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The robustness of interdependent weighted networks

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HIGHLIGHTS

- The interdependent weighted networks are built based on nodes betweens.
- Robustness of network are studied by defining cascading failures of local weighted flow redistribution rule.
- These results indicate that weights should be considered for designing more robustness interdependent networks.

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ABSTRACT

We analysis the cascading failure of interdependent weighted networks based on percolation theory and local weighted flow redistribution rule. The weight of a node is B_i^{θ}, where B_i is the betweenness of the node. We assume that the two networks A and B of size N^A andN^B are BA networks, and suppose two different one-to-one interdependent network, one is random, the other based on nodes' betweenness. Assume that a failed node in network A leads to a redistribution of the flow passing through it to its neighboring nodes and the interdependent node in network B also to be disabled, where the two failure factors happen concurrently, or successively. We find that the one-to-one random interdependent weighted complex network reaches the strongest robustness level when the weight parameter θ =0.5, no matter cascading failure caused by two failure factors happen concurrently, or successively. And when the interdependent relationship based on nodes' betweenness, θ=0.7 to get the strongest robustness level.

1. Introduction

The infrastructures in modern life can be characterized by complex networks [1–6], such as water, electricity, communication systems and airline networks. In the earlier time, research focuses on the limited case of non-interacting network [1,7–13]. But interdependent networks are ubiquitous in our society, the study of interdependent networks' cascading behaviors has attracted more and more attention. Based on percolation theory, Buldyrev et al. [14] developed a framework to study the robustness of interdependent networks. Different from this framework which analysis of fully interdependent networks, Parshani et al. [15] and Dong et al. [16] studied partially interdependent networks. To develop the robustness of interdependent networks with a random number of support and dependence relationships, Shao et al. [17] generalized the framework [14–16] which assume that each node in network A depends on one and only one node in network B and vice versa. In all the previous studies, the dependent pairs of nodes in both networks were chosen randomly, then the theory of

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interdependent networks has been expanded to make the framework closer to reality. Parshani et al. [18] studied numerically interdependent networks with assortativity of dependency links, and then, Buldyrev et al. [19] analytically develop the case of full degree–degree correlation. By using the cavity method, Watanabe et al. [20] develop the general case of degree–degree correlations with connectivity or dependency links. In addition to these interdependent coupled networks, some researchers also studied the percolation of network of network interdependent networks [21–23].

In addition to this, in many real-world networks, there is a "cost" or a "weight" associated with each link or vertex, then a weighted network [24,25] is defined as that the larger the weight on a link or vertex, the harder this link or vertex be disabled. In the scientist collaboration network, Newman [26,27] studied the weight and distribution characteristics of betweenness. Based on this, Barrat et al. [28] took Global airline network as an example, extend clustering coefficient and the average degree to weighted networks. Besides, there are lots of empirical works in Internet network, metabolic network, transport network and so on [29–31]. Motter and Lai [32] put forward a classical model (LA model) based on the linear relationship between node capacity and load in reality, then they used this model to study cascade of overload failures in different networks, which random or intentional attack a vertex. In order to capture the relationship between the weighted feature and the cascading failure, Wang et al. [33] put forward a local weighted flow redistribution rule, and constructed a cascading failure model among edges for a weighted network. Then in Ref. [34], Wu et al. generalized this model of the vertex to study cascading failure in BA network by the load on nodes. Recently, S.Muldoon [35] et al. generated weighted small-world networks and Lu [36] et al. researched algorithms for community detection in weighted networks.

In this paper, we study the robustness against cascading failure of interdependent weighted network, and weight of nodes in this network depend on their betweenness. Cascading failure triggered by small nodes attacking on one interdependent weighted network. We define the stable state is that when the cascade of interdependent failures and overload failures both end.

