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Event-triggered control of continuous-time switched linear systems

Xiaoqing Xiao, Lei Zhou, Daniel W. C. Ho, *IEEE Fellow*, Guoping Lu

***Abstract*—In this paper, we consider the event-triggered control problem for continuous-time switched linear systems. It is assumed that only the sampled information of system state and switching signal is available to the controller at each sampling instant. Based on a mode dependent event-triggered transmission scheme, the closed-loop system is modeled as a switched system with delayed state and augmented switching signal. Then, an exponential stability condition characterized by the dwell time and average dwell time of the switching signal is obtained. The condition presents an extension of the multiple Lyapunov functional method based stability analysis for sampled-data control of non-switched system. Consequently, the design methods for state feedback controller gains and event-triggered parameters are then formulated by the properly selected quadratic Lyapunov functional. The analysis results are significant and also leads an important step to study the event-triggered control for switched system. Finally, base on the definition of event-trigger efficiency, the effectiveness and improvement of the proposed approach are illustrated by two numerical examples.**

***Keywords*-Switched system, event-triggered control, average dwell time, exponential stability**

I. Introduction

A switched system is a hybrid dynamical system that consists of a finite number of subsystems and a logical rule that orchestrates switching between them. The stability problem of switched systems has been studied extensively in the past few decades, see the survey papers [1], [2], the recent book [3] and references therein. On the other hand, with the development of computer technology, microelectronics and communication networks, sampled-data control systems have been the subject of many researches in recent years. Among many methods to derive the stability criteria for sampled-data control systems, input delay approach was widely investigated, where the closed-loop system is modeled as a continuous time system with the delayed control input and Lyapunov functionals are utilized to investigate the problem [4], [5], [6], [7].

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In sampled-data control of switched systems, the controller receives the sampled information of the system state and switching signal at each sampling instant. The synchronous between the controller mode and subsystem mode cannot be guaranteed if switching occurs between sampling intervals. Therefore, the resulted closed-loop system is an asynchronously switched control system with sampled state, which may become unstable due to mode mismatches. The asynchronous controller parameters and sampled state can be viewed as a perturbation on the closed-loop system without sampling, then robust analysis approach and Lyapunov stability theory can be applied to establish the stability criteria [8]. Stabilization of switched systems with dynamic and static quantizers has been addressed in [9] and [10], respectively. Since the mode mismatch occurs arbitrarily, it is quite difficult to estimate the reachable sets or ultimate bounds of the state trajectories during sampling intervals. Based on the assumption of that the state is sampled and the system is switched simultaneously with fixed period, stabilization of networked switched linear systems subjected to network-induced delays is considered in [11]. However, many switching signals, such as the switching signals with (average) dwell time constraints, do not always meet this assumption.

Furthermore, the data sampling in previous studies is usually periodic time-triggered. It is obviously inadvisable when the system is operating desirably and the control signal is transmitted through a network with constrained bandwidth. Then, it is of interest to consider other more efficient paradigm to mitigate the unnecessary data sampling and waste of communication resources. Among which the event-triggered scheme, which has been proposed in the late nineties [12], [13], was widely utilized in various fields, such as networked control [14], [15], [16], networked filtering and estimation [17], [18], [19], [20], consensus of multi-agent systems [21], [22]. Recently, the event-triggered control problem for switched systems has been reported in [23], [24]. However, the controller is assumed to have the full information of the system mode. Therefore, the switching between the system and controller is synchronous and it requires monitoring of the event-triggering conditions continuously. The stability and L_2-gain of hybrid systems with linear flow dynamics periodic time-triggered jumps and nonlinear jump maps are investigated in [25], which shows its applications in sampled-data systems with arbitrarily switching controllers. However, the controlled plant has only one mode, so there exists no mismatch between the system mode and controller mode. Therefore, the event-triggered control problem for switched systems has not been fully investigated, which motivates the present study.

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In this paper, we assume that only the sampled-data of active mode and state of the switched system is available to the switched controllers, which might lead to the mismatch between the system mode and controller mode and make the analysis more difficult as compared with [11], [23], [24]. To reflect the basic features of switched systems, a new event-triggered transmission scheme with both mode dependent parameters and dwell time constraint is proposed. Then the closed-loop system is modeled as a switched system with delayed state and augmented switching signal, which is generated by merging the switching signal with its event-triggered sampled switching signal. By using of a multiple Lyapunov functional method, sufficient exponential stability conditions are obtained. Meanwhile, the design methods for state feedback controller gains and event-triggered parameters are proposed based on matrix inequalities. The proposed method extends the input delay approach of sampled-data control systems [5] and event-triggered transmission scheme [18], [20] to event-triggered sampled-data control of switched systems. Furthermore, the Zeno-like behaviors [21] can be avoided by selecting a sufficiently small sampling period. It is expected that the contribution of this work will be useful for further development for the sampled-data control of switched systems, such as L_2 control, quantized sampled-data control and other applications.

Notation: Throughout this paper, $\mathbf{R}^+$ denotes the set of positive real number. $\mathbf{R}^n$ denotes the $n-$dimensional Euclidean space. I is the appropriately dimensioned identity matrix, W^{-1} denotes inverse of matrix W, W^T denotes transpose of matrix W, W^{-T} denotes transpose of matrix W^{-1}, $W > 0$ means that W is positive definite. δ_{pq} is the standard Kronecker delta function. Asterisk '$*$' in a symmetric matrix denotes the entry implied by symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

II. PROBLEM FORMULATION

A. Switched systems

In this paper, we consider the following continuous time switched linear systems:

$$\dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \tag{1}$$

where $x(t) \in \mathbf{R}^n$ and $u(t) \in \mathbf{R}^m$ are the system state and control input, respectively. The switching law $\sigma(t) : \mathbf{R}^+ \to \mathcal{M} = \{1, 2, \dots, s\}$ is a piecewise constant and right continuous function, s is the number of modes of the overall switched system. For every $p \in \mathcal{M}$, A_p and B_p are constant matrices. For simplicity, we use (A_p, B_p) to denotes the p-th subsystem.

The following definition and assumptions are necessary in this paper.

Definition 2.1: [3], [26] (1) Let $N_\sigma(\tau, t)$ be the number of switches of $\sigma(t)$ in the interval (τ, t). If there exist a constant $\tau_a > 0$ and positive number N_0 such that $N_\sigma(\tau, t) \leq N_0 + (t-\tau)/\tau_a$ for all $t \geq \tau \geq 0$, then we call $\sigma(t)$ has an average dwell time τ_a and chatter bound N_0. For simplicity, we denote $\sigma(t) \in \mathcal{S}[\tau_a, N_0]$.

(2) If there exist a constant $\tau_d > 0$ and $N_\sigma(\tau, t) \leq 1$ when $t - \tau \leq \tau_d$, then we call $\sigma(t)$ has a dwell time τ_d.

