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Brief paper

The partial pinning control strategy for large complex networks ☆

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ABSTRACT

In large directed complex networks, it may result unfeasible to successfully pinning control the whole network. Indeed, when the pinner node can be connected only to a limited number of nodes, it may be impossible to guarantee pinning controllability of all the network nodes. In this paper, we introduce the partial pinning control problem, which consists in determining the optimal selection of the nodes to be pinned so as to maximize the fraction of nodes of the whole network that can be asymptotically controlled to the pinner's trajectory. A suboptimal solution to this problem is provided for a class of nonlinear node dynamics, together with the bounds on the minimum coupling and control gains required to "partially control" the network. The theoretical analysis is translated into an integer linear program (ILP), which is solved on a testbed network of 688 nodes.

1. Introduction

An increasing number of complex control systems in applications can be modeled as networks of nonlinear dynamical agents (the nodes), communicating with the others via a communication protocol defined on the network edges. Researchers in different areas of applied science and engineering have been addressing the problem of selecting the network topology and the communication protocols among agents in order for the complex network to perform a desired function. Examples include rendezvous and flocking problems in robotics (Cortes, Martinez, & Bullo, 2006; Han & Ge, 2015; Tanner, Jadbabaie, & Pappas, 2007), synchronization of sensor networks (An et al., 2011), consensus and multi-agent coordination problems in control theory (DeLellis, di Bernardo, Gorochowski, & Russo, 2010; Li, Wen, Duan, & Ren, 2015), and the emergence of coordinated motion in biological settings (Ghosh, Rangarajan, & Sinha, 2010; Paley, Leonard, Sepulchre, Grunbaum, & Parrish, 2007).

Substantial attention has been devoted on synchronization (Belykh, Belykh, & Hasler, 2006; Gao, Meng, Chen, & Lam, 2010; Liu & Chen, 2015; Pecora & Caroll, 1990) and consensus of complex networks (DeLellis, diBernardo, Garofalo, & Liuzza, 2010; Li, Chen, Su, & Li, 2016). The idea is to find strategies to regulate the behavior of large ensembles of interacting agents that ensure all systems in the network evolve towards the same asymptotic trajectory (Vishnampet, 1993). At first, diffusively coupled identical nonlinear systems were considered: given the node dynamics, the problem becomes that of determining the range of the values of the coupling gains for which the network synchronizes. This *synchronizability problem*has been solved mainly by using the so-called Master Stability Function approach (Pecora & Caroll, 1998), contraction theory (Lohmiller & Slotine, 1998; Russo & di Bernardo, 2009), and passivity tools (Gao, Chen, & Chai, 2007).

Although analytical conditions for synchronizing all nodes towards an asymptotic solution were obtained, a major problem still remains from a control viewpoint. Indeed, such common solution, if it exists, cannot be arbitrarily imposed. A possible strategy to achieve this goal would be to directly add some feedback control input on each of the systems in the network so to steer the dynamics of each agent towards the desired trajectory. In practice, when more than a handful of agents are considered, this approach is not viable. A feasible alternative is represented by *Pinning Control* (Huang & Manton, 2009; Li, Sun, Small, & Fu, 2015; Wang & Chen, 2002), where the control action is exerted through an additional node, the *pinner*, which is directly connected only to a subset of the network nodes, the *pinned nodes*. In this scenario, the problem consists not only in designing the strength and form of the control action to be exerted by the pinner, but also in determining how many, and which pinned nodes need to be selected to achieve the control objective (Porfiri & di Bernardo, 2008; Sorrentino, di Bernardo, Garofalo, & Chen, 2007). In the recent literature, an optimal location of the pinned nodes is sought, so as to guarantee that all the network nodes asymptotically follow the reference trajectory imposed by the pinner. To control directed networks,

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under suitable assumptions on the individual dynamics, if the network graph admits a spanning tree, it suffices to pin the root of this tree (Chen, Liu, & Lu, 2007). Otherwise, it is necessary to pin at least one node in each root strongly connected component (RSCC) of the network (Lu, Li, & Rong, 2010), that is, each SCC the nodes of which have incoming edges only from nodes of the same SCC.

In this paper, we take a different point of view with respect to Refs. Chen et al. (2007) and Lu et al. (2010). Inspired by the work on controllability of large networks (DeLellis, Garofalo, & Lo Iudice, 2016; Gao, Liu, D'Souza, & Barabási, 2014; Lo Iudice, Garofalo, & Sorrentino, 2015), we consider the case in which, for technological or economic reasons, the pinner signal can only reach a limited number of nodes belonging to a given set. Moreover, to allow coping with the limitations arising when dealing with non-ideal actuators, see e.g. Ref. Ocampo-Martinez, Puig, Cembrano, and Quevedo (2013), we assume that constraints exist on the value of the coupling and control gains which could lead to pinning nodes in non root SCCs to control the whole network. As these restrictions may not allow complete pinning controllability, a question naturally arises: which nodes must be pinned to drag the greatest number of nodes to the pinner's trajectory? We call this the *partial pinning control* problem and, after providing an analytic solution, we translate it into an integer linear program (ILP). Moreover, an optimization problem is formulated and solved to select the pinning and coupling gains. The effectiveness of the approach is then illustrated on a testbed example.

