Flocking Algorithm for Directed Multi-Agent Networks via Pinning Control

MEI Yanhong[1], QIU Yun[1], CHEN Shiming[1]
1. School of Electrical and Electronic Engineering, East China Jiaotong University, Nanchang, 330013, China
E-mail: frankmyhong@163.com, c1977318@hotmail.com

Abstract: In this paper, an effective pinning strategy for directed multi-agent network to improve the control efficiency is proposed, and combining the theory of graph and analysis of control theory method. The algorithm based on information transfer matrix of the multi-agent network topology structure and according to maximum multiplicity of its eigenvalue, the number of pinning nodes is obtained when the network controllable, then, using the control theory of PBH criterion to identify the specific pinned node and the pinning flocking control law is designed. Furthermore, analyze different combination of pinned nodes and topological properties for pinned node is obtained. Simulation results demonstrate that the proposed algorithm is effective in improving the control efficiency of the entire system

Key Words: Directed network, Multi-agent, Pinning-flocking control, PBH criterion

1 Introduction

In 1986, Reynolds introduced Boid model for simulation group behaviors that satisfy the three flocking rules: flock centering, collision avoidance, velocity matching rules, and the classic flocking algorithm is proposed [1]. With the development of computer technology and artificial intelligence, the multi-agent coordination control has become a hot topic in the field of complexity science and made a lot of achievements [2-4].

Considering the classic flocking algorithm may have a “split” phenomenon, a local flocking algorithm based on the agent visual field is proposed in our previous work, which used flocking pursuing mode and anti-flocking searching mode between the agent and the target to achieve a multi-target tracking [5]. Olfati-Saber put forward a flocking control algorithm with a virtual leader, and assumed that the group of all agents can obtain the state of the virtual leader information [6]. For practical applications, this control method is difficult to achieve, with the deepening research, the concept of pinning control of complex networks is considered [7-8], through a fraction agents of the network is directly applied to the input control to achieve the overall flocking behavior. On the basis of Sabers’ research, Su H S *et al.* further studied only a small fraction of agents knows the virtual leader information, and ultimately achieve the whole flocking behavior, but without research the pinned node selection strategy [9], Wan X F *et al.* researched with time varying topology of flocking control for multi-agent network based on pinning control, and only a small fraction of agents have access to the reference state, but the pinning control strategy is determined to the randomly selected [10]. Pinning strategy is largely reduced the control costs, however, randomly pinning are uncertainty, the system control efficiency is low.

Further research, the specifically pinning control still developed, Hou B *et al.* studied the cluster consensus problem of high-order multi-agent systems with an observer-based control scheme [11], Ma Q *et al.* researched the group consensus problem of second-order nonlinear multi-agent systems through leader-following approach and pinning control [12], In [13], a distributed cooperative control protocols is proposed, which used an inverse optimality approach together with partial stability to consider the cooperative consensus and pinning control. In [11-13], merely use the concept of pinning control, not in-depth analysis it. In this paper, an effective pinning strategy for directed multi-agent network to improve the control efficiency is proposed. The algorithm based on information transfer matrix of the directed multi-agent network topology structure and according to maximum multiplicity of its eigenvalue, the number of pinning nodes is obtained when the network controllable, then, using the control theory of PBH criterion to identify the specific pinned node and the pinning flocking control law is designed. Taking the pinned node into account it is not unique, furthermore, analyze different combination of pinned nodes and topological properties for pinned node is obtained. Simulation results demonstrate that the proposed algorithm is effective in improving the control efficiency of the entire system.

2 Preliminaries

Definition 1: A graph $G=(V,E)$ that consists of a set of vertices $V=\{1,2,...,N\}$, whose elements represent agents in the system, and a set of edges $E=\{e_{ij}=(i,j):i,j\in V\}$, which represent neighboring relations. If at any point (i,j) and (j,i) correspondence with the same edge, the graph is called undirected graph, otherwise known as a directed graph.

Definition 2: The adjacency matrix $A=(a_{ij})_{N\times N}$ of graph $G=(V,E)$ is a matrix with nonzero elements satisfying the property $a_{ij}=1$ which represent a communication between the node i and j, otherwise $a_{ij}=0$. The graph Laplacian matrix can be expressed as: $L=D-A$, the degree matrix $D=diag(d_i)$ is the diagonal matrix which composed of nodes degree, $d_i=\sum_{j=1}^{N}a_{ij}$ is the degree of node i.