2. The model

Here, we propose a cascading model with nodes' weighted flow redistribution rule and percolation theory on interdependent weighted networks. We assume two networks A and B of sizes N_A and N_B and with given mean degree $\langle k_A \rangle$ and $\langle k_B \rangle$ are typical networks, i.e., the Barabási–Albert (BA) SF networks [37]. Each connectivity link connect nodes in the same network, and interdependent link connected by two nodes come from different n networks. For model interdependent network, we consider two different dependency relations. (1) A one-to-one random interdependent relationship of nodes in network A and network B. (2) A one-to-one interdependent relationship based on betweenness of each node in network A and network B.

Now, we briefly describe the cascading model under redistribution rule and percolation theory. We assume the weight of a node i to be $w_i = B_i^\theta$, where θ is a tunable weight parameter, and B_i is the betweenness of node i. Moreover, Refs. [33,34] show that degree-dependent weighting of nodes in single networks, which the weight of a node i is $w_i = k_i^\theta$. In this sense, our assumption on the weights is in accordance with the previous load-based model but can reflect the importance of the node which has the small degree but passed through by many shortest paths. We assume that each node j has an initial load L_j, and $L_j = w_j$. Besides, following previous models, each node has a threshold value C_j, which is the maximum flow that the node can transmit. According to usual practice [33,34], assume that the threshold value of node j is proportional to its weight (initial load):

$$C_j = \beta w_j, \tag{1}$$

where $\beta \geq 1$ is a threshold parameter. In a single weighted network, based on redistribution rule, the flow in disabled node i will be redistributed to its neighbor nodes. The additional flow ΔL_j received by node i is proportional to its weight, i.e.

$$\Delta L_j = L_i \frac{w_j}{\Sigma_{l \in \Gamma_i} w_l}, \tag{2}$$

where Γ_i is the sets of neighbor node of i [33,34]. If

$$L_j + \Delta L_j > C_j = \beta w_j, \tag{3}$$

then node i will be disabled and induce further redistribution of flow $L_j + \Delta L_j$ and potentially further nodes will be disabled. If all nodes' flow less than their threshold, the cascading failure stops. Meanwhile, for interdependent networks without weighting [14–17], a random attack on network A will cause a process of cascading failure and a node to be functional in network A must (i) have at least one functional support node in network B and (ii) belong to the giant component of functional nodes in network A. Similarly, nodes in network B must satisfy similar conditions to be functional.

In interdependent weighted network, the failure on a node, may be cause a cascading failure not only on account for the interdependent relationship but also due to the redistribution rule. So, We assume two different failure mechanisms. In failure mechanism (I), we define that interdependent relationship and redistribution rule will cause the cascading failure at the same time, but in the failure mechanism (II), we assume that cascading failure caused by interdependent relationship happens after the cascading failure caused by redistribution rule ends. The attack on the coupled network system is represented by a random removal of a node in network A. Until no further nodes failure in both networks under two different

