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Adaptive leader-following rendezvous and flocking for a class of uncertain second-order nonlinear multi-agent systems

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Abstract

In this paper, we study the leader-following rendezvous and flocking problems for a class of second-order nonlinear multi-agent systems, which contain both external disturbances and plant uncertainties. What differs our problems from the conventional leader-following consensus problem is that we need to preserve the connectivity of the communication graph instead of assuming the connectivity of the communication graph. By integrating the adaptive control technique, the distributed observer method and the potential function method, the two problems are both solved. Finally, we apply our results to a group of van der Pol oscillators.

Keywords: Adaptive control, connectivity preservation, multi-agent systems, nonlinear systems

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1 Introduction

Over the past few years, the study of cooperative control problems for multi-agent systems has attracted extensive attention. In many cooperative control problems such as the consensus problem, the communication graph is predefined and has to satisfy certain connectivity assumption [1–5]. However, in some real applications such as rendezvous problem and flocking problem, the communication graph is defined by the distance of various agents, and is thus state-dependent. It is more practical to enable a control law to not only achieve consensus but also preserve the connectivity of the graph instead of assuming the connectivity of the graph. Such a problem is called rendezvous with connectivity preservation problem. If the objective of collision avoidance is also imposed, then the problem can be further called flocking.

Depending on whether or not a multi-agent system has a leader, the rendezvous/flocking problem can be further divided into two classes: leaderless and leader-

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following. The leaderless rendezvous/flocking problem aims to make the state (or partial state) of all agents approach a same location, while the leader-following rendezvous/flocking problem further requires the state (or partial state) of all agents to track a desired trajectory generated by some leader system. The leaderless rendezvous/flocking problem has been studied for single-integrator multi-agent systems in [6–8] and double-integrator multi-agent systems in [9–11] while the leader-following rendezvous/flocking problem has also been studied for single-integrator multi-agent systems in [12, 13] and double-integrator multi-agent systems in [10, 13–16].

More recently, the leader-following rendezvous/flocking problem has been further studied for some second-order nonlinear multi-agent systems under various assumptions in [17–20]. Specifically, in [17], the connectivity preserving leader-following consensus problem for uncertain Euler-Lagrange multi-agent systems is studied. In [18], the differences between the nonlinear functions of all agents are assumed to be bounded for all time. In [19, 20], the nonlinear functions are assumed to satisfy global Lipschitz-like condition and all followers know the information of the virtual leader.

In this paper, we will study both the leader-following rendezvous problem and the leader-following flocking problem for a class of second-order nonlinear multi-agent systems by a distributed state feedback control law with different potential functions. Our problems differ from existing works in at least two aspects. First, our system as given in next section is subject to not only external disturbances but also plant uncertainties. Second, the nonlinear functions in our system do not have to satisfy some bounded condition or global Lipschitz-like condition. To overcome these difficulties, we need to combine the adaptive control technique, the distributed observer method and the potential function method to solve our problems.

The rest of this paper is organized as follows. In Section 2, we give two problem formulations and some preliminaries. In Sections 3 and 4, we give the main results. In Section 5, we provide an example to illustrate our design. Finally, in Section 6, we conclude the paper with some remarks. It is noted that the preliminary version of this paper without any proof was presented in [21].

Notation For any column vectors a_i, $i = 1, \dots, s$, denote $\mathrm{col}(a_1, \dots, a_s) = [a_1^{\mathrm{T}}, \dots, a_s^{\mathrm{T}}]^{\mathrm{T}}$. $\otimes$ denotes the Kronecker product of matrices. $\|x\|$ denotes the Euclidean norm of vector x. $\|A\|$ denotes the induced norm of matrix A by the Euclidean norm. For any real symmetric matrix A, $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and maximum eigenvalues of A, respectively. For any two symmetric matrices A and B, the symbol $A \geqslant B$ means the matrix $A - B$ is positive semi-definite.

2 Problem formulation

Consider a class of second-order nonlinear multi-agent systems as follows:

$$\begin{cases} \dot{q}_i = p_i, \\ \dot{p}_i = f_i^{\mathrm{T}}(q_i, p_i)\theta_i + d_i(w) + u_i, \quad i = 1, \dots, N, \end{cases} \tag{1}$$

where $q_i, p_i \in \mathbb{R}^n$ are the states, $u_i \in \mathbb{R}^n$ is the input, $f_i(q_i, p_i) \in \mathbb{R}^{m \times n}$ is a known matrix with every element being continuous function, $\theta_i \in \mathbb{R}^m$ is an unknown constant parameter vector, $d_i(w) \in \mathbb{R}^n$ denotes the disturbance with $d_i(\cdot)$ being some C^1 function, and w is generated by the linear exosystem as follows:

$$\dot{w} = S_{\mathrm{b}} w \tag{2}$$

with $w \in \mathbb{R}^{n_w}$ and $S_{\mathrm{b}} \in \mathbb{R}^{n_w \times n_w}$. It is assumed that the reference signal is generated by the following linear exosystem

$$\dot{x}_0 = S_{\mathrm{a}} x_0, \tag{3}$$

where $S_{\mathrm{a}} = \begin{bmatrix} 0_{n\times n} & I_n \\ S_{\mathrm{a1}} & S_{\mathrm{a2}} \end{bmatrix} \in \mathbb{R}^{2n\times 2n}$ with $S_{\mathrm{a1}}, S_{\mathrm{a2}} \in \mathbb{R}^{n\times n}$ and $x_0 = \mathrm{col}(q_0, p_0)$ with $q_0, p_0 \in \mathbb{R}^n$. Let $v = \mathrm{col}(x_0, w) \in \mathbb{R}^{n_v}$ and $S = \mathrm{diag}\{S_{\mathrm{a}}, S_{\mathrm{b}}\} \in \mathbb{R}^{n_v \times n_v}$ with $n_v = 2n + n_w$. Then, we can put (2) and (3) into the following form:

$$\dot{v} = Sv. \tag{4}$$

The plant (1) and the exosystem (4) together can be viewed as a multi-agent system of $(N + 1)$ agents with (4) as the leader and the N subsystems of (1) as N followers. As in [15, 17], define a time-varying graph $\bar{\mathcal{G}}(t) = (\bar{\mathcal{V}}, \bar{\mathcal{E}}(t))$ with respect to (1) and (4), where $\bar{\mathcal{V}} = \{0, 1, \dots, N\}$ with 0 associated with the leader system and with $i = 1, \dots, N$ associated with the N followers, respectively, and $\bar{\mathcal{E}}(t) \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$ is defined by different rules for rendezvous and flocking problem. The graph $\bar{\mathcal{G}}(t)$ is said to be connected at time t if there is a directed path from node 0 to every other node.