2. Mathematical preliminaries and notation

Let us consider a directed graph (digraph) $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V}$ and $\mathcal{E}$ are the set of vertexes and edges of $\mathcal{G}$, respectively. A is the adjacency matrix of $\mathcal{G}$, and its ijth element a_{ij} is greater than zero if there exists an edge from j to i, while it is zero otherwise. Moreover, the ijth element ℓ_{ij} of the Laplacian matrix L of the digraph is equal to $-a_{ij}$ if $j \neq i$ while it is $\sum_{j=1}^{N} a_{ij}$ if $j = i$. Any digraph $\mathcal{G}$ can be decomposed in its σ SCCs $\mathcal{G}_i = (\mathcal{V}_i, \mathcal{E}_i)$, where $\mathcal{V}_i$ is the set of nodes of $\mathcal{G}_i$ and $\mathcal{E}_i = \{(l, m) \in \mathcal{E} : l, m \in \mathcal{V}_i\}$ the set of edges, and we label the SCCs so that only the first $\rho \geq 1$ are also RSCCs. The Directed Acyclic Graph (DAG) condensation $\mathcal{G}^D = (\mathcal{V}^D, \mathcal{E}^D)$ of $\mathcal{G}$ is a graph whose nodes represent the SCCs of $\mathcal{G}$ while $(i, j) \in \mathcal{E}^D$ if, in $\mathcal{G}$, there exists at least an edge connecting a node of $\mathcal{V}_j$ to one of $\mathcal{V}_i$. Every node i of $\mathcal{G}$ has a set of nodes in its downstream, as we say that node j_l is in the downstream of node j_1 (j_1 is in the upstream of j_l) if there exists a sequence $\{\ell_{j_{i+1} j_i}\}_{i=1}^{l-1}$ of nonzero entries of the Laplacian L, that is, if there exists a directed path from node j_1 to node j_l. We denote by $\Gamma(\mathcal{G}_i)$ ($\Gamma^T(\mathcal{G}_i)$) the set of nodes of $\mathcal{G}$ that are only in the downstream (upstream) of the nodes in $\mathcal{V}_i$, including the nodes in $\mathcal{V}_i$ itself. Leveraging the decomposition in layers of a DAG (Liu, Slotine, & Barabási, 2012), we can now define the levels of a graph $\mathcal{G}$.

Definition 1. Given $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ and a set of nodes $\Gamma \subseteq \mathcal{V}$, the mth level of Γ is

$$\mathcal{R}_m := \{\mathcal{V}_l \subseteq \Gamma : \exists (i, j) \in \mathcal{V}_l \times \mathcal{R}_{m-1}, \nexists (i, j) \in \mathcal{V}_l \times \mathcal{B}_m\},$$

with $\mathcal{B}_m := \{\mathcal{V}_l \subseteq (\Gamma - \cup_{k=0}^{m-1} \mathcal{R}_k)\}$, $\mathcal{R}_0 := \{\mathcal{V}_l \subseteq \Gamma : \nexists (i, j) \in \mathcal{V}_l \times (\Gamma - \mathcal{V}_l)\}$; b is the smallest integer such that $\mathcal{R}_{b+1} = \emptyset$.

In what follows, when referring to an SCC, we will use a double subscript to identify the specific SCC the nodes of which are in a given level of Γ. For instance, $\mathcal{V}_{ij}$ will identify the set of nodes of the ith SCC belonging to $\mathcal{R}_j$, see Fig. 1. Finally, given a set χ, we denote its cardinality by $|\chi|$. $\mathrm{diag}\{d_1, \ldots, d_m\}$ denotes the $m \times m$ diagonal matrix with diagonal elements $d_1, \ldots, d_m$, while 1_m is an m-dimensional vector of ones, and I_m is the $m \times m$ identity matrix. $\mathcal{D}_m^+$ is the set of positive definite diagonal matrices in $\mathbb{R}^{m\times m}$. Given a square matrix $M \in \mathbb{R}^{m\times m}$, we denote its symmetric part $M_{\mathrm{sym}} := 0.5(M + M^T)$ and its eigenvalues as $\lambda_1(M), \ldots, \lambda_m(M)$.

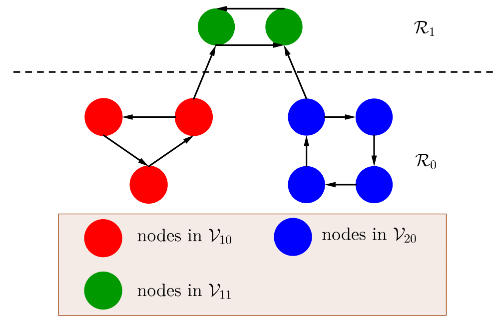

Fig. 1. Representation of the sets $\mathcal{R}_0, \mathcal{R}_1, \ldots, \mathcal{R}_b$ on a sample DAG condensation $\mathcal{G}^D$ of a network graph $\mathcal{G}$.

3. Partial pinning control

3.1. Problem formulation

We consider a linearly coupled network described by a digraph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where the $N = |\mathcal{V}|$ nodes are nonlinear dynamical systems with state $x_i \in \mathbb{R}^n$, while the edges describe the interconnections among the nodes. An extra node, the pinner, with state $s \in \mathbb{R}^n$ and identical dynamics but different initial conditions, is added to the network. A subset of $\mathcal{V}$, say $\mathcal{C}$, is the set of *pinnable nodes* that can be directly controlled by the pinner. We call *pinned* the nodes in $\mathcal{P} \subseteq \mathcal{C}$ that actually receive an input from the pinner. The network dynamics are

$$\dot{x}_i = f(x_i, t) + c \sum_{j=1}^{N} a_{ij} H(x_j - x_i) - \kappa \delta_i H(x_i - s), \tag{1}$$

for $i \in \mathcal{V}$, where $\dot{s} = f(s, t)$ are the pinner's dynamics, $f : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ is the nonlinear vector field describing the node dynamics, $c, \kappa \in \mathbb{R}$ are the coupling and control gains, respectively, $H \in \mathbb{R}^{n\times n}$ is the inner coupling matrix describing the information exchanged among neighboring nodes, and $\delta_i = 1$ if $i \in \mathcal{P}$, and 0 otherwise.