Assumption 2.1: For each $p \in \mathcal{M}$, the subsystem (A_p, B_p) is stabilizable. Moreover, there exists a state feedback gain matrix K_p such that $A_p + B_pK_p$ is Hurwitz stable.

Assumption 2.2: Assume that $\sigma(t)$ has a dwell time τ_d and $\sigma(t) \in \mathcal{S}[\tau_a, N_0]$.

Based on Assumption 2.1, the closed-loop system under switched state feedback controller $u(t) = K_{\sigma(t)}x(t)$ is exponential stable with slow switching characterized by dwell time or average dwell time.

B. Event-triggered transmission scheme based on sampled-data

The main purpose of this paper is to investigate how to improve the event-triggered scheme for non-switched systems to switched systems. In addition, some sufficient conditions are derived such that the resulted system is asymptotically stable. Different from [23], [24], we assume that both the system state and mode are sampled at discrete times $t_k = kh$, where h is the fixed sampling period. We mention that this assumption is quite natural, and yet has not been fully considered in existing literatures. To guarantee that there exists at least one sampling within each switching interval of $\sigma(t)$, we assume that the sampling period $h \leq \tau_d$ in this paper.

To determine the current sampled data should be transmitted or not, a mode dependent event-triggered transmission scheme is proposed to generate the transmission sequence $\{t_{s_k}\}_{k\geq 0}$ with $t_{s_0} = 0$ as follows.

$$t_{s_{k+1}} = \min\left\{t'_{s_{k+1}}, t_{s_k} + \left[\frac{\tau_d}{h}\right]h\right\}, k \geq 0, \tag{2}$$

and

$$t'_{s_{k+1}} = \min_{t_j > t_{s_k}} \{t_j | \varphi(e(t_j), x(t_{s_k})) \geq 0\}, \tag{3}$$

where $e(t_j) = x(t_j) - x(t_{s_k})$ denotes the sampled-data error at new sampling instant $t_j > t_{s_k}$, $[\frac{\tau_d}{h}]$ denotes the nearest integer less than or equal to $\frac{\tau_d}{h}$, $\varphi(e(t_j), x(t_{s_k})) = e^T(t_j)\Phi_{\sigma(t_{s_k})}e(t_j) - \nu_{\sigma(t_{s_k})}x^T(t_{s_k})\Phi_{\sigma(t_{s_k})}x(t_{s_k})$. The positive defined matrix Φ_p and positive scale ν_p are event-triggered parameters associated with the p-th subsystem (A_p, B_p).We mention that the event-triggered scheme (2) and (3) is quite general and can recover the event-triggered scheme in [18], [20] for non-switched systems by letting $\tau_d \to \infty$.

Remark 2.1: For simplicity, we call $[t_{s_k}, t_{s_{k+1}})$ a holding interval of the event-triggered transmission scheme (2) and (3). Condition (2) is given to make $t_{s_{k+1}} - t_{s_k} \leq \tau_d$, which implies that there is at most one switch within each holding interval $[t_{s_k}, t_{s_{k+1}})$. Or equivalently, there is at least one sampling within each switching interval of $\sigma(t)$. This assumption is made due to the following two aspects. Firstly, the modes of both the controller and event-triggered parameters are not consistent with the system mode once switching occurs within the holding interval. Then the closed-loop system may become unstable and the event-triggered condition may lose its effectiveness. Therefore, it is reasonable to transmit the sampled-data once the system switches. Secondly, similar

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to the periodically time-triggered sampled-data control for switched system [9], [10], this assumption can make stability analysis simple. In fact, the assumption can be removed and the event instant $t_{s_{k+1}}$ is determined by the right side of (3) directly. However, the analysis will then be more complicated and conservative.

C. Augmented switching signal

The main idea of this subsection comes from the input delay approach in [5] and the merging switching signal technique in [27]. To this end, let $\tau(t) = t - t_{s_k}$ for $t \in [t_{s_k},\ t_{s_{k+1}})$ and rewrite $\sigma(t_{s_k})$ as $\sigma'(t) = \sigma(t - \tau(t))$. We call $\sigma'(t)$ the event-triggered scheme induced switching signal from $\sigma(t)$, which can also be viewed as the delayed switching signal of $\sigma(t)$. The augmented switching signal for $\sigma(t)$ is defined by merging with $\sigma'(t)$ as $\tilde{\sigma}(t) = (\sigma(t), \sigma'(t))$. In Fig. 1, we give an example to illustrate the merging time evolution for the event-triggered transmission and switching.

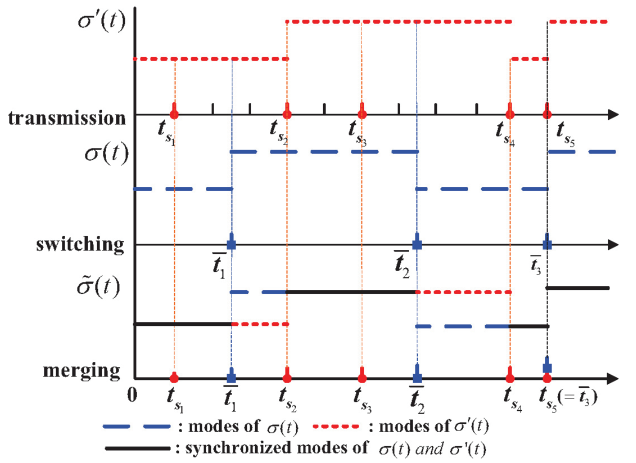

Fig. 1. An example of the merging time sequences and modes of $\tilde{\sigma}(t)$

It follows from (2) that there are at most $[\frac{\tau_d}{h}]$ sampling intervals in each holding interval. Therefore, $\tau(t) \in [0, \tau_d)$. Then, it follows from Lemma 2 in [27] that $\sigma'(t) \in S[\tau_a, N_0 + \tau_d/\tau_a]$. Then, by Lemma 1 in [27], we have

Lemma 2.1: Considering the event-triggered transmission scheme (2) and switching signal $\sigma(t) \in \mathcal{S}[\tau_a, N_0]$, then $\tilde{\sigma}(t) \in \mathcal{S}[\frac{\tau_a}{2}, 2N_0 + \tau_d/\tau_a]$.

Given an interval $[\tau, t)$, let $T_s(\tau, t)$ be the total time in $[\tau, t)$ that switching signals $\sigma(t)$ and $\sigma'(t)$ are synchronous, and let $T_{as}(\tau, t) = t - \tau - T_s(\tau, t)$ denote their total asynchronous time in $[\tau, t)$. Assume that $[\bar{t}_k, \bar{t}_{k+1})$ is an arbitrary switching interval of $\sigma(t)$. Note that there is at least one sampling within $[\bar{t}_k, \bar{t}_{k+1})$ when Assumption 2.2 is satisfied, then the total asynchronous time of $\sigma(t)$ and $\sigma'(t)$ in $[\bar{t}_k, \bar{t}_{k+1})$ is at most τ_d. In other words, the synchronous time of $\sigma(t)$ and $\sigma'(t)$ is at least $\bar{t}_{k+1} - \bar{t}_k - \tau_d$. Then, by the similar analysis of Lemma 3 in [27], we obtain the following conclusion.

