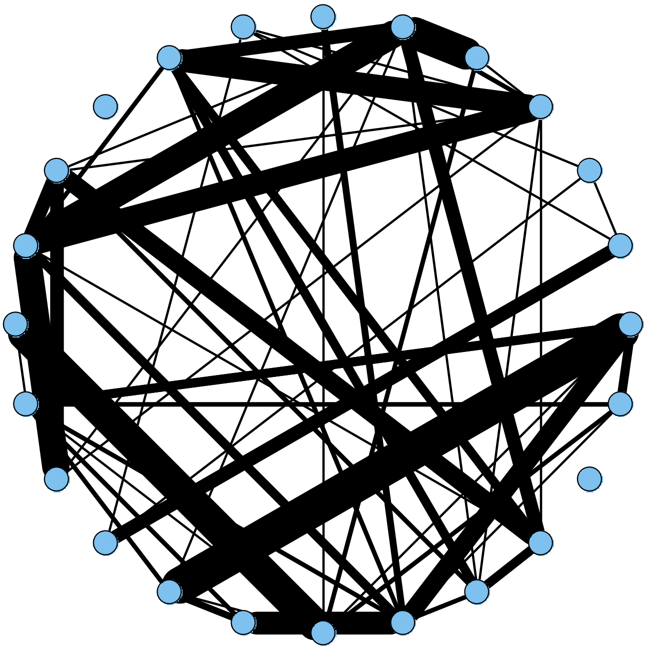


Fig. 1. UK faculty ($N = 81$, $L = 817$) real-world network weighted subgraph (24 nodes); the thickness of the link indicates the weight associated to it. The figure shows the wide heterogeneity among links in the UK faculty real-world complex network.

1. Introduction

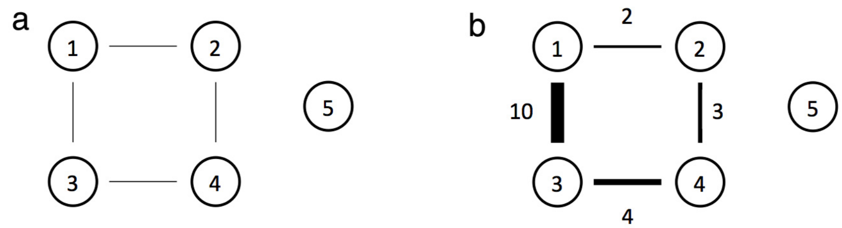
The resilience of real-world complex networks, such as Internet, electrical power grids, airline routes, ecological and biological networks [1–7] to "node failure" (i.e. node malfunctioning or removal) is a topic of fundamental importance for both theoretical and applied network science. Node loss (failure) can eliminate focal component in the system compromising the functioning: for example nodes loss can reduce the transport properties of road networks removing important connection among nodes and reducing the reachability of road network components [1,2]; in Internet network node loss can represent the malfunctioning of a router connecting couples of routers with negative effects on the information delivery [1,2,8]; in ecological food web networks the node loss represent the extinction of species acting as energy resources for other consumer species [5]. In the last years, several studies have investigated network response to nodes removal [1–4,8–11]. The majority of these studies considered binary networks where the intensity (weight) of the links is not accounted for, i.e. a link is either present or absent. But in real-world networks the weight of connections, and thus the importance for network functioning, can be largely different. With a complex topological structure, real world networks display a large heterogeneity in the capacity and intensity of the connections [12,13]. See Fig. 1 for graphical representation of a real-world weighted complex network and the wide difference in the weight of the links. In airport networks, links represent fly routes connecting airports and the link's weight indicates the number of passengers between airports [13]; in neural networks, link's weight accounts for the number of synapses and gap junctions among neurons [14]; in immune networks the weight of the links indicate the amount of exchanging chemical signals between lymphocytes and proteins such as cytokines or antibodies [15,16]; in food webs, i.e. ecological networks describing species and trophic interactions among them, link's weight indicates the amount of energy and material passing among species [5,17–19]. Bellingeri and Bodini [5] showed how including the weight of the links in food webs simulation scenarios to forecast secondary extinction may change the response of these systems to node loss. For these reasons, neglects links weight may be an oversimplification of the real complex system and may produce misleading results when forecasting network reaction to node loss.