Definition 2. Network (1) is q-partially pinning controlled to the pinner's trajectory when

$$\lim_{t\to+\infty} \|x_i(t) - s(t)\| = 0, \quad i \in \mathcal{Q}, \tag{2}$$

where $\mathcal{Q} \subseteq \mathcal{V}$ and $q = |\mathcal{Q}|$; when $q = N$, network (3) is fully pinning controlled, while when $\mathcal{V}_i \subseteq \mathcal{Q}$, then we say that the SCC $\mathcal{G}_i$ is pinning controlled.

Problem 1. Partial pinning control.

$$\begin{aligned} &q^* = \max_{\mathcal{P}\subseteq\mathcal{C}} |\mathcal{Q}| \\ &|\mathcal{P}| = p \\ &c \leq c_M, \ \kappa \leq \kappa_M. \end{aligned} \tag{3}$$

Before illustrating the problem solution, we need to give the following definition:

Definition 3 (DeLellis, di Bernardo, & Russo, 2011; Lu & Chen, 2006). Given two $n \times n$ matrices $V > 0$ and W, a vector field $g : \mathbb{R}^n \times \mathbb{R}^+ \to \mathbb{R}^n$ is QUAD(V, W) if $(x - s(t))V(g(x, t) - g(s(t), t)) \leq (x - s(t))^T W(x - s(t))$, for all $x \in \mathbb{R}^n$, $t \in \mathbb{R}^+$, where $s(t)$ is the pinner trajectory.

3.2. Problem solution

Here, we give the conditions ensuring pinning controllability of any given SCC to then translate Problem 1 into the maximization of the number of nodes encompassed in the pinning controllable SCCs. Therefore, our analysis requires finding all the SCCs of $\mathcal{G}$. From a computational perspective, this task can be performed by Tarjan's algorithm (Tarjan, 1972) in a run time which is $\mathcal{O}(|\mathcal{V}|+|\mathcal{E}|)$, i.e. linear in the number of the network nodes and edges. Before giving our main results, we start by stating our main assumption on the node dynamics, and then give some notation pertinent to the partition in levels introduced in Definition 1.

Assumption 1. There exists a scalar μ such that the vector field f is QUAD(V, W), with $W-\mu VH<-\tau I_n$, $\tau\in\mathbb{R}^+$, and V such that $VH=H^TV^T\geq 0$;

Now, following Definition 1, the upstream $\Gamma^T(\mathcal{G}_l)$ of an SCC $\mathcal{G}_l$ can be decomposed in $b+1$ levels. Let us denote by m_j the number of SCCs in $\mathcal{R}_j$, $j=0,\dots,b$. From the definition of upstream, $m_b=1$, that is, $\mathcal{G}_l$ is the only SCC in $\mathcal{R}_b$, and thus $\mathcal{G}_l=\mathcal{G}_{1b}$. Further, for all $i=1,\dots,m_j$, $j=0,\dots,b$, we sort the nodes in $\mathcal{V}_{ij}$ so that the $p_{ij}:=|\mathcal{P}\cap\mathcal{V}_{ij}|$ pinned nodes precede the others, and decompose the associated Laplacian matrix L_{ij} as

$$\begin{bmatrix} L_{ij}^{11} & L_{ij}^{12}\\ L_{ij}^{21} & L_{ij}^{22}\end{bmatrix},$$

where $L_{ij}^{11}\in\mathbb{R}^{p_{ij}\times p_{ij}}$. Notice that, when $p_{ij}=0$, $L_{ij}=L_{ij}^{22}$. Moreover, we define a diagonal matrix M_{ij}, whose γth diagonal element is defined as $m_{ij}^{\gamma}=\sum_{s\notin\mathcal{V}_{ij}}a_{rs}$, where r is the label the γth node in $\mathcal{V}_{ij}$ has in $\mathcal{V}$. We then define $\bar{L}_{ij}:=L_{ij}+M_{ij}$, which can be decomposed in four blocks as L_{ij}. Moreover, for $i=1,\dots,m_j$, $j=0,\dots,b$, and given a $|\mathcal{V}_{ij}|\times|\mathcal{V}_{ij}|$ positive definite diagonal matrix D_{ij} to be selected, we define matrix Υ_{ij} as

$$\begin{bmatrix} \Upsilon_{ij}^{11} & \Upsilon_{ij}^{12}\\ \Upsilon_{ij}^{21} & \Upsilon_{ij}^{22}\end{bmatrix}, \tag{4}$$