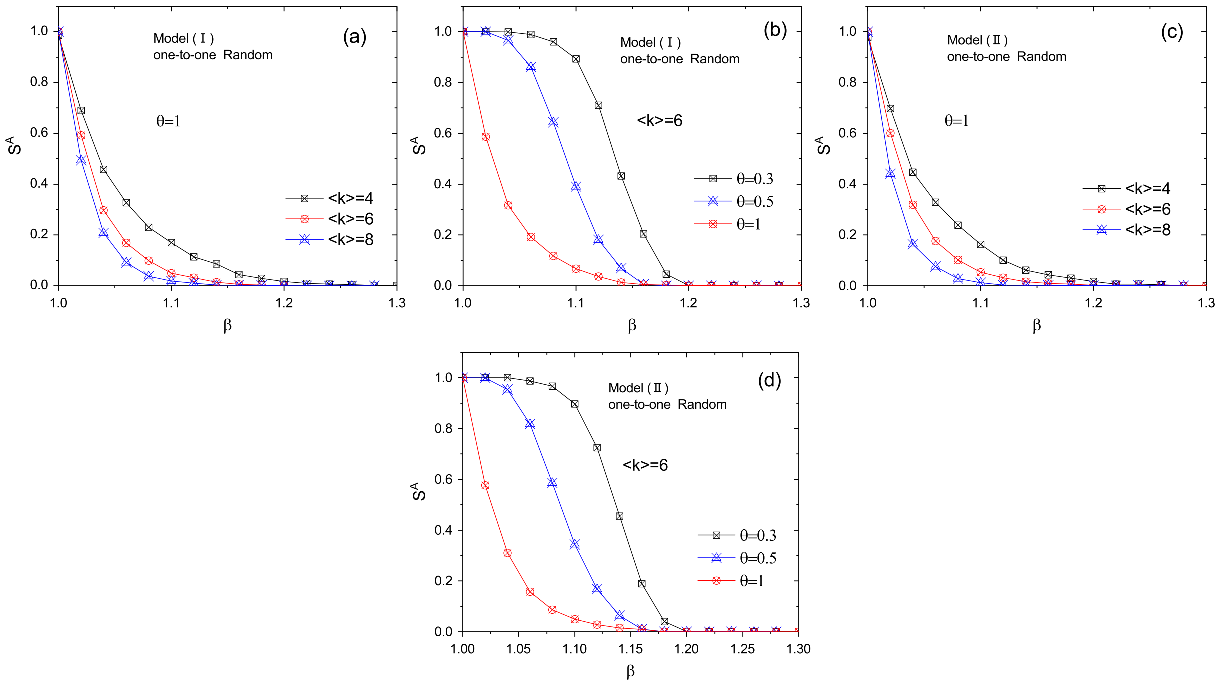

Fig. 1. The case of coupled BA networks with one-to-one random interdependent nodes of network A and network B. The numerical simulation with $N_A = N_B = 10\,000$, $\langle k_A \rangle = \langle k_B \rangle$. (a) and (b) show the case under failure mechanism, S_A as a function of threshold parameter β, while the tunable weight parameter θ and the mean degree of coupled BA networks stay the same respectively. (a) shows the different mean degree of coupled BA networks as $\langle k \rangle = 4$, $\langle k \rangle = 6$, $\langle k \rangle = 8$ meanwhile $\theta = 1$. (b) shows the different θ as $\theta = 0.3, 0.5, 1$ and keep $\langle k \rangle = 6$. (c) and (d) under the case, apart from the failure mechanism, (c) and (d) show the exactly same as (a) and (b). We averaged over 100 realizations.

failure mechanisms, which the stage that the nodes in coupled network satisfy conditions (i), (ii) and (iii) their flow must less than their threshold value, the cascading failure end.

To explore the effect of a small initial attack on our cascading model, we attack only a node i initially to network A and calculate S_i^A, which means the number of disabled nodes in network A after the cascading process based on interdependent relationship and redistribution rule are both over. To quantify the robustness of network A, we adopt the normalized avalanche size

$$S^A = \frac{\Sigma i \in NS_i^A}{N_A(N_A - 1)} \tag{4}$$

Fig. 1 shows S^A as a function of threshold parameter β, we compare the case of one-to-one random interdependent nodes of network A and network B under and failure mechanism. We use $N_A = N_B = 10\,000$, and for simplicity, we set $\langle k_A \rangle = \langle k_B \rangle = \langle k \rangle$. In Fig. 1(a), under failure mechanism, we keep the tunable weight parameter $\theta = 1$ and set the mean degree of BA networks as $\langle k \rangle = 4$, $\langle k \rangle = 6$, $\langle k \rangle = 8$. We can see that, the mean degree is larger then the disabled nodes are fewer. And in Fig. 1(b), we can find that the θ is bigger than the S^A is smaller as we set $\langle k \rangle = 6$ and change θ. Similarly, we just change the failure mechanism as and keep other conditions unchanged (shown in Fig. 1(c)(d)), we can get the similar results. Compare Fig. 1(a) and (c), (b) and (d), we can find that different failure mechanism do not influence the robustness of one-to-one interdependent weighted networks.