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Definition 3: The second smallest eigenvalue λ_2 of Laplacian matrix is the algebraic connectivity of graph G, which represents network connectivity, the larger λ_2 of Laplacian imply better performance of the network connectivity, conversely the smaller value of λ_2, the worse performance of the network connectivity [14-15].

Definition 4: Assuming the sensing radius $r(>0)$ denote the interaction range between two agents, on a two-dimensional space, the agent i as the center, an open ball with radius r determines the set of spatial neighbors of agent i, namely, $N_i(t) = \left\{ j \in V : \left\| q_i - q_j \right\| < r \right\}$, where $\|.\|$ is the Euclidean norm.

In the multi-agent network model, we assume that the system not exist an isolation agent. Consider a group of N dynamic agents with equation of motion:

$$\begin{cases} \dot{q}_i(t) = p_i(t) \\ \dot{p}_i(t) = u_i(t) \end{cases} \tag{1}$$

where $i = 1, 2, ..., N$, q_i and p_i denote the position and velocity vectors of agent i, respectively. u_i denote the control input (acceleration) of agent i, $q_i, p_i, u_i \in R^2$. The virtual leader with equation of motion:

$$\begin{cases} \dot{q}_\gamma(t) = p_\gamma(t) \\ \dot{p}_\gamma(t) = f_\gamma\left(q_\gamma(t), p_\gamma(t)\right) \end{cases} \tag{2}$$

where q_γ and p_γ denote the position and velocity vectors of the virtual leader, respectively.

Definition 5: For any agent i, $i = 1, 2, ..., N$, $\lim_{t\to\infty} \left\| p_i(t) - p_\gamma(t) \right\| = 0$, that is, the velocity between each agent and the virtual leader asymptotically consistent.

3 Flocking Algorithm of Multi-agent System

3.1 Pinning Strategy

In this paper, considering the consensus protocol, agent status depends on neighbors and their own information, specific pinned nodes selection method as follows:

Step1: According to the sensing radius r, the adjacency matrix A of graph G is obtained;

Step2: Find the eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_N$ of information transfer matrix $A + I_N$, to identify the eigenvalue λ^M of the maximum multiplicity for all eigenvalues and corresponding the maximum geometric multiplicity $\delta(\lambda^M)$;

Step3: Elementary transformation row for the matrix $A + I_N - \lambda^M I_N$ to identify the maximum linearly independent of the columns;

Step4: The set of the network controlled nodes corresponding to the linearly dependent columns of the matrix $A + I_N - \lambda^M I_N$ are the pinned nodes, which is the maximum multiplicity $\delta(\lambda^M)$.

The computation complexity of the algorithm is $O(N^2(\log N)^2)$, where N is the number of nodes in network. If $rank(A + I_N - \lambda_i I_N) = N$, then only one pinned node.

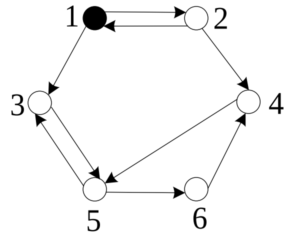


Fig. 1: The pinned nodes selection of directed network

Fig. 1 is a simple directed network as an example, the solving process of pinned nodes is described, where the solid black node is the pinned nodes, the matrix $A + I_N$ and eigenvalues of Fig.1 is, respectively.

$$A + I_N = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}, \lambda = \begin{bmatrix} 2.0000 + 0.0000i \\ 0.0000 + 0.0000i \\ 2.3247 + 0.0000i \\ 0.3376 + 0.5623i \\ 0.3376 - 0.5623i \\ 1.0000 + 0.0000i \end{bmatrix}$$

The distinct eigenvalues of information transfer matrix $A + I_N$ as shown in Fig. 1, the geometric multiplicity $\delta(\lambda^M) = 1$. Thus depending on step 2, Fig. 1 contain one pinned node, according to step 3, the pinned node of Fig. 1 is the node of 1 or 2. After definite $\delta(\lambda^M)$ pinned individuals, the node's state is governed by the following equation:

$$\dot{q}(t) = Aq(t) + Bu(t), q \in R^N, u \in R^M \tag{3}$$

where the vector $q(t)$ is the state of N nodes at time t, the adjacency matrix A denote the interaction strength between two nodes, $B = [b_{ij}]_{N\times M}$ is input matrix, which represent how the external input signals are connected to the nodes of networks, $u(t) = (u_1(t), u_2(t), \cdots u_{\delta(\lambda^M)}(t))$ is the vector of $\delta(\lambda^M)$ pinned nodes' control input at time t.