Lemma 2.2: For some positive constants λ_s, λ_u and $\lambda \in (0, \lambda_s)$, if $(\lambda_s + \lambda_u)\tau_d \le (\lambda_s - \lambda)\tau_a$, then

$$-\lambda_s T_s(\tau, t) + \lambda_u T_{as}(\tau, t) \le c - \lambda(t - \tau),$$

where $c = (\lambda_s + \lambda_u)N_0\tau_d$.

D. Event-triggered sampled-data control system

Let the control input $u(t)$ be the output of the zero-order hold (ZOH) within event-triggered interval $[t_{s_k},\ t_{s_{k+1}})$, which is given by transmitted sampled-data of the system state and system mode as

$$u(t) = K_{\sigma(t_{s_k})}x(t_{s_k}),\quad t \in [t_{s_k},\ t_{s_{k+1}}). \tag{4}$$

It is obvious that $\Omega = [t_{s_k}, t_{s_{k+1}})$ contains l_k sampling intervals with $l_k = s_{k+1} - s_k$. We can divide Ω into l_k subintervals as $\Omega = \cup_l \Omega_{j_l} = \cup_l [t_{j_l}, t_{j_l+1})$ with $j_l = s_k + l$ and $l = 0, 1, \cdots, l_k - 1$. Note that $e(t_{j_l}) = x(t_{j_l}) - x(t_{s_k})$, then we get $x(t_{s_k}) = x(t_{j_l}) - e(t_{j_l})$. For $t \in \Omega_{j_l}$, we define the sampling induced delay $d(t) = t - t_{j_l}$, which is of sawtooth type and bounded by the sampling period h.

Then, based on the definition of augmented switching signal $\tilde{\sigma}(t)$, the closed-loop for switched system (1) with the event-triggered sampled-data controller (4) can be formulated as

$$\dot{x}(t) = A_{\sigma(t)}x(t) + \widetilde{B}_{\tilde{\sigma}(t)}(x(t - d(t)) - e(t_{j_l})), t \in \Omega_{j_l} \subseteq \Omega, \tag{5}$$

where $\widetilde{B}_{\tilde{\sigma}(t)} = B_{\sigma(t)}K_{\sigma'(t)}$.

Note that the sampled-data $x(t_{s_k})$ is not transmitted for every sampling instant $t_{j_l} \in \Omega$, then

$$e^T(t_{j_l})\Phi_{\sigma(t_{s_k})}e(t_{j_l}) < \nu_{\sigma(t_{s_k})}x^T(t_{s_k})\Phi_{\sigma(t_{s_k})}x(t_{s_k}). \tag{6}$$

Remark 2.2: The closed-loop system of event-triggered sampled-data control for (1) is modeled as a switched system with delayed state and augmented switching signal. Delay system based approach has been widely used for analysis of sampled-data control for non-switched systems [4], [5], [6]. However, to the best of our knowledge, similar application has not been found in handling of the sampled switching signal and sampled-data based event-triggered control of switched systems. Furthermore, system (5) is a continuous-time switched system with time-varying state delay. It should be noted that $d(t) = t - t_{j_l}$ in (5) is piecewise differentiable with $\dot{d}(t) = 1$. However, this distinct feature of sampling induced delay has not been considered in [27], [28].

Remark 2.3: The obtained closed-loop system (5) is different from the recent results on networked control for switched systems [11], [23], [24], [29]. The switching and sampling are assumed to be simultaneous for fixed period in [11], however, many switching signals, such as the switching signals with (average) dwell time constraints, do not satisfy this assumption. In [23], [24], the controller is assumed to have the full information of the system mode and the event-triggering condition is given by the system state instead of its sampled-data. Note that the switching signal is assumed to be known to the controller and also the event-driven communication scheme is system mode independent in [29].

Remark 2.4: We mention that the proposed event-triggered mechanism (2) and (3) belongs to the class of periodic event-triggered control (PETC) [15] and the minimum inter-event time is at least one sampling period, which satisfies the event-separation properties [32], [33] naturally. Furthermore, as shown in the Table I of Example 5.1, the Zeno-like behaviors defined in [21] can be avoided by selecting a sufficiently small h such that the inter-event intervals are strictly greater than at

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least one sampling period.

In fact, let t_{s_k} be any given transmission instant and $\sigma(t_{s_k}) = p$. For simplicity, assume that there is no switching during the interval $[t_{s_k}, t_{s_k} + h)$. Denote $\Psi(h) = e^{A_p h} + \int_0^h e^{A_p \tau} d\tau B_p K_p - I$, then we have

$$\varphi(e(t_{s_k}+h), x(t_{s_k})) = x^T(t_{s_k}) \left(\Psi^T(h)\Phi_p\Psi(h) - \nu_p\Phi_p\right) x(t_{s_k}).$$

Note that $\lim_{h\to 0}\Psi(h) = 0$, then for the given event-triggered parameters Φ_p and ν_p, there must exist a sufficient small h_0 such that $\Psi^T(h)\Phi_p\Psi(h) - \nu_p\Phi_p < 0$ for $h < h_0$. Consequently, $\varphi(e(t_{s_k} + h), x(t_{s_k})) < 0$ when $h < h_0$.

Thus, the minimum inter-event time for the proposed event-triggered transmission scheme will be strictly larger than one sampling period by selecting a sufficiently small h.

III. Stability analysis

A. Multiple Lyapunov functional method

Consider the following continuous-time switched system with state delay

$$\dot{x}(t) = f_{\tilde{\sigma}(t)}(x(t), x(t - d(t))), \tag{7}$$

where $\tilde{\sigma}(t)$ is the augmented switching signal, $d(t) \in [0, h]$ is the state delay. The initial condition for system (7) is assume to be $x(t) = \phi(t),\ t \in [-h, 0]$. Suppose that the switching instants for $\tilde{\sigma}(t)$ are $\{\tau_k\}_{k\geq 0}$.

Denote $x_t = x_t(\theta) = x(t + \theta),\ \theta \in [-h, 0]$,

$$\|x_t(\theta)\|_h = \max_{\theta\in[-h,0]} \|x_t(\theta)\| + \left(\int_{-h}^0 \|\dot{x}_t(s)\|^2 ds\right)^{\frac{1}{2}}.$$

We present the following time-dependent multiple Lyapunov functional method for the exponential stability analysis of system (7).

Theorem 3.1: Suppose that there exist functionals V_{pq} : $\mathbf{R} \times \mathcal{C}([-d, 0], \mathbf{R}^n) \to [0, \infty), p, q \in \mathcal{M}$, positive constants a, b such that $a|x_t(0)|^2 \leq V_{pq}(t, x_t) \leq b\|x_t(\theta)\|_h^2$. Assume that $V_{pq}(t) = V_{pq}(t, x_t)$ is continuous from the right for x_t satisfying (7) and absolutely continuous for $t \neq \tau_k$.

