Second-Order Consensus in Multiagent Systems via Distributed Sliding Mode Control

Wenwu Yu, *Senior Member, IEEE*, He Wang, Fei Cheng, Xinghuo Yu, *Fellow, IEEE*, and Guanghui Wen, *Member, IEEE*

***Abstract*—In this paper, the new decoupled distributed sliding-mode control (DSMC) is first proposed for second-order consensus in multiagent systems, which finally solves the fundamental unknown problem for sliding-mode control (SMC) design of coupled networked systems. A distributed full-order sliding-mode surface is designed based on the homogeneity with dilation for reaching second-order consensus in multiagent systems, under which the sliding-mode states are decoupled. Then, the SMC is applied to the decoupled sliding-mode states to reach their origin in finite time, which is the sliding-mode surface. The states of agents can first reach the designed sliding-mode surface in finite time and then move to the second-order consensus state along the surface in finite time as well. The DSMC designed in this paper can eliminate the influence of singularity problems and weaken the influence of chattering, which is still very difficult in the SMC systems. In addition, DSMC proposes a general decoupling framework for designing SMC in networked multiagent systems. Simulations are presented to verify the theoretical results in this paper.**

***Index Terms*—Distributed sliding-mode control (DSMC), finite-time consensus, homogeneity, multiagent systems.**

I. Introduction

COOPERATIVE control in multiagent systems has recently become a popular topic due to its wide applications in many areas, such as power systems, unmaned air vehicles, sensor networks, smart grids, biological systems, robotic teams, formation control, etc. For reaching a global goal, the main idea for cooperative control in multiagent systems is to design the distributed controllers on each agent by using its local neighboring information. That is, under a distributed local protocol, the agents can work cooperatively to achieve a global goal. In particular, under the idea of cooperation, the agents in multiagent systems only share information with their neighbors locally and try to reach an agreement to a certain degree.

Typical collective global behaviors under cooperative control include consensus [1]–[10], synchronization [11]–[17], flocking [18]–[20], and swarming [21], and much progress has been already achieved. The consensus problem usually concerns about how a group of autonomous agents can reach an agreement on position, velocity, or other certain quantity of criteria. First-order consensus has been widely discussed in the recent decade [1], [3], where consensus in a network with a switching topology can be reached if the network is jointly connected frequently enough as the network evolves with time.

Actually, in many real-world applications, agents are governed by both position and velocity states, which force the researchers to study consensus in multiagent systems with second-order dynamics. In recent years, many good results have been devoted to studying second-order consensus in multiagent systems [22]–[25]. A necessary and sufficient condition for second-order consensus in linear multiagent systems was given in [23], which depends on the eigenvalues of the Laplacian matrix and the control gains of the network. In order to study some practical systems with second-order dynamics, the nonlinear multiagent system was considered [24]. However, in many practical applications, finite-time consensus is more desirable for high accuracy and fast convergence. Recently, finite-time consensus in multiagent systems has received increasing attention [26], [27]. Due to some practical applications, finite-time consensus in multiagent systems with second-order dynamics have been widely investigated and typical approaches include homogeneous method [28]–[30], power integrator method [31], and sliding-mode control (SMC) [32]. Among these approaches, SMC was regarded to have easy implementation and high robustness against uncertain disturbances.

While the theoretical analysis on ideal models develops rapidly, there are some gaps between mathematical models and actual plant dynamics, which now are commonly considered as uncertainties. Such mismatches usually come from the unknown plant dynamics, external disturbances and so on. So people have come up with robust control to deal with such problems. Among these technologies, SMC remains to be one of the most efficient approaches in settling bounded

Manuscript received September 6, 2016; accepted October 29, 2016. Date of publication November 22, 2016; date of current version July 17, 2017. This work was supported in part by the National Natural Science Foundation of China under Grant 61322302, Grant 61673107, and Grant 61673104, in part by the Six Talent Peaks of Jiangsu Province of China under Grant 2014-DZXX-004, in part by the National Ten Thousand Talent Program for Young Top-Notch Talents, in part by the Australian Research Council Discovery Scheme under Grant DP140100544, and in part by the Fundamental Research Funds for the Central Universities of China under Grant 2242016K41058 and Grant 2242016K41030. This paper was recommended by Associate Editor G.-P. Liu.

W. Yu, H. Wang, F. Cheng, and G. Wen are with the Department of Mathematics, Southeast University, Nanjing 210096, China (e-mail: wwyu@seu.edu.cn; wanghemath2014@163.com; chengfeiseu@163.com; wenguanghui@gmail.com).

X. Yu is with the School of Engineering, RMIT University, Melbourne, VIC 3001, Australia (e-mail: x.yu@rmit.edu.au).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TCYB.2016.2623901

disturbances and parameter variations [33]–[35]. Typical SMC includes linear sliding-mode (LSM) control and terminal sliding-mode (TSM) control [36], [37], where the former one is asymptotically stable while the latter one is finite-time stable. The key idea for SMC is to design a sliding-mode surface such that the states can first reach this surface and then keep on it for achieving some goal. Thus, the study of SMC usually contains two steps: 1) the design of sliding-mode surface and 2) sliding-mode controller. On the predesigned surface, the states can move to the expected goal, and the controller makes sure that the states can reach the surface. Compared with LSM, TSM has finite-time convergence and smaller steady-state tracking errors [37]. The singularity and chattering problems were both solved in [38]. However, in the existing literature, TSM control was discussed for tracking consensus in multiagent manipulators [32], where the neighboring control inputs were utilized which are not fully distributed. In addition, the chattering behavior cannot be avoided.

Thus, in this paper, we aim to study SMC design for multiagent systems with second-order dynamics. The main contributions are twofold. First, a new decoupled DSMC is first proposed for second-order consensus in multiagent systems, which finally solves the fundamental unknown and difficult problem for SMC design of coupled networked systems. Second, a general framework for studying consensus in multiagent systems is established where the sliding-mode states are introduced to decouple the states of agents. Under this framework, the states of agents are coupled on the sliding-mode surface and the sliding-mode states are decoupled out of this surface, which makes the design of sliding-mode controllers independent. Then, consensus for many complicated multiagent systems with uncertain disturbances and nonlinear dynamics can be discussed.

The rest of this paper is arranged as follows. In Section II, some basic ideas of multiagent systems and other useful concepts are stated. The sliding-mode surface and SMC are designed in Sections III and IV, respectively. In Section V, some simulation examples are used to demonstrate the effectiveness of the results in this paper. Finally, the conclusion is drawn in Section VI.

II. Preliminaries

Graph theory plays a very important role in the study of multiagent systems. Some basic concepts and results about graph theory are introduced briefly in the following.

A weighted undirected network $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ with order N consists of a set of nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, a set of undirected edges $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$, and a weighted adjacency matrix $A = (a_{ij})_{N\times N}$. An undirected edge $\mathcal{E}_{ij}$ in a weighted undirected network $\mathcal{G}$ is denoted by the unordered pair of nodes (v_i, v_j), which means that nodes v_i and v_j can exchange information with each other. According to the definition of adjacency matrices, weights $a_{ij} = a_{ji} > 0$ are positive if and only if there is an edge (v_i, v_j) in $\mathcal{G}$. $A = (a_{ij})_{N\times N}$ is the coupling configuration matrix representing the structure of the network, which is also called the weighted adjacency matrix of the network. The Laplacian matrix $L = (l_{ij})_{N\times N}$ can be denoted as

$$l_{ii} = -\sum_{j=1, j\neq i}^{N} l_{ij}\,, l_{ij} = -a_{ij},\ i \neq j$$

which ensures the diffusion property that $\sum_{j=1}^{N} l_{ij} = 0$.

The multiagent system discussed in this paper has the following form:

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = f(x_i, v_i, t) + d_i(x_i, v_i, t) + b_i(x_i, v_i, t)u_i(t) \\ \qquad\qquad\qquad\qquad i = 1, \ldots, N \end{cases} \tag{1}$$

where $x_i \in R^n$ and $v_i \in R^n$ represent the position and the velocity states of the ith agent, $f(x_i, v_i, t) \in R^n$ and $b_i(x_i, v_i, t) \in R \neq 0$ are two known smooth nonlinear functions of $x_i(t)$ and $v_i(t)$, $u_i(t)$ is the control input designed later, and the partially known function $d_i(x_i, v_i, t) \in R^n$ stands for the parameter uncertainties and the external disturbances in system (1), $i = 1, 2, \ldots, N$. Let $x = [x_1^T, x_2^T, \ldots, x_N^T]^T$, and $v = [v_1^T, v_2^T, \ldots, v_N^T]^T$. The main purpose of this paper is to design a sliding-mode surface, along which the system trajectory can move to the consensus state in finite time, and an SMC which can force the system trajectory to reach the designed sliding-mode surface in finite time for system (1). The approach discussed in this paper can be regarded as distributed SMC (DSMC), where the states of agents are coupled on the sliding-mode surface and the sliding-mode states are decoupled out of this surface.

