Model-Based Edge-Event-Triggered Containment Control Under Directed Topologies

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***Abstract*—This paper investigates the containment control problem of multiagent systems with double integrator dynamics under directed topologies. A model-based edge-event-triggered control protocol is proposed, in which the control input to each agent only contains edge information and its own velocity information. Continuous detection is avoided by the establishment of a predictive model and each controller is only updated at its own event time instants. The theoretical results show that, under our control protocol, the containment control problem can be solved and the Zeno behavior is excluded. The effectiveness is further illustrated by simulation results.**

***Index Terms*—Containment problem, event-triggered control, multiagent systems (MASs).**

I. Introduction

DISTRIBUTED cooperative control of multiagent systems (MASs) has become a newly developed research area in the last decade due to its superior performance in collaborative operations [1]–[11]. Consensus as a fundamental problem in this area has been widely studied in [12]–[22]. Formation and flocking as closely related problems developed based on consensus have been reported in [23]–[28], respectively. According to the existence of leaders in MASs, the problem can fall into three categories, namely, leaderless case, single leader case and multileader case. In this paper, we focus on the problem in multileader case, defined as a containment control problem, in which followers will be driven to a convex hull spanned by leaders. Compared with traditional leader-following consensus, in which the dynamics of the single leader is determined and all other agents in MASs will track its trajectory, containment control is more robust in the case of leader failure. Furthermore, in some special circumstances, containment control is more suitable. For example, a group of unmanned vehicles need to maneuver in a hazardous environment, while only a part of them, namely, leaders, are equipped with necessary sensors to detect the threat in the area, and others can only safely maneuver by remaining in the convex hull spanned by the leaders. Under such a background, containment control has drawn a lot of attention in recent years (see [29]–[31] for single or double integrator cases; [32]–[34] for general linear systems; and [35], [36] for Lagrange systems). In this paper, the subsystems are modeled as double integrators, which are applicable in practical situations, since most of the dynamic systems can be transformed into double integrators by feedback linearization.

The implementation of distributed control depends on the development of embedded microprocessors, which only have limited energy resources. Thus, wasted energy consumption would shorten the lifespan of the MASs. Under this background, the scheduling of sensors and actuators becomes a critical issue. In the traditional time-driven fashion, sampling periods are fixed during life cycles, which are usually too conservative and have redundant sampling when adjacent sampling instants are too close and there is only little change of states between the adjacent measurements. As a development of the traditional time-driven fashion, the idea of event-driven fashion is to sample the sates of systems only when measurement errors exceed certain thresholds. This event-triggered scheduling strategy is more suitable when limited resources are under consideration.

With respect to the thresholds of trigger functions, the event-triggered control can be categorized as state-dependent [37]–[39], in which an event will be triggered when a certain error exceeds a norm related to the state, and time-dependent [40]–[42]. For the second class, Seyboth *et al.* [40] proposed an exponentially decaying threshold with respect to time, which guarantees the asymptotic convergence of the system under undirected topologies. Thereafter, this kind of time-dependent event-triggered control was applied in consensus control [42] and containment control [41]. It should be noted that, in the aforementioned literature, controllers are not only updated at their own event instants, but also at their neighbors' event time instants, which we define as a push-based event-triggered mechanism. In contrast to that, in [43]–[46], a pull-based event-triggered mechanism was proposed, in which each agent only updates its controller at its own event instants, with the price of continuously monitoring its neighbors' states, which increases the occupation of sensor resources. This tradeoff between continuous detection on sensors and frequent updates of controllers was well explained in [47]. In [48]–[55], this

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problem was addressed with a hybrid strategy of time- and event-triggered sampling, which avoids continuous data transmission by examining the event-triggered condition only at periodic sampling instants; in [37], [56], and [57], a self-triggered strategy was proposed, in which each agent decides its next event instant by using the previously received information. Thus, the MASs only need to communicate at each predetermined time instant. In addition, in [58] and [59], this issue was tackled by a model-based event-triggered strategy, where a predictive model of measurement errors is established and the state measurement actions are only carried out when the estimated measurement error exceeds some threshold. Some recent results on event-triggered MASs can be found in [60]–[65] and the references there.

Inspired by Liuzza *et al.* [58], in this paper, we propose a model-based edge-event-triggered containment control protocol for MASs under directed topologies. It should be noted that, different from [58], the interaction topologies discussed in this paper are directed. In addition, different from the aforementioned literature, under our control protocol, only edge information, i.e., relative distances between agents, is required to solve the containment problem [44], [49], [66], [67]. Compared with node information, i.e., absolute positions of agents, edge information is easier to be acquired in practical situations, since most of the onboard sensors, like laser or radar sensors, only have the capability of measuring the relative states between agents. Furthermore, compared with the control protocols based on node information, our strategy is superior with less information request at each event instant. Since whenever an edge occurs, the involved agent only needs to request the information from the agent connected by the edge, instead of the information from all its neighbors.

Taking the advantage of edge information in MASs, we propose a model-based edge-event-triggered control protocol. The main contributions of this paper are as follows.

1) Most of the existing results about containment control problems only consider whether followers can coverage to the convex hull spanned by leaders, ignoring the behavior of leaders. While in this paper, inspired by Liu *et al.* [41] and Zhang *et al.* [45], we consider a combined problem of formation and containment. More specifically, the leaders will converge to a formation and the followers will coverage to the convex hull spanned by leaders. Both followers and leaders can communicate with their neighbors during the maneuver.
2) Inspired by Liuzza *et al.* [58] and Xu *et al.* [59], we propose a model-based event-triggered protocol, by which the continuously monitoring can be avoided. At the same time, the controller of each agent will not be updated at its neighbors' event instants, which solves the contradictive tradeoff of resource saving between sensors and actuators. In addition, with the edge-event-triggered strategy, at every event instant, the agent only needs to request information from only one of its neighbors, by which, the sensor resources are further saved.
3) Compared with the aforementioned literature, the control input to each agent in this paper only contains the information of relative distances between agents and each agent's own velocity. Both of them are easier to be acquired by the onboard sensors, compared with node information, like absolute positions, which are needed in the traditional node-triggered control strategies.

The rest of this paper is organized as follows. Section II introduces some preliminaries and formulates the problem. The edge-event-triggered containment control is investigated in Section III. To avoid continuous monitoring, a model-based strategy is given in Section IV. The simulation results are given in Section V to show the effectiveness of the control protocol. The conclusions are drawn in Section VI.

II. Preliminary and Problem Formulation

A. Preliminaries

Consider a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, consisting of n nodes $\mathcal{V} = \{v_1, \ldots, v_n\}$ and edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The adjacency matrix $A = [a_{ij}]_{n\times n}$ is a non-negative matrix, in which $a_{ij} = 1$ if and only if $(v_j, v_i) \in \mathcal{E}$, and $a_{ij} = 0$, otherwise. Assume the graph containing no self-loop, thus, $a_{ii} = 0$. Furthermore, when $a_{ij} = 1$, v_j is called a neighbor of v_i, which means v_i can receive information from v_j. The Laplacian matrix of $\mathcal{G}$ is defined as $\mathcal{L} = [l_{ij}]$, where $l_{ii} = \sum_{j=1}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$. The Laplacian matrix can be written in the Jordan canonical form $\mathcal{L} = PJP^{-1}$, where $J = \mathrm{diag}[J_0, \ldots, J_s]$, with J_g represented as

$$J_g = \begin{bmatrix} \lambda_g & 1 & 0 & \cdots & 0 \\ 0 & \lambda_g & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & 1 \\ 0 & 0 & \cdots & 0 & \lambda_g \end{bmatrix}_{n_g\times n_g}. \tag{1}$$

In (1), λ_g is the eigenvalue of $\mathcal{L}$ with algebraic multiplicity n_g. The transformation matrix P is denoted by $P = [p_1, p_2, \ldots, p_n]$ and $P^{-1} = [q_1, q_2, \ldots, q_n]^T$, where p_i is the right eigenvector or generalized right eigenvector of $\mathcal{L}$ and q_i is the left eigenvector or generalized left eigenvector of $\mathcal{L}$.

Suppose there are p directed edges in $\mathcal{G}$, labeled from 1 to p. Then, the incidence matrix D is defined as $D = [d_{il}]$, where $d_{il} = 1$, if v_i is the head of the directed edge l; $d_{il} = -1$, if v_i is the tail of the directed edge l; and $d_{il} = 0$, otherwise. Furthermore, the in-incidence matrix $\overline{D}$ is denoted by $\overline{D} = [\bar{d}_{il}]$, where $\bar{d}_{il} = 1$, when $d_{il} = 1$, and $\bar{d}_{il} = 0$, otherwise.