In this study, we analyzed the response of real world and model networks to node loss accounting for links weight in the model. We removed nodes following well known binary removal criteria [3,8] and introduced a weighted measure of node's importance: the node strength [20,21]. We measured the damage induced in the network after the removal of nodes using the size of the largest connected cluster [3,8]. To describe the weighted structure of the network subjected to nodes removal, we used a measure that takes into account the weight of the links: the weighted efficiency [20,21].

2. Methods

2.1. The attack strategies

We used attack strategies that have already been described in the literature. Nearest neighbors (*First*): nodes are sequentially removed according to the number of nearest neighbors of each node (i.e. node degree) [3,8]. Next to nearest



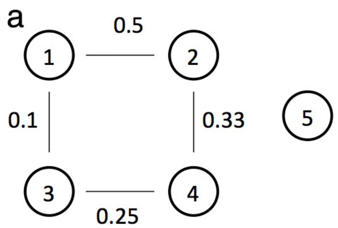
c

	1	2	3	4	5
1	0	1	1	2	Inf
2	1	0	2	1	Inf
3	1	2	0	1	Inf
4	2	1	1	0	Inf
5	Inf	Inf	Inf	Inf	Inf

d

	1	2	3	4	5
1	0	2	10	5	Inf
2	2	0	7	3	Inf
3	10	7	0	4	Inf
4	5	3	4	0	Inf
5	Inf	Inf	Inf	Inf	Inf

Fig. 2. (a) Binary network with no weights associated to the links; (b) weighted networks with links weight; (c) shortest paths of the binary network in (a); shortest paths of the weighted network in (b).



b

	1	2	3	4	5
1	0	0.5	0.1	0.35	Inf
2	0.5	0	0.58	0.33	Inf
3	0.1	0.58	0	0.25	Inf
4	0.35	0.33	0.25	0	Inf
5	Inf	Inf	Inf	Inf	Inf

c

	1	2	3	4	5
1	0	2	10	2.87	0
2	2	0	1.7	3	0
3	10	1.7	0	4	0
4	2.86	3	4	0	0
5	0	0	0	0	0

Fig. 3. (a) The network ($N = 5,\ L = 4$) of Fig. 2 with the reciprocal of the links weight; (b) the weighted shortest paths computed for every pairs of nodes; (c) the efficiencies computed for each pairs of nodes.

neighbors (*Sec*): nodes are removed according the number of nodes connected to the nearest neighbors of that node. Next to nearest neighbors plus nearest neighbors ($F + S$): nodes are removed according to the number of nearest neighbors plus nodes connected to the nearest neighbors of that node. Nodes betweenness centrality (*Bet*): nodes are sequentially removed according to their betweenness centrality, which is the number of shortest paths from all vertices to all others that pass through that node [13]. Google Page Rank (*Goog*): nodes are deleted according to the rank produced by the algorithm used by Google to rank their search engine results [22]. For all the attack strategies in the case of ties (i.e. nodes with the same degree), the sequence of nodes removal is randomly chosen. Random removal (*Rand*): nodes are randomly removed. Then we introduced a new strategy: the strength attack (*Str*): nodes are removed according to the vertex strength. The vertex strength of a certain node is the sum of the weight of the links incident to the node. When the network is binary (i.e. no weight associated to the links), the vertex strength equals the vertex degree; when links are weighted, the two ranks may differ.

