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Distributed event-triggered control of multi-agent systems with combinational measurements[☆]

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ABSTRACT

This paper studies the distributed rendezvous problem of multi-agent systems with novel event-triggered controllers. We have proposed a combinational measurement approach to event design and developed the basic event-triggered control algorithm. As a result, control of agents is only triggered at their own event time, which reduces the amount of communication and lowers the frequency of controller updates in practice. Furthermore, based on the convergence analysis of the basic algorithm, we have proposed a new iterative event-triggered algorithm where continuous measurement of the neighbor states is avoided. It is noted that the amount of communication among agents has been significantly reduced without obvious negative effects on the control performances. The effectiveness of the proposed strategies is illustrated by numerical examples in 3D spaces.

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1. Introduction

Nowadays many applications require large sets of robots, vehicles or mobile sensors to work cooperatively and accomplish complex tasks. In view of this, many researchers have devoted themselves to the study of coordination control of multi-agent systems. Typical research in this field includes the problems of consensus (Olfati-Saber, Fax, & Murray, 2007; Ren & Beard, 2005), formation control (Fax & Murray, 2004), rendezvous (Cortés, Martínez, & Bullo, 2006; Fan, Feng, Wang, & Qiu, 2011; Lin, Morse, & Anderson, 2003, 2007a,b), agent flocking (Tanner, Jadbabaie, & Pappas, 2007), and deployment (Nowzari & Cortés, 2012; Song, Feng, Fan, & Wang, 2011). To lower the cost, each agent may equip with a small embedded micro-processor and capability-limited onboard communication and actuation modules, which usually have only limited energy resources and computing capabilities. These factors motivate researchers to develop event-triggered control schemes for digital platforms, see, for example, De Persis, Sailer, and Wirth (2011), and Wang and Lemmon (2011). For multi-agent systems, the authors in Dimarogonas, Frazzoli, and Johansson (2012) employ the deterministic event-triggered strategy introduced in Tabuada (2007) to develop consensus control algorithms. Furthermore, the lower bounds for the inter-event time are provided to ensure there is no Zeno behavior (Dimarogonas et al., 2012). Event-triggered control has also been addressed in decentralized control over wireless sensor/actuator networks (Mazo & Tabuada, 2011) and multi-agent systems with event-based communication (Seyboth, Dimarogonas, & Johansson, 2012).

In event-triggered control, the measurement error plays an essential role in the event design. When its magnitude reaches the prescribed threshold, an event is triggered and the controller is updated. Generally speaking, the threshold can be state-independent (Mazo & Tabuada, 2011; Seyboth et al., 2012) and state-dependent (Dimarogonas et al., 2012). In the former case, the threshold can be a constant or a time-dependent variable. It is noted that the state-dependent threshold is better and more natural since in the constant threshold case the agent system cannot achieve asymptotic convergence and in the time-dependent variable case the convergence rate is governed by an external signal (Seyboth et al., 2012). However, the drawback of the existing controllers with state-dependent thresholds lies in that each agent is required to be triggered at the neighbors' event time. This increases the load of communication and brings higher frequency of controller updates.

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In this paper we firstly propose a combinational measuring approach to event-design and develop a basic event-triggered control algorithm for rendezvous. In this approach the measurement error of each agent is determined by a convex combination of its neighbors' states rather than by measuring the agents' own states. By categorizing the triggering executions, it is proven that each agent will be triggered regularly and the group will asymptomatically achieve rendezvous. Then based on the convergence analysis, we have developed a new iterative event-triggered control algorithm without continuous measuring of the neighbor states to further reduce inter-agent communication. The contribution of this work are as follows. Firstly, by the combinational measuring approach, the controller of each agent is allowed to be triggered only at its own triggering time instants, which reduces the amount of communication and the frequency of controller updates. Secondly, the categorization of triggering executions help investigate the behaviors of the agent system in a solid manner. Thirdly, in the iterative algorithm without continuous measurement, each agent does both of the actions of measuring the neighbor states and sending combined state signals only at its own triggering time instants, which further reduces communication significantly. Compared with the existing work in Dimarogonas et al. (2012), the proposed controller achieves nearly the same convergence performances with much less amount of communication and controller updates.

