Sliding Mode Control for a Class of Nonlinear Multi-agent System with Time-delay and Uncertainties

Jie Zhang, Ming Lyu, Tianfeng Shen, Lei Liu, Yuming Bo

***Abstract*—In this paper, a type of multi-agent system is adopted in the practical project with time-delay, uncertainties and linear feedback. In addition, sliding mode control is used to ensure the robust stability of the system since it is insensitive to parameter change and interference. For the system in three different conditions, namely fixed structure, every agent being only influenced by single time-delay and each agent being affected by multiple time-delay, the corresponding sliding surface and the control law are improved. The reaction factors are proven by Lyapunov functions and the LMI approach is taken to guarantee the robust stability of the sliding surface. To prove the effectiveness of the conclusion, experiments on each condition are conducted. Besides, in the last part, a simple application is applied and proved to be effective.**

***Index Terms*—Multi-agent system , sliding mode control , time-delay system , uncertainty.**

I. INTRODUCTION

SINCE its beginning in the mid-1980s, the multi-agent system had gained considerable attention for its advantages in many fields, such as distributed sensor networks, cooperative control of unmanned vehicles, flocking and formation control [1].

However, with the ever-increasing application of the multi-agent system, there appeared an increasing number of problems, including communication time-delay, random packet loss, uncertainty of the system mode and unknown interference. These problems may cause communication time-delay or the channel jam, thus reducing the communication quality. Some serious problems may even cause the system breakdown [2]–[6].

Many theories are proposed to solve the problems. A novel control strategy for multi-agent coordination with event-based broadcasting is presented in this paper [7]. To solve the problem of consensus tracking without the assumption that the topology among followers is strongly connected and fixed, distributed consensus tracking is addressed in [8] for multi-agent systems with Lipschitz-type node dynamics. [9] considers the problems of containment control occurred in the application of both continuous-time and discrete-time multi-agent systems with general linear dynamics under directed communication topologies. Many researchers have put forward various control theories for numerous conditions of linear cases.

Most previous works focused only on linear systems. Recently, an increasing number of researchers have put their effort on the nonlinear system since it has many advantages in practical application. Based on a representative general nonlinear model, [10] obtains several basic criteria for the consensus of the multi-agent system. An adaptive neural network (NN) consensus control method for a class of nonlinear multi-agent systems with state time-delay is proposed in [11]. Distributed finite-time tracking control for nonlinear multi-agent systems subject to external disturbances is also investigated [12]. In addition, some other investigations are also gained for the nonlinear condition [13].

Various types of control theories are adopted in the control of the multi-agent system, such as distributed tracking control and leader-following control. Compared with other control methods, sliding mode control is more stable under internal and external interferences. Besides, it is superior in its application in dealing with the uncertainties of the system and its simplified realization [14]. In [15], the fast finite-time Lyapunov stability theorem and the terminal sliding control technique are adopted to achieve fast finite-time leader-follower consensus with high order SISO uncertain nonlinear agents. [16] combines the approach of the second-order sliding mode observer with state feedback analysis to tract the target location in finite time. In [17], [18], sliding mode control is used to estimate robust fault in consensus tracking. Time-delay and uncertainties are the most serious problems in the application of the multi-agent system in the fields like unmanned vehicle control and aircraft [19]–[21]. Many researches have focused on solving such problems. In [22], researchers employ the Lyapunov stability theory and linear matrix inequality (LMI) technique to deal with the uncertainties. In [23], the impulsive consensus of multi-agent nonlinear systems with control gain error is further studied. In [24], the sliding surface is designed to maximize the calculable set of admissible delays. In [25], elayed full-state feedback

Manuscript received Month xx, 2xxx; revised Month xx, xxxx; accepted Month x, xxxx. This work was supported in part by Science and Technology Program of Lianyungang under Grant SH1441 and in part by The Innovation Project of Cultivating Graduate Student in Jiangsu Province under Grant KYLX16_0455.

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This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TIE.2017.2701777, IEEE Transactions on Industrial Electronics

control is used to deal with the occasional time-delay.

The multi-agent system includes vast amounts of various conditions [26], [27]. Although much work has been done, to the best of our knowledge, a large number of researches on the multi-agent system under uncertainties and time-delay with the nonlinear characteristic are with relatively low quality. As a result, in this class of multi-agent system, sliding mode control is adopted in order to ensure its robust stability and the simplicity of the application. Three different models are investigated in this paper. Firstly, the time-delay independent system is without uncertainties. Under such a condition, a virtual feedback is used to construct a quadratically stable sliding surface. Secondly, the uncertain system with single time-delay is investigated. In this condition, the non-singular transformation is used to build a quadratically stable sliding surface which can make the system meet the trigger condition. Thirdly, the multiple time-delay system with uncertainties is analyzed. An equivalent reduced order system is applied so as to construct a quadratically stable sliding surface which can support the system to reach the sliding surface in a limited time. For all of the three conditions, the corresponding control laws are given. Both the approaching and sliding motions have excellent robust stability under uncertainties and external interrupts. The simulation experiments are conducted to test and verify its correctness. In the last part, a practical application is put forward.

II. Problem Formulation

For the multi-agent system, the flow of information is transmitted between each agent with directions. In this paper, $G = \{v, e\}$ is used to represent a directed graph, while n represents the set of agents $v = \{1, 2, \cdots, n\}$, $e \subseteq v \times v$ represents the set of edges, while edge $(i, j) \in e$ indicates that agent j is able to receive information from agent i. The adjacent matrix $A = [a_{ij}] \in R^{n\times n}$ of directed graph G is defined as follows: if any flow of information exists between two agents i and j, then a_{ij} denotes the flow of information from i to j. What needs to be further explained is that while agent i sends information to j, it may also receives information from j at the same time. Moreover, a_{ij} is unnecessarily equal to a_{ji}. If the multi-agent system is static, the system can be considered as a static system without uncertainties. However, the time-delay still exists because of the communication among all agents. The model is shown below:

$$\begin{cases} \dot{x}(t) = Ax(t) + A_d x(t-\tau) + B(u(t) + f(t)) + g(x(t)) \\ x(t) = \Psi(t) \qquad t \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ refers to input of the system, $f(t)$ is disturbance, while $g(x(t))$ is the continuous function of the system state. In addition, $x(t)$,A, $A_d(t)$, and B are constant matrices with appropriate dimensions.

However, in the real condition, multi-agents can not always be in a static state. The connection of agents within the communication range may be suspended by the increase of distance. In addition, the agents which are not connected may get into connection due to the variation of their location. In the system, if the state change is only limited to a particular agent, the model is shown below:

$$\begin{cases} \dot{x}(t) = (A + \Delta A(t))x(t) + (A_d + \Delta A_d(t))x(t-\tau) \\ \qquad + B(u(t) + f(t)) + g(x(t)) \\ x(t) = \Psi(t) \qquad t \in [-\tau, 0] \end{cases} \tag{2}$$

where $x(t) \in R^n$ denotes the system state, $u(t) \in R^m$ refers to current input, $\Delta A(t)$ and $\Delta A_d(t)$ are internal parameter perturbations caused by uncertainties; $f(t)$ is external disturbance, $g(x(t))$ is the continuous function of the system state; $x(t)$, A, $A_d(t)$, and B are constant matrices featured with appropriate dimensions.