B. Problem Formulation

Consider a MAS containing n identical agents. Each agent is modeled by a double integrator

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = u_i(t) \end{cases} \quad i = 1, 2, \ldots, n \tag{2}$$

and represented by a node in $\mathcal{G}$, where $x_i \in \mathbb{R}$, $v_i \in \mathbb{R}$, and $u_i \in \mathbb{R}$, represent the position state, velocity state, and control input of agent i. For convenience, we only discuss the situation when x_i, v_i, u_i, are 1-D vectors in this paper, but it is easy to generalize the results into multidimensional cases by matrix extension using Kronecker product. All agents in the MAS are divided into two groups, which are $m(m < n)$ leaders and $n'(n' = n - m)$ followers. Denote the leader set and follower set by $\mathcal{R}$ and $\mathcal{F}$, respectively. The leaders have access to the global information $h = [h_1, h_2, \ldots, h_m]^T$, $h \in \mathbf{R}^m$, which represents the expected formation of the MAS; the followers can only receive relative information from their leader or follower neighbors. The interaction topology of the MAS is represented by a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, and the Laplacian matrix by $\mathcal{L} = [l_{ij}]_{n\times n}$. Since leaders cannot receive information from followers, the Laplacian matrix $\mathcal{L}$ can be partitioned as

$$L = \begin{bmatrix} L_r & 0_{n'} \\ L_{fr} & L_f \end{bmatrix}. \tag{3}$$

The following assumption is imposed.

Assumption 1: The graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ contains a directed spanning tree.

Lemma 1 [41]: Under Assumption 1, the matrix $-L_r$ only has one zero eigenvalue and all other eigenvalues have negative real parts, all the eigenvalues of matrix $-L_f$ have negative real parts. Further more, each element of $-L_f^{-1}L_{fr}$ is non-negative and the sum of each row is 1.

Suppose there are p_r edges among leaders, p_{f_1} edges directed from leaders to followers and p_{f_2} edges among followers, where $p_{f_1} + p_{f_2} = p_f$, $p_r + p_f = p$, represent the total edges directed to followers and the total edges among the MAS, respectively. For edge l, the edge state $y_l(t)$ is defined by $y_l(t) = x_i(t) - x_j(t)$, if agent i can receive information from agent j through edge l. In addition, relative formation information h_l is defined by $h_l = h_i - h_j$ for leaders, if leader i can receive information from leader j through edge l. The incidence and in-incidence matrices are denoted by D and $\bar{D}$, respectively. Furthermore, according to the edges belonging to leader set $\mathcal{R}$; follower set $\mathcal{F}$; and the edges directed from leaders to followers, divide D and $\bar{D}$ as follows:

$$D = \left[\begin{array}{c:c:c} D_r & \multirow{2}{*}{D_{fr}} & 0_{m\times p_{f_2}} \\ 0_{n'\times p_r} & & D_f \end{array}\right]$$

$$\bar{D} = \begin{bmatrix} \bar{D}_r & 0_{m\times p_{f_1}} & 0_{m\times p_{f_2}} \\ 0_{n'\times p_r} & \bar{D}'_{fr} & \bar{D}_f \end{bmatrix} \tag{4}$$

where $\bar{D}_r = [\bar{d}^r_{il}]_{m\times p_r}$ and $\bar{D}_f = [\bar{d}^f_{il}]_{n'\times p_{f_2}}$ correspond to D_r and D_f in incidence matrix D, with the elements satisfying $\bar{d}^r_{il} = 1$ ($\bar{d}^f_{il} = 1$) when the corresponding elements in D_r (D_f) are 1, and $\bar{d}^r_{il} = 0$ ($\bar{d}^f_{il} = 0$), otherwise; $\bar{D}'_{fr} = [\bar{d}^{fr}_{il}]_{n'\times p_{f_1}}$, corresponds to the $(m+1)$th to nth rows in D_{fr}, with the elements satisfying $\bar{d}^{fr}_{il} = 1$ when the corresponding element in D_{fr} is 1, and $\bar{d}^{fr}_{il} = 0$, otherwise.

The event time instants of agent i are denoted by $t_0^{v_i}, t_1^{v_i}, \ldots$ Then, $\hat{v}_i(t) = v_i(t_k^{v_i})$, $t \in [t_k^{v_i}, t_{k+1}^{v_i})$ represents the velocity of agent i at the latest event time instant before t, and $e_i^v(t) = \hat{v}_i(t) - v_i(t)$ represents the measurement error which records the change of velocity between t and the latest event instant $t_k^{v_i}$. Similarly, the event instants of edge l are denoted by $t_0^{\epsilon_l}, t_1^{\epsilon_l}, \ldots$ Then $\hat{y}_l(t) = y_l(t_k^{\epsilon_l})$, $t \in [t_k^{\epsilon_l}, t_{k+1}^{\epsilon_l})$, represents the state of edge l at the latest event instant before t, and $e_l^\epsilon(t) = \hat{y}_l(t) - y_l(t)$ represents the measurement error. For a better understanding of the information transmission among agents under edge-event triggered mechanism, a schematic diagram is given as in Fig. 1.

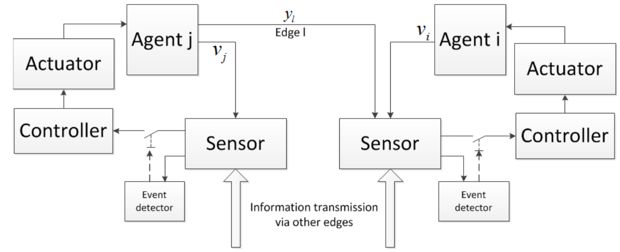

Fig. 1. Information transmission among agents under the edge-event triggered mechanism. Here, assume agent i can receive information from agent j through edge l.

Now we propose a control protocol based on edge state and self-velocity state of each agent under event-triggered strategy.

The control input to the leaders and followers are given as follows:

$$u_i(t) = -\sum_{l=1}^{p_r} \bar{d}_{il}\big(\hat{y}_l(t) - h_l\big) - k_r\hat{v}_i(t), \quad i \in \mathcal{R}$$

$$u_i(t) = -\sum_{l=p_r+1}^{p} \bar{d}_{il}\hat{y}_l(t) - k_f\hat{v}_i(t), \quad i \in \mathcal{F}. \tag{5}$$

Define the trigger functions for edge l and agent i as follows:

$$f_l^\epsilon\big(t, e_l^\epsilon(t)\big) = \big|e_l^\epsilon(t)\big| - \sigma_l^\epsilon e^{\gamma t} \tag{6}$$

$$f_i^v\big(t, e_i^v(t)\big) = \big|e_i^v(t)\big| - \sigma_i^v e^{\gamma t} \tag{7}$$

where σ_l^ϵ, σ_i^v, and γ are constants, and their value ranges will be determined later.

Based on (6) and (7), the occurrence of an event depends on the violation of inequalities

$$f_l^\epsilon\big(t, e_l^\epsilon(t)\big) \le 0 \tag{8}$$

or

$$f_i^v\big(t, e_i^v(t)\big) \le 0 \tag{9}$$

which acts as the trigger condition of our control protocol.

The main purpose of this paper is to prove that the MAS will coverage asymptotically to the expected state under the proposed protocol, in which the leaders will coverage to a formation and the followers will coverage to the convex hull spanned by leaders, and the MAS can keep the formation structure.

III. Edge-Event-Triggered Containment Control

In this section, we will study the dynamic performance of the MAS under edge-event-triggered control protocol. Each agent can receive relative information through edge l and monitor its own velocity continuously, but its controller is updated only at the instant when measurement error achieves the threshold.

Denote $x_r = [x_1^r, \ldots, x_m^r]^T$, $x_r \in \mathbf{R}^m$, as the position states of leaders, where x_i^r represents the ith leader's position state. Similarly, denote $v_r = [v_1^r, \ldots, v_m^r]^T$, $v_r \in \mathbf{R}^m$ as the velocities of leaders, and $x_f = [x_1^f, \dots, x_{n'}^f]^T$, $x_f \in \mathbf{R}^{n'}$; $v_f = [v_1^f, \dots, v_{n'}^f]^T$, $v_f \in \mathbf{R}^{n'}$ as position and velocity states of the followers. Let $e_L^r = [e_1^r, \dots, e_{p_r}^r]^T$, $e_L^r \in \mathbf{R}^{p_r}$; $e_L^{fr} = [e_1^{fr}, \dots, e_{p_{f_1}}^{fr}]^T$, $e_L^{fr} \in \mathbf{R}^{p_{f_1}}$; and $e_L^f = [e_1^f, \dots, e_{p_{f_2}}^f]^T$, $e_L^f \in \mathbf{R}^{p_{f_2}}$ represent measurement errors of the edges among leaders, directed from leaders to followers and among followers, respectively; $e_v^r = [e_1^{rv}, \dots, e_m^{rv}]^T$, $e_v^r \in \mathbf{R}^m$ and $e_v^f = [e_1^{fv}, \dots, e_{n'}^{fv}]^T$, $e_v^f \in \mathbf{R}^{n'}$, represent leaders and followers own measurement errors of velocity.