Remark 1: As mentioned above, the main concern here is the DSMC design and disturbance rejection. Besides, very few existing works have solved the second-order finite-time consensus with both nonlinear terms and disturbances. Therefore, in this paper, the nonlinear term is assumed to be known beforehand.

Before presenting the main result, several assumptions and notations will be introduced as follows.

Assumption 1: The nonlinear function $d_i(x_i, v_i, t)$ representing the parameter uncertainties and the external disturbances in multiagent system (1) is assumed to satisfy the following condition:

$$\|d_i(x_i, v_i, t)\|_\infty \leq \bar{\alpha}_i \tag{2}$$

where $\bar{\alpha}_i > 0$ is a constant and $\| \centerdot \|_\infty$ is the infinite norm of a vector, $i = 1, 2, \ldots, N$.

Assumption 2: The derivative of $d_i(x_i, v_i, t)$ in system (1) is assumed to satisfy the following condition:

$$\left\|\dot{d}_i(x_i, v_i, t)\right\|_\infty \leq \beta_i \tag{3}$$

where $\beta_i > 0$ is a constant and $\| \centerdot \|_\infty$ is the infinite norm of a vector, $i = 1, 2, \ldots, N$.

In the following part, the homogeneity with dilation for finite-time convergence analysis will be introduced [28]–[30].

Consider a k-dimensional system

$$\dot{y} = g(y) \tag{4}$$

where $y = (y_1, \ldots, y_k)^T$ and $g(y) = (g_1(y), \ldots, g_k(y))^T$.

Definition 1: $g(y)$ is called homogeneous of degree $\sigma \in R$ with dilation $(r_1, \ldots, r_k)$, if

$$g_i\big(\varepsilon^{r_1} y_1, \ldots, \varepsilon^{r_k} y_k\big) = \varepsilon^{\sigma + r_i} g_i(y),\ \ i = 1, \ldots, k, \text{ for any } \varepsilon > 0.$$

Definition 2 [29]: System (4) is called homogeneous if its vector field $g(y)$ is homogeneous. Moreover

$$\dot{y} = g(y) + \hat{g}(y),\ \ \hat{g}(0) = 0,\ \ y \in R^k \tag{5}$$

is called locally homogeneous if g is homogeneous of degree $\sigma \in R$ with dilation $(r_1, \ldots, r_k)$, and $\hat{g}(y) = (\hat{g}_1(y), \ldots, \hat{g}_k(y))^T$ is a continuous vector field satisfying

$$\lim_{\epsilon \to 0} \frac{\hat{g}_i(\epsilon^{r_1} x_1, \ldots, \epsilon^{r_k} x_k)}{\epsilon^{\sigma + r_i}} = 0,\ \ \forall x \neq 0, i = 1, \ldots, k. \tag{6}$$

Definition 3: $\mathrm{sig}(\cdot) : R^k \to R^k$, is an odd function that is defined as

$$\mathrm{sig}(y)^\alpha = \big(|y_1|^\alpha \mathrm{sgn}(y_1), \ldots, |y_k|^\alpha \mathrm{sgn}(y_k)\big)^T, \alpha > 0$$

where $|y_i|$ is the absolute value of y_i, $\mathrm{sgn}(\cdot)$ is the sign function, and $y = (y_1, y_2, \ldots, y_k)^T$ for $i = 1, 2, \ldots, k$.

Definition 4: Second-order consensus in multiagent system (1) is said to be achieved if for any initial conditions

$$\lim_{t \to \infty} \|x_i(t) - x_j(t)\| = 0, \quad \lim_{t \to \infty} \|v_i(t) - v_j(t)\| = 0 \quad \forall i, j = 1, 2, \ldots, N.$$

Definition 5 [41]: The upper right-hand derivative $D^+ v(t)$ is defined by

$$D^+ v(t) = \limsup_{h \to 0^+} \frac{v(t+h) - v(t)}{h}.$$

Definition 6 [41]: A point p is said to be a *positive limit point* of $x(t)$ if there is a sequence t_n, with $t_n \to \infty$ as $n \to \infty$, such that $x(t_n) \to p$ as $n \to \infty$. The set of all positive limit points of $x(t)$ is called the *positive limit set* of $x(t)$.

Lemma 1 [28]–[30]: Suppose system (4) is homogeneous of degree σ with dilation $(r_1, \ldots, r_k)$, nonlinear function g is continuous, and $y = 0$ is its asymptotically stable equilibrium. If homogeneity degree $\sigma < 0$, the equilibrium of system (4) is globally finite-time stable. Besides, if (6) holds, then the equilibrium of system (5) is locally finite-time stable. Moreover, if the equilibrium of system (5) is globally asymptotically stable and locally finite-time convergent, it is globally finite-time stable.

Lemma 2 [39]: Let $y(t)$ be a solution of $\dot{y} = f(y)$ with $y(0) = y_0 \in R^k$, where $f : U \to R^k$ is continuous with an open subset U of R^k, and let $V : U \to R$ be a locally Lipschitz function such that $D^+ V(x(t)) \leq 0$, where D^+ denotes the upper Dini derivative. Then, with denoting the positive limit set as $\Lambda^+(x_0)$, $\Lambda^+(x_0) \cap U$ is contained in the union of all solutions that remain in $S = \{x \in U : D^+ V(x(t)) = 0\}$.

The above lemma is an extended version of LaSalle invariance principle under the nonsmooth case.

Lemma 3 [40]: If $a_1, a_2, \ldots, a_n \geq 0$ and $0 < p < q$, then

$$\left(\sum_{i=1}^n a_i^q\right)^{1/q} \leq \left(\sum_{i=1}^n a_i^p\right)^{1/p}.$$

III. Design of the Sliding-Mode Surface

The sliding-mode variables or states can be designed in the form as follows:

$$s_i = \ddot{x}_i - \sum_{j=1}^N a_{ij}\big[\psi_1\big(\mathrm{sig}\big(x_j - x_i\big)^{\alpha_1}\big) + \psi_2\big(\mathrm{sig}\big(\dot{x}_j - \dot{x}_i\big)^{\alpha_2}\big)\big]$$

where α_1 and α_2 satisfy $0 < \alpha_1 < 1$, $\alpha_2 = [2\alpha_1/(1+\alpha_1)]$, and $\psi_1, \psi_2 : R^n \to R^n$, are two continuous odd functions satisfying $y_i^T \psi_k(y_i) > 0$ when $y_i \neq 0$ and $\psi_k(y_i) = c_{ki} y_i + o(y_i)$ around $y_i = 0$ with $c_{ki} > 0$, $k = 1, 2$; $i = 1, 2, \ldots, N$. What is more, we denote $\psi_k(y_i) = [\psi_k(y_{i1}), \psi_k(y_{i2}), \ldots, \psi_k(y_{in})]^T$, if $y_i = (y_{i1}, \ldots, y_{in})^T \in R^n$, for simplification.

According to system (1), the designed sliding-mode states can be rewritten into an equivalent form

$$s_i = \ddot{x}_i - \sum_{j=1}^N a_{ij}\big[\psi_1\big(\mathrm{sig}\big(x_j - x_i\big)^{\alpha_1}\big) + \psi_2\big(\mathrm{sig}\big(v_j - v_i\big)^{\alpha_2}\big)\big] \tag{7}$$

and accordingly, the sliding-mode surface is

$$\Big\{\big(x_1^T, \ldots, x_N^T, v_1^T, \ldots, v_N^T\big)^T | s_i = 0,\ \forall i = 1, 2, \ldots, N\Big\}. \tag{8}$$

Theorem 1: If the states in system (1) can reach the designed sliding-mode surface in (8), then second-order consensus in multiagent system (1) can be achieved in finite time.