If the state change of an agent is influenced by time-delays of multiple agents, the model is shown as follows:

$$\begin{cases} \dot{x}(t) = (A + \Delta A(t))x(t) + \sum_{i=1}^{N}(A_d + \\ \qquad \Delta A_d(t))x_i(t-\tau_i) + B(u(t) + f(t)) + g(x(t)) \\ x(t) = \Psi(t) \qquad t \in [-\tau, 0] \end{cases} \tag{3}$$

where $x(t) \in R^n$ denotes the system state, $u(t) \in R^m$ refers to current input, $\Delta A(t)$ and $\Delta A_d(t)$ are internal parameter perturbations caused by uncertainties; $f(t)$ refers to external disturbance, $g(x(t))$ is the continuous function of the system state $x(t)$, A, $A_d(t)$, and B are constant matrices with appropriate dimensions, while N is the number of motors in the system.

For the systems shown in (1), (2) and (3), the following assumptions are put forward:

Assumption 1. Suppose (A, B) could be stabilized, then there is a matrix K makes $\bar{A} = A - BK$ stabilizing and $rank(B) = m \leq n$.

Assumption 2. The disturbance has an upper bound, which meets $\|f(t)\| \leq \delta_f$.

Assumption 3. Time delay τ_i is boundary, and $\| \underset{i\in N}{\tau_i} \| \leq \tau$.

Assumption 4. The perturbation parameter of the system satisfies

$$[\Delta A \quad \Delta A_d] = GD(t)[H \quad H_d] \tag{4}$$

Assumption 5. Suppose $g(x(t))$ is the continuous function of the system state $x(t)$, then there is a positive constant ρ that makes $g(x(t))^T g(x(t)) \leq \rho x(t)^T x(t)$.

Respectively, G, H and H_d, are known constant matrix; $D(t)$ is uncertain matrix as a result of, yet Lebesgue-measurable, and $D^T(t)D(t) \leq I$.

Generally speaking, two steps should be taken to design the sliding mode control. Firstly, a stable sliding surface should be chosen; secondly, a suitable control law should be design.

Some lemmas which will be used in the proof below are presented here.

Lemma 1: Let $Y = Y^T$, D, E, and $F(t)$ be real matrix of proper dimensions, and $F^T(t)F(t) \leq I$, then inequality $Y + DFE + (DFE)^T < 0$ holds if there exists a constant ε, which makes the following equation holds.

$$Y + \varepsilon DD^T + \varepsilon^{-1}E^T E < 0$$

Lemma 2: Schur complement Given a symmetric matrix $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, where S_{11} is $r \times r$ dimensional, the following three conditions are equivalent:

(1) $S < 0$;
(2) $S_{11} < 0$, $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;
(3) $S_{22} < 0$, $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$;

III. Main Results

A. Time-delay independent system without uncertainties

In this section, the sliding surface and the control law in time-delay independent system without uncertainties are investigated.

Theorem 1: For system (1) that meets assumption 1 , 2 and 5, the system of any initial condition will reach the sliding surface in finite time, or in other words, it will gradually be driven onto the sliding surface $S(t) = B^T Px(t)$ according to the control law

$$u(t) = u_{eq}(t) + u_{sw}(t) \tag{5}$$

where the equivalent control is $u_{eq}(t) = -(B^T PB)^{-1}[B^T PAx(t) + B^T PA_d x(t-\tau) + B^T Pg(x(t))]$, while nonlinear switch control is $u_{sw}(t) = -(B^T PB)^{-1}\{[|B^T PB|\delta_f + \varepsilon]sgn(S)\}$, in which ε is a positive constant.

Proof 1: Choosing a Lyapunov function as

$$V = \frac{1}{2}S^T S \tag{6}$$

Obviously, for all $S(x,t) \neq 0$, Lyapunov function V is positive-definite. Taking the derivative of (6) and substituting (1) for it , then

$$\dot{V} = S\dot{S}$$

and

$$\begin{aligned}\dot{S} &= B^T P\dot{x}\\ &= B^T PAx(t) + B^T PA_d x(t-\tau) + B^T PBu(t) + B^T PBf(t)\\ &= B^T PAx(t) + B^T PA_d x(t-\tau) + B^T PB[u_{eq}(t) + u_{sw}(t)]\\ &+ B^T PBf(t)\\ &= B^T PBu_{sw}(t) + B^T PBf(t)\\ &\leq -\varepsilon sgn(S)\end{aligned}$$

So there is

$$\dot{V} \quad = S\dot{S} \leq -\varepsilon|S| \leq 0$$

It is proved that the system can meet the reaching condition of the sliding mode regardless of the initial condition. Thus, the proof of Theorem 1 is completed.

Theorem 2: If the matrix (7) exists, then X and V are symmetric positive-definite matrices, and matrix L has an appropriate dimension. In this case, system (1) is quadratically stable under the control law (5) and the sliding surface of system (1) is $S(t) = B^T Px(t)$, in which $P = X^{-1}$.

$$\begin{bmatrix} AX+XA^T-BL-L^TB^T+I & A_dV & X & X\\ VA_d^T & -V & 0 & 0\\ X & 0 & -V & 0\\ X & 0 & 0 & -\frac{1}{\rho}I\end{bmatrix} < 0 \tag{7}$$

Proof 2: First, we set (5) as a virtual feedback form

$$u(t) = -Kx(t) + v(t) \tag{8}$$

where $v(t) = Kx(t) + u_{eq}(t) + u_{sw}(t)$. Substituting equation (1) into (8), then the function of the system can be written as the following:

$$\dot{x}(t) = \bar{A}x(t) + A_d x(t-\tau) + B(v(t) + f(t)) + g(x(t)) \tag{9}$$

where $\bar{A} = A - BK$. Choosing Lyapunov function as

$$V(x,t) = x^T(t)Px(t) + \int_{t-\tau}^{t} x^T(s)Qx(s)ds \tag{10}$$

Taking derivative of (10) and then substituting (9) into it

$$\begin{aligned}\dot{V} &= 2x^T P[\bar{A}x + A_d x(t-\tau) + B(v+f) + g(x(t))]\\ &+ x^T Qx - x^T(t-\tau)Qx(t-\tau)\end{aligned}$$

From Theorem 1, the system can meet the trigger condition of the sliding mode with the control law(7), or in other words, the system can reach the sliding surface in finite time regardless of the initial condition. Therefore, if $S = B^T Px = 0$ is valid, $S^T = x^T P^T B = 0$ stands as well. The above equation can be transformed into

$$\begin{aligned}\dot{V} =& 2x^T P\bar{A}x + 2x^T PA_d x(t-\tau) + 2x^T Pg(x(t))\\ &+ x^T Qx - x^T(t-\tau)Qx(t-\tau)\end{aligned}$$