(1) There exists some constant $\mu \geq 1$ such that

$$V_{\tilde{\sigma}(\tau_k)}(\tau_k) \leq \mu V_{\tilde{\sigma}(\tau_k^-)}(\tau_k^-), \tag{8}$$

(2) There exist positive constants λ_s and λ_u such that along system (7), the following estimation hold.

$$\dot{V}_{\tilde{\sigma}(t)}(t) + 2\lambda_{\tilde{\sigma}(t)} V_{\tilde{\sigma}(t)}(t) \leq 0,\ t \in [\tau_k, \tau_{k+1}), \tag{9}$$

where $\lambda_{\tilde{\sigma}(t)} = \lambda_s$ when $t \in T_s(\tau_k, \tau_{k+1})$ and $\lambda_{\tilde{\sigma}(t)} = -\lambda_u$ if $t \in T_{as}(\tau_k, \tau_{k+1})$.

Then, the system (7) is exponentially stable if

$$\lambda_s\tau_a > \ln\mu + \tau_d(\lambda_s + \lambda_u). \tag{10}$$

Proof: For any $t > 0$, let $\tau_1, \ldots, \tau_{N_{\tilde{\sigma}}(0,t)}$ be the switching instants for $\tilde{\sigma}(t)$ in interval $(0, t)$. Without loss of generality, we assume $\tau_1 > 0$ and $\tau_{N_{\tilde{\sigma}}(0,t)} < t$. Let $\tau_0 = 0$ and $\tau_{N_{\tilde{\sigma}}(0,t)+1} = t$, then from (8) and (9), we have

$$V_{\tilde{\sigma}(t)}(t) \leq \mu^{N_{\tilde{\sigma}}(0,t)} \exp\left(\sum_{i=0}^{N_{\tilde{\sigma}}(0,t)} -2\lambda_{\tilde{\sigma}(\tau_i)}(\tau_{i+1} - \tau_i)\right) V_{\tilde{\sigma}(0)}(0).$$

It follows from Lemma 2.1 that $N_{\tilde{\sigma}}(0, t) \leq 2N_0 + \tau_d/\tau_a + 2t/\tau_a$. Considering (10), for any $\lambda \in (\frac{\ln\mu}{\tau_a}, \lambda_s - \frac{\tau_d}{\tau_a}(\lambda_s + \lambda_u))$, we get $\gamma = \lambda - (\ln\mu)/\tau_a > 0$ and $(\lambda_s + \lambda_u)\tau_d \leq (\lambda_s - \lambda)\tau_a$. By applying Lemma 2.2 and the notation of $\lambda_{\tilde{\sigma}(t)}$, we have

$$\begin{aligned} V_{\tilde{\sigma}(t)}(t) &\leq \mu^{N_{\tilde{\sigma}}(0,t)} \exp(-2\lambda_s T_s(0, t) + 2\lambda_u T_{as}(0, t)) V_{\tilde{\sigma}(0)}(0) \\ &\leq \mu^{N_{\tilde{\sigma}}(0,t)} \exp(2c - 2\lambda t) V_{\tilde{\sigma}(0)}(0) \\ &= \mu^{(2N_0+\tau_d/\tau_a)} e^{2c} \exp(-2(\lambda - (\ln\mu)/\tau_a)t) V_{\tilde{\sigma}(0)}(0) \\ &= \alpha e^{-2\gamma t} V_{\tilde{\sigma}(0)}(0), \end{aligned}$$

where $\alpha = \mu^{(2N_0+\tau_d/\tau_a)} e^{2c}$. Then the exponentially stability of system (5) is derived. This completes the proof.

Remark 3.1: Obviously, Theorem 3.1 presents an extension of time-dependent Lyapunov functional based stability analysis method proposed in [5] for sampled-data control of non-switched system. In addition, different from the multiple Lyapunov function techniques used in [27], [30], we utilize a multiple Lyapunov functional based approach. Moreover, since the error between state and its delayed information was modeled as a bounded perturbation and therefore, asymptotical stability can only be guaranteed for sufficient small state delay and switching delay in [27].

Remark 3.2: In [30], the authors consider the asynchronously switched control problem for switched linear systems with the fixed time lag of switched controllers to system modes, and present that the system is asymptotically stable if the average dwell time satisfies

$$\tau_a > \tau_a^* = [\mathcal{T}_{\max}(\lambda_s + \lambda_u) + \ln\mu]/\lambda_s, \tag{11}$$

where $\mathcal{T}_{\max}$, the maximal delay of asynchronous switching, is assumed to be a priori since it is hard to know in advance. It is interesting to note that average dwell time condition (10) presents an explicit counterpart of (11). The dwell time τ_d in (10) (corresponding to the $\mathcal{T}_{\max}$ in (11)) is the maximal estimation for asynchronous time of $\sigma(t)$ and $\sigma'(t)$ during the switching interval of $\sigma(t)$, which is easy to get from the switching signal $\sigma(t)$. The main motivation for this formulation is that the mismatch between the control mode and system mode is caused by switching within holding intervals. Owing to the carefully designed event-triggered transmission scheme (2) and (3), we can reduce asynchronous time to dwell time condition of the switching signal.

Remark 3.3: In this paper, the asynchronous time of $\sigma(t)$ and $\sigma'(t)$ is estimated in the worst case. Therefore, the resulted average dwell time condition (10) has certain conservativeness. In fact, in the special case of $\sigma'(t) = \sigma(t)$, that is, $\sigma(t)$ is only switched at transmission instants, then $\sigma(t)$ and $\sigma'(t)$ are synchronous and the asynchronous time is equal to zero. In this case, the condition (10) reduces to $\lambda_s\tau_a > \ln\mu$ regardless of any given τ_d, which recovers the basic stability condition for continuous switched systems in [3], [26].

B. Stability condition based on quadratic Lyapunov functional

In this subsection, we will utilize Theorem 3.1 to present a matrix inequality based criteria for the exponential stability of the closed-loop system (5) by construction of a proper quadratic Lyapunov functional.