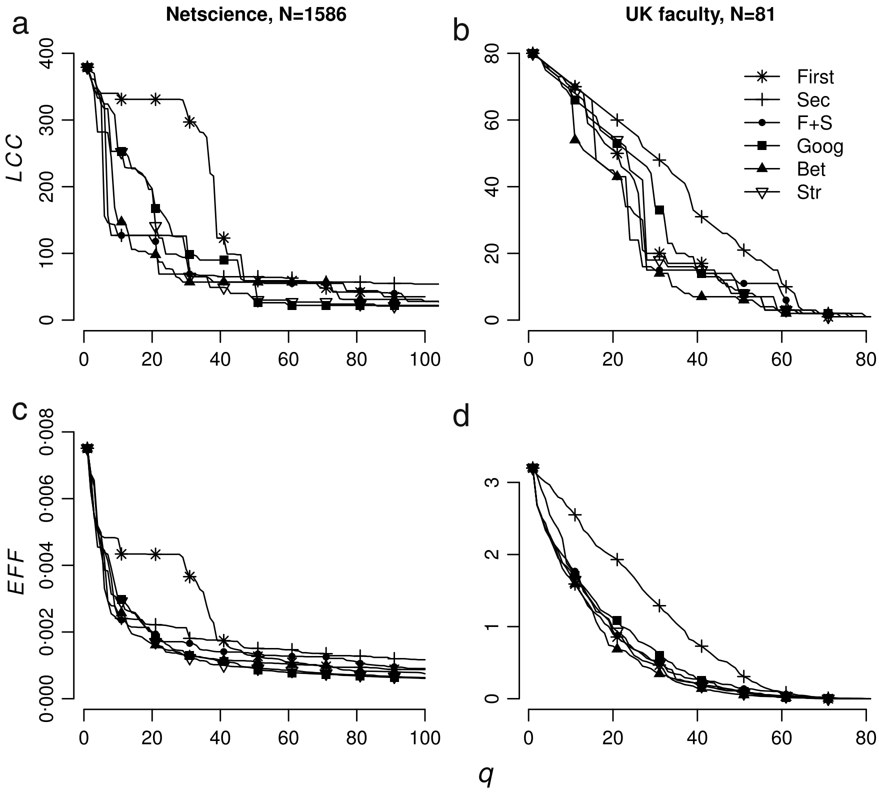


Fig. 4. LCC and Weighted efficiency (EFF) for the UK faculty ($N = 81$, $L = 817$) and Netscience ($N = 1589$, $L = 1742$) real-world networks as a function of the nodes removals for the 7 attack strategies (*Rand* excluded). For the Netscience network are plotted the patterns for the first fraction of removals ($q = 100$).

2.2. The random weighted ER graph

We tested the attack strategies on a weighted version of the Erdös–Rényi random graphs [23]. First we produced a binary ER graphs with N nodes connected by L links, which are chosen randomly with an occupation probability p from $L\max = N(N-1)/2$ possible links [23]. We analyzed ER graph with $N = 500$ and $p = 0.008$. Once the ER graph is produced, we associated weights to the links randomly drawing the intensity of the links from a uniform distribution in the interval (1100).

2.3. The largest connected cluster (LCC)

Several indexes and measures have been introduced to describe binary network damage. We use the size of the largest connected component (LCC), i.e. the size of the largest connected sub-graph in the network [2,3,8], as a measure of network damage during the attack, where a faster decrease in the size of the LCC indicates a more efficient attack strategy. The LCC represents the binary measure of the network damage, not accounting for links weight. We measured LCC along the entire removal process up to reach the percolation threshold p_c, i.e. fraction of nodes removed producing the value of zero of the LCC [2,5].

2.4. The weighted efficiency (EFF)

We measured the efficiency of the network during the node removal process. We used "weighted efficiency" (EFF) as a measure of the network efficiency [20]. Weighted efficiency is based on the weighted shortest paths notion. The shortest path measure of a networks is the average of the minimum number of links needed to travel among each couple of nodes [21]. In a binary network, the shortest path between two nodes is an integer number indicating the minimum amount of links necessary to travel from a node to the other. In a weighted network, a path between two nodes is the sum of the weight associated to the links necessary to travel between the nodes; consequently, the weighted shortest path is the minimum sum of the weights necessary to travel between nodes [21]. See Fig. 2 for example of the binary and weighted networks and the associated shortest paths.