According to assumption 5

$$2x^T Pg = 2(PT)^T g(x(t)) \leq x(t)^T PPx(t) + \rho x(t)^T x(t)$$

Then

$$\begin{aligned}\dot{V} &= 2x^T P\bar{A}x + 2x^T PA_d x(t-\tau) + x(t)^t PPx(t)\\ &+ \rho x(t)^T x(t) + x^T Qx - x^T(t-\tau)Qx(t-\tau)\\ &= \begin{bmatrix}x^T(t) & x^T(t-\tau)\end{bmatrix} M \begin{bmatrix}x(t)\\ x(t-\tau)\end{bmatrix}\end{aligned} \tag{11}$$

where

$$M = \begin{bmatrix}\bar{A}^T P + P\bar{A} + Q + PP + \rho I & PA_d\\ A_d^T P & -Q\end{bmatrix} \tag{12}$$

If system (1) is quadratically stable, $M < 0$ will always be established. Multiply $diag\{P^{-1} \quad I\}$ in both sides of equation (12), then

$$\begin{bmatrix}P^{-1}\bar{A}^T + \bar{A}P^{-1} + P^{-1}QP^{-1} + I + \rho P^{-1}P^{-1} & A_d\\ A_d^T & -Q\end{bmatrix} < 0$$

Setting $P^{-1} = X$, $L = KX$, then

$$\begin{bmatrix}XA^T + AX - BL - L^TB^T + XQX + I + \rho XX & A_d\\ A_d^T & -Q\end{bmatrix} < 0$$

lemma 2 is used twice, and then the matrix can be transformed into

$$\begin{bmatrix}XA^T+AX-BL-L^TB^T+I & A_d & X & X\\ A_d^T & -Q & 0 & 0\\ X & 0 & -Q^{-1} & 0\\ X & 0 & 0 & -\frac{1}{\rho}I\end{bmatrix} < 0 \tag{13}$$

multiply $diag\{I \quad Q^{-1} \quad I \quad I\}$ on both sides of equation (13), then

$$\begin{bmatrix}XA^T+AX-BL-L^TB^T+I & A_dQ^{-1} & X & X\\ Q^{-1}A_d^T & -Q^{-1} & 0 & 0\\ X & 0 & -Q^{-1} & 0\\ X & 0 & 0 & -\frac{1}{\rho}I\end{bmatrix} < 0$$

Letting $Q^{-1} = V$, so the Theorem 2 is proved.

Remark 1. According to the proof Theorem 1 and 2, the stability of the sliding mode control of system (1) can be guaranteed. Theorem 1 proves that the system can reach the sliding surface in finite time. During process of proving theorem 2, a virtual feedback which transforms control law (5) to (8) is used to simplify the proof that the sliding surface is quadratically stable.

B. Uncertain system with single time-delay

As the structure of the system may vary from time to time, internal parameter perturbations may be caused. In this section, the uncertain system with single time-delay will be investigated, which is shown as (2) .

In order to study the system, a nonsingular matrix T can be chosen by the assumption, so that $TB = \begin{bmatrix}0_{(n-m)\times m}\\ B_m\end{bmatrix}$, where B_m is nonsingular with $rank(B_m) = m$. According to the previous investigation , we set $T = \begin{bmatrix}U_2^T\\ U_1^T\end{bmatrix}$, where $U_1 \in$

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TIE.2017.2701777, IEEE Transactions on Industrial Electronics

$R^{n\times m}$ and $U_2 \in R^{n\times(n-m)}$ are two unitary matrices defined by singular value decomposition of matrix B, then:

$$B = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma \\ 0_{(n-m)\times m} \end{bmatrix} V^T$$

where $\Sigma \in R^{m\times m}$ is a symmetric positive-definite matrix; and $V \in R^{m\times m}$ is a unitary matrix. According to the state transformation $y = Tx$, the system (2) can be transformed into

$$\begin{cases} \dot{y}(t) = (\bar{A} + \Delta\bar{A}(t))y(t) + (\bar{A}_d + \Delta\bar{A}_d(t))y(t-\tau) \\ \qquad + \begin{bmatrix} 0_{(n-m)\times m} \\ B_m \end{bmatrix} (u(t) + f(t)) + Tg(x(t)) \\ y(t) = \Phi(t) \qquad t \in [-\tau, 0] \end{cases} \tag{14}$$

where $\bar{A} = TAT^{-1}$, $\bar{A}_d = TA_dT^{-1}$, $\Delta\bar{A}(t) = T\Delta A(t)T^{-1}$, $\Delta\bar{A}_d(t) = T\Delta A_d(t)T^{-1}$ and $\Phi(t) = T\Psi(t)$.

The control law of the system is presented below and the sliding surface can be chosen as

$$S(t) = Ry = Cy_1 + y_2 = 0 \tag{15}$$

where $R = [C \quad I]$, $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and $C \in R^{m\times(n-m)}$.

Theorem 3: For system (2), which meets assumption1, 2, 4 and 5, the sliding surface is (15). Then the system the trigger condition in accordance with the control law below

$$\begin{aligned} u(t) = &- B_m^{-1}[\Phi S + G(\bar{A} + \Delta\bar{A})y(t) + G(\bar{A}_d \\ &+ \Delta\bar{A}_d)y(t-\tau) + B_m\delta_f + GTg(x(t))] \end{aligned} \tag{16}$$

Proof 3: Choosing Lyapunov function as

$$V = \frac{1}{2}S^TS$$

The function below derives from the Lyapunov function and it has substituted (14)

$$\begin{aligned} \dot{V} &= S^T\dot{S} = S^TG\dot{y} \\ &= S^T[G\bar{A}y(t) + G\Delta\bar{A}y(t) \\ &+ G(\bar{A}_d + \Delta\bar{A}_d)y(t-\tau) \\ &+ B_m(u(t) + f(t)) + Tg(x(t))] \end{aligned} \tag{17}$$

Taking control law (16) into account and considering assumption 2, then

$$\dot{V} \le -S^T\Phi S < 0$$

The proof of Theorem 3 is complete.

In the next section, the quadratical stability of multiple time-delay condition will be investigated. Besides, (2) can be considered as a special case ($N = 1$). Then this case can be proved easily. For brevity, the result and the proof are omitted here.

Remark 2. According to previous research, the approaching exponent depends on the value of Φ when the sliding function is approaching the sliding surface. The speed of approaching motion is proportional to $\lambda_{min}(\Phi)$. However, it is difficult to acquire the ideal motion. In addition, the real motion can only converge to a region extremely close to the sliding surface, and then move back and forth across the sliding surface repeatedly.