Under control input (5), the dynamic equations of system (2) can be written as

$$\begin{bmatrix} \dot{x}_r - h \\ \dot{v}_r \end{bmatrix} = A_0 \begin{bmatrix} x_r - h \\ v_r \end{bmatrix} + B_1 \begin{bmatrix} e_L^r \\ e_v^r \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} \dot{x}_f \\ \dot{v}_f \end{bmatrix} = A_1 \begin{bmatrix} x_r \\ v_r \end{bmatrix} + A_2 \begin{bmatrix} x_f \\ v_f \end{bmatrix} + B_2 \begin{bmatrix} e_L^{fr} \\ e_L^f \\ e_v^f \end{bmatrix} \tag{11}$$

where

$$A_0 = \begin{bmatrix} 0_m & I_m \\ -L_r & -k_r I_m \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0_{m \times p_r} & 0_m \\ -\overline{D}_r & -k_r I_m \end{bmatrix}$$
$$A_1 = \begin{bmatrix} 0_{n' \times m} & 0_{n' \times m} \\ -L_{fr} & 0_{n' \times m} \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0_{n'} & I_{n'} \\ -L_f & -k_f I_{n'} \end{bmatrix}$$
$$B_2 = \begin{bmatrix} 0_{n' \times p_{f1}} & 0_{n' \times p_{f2}} & 0_{n'} \\ -\overline{D}_{fr}' & -\overline{D}_f & -k_f I_{n'} \end{bmatrix}.$$

Lemma 2: Under Assumption 1, matrix A_0 only has one zero eigenvalue, with the associated right eigenvector $[\mathbf{1}_m^T, \mathbf{0}_m^T]^T$ and left eigenvector $[k_r \eta^T, \eta^T]^T$, where η is the left eigenvector associated with zero eigenvalue of $-L_r$. Furthermore, all other eigenvalues of A_0 have negative real parts if $k_r > \max_{2 \le i \le m} |\mu_i| \sqrt{[2/\mathrm{Re}(\mu_i)]}$, where μ_i represents the nonzero eigenvalue of $-L_r$.

Note that, the properties of system matrix and feedback gain of closed-loop system under undirected connected topologies and control input (5) are given in [14, Th. 1], while the results with the topologies under Assumption 1 and control input containing relative distance and relative velocity information are given in [68, Lemma 4.10]. Applying similar methods of proof, the conclusion in Lemma 2 in this paper can be easily reached.

Theorem 1: Under Assumption 1, suppose that the feedback gains k_r and k_f in control input (5) satisfy $k_r > \max_{2 \le i \le m} |\mu_i| \sqrt{[2/\mathrm{Re}(\mu_i)]}$, and $k_f > \max_{1 \le j \le n'} |\nu_j| \sqrt{[2/\mathrm{Re}(\nu_j)]}$, where μ_i and ν_j represent the nonzero eigenvalues of $-L_r$ and $-L_f$, respectively. The constants in the trigger function (6) satisfy σ_l^{ϵ}, $\sigma_i^v \ge 1$, parameter γ satisfies $0 \ge \gamma \ge \beta$, where $\beta = \max(\max_{2 \le i \le 2m} [\mathrm{Re}(\lambda_i)/2], \max_{1 \le j \le 2n'} [\mathrm{Re}(\lambda_j')/2])$, with λ_i the eigenvalue of A_0, and λ_j' the eigenvalue of A_2. Then the MAS, described in (2) under control input (5) and trigger condition (8) and (9), will coverage to the expected states asymptotically. More specifically, the leaders will coverage to a formation, and the followers will coverage to the convex hull spanned by the leaders. Furthermore, the MAS does not exhibit Zeno behavior.

Proof: In the following, we will divide the proof into three steps. First, we show that the leaders will coverage to a formation. After that, we prove that the followers will coverage to the convex hull spanned by the leaders. Finally, a lower bound of interevent times is given which implies that the Zeno behavior can be avoided in the system.

Step I: In this step, we will prove the convergence of leaders.

Let $J = \mathrm{diag}[J_1, \dots, J_s]$ be the Jordan canonical of A_0, transformed by $A_0 = PJP^{-1}$, where $P = [p_1, \dots, p_{2m}]$ and $P^{-1} = [q_1, \dots, q_{2m}]^T$, with p_i chosen from the right eigenvectors or generalized right eigenvectors of A_0, and q_i chosen from the left eigenvectors or generalized left eigenvectors of A_0. By Lemma 2 and $k_r > \max_{2 \le i \le m} |\mu_i| \sqrt{[2/\mathrm{Re}(\mu_i)]}$, A_0 has only one zero eigenvalue and the real parts of other eigenvalues are negative. Thus, we have $J_1 = 0$ and J_g, $g \in \{2, \dots, s\}$ have the form like (1).

Define an auxiliary variable z as

$$z = P^{-1} \begin{bmatrix} x_r - h \\ v_r \end{bmatrix}. \tag{12}$$

By (10), the dynamics of z is given by

$$\dot{z} = Jz + P^{-1} B_1 \begin{bmatrix} e_L^r \\ e_v^r \end{bmatrix}. \tag{13}$$

By Lemma 2, we have $p_1 = [\mathbf{1}_m^T, \mathbf{0}_m^T]^T$, $q_1 = [k_r \eta^T, \eta^T]^T$. Denote $z = [z_1, \tilde{z}_2^T, \dots, \tilde{z}_s^T]^T$, $P = [p_1, \tilde{p}_2, \dots, \tilde{p}_s]$, $P^{-1} = [q_1, \tilde{q}_2, \dots, \tilde{q}_s]^T$. For $g \in \{2, \dots, s\}$, we denote $\tilde{p}_g = [p_{g_1}, \dots, p_{g_{n_g}}]$ and $\tilde{q}_g = [q_{g_1}, \dots, q_{g_{n_g}}]$, in which p_{g_i} and q_{g_i} are chosen from the right or generalized right eigenvectors and left or generalized left eigenvectors of A_0 associated with λ_g, respectively; let $\tilde{z}_g = [z_{g_1}, \dots, z_{g_{n_g}}]^T$ represents the states associated with λ_g. Then, we have that $\dot{z}_1 = -\eta^T \overline{D}_r e_L^r - k_r \eta^T I_m e_v^r$, and $\dot{\tilde{z}}_g = J_g \tilde{z}_g + \tilde{q}_g^T B_1 \begin{bmatrix} e_L^r \\ e_v^r \end{bmatrix}$. Furthermore, for each J_g, $g \in \{2, \dots, s\}$, there exists a postive constant c_g, such that $\|e^{J_g t}\| \le c_g e^{[\mathrm{Re}(\lambda_g)/2]t}$. Integrating $\dot{\tilde{z}}_g$ against t, we have

$$\tilde{z}_g(t) = e^{J_g t} \tilde{z}_g(0) + \int_0^t e^{J_g(t-s)} \tilde{q}_g^T B_1 \begin{bmatrix} e_L^r \\ e_v^r \end{bmatrix} ds$$
$$\begin{aligned} \|\tilde{z}_g(t)\| \le\ & c_g e^{\frac{\mathrm{Re}(\lambda_g)}{2} t} \|\tilde{z}_g(0)\| \\ & + \left\|\tilde{q}_g^T B_1\right\| c_g \int_0^t e^{\frac{\mathrm{Re}(\lambda_g)}{2}(t-s)} \left\| \begin{bmatrix} e_L^r \\ e_v^r \end{bmatrix} \right\| ds. \end{aligned} \tag{14}$$

By the restriction of trigger condition, we have

$$\begin{aligned} \|e_L^r(s)\| &\le \sum_{l=1}^{p_r} |e_l^r(s)| \le \sum_{l=1}^{p_r} \sigma_l^{\epsilon} e^{\gamma s} \\ \|e_v^r(s)\| &\le \sum_{i=1}^{m} |e_i^{rv}(s)| \le \sum_{i=1}^{m} \sigma_i^v e^{\gamma s}. \end{aligned} \tag{15}$$