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Theorem 3.2: For the given sampling period $h > 0$ and state feedback gain matrix K_p, if there exist positive scales λ_s, λ_u, ν_q, $\mu \geq 1$, positive definite matrices P_{pq}, Q_{pq}, Φ_q, and matrices N_{pq}, such that the following matrix inequalities hold for all $p, q \in \mathcal{M}$.

$$\begin{aligned} &\mu^{-1} P_{pp} \leq P_{pq} \leq \mu P_{qq}, \ P_{pp} \leq \mu P_{qq}, \\ &e^{2(\lambda_s+\lambda_u)h} Q_{pq} \leq \mu Q_{qq}, \ \forall p \neq q, \end{aligned} \tag{12}$$

$$\Omega_{pq} + h\mathcal{A}_{pq}^T Q_{pq} \mathcal{A}_{pq} < 0, \tag{13}$$

$$\Omega_{pq} + he^{2\tilde{\lambda}_{pq}h} N_{pq}^T Q_{pq}^{-1} N_{pq} < 0, \tag{14}$$

where $\tilde{\lambda}_{pq} = \delta_{pq}\lambda_s, \ \lambda_{pq} = \delta_{pq}\lambda_s + (\delta_{pq} - 1)\lambda_u$,

$$\begin{aligned} \mathcal{A}_{pq} &= \begin{bmatrix} A_p & \widetilde{B}_{pq} & -\widetilde{B}_{pq} \end{bmatrix}, \\ \Omega_{pq} &= \begin{bmatrix} P_{pq}A_p + A_p^T P_{pq} + 2\lambda_{pq}P_{pq} & P_{pq}\widetilde{B}_{pq} & -P_{pq}\widetilde{B}_{pq} \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} \\ &- N_{pq}^T \begin{bmatrix} I & -I & 0 \end{bmatrix} - \begin{bmatrix} I & -I & 0 \end{bmatrix}^T N_{pq} \\ &+ \nu_q \begin{bmatrix} 0 & I & -I \end{bmatrix}^T \Phi_q \begin{bmatrix} 0 & I & -I \end{bmatrix} \\ &- \begin{bmatrix} 0 & 0 & I \end{bmatrix}^T \Phi_q \begin{bmatrix} 0 & 0 & I \end{bmatrix}. \end{aligned}$$

Then the closed-loop system (5) is exponentially stable if the switching signal satisfies (10) in Theorem 3.1.

Proof: Consider the following piece-wise Lyapunov functional:

$$V_{\tilde{\sigma}(t)}(t) = x^T(t) P_{\tilde{\sigma}(t)} x(t) + (h - d(t)) W_{\tilde{\sigma}(t)}(t), \tag{15}$$

with $W_{\tilde{\sigma}(t)}(t) = \int_{t-d(t)}^{t} e^{2\lambda_{\tilde{\sigma}(t)}(s-t)} \dot{x}^T(s) Q_{\tilde{\sigma}(t)} \dot{x}(s) ds$.

Then, it remains to prove that the above defined functional $V_{\tilde{\sigma}(t)}(t)$ satisfies conditions in Theorem 3.1, which contains a lengthy but straightforward calculation; see Appendix VII.

IV. Controller design

The purpose of this section is to obtain a design method for the stabilizing controller gains based on Theorem 3.2.

Set $P_{pq} = P_q$, $\forall p \in \mathcal{M}$. Firstly, pre-and post-multiply both sides of (13) with $\mathrm{diag}(P_{pq}^{-1}, P_{pq}^{-1}, P_{pq}^{-1})$ and then by Schur Complement Lemma, we get

$$\left[\begin{array}{c:c} \widetilde{\Omega}_{pq} & \begin{matrix} hP_q^{-1}A_p^T \\ hY_q^T B_p^T \\ -hY_q^T B_p^T \end{matrix} \\ \hdashline * & -hQ_{pq}^{-1} \end{array}\right] < 0, \tag{16}$$

where

$$\begin{aligned} Y_q &= K_q P_q^{-1}, \ \widetilde{\Phi}_q = P_q^{-1} \Phi_q P_q^{-1}, \\ M_{pq} &= P_q^{-1} N_{pq} \mathrm{diag}(P_q^{-1}, P_q^{-1}, P_q^{-1}), \end{aligned}$$

$$\begin{aligned} \widetilde{\Omega}_{pq} &= \begin{bmatrix} A_p P_q^{-1} + P_q^{-1} A_p^T + 2\lambda_{pq} P_q^{-1} & B_p Y_q & -B_p Y_q \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} \\ &- M_{pq}^T \begin{bmatrix} I & -I & 0 \end{bmatrix} - \begin{bmatrix} I & -I & 0 \end{bmatrix}^T M_{pq} \\ &+ \nu_q \begin{bmatrix} 0 & I & -I \end{bmatrix}^T \widetilde{\Phi}_q \begin{bmatrix} 0 & I & -I \end{bmatrix} \\ &- \begin{bmatrix} 0 & 0 & I \end{bmatrix}^T \widetilde{\Phi}_q \begin{bmatrix} 0 & 0 & I \end{bmatrix} < 0. \end{aligned}$$

Similarly, we can obtain from (14) that

$$\left[\begin{array}{c:c} \widetilde{\Omega}_{pq} & hM_{pq}^T \\ \hdashline * & -he^{-2\tilde{\lambda}_{pq}h} P_q^{-1} Q_{pq} P_q^{-1} \end{array}\right] < 0. \tag{17}$$

Note for any constant β_{pq},

$$-P_q^{-1} Q_{pq} P_q^{-1} \leq -2\beta_{pq} P_q^{-1} + \beta_{pq}^2 Q_{pq}^{-1}. \tag{18}$$

Then we obtain the following state feedback gain design approach.

Theorem 4.1: For the given sampling period $h > 0$, if there exist positive scales λ_s, λ_u, ν_q, $\mu \geq 1$ and a scale β_{pq}, positive definite matrices $\mathcal{P}_q$, $\mathcal{Q}_{pq}$, $\widetilde{\Phi}_q$ and matrices M_{pq}, Y_q, such that the following matrix inequalities hold for all $p, q \in \mathcal{M}$.

$$\mathcal{P}_q \leq \mu \mathcal{P}_p, \ e^{2(\lambda_s+\lambda_u)h} \mathcal{Q}_{qq} \leq \mu \mathcal{Q}_{pq}, \ \forall p \neq q, \tag{19}$$

$$\left[\begin{array}{c:c} \Xi_{pq} & \begin{matrix} h\mathcal{P}_q A_p^T \\ hY_q^T B_p^T \\ -hY_q^T B_p^T \end{matrix} \\ \hdashline * & -h\mathcal{Q}_{pq} \end{array}\right] < 0, \tag{20}$$

$$\left[\begin{array}{c:c} \Xi_{pq} & hM_{pq}^T \\ \hdashline * & -he^{-2\tilde{\lambda}_{pq}h}(2\beta_{pq}\mathcal{P}_q - \beta_{pq}^2 \mathcal{Q}_{pq}) \end{array}\right] < 0, \tag{21}$$

where $\tilde{\lambda}_{pq} = \delta_{pq}\lambda_s, \ \lambda_{pq} = \delta_{pq}\lambda_s + (\delta_{pq} - 1)\lambda_u$,

$$\begin{aligned} \Xi_{pq} &= \begin{bmatrix} A_p \mathcal{P}_q + \mathcal{P}_q A_p^T + 2\lambda_{pq}\mathcal{P}_q & B_p Y_q & -B_p Y_q \\ * & 0 & 0 \\ * & * & 0 \end{bmatrix} \\ &- M_{pq}^T \begin{bmatrix} I & -I & 0 \end{bmatrix} - \begin{bmatrix} I & -I & 0 \end{bmatrix}^T M_{pq} \\ &+ \nu_q \begin{bmatrix} 0 & I & -I \end{bmatrix}^T \widetilde{\Phi}_q \begin{bmatrix} 0 & I & -I \end{bmatrix} \\ &- \begin{bmatrix} 0 & 0 & I \end{bmatrix}^T \widetilde{\Phi}_q \begin{bmatrix} 0 & 0 & I \end{bmatrix}. \end{aligned}$$

Then the closed-loop system (5) is exponentially stable with the decay rate γ, if the inequality (10) in Theorem 3.1 is satisfied. The corresponding state feedback gain $K_q = Y_q \mathcal{P}_q^{-1}$ and event-triggered parameters are ν_q and $\Phi_q = \mathcal{P}_q^{-1} \widetilde{\Phi}_q \mathcal{P}_q^{-1}$.