C. Multiple time-delay system with uncertainties

The previous section focuses on the influence of singe time-delay on the system that is susceptible to internal parameter perturbations. However, in many conditions, one single agent may be influenced by multiple time-delays with parameter perturbation. The system function is shown in (3).

According to non-singular transformation and assumptions 2 to 5 , system (3) can be transformed to:

$$\begin{cases} \dot{y}(t) = (\bar{A} + \Delta\bar{A}(t))y(t) \\ \qquad + \sum_{i=1}^{N}(\bar{A}_d + \Delta\bar{A}_d(t))y_i(t-\tau_i) \\ \qquad + \begin{bmatrix} 0_{(n-m)\times m} \\ B_m \end{bmatrix} (u(t) + f(t)) + \begin{bmatrix} g_1(x(t)) \\ g_2(x(t)) \end{bmatrix} \\ y(t) = \Phi(t) \qquad t \in [-\tau, 0] \end{cases} \tag{18}$$

where $\bar{A} = TAT^{-1}$, $\bar{A}_d = TA_dT^{-1}$, $\Delta\bar{A}(t) = T\Delta A(t)T^{-1}$, $\Delta\bar{A}_d(t) = T\Delta A_d(t)T^{-1}$, $\Phi(t) = T\Psi(t)$, $g_1 = U_2^Tg(x(t))$, $g_2 = U_1^Tg(x(t))$, in addition we have

$$\begin{cases} \dot{y}_1(t) = (\bar{A}_{11} + \Delta\bar{A}_{11}(t))y_1(t) \\ \qquad + \sum_{i=1}^{N}(\bar{A}_{d11} + \Delta\bar{A}_{d11}(t))y_{i1}(t-\tau_i) \\ \qquad + (\bar{A}_{11} + \Delta\bar{A}_{12}(t))y_2(t) \\ \qquad + \sum_{i=1}^{N}\bar{A}_{d12} + \Delta\bar{A}_{d12}(t))y_{i2}(t-\tau_i) + g_1 \\ \dot{y}_2(t) = (\bar{A}_{21} + \Delta\bar{A}_{21}(t))y_1(t) \\ \qquad + \sum_{i=1}^{N}(\bar{A}_{d21} + \Delta\bar{A}_{d21}(t))y_{i1}(t-\tau_i) + (\bar{A}_{22} \\ \qquad + \Delta\bar{A}_{22}(t))y_2(t) \\ \qquad + \sum_{i=1}^{N}\bar{A}_{d22} + \Delta\bar{A}_{d22}(t))y_{i2}(t-\tau_i) \\ \qquad + B_m(u(t) + f(t)) + g_2 \\ y_1(t) = \Phi_1(t) \qquad t \in [-\tau, 0] \\ y_2(t) = \Phi_2(t) \qquad t \in [-\tau, 0] \end{cases} \tag{19}$$

where $y_1 \in R^{(n-m)}$, $y_2 \in R^m$, $B_m = \sum V^T$, $\bar{A}_{11} = U_2^TAU_2$, $\bar{A}_{12} = U_2^TAU_1$, $\bar{A}_{d11} = U_2^TA_dU_2$, $\bar{A}_{d12} = U_2^TA_dU_1$, $\Delta\bar{A}_{11} = U_2^TGD(t)HU_2$, $\Delta\bar{A}_{d11} = U_2^TGD(t)H_dU_2$, $\Delta\bar{A}_{12} = U_2^TGD(t)H_dU_1$, $\Delta\bar{A}_{d12} = U_2^TGD(t)H_dU_1$, $\Phi_1(t) \in R^{(n-m)}$ and $\Phi_2(t) \in R^m$ are the sub-blocks of $\Phi(t)$. It is easy to infer that the first equation of (19) represents the dynamics of sliding motion (18), and therefore, the sliding surface can be designed as

$$S = Cy_1 + y_2 = [C \quad I]y = 0 \tag{20}$$

where $C \in R^{m\times(n-m)}$ and $y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$, substituting $y_2 = -Cy_1$ to the first equation of (19), then the sliding motion is written as

$$\begin{cases} \dot{y}_1(t) = (\bar{A}_{11} + \Delta\bar{A}_{11} - \bar{A}_{12}C - \Delta\bar{A}_{12}C)y_1(t) \\ \qquad + \sum_{i=1}^{N}(\bar{A}_{d11} + \Delta\bar{A}_{d11} - bar A_{d12}C \\ \qquad - \Delta\bar{A}_{d12}C)y_{i1}(t-\tau_i) + g_1 \\ y_1(t) = \Phi_1(t) \qquad t \in [-\tau, 0] \end{cases} \tag{21}$$

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Definition The definition of previous researches demonstrates that if there exists symmetric positive-definite matrix P, $Q \in R^{(n-m)\times(n-m)}$, the uncertain sliding motion (21) is believed to be quadratically stable

$$V(y_1(t),t) = y_1^T(t)Py_1(t) + \int_{t-\tau}^{t} y_1^T(s)Qy_1(s)ds \tag{22}$$

Time t satisfies

$$L(y_1(t),t) = \dot{V}(y_1(t),t) \le 0 \tag{23}$$

for all pairs $(y_1(t),t) \in R^{(n-m)} \times R$.

According to this definition, a constant $C \in R^{m\times(n-m)}$ can be gained and a suitable control law $u(t)$ should be proposed to ensure the quadratical stability of the system and the asymptotic stabilization of the sliding motion.

Firstly, a suitable sliding surface is proposed.

Theorem 4: For the system (3), if there exists symmetric positive-definite matrixs $Z \in R^{(n-m)\times(n-m)}$, $L \in R^{(n-m)\times n}$, $W \in R^{(n-m)\times n}$ and a general matrix $K \in R^{(n-m)\times(n-m)}$ as well as a positive constant ε making (26) holds, where $\Lambda = Z\bar{A}_{11}^T - K^T\bar{A}_{12} + \bar{A}_{11}Z - \bar{A}_{12}K + \varepsilon U_2^T GG^T U_2 + I$, $\eta = \frac{1}{\sqrt{N}}\bar{A}_{d11}L - \frac{1}{\sqrt{N}}\bar{A}_{d12}W$, $\delta = ZU_2^T H^T - KU_1^T H^T$, $\theta = \frac{1}{\sqrt{N}}LU_2^T H_d^T - \frac{1}{\sqrt{N}}WU_1^T H_d^T$.$\kappa = Z\left[[I - C]\right]T^{-T}$. The system is quadratically stable. And its sliding surface is $S = Cy_1(t) + y_2(t) = [C \quad I]Y = 0$.

$$\begin{bmatrix} \Lambda & \eta & \delta & I & \kappa \\ * & -V & \theta & 0 & 0 \\ * & * & -\varepsilon I & 0 & 0 \\ * & * & * & -V & 0 \\ * & * & * & * & -\frac{1}{\rho}I \end{bmatrix} < 0 \tag{24}$$

where $*$ are matrixes acquired based on matrix symmetry.