Thus,

$$\begin{aligned} &\int_0^t e^{\frac{\mathrm{Re}(\lambda_g)}{2}(t-s)} \left\| \begin{bmatrix} e_L^r \\ e_v^r \end{bmatrix} \right\| ds \\ &\le \int_0^t \left(e^{\frac{\mathrm{Re}(\lambda_g)}{2}(t-s)} \|e_L^r\| + e^{\frac{\mathrm{Re}(\lambda_g)}{2}(t-s)} \|e_v^r\| \right) ds \\ &\le \int_0^t e^{\frac{\mathrm{Re}(\lambda_g)}{2}(t-s)} \cdot \sum_{l=1}^{p_r} \sigma_l^{\epsilon} e^{\gamma s} ds + \int_0^t e^{\frac{\mathrm{Re}(\lambda_g)}{2}(t-s)} \cdot \sum_{i=1}^{m} \sigma_i^v e^{\gamma s} ds \end{aligned}$$

$$= \sum_{l=1}^{p_r} \sigma_l^{\epsilon} \frac{e^{\gamma t} - e^{\frac{\mathrm{Re}(\lambda_g)}{2}t}}{\gamma - \frac{\mathrm{Re}(\lambda_g)}{2}} + \sum_{i=1}^{m} \sigma_i^{\nu} \frac{e^{\gamma t} - e^{\frac{\mathrm{Re}(\lambda_g)}{2}t}}{\gamma - \frac{\mathrm{Re}(\lambda_g)}{2}}. \tag{16}$$

Substituting (16) into (14), we have

$$\begin{aligned}\left\|\tilde{z}_g(t)\right\| \le\ & c_g e^{\frac{\mathrm{Re}(\lambda_g)}{2}t}\left\|\tilde{z}_g(0)\right\| \\ & + c_g\left\|\tilde{q}_g^T B_1\right\|\left(\sum_{l=1}^{p_r}\Phi_l^r\left(e^{\gamma t} - e^{\frac{\mathrm{Re}(\lambda_g)}{2}t}\right)\right. \\ & \left. + \sum_{i=1}^{m}\Phi_i^r\left(e^{\gamma t} - e^{\frac{\mathrm{Re}(\lambda_g)}{2}t}\right)\right)\end{aligned} \tag{17}$$

where $\Phi_l^r = [\sigma_l^{\epsilon}/(\gamma - [\mathrm{Re}(\lambda_g)/2])]$ and $\Phi_i^r = [\sigma_i^{\nu}/(\gamma - [\mathrm{Re}(\lambda_g)/2])]$. Thus, when time t approaches infinite, we have that

$$\lim_{t\to\infty}\left\|\tilde{z}_g(t)\right\| \to 0. \tag{18}$$

Let $\begin{bmatrix} x_r - h \\ v_r \end{bmatrix} = \xi_r$, and $\delta_r = \xi_r - p_1 z_1$. We have

$$\begin{aligned}\delta_r &= Pz(t) - p_1 z_1 \\ &= p_1 z_1 + \tilde{p}_2\tilde{z}_2 + \cdots + \tilde{p}_s\tilde{z}_s - p_1 z_1 \\ \|\delta_r\| &\le \|\tilde{p}_2\|\|\tilde{z}_2\| + \cdots + \|\tilde{p}_s\|\|\tilde{z}_s\|.\end{aligned} \tag{19}$$

When time t approches infinite, we have $\lim_{t\to\infty}\|\delta_r\| \to 0$, which means $\lim_{t\to\infty}\xi_r \to p_1 z_1$. Thus, we have $\lim_{t\to\infty}[x_r(t)^T, v_r(t)^T] = [z_1\mathbf{1}_m^T + h^T, \mathbf{0}_m^T]$. Furthermore, since $\dot{z}_1 = -\eta^T\overline{D}_r e_L^r - k_r\eta^T I_m e_v^r$, by (15), we have $\lim_{t\to\infty}\dot{z}_1 \to 0$, which leads to the conclusion that leaders can converge to a formation and remain steady.

Step II: Now, we will prove the convergence of the followers.

Let $\zeta_r = [x_r^T, v_r^T]^T$, $\zeta_f = [x_f^T, v_f^T]^T$, $e_f = [e_L^{fr\,T}, e_L^{f\,T}, e_v^{f\,T}]^T$, $e_r = [e_L^{r\,T}, e_v^{r\,T}]^T$, and $\zeta = A_1\zeta_r + A_2\zeta_f$. Then, the derivative of ζ can be represented by

$$\begin{aligned}\dot{\zeta} &= A_1A_0\left(\zeta_r - [h^T, \mathbf{0}_m^T]^T\right) + A_2A_1\zeta_r + A_2^2\zeta_f + A_1B_1e_r \\ &\quad + A_2B_2e_f \\ &= A_2\zeta + A_1A_0\left(\zeta_r - [h^T, \mathbf{0}_m^T]^T\right) + A_1B_1e_r + A_2B_2e_f.\end{aligned} \tag{20}$$

Integrating $\dot{\zeta}$ against t, we have

$$\begin{aligned}\zeta &= e^{A_2t}\zeta + \int_0^t e^{A_2(t-s)}A_1A_0\left(\zeta_r - \left[h^T, \mathbf{0}_m^T\right]^T\right)ds \\ &\quad + \int_0^t e^{A_2(t-s)}A_1B_1e_r ds + \int_0^t e^{A_2(t-s)}A_2B_2e_f ds.\end{aligned} \tag{21}$$

By Lemma 1, all the eigenvalues of $-L_f$ have negative real parts. Since $k_f > \max_{1\le j\le n'}|\nu_j|\sqrt{[2/\mathrm{Re}(\nu_j)]}$, by Lemma 2, it is easy to come to the conclusion that all the eigenvalues of A_2 have negative real parts. Similar to the proof of the leaders, we know that

$$\lim_{t\to\infty} e^{A_2t}\zeta \to 0 \tag{22}$$

and there exist positive constants c_1 and a, such that $\|e^{A_2t}\| \le c_1e^{at}$.

For the second term in (21), we have

$$\begin{aligned}&\left\|\int_0^t e^{A_2(t-s)}A_1A_0\left(\zeta_r - \left[h^T, \mathbf{0}_m^T\right]^T\right)ds\right\| \\ &= \left\|\int_0^t e^{A_2(t-s)}\begin{bmatrix} 0_{n'\times m} & 0_{n'\times m} \\ 0_{n'\times m} & -L_{fr}\end{bmatrix}\left(\zeta_r - \left[h^T, \mathbf{0}_m^T\right]^T\right)ds\right\| \\ &= \left\|\int_0^t e^{A_2(t-s)}\begin{bmatrix} 0_{n'\times m} & 0_{n'\times m} \\ 0_{n'\times m} & -L_{fr}\end{bmatrix}\right. \\ &\qquad \left.\times\left(p_1z_1 + \tilde{p}_2\tilde{z}_2 + \cdots + \tilde{p}_s\tilde{z}_s - \left[h^T, \mathbf{0}_m^T\right]^T\right)ds\right\| \\ &= \left\|\sum_{g=2}^{s}\int_0^t e^{A_2(t-s)}\begin{bmatrix} 0_{n'\times m} & 0_{n'\times m} \\ 0_{n'\times m} & -L_{fr}\end{bmatrix}\tilde{p}_g\tilde{z}_g ds\right\|.\end{aligned}$$

With (17), we have

$$\begin{aligned}&\left\|\int_0^t e^{A_2(t-s)}A_1A_0\left(\zeta_r - \left[h^T, \mathbf{0}_m^T\right]^T\right)ds\right\| \\ &\le c_1\left\|L_{fr}\right\|\sum_{g=2}^{s}\left\|\tilde{p}_g\right\|\int_0^t e^{a(t-s)}\left\|\tilde{z}_g\right\|ds \\ &\le c_1\left\|L_{fr}\right\|\sum_{g=2}^{s}\left\|\tilde{p}_g\right\|\int_0^t e^{a(t-s)}\left(c_g e^{\frac{\mathrm{Re}(\lambda_g)}{2}s}\left\|\tilde{z}_g(0)\right\|\right. \\ &\quad + c_g\left\|\tilde{q}_g^T B_1\right\|\left(\sum_{l=1}^{p_r}\sigma_l^{\epsilon}\frac{e^{\gamma s} - e^{\frac{\mathrm{Re}(\lambda_g)}{2}s}}{\gamma - \frac{\mathrm{Re}(\lambda_g)}{2}}\right. \\ &\quad \left.\left. + \sum_{i=1}^{m}\sigma_i^{\nu}\frac{e^{\gamma s} - e^{\frac{\mathrm{Re}(\lambda_g)}{2}s}}{\gamma - \frac{\mathrm{Re}(\lambda_g)}{2}}\right)\right)ds \\ &= c_1\left\|L_{fr}\right\|\sum_{g=2}^{s}\left(\Phi_g^f\left(e^{\frac{\mathrm{Re}(\lambda_g)}{2}t} - e^{at}\right) + \Psi_g\left(\sum_{l=1}^{p_r}\Phi_l^f\left(e^{\gamma t} - e^{at}\right)\right.\right. \\ &\quad \left.\left. + \sum_{i=1}^{m}\Phi_i^f\left(e^{\gamma t} - e^{at}\right)\right)\right)\end{aligned}$$

where $\Phi_g^f = ([c_g\|\tilde{p}_g\|\|z_g(0)\|]/[\mathrm{Re}(\lambda_g)/2 - a])$, $\Phi_l^f = (\sigma_l^{\epsilon}/[(\gamma - \mathrm{Re}(\lambda_g)/2)^2(\gamma - a)])$, $\Phi_i^f = (\sigma_i^{\nu}/[(\gamma - \mathrm{Re}(\lambda_g)/2)^2(\gamma - a)])$, and $\Psi_g = c_g\|\tilde{p}_g\|\|\tilde{q}_g^T B_1\|$.