The rest of this paper is organized as follows. Sections 2 and 3 present the basic event-triggered controller design and the convergence analysis. Section 4 presents the iterative event-triggered algorithm without continuous measurement. In Section 5, numerical simulations are provided. Finally the paper is concluded in Section 6.

2. Basic event-triggered algorithm

Consider a multi-agent system with N agents, labeled by $1, 2, \ldots, N$. The agents are able to move, compute and communicate, and required to achieve the rendezvous task. Their positions at time t are represented by $x_i(t) \in \mathbb{R}^n (n = 2, 3)$ with kinematics being

$$\dot{x}_i(t) = u_i(t), \quad i = 1, \ldots, N. \tag{1}$$

The communication topology of the system is represented by an undirected graph $G = (V, E)$, where V is the vertex set and E is the edge set. The neighbor set of agent i is denoted by $N_i = \{j \in V \mid (j, i) \in E\}$. To introduce the event-triggered scheme, assume that the triggering time sequence of agent i is $t_0^i = 0, t_1^i, \ldots, t_k^i, \ldots$ and each agent can obtain the neighbors' states. In the existing works, each agent measures its own state $x_i(t)$ and its controller has to be triggered at the neighbors' triggering time (Dimarogonas et al., 2012). In this work we use the combinational measurement of the neighbors' states instead. The real-time average position of agent i and its neighbors in agent i's local coordinate system is $q_i(t) = 1/(n_i + 1) \sum_{j \in N_i} (x_j(t) - x_i(t))$. At time t_k^i, agent i measures $q_i(t)$ and takes the measurement as its target point, which will remain unchanged until t_{k+1}^i comes. Thus the measurement of agent i in the time interval $[t_k^i, t_{k+1}^i)$ is $q_i(t_k^i)$ and the measurement error is

$$e_i(t) = q_i(t_k^i) - q_i(t). \tag{2}$$

We propose the control law for agent i as follows,

$$u_i(t) = \xi_i q_i(t_k^i), \quad t \in [t_k^i, t_{k+1}^i), \tag{3}$$

where $\xi_i > 0$. Note that this controller only depends on the agent states at time t_k^i. Thus each agent is only triggered at its own event time.

In the event design, the strategy in Tabuada (2007) is used. That is, when the error norm $\|e_i(t)\|$ reaches a given threshold, an event occurs and the controller is updated. Let the threshold be $\beta_i \|q_i(t)\|$ with $\beta_i > 0$. Then an event is triggered if and only if

$$g\big(e_i(t), q_i(t)\big) = \|e_i(t)\| - \beta_i \|q_i(t)\| = 0. \tag{4}$$

Thus at $t = t_k^i$, $g\big(e_i(t_k^i), q_i(t_k^i)\big) = 0$ and $e_i(t)$ is reset to 0 automatically. We have the following lemma.

Lemma 1. *Consider an agent system with kinematics (1) under controllers (3). Assume that G is connected and agents are triggered by (4). Then the function*

$$\mathbf{V}(t) = \frac{1}{2} x^T(t)(L \otimes I_n) x(t) \tag{5}$$

is non-increasing if $0 < \beta_i < 1$ for all $i = 1, \ldots, N$, where $x(t) = (x_1^T(t), \ldots, x_N^T(t))^T$ denotes the augmented state and L is the Laplacian matrix of G.

Proof. Let $z_i(t) = \sum_{j \in N_i} (x_i(t) - x_j(t))$. Since L is symmetric, by some operations one will have

$$\dot{\mathbf{V}}(t) = -\sum_{i=1}^{N} \frac{\xi_i}{n_i + 1} \|z_i(t)\|^2 + \sum_{i=1}^{N} \xi_i z_i^T(t) e_i(t). \tag{6}$$

For any $a > 0$ and any $x, y \in \mathbb{R}^n$, $|x^T y| \le a/2 \|x\|^2 + 1/(2a) \|y\|^2$. Thus if let

$$\|e_i(t)\| \le \eta_i \|z_i(t)\| \tag{7}$$

one will have

$$\dot{\mathbf{V}}(t) \le -\sum_{i=1}^{N} \left(\frac{\xi_i}{n_i + 1} - \frac{a_i \xi_i}{2} - \frac{\xi_i \eta_i^2}{2 a_i} \right) \|z_i(t)\|^2. \tag{8}$$