Remark 4.1: To find the upper bound of h, we can firstly choose a sufficient small $h = h_0$ such that there exist feasible solutions for (19), (20) and (21) in Theorem 4.1. Let $\varepsilon > 0$ be a sufficiently small step increment and $h_k = h_0 + k\varepsilon$ $(k \geq 1)$. If (19), (20) and (21) are feasible for h_k while not feasible for h_{k+1}, then the upper bound of h can be estimated as h_k.

V. Numerical examples

Example 5.1: Consider the following railway system model studied in [8], which can be formulated as the switched system described by (1) with $\mathcal{M} = \{1, 2\}$ and system parameters as

$$A_1 = \begin{bmatrix} 0 & 1 \\ 0 & -3.2 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0.6 \end{bmatrix}, \ A_2 = \begin{bmatrix} 0 & 1 \\ 0 & -0.5 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix},$$

For a given dwell time $\tau_d = 1$ and a pair of controller gains, the upper bound of the sampling period can be estimated as $h_{\max} < 0.0384$ in [8]. To show the less conservativeness of the proposed approach, let $\nu_1 = 0.15$, $\nu_2 = 0.1$, $\beta_{pq} = 1$, $\mu = 1.1$, $\lambda_s = 0.25$ and $\lambda_u = 0.1$. It can be verified that (19),

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(20) and (21) are solvable for $h = 0.3$. The corresponding state feedback gains and event-triggered parameters are $K_1 = \begin{bmatrix} -3.6491 & -2.7685 \end{bmatrix}$, $K_2 = \begin{bmatrix} -4.4625 & -4.7156 \end{bmatrix}$ and

$$\Phi_1 = \begin{bmatrix} 0.6215 & 0.6176 \\ 0.6176 & 0.8318 \end{bmatrix}, \ \Phi_2 = \begin{bmatrix} 0.8131 & 0.8417 \\ 0.8417 & 0.9881 \end{bmatrix}.$$

Then the closed-loop system is exponentially stable if the average dwell time for the switching signal satisfies $\tau_a > ((\lambda_s + \lambda_u)\tau_d + \ln \mu)/\lambda_s = 0.8432$ by selecting $\tau_d = 0.33$. Furthermore, to find the upper bound of h, let $h_0 = 0.3$ and $h_k = h_0 + k\varepsilon$ with the step increment $\varepsilon = 0.0001$. It can be verified that (19), (20) and (21) are feasible for h_{117} while not feasible for h_{118}, then according to Remark 4.1, the upper bound of h can be estimated as $h_{117} = 0.3117$.

The simulations of state trajectories for the closed-loop system with $h = 0.05$ and $\tau_a = 0.9$ are shown in Fig. 2, where x_1 and x_2 denote the train's position and velocity, respectively and the dashed lines represent the state trajectories of periodic time-triggered sampled-data control system. The corresponding switching signal $\sigma(t)$ and the event-triggered transmitted sampled switching signal are illustrated in Fig. 3. The length of holding intervals is shown in Fig. 4.

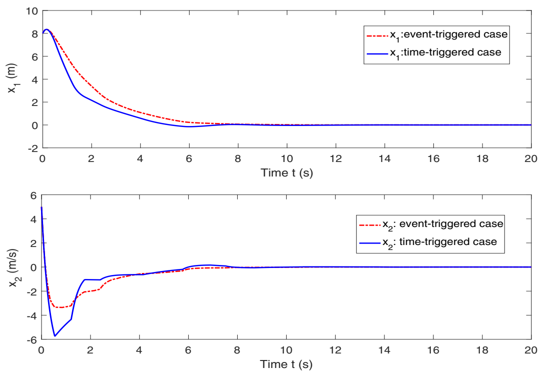

Fig. 2. The system state with $h = 0.05$ and $\tau_a = 0.9$.

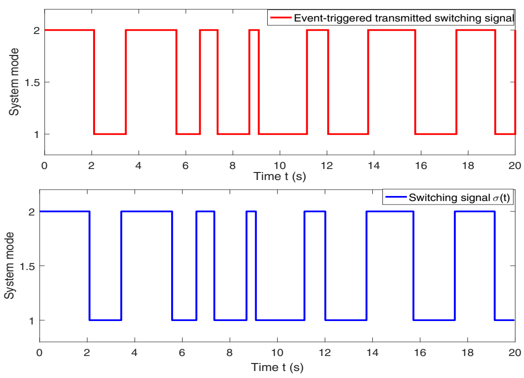

Fig. 3. The switching signal and event-triggered transmitted switching signal.

To further describe effectiveness of the proposed event-triggered control mechanism, let $N[t_0, t_f]$ be the number of event-triggered transmissions in $[t_0, t_f]$ and define the event-

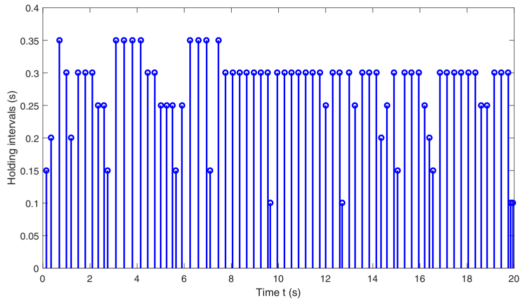

Fig. 4. The holding intervals of the event-triggered scheme.

TABLE I
THE EVENT-TRIGGER EFFICIENCY WITH DIFFERENT h.

h	h_m	h_M	$h^a_{[0,20]}$	$\eta_{[0,20]}$
0.01	0.04	0.34	0.2381	22.810
0.05	0.1	0.35	0.2564	4.1280
0.1	0.1	0.4	0.2985	1.9850
0.2	0.2	0.4	0.3030	0.5150
0.3	0.3	0.6	0.3226	0.0753

trigger efficiency $\eta_{[t_0,t_f]}$ during the interval $[t_0, t_f]$ as

$$\eta_{[t_0,t_f]} = \frac{h^a_{[t_0,t_f]} - h}{h},$$

where $h^a_{[t_0,t_f]} = \frac{t_f - t_0}{N[t_0,t_f]}$ denotes the average inter-event time.