In Fig. 2, we can see the similar result to Fig. 1, which shows S^A as a function of threshold parameter β with (2) interdependent relationship. And we can find that, in this condition, the mean degree of BA networks and the tunable weight parameter θ have the same effect on S^A with (1) interdependent relationship. Comparing Fig. 2(a)(c) and Fig. 2(b)(d), we can confirm that again, different failure mechanisms have no effect on interdependent weighted networks, no matter cascading failure caused by interdependent relationship and redistribution rule occurs concurrently, or successively. In Figs. 1 and 2, we can also clearly identify the critical β_c, which is the least value of protection strength to avoid cascading failure. Apparently, the lower the value of β_c, the stronger the robustness of the interdependent weighted network against cascading failure caused by redistribution rule.

Then we investigate the relationship between θ, $\langle k \rangle$ and β_c of coupled BA networks. As shown in Fig. 3(a)(c), both these networks display the strongest robustness level at $\theta = 0.5$ for all different average degrees. When β_c as a function of $\langle k \rangle$, in Fig. 3(b)(d), we can find that for different θ, $\langle k \rangle$ is bigger then the robustness of networks is stronger. Fig. 3 shows the results of coupled BA network with one-to-one random interdependent relationship, and Fig. 4 shows the results under

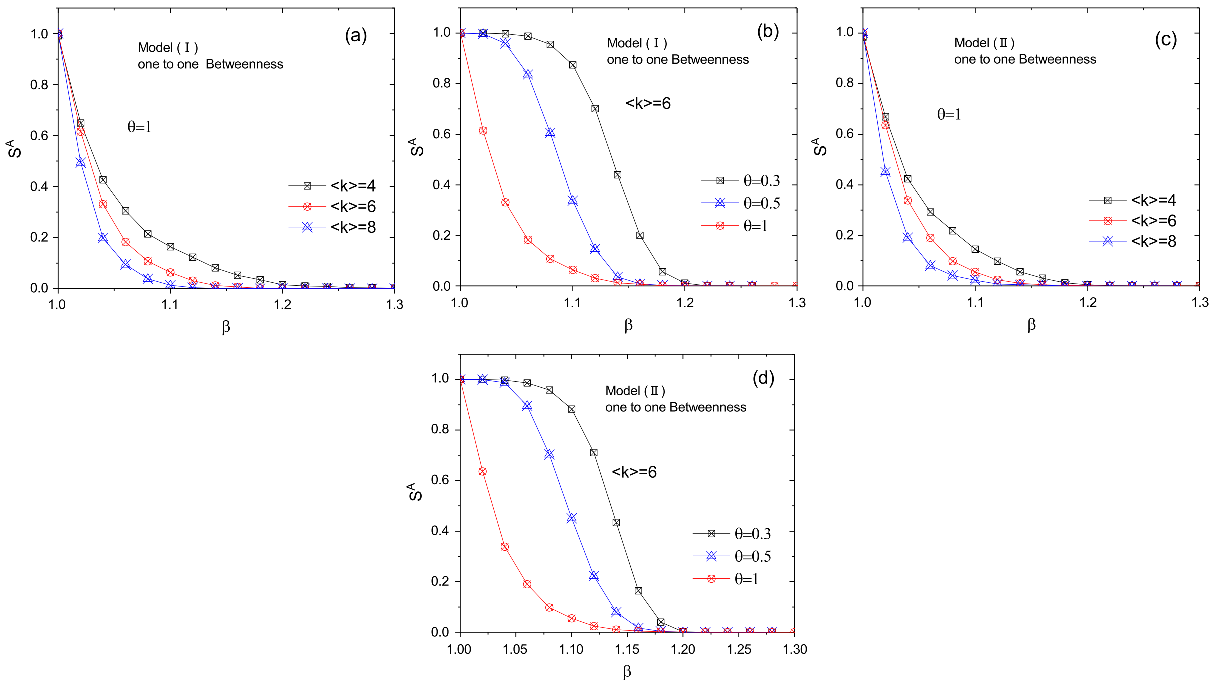