Proof: Some simplified analysis can be found in [29] for $n = 1$. Consider the states on the sliding-mode surface, one has

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = \hat{u}_i, \end{cases} \qquad i = 1, \ldots, N \tag{9}$$

where $\hat{u}_i = \sum_{j=1}^N a_{ij}[\psi_1(\mathrm{sig}(x_j - x_i)^{\alpha_1}) + \psi_2(\mathrm{sig}(v_j - v_i)^{\alpha_2})]$. The finite-time consensus problem of system (9) is equivalent to the finite-time stable problem of the system as follows:

$$\begin{cases} \dot{e}_i^1 = e_i^2, \\ \dot{e}_i^2 = \hat{u}_i - \bar{u}, \end{cases} \qquad i = 1, \ldots, N \tag{10}$$

where $e_i^1 \triangleq (e_{i1}^1, \ldots, e_{in}^1)^T = x_i - (1/N)\sum_{j=1}^N x_j$, $e_i^2 \triangleq (e_{i1}^2, \ldots, e_{in}^2)^T = v_i - (1/N)\sum_{j=1}^N v_j$, and $\bar{u} = (1/N)\sum_{j=1}^N \hat{u}_j$, $i = 1, \ldots, N$. Under the error system (10), the control $\hat{u}_i$ can be written as

$$\hat{u}_i = \sum_{j=1}^N a_{ij}\Big[\psi_1\Big(\mathrm{sig}\Big(e_j^1 - e_i^1\Big)^{\alpha_1}\Big) + \psi_2\Big(\mathrm{sig}\Big(e_j^2 - e_i^2\Big)^{\alpha_2}\Big)\Big].$$

In addition, since the network is undirected and ψ_1, ψ_2 are two continuous odd functions, one has

$$\bar{u} = \frac{1}{N}\sum_{i=1}^N \sum_{j=1}^N a_{ij}\big[\psi_1\big(\mathrm{sig}\big(x_j - x_i\big)^{\alpha_1}\big) + \psi_2\big(\mathrm{sig}\big(v_j - v_i\big)^{\alpha_2}\big)\big] = 0.$$

A Lyapunov function is constructed in the following form:

$$V=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{n}\int_{0}^{e_{ik}^{1}-e_{jk}^{1}} a_{ij}\psi_1\big(\mathrm{sig}(y)^{\alpha_1}\big)dy + \frac{1}{2}\sum_{i=1}^{N}\Big(e_i^2\Big)^T e_i^2. \quad (11)$$

By noticing that $a_{ij}=a_{ji}$ and $\psi_1(\mathrm{sig}(e_{ik}^1-e_{jk}^1)^{\alpha_1}) = -\psi_1(\mathrm{sig}(e_{jk}^1-e_{ik}^1)^{\alpha_1})$, one can calculate the derivative of the Lyapunov function as follows:

$$\begin{aligned}\dot V &= \sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}\Big(e_i^2\Big)^T\psi_1\Big(\mathrm{sig}\Big(e_i^1-e_j^1\Big)^{\alpha_1}\Big)+\sum_{i=1}^{N}\Big(e_i^2\Big)^T\dot e_i^2\\
&=\sum_{i=1}^{N}\Big(e_i^2\Big)^T\sum_{j=1}^{n}a_{ij}\psi_1\Big(\mathrm{sig}\Big(e_i^1-e_j^1\Big)^{\alpha_1}\Big)+\sum_{i=1}^{N}\Big(e_i^2\Big)^T\\
&\quad\times\sum_{j=1}^{N}a_{ij}\Big[\psi_1\Big(\mathrm{sig}\Big(e_j^1-e_i^1\Big)^{\alpha_1}\Big)+\psi_2\Big(\mathrm{sig}\Big(e_j^2-e_i^2\Big)^{\alpha_2}\Big)\Big]\\
&=\sum_{i=1}^{N}\sum_{j=1}^{N}\Big(e_i^2\Big)^T a_{ij}\psi_2\Big(\mathrm{sig}\Big(e_j^2-e_i^2\Big)^{\alpha_2}\Big)\\
&=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\Big[\Big(a_{ij}+a_{ji}\Big)\Big(e_i^2\Big)^T\psi_2\Big(\mathrm{sig}\Big(e_j^2-e_i^2\Big)^{\alpha_2}\Big)\Big]\\
&=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\Big[\Big(e_i^2-e_j^2\Big)^T a_{ij}\psi_2\Big(\mathrm{sig}\Big(e_j^2-e_i^2\Big)^{\alpha_2}\Big)\Big]\\
&=\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{n}\Big[\Big(e_{ik}^2-e_{jk}^2\Big)^T a_{ij}\psi_2\Big(\mathrm{sig}\Big(e_{jk}^2-e_{ik}^2\Big)^{\alpha_2}\Big)\Big].\end{aligned} \quad (12)$$

Since ψ_2 is a continuous odd function, one has

1) $\forall e_{jk}^2-e_{ik}^2\neq 0\in R$

$$\Big(e_{ik}^2-e_{jk}^2\Big)\mathrm{sig}\Big(e_{jk}^2-e_{ik}^2\Big)^{\alpha_2}=-|e_{jk}^2-e_{ik}^2|^{1+\alpha_2}<0.$$

2) Around $e_{jk}^2-e_{ik}^2=0$

$$\Big(e_{ik}^2-e_{jk}^2\Big)\Big[c_i\mathrm{sig}\Big(e_{jk}^2-e_{ik}^2\Big)^{\alpha_2}+o\Big(e_{jk}^2-e_{ik}^2\Big)\Big]<0.$$

According to the above results, the derivative of V satisfies

$$\dot V\le 0$$

which means $V(t)\le V(0)$. Since $V(t)$ is radially unbounded over e_i^1 and e_i^2, it can be obtained from (11) that e_i^1 and e_i^2 are bounded. Then, one can analyze the second-order consensus by using LaSalle invariance principle from Lemma 2. Since the Lyapunov function constructed in (11) is smooth, the Dini derivative here is the regular derivative. Set the invariant set $S=\{((e_1^1)^T,\ldots,(e_N^1)^T,(e_1^2)^T,\ldots,(e_N^2)^T)^T|\dot V=0\}$. As the undirected graph is connected, if $\dot V\equiv 0$, e_i^2 must be equal to e_j^2, $\forall j\neq i$. Under the condition that $e_i^2\equiv e_j^2$, $\forall j\neq i$, the control law becomes

$$\hat u_i=\sum_{j=1}^{N}a_{ij}\psi_1\Big(\mathrm{sig}\Big(e_j^1-e_i^1\Big)^{\alpha_1}\Big).$$

In addition, one has that $\sum_{i=1}^N\hat u_i\equiv 0$ as $a_{ij}=a_{ji}$. Then one has $\sum_{i=1}^N(e_i^1)^T\sum_{j=1}^N a_{ij}\psi_1(\mathrm{sig}(e_j^1-e_i^1)^{\alpha_1}))\equiv 0$, which turns to be $(1/2)\sum_{i=1}^N(e_j^1-e_i^1)^T\sum_{j=1}^N a_{ij}\psi_1(\mathrm{sig}(e_j^1-e_i^1)^{\alpha_1}))\equiv 0$.

Since the graph is connected, one can get that $e_j^1=e_i^1$, $\forall j\neq i$. Thus, we find that $e_j^1=e_i^1$ and $e_j^2=e_i^2$. By using Lemma 2, $e_i^1-e_j^1\to 0$, $e_i^2-e_j^2\to 0$ as $t\to\infty$, which is equivalent to $x_i-x_j\to 0$, $v_i-v_j\to 0$ as $t\to\infty$, $\forall i, j=1,2,\ldots,N$.

Therefore, one finally has that the error system (10) is asymptotically stable. In order to reach finite-time consensus, the homogeneity with dilation will be used according to Lemma 1 in the next part.

From the assumptions of the odd functions ψ_1 and ψ_2 ($\psi_i(y)=c_{ki}y+o(y)$ around $y=0$ with $c_{ki}>0$ for $k=1,2; i=1,2,\ldots,N$), the protocol $\hat u_i$ in system (9) can be rewritten as follows:

$$\hat u_i^1=\sum_{j=1}^{N}a_{ij}\Big[c_{1i}\mathrm{sig}\Big(e_j^1-e_i^1\Big)^{\alpha_1}+c_{2i}\mathrm{sig}\Big(e_j^2-e_i^2\Big)^{\alpha_2}\Big]$$

$$\hat u_i^2=\sum_{j=1}^{N}a_{ij}\Big[o\Big(\mathrm{sig}\Big(e_j^1-e_i^1\Big)^{\alpha_1}\Big)+o\Big(\mathrm{sig}\Big(e_j^2-e_i^2\Big)^{\alpha_2}\Big)\Big]$$

where $\hat u_i=\hat u_i^1+\hat u_i^2$ for $i=1,2,\ldots,N$.

It is easy to find that system (9) with variables $((x_1)^T,\ldots,(x_N)^T,(v_1)^T,\ldots,(v_N)^T)^T$ is homogeneous of degree $\sigma=\alpha_1-1<0$ with dilation $(\underbrace{2,..,2}_{nN},\underbrace{1+\alpha_1,\ldots,1+\alpha_1}_{nN})$. Thus, system (9) is locally homogeneous of degree σ with the same dilation under the protocol $\hat u_i=\hat u_i^1+\hat u_i^2$. Through Lemma 1, system (10) with variables $((e_1^1)^T,\ldots,(e_N^1)^T,(e_1^2)^T,\ldots,(e_N^2)^T)^T$ is also locally homogeneous of degree σ with dilation $(\underbrace{2,..,2}_{nN},\underbrace{1+\alpha_1,\ldots,1+\alpha_1}_{nN})$.