Proof 4: Taking symmetric positive-definite matrix P and $Q \in R^{(n-m)\times(n-m)}$ to construct the following Lyapunov function

$$V(y_1(t),t) = y_1^T(t)Py_1(t) + \sum_{i=1}^{N}\int_{t-\tau_i}^{t} y_1^T(s)Qy_1(s)ds \tag{25}$$

For all $t \in [-\tau, 0]$, $y_1(t) \neq 0$. Taking the derivative of (25) through (21) , we have

$$\begin{aligned}\dot{V}(y_1(t),t) =& 2y_1^T(t)P\dot{y_1}(t) + Ny_1^TQy_1(t) \\ &- \sum_{i=1}^{N} y_1^T(t-\tau_i)Qy_1(t-\tau_i)\end{aligned}$$

Letting $\tilde{A}_{11} = \bar{A}_{11} - \bar{A}_{12}C$, $\Delta\tilde{A}_{11} = \Delta\bar{A}_{11} - \Delta\bar{A}_{12}C = U_2^T GD(t)H(U_2 - U_1C)$, $\tilde{A}_{d11} = \bar{A}_{d11} - \bar{A}_{d12}C$, $\Delta\tilde{A}_{d11} = \Delta\bar{A}_{d11} - \Delta\bar{A}_{d12}C = U_2^T GD(t)H_d(U_2 - U_1C)$, Then substituting (21) in

$$\begin{aligned}\dot{V}(y_1(t),t) \le& 2y_1P(\tilde{A}_{11} + \Delta\tilde{A}_{11})y_1 \\ &+ 2\sum_{i=1}^{N} y_1P(\tilde{A}_{d11} + \Delta\tilde{A}_{d11})y_i(t-\tau_i) + y_1^T NQy_1 \\ &+ \sum_{i=1}^{N} y_i^T Qy_i(t-\tau_i) + y_1^T PPy_1 + g^T U_2U_2^T g\end{aligned}$$

Because U_2 is an unitary matrix,$U_2^TU_2 = E$. And according to assumption 5

$$\begin{aligned}\dot{V}(y_1(t),t) \le& 2y_1P(\tilde{A}_{11} + \Delta\tilde{A}_{11})y_1 \\ &+ 2\sum_{i=1}^{N} y_1P(\tilde{A}_{d11} + \Delta\tilde{A}_{d11})y_i(t-\tau_i) + y_1^T NQy_1 \\ &+ \sum_{i=1}^{N} y_i^T Qy_i(t-\tau_i) + y_1^T PPy_1 \\ &+ \rho y_1^T \begin{bmatrix} I & -C \end{bmatrix} T^{-T}T^{-1}\begin{bmatrix} I \\ -C \end{bmatrix} y_1\end{aligned}$$

Letting $F = \begin{bmatrix} I & -C \end{bmatrix} T^{-T}$, $Y(t-\tau) = \begin{bmatrix} y_1(t-\tau_1) & \dots & y_N(t-\tau_N) \end{bmatrix}^T$. So there is

$$\dot{V}(y_1(t),t) =$$

$$\begin{bmatrix} y_1(t) \\ y_1(t-\tau_1) \\ \vdots \\ y_N(t-\tau_N) \end{bmatrix}^T \begin{bmatrix} (\bar{A}_{11}+\Delta\tilde{A}_{11})^TP+P(\bar{A}_{11}+\Delta\tilde{A}_{11})+NQ+PP+\rho FF^T & P(\tilde{A}_{d11}+\Delta\tilde{A}_{d11}) \\ (\tilde{A}_{d11}+\Delta\tilde{A}_{d11})^TP & -Q \end{bmatrix} \begin{bmatrix} y_1(t) \\ y_1(t-\tau_1) \\ \vdots \\ y_N(t-\tau_N) \end{bmatrix}$$

$$= \begin{bmatrix} y_1(t) \\ Y(t-\tau) \end{bmatrix}^T \begin{bmatrix} (\bar{A}_{11}+\Delta\tilde{A}_{11})^TP+P(\bar{A}_{11}+\Delta\tilde{A}_{11})+NQ+PP+\rho FF^T & P(\tilde{A}_{d11}+\Delta\tilde{A}_{d11}) \\ (\tilde{A}_{d11}+\Delta\tilde{A}_{d11})^TP & -Q \end{bmatrix} \begin{bmatrix} y_1(t) \\ Y(t-\tau) \end{bmatrix}$$

Setting $Z = P^{-1}$, $J = ZQZ$, equation (26) can be written as

$$\dot{V}(y_1(t),t) = \begin{bmatrix} Py_1(t) \\ PY(t-\tau) \end{bmatrix}^T M \begin{bmatrix} Py_1(t) \\ PY(t-\tau) \end{bmatrix} \tag{26}$$

where

$$M = \begin{bmatrix} Z(\tilde{A}_{11}+\Delta\tilde{A}_{11})^T+(\tilde{A}_{11}+\Delta\tilde{A}_{11})Z+NJ+I+\rho ZFF^TZ & (\tilde{A}_{d11}+\Delta\tilde{A}_{d11})Z \\ Z(\tilde{A}_{d11}+\Delta\tilde{A}_{d11})^T & -J \end{bmatrix} \tag{27}$$

Substituting (4) to (28), and following Lemma 1 that

$$\begin{aligned}M =& \begin{bmatrix} Z\tilde{A}_{11}^T+\tilde{A}_{11}Z+NJ+I+\rho ZFF^TZ & \tilde{A}_{d11}Z \\ Z\tilde{A}_{d11}^T & -J \end{bmatrix} \\ &+ \begin{bmatrix} U_2^TG \\ 0 \end{bmatrix} D(t) \begin{bmatrix} H(U_2-U_1C)Z & H_d(U_2-U_1C)Z \end{bmatrix} \\ &+ \begin{bmatrix} H(U_2-U_1C)Z & H_d(U_2-U_1C)Z \end{bmatrix}^T D^T(t) \begin{bmatrix} U_2^TG \\ 0 \end{bmatrix}^T\end{aligned} \tag{28}$$

Using Lemma 2 twice, $M < 0$ is equivalent to

$$\begin{bmatrix} Z\tilde{A}_{11}^T+\tilde{A}_{11}Z+\varepsilon U_2^TGG^TU_2+I & \tilde{A}_{d11}Z & Z(U_2-U_1C)^TH^T & I & ZF \\ Z\tilde{A}_{d11}^T & -J & Z(U_2-U_1C)^TH_d^T & 0 & 0 \\ H(U_2-U_1C)Z & H_d(U_2-U_1C)Z & -\varepsilon I & 0 & 0 \\ I & 0 & 0 & -(NJ)^{-1} & 0 \\ F^TZ & 0 & 0 & 0 & -\frac{1}{\rho}I \end{bmatrix} < 0 \tag{29}$$