Remark 1 Compared with the second-order nonlinear systems studied in [18–20], our system contains not only the external disturbances but also the parameter uncertainties, and the boundaries of the uncertainties are allowed to be arbitrarily large, while the systems in [18, 20] contain neither external disturbances nor plant uncertainties, and the system in [19] contains only plant uncertainties but no external disturbances. Moreover, the nonlinear function f_i in (1) does not need to be bounded as assumed in [18], or satisfy the global Lipschitz-like condition as assumed in [19, 20].

2.1 Leader-Following Rendezvous Problem

For leader-following rendezvous problem, $\bar{\mathcal{E}}(t)$ is defined by the following rules: Given any $r > 0$ and $\epsilon \in (0, r)$, for any $t \geqslant 0$, $\bar{\mathcal{E}}(t) = \{(i, j) \,|\, i, j \in \bar{\mathcal{V}}, i \neq j\}$ is defined such that

1) $\bar{\mathcal{E}}(0) = \{(i, j) \,|\, \|q_i(0) - q_j(0)\| < r - \epsilon, i = 0, 1, \dots, N, j = 1, \dots, N\}$;

2) for $i = 0, 1, \dots, N$, $j = 1, \dots, N$, if $\|q_i(t) - q_j(t)\| \geqslant r$, then $(i, j) \notin \bar{\mathcal{E}}(t)$;

3) for $i = 0, 1, \dots, N$, $(i, 0) \notin \bar{\mathcal{E}}(t)$;

4) for $i = 0, 1, \dots, N$, $j = 1, \dots, N$, if $(i, j) \notin \bar{\mathcal{E}}(t^-)$ and $\|q_i(t) - q_j(t)\| < r - \epsilon$, then $(i, j) \in \bar{\mathcal{E}}(t)$;

5) for $i = 0, 1, \dots, N$, $j = 1, \dots, N$, if $(i, j) \in \bar{\mathcal{E}}(t^-)$ and $\|q_i(t) - q_j(t)\| < r$, then $(i, j) \in \bar{\mathcal{E}}(t)$.

Note that the above rules are similar to those in [15]. We denote the neighbor set of the ith agent at time t by $\bar{\mathcal{N}}_i(t) = \{j \,|\, (j, i) \in \bar{\mathcal{E}}(t)\}$. Then, we consider a control law of the following form:

$$\begin{cases} u_i = h_i(q_i, p_i, \zeta_i, q_j, p_j, \zeta_j, j \in \bar{\mathcal{N}}_i(t)), \\ \dot{\zeta}_i = l_i(q_i, p_i, \zeta_i, q_j, p_j, \zeta_j, j \in \bar{\mathcal{N}}_i(t)), \end{cases} \tag{5}$$

where h_i and l_i are some nonlinear functions, and $\zeta_i \in \mathbb{R}^{n_{\zeta_i}}$ with n_{ζ_i} to be defined later. A control law of the form (5) is called a dynamic distributed state feedback control law, since u_i only depends on the state information of its neighbors and itself. Then, we define the leader-following rendezvous problem for system (1) as follows.

Problem 1 Given the plant (1), the exosystem (4), any $r > 0$ and $\epsilon \in (0, r)$, find a distributed control law of the form (5), such that, for any $w \in \mathscr{W}$ with $\mathscr{W}$ being some compact subset of $\mathbb{R}^{n_w}$ and any initial condition $q_i(0), i = 0, 1, \dots, N$, making $\bar{\mathcal{G}}(0)$ connected, the closed-loop system composed of (1) and (5) has the following properties:

1) $\bar{\mathcal{G}}(t)$ is connected for all $t \geqslant 0$;

2) $\lim\limits_{t\to\infty}(q_i - q_0) = 0$ and $\lim\limits_{t\to\infty}(p_i - p_0) = 0$ for $i = 1, \dots, N$.

2.2 Leader-following flocking problem

For leader-following flocking problem, $\bar{\mathcal{E}}(t)$ is defined by the following rules: Given any $r > 0$, $\epsilon \in (0, r)$ and $R \in [0, r - \epsilon)$, for any $t \geqslant 0$, $\bar{\mathcal{E}}(t) = \{(i, j) | i, j \in \bar{\mathcal{V}}, i \neq j\}$ is defined such that

1) $\bar{\mathcal{E}}(0) = \{(i, j) \,|\, R < \|q_i(0) - q_j(0)\| < r - \epsilon, i = 0, 1, \dots, N, j = 1, \dots, N\}$;

2) for $i = 0, 1, \dots, N$, $j = 1, \dots, N$, if $\|q_i(t) - q_j(t)\| \geqslant r$, then $(i, j) \notin \bar{\mathcal{E}}(t)$;

3) for $i = 0, 1, \dots, N$, $(i, 0) \notin \bar{\mathcal{E}}(t)$;

4) for $i = 0, 1, \dots, N$, $j = 1, \dots, N$, if $(i, j) \notin \bar{\mathcal{E}}(t^-)$ and $R < \|q_i(t) - q_j(t)\| < r - \epsilon$, then $(i, j) \in \bar{\mathcal{E}}(t)$;

5) for $i = 0, 1, \dots, N$, $j = 1, \dots, N$, if $(i, j) \in \bar{\mathcal{E}}(t^-)$ and $R < \|q_i(t) - q_j(t)\| < r$, then $(i, j) \in \bar{\mathcal{E}}(t)$.

Note that the above rules are similar to those in Section IV of [17]. Then, we define the leader-following flocking problem for system (1) as follows.

Problem 2 Given the plant (1), the exosystem (4), any $r > 0$, $\epsilon \in (0, r)$ and $R \in [0, r - \epsilon)$, find a distributed control law of the form (5), such that, for any $w \in \mathscr{W}$ with $\mathscr{W}$ being some compact subset of $\mathbb{R}^{n_w}$ and any initial condition $q_i(0)$, $i = 0, 1, \dots, N$, satisfying $\|q_i(0) - q_j(0)\| > R$ for $i \neq j$, $i, j = 0, 1, \dots, N$, and making $\bar{\mathcal{G}}(0)$ connected, the closed-loop system composed of (1) and (5) has the following properties:

1) $\bar{\mathcal{G}}(t)$ is connected for all $t \geqslant 0$;

2) $\lim\limits_{t\to\infty}(p_i - p_0) = 0$ for $i = 1, \dots, N$;

3) Collision can be avoided among all agents, that is $\|q_i(t) - q_j(t)\| > R$ for $i, j = 0, 1, \dots, N$, $i \neq j$ and all $t \geqslant 0$.

2.3 One assumption

To solve the above two problems, we need one assumption as follows.

Assumption 1 The exosystem (4) is neutrally stable, i.e., all the eigenvalues of S are semi-simple with zero real parts.