Since system (10) is globally asymptotically stable and locally homogeneous of degree $\sigma<0$, by Lemma 1, system (10) is globally finite-time stable, which indicates that second-order consensus in system (1) can be reached in finite time. Therefore, system (1) can reach second-order consensus along the designed sliding-mode surface in (8) within finite time. ■

Remark 2: In this section, a new distributed full-order sliding-mode surface is designed in (8), which is based on the homogeneity with dilation. Once the states reach the sliding-mode surface, then second-order consensus in multiagent system (1) can be achieved in finite time. The next objective is to design an SMC such that the states of agents can reach the designed sliding-mode surface in finite time. Interestingly, it will be shown that the sliding-mode states are decoupled which indicates that the SMC designed in this paper is distributed. Thus, under the DSMC, the states of agents can first reach the designed sliding-mode surface in finite time and then move to the second-order consensus state along the surface in finite time as well.

IV. Design of the Sliding-Mode Control

In this section, an SMC is designed to force the system trajectory to reach the sliding-mode surface (8) in finite time. From (7), the sliding-mode state s_i for agent i only depends on the relative position and velocity states between the neighboring agents and its own, which is a distributed variable, $i = 1, 2, \ldots, N$. Next, by using the sliding-mode states s_i, a new DSMC is designed as follows where the sliding-mode states are fully decoupled, $i = 1, 2, \ldots, N$. Let the control input in system (1) be designed by

$$u_i(t) = b_i^{-1}(x_i, v_i, t)\big(u_{eqi}(t) + u_{ni}(t)\big) \tag{13}$$

where

$$u_{eqi}(t) = -f(x_i, v_i, t) + \sum_{j=1}^{N} a_{ij}\Big[\psi_1\big(sig\big(x_j - x_i\big)^{\alpha_1}\big) + \psi_2\big(sig\big(v_j - v_i\big)^{\alpha_2}\big)\Big] \tag{14}$$

$$\dot{u}_{ni}(t) + p_i u_{ni}(t) = -k_i \mathrm{sgn}(s_i(t)) \tag{15}$$

$p_i > 0$ and $k_i > 0$ are positive, and $k_i > \beta_i + \bar{\alpha}_i p_i + \varepsilon_i$ with $\varepsilon_i > 0$.

Theorem 2: Under Assumptions 1 and 2, the states of multiagent system (1) can reach the sliding-mode surface in (7) and (8) in finite time, if the SMC is designed as (13)–(15). The states of multiagent system (1) will achieve second-order consensus in finite time along the sliding-mode surface (7) and (8).

Proof: The proof will be divided into two parts.

Part I (The Reaching Phase): From multiagent system (1), the sliding-mode variable (7) can be represented in the following form:

$$\begin{aligned} s_i(t) &= \ddot{x}_i - \sum_{j=1}^{N} a_{ij}\big[\psi_1\big(\mathrm{sig}\big(x_j - x_i\big)^{\alpha_1}\big) + \psi_2\big(\mathrm{sig}\big(\dot{x}_j - \dot{x}_i\big)^{\alpha_2}\big)\big] \\ &= f(x_i, v_i, t) + d_i(x_i, v_i, t) + b_i(x_i, v_i, t)u_i \\ &\quad - \sum_{j=1}^{N} a_{ij}\big[\psi_1\big(\mathrm{sig}\big(x_j - x_i\big)^{\alpha_1}\big) + \psi_2\big(\mathrm{sig}\big(\dot{x}_j - \dot{x}_i\big)^{\alpha_2}\big)\big]. \end{aligned} \tag{16}$$

Substituting (13) and (14) into (16), one has

$$s_i(t) = d_i(x_i, v_i, t) + u_{ni}(t). \tag{17}$$

Then, by simple calculation, one obtains

$$\begin{aligned} \dot{s}_i(t) &= \dot{d}_i(x_i, v_i, t) + \dot{u}_{ni}(t) \\ &= \dot{d}_i(x_i, v_i, t) + \dot{u}_{ni}(t) + p_i u_{ni}(t) - p_i u_{ni}(t) \\ &= \dot{d}_i(x_i, v_i, t) - k_i \mathrm{sgn}(s_i(t)) - p_i u_{ni}(t). \end{aligned} \tag{18}$$

Let $d_i(t) = d_i(x_i, v_i, t)$ for simplification. Hence

$$s_i^T(t)\dot{s}_i(t) = s_i^T(t)\dot{d}_i(t) - k_i|s_i(t)| - p_i s_i^T(t)u_{ni}(t).$$

In the following, the relation between $\bar{\alpha}_i$ and $u_{ni}(t)$ is discussed.

From (15) and (17), one obtains

$$\dot{u}_{ni}(t) + p_i u_{ni}(t) = -k_i \mathrm{sgn}(u_{ni}(t) + d_i(t)). \tag{19}$$

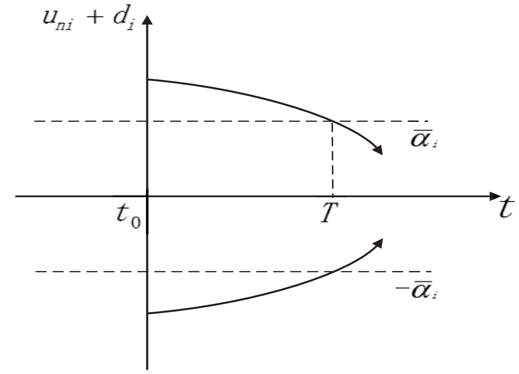


Fig. 1. Movement trajectory of $u_{ni} + d_i$ with time t.

Add $\dot{d}_i + p_i d_i$ into the two sides of (19), one has

$$(u_{ni} + d_i)' + p_i(u_{ni} + d_i) = -k_i \mathrm{sgn}(s_i) + \dot{d}_i + p_i d_i \tag{20}$$

which is

$$\big(e^{p_i t}(u_{ni} + d_i)\big)' = e^{p_i t}\big(-k_i \mathrm{sgn}(u_{ni} + d_i) + \dot{d}_i + p_i d_i\big).$$

Let $(u_{ni} + d_i)_k$ be the kth element of vector $u_{ni} + d_i$, $k = 1, 2, \ldots, n$. Then, for any k, three cases will be analyzed in the following.

Case 1: If $s_{ik} = (u_{ni} + d_i)_k > 0$

$$\mathrm{sgn}(u_{ni} + d_i)_k = 1.$$

Since $k_i > \beta_i + p_i\bar{\alpha}_i$, one has

$$-k_i \mathrm{sgn}(u_{ni} + d_i)_k + \big(\dot{d}_i\big)_k + p_i(d_i)_k < 0.$$

Hence

$$\big(e^{p_i t}(u_{ni} + d_i)_k\big)' < 0$$

which indicates that $e^{p_i t}(u_{ni} + d_i)_k$ decreases in terms of time t if $s_{ik} = (u_{ni} + d_i)_k > 0$. Since $e^{p_i t}$ is always positive and increases in terms of t, then one has that $(u_{ni} + d_i)_k$ decreases once $s_{ik} = (u_{ni} + d_i)_k > 0$.

Case 2: If $s_{ik} = (u_{ni} + d_i)_k < 0$

$$\mathrm{sgn}(u_{ni} + d_i)_k = -1.$$

Since $k_i > \beta_i + p_i\bar{\alpha}_i$, one has

$$-k_i \mathrm{sgn}(u_{ni} + d_i)_k + \big(\dot{d}_i\big)_k + p_i(d_i)_k > 0.$$

Hence

$$\big(e^{p_i t}(u_{ni} + d_i)_k\big)' > 0.$$

Since $s_{ik} = (u_{ni} + d_i)_k < 0$, similarly as case 1, $(u_{ni} + d_i)_k$ increases once $s_{ik} = (u_{ni} + d_i)_k < 0$.

Case 3: If $s_{ik} = (u_{ni} + d_i)_k = 0$, then the sliding-mode surface in (8) is reached, which solves the problem.

Next, one aims to prove that for any $k = 1, 2, \ldots, n$, there exists a time T, such that $|(u_{ni}(t))_k| \le \bar{\alpha}_i$ for all $t > T$. Suppose that $(u_{ni}(t_0))_k > \bar{\alpha}_i$ at some time t_0. Then, according to $|(d_i(t_0))_k| \le \bar{\alpha}_i$, one has $(u_{ni}(t_0) + d_i(t_0))_k > 0$. From case 1 and (20), one obtains $(u_{ni} + d_i)_k$ decreases and $\dot{u}_{ni} \le -k_i$ once $(u_{ni}(t_0))_k > \bar{\alpha}_i$ at time t_0, which indicates that u_{ni} can reach $\bar{\alpha}_i$ at some finite time T, that is, $u_{ni}(T) = \bar{\alpha}_i$. For the time

$t > T$, the trajectory will be discussed with the help of Fig. 1 in the following.