Fig. 2. The case of coupled BA networks with one-to-one interdependent relationship based on nodes' betweenness of network A and network B. The numerical simulation with $N_A = N_B = 10\,000$, $\langle k_A \rangle = \langle k_B \rangle$. (a) and (b) show the case under failure mechanism, S_A as a function of threshold parameter β, while the tunable weight parameter θ and the mean degree of coupled BA networks stay the same respectively. (a) shows the different mean degree of coupled BA networks as $\langle k \rangle = 4$, $\langle k \rangle = 6$, $\langle k \rangle = 8$ meanwhile $\theta = 1$. (b) shows the different θ as $\theta = 0.3, 0.5, 1$ and keep $\langle k \rangle = 6$ with one-to-one interdependent relationship under failure mechanism. (c) and (d) under the case, apart from the failure mechanism, (c) and (d) show the exactly same as (a) and (b). The results are averaged over 100 realizations.

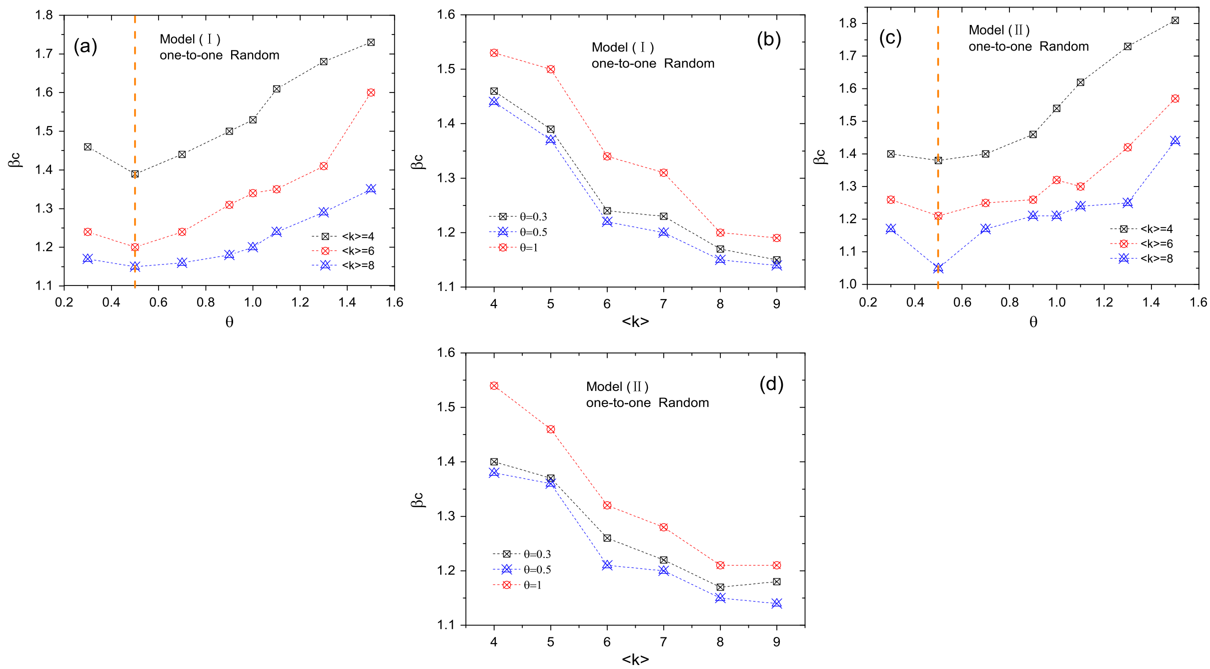

Fig. 3. The case of coupled BA networks with one-to-one random interdependent nodes of network A and network B. The numerical simulation with $N_A = N_B = 10\,000$, $\langle k_A \rangle = \langle k_B \rangle$, (a) and (b) under failure mechanism, (c) and (d) under failure mechanism. (a) and (c) show the β_c as a function of the tunable weight parameter θ, for different average degrees of BA networks. (b) and (d) show the β_c as a function of the mean degree $\langle k \rangle$ for different θ. The results are averaged over 100 realizations.