Remark 2 Under Assumption 1, the exosystem (4) can generate some fundamental types of reference signals and disturbance signals such as step signals, sinusoidal signals and their finite combinations. Moreover, under Assumption 1, given any compact set $\mathscr{V}_0$, there exists a compact set $\mathscr{V}$ such that, for any $v(0) \in \mathscr{V}_0$, the trajectory $v(t)$ of the exosystem (4) remains in $\mathscr{V}$ for all $t \geqslant 0$.

3 Leader-following rendezvous

In this section, we will consider the leader-following rendezvous problem. We first recall the concept of the distributed observer for the leader system (4) proposed in [22] as follows:

$$\dot{\hat{v}}_i = S\hat{v}_i + \mu_0 \sum_{j=0}^{N} \bar{a}_{ij}(t)(\hat{v}_j - \hat{v}_i), \quad i = 1, \dots, N, \tag{6}$$

where $\hat{v}_0 = v$, $\hat{v}_i \in \mathbb{R}^{n_v}$ for $i = 1, \dots, N$, μ_0 is some positive constant to be determined later, and $\bar{a}_{ij}(t) = 1$ if $(j, i) \in \bar{\mathcal{E}}(t)$, $a_{ij} = 0$ if otherwise. Let $\tilde{v}_i = \hat{v}_i - v$ for $i = 0, 1, \dots, N$. Then,

$$\dot{\tilde{v}}_i = S\tilde{v}_i + \mu_0 \sum_{j=0}^{N} \bar{a}_{ij}(t)(\tilde{v}_j - \tilde{v}_i), \quad i = 1, \dots, N. \tag{7}$$

Let $\tilde{v} = \mathrm{col}(\tilde{v}_1, \dots, \tilde{v}_N)$ and $\hat{v} = \mathrm{col}(\hat{v}_1, \dots, \hat{v}_N)$. Then, (7) can be further put into the following compact form

$$\dot{\tilde{v}} = (I_N \otimes S - \mu_0(H(t) \otimes I_{n_v}))\tilde{v}, \tag{8}$$

where $H(t) = [h_{ij}(t)]_{i,j=1}^{N}$ with $h_{ij}(t) = -\bar{a}_{ij}(t)$ for $i \neq j$ and $h_{ii}(t) = \sum_{j=0}^{N} \bar{a}_{ij}(t)$. Since, for $i, j = 1, 2, \dots, N$, $\bar{a}_{ij}(t) = \bar{a}_{ji}(t)$ for all $t \geqslant 0$ under the rules defined in Section 2, $H(t)$ is symmetric. Moreover, by Lemma 4 of [3] or Lemma 1 in [22], $H(t)$ is positive definite if $\bar{\mathcal{G}}(t)$ is connected.

By Theorem 1 and Remark 4 of [22], under Assumption 1 and the condition that the graph $\bar{\mathcal{G}}(t)$ is fixed and connected, we have $\lim_{t\to\infty} \tilde{v}(t) = 0$ exponentially. That is why (6) is called the distributed observer for (4).

To achieve connectivity preservation, we will adopt the same potential function used in [17] as follows:

$$\psi(s) = \frac{1}{2(r^2 - s^2)}, \quad 0 \leqslant s < r. \tag{9}$$

Since $\dfrac{\mathrm{d}\psi(s)}{\mathrm{d}s} = \dfrac{s}{(r^2 - s^2)^2}$ for $0 \leqslant s < r$, we have $\nabla_{q_i} \psi(\|q_i - q_j\|) = \dfrac{q_i - q_j}{(r^2 - \|q_i - q_j\|^2)^2}$ and $\nabla_{q_j} \psi(\|q_i - q_j\|) = \dfrac{q_j - q_i}{(r^2 - \|q_i - q_j\|^2)^2}$.

Now we propose our distributed dynamic control law as follows:

$$\begin{cases} u_i = -f_i^{\mathrm{T}}(q_i, p_i)\hat{\theta}_i - d_i(\hat{w}_i) - k_i s_i + \dot{p}_{\mathrm{r}i}, \\ \dot{\hat{\theta}}_i = f_i(q_i, p_i)s_i, \\ \dot{\hat{v}}_i = S\hat{v}_i + \mu_0 \sum_{j=0}^{N} \bar{a}_{ij}(t)(\hat{v}_j - \hat{v}_i), \quad i = 1, \dots, N, \end{cases} \tag{10}$$

where k_i is some positive constant, and

$$\begin{cases} \hat{w}_i = C_1\hat{v}_i, \\ p_{\mathrm{r}i} = C_2\hat{v}_i - \sum_{j=0}^{N} \bar{a}_{ij}(t) \nabla_{q_i} \psi(\|q_i - q_j\|), \\ s_i = p_i - p_{\mathrm{r}i} \end{cases} \tag{11}$$

with $C_1 = [0_{n_w \times 2n} \ \ I_{n_w}]$ and $C_2 = [0_{n\times n} \ \ I_n \ \ 0_{n \times n_w}]$.

Let $\bar{q}_i = q_i - q_0$ and $\bar{p}_i = p_i - p_0$ for $i = 0, 1, \dots, N$. Note that $\bar{q}_i - \bar{q}_j = q_i - q_j$ and $\bar{p}_i - \bar{p}_j = p_i - p_j$. Thus, for $i = 1, \dots, N$, we have

$$p_{\mathrm{r}i} = C_2\hat{v}_i - \sum_{j=0}^{N} \bar{a}_{ij}(t) \nabla_{\bar{q}_i} \psi(\|\bar{q}_i - \bar{q}_j\|), \tag{12}$$

which implies

$$\begin{aligned} &\sum_{j=0}^{N} \bar{a}_{ij}(t) \nabla_{\bar{q}_i} \psi(\|\bar{q}_i - \bar{q}_j\|) \\ &= C_2\hat{v}_i - p_{\mathrm{r}i} = C_2\tilde{v}_i + C_2 v - p_i + s_i \\ &= C_2\tilde{v}_i + p_0 - p_i + s_i = C_2\tilde{v}_i - \bar{p}_i + s_i. \end{aligned} \tag{13}$$