with $\Upsilon_{ij}^{11}=c(D_{ij}^{11}\bar{L}_{ij}^{11}+(D_{ij}^{11}\bar{L}_{ij}^{11})^T)+2\kappa D_{ij}^{11}-2\mu D_{ij}^{11}$, $\Upsilon_{ij}^{12}=c(D_{ij}^{11}L_{ij}^{12}+L_{ij}^{21}D_{ij}^{22})$, and $\Upsilon_{ij}^{22}=c(D_{ij}^{22}\bar{L}_{ij}^{22}+(D_{ij}^{22}\bar{L}_{ij}^{22})^T)-2\mu D_{ij}^{22}$. We can now define

$$\phi_{ij}:=\min_{k=1,\dots,|\mathcal{V}_{ij}|-p_{ij}}\Re(\lambda_k(\bar{L}_{ij}^{22}))>0, \tag{5}$$

$$\kappa_{ij}:=\min_{\kappa}\left\{\kappa:\inf_{D_{ij}^{11}\in\mathcal{D}_{p_{ij}}^{+}}\left(\lambda_{\min}(\Upsilon_{ij}/\Upsilon_{ij}^{22})\right)\geq 0\right\}, \tag{6}$$

for all $i=1,\dots,m_j$, $j=0,\dots,b$, where we set $\Upsilon_{ij}/\Upsilon_{ij}^{22}=0$ for the case $p_{ij}=0$ in which $\Upsilon_{ij}=\Upsilon_{ij}^{22}$. For any level $j\leq b$, we define $\phi_j:=\min_{i=1,\dots,m_j}\phi_{ij}$, and $\kappa_j:=\max_{i=1,\dots,m_j}\kappa_{ij}$. Finally, we define

$$\bar{c}_l:=1/\min_{j\leq b}\phi_j,\quad \bar{\kappa}_l:=\max_{j\leq b}\kappa_j. \tag{7}$$

Theorem 1. *A network SCC $\mathcal{G}_l$ is pinning controlled if (i) $\mathcal{P}\cap\mathcal{V}_i\neq\emptyset$ $\forall i\leq\rho:\mathcal{V}_i\in\Gamma^T(\mathcal{G}_l)$, and (ii) $c>\bar{c}_l$, $\kappa>\bar{\kappa}_l$.*

Proof. We decompose $\Gamma^T(\mathcal{G}_l)$ in $b+1$ levels following Definition 1 and then split the proof in two steps.

(i) Pinning controllability of the nodes in $\mathcal{R}_0$. For all nodes in $\mathcal{R}_0$, Eq. (1) becomes

$$\dot{x}_i=f(x_i,t)-c\sum_{j=1}^{|\mathcal{V}_{k0}|}\ell_{ij}^{k0}Hx_j-\kappa\delta_iH(x_i-s), \tag{8}$$

for all $i\in\mathcal{V}_{k0}$, $k=1,\dots,m_0$. Therefore, we obtain m_0 decoupled sets of differential equations, that can be analyzed separately. The error equations can be written as $\dot{e}_i=f(x_i,t)-f(s,t)-c\sum_{j=1}^{|\mathcal{V}_{k0}|}\ell_{ij}^{k0}He_j-\kappa\delta_iHe_i$, for all $i\in\mathcal{V}_{k0}$, $k=1,\dots,m_0$.[1] In matrix form, we have

$$\begin{aligned}\dot{\eta}_{k0}&=F(\xi_{k0},t)-F(1_{|\mathcal{V}_{k0}|}\otimes s,t)\\&\quad-c(L_{k0}\otimes H)\eta_{k0}-\kappa(\Delta_{k0}\otimes H)\eta_{k0},\end{aligned} \tag{9}$$

where η_{k0} and ξ_{k0} are the stack vectors of all the e_i and x_i such that $i\in\mathcal{V}_{k0}$, respectively, $F(\xi_{k0},t)=[f(\xi_{k0}^1,t)^T,\dots,f(\xi_{k0}^{|\mathcal{V}_{k0}|},t)^T]^T$, and Δ_{k0} is the diagonal matrix with diagonal elements δ_i, $i\in\mathcal{V}_{k0}$. W.l.o.g., the nodes of $\mathcal{V}_{k0}$ are sorted so that only the first p_{k0} diagonal elements of Δ_{k0} are 1. For this subnetwork, we build the following candidate Lyapunov function:

$$U_{k0}=\frac{1}{2}\eta_{k0}^T(D_{k0}\otimes V)\eta_{k0}, \tag{10}$$

where D_{k0} is a $|\mathcal{V}_{k0}|\times|\mathcal{V}_{k0}|$ positive definite diagonal matrix to be selected. Differentiating U_{k0} yields

$$\begin{aligned}\dot{U}_{k0}&=\eta_{k0}^T(D_{k0}\otimes V)(F(\xi_{k0},t)-F(1_{|\mathcal{V}_{k0}|}s,t))\\&\quad-\eta_{k0}^T(D_{k0}\otimes V)(c(L_{k0}\otimes H)+\kappa(\Delta_{k0}\otimes H))\eta_{k0}.\end{aligned}$$

Exploiting the hypothesis on the vector field f, and as VH is symmetric, we can write

$$\begin{aligned}\dot{U}_{k0}&\leq-\tau\eta_{k0}^T(D_{k0}\otimes I_n)\eta_{k0}\\&\quad-\eta_{k0}^T\left(D_{k0}(cL_{k0}-\mu I_{|\mathcal{V}_{k0}|}+\kappa\Delta_{k0})\otimes VH\right)\eta_{k0}\\&=-\tau\eta_{k0}^T(D_{k0}\otimes I_n)\eta_{k0}-\eta_{k0}^T\left(\Upsilon_{k0}\otimes VH\right)\eta_{k0}.\end{aligned}$$