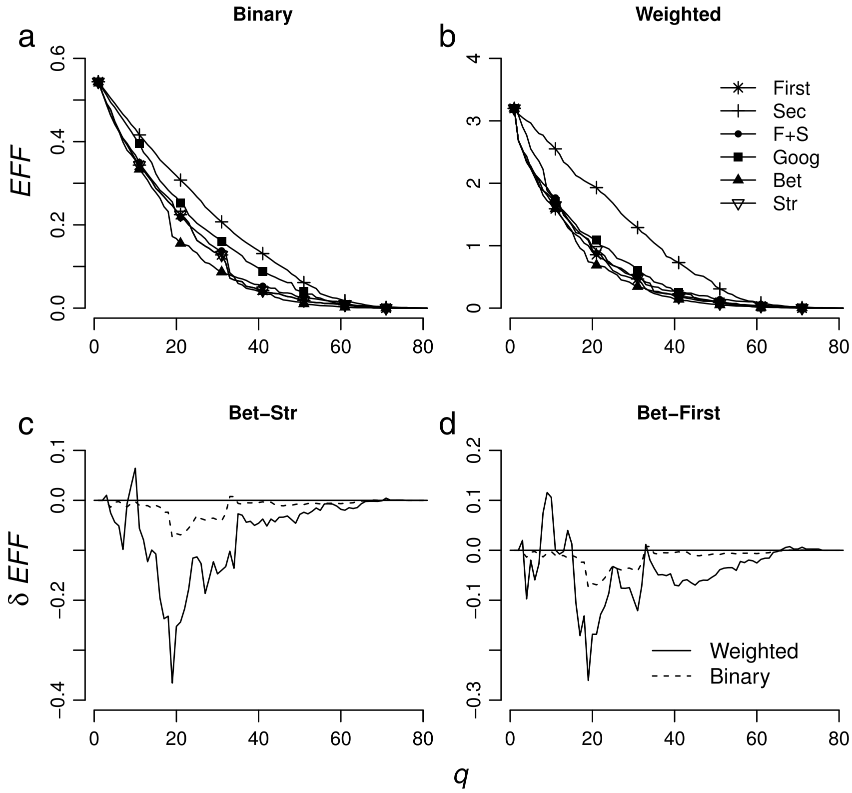


Fig. 5. Binary and weighted efficiency (EFF) of the UK faculty ($N = 81$, $L = 817$) real-world network as a function of the nodes removals for the 7 attack strategies (*Rand* excluded). We plot δ EFF at each removal showing the difference between the efficiency produced by the two attack strategies; the horizontal line for δ EFF $= 0$ indicates the same efficiency for the compared strategies (panels (c) and (d)). When the two functions describing δ EFF are at the opposite side of the horizontal line for δ EFF $= 0$, it means that binary and weighted simulation return different rank for the attack strategies. For example, in the first fraction of removals in the case we rank the efficiency with weighted measure *Bet* is more efficient than *Str* (dashed line, panel (c), the function is above the horizontal line for δ EFF $= 0$), when the efficiency is binary the *Str* is better than *Bet* (solid line, panel (c), the function is below the horizontal line indicating no difference in efficiency).

In order to assign more importance to the bigger links, we compute the reciprocal of the link weight.
To compute the weighted shortest paths (WSP), we:

(i) Compute the reciprocal of each link weight. This way, links with higher weight value represent "wider and faster routes" or, in terms of distance, "shorter routes", producing lower WSP. See Fig. 2(a) for a representation of the weighted network in Fig. 2(b).
(ii) Compute the WSP for each node pairs. See Fig. 3(b).
(iii) Compute the efficiency for each pair of nodes by producing the reciprocal of the WSP (See Fig. 3(c)). The higher WSP between two nodes, the lower the efficiency.
(iv) We sum the efficiency values and we average on the total number of node pairs (i.e. N (N-1)/2). The average value of the nodes efficiency is the network weighted efficiency (EFF).

Computing the reciprocal means that shorter WSPs increase the efficiency of the network and *vice versa*. Further, in the case two nodes are not joined by paths (as in the case of isolated clusters), the WSPs return the infinite value for the nodes pairs. Computing the reciprocal of the efficiency for a disjoined couple of nodes equals zero (lowest efficiency), maintaining the general idea about network efficiency.

2.5. The real world networks

We analyzed the response to nodes removal for two real-world networks. We studied the personal friendship network of a faculty of a UK university (UK faculty). UK faculty is the social network of the academic staff of a given Faculty of a UK university consisting of three separate schools and network structure was constructed from tie-strength measured with a questionnaire. The network consists of 81 nodes–vertices (individuals) and 817 directed and weighted links or connections [24]. The school affiliation of each individual is stored as a vertex attribute and the dataset served as a testbed for community detection algorithms.