For $\tau_d = 0.33$ and $\tau_a = 0.9$, the numerical simulation results for the minimum / maximum inter-event time h_m / h_M, the average inter-event time $h^a_{[0,20]}$ and event-trigger efficiency $\eta_{[0,20]}$ for different h are listed in Table I, which shows that Zeno-like behaviors can be avoided when $h \leq 0.05$. Furthermore, the larger sampling period h may lead to the larger average length of the inter-event time and the smaller event-trigger efficiency, therefore, the effectiveness of the event-triggered control mechanism will be decreased. On the other hand, the event-triggered mechanism with smaller h usually has the higher efficiency but the smaller average length of the inter-event time. That means, more data sampling will be carried out and the calculation burden will be increased since the event-triggered condition is verified more frequently. Therefore, the event-triggered mechanism with the properly large h may be more practically implementable even though the Zeno-like behavior possibly exists.

Example 5.2: Consider the switched linear systems with

$$A_1 = \begin{bmatrix} -2.9 & 1.5 \\ 2.5 & 1.4 \end{bmatrix}, B_1 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1.1 & -2.5 \\ -1.9 & -2.0 \end{bmatrix}, B_2 = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}.$$

Without considering the data sampling and event-triggered control scheme, the synchronously switched control problem with mode delay $\mathcal{T}_{\max}$ was considered in [30]. Obviously, $\mathcal{T}_{\max}$ corresponds to τ_d in the framework of this paper according to Remark 3.2. To illustrate the advantage of the proposed method, the comparative computation results with different τ_d and $\mathcal{T}_{\max}$ are listed in Table II, where we select

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TABLE II
COMPARISON OF τ_a^* WITH DIFFERENT τ_d ($\mathcal{T}_{\max}$).

$\tau_d(\mathcal{T}_{\max})$	Theorem 4.1 ($\mu=\lambda_s=1.1, \lambda_u=0.7$)	Theorem 1 in [30] ($\lambda=\beta=\mu=2$)
1	$\tau_a^*=1.72$	$\tau_a^*=2.35$
2	$\tau_a^*=3.36$	$\tau_a^*=4.35$
3	$\tau_a^*=4.99$	$\tau_a^*=6.35$

$h=0.001, \nu_1=\nu_2=0.01, \beta_{pq}=1$. It shows that even in the framework of event-triggered control with sampled data, the proposed approach yields a smaller lower bound τ_a^* for the average dwell time, which indicates system can still be stabilized with faster switching. In addition, the lower bound $\tau_a^*=3.42$ is obtained in [31] when the mode delay is selected as $1, 2$ and 3. Though it is insensitive to the bound of delay, there still exists some conservativeness when the mode delay is relatively small.

VI. CONCLUSIONS

The event-triggered control problem for continuous-time switched linear systems is investigated in this paper. Based on the switching signal merging technique, the input delay system approach and mode dependent event-triggered transmission scheme, the closed-loop system is modeled as a switched system with delayed state and augmented switching signal. Then, sufficient exponential stability conditions are obtained by using of multiple Lyapunov functional method. State feedback controllers can also be designed by solving a set of linear matrix inequalities. The simulation result shows that the event-triggered mechanism has high efficiency and can avoid Zeno-like behaviors by selecting a sufficiently small sampling period. We mention that the proposed approach can be generalized to many applications. For example, L_2 control problem when the system is subject to the exogenous disturbance; quantized sampled-data control problems for switched systems via a communication channel with limited capacity.

VII. APPENDIX

Proof of Theorem 3.2: Let τ_k be any switching instant of $\tilde{\sigma}(t)$. Obviously, τ_k should be either some switching instant of $\sigma(t)$ or event-triggered transmission instant. It is easy to verify that the functional $V_{\tilde{\sigma}(t)}(t)$ defined in (15) is continuous for $t \neq \tau_k$. We first show that matrix constraints (12) imply that $V_{\tilde{\sigma}(t)}(t)$ satisfies (8) in Theorem 3.1.

Case 1. Suppose that the switching instant of $\tilde{\sigma}(t)$ is some switching instant of $\sigma(t)$. Without loss of generality, let $\tau_k = \bar{t}_l = t_{s_k}+\bar{t}$, where $\bar{t} \in (0, \tau_d)$. Denote $\sigma(t_{s_k}) = q$ and $\sigma(\bar{t}_l) = p$. Note that t_{s_k} is the sampling instant, then $\tilde{\sigma}(t) = (q, q)$ when $t \in [t_{s_k}, \bar{t}_l)$ and $\tilde{\sigma}(t) = (p, q)$ for $t \in [\bar{t}_l, t_{s_{k+1}})$. It follows from (12) that

$$V_{\tilde{\sigma}(\bar{t}_l)}(\bar{t}_l) \leq \mu x^T(\bar{t}_l)P_{qq}x(\bar{t}_l) + (h - d(\bar{t}_l))W_{\tilde{\sigma}(\bar{t}_l)}(\bar{t}_l), \quad (22)$$

and

$$\begin{aligned} W_{\tilde{\sigma}(\bar{t}_l)}(\bar{t}_l) &= \int_{\bar{t}_l-d(\bar{t}_l)}^{\bar{t}_l} e^{-2\lambda_u(s-\bar{t}_l)}\dot{x}^T(s)Q_{pq}\dot{x}(s)ds \\ &\leq \mu \int_{\bar{t}_l-d(\bar{t}_l)}^{\bar{t}_l} e^{2\lambda_s(s-\bar{t}_l)}\dot{x}^T(s)Q_{qq}\dot{x}(s)ds. \end{aligned} \quad (23)$$

Therefore, $V_{\tilde{\sigma}(\tau_k)}(\tau_k) = V_{\tilde{\sigma}(\bar{t}_l)}(\bar{t}_l) \leq \mu V_{\tilde{\sigma}(\bar{t}_l^-)}(\bar{t}_l^-) = \mu V_{\tilde{\sigma}(\tau_k^-)}(\tau_k^-)$.

Case 2. Suppose that the switching instant of $\tilde{\sigma}(t)$ is some sampling instant. For simplicity, we suppose that $\tau_k = t_{s_{k+1}}$. Noting that $d(t_{s_{k+1}}^-) = h$ and $d(t_{s_{k+1}}) = 0$, therefore,

$$\begin{aligned} V_{\tilde{\sigma}(\tau_k)}(\tau_k) = x^T(\tau_k)P_{pp}x(\tau_k) &\leq \mu x^T(\tau_k)P_{pq}x(\tau_k) \\ &= \mu V_{\tilde{\sigma}(\tau_k^-)}(\tau_k^-). \end{aligned} \quad (24)$$

In summary, $V_{\tilde{\sigma}(t)}(t)$ defined in (15) satisfies (8) in Theorem 3.1. Now it only needs to verify that condition (9) in Theorem 3.1 is satisfied. Due to Assumption 2.1, two situations are considered respectively.