Then $\dot{\mathbf{V}}(t) \le 0$ if

$$a_i < \frac{2}{n_i + 1} \quad \text{and} \quad \eta_i < \sqrt{\frac{2 a_i}{n_i + 1} - a_i^2}. \tag{9}$$

Since $z_i(t) = -(n_i + 1) q_i(t)$ and $\sup \eta_i = 1/(n_i + 1)$, by some manipulations (7) becomes

$$\|e_i(t)\| \le \beta_i \|q_i(t)\|, \quad 0 < \beta_i < 1. \tag{10}$$

Then (8) can be re-written as

$$\dot{\mathbf{V}}(t) \le -\sum_{i=1}^{N} \frac{1}{2} \xi_i (n_i + 1)(1 - \beta_i^2) \|q_i(t)\|^2, \tag{11}$$

which implies $\mathbf{V}(t)$ is non-increasing. □

3. Rendezvous analysis

Consider a system $\dot{x}(t) = f(x(t), u(t)), x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^p$ under an event-triggered controller $u(t) = u(t_k), t \in [t_k, t_k + 1)$. Events are designed based on an event-related vector $\mu(t) = (\mu_1(t), \ldots, \mu_m(t))^T \in \mathbb{R}^m$, where $\mu_1(t), \mu_2(t), \ldots, \mu_m(t)$ are constant or time-dependent variables or variables dependent on the state $x(t)$ (Seyboth et al., 2012). Let $\mathbb{E} \subset \mathbb{R}^m$ be a closed set containing all feasible event-related vectors, which can be partitioned into two subsets: the triggering event set $\mathcal{E}$ and the non-event set $\bar{\mathcal{E}} = \mathbb{E} \setminus \mathcal{E}$. Then an event is triggered at time t_k if and only if $\mu(t_k) \in \mathcal{E}$. With the initial state $x(0) = x_0$, the system execution may have the following three cases.

Case 1. (singular triggering) $\exists t_1 \ge 0$ such that $\mu(t_1) \in \mathcal{E}$, and $\mu(t) \notin \mathcal{E}$ for all $t \ge t_1$ (or $\mu(t) \notin \mathcal{E}$ for all $t \ge 0$ if $t = 0$ is not a default triggering time instant);

Case 2. (continuous triggering) $\exists \tau \geq 0$ such that $\mu(t) \in \mathcal{E}$ for all $t \geq \tau$;

Case 3. There exists an infinite sequence $t_1, \ldots, t_k, \ldots$ such that $\mu(t) \in \mathcal{E}$ for all $t = t_k$ and $\mu(t) \notin \mathcal{E}$ for all $t \neq t_k$.

And case 3 can further be sorted into two sub-cases:

Case 3-1. (Zeno triggering) $\lim_{k\to\infty} t_k < \infty$;

Case 3-2. (regular triggering) $\lim_{k\to\infty} t_k = \infty$.

Generally speaking, one may expect the system always behaves as in Case 3-2. We have the following lemmas.

Lemma 2 (*No Singular Triggering*). *Consider an agent i with a non-empty neighbor set N_i. Its kinematic is (1) and its controller is (3), with (4) being the triggering condition. If there exists t_k^i such that $q_i(t_k^i) \neq 0$, then agent i will not exhibit singular triggering for all $t > t_k^i$.*

Proof. We only need to prove t_{k+1}^i exists and $q_i(t_{k+1}^i) \neq 0$. Firstly, from (2) and (4), one observes that $\|q_i(t)\|$ decreasing to $r_1 = 1/(1+\beta_i)\|q_i(t_k^i)\|$ and $\|q_i(t)\|$ increasing to $r_2 = 1/(1-\beta_i)\|q_i(t_k^i)\|$ are the two "worst" cases when events occur. Then t_{k+1}^i exists and $\|q_i(t_{k+1}^i)\| > 0$. Secondly, we will prove that $\|q_i(t)\|$ cannot always stay in $\mathbb{R}_k^i = (r_1, r_2)$ with no event triggering. Notice that $\|x_i(t) - x_j(t)\|$ will always be upper bounded by $\sqrt{\mathbf{V}(t)}$ for all $t \geq 0$. Then one has

$$\|q_i(t)\| \leq \frac{1}{n_i+1} \sum_{j\in N_i} \|x_i(t) - x_j(t)\| < \sqrt{\mathbf{V}(t)}. \tag{12}$$