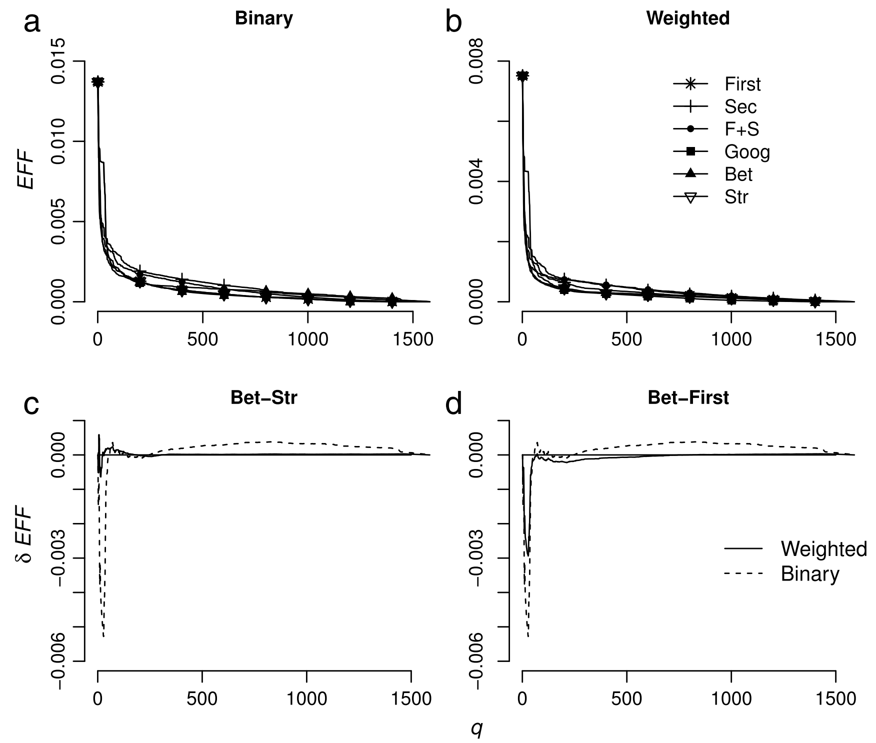


Fig. 6. Binary and weighted efficiency (EFF) of the Netscience ($N = 1589,\ L = 1742$) real-world network as a function of the nodes removals for the 7 attack strategies (*Rand* excluded). We plot δ EFF at each removal showing the difference between the efficiency produced by the two attack strategies (panels (c) and (d)); the horizontal line for δ EFF $= 0$ indicates the same efficiency for the compared strategies. When the two functions describing δ EFF are at the opposite side of the horizontal line for δ EFF $= 0$, it means that binary and weighted simulation return different rank for the attack strategies.

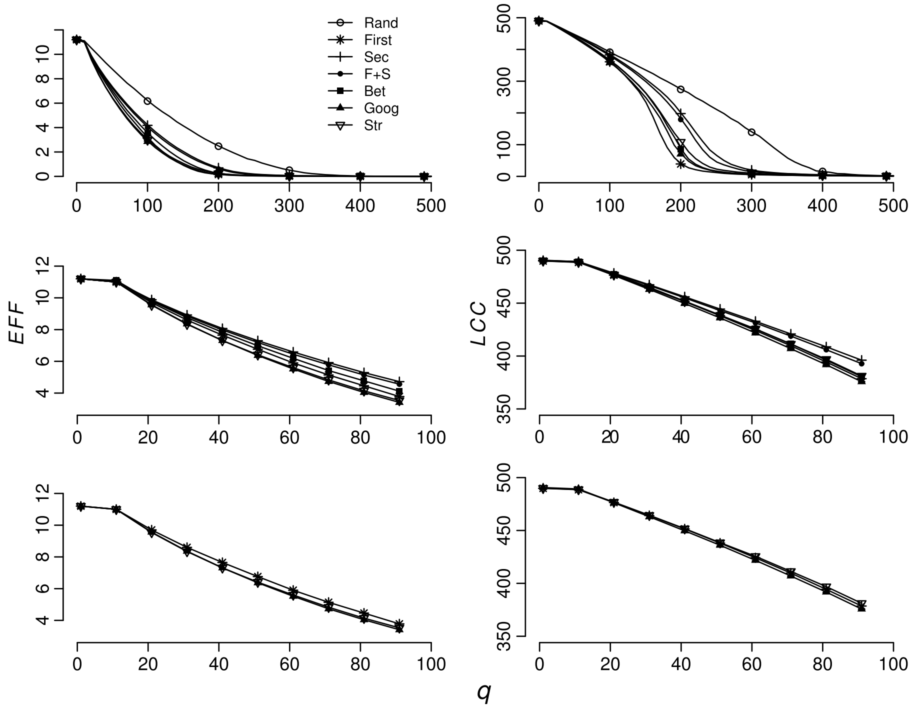


Fig. 7. EFF and LCC as a function of the nodes removals for the ER weighted random graph ($N = 500,\ p = 0.008,\ W\max = 100$). First row shows the results from the 8 strategies. Second row show the results excluding the *Rand* strategy up to reach $q = 100$. Third row depicts the three more efficient strategies, i.e. *First, Goog, Str*.