1) If $(u_{ni}(t) + d_i(t))_k > 0$ for $t > T$, it will decrease which means its trajectory will move down at the next moment.
2) If $(u_{ni}(t) + d_i(t))_k < 0$ for $t > T$, it will increase which means its trajectory will move up at the next moment.

Once $u_{ni}(t)$ approaches the bound $\bar{\alpha}_i$ or $-\bar{\alpha}_i$, it will decrease or increase according to (19). Thus, $|(u_{ni}(t))_k| \leq \bar{\alpha}_i$ for all $t > T$. To conclude, $u_{ni}(t)$ satisfies $|(u_{ni}(t))_k| \leq \bar{\alpha}_i$ for all $t > T$.

According to (2), (3), and (15), the following relationship can be obtained:

$$\begin{aligned} s_i^T(t)\dot{s}_i(t) &= s_i^T(t)\dot{d}_i(t) - k_i 1_n^T|s_i(t)| - p_i s_i^T(t)u_{ni}(t) \\ &\leq \left(s_i^T(t)\dot{d}_i(t) - \beta_i 1_n^T|s_i(t)|\right) \\ &\quad + \left(-p_i s_i^T(t)u_{ni}(t) + p_i\bar{\alpha}_i 1_n^T|s_i(t)|\right) \\ &\quad - (k_i - \beta_i - \bar{\alpha}_i p_i)1_n^T|s_i(t)| \end{aligned} \tag{21}$$

where 1_n^T is a vector with all the elements being 1.

A Lyapunov function can be represented in the form: $V(t) = (1/2)s^T s$, where $s = (s_1{}^T, s_2{}^T, \ldots, s_n{}^T)^T$. Under Assumptions 1 and 2 as well as the condition $k_i > \beta_i + \bar{\alpha}_i p_i + \varepsilon_i$, the derivative of $V(t)$ with respect to time t can be derived as follows from Lemma 3:

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{N} s_i^T(t)\dot{s}_i(t) \\ &\leq -\sum_{i=1}^{N} \varepsilon_i 1_n^T|s_i(t)| \\ &\leq -\varepsilon_{\min}\sum_{i=1}^{N}\sum_{i=1}^{N}\left|s_{ij}(t)\right| \\ &\leq -\varepsilon_{\min}\sqrt{V(t)} \end{aligned}$$

which means that the states of multiagent system (1) can reach the sliding-mode surface in (7) and (8) in finite time, if the SMC is designed as (13)–(15).

Part II (On Sliding Surface): After the sliding surface (7) and (8) is reached, the states of multiagent system (1) will stay on this sliding surface henceforward. Then, the second-order consensus can be achieved in finite time followed directly by Theorem 1. ■

Remark 3: Note that in the design of DSMC (13)–(15), $u_{eqi}(t)$ and $u_{ni}(t)$ are both continuous since their derivatives exist, which indicate that $u_i(t)$ in (13) is also continuous. In addition, from the design of the sliding-mode surface (7) and (8), the calculation of the derivative of $s_i(t)$ is prevented as in (18) for $i = 1, 2, \ldots, N$. Thus, the DSMC here in this paper can avoid singularity problems and can weaken the influence caused by chattering.

A simplified version is also given where $p_i = 0$.

Corollary 1: Under Assumptions 1 and 2, the states of multiagent system (1) can reach the sliding-mode surface in (7) and (8) in finite time, if the SMC is designed as (13), (14), and

$$\dot{u}_{ni}(t) = -k_i \operatorname{sgn}(s_i(t)) \tag{22}$$

where $k_i > 0$ is positive, and $k_i > \beta_i + \varepsilon_i$ with $\varepsilon_i > 0$.

Proof: Let $p_i = 0$ in (15) of Theorem 2, one can easily get the result. ■

Note that from (18), $\dot{s}_i(t) = \dot{d}_i(x_i, v_i, t) - k_i \operatorname{sgn}(s_i(t)) - p_i u_{ni}(t)$, where the negative feedback for $u_{ni}(t)$ may not be good for control. Next, the term $s_i(t)$ is used instead of $u_{ni}(t)$.

Corollary 2: Under Assumptions 1 and 2, the states of multiagent system (1) can reach the sliding-mode surface in (7) and (8) in finite time, if the SMC is designed as (13), (14), and

$$\dot{u}_{ni}(t) = -q_i s_i(t) - k_i \operatorname{sgn}(s_i(t)) \tag{23}$$

where $q_i > 0$ and $k_i > 0$ are positive, and $k_i > \beta_i + \varepsilon_i$ with $\varepsilon_i > 0$.

Proof: The result can be easily proved as follows:

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{N} s_i^T(t)\dot{s}_i(t) \\ &\leq -\sum_{i=1}^{N} \varepsilon_i 1_n^T|s_i(t)| - q_i s_i^T(t)s_i(t) \\ &\leq -\varepsilon_{\min}\sqrt{V(t)}. \end{aligned} \tag{24}$$

■

Remark 4: The designed SMC in (13)–(15) is distributed, where only local relative position $(x_i(t) - x_j(t))$ and velocity $(\dot{x}_i(t) - \dot{x}_j(t))$ information of neighboring agents as well as $\operatorname{sgn}(s_i(t))$ are utilized. Here, s_i depends on the variable $\ddot{x}_i$, which should be known. However, only $\operatorname{sgn}(s_i(t))$ is used rather than the exact value $s_i(t)$, which can be calculated by defining a new function $g_i(t)$ [38]

$$\begin{aligned} g_i(t) &= \int_0^t s_i(t)dt \\ &= \dot{x}_i \\ &\quad - \int_0^t \sum_{j=1}^{N} a_{ij}\left[\psi_1\left(\operatorname{sig}\left(x_j - x_i\right)^{\alpha_1}\right) + \psi_2\left(\operatorname{sig}\left(v_j - v_i\right)^{\alpha_2}\right)\right]dt. \end{aligned}$$

Then, the value of $\operatorname{sgn}(s_i(t))$ can be calculated from the following equation:

$$\operatorname{sgn}(s_i) = \operatorname{sgn}(g_i(t) - g_i(t - \tau)) \tag{25}$$

where τ represents a time delay. This is reasonable since $s_i(t) = \lim_{\tau \to 0^+} {(g_i(t) - g_i(t-\tau))}/{\tau}$. Thus, instead of knowing the exact value of s_i, we only needed to see whether g_i increases or decreases. Usually, it is easier to obtain $\operatorname{sgn}(s_i(t))$ than $s_i(t)$.

V. Simulation Examples

In this section, some simulation examples are given to verify the theoretical analysis in this paper.

Consider a multiagent system with five agents, and the corresponding graph is shown in Fig. 2. The adjacency matrix is defined as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

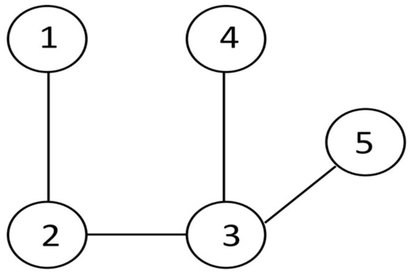


Fig. 2. Undirected graph with five agents.

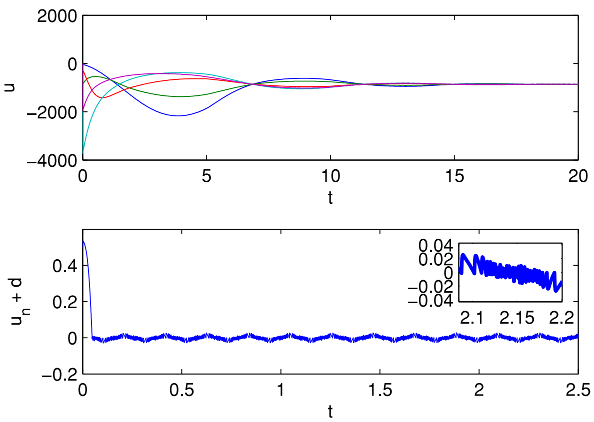


Fig. 3. Trajectory of $u_{ni}+d_i$ with time t when $p_i = 0$, using (13)–(15) under LAHF disturbances

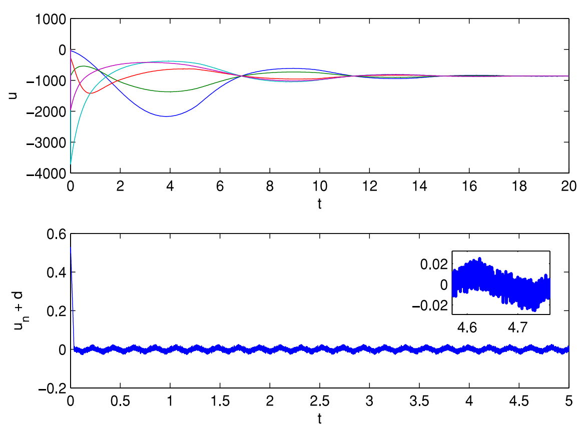


Fig. 4. Trajectory of $u_{ni} + d_i$ with time t when $p_i = 10$, using (13)–(15) under LAHF disturbances.