Multiply $diag\{I \quad \frac{1}{\sqrt{N}}J^{-1} \quad I \quad I \quad I\}$ on both sides of the matrix, then

$$\begin{bmatrix} Z\tilde{A}_{11}^T+\tilde{A}_{11}Z+\varepsilon U_2^TGG^TU_2+I & \frac{1}{\sqrt{N}}\tilde{A}_{d11}ZJ^{-1} & Z(U_2-U_1C)^TH^T & I & ZF \\ \frac{1}{\sqrt{N}}J^{-1}Z\tilde{A}_{d11}^T & -(NJ)^{-1} & \frac{1}{\sqrt{N}}J^{-1}Z(U_2-U_1C)^TH_d^T & 0 & 0 \\ H(U_2-U_1C)Z & \frac{1}{\sqrt{N}}J^{-1}H_d(U_2-U_1C)Z & 0 & -\varepsilon I & 0 \\ I & 0 & 0 & 0 & -(NJ)^{-1} \\ F^TZ & 0 & 0 & 0 & -\frac{1}{\rho}I \end{bmatrix} < 0 \tag{30}$$

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Letting $CZ = K$, $Q^{-1}Z^{-1} = L$, $CQ^{-1}Z^{-1} = W$ and $(NJ)^{-1} = V$, then equation (33) can be expressed as

$$\begin{bmatrix} \Lambda & \eta & \delta & I & \kappa \\ * & -V & \theta & 0 & 0 \\ * & * & -\varepsilon I & 0 & 0 \\ * & * & * & -V & 0 \\ * & * & * & * & -\frac{1}{\rho}I \end{bmatrix} < 0$$

where $\Lambda = Z\bar{A}_{11}^T - K^T\bar{A}_{12} + \bar{A}_{11}Z - \bar{A}_{12}K + \varepsilon U_2^T GG^T U_2 + I$, $\eta = \frac{1}{\sqrt{N}}\bar{A}_{d11}L - \frac{1}{\sqrt{N}}\bar{A}_{d12}W$, $\delta = ZU_2^T H^T - KU_1^T H^T$, $\theta = \frac{1}{\sqrt{N}}LU_2^T H_d^T - \frac{1}{\sqrt{N}}WU_1^T H_d^T$.

So the proof of Theorem 4 is complete.

Next, a simulation is conducted to prove the proposed control law. **Theorem 5**: For a class of multi-agent system like (3), which satisfies assumption 2 to 5, the system can reaching sliding surface (31) in finite time according to control law (32) regardless of its initial condition.

$$S = [C \quad I]Tx(t) = Rx(t) = 0 \tag{31}$$

$$u(t) = u_{eq} + u_{sw} \tag{32}$$

where equivalent control is $u_{eq} = -(B^TB)^{-1}[B^TAx(t) + B^TA_dN\chi + B^Tg(x(t))]$, nonlinear switch control $u_{sw} = -(B^TB)^{-1}[B^TG(Hx(t) + H_dN\chi) + (B^TB)\delta_f + \varepsilon]$, in which ε is a positive constant and $\|\underset{i\in N}{x_i(t-\tau_i)}\| \leq \chi$.

Proof 5: Considering the following Lyapunov function

$$V(t) = \frac{1}{2}S^TS \tag{33}$$

Obviously, only if $S(x,t) = 0$, V is positive-definite. Taking the derivative of (33) and substituting (3) in it, then

$$\begin{aligned} \dot{V} &= S^T\dot{S} \\ &\leq S^TRAx + S^TRGHx + S^TR(A_d + GH_d)N\chi \\ &+ S^TB_m(u+f) + S^TRg \end{aligned} \tag{34}$$

Considering control law (32), then

$$\begin{aligned} \dot{V} &\leq S^T[(-B_m\delta_f - \varepsilon)sgn(S) + B_m\delta_f] \\ &\leq -\varepsilon S^T sgn(S) \\ &\leq 0 \end{aligned} \tag{35}$$

It is ensured that the system can meet the trigger condition, and the proof of Theorem 5 is complete. .

Remark 3. To reduce the difficulty in the designing of the sliding mode control system, non-linear transformation is used, yet the multiple time-delay system should reinforce control (32), in order to keep the system on the surface.

IV. Numerical simulations

In this section, two simulations are conducted to test the theorems developed in this paper. Consider multi-agent system (1), transforming is from case on [24] where

$$A = \begin{bmatrix} -1.8 & 0.3 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0.5 & -2 \end{bmatrix}, A_d = \begin{bmatrix} -1 & 0 & 0 \\ -0.2 & 0.5 & 0 \\ 0 & 0.2 & 0.6 \end{bmatrix}$$

$$f(t) = 0.2\sin(t), \tau = 0.5s \ and \ B = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T.$$

$$g(x(t)) = \begin{bmatrix} \frac{0.08x_1(t)}{2x_2^2+1} \\ 0.1x_2(t)\sin(x_3(t)) \\ 0.1x_3(t)\cos(x_3(t)) \end{bmatrix}, \rho = 0.2$$

Where A, A_d and B are matrices which are transformed from the citation above. $f(t)$ refers to system disturbance; τ is the maximum of time-delay and $g(x(t))$ is the nonlinear characteristic of the system. The initial states are: $x(t) = [1.5 \quad 0.4 \quad -1]^T$, $t \in [-0.5 \quad 0]$. Based on Theorem 2, we have

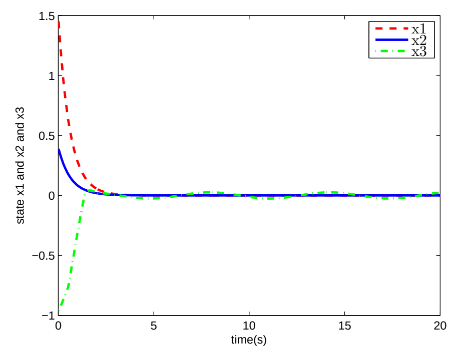
$$X = \begin{bmatrix} 1.4129 & 0.0840 & -0.0000 \\ 0.0840 & 1.5475 & 0.0000 \\ -0.0000 & 0.0000 & 0.9389 \end{bmatrix}, V = \begin{bmatrix} 2.7784 & -0.1559 & 0.0101 \\ -0.1559 & 3.9596 & -0.2555 \\ 0.0101 & -0.2555 & 3.8022 \end{bmatrix}$$
$$L = [0.0420 \quad 0.7738 \quad 0.9694]$$

K is Designed to make (A, B) stabilization, $K = [0.0000 \quad 0.5000 \quad 1.0326]$.