The second real world system is the coauthorship network of scientists working on network theory and experiment (Netscience) with 1589 nodes and 2742 weighted links [25]. The network was compiled from the bibliographies of two review articles on networks, Newman [26] and Boccaletti et al. [27], with a few additional references added by hand [25]. Nodes represent authors and link's weight represents the number of common papers and the number of authors of these papers. See the Newman [26] for the definition of link weight.

2.6. The difference in efficiency: δ EFF

In order to compare the attack strategies we measured the difference in efficiency (EFF) for each node removals along the removal sequence. This difference in efficiency is named δ EFF. A positive δ EFF between strategies 1 and 2 indicates higher network efficiency when we used strategy 1, i.e. the strategy 1 is less efficient to damage the network removing nodes. We computed δ EFF along the entire nodes removal. We computed δ EFF both for binary and weighted efficiency.

3. Results

3.1. Real world networks

3.1.1. UK Faculty

The efficacy of the attack strategies can change during the removal process, both for LCC and EFF. The betweenness (*Bet*) centrality is the best strategy for reducing LCC for most part along the removal sequence and is the faster strategy to vanish LCC (Fig. 4, Table 1A in the Appendix). But in the first fraction *Goog* and *Str* are better than *Bet* to reduce LCC (up to $q = 10$, Fig. 4). The *Bet* strategy was the most efficient strategy to decrease the weighted efficiency (EFF) of the network . The random removal (*Rand*) was the less efficient strategy for reducing the weighted efficiency (EFF) and the largest connected cluster (LCC) in the UK Faculty network (Fig. 1A Appendix). The *Rand* showed roughly the same efficacy of the *Sec* strategy for reducing the LCC, with several crossing functions along the nodes removal (Fig. 1A Appendix). For the UK Faculty network, the *Rand* strategy was more efficient than *Sec* to decrease the EFF in the network (Fig. 1A Appendix, last panel).

When comparing the efficiency result of the binary and weighted simulations we found that, in general, *Bet* is the best strategy followed by *First* strategy for both the measures. When the damage is measured by the binary efficiency the *First* degree returns the same outcome of the *Str* strategy. The *Goog* strategy is more efficient for the weighted analyses. *Sec* strategy is the less efficient for both the measures. When we plot the difference in efficiency along the removal sequence (δ EFF) for *Bet*, *Str* and *First* strategies we observe several differences in ranking the efficiency between binary and weighted simulations. In Fig. 5 panels (c) and (d) we plot the difference in efficiency (δ EFF) between binary and weighted simulations.

3.1.2. Netscience

Sec and $F + S$ strategies showed the higher efficacy to reduce LCC in the first fraction of removals in Netscience; around $q = 100$ *Goog* is the best strategy for reducing LCC (Fig. 4); *First* is the best strategy to vanish the LCC (Table 1A Appendix). *Bet* was the best strategy to reduce EFF in Netscience in the earlier fraction of removals (for $q<10$), then *Str* strategy became the most efficient (for $10<q<1500$) (Fig. 4). The *Rand* strategy was the worst attack for both measures (Fig. 1A Appendix).

Similarly to what we observed with the UK faculty network, we found several differences in ranking the efficiency between binary and weighted simulations. *Bet* is the best strategy for both the measures, but in the first fraction of removals, for $q<60$, *Str* performed better than *Bet* only for the weighted measure. Comparing δ EFF for *Bet* and *First* we found that for $q \cong 90$ *First* performed better than *Bet* in the case the measure is weighted. On the contrary, in the same range of removal sequence when the measure is binary we found *Bet* performs better than *First*. In Fig. 6 panels (c) and (d) we plot the difference in efficiency (δ EFF).

3.1.3. The ER random weighted graph

Goog was the most efficient strategies to reduce the LCC up to $q \cong 0.2$. For $q > 0.2$, *First* became more efficient than *Goog* for reducing LCC (Table 1A in the Appendix). *Sec* and *Rand* were the less efficient strategies for both measures. *Str* is the best strategy for reducing the weighted efficiency EFF for the early fraction of removals ($q<0.1$); after this threshold of removals, the *Goog* strategy became the most efficient attack to decrease EFF in ER random weighted graph (Fig. 7).