Thus, when time t approaches infinite, we have

$$\lim_{t\to 0}\left\|\int_0^t e^{A_2(t-s)}A_1A_0\left(\zeta_r - \left[h^T, \mathbf{0}_m^T\right]^T\right)ds\right\| \to 0 \tag{23}$$

similar arguments, it is easy to get that

$$\begin{aligned}&\lim_{t\to 0}\left\|\int_0^t e^{A_2(t-s)}A_1B_1e_r ds\right\| \to 0 \\ &\lim_{t\to 0}\left\|\int_0^t e^{A_2(t-s)}A_2B_2e_f ds\right\| \to 0.\end{aligned} \tag{24}$$

Substituting (22)–(24) into (21), we have $\lim_{t\to\infty}\|\zeta\| \to 0$. Thus, $\lim_{t\to\infty}\zeta_f \to -A_2^{-1}A_1\zeta_r$, which can also be written in the following form as:

$$\lim_{t\to\infty}\left(\begin{bmatrix} x_f \\ v_f\end{bmatrix} - \begin{bmatrix} -L_f^{-1}L_{fr} & 0_{n'\times m} \\ 0_{n'\times n'} & 0_{m\times m}\end{bmatrix}\begin{bmatrix} x_r \\ v_r\end{bmatrix}\right) = 0. \tag{25}$$

By Lemma 1, we come to the conclusion that the followers will coverage to the convex hull spanned by leaders.

This article has been accepted for inclusion in a future issue of this journal. Content is final as presented, with the exception of pagination.

Step III: In the following, we will show the Zeno behavior can be avoided by giving a lower bound of the interevent times.

First, we show that the interevent time of edge l among leaders is lower-bounded by a positive constant τ. Assuming two adjacent event time instants of edge l are t^* and t', we have $e_l^r(t^*) = 0$, $e_l^r(t') = 0$, and $f_l^\epsilon(t, e_l^\epsilon(t)) \leq 0$ for $t \in [t^*, t')$. Since $e_l^r(t) = \hat{y}_l(t) - y_l(t)$, between two adjacent event time instants, we have the dynamics of the measurement error

$$\dot{e}_l^r(t) = -\dot{y}_l(t).$$

Since $y_l(t) = D_{rl}^T x_r(t)$, where D_{rl} represents the lth column of D_r, by (10), we have

$$\dot{e}_l^r(t) = -D_{rl}^T [0_m \ I_m] \xi_r. \tag{26}$$

Substitute (12) into (26), we have

$$\dot{e}_l^r(t) = -D_{rl}^T [0_m \ I_m] \big[p_1 z_1 + \tilde{p}_2 \tilde{z}_2 + \cdots + \tilde{p}_s \tilde{z}_s\big]. \tag{27}$$

Since $p_1 = [\mathbf{1}_m^T, \mathbf{0}_m^T]^T$, $p_1 z_1$ in (27) can be eliminated. By (17), we have

$$\begin{aligned} \left\|\dot{e}_l^r(t)\right\| &\leq \|D_{rl}\| \sum_{g=2}^{s} \left\|\tilde{p}_g\right\| \left\|\tilde{z}_g\right\| \\ &= \|D_{rl}\| \sum_{g=2}^{s} c_g \left\|\tilde{p}_g\right\| \Bigg(e^{\frac{\mathrm{Re}(\lambda_g)}{2} t} \left\|\tilde{z}_g(0)\right\| + \left\|\tilde{q}_g^T B_1\right\| \\ &\qquad \times \Bigg(\sum_{l=1}^{p_r} \Phi_l^r \Big(e^{\gamma t} - e^{\frac{\mathrm{Re}(\lambda_g)}{2} t} \Big) \\ &\qquad + \sum_{i=1}^{m} \Phi_i^r \Big(e^{\gamma t} - e^{\frac{\mathrm{Re}(\lambda_g)}{2} t} \Big) \Bigg) \Bigg) \\ &\leq \rho e^{\gamma t} \end{aligned} \tag{28}$$

where $\rho = \|D_{rl}\| \sum_{g=2}^{s} c_g \|\tilde{p}_g\| (\|\tilde{z}_g(0)\| + \|\tilde{q}_g B_1\| (\sum_{l=1}^{p_r} \Phi_l^r + \sum_{i=1}^{m} \Phi_i^r))$. Furthermore, for $t \in [t^*, t')$, we have

$$\left\|e_l^r(t)\right\| \leq \int_{t^*}^{t} \left\|\dot{e}_l^r(s)\right\| ds \leq \int_{t^*}^{t} \rho e^{\gamma s} ds. \tag{29}$$

The next event will not be triggered until $\|e_l^r(t)\| \geq \sigma_l^\epsilon e^{\gamma t}$, combining with (29), a lower bound of the interevent time can be given by solving the equation

$$\int_{t^*}^{t^*+\tau} \rho e^{\gamma s} ds = \sigma_l^\epsilon e^{\gamma (t^* + \tau)}$$

which gives $\tau = ([\ln \rho\gamma - \ln(\rho\gamma - \sigma_l^\epsilon)]/\gamma)$. Note that, τ is independent of the event time t^*. Thus, for all $t > 0$, the interevent time of edge l among leaders is lower bounded by a positive constant, which means that Zeno behavior can be avoided.

We now give a lower bound of the interevent time of velocity measurement errors of leaders. Without loss of generality, we take the example of agent i. Assuming two adjacent event instants of leader i are t^* and t', similar to the discussion of edge l, we have the dynamics of the measurement error

$$\dot{e}_i^{rv}(t) = -\dot{v}_i^r(t). \tag{30}$$

Substituting (10) and (12) into (30), we have

$$\begin{aligned} \dot{e}_i^{rv}(t) = {}& [-L_{ri}, -k_r I_{mi}] \big[p_1 z_1 + \cdots + \tilde{p}_s \tilde{z}_s\big] \\ & + \big[-\overline{D}_{ri}, -k_r I_{mi}\big] e_r \end{aligned} \tag{31}$$

where L_{ri}, $\overline{D}_{ri}$, and I_{mi} represent the ith row of L_r, $\overline{D}_r$, and I_m, respectively. Since the row sum of L_r is zero, and $p_1 = [\mathbf{1}_m^T, \mathbf{0}_m^T]$, $p_1 z_1$ can be eliminated in (31). By (17) and trigger condition (9), we have

$$\begin{aligned} \left\|\dot{e}_i^{rv}(t)\right\| \leq {}& \|L_{ri}, k_r I_{mi}\| \sum_{g=2}^{s} \left\|\tilde{p}_g\right\| \left\|\tilde{z}_g\right\| \\ & + \left\|\overline{D}_{ri}, k_r I_{mi}\right\| \big(\left\|e_L^r\right\| + \left\|e_v^r\right\| \big). \end{aligned} \tag{32}$$

Substituting (15) into (32), we have

$$\begin{aligned} \left\|\dot{e}_i^{rv}(t)\right\| &\leq \|L_{ri}, k_r I_{mi}\| \sum_{g=2}^{s} \left\|\tilde{p}_g\right\| \left\|\tilde{z}_g\right\| \\ &\quad + \left\|\overline{D}_{ri}, k_r I_{mi}\right\| \left(\sum_{l=1}^{p_r} \sigma_l^\epsilon e^{\gamma s} + \sum_{i=1}^{m} \sigma_i^v e^{\gamma s} \right) \\ &\leq \rho_v e^{\gamma t} \end{aligned}$$

where

$$\begin{aligned} \rho_v = {}& \|L_{ri}, k_r I_{mi}\| \\ & \times \sum_{g=2}^{s} c_g \left\|\tilde{p}_g\right\| \left(\left\|\tilde{z}_g(0)\right\| + \left\|\tilde{q}_g^T B_1\right\| \left(\sum_{l=1}^{p_r} \Phi_l^r + \sum_{i=1}^{m} \Phi_i^r \right) \right) \\ & + \left\|\overline{D}_{ri}, k_r I_{mi}\right\| \left(\sum_{l=1}^{p_r} \sigma_l^\epsilon + \sum_{i=1}^{m} \sigma_i^v \right). \end{aligned}$$

Similar to the proof for edge l, the lower bound for agent i can be given by $\tau = ([\ln \rho_v\gamma - \ln(\rho_v\gamma - \sigma_i^v)]/\gamma)$. Thus, we come to the conclusion that the trigger condition of the velocity measurement error of leaders under control protocol (5) will not cause Zeno behavior. Similarly, the lower bound of the interevent times for followers can be acquired.