To ensure that $-\eta_{k0}^T(\Upsilon_{k0}\otimes VH)\eta_{k0}\leq 0$, we show that $\Upsilon_{k0}=\left(D_{k0}(cL_{k0}-\mu I_{|\mathcal{V}_{k0}|}+\kappa\Delta_{k0})\right)_{\text{sym}}\geq 0$. Notice that Υ_{k0} can be written in four blocks as in (4). From the properties of the Schur complement (Zhang, 2006), we have that, if $\Upsilon_{k0}^{22}>0$, and $\Upsilon_{k0}/\Upsilon_{k0}^{22}=\Upsilon_{k0}^{11}-\Upsilon_{k0}^{12}\Upsilon_{k0}^{22^{-1}}\Upsilon_{k0}^{12^T}\geq 0$, then $\Upsilon_{k0}\geq 0$. As we are considering a RSCC, the set of pinned nodes can access all the other nodes in $\mathcal{V}_{k0}$. Then, from Lu et al. (2010) and Wu (2007), we have

$$\phi_{k0}=\min_{i=1,\dots,|\mathcal{V}_{k0}|-p_{k0}}\Re(\lambda_i(L_{k0}^{22}))>0,$$

which implies that, if $c>\frac{\mu}{\phi_{k0}}$, then $cL_{k0}^{22}-\mu I_{|\mathcal{V}_{k0}|-p_{k0}}$ is an M-matrix (Fujimoto & Ranade, 2004). Therefore, there exists a $D_{k0}^{22}:\Upsilon_{k0}^{22}>0$. Moreover, the term $2\kappa D_{k0}^{11}$ in Υ_{k0}^{11} ensures that, for all $D_{k0}^{11}\in\mathcal{D}_{p_{k0}}^{+}$, there exists a finite $\kappa:\Upsilon_{k0}/\Upsilon_{k0}^{22}>0$, and then $\Upsilon_{k0}>0$. Then, given that $c>\mu/\phi_{k0}$, the kth RSCC in $\mathcal{R}_0$ is pinning controlled if

$$\kappa>\kappa_{k0}:=\min_{\kappa}\left\{\kappa:\inf_{D_{k0}^{11}\in\mathcal{D}_{p_{k0}}^{+}}\left(\lambda_{\min}(\Upsilon_{k0}/\Upsilon_{k0}^{22})\right)\geq 0\right\}.$$

Then, all the nodes in $\mathcal{R}_0$ are pinning controlled if

$$(c,\kappa)\in\left\{c>\frac{\mu}{\phi_0},\kappa>\kappa_0\right\}. \tag{11}$$

(ii) Pinning controllability of the nodes in $\mathcal{R}_k$, $k=1,\dots,b$. For the remaining SCCs we can write

$$\begin{aligned}\dot{e}_i&=f(x_i,t)-f(s,t)-c\sum_{j=1}^{|\mathcal{V}_{kh}|}\ell_{ij}^{kh}He_j\\&\quad-c\sum_{j\in\mathcal{A}_i}a_{ij}H(x_i-x_j)-\kappa\delta_iHe_i,\end{aligned} \tag{12}$$

for all $i\in\mathcal{V}_{kh}$, $k=1,\dots,m_h$, $h=1,\dots,b$, where $\mathcal{A}_i=\cup_{m=0}^{i-1}\mathcal{R}_m$.

(ii-a) Nodes in $\mathcal{R}_1$. As $\mathcal{A}_1=\mathcal{R}_0$, trivial algebraic manipulations yield that, for all nodes in $\mathcal{V}_{l1}$, $l=1,\dots,m_1$, the term

[1] For conciseness, in what follows we refer to the kth RSCC in $\mathcal{R}_0$, but we mean that all the derivation applies for any k.

$c\sum_{j\in\mathcal{A}_i} a_{ij}H(x_i - x_j)$ in (12) can be written as

$$c\sum_{j\in\mathcal{R}_0} a_{ij}H(x_i - s) - c\sum_{j\in\mathcal{R}_0} a_{ij}H(x_j - s). \tag{13}$$

From step (i), the origin of system (9) is globally exponentially stable. This means that

$$\left\| c\sum_{j\in\mathcal{R}_0} a_{ij}H(x_j - s)\right\| \leq \alpha_1 e^{\alpha_2 t}, \tag{14}$$

for some finite scalars $\alpha_1 \geq 0$, $\alpha_2 \leq \bar{\alpha} < 0$. Following the classical results in Khalil (2001), the global asymptotic stability of (12) can be studied by neglecting the bounded and asymptotically vanishing term $c\sum_{j\in\mathcal{R}_0} a_{ij}H(x_j - s)$, thus yielding

$$\begin{aligned}\dot{e}_i &= f(x_i, t) - f(s, t) - c\sum_{j=1}^{|\mathcal{V}_{k1}|} \ell_{ij}^{k1} He_j \\ &\quad - c\sum_{j\in\mathcal{R}_0} a_{ij}H(x_i - s) - \kappa\delta_i He_i,\end{aligned} \tag{15}$$