Since $\|q_i(t)\| \in \mathbb{R}_k^i$, from (11) one yields that $\mathbf{V}(t)$ strictly decreases. Thus $\|q_i(t)\|$ will eventually decrease because of (12), and at least an event will be triggered when $\|q_i(t)\|$ decreases to r_1. □

Lemma 3 (*No Continuous Triggering*). *Consider an agent i with the same conditions as those in Lemma 2. If t_k^i exists and $q_i(t_k^i) \neq 0$, agent i will not exhibit continuous triggering for all $t > t_k^i$.*

Proof. Since $q_i(t)$ is continuous and an event is triggered if and only if $\|e_i(t)\| = \beta_i\|q_i(t)\|$, this lemma straightforwardly follows from the proof of Lemma 2, where all scenarios of the evolution of $\|q_i(t)\|$ are addressed. □

Lemma 4 (*No Zeno Triggering*). *Consider an agent i with the same conditions as those in Lemma 2. If t_k^i exists and $q_i(t_k^i) \neq 0$, agent i will not exhibit Zeno triggering for all $t > t_k^i$.*

Proof. From (2) one has $\dot{e}_i(t) = -\dot{q}_i(t)$. Thus one can yield $\frac{d}{dt}\|e_i(t)\| \leq \|\dot{q}_i(t)\|$. Let

$$k_i(t) = \arg\max_{k\in\mathbb{N}}\{t_k^i | t_k^i \leq t\}. \tag{13}$$

Then $t_{k_i(t)}^i$ is the latest event time instant of agent i before the current time t. One has

$$\begin{aligned} \frac{d}{dt}\|e_i(t)\| &\leq \left\| \frac{1}{n_i+1} \sum_{j\in N_i} (\xi_i q_i(t_{k_i(t)}^i) - \xi_j q_j(t_{k_j(t)}^j)) \right\| \\ &\leq \left\| \frac{\xi_i n_i}{n_i+1} q_i(t_{k_i(t)}^i) \right\| + \left\| \frac{1}{n_i+1} \sum_{j\in N_i} \xi_j q_j(t_{k_j(t)}^j) \right\|. \end{aligned} \tag{14}$$

Since $\mathbf{V}(t)$ is non-increasing, from (12) one has that for each agent $l \in N_i \cup \{i\}$, $\|q_l(t_{k_l(t)}^l)\| < \sqrt{\mathbf{V}(t_k^i)}$ for all $t > t_k^i$. Denote $\zeta_i = \max_{l\in N_i\cup\{i\}}\{\xi_l\}$. From (14) one has $\frac{d}{dt}\|e_i(t)\| < 2\zeta_i\sqrt{\mathbf{V}(t_k^i)}$. Since (2) holds, a sufficient condition to guarantee (10) is

$$\|e_i(t)\| \leq \frac{\beta_i}{1+\beta_i}\|q_i(t_k^i)\|. \tag{15}$$

Thus if $q_i(t_k^i) \neq 0$, $T_k^i = t_{k+1}^i - t_k^i$ is lower bounded by a strictly positive value

$$\tau_k^i = \frac{\beta_i\|q_i(t_k^i)\|}{2\zeta_i(1+\beta_i)\sqrt{\mathbf{V}(t_k^i)}}, \tag{16}$$

which implies that agent i will not exhibit Zeno triggering. □

Theorem 5. *Consider a group of N agents moving in the working space $\mathbb{R}^n$ with the kinematics (1). Assume that the communication graph G is connected. If no agent is initially located at the center of its neighbors, the group will achieve rendezvous asymptotically under the event-triggered control law (3) with the triggering condition (4).*