The closed-loop system composed of (1) and (10) is as follows:

$$\begin{cases} \dot{q}_i = p_i, \quad i = 1, \dots, N, \\ \dot{p}_i = -f_i^{\mathrm{T}}(q_i, p_i)\tilde{\theta}_i + \tilde{d}_i(\tilde{w}_i, w) - k_i s_i + \dot{p}_{\mathrm{r}i}, \\ \dot{\hat{\theta}}_i = f_i(q_i, p_i)s_i, \\ \dot{\tilde{v}}_i = S\tilde{v}_i + \mu_0 \sum_{j=0}^{N} \bar{a}_{ij}(t)(\tilde{v}_j - \tilde{v}_i), \end{cases} \tag{14}$$

where $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$, $\tilde{w}_i = \hat{w}_i - w$ and $\tilde{d}_i(\tilde{w}_i, w) = d_i(w) - d_i(\hat{w}_i) = d_i(w) - d_i(\tilde{w}_i + w)$. It is easy to see that $\tilde{d}_i(0, w) = 0$ for all $w \in \mathbb{R}^{n_w}$. Since $d_i(\cdot)$ is a C^1 function, by Lemma 11.1 of [23], there exists some smooth function $\bar{d}_i(\tilde{w}_i, w) \geqslant 0$ such that, for all $w \in \mathbb{R}^{n_w}$,

$$\|\tilde{d}_i(\tilde{w}_i, w)\|^2 \leqslant \bar{d}_i(\tilde{w}_i, w)\|\tilde{w}_i\|^2. \tag{15}$$

Under Assumption 1, by Remark 2, $w \in \mathscr{W}$ for all $t \geqslant 0$ with $\mathscr{W}$ being some compact subset of $\mathbb{R}^{n_w}$. Together with $\tilde{w}_i = C_1\tilde{v}_i$, we can conclude that there exists some smooth function $\hat{d}(\tilde{v}) \geqslant 0$ such that, for all $w \in \mathscr{W}$,

$$\sum_{i=1}^{N} \tilde{d}_i(\tilde{w}_i, w)\|\tilde{w}_i\|^2 \leqslant \hat{d}(\tilde{v})\|\tilde{v}\|^2. \tag{16}$$

Now we give our result as follows.

Theorem 1 Under Assumption 1, the leader-following rendezvous problem for the multi-agent system composed of (1) and (4) is solvable by the distributed state feedback control law (10) with the potential function (9).

Proof By the continuity of the solution of the closed-loop system (14), there exists $0 < t_1 \leqslant +\infty$ such that $\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(0)$ for all $0 \leqslant t < t_1$. Thus, $\bar{a}_{ij}(t) = \bar{a}_{ij}(0)$ and $H(t) = H(0)$ for all $0 \leqslant t < t_1$. Let

$$V_1 = \sum_{i=1}^{N}\Big(\frac{1}{2}\sum_{j=1}^{N} \bar{a}_{ij}(t)\psi(\|\bar{q}_i - \bar{q}_j\|) + \bar{a}_{i0}(t)\psi(\|\bar{q}_i\|)\Big). \tag{17}$$

By the rules of $\bar{\mathcal{E}}(t)$ defined in Section 2, $\bar{a}_{ij}(t) = \bar{a}_{ji}(t)$ for $i, j = 1, \dots, N$. Then, from (13), along the trajectory of the closed-loop system (14), for $0 \leqslant t < t_1$, we have

$$\begin{aligned}
\dot{V}_1 &= \sum_{i=1}^{N}\Big(\frac{1}{2}\sum_{j=1}^{N} \bar{a}_{ij}(t)(\dot{\bar{q}}_i^{\mathrm{T}} \nabla_{\bar{q}_i} \psi(\|\bar{q}_i - \bar{q}_j\|)\\
&\quad + \dot{\bar{q}}_j^{\mathrm{T}} \nabla_{\bar{q}_j} \psi(\|\bar{q}_i - \bar{q}_j\|))\\
&\quad + \bar{a}_{i0}(t)\dot{\bar{q}}_i^{\mathrm{T}} \nabla_{\bar{q}_i} \psi(\|\bar{q}_i\|)\Big)\\
&= \sum_{i=1}^{N}\Big(\sum_{j=1}^{N} \bar{a}_{ij}(t)\dot{\bar{q}}_i^{\mathrm{T}} \nabla_{\bar{q}_i} \psi(\|\bar{q}_i - \bar{q}_j\|)\\
&\quad + \bar{a}_{i0}(t)\dot{\bar{q}}_i^{\mathrm{T}} \nabla_{\bar{q}_i} \psi(\|\bar{q}_i\|)\Big)\\
&= \sum_{i=1}^{N} \bar{p}_i^{\mathrm{T}}\Big(\sum_{j=0}^{N} \bar{a}_{ij}(t) \nabla_{\bar{q}_i} \psi(\|\bar{q}_i - \bar{q}_j\|)\Big)\\
&= \sum_{i=1}^{N} \bar{p}_i^{\mathrm{T}}(C_2\tilde{v}_i - \bar{p}_i + s_i)\\
&\leqslant \sum_{i=1}^{N}\Big(\frac{1}{4}\|\bar{p}_i\|^2 + \|C_2\tilde{v}_i\|^2 - \|\bar{p}_i\|^2\\
&\quad + \frac{1}{4}\|\bar{p}_i\|^2 + \|s_i\|^2\Big)\\
&\leqslant \sum_{i=1}^{N}\Big(-\frac{1}{2}\|\bar{p}_i\|^2 + \|s_i\|^2\Big) + \|C_2\|^2\|\tilde{v}\|^2.
\end{aligned} \tag{18}$$

Let

$$V_2 = \frac{1}{2}\sum_{i=1}^{N}(s_i^{\mathrm{T}} s_i + \tilde{\theta}_i^{\mathrm{T}}\tilde{\theta}_i). \tag{19}$$

Then, from (15) and (16), along the trajectory of the closed-loop system (14), for $0 \leqslant t < t_1$, we have

$$\begin{aligned}
\dot{V}_2 &= \sum_{i=1}^{N} s_i^{\mathrm{T}}(\dot{p}_i - \dot{p}_{\mathrm{r}i}) + \sum_{i=1}^{N} \tilde{\theta}_i^{\mathrm{T}}\dot{\hat{\theta}}_i\\
&= \sum_{i=1}^{N} s_i^{\mathrm{T}}(-f_i^{\mathrm{T}}(q_i, p_i)\tilde{\theta}_i + \tilde{d}_i(\tilde{w}_i, w) - k_i s_i)\\
&\quad + \sum_{i=1}^{N} \tilde{\theta}_i^{\mathrm{T}} f_i(q_i, p_i)s_i\\
&= \sum_{i=1}^{N} s_i^{\mathrm{T}}(\tilde{d}_i(\tilde{w}_i, w) - k_i s_i)\\
&\leqslant \sum_{i=1}^{N}\Big(\frac{1}{4}\|s_i\|^2 + \|\tilde{d}_i(\tilde{w}_i, w)\|^2 - k_i\|s_i\|^2\Big)\\
&\leqslant \sum_{i=1}^{N}\Big(-(k_i - \frac{1}{4})\|s_i\|^2\Big) + \hat{d}(\tilde{v})\|\tilde{v}\|^2.
\end{aligned} \tag{20}$$