Examples 1 and 2 are provided to show the role of the parameter p_i in the design of the controller $u_{ni}(t)$ based on Theorem 2 and Corollary 1, and Example 3 is given to show another design for $u_{ni}(t)$ as in Corollary 2.

A. Example 1

Consider the dynamics of system (1) below

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = v_i^3 + \sin(30t) + u_i(t), \end{cases} \quad i = 1, \ldots, N \tag{26}$$

where $d_i(x_i, v_i, t) = \sin(30t)$ and $b_i(t) = 1$. Here, $u_i(t) = u_{eqi}(t) + u_{ni}(t)$ and the controller $u_{ni}(t)$ is designed in (15) as follows:

$$\dot{u}_{ni} + p_i u_{ni} = -k_i \text{sgn}(s_i)$$

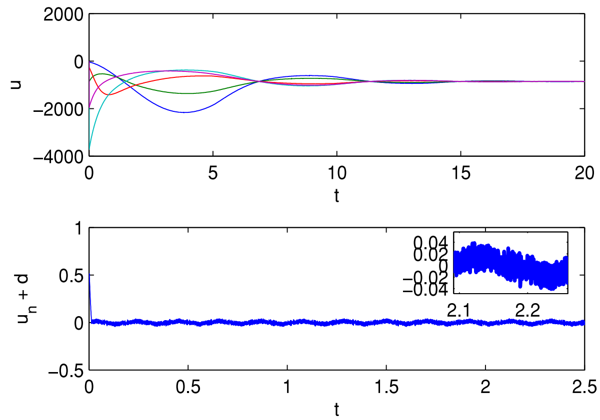


Fig. 5. Trajectory of $u_{ni} + d_i$ with time t when $p_i = 40$.

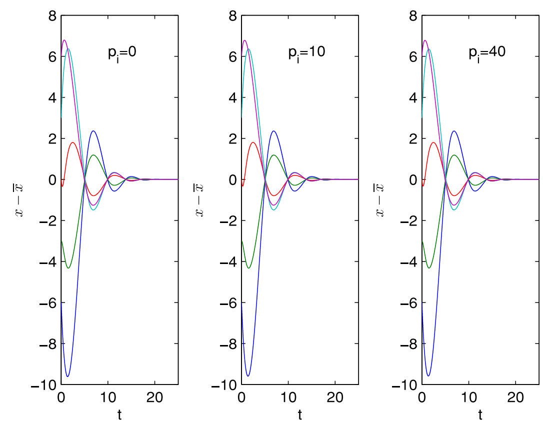


Fig. 6. Trajectory of state error $x_i - \bar{x}$ with time t with different p_i, using (13)–(15) under LAHF disturbances.

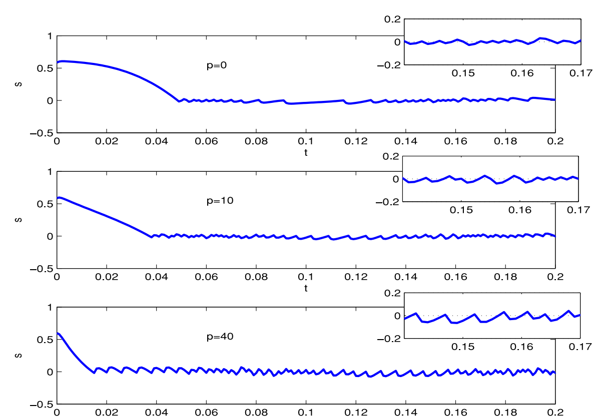


Fig. 7. Trajectory of sliding variable s_i with time t with different p_i.

where $k_i > \beta_i + \bar{\alpha}_i p_i + \varepsilon_i$, with $\bar{\alpha}_i = 1$ and $\beta_i = 30$ because $|d_i(x_i, v_i, t)| \leq 1$ and $|\dot{d}(x_i, v_i, t)| \leq 30$. Choose $\varepsilon_i = 1$ and set $k_i = \beta_i + \bar{\alpha}_i p_i + \varepsilon_i + 1$. Based on Theorem 2, one can choose three different parameters p_i to find out how p_i influences the controller $u_{ni}(t)$: 1) $p_i = 0$; 2) $p_i = 10$; and 3) $p_i = 40$.

The simulation results are illustrated in Figs. 3–7, among which, Figs. 3–5 show the trajectories of $u_{ni} + d_i$ with time t corresponding to $p_i = 0$, $p_i = 10$ and $p_i = 40$, respectively, and Figs. 6 and 7 depict the trajectory of state error $x_i - \bar{x}$ and sliding variable s_i with time t for different p_i,

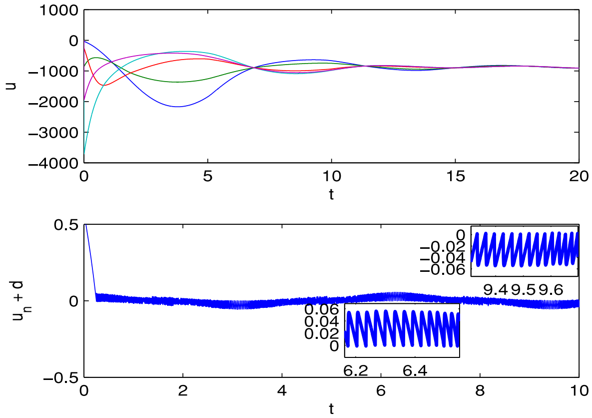


Fig. 8. Trajectory of $u_{ni}+d_i$ with time t when $p_i = 0$, using (13)–(15) under HALF disturbances.

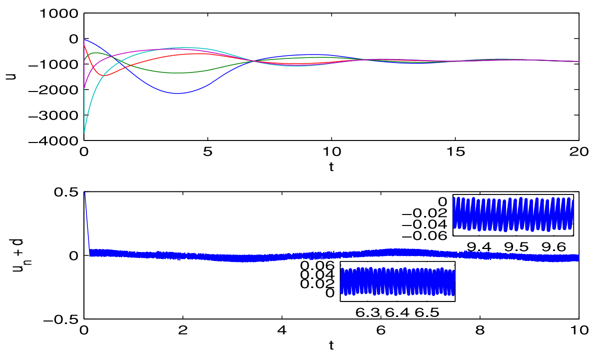


Fig. 9. Trajectory of $u_{ni} + d_i$ with time t when $p_i = 0.1$, using (13)–(15) under HALF disturbances.

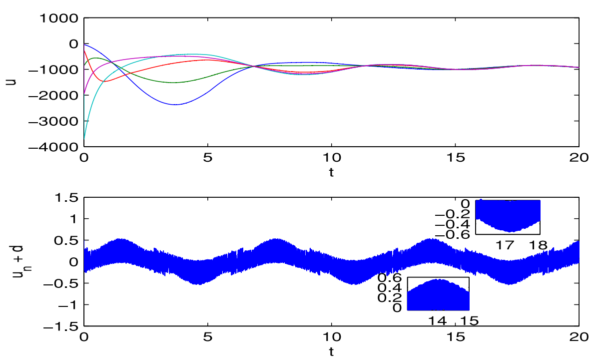


Fig. 10. Trajectory of $u_{ni} + d_i$ with time t when $p_i = 10$, using (13)–(15) under HALF disturbances.

respectively. Here, $\bar{x} = \sum_{j=1}^{N} x_j/N$. One can find that the state of $u_{ni} + d_i$ changes little even when p_i takes different values. What is more, Fig. 6 shows that the time for reaching consensus is almost the same for different p_i. Thus, if d_i takes a small value but a large derivative $\dot{d}_i$, u_{ni} is not sensitive to the value of p_i, which doesn't influence the consensus much. From Figs. 3–5, one can see that the states of control input $u(t) = (u_1^T(t), u_2^T(t), \ldots, u_N^T(t))^T$ are continuous, which can weaken chattering behavior essentially.