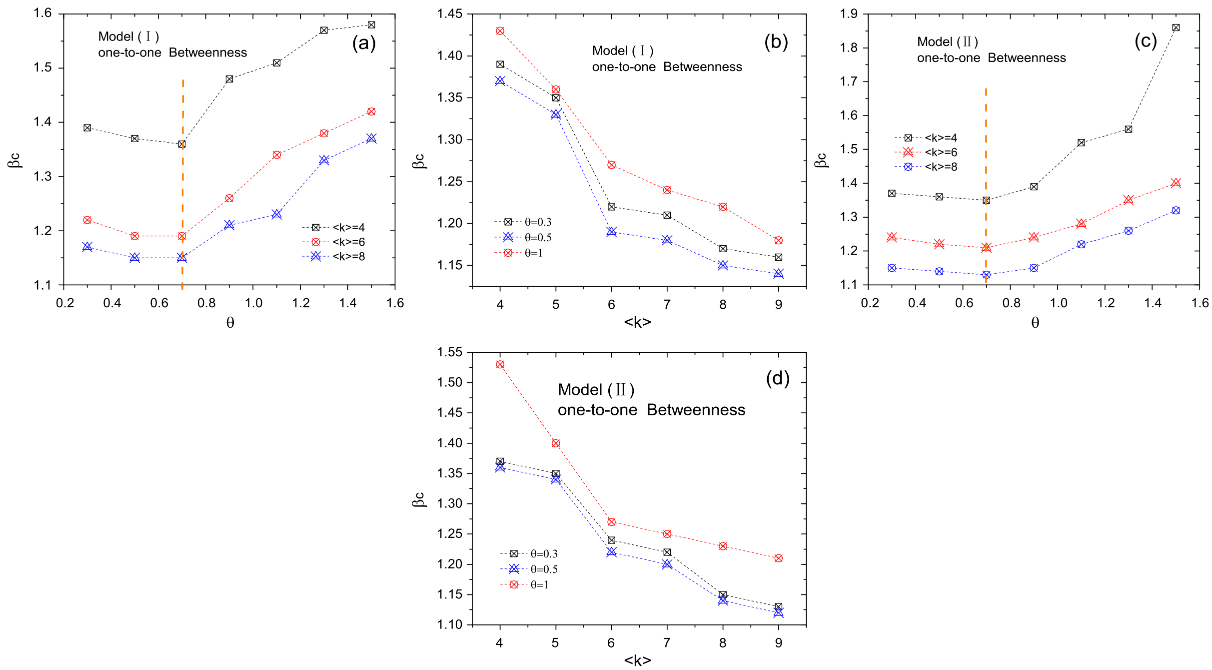

Fig. 4. The case of coupled BA networks with one-to-one interdependence relation based on nodes' betweenness of network A and network B. The numerical simulation with $N_A = N_B = 10\,000$, $\langle k_A \rangle = \langle k_B \rangle$, (a) and (b) under failure mechanism, (c) and (d) under failure mechanism. (a) and (c) show the β_c as a function of the tunable weight parameter θ, for different average degrees of BA networks. (b) and (d) show the β_c as a function of the mean degree $\langle k \rangle$ for different θ. The results are averaged over 100 realizations.

the interdependent relationship based on nodes' betweenness. In Fig. 4(a)(c), we can find that $\theta = 0.7$ is optimum for synchronization in interdependent weighted networks against cascading failures. Fig. 4(b)(d) show the similar results to Fig. 3(b)(d). Under the same conditions, for one-to-one interdependent networks, random interdependent relationship has the stronger robustness than the relationship based on nodes' betweenness.

3. Conclusions

In conclusion, we have investigated the cascading reaction behaviors on interdependent weighted networks with small vertex-based initial attacks. We found that one-to-one random interdependent weighted networks reach their strongest robustness level when the weight parameter equals to 0.5, but when the relationship based on nodes' betweenness, $\theta = 0.7$. Different failure machines do not affect on the robustness of coupled BA networks. These results indicate that the significant roles of weights and interdependent relationship in interdependent complex networks for designing protection strategies against cascading failures.

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Further Reading

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