Proof. From Lemmas 2 to 4, each agent will only exhibit regular triggering for all time. Then from (11) $\mathbf{V}(t)$ asymptotically converges to 0. Since the null space of L is *span*$\{\mathbf{1}_N\}$ (Olfati-Saber et al., 2007), all the agents will asymptotically achieve rendezvous as time goes to infinity. □

4. Iterative event-triggered algorithm

A significant problem of the basic event-triggered algorithm is that each agent still needs to continuously measure the states of all its neighbors. See the triggering condition (4). In this section, we will propose a distributed iterative algorithm to deal with this problem. As a result, continuous measurement of the neighbor states can be avoided.

There are two major features in the new triggering approach. First, only at each triggering time instant t_k^i, agent i measures the combined state $q_i(t)$ and then sends $q_i(t_k^i)$ to its neighbors. Second, the next triggering time instant t_{k+1}^i is determined based on $q_i(t_k^i)$ and the previously received combined states from its neighbors. Let t_{k+1}^{i*} be the next triggering time instant determined in the basic algorithm. By continuity of $e_i(t)$ and $q_i(t)$, t_{k+1}^{i*} is the first time instant after t_k^i such that $\|e_i(t)\| = \beta_i\|q_i(t)\|$. Thus for any $\tau \in (t_k^i, t_{k+1}^{i*})$, $\|e_i(t)\|$ can only increase to a level which is smaller than $\beta_i\|q_i(t)\|$ from time t_k^i to τ, which implies (10) holds and thus $\dot{\mathbf{V}}(t) \leq 0$ in $[t_k^i, \tau)$. This means τ is a valid choice for the next triggering time instant t_{k+1}^i. Following this idea, to obtain a valid τ, two sets of information are memorized by agent i during the evolution: the combined states $\mathcal{N}_i(t) = \{q_l(t_{k_l(t)}^l) | l \in N_i \cup \{i\}\}$ and the time instants when these states are received $\mathcal{T}_i(t) = \{t_{k_l(t)}^l | l \in N_i \cup \{i\}\}$. Denote $s_0 = \beta_i/(1+\beta_i)\|q_i(t_k^i)\|$ and

$$\eta_i(t) = \left\| \frac{1}{n_i+1} \sum_{j\in N_i} (\xi_i q_i(t_{k_i(t)}^i) - \xi_j q_j(t_{k_j(t)}^j)) \right\|. \tag{17}$$

Let τ' be the accurate time consumed for $\|e_i(t)\|$ to increase from 0 to s_0. Since (15) is sufficient for (10), by continuity of $e_i(t)$, one has $\tau' < t_{k+1}^{i*}$ and thus $\dot{\mathbf{V}}(t) \leq 0$ in $[t_k^i, \tau')$. From (14), the increasing rate of $\|e_i(t)\|$ is lower than $\eta_0 = \eta_i(t_k^i)$. Then $\tau_0 = t_k^i + s_0/\eta_0 < \tau' < t_{k+1}^{i*}$. Thus τ_0 will be a feasible choice for $\tau = t_{k+1}^i$ and $\dot{\mathbf{V}}(t) \leq 0$ in $[t_k^i, \tau_0)$. Notice that in determination of τ_0, no continuous measurement of $q_i(t)$ is needed.

However, directly using $t_{k+1}^i = \tau_0$ is rather conservative since from (14) one can notice that the upper bound $\eta_i(t)$ for the increasing rate of $\|e_i(t)\|$ may change when agent i receives a combined state from any neighbor in the time interval (t_k^i, τ_0). Thus we develop an iterative algorithm to determine t_{k+1}^i. If no combined state is received in (t_k^i, τ_0), then $t_{k+1}^i = \tau_0$. Otherwise, if a combined state is received from any neighbor at $t' \in (t_k^i, \tau_0)$, the sets $\mathcal{N}_i(t)$, $\mathcal{T}_i(t)$ and $\eta_i(t)$ are changed and t_{k+1}^i is updated by a new selection. Intuitively speaking, this new selection is determined based on how