Since the number of agents is finite, the number of connected graphs associated with these $N+1$ agents is also finite. Denote all connected graphs by $\{\bar{\mathcal{G}}_1, \dots, \bar{\mathcal{G}}_{n_0}\}$ and denote the H matrix associated with these connected graphs by $\{H_1, \dots, H_{n_0}\}$ which are all symmetric and positive definite. Let

$$V_3 = \tilde{v}^{\mathrm{T}}\tilde{v}. \tag{21}$$

Then, along the trajectory of the distributed observer (8), for $0 \leqslant t < t_1$, we have

$$\begin{aligned}
\dot{V}_3 &= \tilde{v}^{\mathrm{T}}(I_N \otimes (S + S^{\mathrm{T}}) - 2\mu_0(H(t) \otimes I_{n_v}))\tilde{v}\\
&\leqslant \lambda_1\|\tilde{v}\|^2 - 2\mu_0\lambda_2\|\tilde{v}\|^2,
\end{aligned} \tag{22}$$

where $\lambda_1 = \lambda_{\max}(S^{\mathrm{T}} + S)$ and $\lambda_2 = \min\{\lambda_{\min}(H_1), \dots, \lambda_{\min}(H_{n_0})\}$. Choose $\mu_0 \geqslant \frac{1}{2\lambda_2}(\lambda_1 + 1)$. Then, for $0 \leqslant t < t_1$, we have

$$\dot{V}_3 \leqslant -\|\tilde{v}\|^2. \tag{23}$$

Choose some smooth function $\rho(\|\tilde{v}\|^2) \geqslant \|C_2\|^2 + \hat{d}(\tilde{v}) + 1$. Let

$$\bar{V}_3 = \int_0^{V_3} \rho(s)\mathrm{d}s. \tag{24}$$

Then, from (23) and (24), for $0 \leqslant t < t_1$, we have

$$\dot{\bar{V}}_3 = \rho(V_3)\dot{V}_3 \leqslant -(\|C_2\|^2 + \hat{d}(\tilde{v}) + 1)\|\tilde{v}\|^2. \tag{25}$$

Finally, let

$$V = V_1 + V_2 + \bar{V}_3. \tag{26}$$

Then, it can be seen that for all initial condition $q_i(0)$, $i = 0, 1, \dots, N$, that makes $\bar{\mathcal{G}}(0)$ connected,

$$V(0) < +\infty. \tag{27}$$

Choose $k_i \geqslant \frac{7}{4}$. Then, from (18), (20) and (25), along the trajectory of the closed-loop system (14), for $0 \leqslant t < t_1$, we have

$$\begin{aligned}\dot{V} \leqslant & \sum_{i=1}^{N}(-\frac{1}{2}\|\bar{p}_i\|^2 + \|s_i\|^2) + \|C_2\|^2\|\tilde{v}\|^2 \\ & + \sum_{i=1}^{N}(-(k_i - \frac{1}{4})\|s_i\|^2) + \hat{d}(\tilde{v})\|\tilde{v}\|^2 \\ & - (\|C_2\|^2 + \hat{d}(\tilde{v}) + 1)\|\tilde{v}\|^2 \\ = & \sum_{i=1}^{N}(-\frac{1}{2}\|\bar{p}_i\|^2 - (k_i - \frac{5}{4})\|s_i\|^2) - \|\tilde{v}\|^2 \\ \leqslant & -\frac{1}{2}\sum_{i=1}^{N}(\|\bar{p}_i\|^2 + \|s_i\|^2) - \|\tilde{v}\|^2. \end{aligned}\tag{28}$$

If $t_1 = +\infty$, then $\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(0)$ for all $t \geqslant 0$, and thus (28) holds for all $t \geqslant 0$.

If $\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(0)$ does not hold for all $t \geqslant 0$, then, by the same argument as used in the proof of Theorem 3.1 of [15], there exists a finite integer $k > 0$ such that

$$\begin{cases}\bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(0), & t \in [0, t_1), \\ \bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(t_i) \supset \bar{\mathcal{G}}(t_{i-1}), & t \in [t_i, t_{i+1}),\ i = 1, \dots, k-1, \\ \bar{\mathcal{G}}(t) = \bar{\mathcal{G}}(t_k) \supset \bar{\mathcal{G}}(t_{k-1}), & t \in [t_k, +\infty).\end{cases}\tag{29}$$

Thus $\bar{\mathcal{G}}(t) \supseteq \bar{\mathcal{G}}(0)$ for all $t \geqslant 0$ which implies $\bar{\mathcal{G}}(t)$ is connected for all $t \geqslant 0$ and

$$\dot{V} \leqslant -\frac{1}{2}\sum_{i=1}^{N}(\|\bar{p}_i\|^2 + \|s_i\|^2) - \|\tilde{v}\|^2 \tag{30}$$

for any $t \in [t_i, t_{i+1})$ with $i = 0, 1, \dots, k$, $t_0 = 0$ and $t_{k+1} = +\infty$.

Since $V(t) \geqslant 0$ is lower bounded, by (30), $\lim\limits_{t\to\infty} V(t)$ exists and for $i = 1, \dots, N$, s_i, $\tilde{\theta}_i$, $\tilde{v}_i$ as well as $\bar{q}_i - \bar{q}_j$ with $j \in \bar{\mathcal{N}}_i(t_k)$ are bounded. Since the graph $\bar{\mathcal{G}}(t)$ is connected for all $t \geqslant t_k$, $q_i - q_j$ with $j \in \bar{\mathcal{N}}_i(t_k)$ are bounded and q_0 is bounded by Remark 2, we can easily obtain that q_i is bounded for $i = 0, 1, \dots, N$. By Remark 2, v is bounded, thus $\hat{v}_i = v + \tilde{v}_i$ is also bounded. From the second equation of (11), $p_{\mathrm{r}i}$ is bounded for $i = 0, 1, \dots, N$. Then, from the third equation of (11), p_i is also bounded. By Remark 2, p_0 is bounded and thus $\bar{p}_i = p_i - p_0$ is bounded.