3.2 Pinning Flocking Algorithm of Multi-agent System

In this algorithm, each agent has a control input that composed of three portions:

$$u_i(t) = \alpha_i + \beta_i + \gamma_i \tag{4}$$

$$\begin{cases} \alpha_i = -\sum_{j\in N_i(t)} \left(p_i - p_j\right) \\ \beta_i = -\sum_{j\in N_i(t)} \nabla_{q_i} V_{ij} \\ \gamma_i = -k_i\left[c_1(q_i - q_\gamma) + c_2(p_i - p_\gamma)\right] \end{cases} \tag{5}$$

where α_i is a velocity consensus algorithm, which is used to match velocity with nearby neighborhoods, β_i is a gradient-based position, which is used to accomplish the rules of cohesion, and γ_i is a navigational feedback which lie on a group objective, where $k_i = 1$, if the agent i can obtain information from virtual leader, otherwise $k_i = 0$. The potential function is the attraction/repulsion function $g(q)$ [16], which is expressed as:

$$g(q) = -q\left[a - be^{-\|q\|^2/c}\right] \tag{6}$$

where $\| q \| = \sqrt{q^T q}$ is 2-norm, a, b and c are positive constants, $b > a$. When $q = 0$ or $q = \delta = \sqrt{c \ln(b/a)}$, $g(q) = 0$, attraction and repulsion forces are equal, δ is the equilibrium distance between agents. The negative sign of attraction function $g_a(q) = -aq$ means the movement direction of the agent, the parametric a represent the agent attractive force, the parametric b of repulsion function $g_r(q) = bqe^{-\|q\|^2/c}$ is used to adjust equilibrium position among agents. The attraction-repulsion function $g(q)$ is attraction to the other individuals on long distances and repulsion from the other individuals on short distances. Therefore, avoid the agents collide due to excessive proximity and maintaining aggregation groups behavior. In a summary

$$\begin{aligned} V_i(q) &= \sum_{j=1, j \neq i}^{N} g\left(\|q_{ij}\|_\sigma\right) \\ &= \sum_{j \in N_i(t)} g\left(\|q_{ij}\|_\sigma\right) + \sum_{j \notin N_i(t), j \neq i} g\left(\|r\|_\sigma\right) \end{aligned} \tag{7}$$

3.3 Stability Analysis

Theorem 1: Consider the second-order multi-agent system by the dynamics formula (1) and the motion of virtual leader formula (2), the initial position of agent is randomly distributed. Assume that the initial velocity mismatch and the initial energy H_0 are finite, with the control protocols formula (4) and (5), the following statement holds.

i) All agents asymptotically move with the same velocity.

ii) The system is asymptotically converges stability, where have a local minimum energy.

iii) The groups of two agents no collision for all $t \geq 0$.

Proof: Let $\tilde{q}_i = q_i - q_\gamma$, $\tilde{p}_i = p_i - p_\gamma$ denote the relative position and velocity difference between the agent and virtual leader, respectively.

$$\begin{cases} \dot{\tilde{q}}_i(t) = \tilde{p}_i(t) \\ \dot{\tilde{p}}_i(t) = \dot{p}_i(t) = u_i(t) \end{cases} \tag{8}$$

Thus, the control protocol can be rewritten as follows:

$$\begin{aligned} u_i(t) = &-\sum_{j \in N_i(t)} \left(\tilde{p}_i - \tilde{p}_j\right) - \sum_{j \in N_i(t)} \nabla_{\tilde{q}_i} V_{ij} \\ &- k_i \left(c_1 \tilde{q}_i + c_2 \tilde{p}_i\right) \end{aligned} \tag{9}$$

Define the following energy function H as the Lyapunov function, which is the sum of the total potential energy between two agents and the relative potential and kinetic energy between the agents and virtual leader.

$$H(\tilde{q}, \tilde{p}) = \frac{1}{2}\sum_{i=1}^{N} U_i(\tilde{q}) + \frac{1}{2}\sum_{i=1}^{N} \tilde{p}_i^T \tilde{p}_i \tag{10}$$

$$U_i(\tilde{q}) = V_i(\tilde{q}) + k_i c_1 \tilde{q}_i^T \tilde{q}_i \tag{11}$$

According to the symmetry of the attraction/repulsion function $g(q)$ one has

$$\begin{aligned} \frac{1}{2}\sum_{i=1}^{N} \dot{U}_i(\tilde{q}) = &\sum_{i=1}^{N} \sum_{j \in N_i(t)} \tilde{p}_i^T \nabla_{\tilde{q}_i} g\left(\|\tilde{q}_{ij}\|_\sigma\right) \\ &+ k_i c_1 \sum_{i=1}^{N} \tilde{q}_i^T \tilde{p}_i \end{aligned} \tag{12}$$