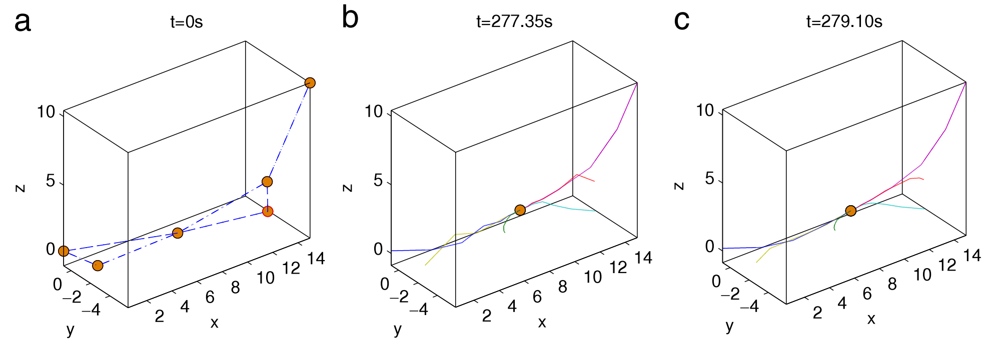

Fig. 1. Communication topology and trajectories of agents under different controllers: (a) initial positions; (b) basic event-triggered algorithm; (c) Algorithm 1.

long it will spend for the increasing of $\|e_i(t)\|$ to cover the rest of s_0 after time t' with the new increasing rate. See Algorithm 1 for details, where τ_p represents the temporary selection for t^i_{k+1} and T represents the prescribed working time or the lifespan of the system. Finally, once t^i_{k+1} is determined, agent i will measure the states of all the neighbors to obtain $q_i(t^i_{k+1})$ and then sends it to all the neighbors. Its control input will immediately be updated by $u_i(t) = \xi_i q_i(t^i_{k+1})$.

Algorithm 1 Determination of Triggering Time t^i_{k+1}.

Require:

$t_0 \leftarrow t^i_k$; $s_0 \leftarrow \frac{\beta_i}{1+\beta_i}\|q_i(t_0)\|$; $\eta_0 \leftarrow \eta_i(t_0)$;

Ensure:

```
p ← 0; // A flag
while t < T do
    τ_p ← t_p + s_p/η_p;
    if a combined state is received in (t_p, τ_p) then
        update T_i(t), N_i(t); p ← p + 1; t_p ← max T_i(t);
        s_p ← s_{p−1} − η_{p−1}(t_p − t_{p−1}); η_p ← η_i(t_p);
        t^i_{k+1} ← T; Continue;
    else
        t^i_{k+1} ← τ_p; Break;
    end if
end while
return t^i_{k+1};
```

Theorem 6. *Consider a group of N agents moving in the working space $\mathbb{R}^n$ with kinematics (1). Assume that the communication graph G is connected. If no agent is initially located at the center of its neighbors, the group will achieve rendezvous asymptotically under the controller (3), where t^i_k is determined by Algorithm 1.*

Proof. From the proof of Lemma 2, t^{i*}_{k+1} determined by (4) exists. Since t^i_{k+1} is always a lower bound of t^{i*}_{k+1}, t^i_{k+1} determined in Algorithm 1 exists and there is no singular triggering. Furthermore, one concludes that $q_i(t^i_{k+1}) \neq 0$ from the proof of Lemma 2 by the continuity of $q_i(t)$. And since $q_i(t^i_k) \neq 0$, t^i_k and t^i_{k+1} are separated at least by a positive number s_0/η_0 and thus there is no continuous triggering and Zeno triggering.