Next, we will show that $\ddot{V}$ is bounded for all $t \geqslant t_k$ which implies that $\dot{V}$ is uniformly continuous for all $t \geqslant t_k$. Note that, for $t \geqslant t_k$

$$\begin{aligned}\ddot{V} =& \ddot{V}_1 + \ddot{V}_2 + \ddot{V}_3 \\ =& \sum_{i=1}^{N}(\dot{\bar{p}}_i^{\mathrm{T}}(C_2\tilde{v}_i - \bar{p}_i + s_i) + \bar{p}_i^{\mathrm{T}}(C_2\dot{\tilde{v}}_i - \dot{\bar{p}}_i + \dot{s}_i) \\ & - 2k_i\dot{s}_i^{\mathrm{T}}s_i + \dot{s}_i^{\mathrm{T}}\tilde{d}_i(\tilde{w}_i, w)) + s_i^{\mathrm{T}}(\frac{\partial\tilde{d}_i(\tilde{w}_i, w)}{\partial\tilde{w}_i}\dot{\tilde{w}}_i \\ & + \frac{\partial\tilde{d}_i(\tilde{w}_i, w)}{\partial w}\dot{w})) + 2\rho(\|\tilde{v}\|^2)\tilde{v}^{\mathrm{T}}(I_N \otimes (S + S^{\mathrm{T}}) \\ & - 2\mu_0(H(t_k) \otimes I_{n_v}))\dot{\tilde{v}} + \frac{\partial\rho(\|\tilde{v}\|^2)}{\partial\|\tilde{v}\|^2}(2\tilde{v}^{\mathrm{T}}\dot{\tilde{v}})^2.\end{aligned}\tag{31}$$

For $i = 1, \dots, N$, from (7), $\dot{\tilde{v}}_i$ and $\dot{\tilde{w}}_i$ are bounded for all $t \geqslant 0$. From (4), $\dot{v}$ is bounded and thus $\dot{p}_0$ and $\dot{w}$ are both bounded. Next, we still need to show $\dot{\bar{p}}_i$ and $\dot{s}_i$ are bounded. For $i = 1, \dots, N$, from (11), for $t \geqslant t_k$, we have

$$\begin{aligned}\dot{p}_{\mathrm{r}i} =& C_2\dot{\hat{v}}_i - \sum_{j=0}^{N}\bar{a}_{ij}(t_k)\frac{\mathrm{d}}{\mathrm{d}t}(\nabla_{q_i}\psi(\|q_i - q_j\|)) \\ =& C_2\dot{\hat{v}}_i - \sum_{j=0}^{N}\bar{a}_{ij}(t_k)(\frac{p_i - p_j}{(r^2 - \|q_i - q_j\|^2)^2} \\ & + \frac{4(q_i - q_j)(p_i - p_j)^{\mathrm{T}}(q_i - q_j)}{(r^2 - \|q_i - q_j\|^2)^3}),\end{aligned}\tag{32}$$

which is also bounded, since $\dot{\hat{v}}_i$, q_i and p_i are bounded. Then, from the second equation of (14), $\dot{p}_i$ is bounded. Thus $\dot{\bar{p}} = \dot{p}_i - \dot{p}_0$ and $\dot{s}_i = \dot{p}_i - \dot{p}_{\mathrm{r}i}$ are also bounded. That is to say, $\ddot{V}(t)$ is bounded. By Barbalat's lemma, we have $\lim\limits_{t\to\infty}\dot{V}(t) = 0$. Together with (30), we have $\lim\limits_{t\to\infty}\bar{p}_i = 0$, $\lim\limits_{t\to\infty}s_i = 0$ and $\lim\limits_{t\to\infty}\tilde{v}_i = 0$ for $i = 1, \dots, N$. Then, from (13), we have

$$\lim_{t\to\infty}\sum_{j=0}^{N}\bar{a}_{ij}(t)\nabla_{\bar{q}_i}\psi(\|\bar{q}_i - \bar{q}_j\|) = 0. \tag{33}$$

Now, for $i = 1, 2, \dots, N$, $j = 0, 1, \dots, N$ and $j \neq i$, and $t \geqslant 0$, let

$$\bar{m}_{ij} = -\frac{\bar{a}_{ij}(0)}{r^4},\ \ \bar{h}_{ij}(t) = -\bar{a}_{ij}(t)\phi_{ij}(t), \tag{34}$$

$$\bar{m}_{ii} = -\sum_{j=0, j\neq i}^{N}\bar{m}_{ij},\ \ \bar{h}_{ii}(t) = -\sum_{j=0, j\neq i}^{N}\bar{h}_{ij}(t), \tag{35}$$

where

$$\phi_{ij}(t) = \frac{1}{(r^2 - \|\bar{q}_i - \bar{q}_j\|^2)^2}. \tag{36}$$

Define $\bar{H}_0 = [\bar{m}_{ij}]_{i,j=1}^N$ and $\bar{H}_1(t) = [\bar{h}_{ij}(t)]_{i,j=1}^N$. Then, from (33), the definition of $\bar{h}_{ij}(t)$ and the fact $\bar{q}_0 = q_0 - q_0 = 0$, we have, for $i = 1, \dots, N$,

$$\begin{aligned}
&\lim_{t\to\infty} \sum_{j=0}^{N} \bar{a}_{ij}(t) \nabla_{\bar{q}_i} \psi(\|\bar{q}_i - \bar{q}_j\|) \\
&= \lim_{t\to\infty} \sum_{j=0}^{N} \bar{a}_{ij}(t)\phi_{ij}(t)(\bar{q}_i - \bar{q}_j) \\
&= \lim_{t\to\infty} \sum_{j=0}^{N} \bar{h}_{ij}(t)\bar{q}_j = \lim_{t\to\infty} \sum_{j=1}^{N} \bar{h}_{ij}(t)\bar{q}_j = 0, \qquad (37)
\end{aligned}$$

which can be further put into the following form:

$$\lim_{t\to\infty}(\bar{H}_1(t) \otimes I_n)\bar{q} = 0. \qquad (38)$$

Since $\bar{\mathcal{G}}(0)$ is connected, by Lemma 4 of [3] or Lemma 1 of [22], the matrix $\bar{H}_0$ is positive definite. Again, by the same argument as used in the proof of Theorem 3.1 of [15], we conclude that the matrix $\bar{H}_1(t) - \bar{H}_0$ is positive semi-definite for all $t \geqslant 0$. This fact together with (38) gives

$$\begin{aligned}
0 &= \lim_{t\to\infty} \bar{q}^{\mathrm{T}}(\bar{H}_1(t) \otimes I_n)\bar{q} \\
&\geqslant \lim_{t\to\infty} \bar{q}^{\mathrm{T}}(\bar{H}_0 \otimes I_n)\bar{q} \geqslant 0. \qquad (39)
\end{aligned}$$

which implies that $\lim_{t\to\infty} \bar{q} = 0$, i.e., $\lim_{t\to\infty}(q_i - q_0) = 0$. □

4 Leader-following flocking

In this section, we will consider the leader-following flocking problem. The technique is similar to that used in Section 3. However, what makes the flocking problem different from rendezvous problem is that we need to avoid collision among agents. For this purpose, we need to use a different potential function as follows:

$$\psi(s) = \frac{1}{2(r^2 - s^2)} + \frac{1}{2(s^2 - R^2)}, \quad R < s < r, \qquad (40)$$

which is similar to that in [9]. Then, we give the result as follows.