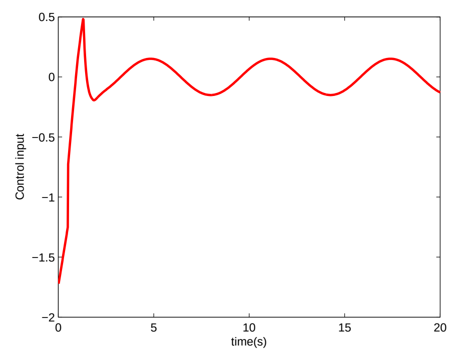
Thus the sliding mode coefficient is acquired as

$$P = X^{-1} = \begin{bmatrix} 0.7101 & -0.0385 & -0.1041 \\ -0.0385 & 0.6483 & -0.0000 \\ -0.1041 & 0.0000 & 1.0651 \end{bmatrix}$$

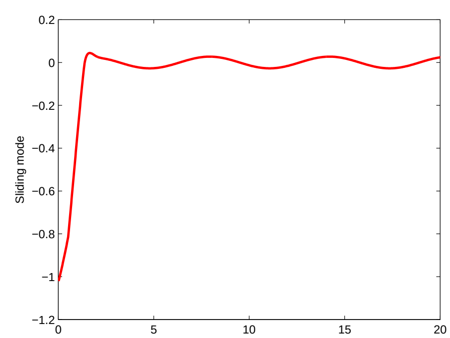
The parameter of controller (5) is $\delta_f = 0.2$; $\varepsilon = 0.15$, and then the simulation result is shown in Fig.1.



(a)



(b)



(c)

Fig. 1. States, control input and sliding mode.

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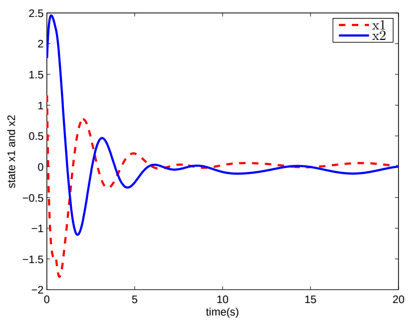
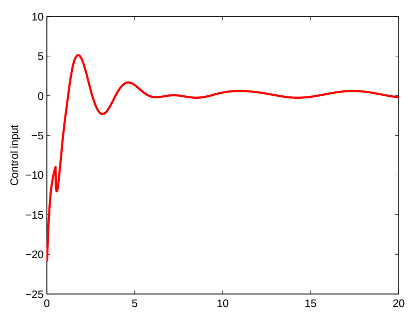
Consider MAS (2), where

$$A=\begin{bmatrix}-2.5 & 1.6\\ 3.5 & 1.2\end{bmatrix},\ G=\begin{bmatrix}0.1 & 0.2\\ 0.3 & 0.2\end{bmatrix},$$
$$D(t)=\begin{bmatrix}0.5\sin(t) & 0\\ 0 & 0.5\sin(t)\end{bmatrix},\ H=\begin{bmatrix}0.5 & 0.2\\ 0.2 & 0.23\end{bmatrix}$$
$$H_d=\begin{bmatrix}0.13 & 0.2\\ 0.25 & 0.3\end{bmatrix},\ B=\begin{bmatrix}1\\ 0\end{bmatrix},\ T=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$
$$g(x(t))=\begin{bmatrix}\frac{0.08x_1(t)}{2x_2^2(t)+1}\\ 0.1x_2(t)\sin(x_2(t))\end{bmatrix},\ f(t)=0.2\sin(t),\ \tau=0.5s.$$

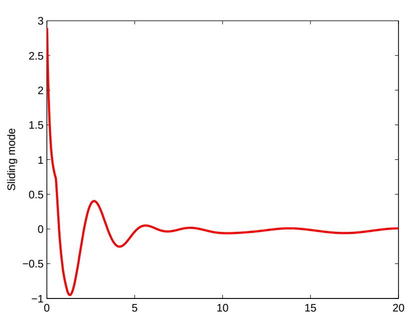
Apart from the parameters introduced in the first simulation, G, $D(t)$, H and H_d are matrices transformed from related references. The initial states are: $x(t)=[1.2 \quad 1.6]^T$, $t\in[-0.5 \quad 0]$, while follows from Theorem 4 while $N=1$, $c=1.64$. The parameters of the controller are: $\Phi=5$ and $\delta_f=0.2$. Besides, the simulation results are shown in Fig.2.



(a)



(b)



(c)

Fig. 2. States, control input and sliding mode.

Compared with other control methods, sliding mode control has its superiority with robust stability. It is less susceptible to the influence of perturbations and uncertainties of the system model. The simulation clearly showed that due to its superiority, the system can be relatively stable in a short period of time regardless of its initial state and perturbation.

V. Application Simulation

In this section, the paper presented one of the practical applications of the MAS system along with the simulation result conducted to verify it.

The car-following system is a field where MAS system can be readily applied. The typical car-following system is proposed by Bando [28] as:

$$\dot{v}_i(t)=k[F(\Delta x_i(t))-v_i(t)] \tag{36}$$

Where, k is the sensitivity coefficient, $\dot{v}_i(t)$, $v_i(t)$ is the acceleration and velocity of the ith vehicle at time t, $\Delta x_i(t)=x_{i+1}(t)-x_i(t)$ is the headway of vehicle i and vehicle i+1 at time t. $F(\Delta x_i(t))$ is the optimal velocity function(OVF)of the vehicle n.

According to [29], while choosing $\hat{x}_i(t)$ and $\hat{v}_i(t)$ to describe the car-following system. The equation (36) can be linearized around steady state as:

$$\begin{cases}\begin{bmatrix}\frac{d\hat{v}_i(t)}{dt}\\ \frac{d(\Delta\hat{x}_i(t))}{dt}\end{bmatrix}=\begin{bmatrix}-k-\lambda & k\mu\\ -1 & 0\end{bmatrix}\begin{bmatrix}\hat{v}_i(t)\\ \Delta\hat{x}_i(t)\end{bmatrix}\\ \qquad +\begin{bmatrix}\lambda\\ 1\end{bmatrix}\hat{v}_{i+1}(t)\\ \hat{v}_i(t)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\hat{v}_i(t)\\ \Delta\hat{x}_i(t)\end{bmatrix}\end{cases} \tag{37}$$

Where

$$\Delta\hat{x}_i(t)=\Delta x_i(t)-d_0, \hat{v}_i(t)=v_i(t)-v_0$$
$$\Delta\hat{v}_i(t)=\hat{v}_{i+1}(t)-\hat{v}_i(t), \mu=\frac{dF(\Delta x_i(t))}{d\Delta x_i(t)}|_{\Delta x_i(t)=d_0}$$

Assuming that

$$z(t)=\begin{bmatrix}\hat{v}_i(t) & \Delta\hat{x}_i(t)\end{bmatrix}^T, A=\begin{bmatrix}-k-\lambda & k\mu\\ -1 & 0\end{bmatrix}$$
$$B=\begin{bmatrix}\lambda & 1\end{bmatrix}^T, C=\begin{bmatrix}1 & 0\end{bmatrix}, u(t)=\hat{v}_{i+1}(t)$$

Then the equation (37) can be written as:

$$\begin{cases}\dot{z}(t)=Az(t)+Bu(t)\\ y(t)=Cz(t)\end{cases} \tag{38}$$

The multi-agent system introduced in this paper can find wide application in the real car-following system that is susceptible to time-delay and uncertainties. This paper used the MAS (3) as an example. The equation (38) can be transformed into:

$$\begin{cases}\dot{x}(t)=(A+\Delta A(t))x(t)+\sum_{i=1}^{N}(A_d+\\ \qquad \Delta A_d(t))x_i(t-\tau_i)+B(u(t)+f(t))+g(x(t))\\ y(t)=Cx(t)\end{cases} \tag{39}$$

Where $x(t)$ is the state of the car including the differences of distance and velocity, $f(t)$ is the perturbation, $g(x(t))$ is the nonlinear characteristic.