for all $i \in \mathcal{V}_{k1}$, $k = 1, \ldots, m_1$. Therefore, for the kth SCC in $\mathcal{R}_1$, we can write the candidate Lyapunov function $U_{k1} = \frac{1}{2}\eta_{k1}^T(D_{k1}\otimes V)\eta_{k1}$, where η_{k1} is the stack vector of all the e_i such that $i \in \mathcal{V}_{k1}$, and D_{k1} is a $|\mathcal{V}_{k1}| \times |\mathcal{V}_{k1}|$ positive definite diagonal matrix to be selected. Following similar arguments as in step (i), we have that $\dot{U}_{k1} \leq -\tau\eta_{k1}^T(D_{k1}\otimes I_n)\eta_{k1} - \eta_{k1}^T(\Upsilon_{k1}\otimes VH)\eta_{k1}$, where matrix $\Upsilon_{k1} = \left(D_{k1}(c(L_{k1} + M_{k1}) - \mu I_{|\mathcal{V}_{k1}|} + \Delta_{k1})\right)_{\text{sym}}$ can be written following (4). Then, if

$$(c, \kappa) \in \left\{c > \frac{\mu}{\phi_1}, \kappa > \kappa_1\right\}, \tag{16}$$

then $\Upsilon_{k1} \geq 0$, which implies $\dot{U}_{k1} \leq -\tau\eta_k^T(D_k \otimes I_n)\eta_k$, for all $k = 1, \ldots, m_1$. Therefore, all the nodes in $\mathcal{R}_1$ are asymptotically pinning controlled if (11) and (16) hold.

(ii-b) Nodes in $\mathcal{R}_k$, $k = 2, \ldots, b$. The same procedure can be iterated for all the nodes in $\mathcal{R}_k$, $k = 2, \ldots, b$, thus yielding that, as $c > \bar{c}_l$, $\kappa > \bar{\kappa}_l$ from the hypotheses, all the nodes in $\Gamma^T(\mathcal{G}_l)$, are asymptotically pinning controlled. Being $\mathcal{V}_l \subseteq \Gamma^T(\mathcal{G}_l)$, the thesis follows.

Exploiting Theorem 1, we translate Problem 1 into maximizing the number of nodes encompassed in SCCs whose pinning controllability can be guaranteed.

Theorem 2. *If Assumption 1 holds and we pin each node of the set*

$$\begin{aligned}\mathcal{P}^* &= \arg\max_{\mathcal{P}:|\mathcal{P}|=p} q(\mathcal{P}) := |\{\mathcal{V}_l \subseteq \mathcal{V} : \mathcal{P}\cap\mathcal{V}_i \neq \emptyset\, \forall i \leq \rho : \\ &\quad \mathcal{V}_i \subseteq \Gamma^T(\mathcal{G}_l) \text{ and } \bar{c}_l \leq c_M,\ \bar{\kappa}_l \leq \kappa_M\}|\end{aligned} \tag{17}$$

then network (1) is $q(\mathcal{P}^*)$ partially pinning controlled.

Proof. The proof is a consequence of Theorem 1.

The set $\mathcal{P}^*$ defined in (17) is not unique as different selections of the pinned nodes within an SCC may yield the same set of pinning controllable nodes, say $\mathcal{Q}^*$. Nevertheless, in Section 4 we discriminate among alternative pinning selections within an SCC, as they can determine substantial changes in the minimum coupling and control gains required for the control task. Namely, we solve

$$\begin{aligned}&\min_{\mathcal{P},c,\kappa} J(c, \kappa) \\ &\mathcal{Q} = \mathcal{Q}^*,\ |\mathcal{P}| = p\end{aligned} \tag{18}$$

where $J(c, \kappa)$ is a cost function which is non-decreasing with respect to c when κ is kept constant and vice versa.

Theorem 2 implies that solving Problem 1 requires pinning at least one node of a certain set of RSCCs. As most generative graph models and real network topologies encompass single node RSCCs, the computation of $\bar{\kappa}_l$ in (7) simplifies by exploiting the following theorem:

Theorem 3. *If $|\mathcal{V}_{ij}| = 1$, the solution of (6) is $\kappa_{ij} = \mu$.*

Proof. *As $p_{ij} = 1$, we have $\Upsilon_{ij}/\Upsilon_{ij}^{22} = \Upsilon_{ij}^{11} = D_{ij}^{11}(2cL_{ij}^{11} + 2\kappa - 2\mu)$. Then, $\lambda_{\min}(\Upsilon_{ij}/\Upsilon_{ij}^{22}) = \Upsilon_{ij}/\Upsilon_{ij}^{22}$, and the minimization in (6), as $D_{ij}^{11} \in \mathbb{R}^+$ and $L_{ij}^{11} = 0$, simplifies to finding the smallest $\kappa : (2\kappa - 2\mu) \geq 0$.*

4. Algorithm for control design

Based on the findings reported in the previous section, here we build an algorithm that (i) optimally selects the SCCs to be pinned, i.e. finds $\mathcal{Q}^*$, and (ii) for each pinned SCC, determines the nodes to be pinned, and computes the optimal coupling and control gains, i.e. finds $\mathcal{P}^*$, c^*, and κ^* in (18).