Theorem 2 Under Assumption 1, the leader-following flocking problem for the multi-agent system composed of (1) and (4) is solvable by the distributed state feedback control law (10) with the potential function (40).

Proof The proof is similar to the proof of Theorem 1, the only difference is that we need to show that the collision can be avoided in the sense that $\|\bar{q}_i(t) - \bar{q}_j(t)\| > R$, $i, j = 0, 1, \dots, N$ and $i \neq j$ for all $t \geqslant 0$.

If the collision happens at a finite time t_l, which implies that $\lim_{t\to t_l^-} \|\bar{q}_i(t) - \bar{q}_j(t)\| \to R$ and thus $\lim_{t\to t_l^-} V(t) = +\infty$. However, by (30), we have $V(t) \leqslant V(0) < +\infty$ for all $t \geqslant 0$, which makes the contradiction. Thus the collision can be avoided in the sense that $\|\bar{q}_i(t) - \bar{q}_j(t)\| > R$, $i, j = 0, 1, \dots, N$ and $i \neq j$ for all $t \geqslant 0$.

Thus the proof is completed. □

5 An example

In this section, we will apply our results to the leader-following rendezvous/flocking problem for a group of van der Pol systems as follows:

$$\begin{cases}
\dot{q}_i = p_i, \quad i = 1, 2, 3, 4, \\
\dot{p}_i = \begin{bmatrix} -q_{1i} & p_{1i}(1 - q_{1i}^2) \\ -q_{2i} & p_{2i}(1 - q_{2i}^2) \end{bmatrix} \begin{bmatrix} \theta_{1i} \\ \theta_{2i} \end{bmatrix} + d_i(w) + u_i,
\end{cases} \qquad (41)$$

where $q_i = [q_{1i}, q_{2i}]^{\mathrm{T}} \in \mathbb{R}^2$ and $p_i = [p_{1i}, p_{2i}]^{\mathrm{T}} \in \mathbb{R}^2$ for $i = 1, \dots, 4$, $w = [w_1, w_2]^{\mathrm{T}}$, and

$$\begin{aligned}
d_1(w) &= \begin{bmatrix} w_1^2 \\ w_2^2 \end{bmatrix}, \quad d_2(w) = \begin{bmatrix} w_1^2 \\ w_1 w_2 \end{bmatrix} \\
d_3(w) &= \begin{bmatrix} w_1 w_2 \\ w_2^2 \end{bmatrix}, \quad d_4(w) = \begin{bmatrix} w_2^2 \\ w_1^2 + w_2^2 \end{bmatrix}.
\end{aligned}$$

Clearly, system (41) is in the form (1) with

$$f_i^{\mathrm{T}}(q_i, p_i) = \begin{bmatrix} -q_{1i} & p_{1i}(1 - q_{1i}^2) \\ -q_{2i} & p_{2i}(1 - q_{2i}^2) \end{bmatrix}, \quad \theta_i = \begin{bmatrix} \theta_{1i} \\ \theta_{2i} \end{bmatrix}.$$

The exosystem is in the form (4) with

$$S_{\mathrm{a}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \otimes I_2, \quad S_{\mathrm{b}} = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}.$$

Clearly, Assumption 1 is satisfied.

The initial communication graph $\bar{\mathcal{G}}(0)$ is described by Fig. 1 where node 0 is associated with the leader and other nodes are associated with the followers.

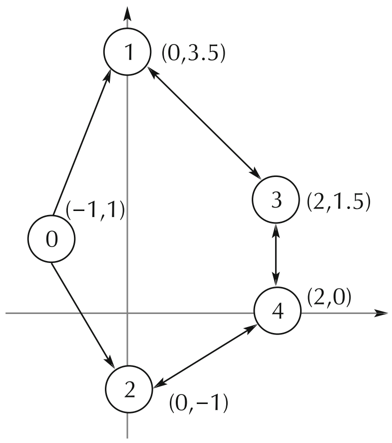

Fig. 1 The initial communication graph.

5.1 Leader-following rendezvous

By Theorem 1, we design a distributed state feedback control law of the form (10) with the potential function given by (9), $r = 3$, $\epsilon = 0.2$, $\mu_0 = 10$ and $k_i = 4$ for $i = 1, 2, 3, 4$.

Simulation is performed with

$$\theta_1 = [2, 4]^{\mathrm{T}}, \quad \theta_2 = [3, 1]^{\mathrm{T}},$$
$$\theta_3 = [5, 2]^{\mathrm{T}}, \quad \theta_4 = [4, 6]^{\mathrm{T}}$$

and the following initial conditions:

$$\begin{aligned}
&q_0(0) = [-1, 1]^{\mathrm{T}}, \quad q_1(0) = [0, 3.5]^{\mathrm{T}},\\
&q_2(0) = [0, -1]^{\mathrm{T}}, \quad q_3(0) = [2, 1.5]^{\mathrm{T}},\\
&q_4(0) = [2, 4]^{\mathrm{T}}, \quad w(0) = [1, 0]^{\mathrm{T}},\\
&p_0(0) = [0, 1]^{\mathrm{T}}, \quad p_1(0) = [2, 4]^{\mathrm{T}},\\
&p_2(0) = [1, 3]^{\mathrm{T}}, \quad p_3(0) = [3, 4]^{\mathrm{T}},\\
&p_4(0) = [2, 1]^{\mathrm{T}}, \quad \hat{\theta}_1(0) = [1, -2]^{\mathrm{T}},\\
&\hat{\theta}_2(0) = [3, -4]^{\mathrm{T}}, \quad \hat{\theta}_3(0) = [5, -2]^{\mathrm{T}},\\
&\hat{\theta}_4(0) = [-3, 4]^{\mathrm{T}},\\
&\hat{v}_1(0) = [-3, 2, 4, -5, 3, 1]^{\mathrm{T}},\\
&\hat{v}_2(0) = [2, 4, -1, 5, 3, 5]^{\mathrm{T}},\\
&\hat{v}_3(0) = [3, 2, 4, -2, 4, -1]^{\mathrm{T}},\\
&\hat{v}_4(0) = [2, -5, 4, -3, 6, 1]^{\mathrm{T}}.
\end{aligned}$$

It is easy to see that the initial diagraph $\bar{\mathcal{G}}(0)$ is connected under the first five rules defined in Section 2.

Figs. 2, 3 and 4 show that all followers approach the position of the leader asymptotically with the same velocity of the leader while preserving the connectivity, that is to say, the leader-following rendezvous problem for system (41) is solved by the distributed state feedback control law of the form(10) with the potential function given by (9).