Derivative the Lyapunov function can be obtained:

$$\begin{aligned} \dot{H} &= \frac{1}{2}\sum_{i=1}^{N} \dot{U}_i(\tilde{q}) + \sum_{i=1}^{N} \tilde{p}_i^T \dot{\tilde{p}}_i \\ &= \sum_{i=1}^{N} \sum_{j \in N_i(t)} \tilde{p}_i^T \nabla_{\tilde{q}_i} g\left(\|\tilde{q}_{ij}\|_\sigma\right) \\ &\quad + k_i c_1 \sum_{i=1}^{N} \tilde{q}_i^T \tilde{p}_i + \sum_{i=1}^{N} \tilde{p}_i^T u_i \end{aligned} \tag{13}$$

Therefore, formula (9) into the formula (13) and simplify

$$\dot{H} = \sum_{j \in N_i(t)} \tilde{p}_i^T \left[-\sum_{j \in N_i(t)} \left(\tilde{p}_i - \tilde{p}_j\right) - k_i c_2 \tilde{p}_i \right] \tag{14}$$

Hence $\sum_{j \in N_i(t)} \left(\tilde{p}_i - \tilde{p}_j\right) = \sum_{j=1}^{N} a_{ij} \left(\tilde{p}_i - \tilde{p}_j\right) = \sum_{j=1}^{N} l_{ij} \tilde{p}_i$, we have

$$\begin{aligned} \dot{H} &= -\sum_{i=1}^{N} \tilde{p}_i^T \left[\sum_{j=1}^{N} l_{ij} + k_i c_2 \right] \tilde{p}_i \\ &= -\tilde{p}^T \left[\left(L(t) + c_2 K(t) \right) \otimes I_N \right] \tilde{p} \end{aligned} \tag{15}$$

where $\tilde{p} = col(\tilde{p}_1, \cdots, \tilde{p}_N)$, $K(t) = diag(k_1, \cdots, k_N)$. Here, $\dot{H} \leq 0$ implies that $L(t)$ and $K(t)$ are positive semi-definite matrix, respectively, $L(t) + c_2 K(t)$ is also positive semi-definite matrix. $\dot{H} \leq 0$ means the energy H is monotonically decreasing for all $t \geq 0$, $H(t) \leq H_0$, $\Omega = \left\{ (\tilde{q}^T, \tilde{p}^T)^T \in R^{2 \times 2N} : H \leq H_0 \right\}$ is an invariant set. From LaSalle's invariance principle, all agents starting in Ω converge to the largest invariant set $S = \left\{ (\tilde{q}^T, \tilde{p}^T)^T \in R^{2 \times 2N} : \dot{H} = 0 \right\}$. Thus, according to formula (15), $\dot{H} = 0$ is equivalent to $\tilde{p} = col(\tilde{p}_1, \tilde{p}_2, ..., \tilde{p}_N) \equiv col(0, 0, ..., 0)$, therefore, $\tilde{p}_1 = \tilde{p}_2 = ... = \tilde{p}_N$. This means that the velocity of all agents asymptotically converge the virtual leader velocity, or $p_1 = p_2 = ... = p_N = p_\gamma$, which proves part i). At the same time, exist a local minimum value $\tilde{q}$ that makes the system asymptotically converges to stable, which have a local minimum energy. Part ii) is proven.

To prove part iii), suppose that there are at least two agents collide at any time $t \geq 0$. This implies the energy of the system at time t is at least H_0, which is contradict with the condition $H(t) \leq H_0$. Finally, no two agents can possibly collide at any time $t \geq 0$.

4 Numerical Simulation and Research

In this section, the effectiveness of the proposed algorithm in this paper is verified by MATLAB simulation. The parameters used in this simulation are specified as follows:

The number of agents $N = 10$, the initial positions are chosen randomly from distributed in the square area of $[0, 20] \times [0, 20]$, the velocities and direction angles are randomly from distributed in the square area of $[1,2]\times[1,2]$ and randomly on the interval $(-\pi,\pi)$, respectively. The virtual leader's position and velocity is $q_\gamma(0)=[10,10]$ and $p_\gamma(0)=0.35\times[1,1]$, respectively. The desired distance $d=7$, the agent of sensing radius $r=1.2d$, the feedback coefficient $c_1=0.6$, $c_2=0.4$, $a=4$, $b=25$, $c=10$.