Finally, we come to the conclusion that Zeno behavior can be avoided in the system. ∎

IV. Discontinuous Detection

In this section, we will propose a model-based event-triggered strategy based on the basic event-triggered strategy discussed previously to avoid continuous detection among the MAS.

According to the energy consumption of the on-board sensors, the control input (5) can be divided into two parts: the one related to edge information, which can only be acquired through active detection of sensors (here, the on-board sensor needs to measure the signal transmitted by itself and reflected by the involved neighbor) and the one related to each agent's own velocity, which only depends on the imbedded speed sensor, sparing the need of active detection. In order to avoid continuous detection, for each edge, a predictive model is established, which can give an estimation of the measurement error based on the previously received information. The measurement action is only carried out at the instant when the estimated errors exceed their thresholds. At the same time, the controller is updated according to the new measurement information, and the broadcasting action is executed to inform other agents about the change in its controller.

For each agent i, define a set $T_i(t) = \{t^{\epsilon_l}_{k_{\epsilon_l}}, t^{v_i}_{k_{v_i}} | k_{\epsilon_l} = 1, 2, \ldots, k_{v_i} = 1, 2, \ldots, \bar{d}_{il} = 1\}$. Arranging the elements in $T_i(t)$ in ascending order, gets a new time sequence t^i_0, $t^i_1, \ldots$, which represents all the event time instants related to agent i. Furthermore, for each unordered pair (i, j), connected by an edge, i.e., $a_{ij} = 1$ and/or $a_{ji} = 1$, define a set $T_{(i,j)}(t) = \{t^i_{k_i}, t^j_{k_j} | k_i = 1, 2, \ldots, k_j = 1, 2, \ldots\}$. Arranging the elements in $T_{(i,j)}(t)$ in ascending order, gets a new time sequence $t^{(i,j)}_0, t^{(i,j)}_1, \ldots$, which represents all the event instants related to agent i or agent j.

Note that the subscript (i, j) does not represent any oriented edge. Since the topology is directed, if agent i and agent j are connected, there might be only one edge or two edges with different directions between them, i.e., $(v_i, v_j) \in \mathcal{E}$ and/or $(v_j, v_i) \in \mathcal{E}$. The unordered pair (i, j) only represents a connected relation between the two agents.

Now, we are ready to present the predictive model for edge measurement error.

For edge l, directed from agents j to i, and for $t \in [t^{(i,j)}_{k'}, t^{(i,j)}_{k'+1})$, define auxiliary variables C_l, V_l as follows:

$$C_l(t) = u_i\left(t^i_{k_i}\right) - u_j\left(t^j_{k_j}\right) \tag{33}$$

$$V_l(t) = V_l\left(t^{(i,j)}_{k'}\right) + C_l\left(t - t^{(i,j)}_{k'}\right). \tag{34}$$

For better understanding, consider C_l as estimated relative acceleration, V_l as estimated relative velocity. By the Newton's law of motion, the estimated change of relative distance can be written as

$$\begin{aligned} E_l(t) = {}& V_l\left(t^{(i,j)}_{k'}\right)\left(t - t^{(i,j)}_{k'}\right) + 1/2 C_l\left(t^{(i,j)}_{k'}\right)\left(t - t^{(i,j)}_{k'}\right)^2 \\ &+ E_l\left(t^{(i,j)}_{k'}\right). \end{aligned} \tag{35}$$

Note that $E_l(t)$ represents the estimated change of relative distance, which is in fact the estimated measurement error. Thus, (35) is the predictive model for edge l we need.

Substitute (35) into (6). During $t \in [t^{(i,j)}_{k'}, t^{(i,j)}_{k'+1})$, the trigger function can be written in the following form:

$$\begin{aligned} f_l(t) = {}& \left| V_l\left(t^{(i,j)}_{k'}\right)\left(t - t^{(i,j)}_{k'}\right) + 1/2 C_l\left(t^{(i,j)}_{k'}\right)\left(t - t^{(i,j)}_{k'}\right)^2 \right. \\ &\left. + E_l\left(t^{(i,j)}_{k'}\right)\right| - \sigma^{\epsilon}_l e^{\gamma t} \end{aligned} \tag{36}$$

and the trigger condition is given by the violation of

$$f_l(t) \leq 0. \tag{37}$$

Based on (37), for edge l, directed from agents j to i, the generation of event instants $t^{\epsilon_l}_{k_{\epsilon_l}}$, $t^{v_i}_{k_{v_i}}$, $t^i_{k_i}$, and $t^{(i,j)}_{k'}$ can be given as in Algorithm 1.

Under Algorithm 1, information only needs to be transmitted to inform others about the change of its dynamics when the input to an agent, for example, agent i, is changed. Every agent considering agent i as a neighbor will receive this information and change the parameters in their predictive model accordingly. Note that, the times of information broadcasting of each agent is exactly the same with the times of its controller update. Fortunately, for each agent i, u_i is piecewise constant, and thus, the data transmission among the MAS is discontinuous. The data transmission only happens when an estimated measurement error exceeds its threshold, which forces u_i to be changed.

More specifically, $t^i_{k_i}$ represents the time instant when u_i is changed. There are two types of events responsible for the change of u_i, namely, the event related to the edge directed to agent i at $t^{\epsilon_w}_{k_{\epsilon_w}}$ or the event related to the velocity measurement error of agent i at $t^{v_i}_{k_{v_i}}$. Meanwhile, $t^{(i,j)}_{k'}$ represents the time

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Algorithm 1 Model-Based Edge-Event-Triggered Control Algorithm for Edge l

Initialization

At time instant $t = 0$

a) Initialize $k_{v_i} = k_{\epsilon_l} = k_i = k' = 0$, $t^{v_i}_0 = t^i_0 = t^{(i,j)}_0 = t^{\epsilon_l}_0 = 0$, $T = t_f$, $E_l(t^{(i,j)}_0) = 0$
b) Measure $v_i(0)$, $v_j(0)$, $\{y_w(0) | \bar{d}_{iw} = 1\}$
c) Compute $u_i(t^i_0)$ by (5), broadcast $u_i(t^i_0)$ and receive $u_j(t^j_0)$
d) Compute $C_l(t^{(i,j)}_0)$, $V_l(t^{(i,j)}_0)$ by (33), (34), respectively.

Iteration

When $0 \leq t < T$ *Step* 1:

a) For agent i, compute $C_l(t)$, $V_l(t)$, and $E_l(t)$ by (33) (34) (35), respectively, while listening to possible information transmitted from its neighbors.
b) If inequality related to the measurement error of edge l (37) is satisfied a1:
   i) If agent i receives information u_j broadcasted by agent j through edge l at time $t^j_{k_j}$, then, $k' = k' + 1$, $t^{(i,j)}_{k'} \leftarrow t^j_{k_j}$; update the parameters in $C_l(t)$, $V_l(t)$ and $E_l(t)$ by (33) (34) and (35), respectively.
   ii) If other edge w directed to agent i is triggered at time $t^{\epsilon_w}_{k_{\epsilon_w}}$, then, $k_i = k_i + 1$, $k' = k' + 1$; $t^i_{k_i} \leftarrow t^{\epsilon_w}_{k_{\epsilon_w}}$, $t^{(i,j)}_{k'} \leftarrow t^{\epsilon_w}_{k_{\epsilon_w}}$; update $u_i(t^i_{k_i})$ by (5), update the parameters in $C_l(t)$, $V_l(t)$, $E_l(t)$ by (33) (34) and (35), respectively.
   iii) If inequality related to velocity measurement error (9) is violated at time t^*, then, $k_{v_i} = k_{v_i} + 1$, $k_i = k_i + 1$, $k' = k' + 1$; $t^{v_i}_{k_{v_i}} \leftarrow t^*$, $t^i_{k_i} \leftarrow t^*$, $t^{(i,j)}_{k'} \leftarrow t^*$; ; update $u_i(t^i_{k_i})$ by (5), broadcast u_i; update the parameters in C_l, V_l, and E_l by (33) (34) and (35), respectively.
c) If inequality related to measurement error of edge l (37) is violated at time t^*, then, $k_{\epsilon_l} = k_{\epsilon_l} + 1$, $k_i = k_i + 1$, $k' = k' + 1$; $t^{\epsilon_l}_{k_{\epsilon_l}} \leftarrow t^*$, $t^i_{k_i} \leftarrow t^*$, $t^{(i,j)}_{k'} \leftarrow t^*$; measure the edge information $y_l(t^{\epsilon_l}_{k_{\epsilon_l}})$; update $u_i(t^i_{k_i})$ by (5); broadcast u_i; reset $E_l(t^{(i,j)}_{k'}) = 0$

Finish

When $t \geq T$, jump out of the algorithm

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instant when the control input to any agent in the connected unordered pair (i, j) is changed, i.e., the time $t^i_{k_i}$ when u_i is changed or the time $t^j_{k_j}$ when u_j is changed.