(i) Selection of the pinned SCCs

Denote by π the scalar $|\mathcal{V}_D| + 1$, representing the pinner. The problem of finding the SCCs to be pinned is translated into an ILP through the following steps:

(1) define a new graph $\mathcal{G}' = (\mathcal{V}', \mathcal{E}')$, where initially $\mathcal{V}' = \mathcal{V}_D \cup \pi$ and $\mathcal{E}' = \mathcal{E}_D \cup \{(i, i) : i \in \mathcal{V}_D\}$;
(2) define a set of variables $Y = \{y_{ij} : (i, j) \in \mathcal{E}'\}$ and a set of weights $W = \{w_{ii} = |\mathcal{V}_i|\ \forall i \in \mathcal{V}'\}$;
(3) denote by $\mathcal{R}_m$ the mth level of $\mathcal{V}_D$;
(4) set $m = 0$;
(5) $\forall \mathcal{V}_l \in \mathcal{R}_m$
   (a) find the minimal set $\mathcal{P}_l$ of nodes of $\mathcal{V}_l$ that must be pinned such that $(\bar{c}_l, \bar{\kappa}_l) \leq (c_M, \kappa_M)$;
   (b) if $\mathcal{P}_l$ exists[2] and $\sum_{j:\mathcal{V}_j\in\Gamma^T(\mathcal{G}_l)} |\mathcal{P}_j| \leq p$, then $(l, \pi) \in \mathcal{E}'$, $y_{l\pi} \in Y$, and $w_{l\pi} = |\mathcal{P}_l| \in W$;
   (c) else $\mathcal{V}' = \mathcal{V}' - \Gamma(\mathcal{G}_l)$, $\mathcal{E}' = \mathcal{E}' - \{(j, k) : j \text{ or } k \in \Gamma(\mathcal{G}_l)\}$, $Y = Y - \{y_{jk} : j \text{ or } k \in \Gamma(\mathcal{G}_l)\}$, $W = W - \{w_{jj} : j \in \Gamma(\mathcal{G}_l)\}$;
(6) if $\mathcal{R}_{m+1} \neq \emptyset$, set $m = m + 1$ and return to step 5. Otherwise, solve the following ILP:

$$\max_Y \sum_{i=1}^{|\mathcal{V}'|} w_{ii}y_{ii} \tag{19}$$

$$\sum_j w_{j\pi}y_{j\pi} \leq p \tag{20}$$

$$k_i^{\text{in}}y_{ii} \leq \sum_{j\neq i:\exists y_{ij}} y_{ij}, \quad \forall i \tag{21}$$

$$k_i^{\text{out}}y_{ii} \geq \sum_{j\neq i:\exists y_{ij}} y_{ji}, \quad \forall i \tag{22}$$

$$y_{ij} \in \{0, 1\}, \qquad \forall i, j. \tag{23}$$

Our procedure starts by building a new graph $\mathcal{G}'$ obtained by adding the pinner to the DAG condensation of the network graph. Moreover, in $\mathcal{G}'$, every SCC is endowed with a self-loop and a variable y_{ij} is associated to each edge in $\mathcal{E}'$. A weight w_{ii}, equal to the number of nodes in the SCC $\mathcal{G}_i$, is associated to each y_{ii} associated to a self-loop. In steps (3) through (6), we hierarchically explore $\mathcal{G}'$ by looking for the SCCs that can be pinning controlled given the constraints on the number of pinned nodes and on the pair (c, κ).

[2] If no pinned nodes are required we say that $\mathcal{P}_l = \emptyset$.

For each SCC $\mathcal{G}_i$ that can be pinning controlled only by pinning one or more of its nodes, we add an edge (i, π) to $\mathcal{E}'$, a decision variable $y_{i\pi}$ to Y, and a weight $w_{i\pi}$ to W that is equal to the number of nodes of $\mathcal{V}_i$ that must be pinned. Moreover, we prune from $\mathcal{G}'$ all the downstream of the SCCs that cannot be pinning controlled. This new graph allows us to translate the optimization problem in (17) into a graph optimization problem, which in turn is translated into the ILP in Eqs. (19)–(23). As $\mathcal{Q}^* = \{\mathcal{V}_i : y_{ii} = 1\}$ and $w_{ii} = |\mathcal{V}_i|$, the ILP maximizes the number of pinning controllable nodes (19). Moreover, as we will select $y_{j\pi}$ pinned nodes in each SCC $\mathcal{G}_j$ such that $y_{j\pi}$ exists and is different from zero, Eq. (20) ensures the pinned nodes be at most p. Finally, Eqs. (21) and (22) ensure that an SCC can be pinning controlled only if its upstream is pinning controlled and a sufficient number of its nodes are pinned.

(ii) Selection of the pinned nodes, and of the coupling and control gains.

Part (i) of the algorithm determines the SCCs to be pinned as, if $y_{i\pi}$ exists, then $|\mathcal{P}_i| = y_{i\pi}$. Here, we provide an optimal but combinatorial strategy to solve (18), that is, to determine the set of pinned nodes $\mathcal{P}^*$ of cardinality p which allows the selection of the pair (c^*, κ^*) that minimizes the cost function J, while guaranteeing the pinning controllability of the nodes in $\mathcal{Q}^*$. To cope with (18) as efficiently as possible, we split the optimization problem in four steps.

(1) Find a lower bound c' for c^* by selecting the maximum among the coupling gains required to pinning control each $\mathcal{G}_l$ such that $y_{ll} = 1$ and $\nexists y_{l\pi}$.
(2) $\forall l : y_{l\pi} \neq 0$, compute (c_l^i, κ_l^i), that is, the values of c and κ that minimize J while allowing to pinning control $\mathcal{G}_l$ when the ith of all combinations of $y_{l\pi}$ of its nodes is pinned.
(3) For all possible sets $\mathcal{P}_m$ of p pinned nodes, $y_{l\pi}$ per SCC $\mathcal{G}_l$ such that $\exists y_{l\pi}$, compute (c_m, κ_m) as
$$(c_m, \kappa_m) = \left(\max\left\{\max_{i,l} c_l^i, c'\right\}, \max_{i,l} \kappa_l^i\right)$$
and the associated cost $J_m := J(c_m, \kappa_m)$.
(4) $(c^*, \kappa^*) = \arg\min_{\mathcal{P}_m} J(c_m, \kappa_m)$.