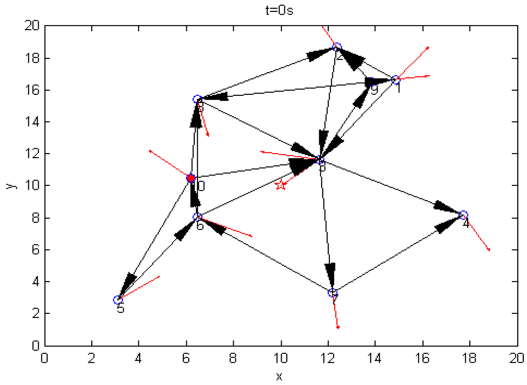
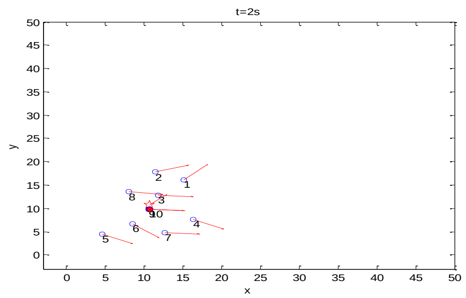


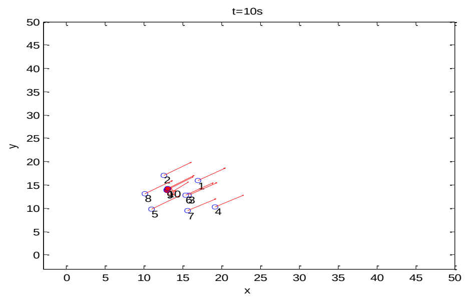
Fig. 2: Initial distribution of multi-agent systems

The initial positions of 10 agents submit to the randomly distribution as shown in Fig. 2, where the hollow circle denote the general agents, the solid red circle denote the pinned agents, the red star represent the virtual leader, the arrows indicate the flow of information transfer. Herein, Pinning strategy identified two pinned nodes and the node is 7/9, 1/5/10.

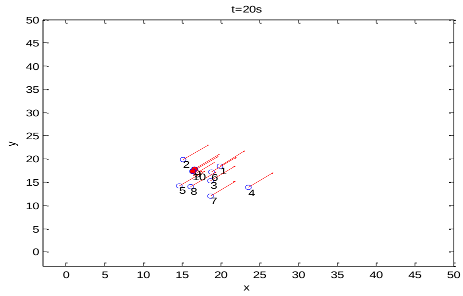
In flocking algorithms (4), the dynamic evolution of multi-agent system as shown in Fig. 3, the pinned node is 9, 10. From Fig. 3 (a)-(d), it can be seen that the dynamic evolution of the system ultimately the consistent. Fig. 4 is the velocity difference between agents and virtual leader, when $t=15.6s$, all agents asymptotically move with the same velocity and the system converges stability.



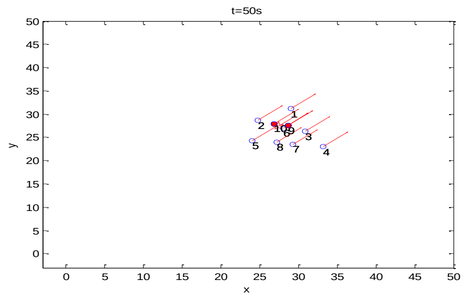
(a)



(b)



(c)



(d)

Fig.3: Evolution of flocking control

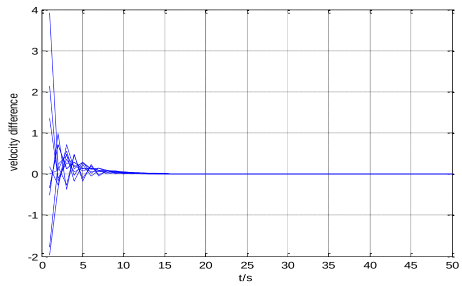


Fig. 4: Velocity difference between agents and virtual leader

In order to analyze the impact of different pinned node for consistency, simulation for all pinned node combination, each experiment is respectively operated independently 100 times, the statistical results of average consensus time are shown in Table 1.