B. Example 2

Consider the dynamics of system (1) below

$$\begin{cases} \dot{x}_i = v_i, \\ \dot{v}_i = v_i^3 + 30\sin(0.1t) + u_i(t), \end{cases} \qquad i = 1, \ldots, N. \qquad (27)$$

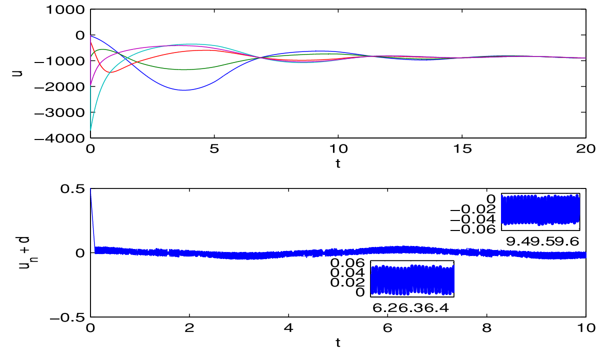


Fig. 11. Trajectory of $u_{ni} + d_i$ with time t when $p_i = 0.1$, using (23) under HALF disturbances.

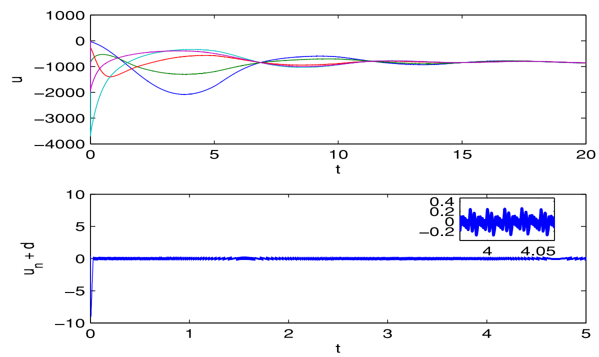


Fig. 12. Trajectory of $u_{ni} + d_i$ with time t when $p_i = 10$, using (23) under HALF disturbances.

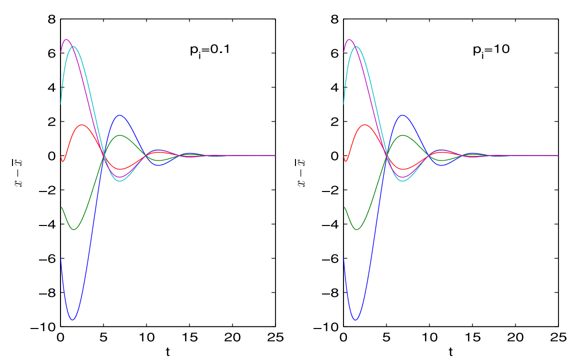


Fig. 13. Trajectory of state error $x_i - \bar{x}$ with time t with different p_i, using (23) under HALF disturbances.

Similar to Example 1, set $k_i = \beta_i + \bar{\alpha}_i p_i + \varepsilon_i + 1$, where $\bar{\alpha}_i = 30$, $\beta_i = 3$, and $\varepsilon_i = 1$. Here, we still choose three different values of p_i: 1) $p_i = 0$; 2) $p_i = 0.1$; and 3) $p_i = 10$. The simulation results are illustrated in Figs. 8–10, which show the trajectories of $u_{ni} + d_i$ with time t corresponding to $p_i = 0$, $p_i = 0.1$, and $p_i = 10$, respectively. One can find that the state of $u_{ni} + d_i$ changes much when p_i takes different values. What is more, when $p_i = 10$, the effect of compensating for d_i is unsatisfactory. Thus, if d_i takes a large value but a small derivative $\dot{d}_i$, u_{ni} is sensitive to the value of p_i.

C. Example 3

Consider the system in Example 2, and u_{ni} is designed as in Corollary 2, where $k_i = \beta_i + \bar{\alpha}_i q_i + \varepsilon_i + 1$ with $\bar{\alpha}_i = 30$, $\beta_i = 3$, and $\varepsilon_i = 1$. Two cases for different q_i are considered: 1) $q_i = 0.1$ and 2) $q_i = 10$. Figs. 11 and 12 show the trajectory of $u_{ni} + d_i$ with time t corresponding to $q_i = 0.1$ and $q_i = 10$, respectively. Fig. 13 shows the trajectory of state error $x_i - \bar{x}$ with time t for different q_i and HALF stands for high amplitude low frequency. One can find that u_{ni} compensates for d_i better when q_i takes a large value. Fig. 13 shows that such a controller design can also achieve consensus, which is easier when computing $u_{ni}(t)$.

VI. Conclusion

In this paper, a new decoupled DSMC has been first proposed for second-order consensus in multiagent systems, which finally solves the fundamental unknown problem for SMC design of coupled networked systems. A distributed full-order sliding-mode surface has been designed based on the homogeneity with dilation for reaching second-order consensus in multiagent systems in finite time, under which the sliding-mode states are decoupled. Then, the SMC has been applied on the decoupled sliding-mode states to reach the sliding-mode surface also in finite time. Thus, the new DSMC can force the states of agents first to reach the designed sliding-mode surface in finite time and then move to the second-order consensus state along the surface in finite time as well. The DSMC designed in this paper can eliminate the influence of singularity problems and can weaken the influence of chattering, which are still very difficult in the SMC design.

The main contribution of this paper is that DSMC proposes a general decoupling framework for designing SMC in networked multiagent systems with uncertainties, which can serve as a basic foundation for finite-time control of multiagent systems. In the future, the applications of the proposed DSMC to multiple Lagrange systems and multivehicle formation for better performance will be investigated. Furthermore, the multiagent systems with higher-order dynamics, nonlinear dynamics, stochastic noises, and some other uncertainties, will be considered as well as the multiagent systems with directed topologies, which is still more challenging nowadays.

References

[1] R. Olfati-Saber and R. M. Murray, “Consensus problems in networks of agents with switching topology and time-delays,” *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[2] M. Cao, A. S. Morse, and B. D. O. Anderson, “Reaching a consensus in a dynamically changing environment: A graphical approach,” *SIAM J. Control Optim.*, vol. 47, no. 2, pp. 575–600, Feb. 2008.

[3] A. Jadbabaie, J. Lin, and A. S. Morse, “Coordination of groups of mobile autonomous agents using nearest neighbor rules,” *IEEE Trans. Autom. Control*, vol. 48, no. 6, pp. 988–1001, Jun. 2003.

[4] W. Ren and R. Beard, *Distributed Consensus in Multi-Vehicle Cooperative Control: Theory and Applications* (Communications and Control Engineering). London, U.K.: Springer-Verlag, 2008.

[5] Y. Cao, W. Yu, W. Ren, and G. Chen, “An overview of recent progress in the study of distributed multi-agent coordination,” *IEEE Trans. Ind. Informat.*, vol. 9, no. 1, pp. 427–438, Feb. 2013.

[6] W. Xiong, W. Yu, J. Lü, and X. Yu, “Fuzzy modelling and consensus of nonlinear multiagent systems with variable structure,” *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 61, no. 4, pp. 1183–1191, Apr. 2014.

[7] G. Wen, Z. Duan, G. Chen, and W. Yu, “Consensus tracking of multi-agent systems with Lipschitz-type node dynamics and switching topologies,” *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 61, no. 2, pp. 499–511, Feb. 2014.

[8] Z. Feng, G. Hu, and G. Wen, “Distributed consensus tracking for multi-agent systems under two types of attacks,” *Int. J. Robust Nonlin. Control*, vol. 26, no. 5, pp. 896–918, 2016.

[9] W. Hu, L. Liu, and G. Feng, “Consensus of linear multi-agent systems by distributed event-triggered strategy,” *IEEE Trans. Cybern.*, vol. 46, no. 1, pp. 148–157, Jan. 2016.

[10] Y. Chen, H. Dong, J. Lü, X. Sun, and K. Liu, “Robust consensus of nonlinear multiagent systems with switching topology and bounded noises,” *IEEE Trans. Cybern.*, vol. 46, no. 6, pp. 1276–1285, Jun. 2016.

[11] A. Arenas, A. Diaz-Guilera, J. Kurths, Y. Moreno, and C. Zhou, “Synchronization in complex networks,” *Phys. Rep.*, vol. 469, no. 3, pp. 93–153, 2008.

[12] J. Lü, X. Yu, G. Chen, and D. Cheng, “Characterizing the synchronizability of small-world dynamical networks,” *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 51, no. 4, pp. 787–796, Apr. 2004.

[13] L. M. Pecora and T. L. Carroll, “Master stability functions for synchronized coupled systems,” *Phys. Rev. Lett.*, vol. 80, no. 10, pp. 2109–2112, 1998.

[14] X. F. Wang and G. Chen, “Synchronization in scale-free dynamical networks: Robustness and fragility,” *IEEE Trans. Circuits Syst. I, Fundam. Theory Appl.*, vol. 49, no. 1, pp. 54–62, Jan. 2002.

[15] W. Yu, J. Cao, and J. Lü, “Global synchronization of linearly hybrid coupled networks with time-varying delay,” *SIAM J. Appl. Dyn. Syst.*, vol. 7, no. 1, pp. 108–133, 2008.

[16] Q. Song, W. Yu, J. Cao, and F. Liu, “Reaching synchronization in networked harmonic oscillators with outdated position data,” *IEEE Trans. Cybern.*, vol. 46, no. 7, pp. 1566–1578, Jul. 2016.