4. Discussion

In this study we found that neglecting the weighted structure of complex networks may produce misleading models to forecast the system response to node failure, i.e. not account the intensity of the links with adapt weighted measures may not unveil the real damage induced in the system. Afterward, once weighted measures are introduced, in order to discover the best attack strategies, and thus the best way to select important vertex, it is important to analyze the network response to node loss using attack strategies accounting the magnitude of the links to the nodes, such as the nodes strength (*Str*). Methods unveiling important nodes in the network are necessary to preserve network functioning [3,6,8,28] and to select important hubs to immunize in the immunization policies [4,9].

The efficiency of the attack strategies changed with the network functioning measures, both for real-world or model networks. For example, in Netscience network the best strategy to reduce LCC in the second fraction of node removals are the *Goog*; but when the networks functioning is measured by the weighted efficiency EFF, the best strategy become *Str*. In this social network the vertex are authors and link weights represents the number of common papers and the number of authors of these papers [25]. When the network functioning is described with the EFF measure that accounts the weight of the links, to use the *Str* strategy considering the magnitude of the links entering to a node produced higher damage when nodes are removed. Further, the best strategy to decrease LCC in ER weighted random graph is the *First*, but when the weighted structure of the network is considered and we measure the damage with the weighted efficiency, the best strategy is *Str*. Further, we found difference in ranking the efficacy of the attack strategies when the network functioning is measured by binary or weighted efficiency (EFF).

Our results corroborate recent analyses comparing binary and weighted response to node loss in food web ecological networks. Food webs are weighted complex networks representing who eats whom in ecosystems, i.e. the transfer of energy and matter among species; in this complex networks node removal represents the species extinction that may trigger further biodiversity loss and many studies analyzed the effect of nodes removal on the secondary extinction [29,30]. These bottom-up "binary topological approaches" considered a species goes extinct after a primary species loss when no resources are available in the consumer diet. More realistic models used weighted food webs accounting for the amount of energy passing in the trophic interaction showing important differences in respect to binary analyses, e.g. a quick raise in the food web secondary extinctions and less complex webs are those that are more robust [5,18,32]. As in this study for model physics and real-world social networks, consider the weighted structure of the food web ecological network may change the response of the system to nodes removal.

In previously binary analyses [3,4], we found a 'transition pattern' for the efficacy of the attack strategies, i.e. the efficacy changed along the removal sequences. For example, in the first steps of a removal process a strategy A can perform better than a strategy B. After this fraction of removals we assist to the efficacy transition and the strategy B becomes more efficient than A. Our results show how the transition pattern discovered for binary analyses held for weighted networks; we found the same efficacy transition along the removal sequence using weighted measure (EFF) for the network damage. This results outline the importance to consider the entire removal process in order to rank the best strategy to decrease the network functioning, both for binary models that for more accurate weighted analyses including links weight.

This research would open straightforward and important extensions.

First it is possible to test the weighted networks reaction to node loss removing nodes according to the weighted betweenness centrality [33], weighted closeness centrality [34] or identifying influential nodes in weighted networks based on evidence theory [35]. Second, to test the complex networks response to links removal using weighted measures of network damage and models considering the weight of the links [36–38], for example analyzing weighted network response to loss of links when links are removed in decreasing order of weight, in increasing order of weight, more central links and according to the degree of the connected nodes (i.e. links connecting high degree nodes are deleted first).

Further, it is extremely interesting understand how the distribution of weights affects the efficacy of the attack strategy, for example testing the difference according to whether the distribution of the weights is normal-like or scale-free; this analyses would be important to unveil real world complex networks functioning since real world biological and ecological networks showed highly skewed distributions of interaction strengths with many weak and few strong interactions [31,39,40] and different physics networks present broad distribution both of vertex strength and links weight [13]. Last, it would be important investigating the effect of the correlation between the weighted degree and the binary degree, on the network response to node loss, and on the efficacy of the attack strategies.

Acknowledgments

We thank Dr. Elena Agliari, Dr. Simone Vincenzi and Dr. Francesco Scotognella for useful comments on a previous version of the manuscript. Many thanks to Giulia Leni for suggestions on the final version of the manuscript.

Appendix A. Supplementary material

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.physa.2017.07.020.

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