Remark 1. If $|\mathcal{V}_l| = 1\ \forall l : y_{l\pi} \neq 0$, the gain c plays no role in synchronizing the nodes of these SCCs, and thus $c^* = c'$. Moreover, from Theorem 3, we have that $\kappa^* = \mu$.

5. Numerical example

We consider a network of coupled Chua's circuits (Bartissol & Chua, 1988), whose individual dynamics can be written as

$$\dot{x}_{i1} = \upsilon_1 \left(-x_{i1} + x_{i2} - \psi(x_{i1})\right),$$
$$\dot{x}_{i2} = x_{i1} - x_{i2} + x_{i3},$$
$$\dot{x}_{i3} = -\upsilon_2 x_{i2},$$

where $\psi(x_{i1}) = \iota_1 x_{i1} + \frac{1}{2}(\iota_2 - \iota_1)(|x_{i1} + 1| - |x_{i1} - 1|)$. We select the parameters $\iota_1 = -3/4$, $\iota_2 = -4/3$, $\upsilon_1 = 10$, and $\upsilon_2 = 18$ and $H = I$ to ensure the existence of the chaotic attractor and fulfill Assumption 1. Hence, the hypotheses of Theorem 2 are satisfied, and we use our algorithm to find the optimal solution to (17). As a directed network topology, we select the actual yeast transcriptional regulatory network of size $N = 688$ (Alon, 0000). We assume $p \leq 15$ and that $c_M = \kappa_M = \infty$. Using the algorithm presented in Section 4, we determine the optimal set of SCCs to be pinned. In the selected scenario, each SCC γ_i such that $y_{i\pi} \neq 0$ only encompasses a single node and thus we fall into the case considered in Remark 1. Hence, $L_{ik} = L_{ik}^{11} = 0$ for all i, and to find the pair (c^*, κ^*) we must solve

$$\min_{\mu, V, W, \tau} \mu \tag{24}$$

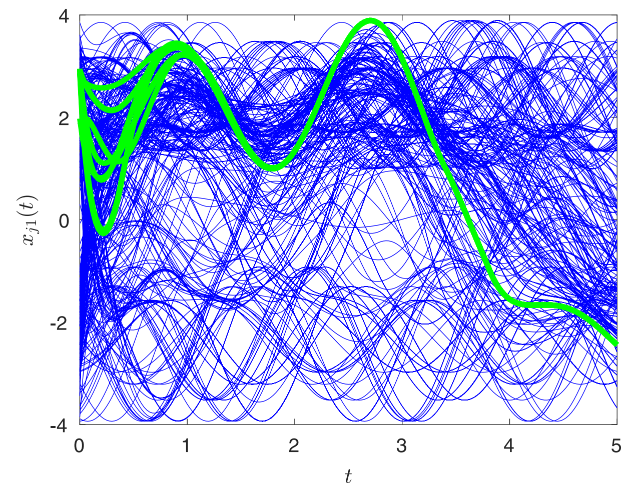

Fig. 2. Evolution of the first state variable of the nodes in $\mathcal{Q}^*$ (green) and in $\mathcal{V} - \mathcal{Q}^*$ (blue). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

$$(x - s)V(f(x, t) - f(s, t)) \leq (x - s)^T W(x - s), \tag{25}$$
$$(W - \mu VH + \tau I_n)_{\text{sym}} < 0 \tag{26}$$
$$\tau \in \mathbb{R}^+, \quad V_{\text{sym}} > 0, \quad VH = H^T V^T \geq 0. \tag{27}$$

The only hurdle in numerically solving this optimization problem might be represented by constraint (25). Nevertheless, it is possible to show that, for any V such that $\upsilon_{21} = \upsilon_{31} = 0$, a matrix $W(V)$ can be easily computed such that (25) holds. Hence, by paying the price of finding a slightly conservative value of the pair (c^*, κ^*), we neglect constraint (25) and select $W = W(V)$ in (26). In our example, this procedure yields the selection $c = \kappa = 4.8$. We test our designed pinning control strategy in a numerical simulation and observe that only the nodes in $\mathcal{Q}^*$ synchronize, as shown in Fig. 2.

6. Conclusions

In this manuscript, we tackled the problem of steering the dynamics of a complex network towards a desired trajectory described by an additional node, the pinner. In particular, we focused on the case of a large directed network, in which the number of pinned nodes is constrained, and it is impossible to control the whole network. In this scenario, we defined and solved the partial pinning control problem, which, for a given number of pinned nodes, consists in the maximization of the number of nodes converging towards the desired trajectory. In particular, exploiting the properties of the DAG condensation of a graph, we derived a set of sufficient conditions ensuring asymptotic control of all the nodes that are only in the downstream of the pinned root strongly connected components of the graph, and provided an estimate for the minimum coupling and control gain required. Exploiting this result, we translated the problem of maximizing the number of controlled nodes into an integer linear programming (ILP) problem, and developed a Matlab routine to solve it. The effectiveness of the approach was then illustrated on a network of 688 coupled Chua's circuits.

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