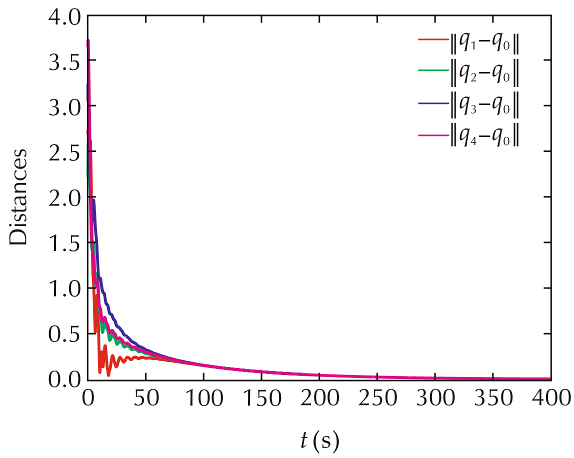

Fig. 2 Distances between leader and all followers.

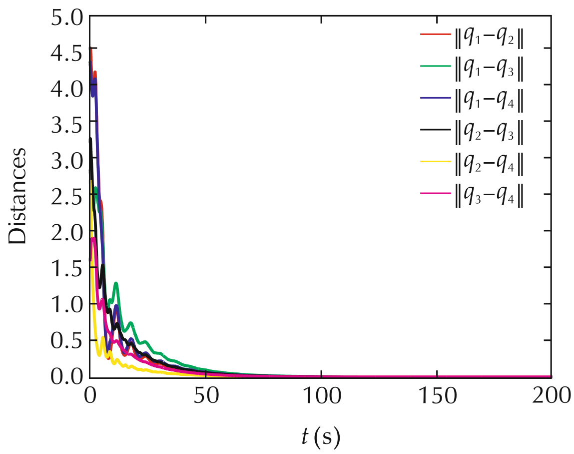

Fig. 3 Distances between all followers.

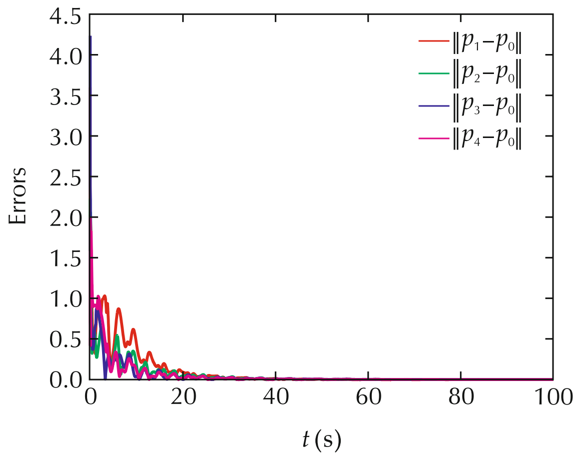

Fig. 4 Velocity errors between leader and all followers.

5.2 Leader-following flocking

By Theorem 2, we design a distributed state feedback control law of the form (10) with the potential function given by (40), $r = 3$, $R = 1$, $\epsilon = 0.2$, $\mu_0 = 10$ and $k_i = 4$ for $i = 1, 2, 3, 4$.

Simulation is performed with the same $\theta_i, i = 1, 2, 3, 4$, and initial conditions as given in the simulation for the leader-following rendezvous problem. It is also easy to see that the initial diagraph $\bar{\mathcal{G}}(0)$ is connected under the second five rules defined in Section 2.

Figs. 5 and 6 show that the connectivity is preserved and the collision is avoided. Fig. 7 further shows that the velocities of all followers approach the velocity of the leader asymptotically. That is to say, the leader-following flocking problem for system (41) is solved by the distributed state feedback control law of the form (10) with the potential function given by (40).

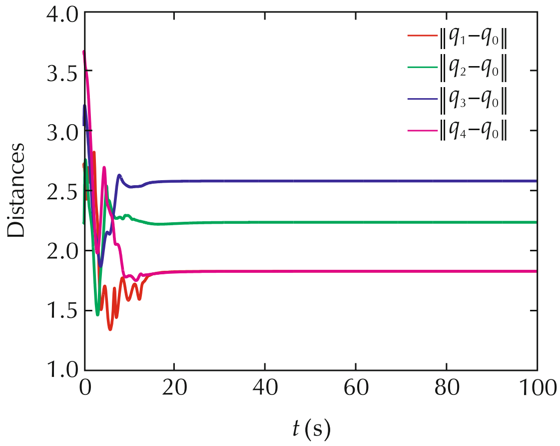

Fig. 5 Distances between leader and all followers.

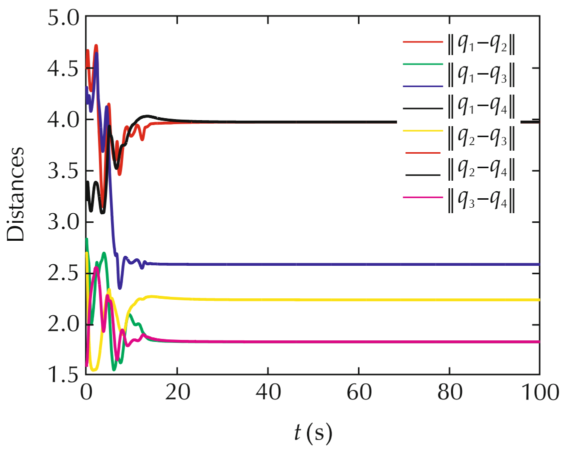

Fig. 6 Distances between all followers.

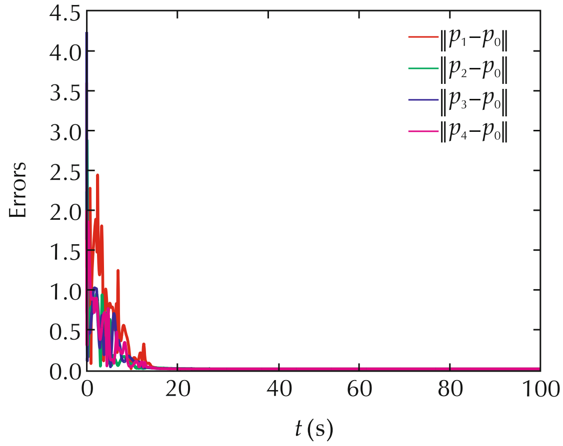

Fig. 7 Velocity errors between leader and all followers.

6 Conclusions

In this paper, we have studied both the leader-following rendezvous problem and flocking problem for a class of second-order nonlinear multi-agent systems. Compared with the existing results, our systems contain not only external disturbances but also parameter uncertainties, and the parameter uncertainties are allowed to be arbitrarily large. By combining the adaptive control technique, the distributed observer method and the potential function method, we have solved the two problems by the distributed state feedback control law.

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