Firstly, we assume that there is one switch in the holding interval $[t_{s_k}, t_{s_{k+1}})$. Similar to the discussion in Case 1 above, suppose the switching instant is $\bar{t}_l = t_{s_k} + \bar{t}$ with $\bar{t} \in (0, \tau_d)$. Let $\sigma(t_{s_k}) = q$ and $\sigma(\bar{t}_l) = p$.

When $t \in [\bar{t}_l, t_{s_{k+1}})$ and $t \in \Omega_{j_l}$, taking the time derivative of the Lyapunov functional (15) along the solution of the system (5), we obtain

$$\begin{aligned} &\dot{V}_{pq}(t) - 2\lambda_u V_{pq}(t) \leq 2x^T P_{pq}\dot{x} - 2\lambda_u x^T P_{pq}x \\ &+ (h - d(t))\dot{x}^T Q_{pq}\dot{x} - \int_{t-d(t)}^{t} \dot{x}^T(s)Q_{pq}\dot{x}(s)ds. \end{aligned} \quad (25)$$

In the following, we simply denote $x = x(t)$ and $x_d = x(t - d(t))$. Since there is no state jumps when the switching occurs, then by the Leibniz-Newton formula, one gets

$$\begin{aligned} 2\xi^T N_{pq}^T(x - x_d) \leq & d(t)\xi^T N_{pq}^T Q_{pq}^{-1} N_{pq}\xi \\ & + \int_{t-d(t)}^{t} \dot{x}^T(s)Q_{pq}\dot{x}(s)ds, \end{aligned} \quad (26)$$

where $\xi = \begin{bmatrix} x^T & x_d^T & e^T(t_{j_l}) \end{bmatrix}^T$.

Furthermore, (6) can be rewritten as

$$e^T(t_{j_l})\Phi_q e(t_{j_l}) < \nu_q(x_d - e(t_{j_l}))^T\Phi_q(x_d - e(t_{j_l})).$$

Therefore,

$$\begin{aligned} &\dot{V}_{pq}(t) - 2\lambda_u V_{pq}(t) \\ &\leq 2x^T P_{pq}\dot{x} - 2\lambda_u x^T P_{pq}x + (h - d(t))\dot{x}^T Q_{pq}\dot{x} \\ &+ d(t)\xi^T N_{pq}^T Q_{pq}^{-1} N_{pq}\xi - 2\xi^T N_{pq}^T(x - x_d) \\ &+ \nu_q(x_d - e(t_{j_l}))^T\Phi_q(x_d - e(t_{j_l})) - e^T(t_{j_l})\Phi_q e(t_{j_l}). \end{aligned} \quad (27)$$

Denote

$$\begin{aligned} \mathcal{A}_{pq} &= \begin{bmatrix} A_p & \widetilde{B}_{pq} & -\widetilde{B}_{pq} \end{bmatrix}, \ X_1 = \begin{bmatrix} I & 0 & 0 \end{bmatrix}, \\ X_2 &= \begin{bmatrix} I & -I & 0 \end{bmatrix}, \ X_3 = \begin{bmatrix} 0 & 0 & I \end{bmatrix}. \end{aligned}$$

Then we can rewritten (27) as

$$\dot{V}_{pq}(t) - 2\lambda_u V_{pq}(t) \leq \xi^T\Gamma_{pq}(d(t))\xi, \ t \in [\bar{t}_l, t_{s_{k+1}}),$$

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where

$$\begin{aligned}\Gamma_{pq}(d(t)) = &X_1^T P_{pq}\mathcal{A}_{pq} + \mathcal{A}_{pq}^T P_{pq} X_1 - 2\lambda_u X_1^T P_{pq} X_1\\ &+ (h - d(t))\mathcal{A}_{pq}^T Q_{pq}\mathcal{A}_{pq} + d(t) N_{pq}^T Q_{pq}^{-1} N_{pq}\\ &+ \nu_q (X_1 - X_2 - X_3)^T \Phi_q (X_1 - X_2 - X_3)\\ &- N_{pq}^T X_2 - X_2^T N_{pq} - X_3^T \Phi_q X_3.\end{aligned}$$

Note that $d(t) \in [0, h)$. Moreover, $\Gamma_{pq}(d(t))$ depends linearly on $d(t)$. Therefore, $\Gamma_{pq}(d(t)) < 0$ if and only if $\Gamma_{pq}(0) < 0$ and $\Gamma_{pq}(h) < 0$ hold simultaneously.

Furthermore, when $t \in [t_{s_k}, \bar{t}_l)$, $\tilde{\sigma}(t) = (q, q)$. Similar to (27), the time derivative of the Lyapunov functional (15) along the solution of the system (5) can be estimated as

$$\dot{V}_{qq}(t) + 2\lambda_s V_{qq}(t) \le \xi^T \Gamma_{qq}(d(t))\xi, \tag{28}$$

where

$$\begin{aligned}\Gamma_{qq}(d(t)) = &X_1^T P_{qq}\mathcal{A}_{qq} + \mathcal{A}_{qq}^T P_{qq} X_1 + 2\lambda_s X_1^T P_{qq} X_1\\ &+ (h - d(t))\mathcal{A}_{qq}^T Q_{qq}\mathcal{A}_{qq} + d(t) e^{2\lambda_s h} N_{qq}^T Q_{qq}^{-1} N_{qq}\\ &+ \nu_q (X_1 - X_2 - X_3)^T \Phi_q (X_1 - X_2 - X_3)\\ &- N_{qq}^T X_2 - X_2^T N_{qq} - X_3^T \Phi_q X_3.\end{aligned}$$

Note that $d(t) \in [0, h)$, then similar to the above analysis, $\Gamma_{qq}(d(t)) < 0$ is equivalent to $\Gamma_{qq}(0) < 0$ and $\Gamma_{qq}(h) < 0$.

When there is no switching in the holding interval $[t_{s_k}, t_{s_{k+1}})$, the proof is omitted since it can be obtained directly by $\bar{t}_l \to t_{s_{k+1}}$, that is, (28) holds for all $t \in [t_{s_k}, t_{s_{k+1}})$.

As a result, we can conclude that

$$\dot{V}_{\tilde{\sigma}(t)}(t) + 2\lambda_{\tilde{\sigma}(t)} V_{\tilde{\sigma}(t)}(t) \le \xi^T \Gamma_{\tilde{\sigma}(t)}(d(t))\xi,\ \ t \in [t_{s_k}, t_{s_{k+1}}).$$

Furthermore, $\Gamma_{\tilde{\sigma}(t)}(d(t)) < 0$ is equivalent to $\Gamma_{\tilde{\sigma}(t)}(0) < 0$ and $\Gamma_{\tilde{\sigma}(t)}(h) < 0$, which can be guaranteed by (13) and (14) respectively. This completes the proof.

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