[17] X. Liu, J. Cao, W. Yu, and Q. Song, “Nonsmooth finite-time synchronization of switched coupled neural networks,” *IEEE Trans. Cybern.*, vol. 46, no. 10, pp. 2360–2371, Oct. 2016.

[18] R. Olfati-Saber, “Flocking for multi-agent dynamic systems: Algorithms and theory,” *IEEE Trans. Autom. Control*, vol. 51, no. 3, pp. 401–420, Mar. 2006.

[19] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, “Novel type of phase transition in a system of self-driven particles,” *Phys. Rev. Lett.*, vol. 75, no. 6, pp. 1226–1229, Aug. 1995.

[20] C. W. Reynolds, “Flocks, herds and schools: A distributed behavioral model,” *Comput. Graph.*, vol. 21, no. 4, pp. 25–34, Jul. 1987.

[21] W. Yu, G. Chen, M. Cao, J. Lü, and H.-T. Zhang, “Swarming behaviors in multi-agent systems with nonlinear dynamics,” *Chaos*, vol. 23, no. 4, 2013, Art. no. 043118.

[22] W. Ren, “On consensus algorithms for double-integrator dynamics,” *IEEE Trans. Autom. Control*, vol. 53, no. 6, pp. 1503–1509, Jul. 2008.

[23] W. Yu, G. Chen, and M. Cao, “Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems,” *Automatica*, vol. 46, no. 6, pp. 1089–1095, 2010.

[24] W. Yu, G. Chen, M. Cao, and J. Kurths, “Second-order consensus for multiagent systems with directed topologies and nonlinear dynamics,” *IEEE Trans. Syst., Man, Cybern. B, Cybern.*, vol. 40, no. 3, pp. 881–891, Jun. 2010.

[25] W. Yu, W. X. Zheng, G. Chen, W. Ren, and J. Cao, “Second-order consensus in multi-agent dynamical systems with sampled position data,” *Automatica*, vol. 47, no. 7, pp. 1496–1503, 2011.

[26] L. Wang and F. Xiao, “Finite-time consensus problems for networks of dynamic agents,” *IEEE Trans. Autom. Control*, vol. 55, no. 4, pp. 950–955, Apr. 2010.

[27] S. Yang, J. Cao, and J. Lu, “A new protocol for finite-time consensus of detail-balanced multi-agent networks,” *Chaos*, vol. 22, no. 4, 2012, Art. no. 043134.

[28] Y. Hong, “Finite-time stabilization and stabilizability of a class of controllable systems,” *Syst. Control Lett.*, vol. 46, no. 4, pp. 231–236, 2002.

[29] X. Wang and Y. Hong, “Finite-time consensus for multi-agent networks with second-order agent dynamics,” in *Proc. 17th World Congr. (IFAC)*, Seoul, South Korea, 2008, pp. 15185–15190.

[30] Y. Zhao, Z. Duan, G. Wen, and Y. Zhang, “Distributed finite-time tracking control for multi-agent systems: An observer-based approach,” *Syst. Control Lett.*, vol. 62, no. 1, pp. 22–28, 2013.

[31] S. Li, H. Du, and X. Lin, “Finite-time consensus algorithm for multi-agent systems with double-integrator dynamics,” *Automatica*, vol. 47, no. 8, pp. 1706–1712, 2011.

[32] S. Khoo, L. Xie, and Z. Man, “Robust finite-time consensus tracking algorithm for multirobot systems,” *IEEE/ASME Trans. Mechatronics*, vol. 14, no. 2, pp. 219–228, Apr. 2009.

[33] C. Edwards and S. K. Spurgeon, *Sliding Mode Control: Theory and Applications*. London, U.K.: Taylor and Francis, 1998.

[34] V. I. Utkin, *Sliding Modes in Control and Optimization*. New York, NY, USA: Springer, 1992.

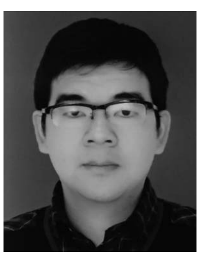
[35] V. I. Utkin, J. Guldner, and J. Shi, *Sliding Mode Control in Electromechanical Systems*. London, U.K.: Taylor and Francis, 1999.

[36] Y. Feng, X. Yu, and Z. Man, “Non-singular terminal sliding mode control of rigid manipulators,” *Automatica*, vol. 38, no. 12, pp. 2159–2167, 2002.
[37] Y. Feng, X. Yu, and F. Han, “On nonsingular terminal sliding-mode control of nonlinear systems,” *Automatica*, vol. 49, no. 6, pp. 1715–1722, 2013.
[38] Y. Feng, F. Han, and X. Yu, “Chattering free full-order sliding-mode control,” *Automatica*, vol. 50, no. 4, pp. 1310–1314, 2014.
[39] N. Rouche, P. Habets, and M. Laloy, *Stability Theory by Liapunov’s Direct Method*. New York, NY, USA: Springer-Verlag, 1977.
[40] G. H. Hardy, J. E. Littlewood, and G. Pólya, *Inequalities*. Cambridge, U.K.: Cambridge Univ. Press, 1988.
[41] H. K. Khalil, *Nonlinear Systems*. Upper Saddle River, NJ, USA: Prentice-Hall, 2002.

Wenwu Yu (S’07–M’12–SM’15) received the B.Sc. degree in information and computing science and the M.Sc. degree in applied mathematics from the Department of Mathematics, Southeast University, Nanjing, China, in 2004 and 2007, respectively, and the Ph.D. degree from the Department of Electronic Engineering, City University of Hong Kong, Hong Kong, in 2010.

He is currently the Founding Director of the Laboratory of Cooperative Control of Complex Systems, an Associate Director with the Research Center for Complex Systems and Network Sciences, an Associate Dean with the Department of Mathematics, and a Full Professor with the Distinguished Young Honor with Southeast University. He held several visiting positions in Australia, China, Germany, Italy, The Netherlands, and the USA. His current research interests include multiagent systems, complex networks and systems, nonlinear dynamics and control, smart grids, neural networks, game theory, robotics, network security, and optimization.

Dr. Yu was a recipient of the Highly Cited Researchers Award in Engineering by Thomson Reuters in 2014–2016, the National Natural Science Fund for Excellent Young Scholars in 2013, and the National Ten Thousand Talent Program for Young Top-Notch Talents in 2014.



He Wang received the B.Sc. degree from Southeast University, Nanjing, China, in 2013, where he is currently pursuing the Ph.D. degree with the Department of Mathematics, Southeast University, Nanjing, China.

He reviewed papers for complex networks 2016. His current research interests include multiagent systems, sliding mode control, finite-time control, and robotics.

Fei Cheng received the B.Sc. degree in applied mathematics and the M.Sc. degree in mathematics from the Department of Mathematics, Southeast University, Nanjing, China, in 2013 and 2015, respectively.

His current research interests include multiagent systems, nonlinear dynamics and control, and intermittent control.

Xinghuo Yu (M’92–SM’98–F’08) received the B.Eng. and M.Eng. degrees in electrical engineering from the University of Science and Technology of China, Hefei, China, in 1982 and 1984, respectively, and the Ph.D. degree from Southeast University, Nanjing, China, in 1988.

He is an Associate Deputy Vice-Chancellor and a Distinguished Professor of the RMIT University (Royal Melbourne Institute of Technology), Melbourne, VIC, Australia. His current research interests include variable structure and nonlinear control, complex and intelligent systems, and smart energy systems.

Dr. Yu was a recipient of the number of awards and honors for his contributions, including the 2013 Dr.-Ing. Eugene Mittelmann Achievement Award of the IEEE Industrial Electronics Society and the 2012 IEEE Industrial Electronics Magazine Best Paper Award. He is the President-Elect of the IEEE Industrial Electronics Society from 2016 to 2017. He served/is serving as an Associate Editor for the IEEE TRANSACTIONS ON AUTOMATIC CONTROL, the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS, the IEEE TRANSACTIONS ON INDUSTRIAL INFORMATICS, and the IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—I: REGULAR PAPERS.

Guanghui Wen (S’11–M’13) received the Ph.D. degree in mechanical systems and control from Peking University, Beijing, China, in 2012.

He is currently the Associate Director of the Laboratory of Cooperative Control of Complex Systems, Southeast University, Nanjing, China, an Associate Professor with the Department of Mathematics, Southeast University. He held several visiting positions in Australia, China, and Qatar. His current research interests include cooperative control of multiagent systems, analysis and synthesis of complex networks, cyber-physical systems, and robust control.

Dr. Wen was a recipient of the Best Student Paper Award in the 6th Chinese Conference on Complex Networks in 2010. He currently serves as an Editorial Board Member and an Associate Editor of the *Asian Journal of Control*.