Therefore, applying the predictive model established in this section, each agent can decide by itself when to broadcast and request information, and the continuous detection can be avoided.

Based on the discussion above, we give the following results on model-based edge-event-triggered containment for the MAS described in (2).

Theorem 2: Under Assumption 1, with control input (5), Algorithm 1 can solve the containment problem of the MAS described in (2), if the feedback gains k_r and k_f in (5) and constants σ^v_i, σ^ϵ_l, and γ in (7), (36) satisfy the conditions in Theorem 1. More specifically, the leaders can converge to a formation, and the followers can converge to the convex hull spanned be leaders. Zeno behavior is excluded from the system.

Proof: The main differences between the MAS under basic event-triggered strategy in Section III and under the model-based strategy in this section are the different mechanisms adopted to generate event instants. Thus, in order to prove the convergence of the MAS under Algorithm 1, we will first show that, the event instants generated by the two mechanisms are exactly the same. Then, by similar arguments to those in Theorem 1, the conclusion can be easily reached.

The generation of an event is decided by the violation of (37) or (9) in Algorithm 1, and it is decided by (8) or (9) based on the basic event-triggered strategy. The identical time sequence generated by the two mechanisms will be guaranteed if we can prove the equivalence of (8) and (37).

Without loss of generality, we take edge l directed from agent j to agent i for an example. For the basic event-triggered strategy, given an initial time instant $t^{\epsilon_l}_0$, for $t \in [t^{\epsilon_l}_0, t^{\epsilon_l}_1)$, under control protocol (5), the dynamics of $e^\epsilon_l(t)$ can be written as

$$\begin{aligned} e^\epsilon_l(t) &= \hat{y}_l(t) - y_l(t) \\ &= y_l\left(t^i_0\right) - \left(\left(x_i\left(t^i_0\right) + \int_{t^i_0}^t v_i(s)ds\right)\right. \\ &\quad \left.- \left(x_j\left(t^j_0\right) + \int_{t^j_0}^t v_j(s)ds\right)\right) \\ &= v_i\left(t^i_0\right)\left(t - t^i_0\right) + \int_{t^i_0}^t \left(\int_{t^i_0}^s u_i\left(s'\right)ds'\right)ds \\ &\quad - \left(v_j\left(t^j_0\right)\left(t - t^j_0\right) + \int_{t^j_0}^t \left(\int_{t^j_0}^s \left(u_j\left(s'\right)ds'\right)ds\right)\right). \end{aligned} \tag{38}$$

Since control input (5) is piecewise constant, with time sequence $t^{(i,j)}_0$, $t^{(i,j)}_1, \cdots$ recording every instant when u_i or u_j changes, without loss of generality, assume u_i and u_j have changed n_l times before next event instant $t^{\epsilon_l}_1$, and all agents and edges are initialized at the same instant $t = 0$. Thus, for $t \in [t^{(i,j)}_0, t^{(i,j)}_1)$, we have

$$\begin{aligned} e^\epsilon_l(t) &= \left(v_i(0) - v_j(0)\right)\left(t - t^{(i,j)}_0\right) \\ &\quad + \frac{1}{2}\left(u_i(0) - u_j(0)\right)\left(t - t^{(i,j)}_0\right)^2. \end{aligned} \tag{39}$$

Substituting (33) and (34) into (39), we have

$$e^\epsilon_l(t) = V_l\left(t^{(i,j)}_0\right)\left(t - t^{(i,j)}_0\right) + 1/2C_l\left(t^{(i,j)}_0\right)\left(t - t^{(i,j)}_0\right)^2. \tag{40}$$

Similarly, before $t^{\epsilon_l}_1$, for $t \in [t^{(i,j)}_{k'}, t^{(i,j)}_{k'+1})$, we have

$$\begin{aligned} e^\epsilon_l(t) &= V_l\left(t^{(i,j)}_{k'}\right)\left(t - t^{(i,j)}_{k'}\right) \\ &\quad + 1/2C_l\left(t^{(i,j)}_{k'}\right)\left(t - t^{(i,j)}_{k'}\right)^2 + e^\epsilon_l\left(t^{(i,j)}_{k'}\right). \end{aligned} \tag{41}$$

When $t = t^{\epsilon_l}_1$, an event of edge l occurs, edge information y_l is measured, and measurement error e_l is reset 0.

With similar analysis, at every time $t \in [t^{\epsilon_l}_{k_{\epsilon_l}}, t^{\epsilon_l}_{k_{\epsilon_l}+1})$, the measurement error has the same expression as in (41).

Substituting (41) into (8), it is easy to find that (8) and (37) share exactly the same dynamics. Thus, the equivalence of the event instants generated by the two mechanisms is guaranteed.

Then, referring to the proof of Theorem 1, the convergence and Zeno-free behavior of the MAS can be guaranteed. ■

Remark 1: Referring to trigger function (6), in order to check the trigger condition, each agent needs to monitor the edge information continuously. To solve this problem, a model-based strategy is introduced, which replaces the continuous detection by discrete-time information broadcasting. It should be noted that the information broadcasting only happens when the control input to an agent is changed. Since the control input is piecewise constant, the broadcasting is discontinuous. Compared to the continuous detection, the energy consumption is reduced.

In order to further reduce energy consumption of the microprocessor, we introduce the following trigger rules.

1) When the edge measurement error function is triggered, update the edge information as well as the velocity information
2) When the velocity measurement error is triggered, only update the velocity information

Combining the trigger rules with Algorithm 1, the times of controller update can be reduced without bringing any extra measurement action.

V. Simulation

In this section, a simulation example will be used to further illustrate the effectiveness of the model-based edge-event-triggered strategy.

Consider a MAS with four leaders labeled with r_1–r_4, and six followers labeled with f_1–f_6. The communication topology of the MAS is shown in Fig. 2. It is obvious that the topology is directed and contains a spanning tree. The dynamics of the system is described as in (2). By Theorem 1, the feedback gains of the control input (5) are chosen as $k_r = 2.5$ and $k_f = 2.5$. For simplicity, the constants in trigger function (6) and (7) are chosen identically as $\sigma^\epsilon_l = \sigma^v_i = 5$ and $\gamma = -0.2$. The expected formation of leaders is $h = [-4, -2, 2, 4]$, and the initial states of the leaders and followers are $x_r = [0, 0, 0, 0]$, $v_r = [0, 0, 0, 0]$, $x_f = [-5, -3, -1, 1, 3, 5]$, and $v_f = [0, 0, 0, 0, 0, 0]$, respectively. Applying Algorithm 1, the

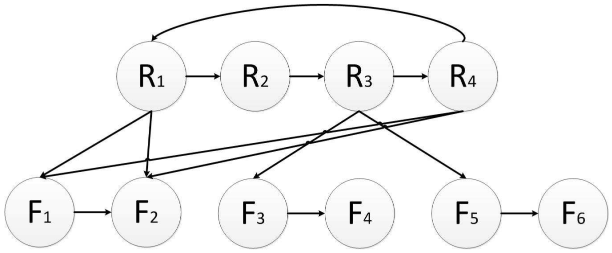

Fig. 2. Communication topology of MAS.