Table 1 shows that difference pinned nodes ultimate achieve consensus time. Under the same scale of network, selected pinned node 9, 10, average convergence time is minimal, because of the node degree distributed of network. Further to research the topology properties of pinned nodes for directed network, this paper analyzes the degree distribution characteristics of pinned nodes, and founded a pinned node out-degree is greater than the in-degree, the zero of in-degree must be pinned node. Randomly given multi-agent network as an example, $N=20$, in Fig. 5, pinned nodes are mainly composed of zero in-degree and low in-degree.

Table 1: Average Consensus Time

Pinned node	Time/s
9,10	15.6
7,10	22.3
1,7	35.5
1,9	24.5
5,7	37.3
5,9	32.5

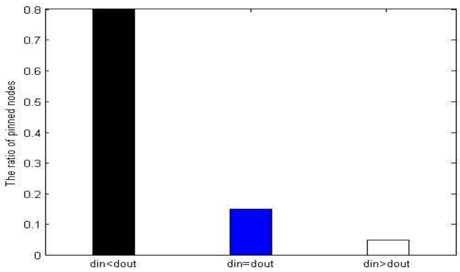


Fig. 5: Relation between ratio of pinned nodes and degree

5 Conclusion

In the research of flocking control for directed multi-agent system, one of the key issues is how to improve the flocking control efficiency. In this paper, an effective pinning strategy for directed multi-agent network to improve the control efficiency is proposed, and combining the theory of graph and analytical of control theory method. Using the control theory of PBH criterion to identify the pinned node, analyze different combination of pinned nodes and topological properties for pinned node is obtained. Simulation results demonstrate that the proposed algorithm is effective in improving the control efficiency of the entire system.

References

[1] C. W. Reynolds, Flocks, herds, and schools: a distributed behavioral model, *Computer Graphics (ACM)*, 21(4): 25-34, 1987.

[2] Z. Lin, L. Wang, Z. Han and M. Fu, Distributed formation control of multi-agent systems using complex laplacian, *IEEE Transactions on Automatic Control*, 59(7): 1765-1777, 2014.

[3] L. Li, D. W. C. Ho and S. Xu, A distributed event-triggered scheme for discrete-time multi-agent consensus with communication delays, *IET Control Theory and Applications*, 8(10): 830-837, 2014.

[4] S. Martin, A. Girard, A. Fazeli and A. Jadbabaie, Multi-agent flocking under general communication rule, *IEEE Transactions on Control of Network Systems*, 1(2): 155-166, 2014.

[5] H. Pei, S. Chen and Q. Lai, A local flocking algorithm of multi-agent dynamic systems, *International Journal of Control*, 88(11): 2242-2249, 2015.

[6] R. Olfati-Saber, Flocking for multi-agent dynamic systems: algorithms and theory, *IEEE Transactions on Automatic Control*, 51(3): 401-420, 2006.

[7] X. Wang and G. Chen, Pinning control of scale-free dynamical networks, *Physica A: Statistical Mechanics and its Applications*, 310(3): 521-531, 2002.

[8] T. Chen, X. Liu and W. Lu, Pinning complex networks by a single controller, *IEEE Transactions on Circuits and Systems-I*, 54(6): 1317-1326, 2007.

[9] H. Su, X. Wang and Z. Lin, Flocking of multi-agents with a virtual leader, *IEEE Transactions on Automatic Control*, 54(2): 293-307, 2009.

[10] X. Wang, X. Li and J. Lu, Control and flocking of networked systems via pinning, *IEEE Circuits and Systems Magazine*, 10(3): 83-91, 2010.

[11] K. H. Movric and F. L. Lewis, Cooperative optimal control for multi-agent systems on directed graph topologies, *IEEE Transactions on Automatic Control*, 59(3): 769-774, 2014.

[12] B. Hou, F. Sun, H. Li, Y. Chen and G. Liu, Observer-based cluster consensus control of high-order multi-agent systems, *Neurocomputing*, 168: 979-982, 2015.

[13] Q. Ma, Z. Wang and G. Miao, Second-order group consensus for multi-agent systems via pinning leader-following approach, *Journal of the Franklin Institute*, 351(3): 1288-1300, 2014.

[14] M. Fiedler, Algebraic connectivity of graphs, *Czechoslovak Mathematical Journal*, 23(2): 298-305, 1973.

[15] Y. Kim and M. Mesbahi, On maximizing the second smallest eigenvalue of a state-dependent graph laplacian, *IEEE Transactions on Automatics Control*, 51(1): 116-120, 2006.

[16] V. Gazi and K. M. Passino, Stability analysis of swarms, *IEEE Transactions on Automatic Control*, 48(4): 692-697, 2003.