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Using smart cars as the experimental platform. The structure of the car-following system is shown in Fig.3.

The experiment system is shown in Fig.4. In this system, the location information is collected by ultrasound and the information is transmitted via Bluetooth. The stable state is shown in Fig.5.

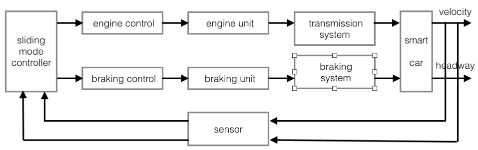


Fig. 3. Structure of the car-following system.

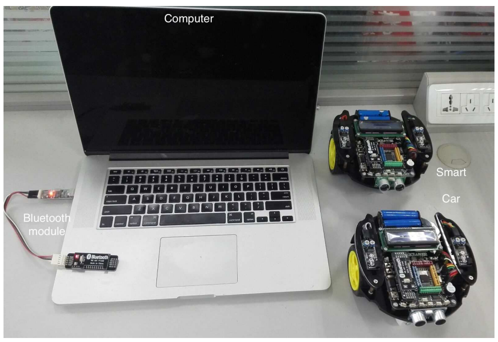


Fig. 4. Experiment system of car-following.

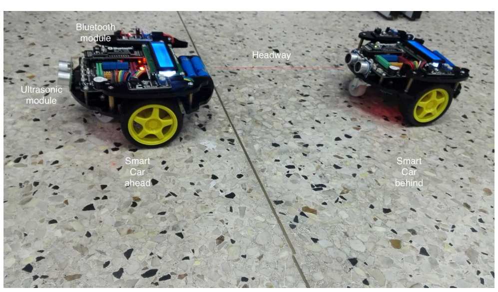


Fig. 5. stable state of two-car situation.

For brevity, the detailed description of specific hardware system is omitted. The numerical information and the result of the experiment is shown below. Transforming form the case on [24], [30]:

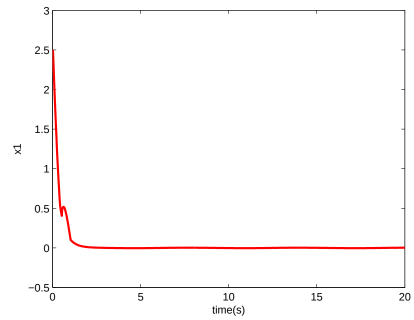
$$A = \begin{bmatrix} -2.2 & 1 \\ -1 & 0 \end{bmatrix}, \; G = \begin{bmatrix} 0.1 & 0.35 \\ 0 & 0.3 \end{bmatrix},$$

$$D(t) = \begin{bmatrix} 0.8\sin(t) & 0 \\ 0 & 0.8\sin(t) \end{bmatrix}, \tau = 0.5s, \; N = 5,$$

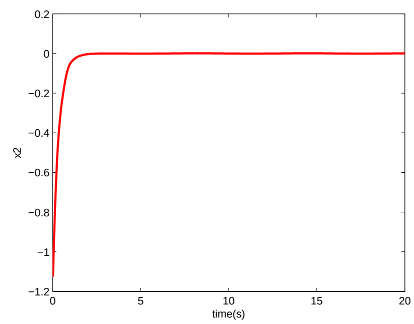
$$H = \begin{bmatrix} 0.35 & 0.5 \\ 0.2 & 2.7 \end{bmatrix}, \; H_d = \begin{bmatrix} 0.55 & 0.3 \\ 0.4 & 0.9 \end{bmatrix},$$

$$A_d = \begin{bmatrix} -0.22 & 0.1 \\ -0.1 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0.2 \\ 1 \end{bmatrix}, \; f(t) = 0.2\sin(t),$$

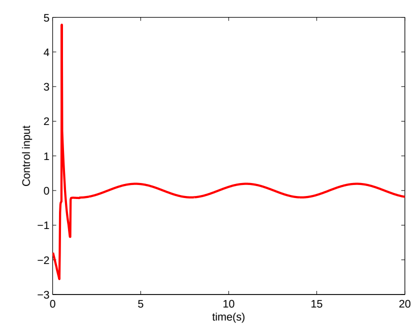
The initial states is: $x(t) = [2.5 \quad -1.2]^T$, $t \in [-0.5 \quad 0]$, and the nonlinear transformation is $T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. According to Theorem 4, $Z = 0.2600$, $L = 2.1232$, $W = 2.4220$, $K = -2.2660$, the controller parameters are: $\delta_f = 0.2$, $\varepsilon = 3.9237$, $\chi = 4$. The results are shown in Fig.6.



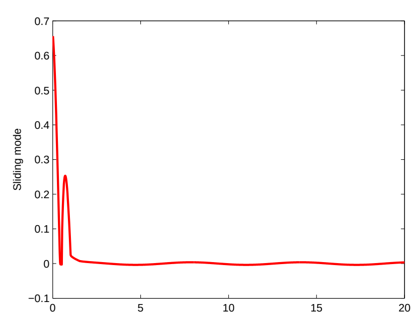
(a)



(b)



(c)



(d)

Fig. 6. headway, speed difference, control input and sliding mode.

We can see from the Fig.6. that the distance between adjacent cars can be swiftly shortened to the safety distance and then kept stable in (a) while the velocity also approaches to be identical with each other in a short time in (b). This proved the application to be practicable.

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VI. Conclusion

The multi-agent system is full of uncertainties in practice. In this paper, a multi-agent system with time-delay, linear feedback and uncertainties is investigated. Sliding surfaces and corresponding control laws are put forward to keep the system stable under both internal and external interferences. The application of sliding surfaces is proposed firstly, and then we use LMI to find out its appropriate parameter. Virtual feedback and nonlinear transformation are used to solve the complex problem of parameter determination. As a result, the system can reach the sliding surface in infinite time and the quadratic stability of the sliding surface can be ensured. However, in practice, sliding motion can only be maintained in a region nearby the sliding surface instead of be kept on it. At present, this issue is inevitable for sliding mode control, and it needs to be further studied.

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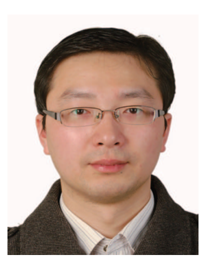

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This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TIE.2017.2701777, IEEE Transactions on Industrial Electronics

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