To prove the convergence of the system, one notices that since $\frac{d}{dt}\|e_i(t)\| \leq \eta_i(t)$, for any $t \in (t^i_k, t^i_{k+1})$,

$$\begin{aligned} \|e_i(t)\| &= \|e_i(t)\| - \|e_i(t^i_k)\| \\ &\leq \int_{t^i_k}^{t} \frac{d}{dt}\|e_i(t)\|dt \leq \int_{t^i_k}^{t} \eta_i(t)dt \\ &\leq \int_{t^i_k}^{t^i_{k+1}} \eta_i(t)dt = s_0 = \frac{\beta_i}{1+\beta_i}\|q_i(t^i_k)\|. \end{aligned} \tag{18}$$

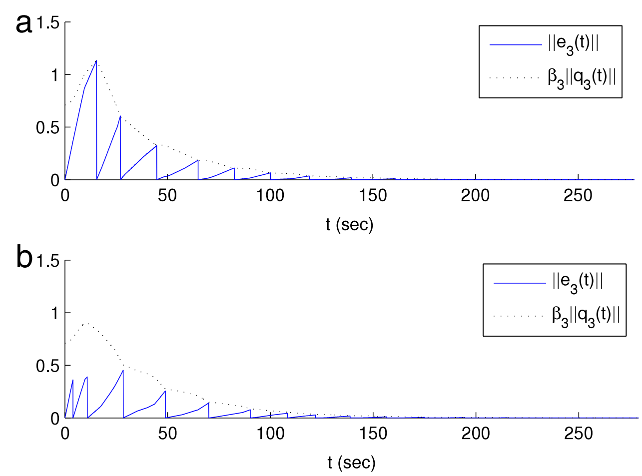

Fig. 2. Error norms of agent 3 under different controllers: (a) basic event-triggered algorithm; (b) Algorithm 1.

Then one has $\|e_i(t)\| \leq \beta_i\|q_i(t^i_k) - e_i(t)\| = \beta_i\|q_i(t)\|$. From the proof of Lemma 1, one concludes that $\dot{\mathbf{V}}(t)$ is non-positive in (t^i_k, t^i_{k+1}). Similar to the proof of Theorem 5, the agent system will asymptotically achieve rendezvous. □

5. Simulations

In this section some simulations are provided to illustrate the proposed algorithms. Consider a group of $N = 6$ agents in $\mathbb{R}^3$. Each agent is governed by the kinematic (1) and the controller (3). The parameters are given by $\xi_i = 0.1$ and $\beta_i = 0.9$ for all agents. The initial positions and the communication graph are shown in Fig. 1(a). When the sum of the distances from the agents to the group center is shorter than 0.01, the group is considered to have achieved rendezvous. The trajectories of agents under different algorithms are shown in Fig. 1. Notice that the agent group eventually achieved rendezvous at $t = 277.35$ s under the basic event-triggered algorithm and $t = 279.10$ s under Algorithm 1, respectively.

Fig. 2 shows the error norms of agent 3 under different algorithms. In the basic event-triggered algorithm, the error norm is reset to 0 once it reaches the threshold $\beta_i\|q_i(t)\|$. See Fig. 2(a). In Algorithm 1, since the next triggering time is selected smaller than t^i_k determined by (4), the error norm will always be strictly smaller than $\beta_i\|q_i(t)\|$. See Fig. 2(b). One can also note that the rendezvous time and triggering number are nearly the same under the two different algorithms.

We also run the simulations with different parameters under different algorithms, and record the rendezvous time and the average number of triggerings in Table 1. From Table 1, one can observe that although Algorithm 1 requires much less

Table 1
Performances with different parameters and controllers.

N	ξ_i	β_i	Controller	Rend. time (s)	Average trig. no.
8	0.3	0.8	Basic alg.	28.23	14.38
8	0.3	0.8	Algorithm 1	28.31	14.88
10	0.4	0.6	Basic alg.	42.67	21.00
10	0.4	0.6	Algorithm 1	42.74	20.70

communication among agents, it has nearly the same performances as those of the basic event-triggered algorithm. Compared with the existing work in Dimarogonas et al. (2012), Algorithm 1 is better at reducing the amount of inter-agent communication and controller updates.

6. Conclusions

In this paper, we have proposed a distributed event-triggered strategy for multi-agent rendezvous where the events are determined by measuring the combinational states of the neighbors. Firstly we have proposed a basic event-triggered algorithm and presented the convergence analysis. Then we have proposed a new iterative event-triggered algorithm where continuous measuring of the neighbor states has been avoided. As a result, communication among agents can be significantly reduced. Future work will include extending the proposed approach to multi-agent systems with communication delays and disturbances, and directed and time-varying networks.

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