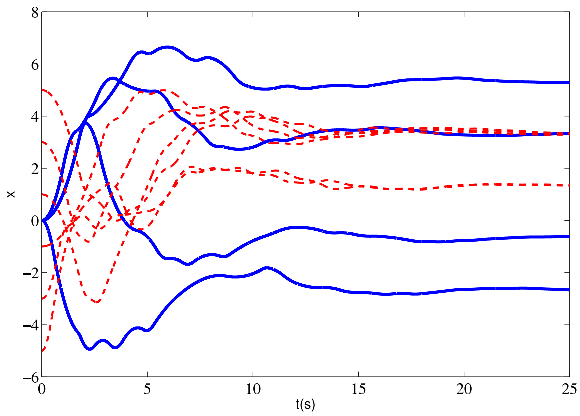

Fig. 3. Position trajectory of each agent under control protocol (5). The solid and dashed lines denote the position trajectories of leaders and followers, respectively.

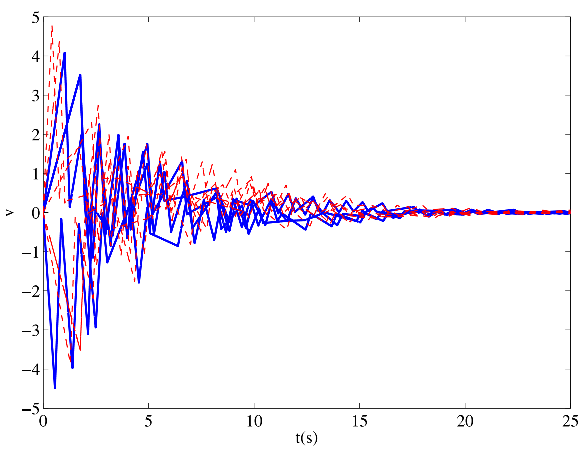

Fig. 4. Velocity of each agent under control protocol (5). The solid and dashed lines denote the velocities of leaders and followers, respectively.

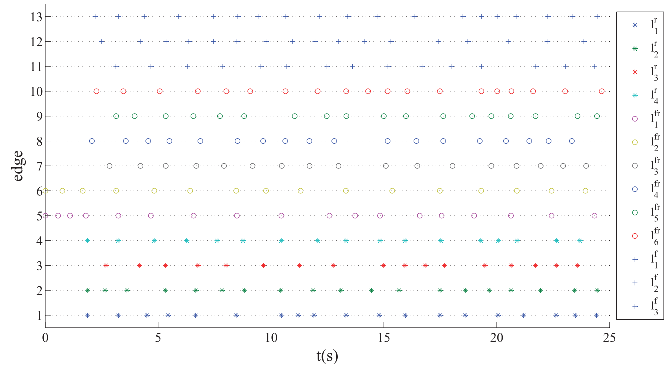

Fig. 5. Event times for each edge generated by edge trigger function under control protocol (5).

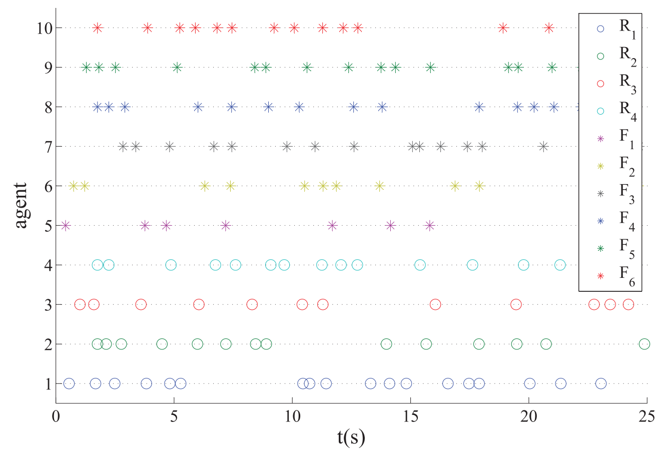

Fig. 6. Event times for each agent generated by velocity trigger function under control protocol (5).

TABLE I
EVENT INTERVALS OF EACH EDGE AMONG LEADERS

Edge	l_1^r	l_2^r	l_3^r	l_4^r
Event times	19	19	17	17
Max interval	2.00	1.82	2.23	1.77
Min interval	0.70	0.77	0.86	0.78
Mean interval	1.25	1.25	1.31	1.36

TABLE II
EVENT INTERVALS OF EACH EDGE DIRECTED FROM LEADERS TO FOLLOWERS

Edge	l_1^{fr}	l_2^{fr}	l_3^{fr}	l_4^{fr}	l_5^{fr}	l_6^{fr}
Event times	18	16	17	18	18	19
Max interval	2.13	2.11	2.10	2.37	2.24	1.84
Min interval	0.53	0.75	0.99	0.86	0.82	0.63
Mean interval	1.43	1.59	1.31	1.25	1.25	1.24

trajectories of agents are depicted in Fig. 3, which illustrates that the containment problem of the MAS has been solved. Fig. 4 shows the trajectories of velocities, in which all the agents asymptotically converge to zero state as time increases. Figs. 5 and 6, respectively, show the edge event times and the velocity event times under control protocol (5). Some statistical properties of the event times are presented in Tables I–V. The existence of a minimal interevent times for each edge and each agent implies that the Zeno behavior is avoided.

Moreover, Fig. 7 shows the control input to each agent. It should be noted that, under Algorithm 1, whenever a controller is updated, the corresponding agent will broadcast information at that time instant.

To further illustrate the exclusion of continuous communication, the information broadcasting times of leaders and followers are shown in Tables VI and VII, respectively.

TABLE III
EVENT INTERVALS OF EACH EDGE AMONG FOLLOWERS

Edge	l_1^f	l_2^f	l_3^f
Event times	15	17	19
Max interval	2.41	2.10	2.16
Min interval	1.13	0.99	0.70
Mean interval	1.52	1.34	1.24

TABLE IV
EVENT INTERVALS OF LEADERS

Agent	r_1	r_2	r_3	r_4
Event times	18	14	12	15
Max interval	5.17	5.08	4.76	2.78
Min interval	0.28	0.37	0.59	0.48
Mean interval	1.32	1.78	2.11	1.59

TABLE V
EVENT INTERVALS OF FOLLOWERS

Agent	f_1	f_2	f_3	f_4	f_5	f_6
Event times	8	12	16	16	16	14
Max interval	8.51	6.01	2.59	4.10	3.30	6.14
Min interval	0.90	0.47	0.30	0.48	0.41	0.62
Mean interval	3.41	2.17	1.45	1.47	1.49	1.71

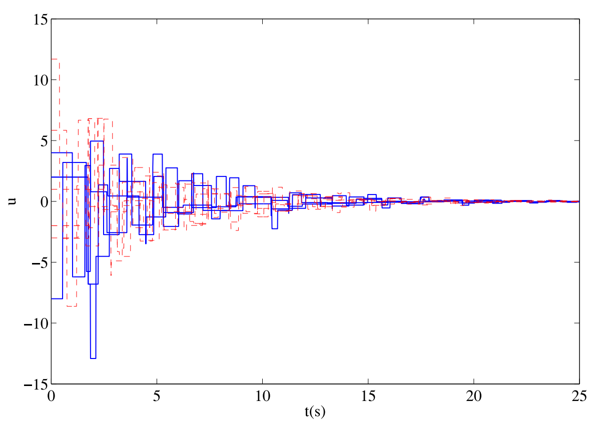

Fig. 7. Control input to each agent under protocol (5). The solid and dashed lines denote the control input to leaders and followers, respectively.

TABLE VI
INFORMATION BROADCASTING TIMES OF LEADERS

Agent	R_1	R_2	R_3	R_4
broadcasting times	35	33	31	32

TABLE VII
INFORMATION BROADCASTING TIMES OF FOLLOWERS

Agent	F_1	F_2	F_3	F_4	F_5	F_6
broadcasting times	59	47	33	33	34	33

VI. CONCLUSION

An edge-event-triggered containment control problem has been studied in this paper, where the behaviors of both leaders and followers have been considered, asymptotic convergence of the MAS can be guaranteed. A control protocol based on edge information has been proposed, under which each agent in the MAS only needs to transmit relative information and the controller only needs to be updated at its own event time. A lower bound of interevent time is given, which implies the Zeno behavior can be excluded. Furthermore, in order to avoid continuous detection, a model-based strategy is introduced, under which, each agent only needs to broadcast information when its control input is changed. Therefore, the contradictive tradeoff of energy conservation between actuators and sensors is solved. The effectiveness of the control protocol has been further illustrated by simulations.

The future work will focus on the event-triggered distributed cooperative control of the systems with time delays and model uncertainty. The robustness of the control protocol will be further studied.

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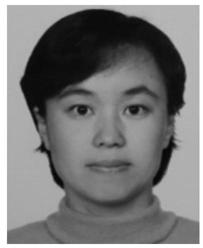

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