[9] S. Pettersson and B. Lennartson, "Stability and robustness for hybrid systems," in *Proc. IEEE Conf. Decis. Control*, Kobe, Japan, 1996, pp. 1202–1207.

[10] P. A. Parrilo, "Structured semidefinite programs and semialgebraic geometry methods in robustness and optimization," Ph.D. dissertation, California Instit. Technol., Pasadena, CA, 2000.

[11] S. Prajna and A. Papachristodoulou, "Analysis of switched and hybrid systems—Beyond piecewise quadratic methods," in *Proc. IEEE Amer. Control Conf.*, Denver, CO, 2003, pp. 2779–2784.

[12] A. Bemporad, G. Bianchini, F. Barbagli, and F. Brogi, "Passivity analysis and passification of discrete-time hybrid systems," presented at the IFAC World Congress, Praha, CZ, 2005.

[13] A. Bemporad, G. Bianchini, F. Brogi, and G. Chesi, "Passivity analysis of discrete-time hybrid systems using piecewise polynomial storage functions," in *Proc. IEEE Conf. Decis. Control*, Sevilla, ES, 2005, pp. 5421–5426.

[14] A. Bemporad, "Efficient conversion of mixed logical dynamical systems into an equivalent piecewise affine form," *IEEE Trans. Autom. Control*, vol. 49, no. 5, pp. 832–838, May 2004.

[15] A. Hassibi and S. Boyd, "Quadratic stabilization and control of piecewise-linear systems," in *Proc. IEEE Amer. Control Conf.*, Philadelphia, PA, 1998, pp. 3659–3664.

[16] D. Mignone, G. Ferrari Trecate, and M. Morari, "Stability and stabilization of piecewise affine and hybrid systems: An LMI approach," in *Proc. 39th IEEE Conf. Decis. Control*, Sydney, Australia, 2000, pp. 504–509.

[17] A. Bemporad, F. Torrisi, and M. Morari, "Optimization-based verification and stability characterization of piecewise affine and hybrid systems," in *Hybrid Systems: Computation and Control* (Lecture Notes in Computer Science), B. Krogh and N. Lynch, Eds., vol. 1790, New York: Springer-Verlag, 2000, pp. 45–58.

[18] F. Cuzzola and M. Morari, "A generalized approach for analysis and control of discrete-time piecewise affine and hybrid systems," in *Hybrid Systems: Computation and Control* (Lecture Notes in Computer Science), A. Sangiovanni-Vincentelli and M. D. Benedetto, Eds. New York: Springer-Verlag, 2001, no. 2034, pp. 189–203.

[19] F. Barbagli, D. Prattichizzo, and K. Salisbury, "Multirate analysis of haptic interaction stability with deformable objects," in *Proc. IEEE Conf. Decis. Control*, Las Vegas, NV, 2002, pp. 917–922.

[20] B. E. Miller, J. E. Colgate, and R. Freeman, "Guaranteed stability of haptic systems with nonlinear virtual environments," *IEEE Trans. Robot. Autom.*, vol. 16, no. 6, pp. 712–719, Dec. 2000.

[21] B. E. Miller, J. E. Colgate, and R. Freeman, "On the role of dissipation in haptic systems," *IEEE Trans. Robot.*, vol. 20, no. 4, pp. 768–771, Aug. 2004.

[22] A. Bemporad, G. Bianchini, and F. Brogi. (2006). Appendix to Passivity Analysis and Passification of Discrete-Time Hybrid Systems [Online]. Available: http://www.dii.unisi.it/~hybrid/passivity

Controllability of a Leader–Follower Dynamic Network With Switching Topology

Bo Liu, Tianguang Chu, Long Wang, and Guangming Xie

***Abstract*—This note studies the controllability of a leader–follower network of dynamic agents linked via neighbor rules. The leader is a particular agent acting as an external input to steer the other member agents. Based on switched control system theory, we derive a simple controllability condition for the network with switching topology, which indicates that the controllability of the whole network does not need to rely on that of the network for every specific topology. This merit provides convenience and flexibility in design and application of multiagent networks. For the fixed topology case, we show that the network is uncontrollable whenever the leader has an unbiased action on every member, regardless of the connectivity of the members themselves. This gives new insight into the relation between the controllability and the connectivity of the leader–follower network. We also give a formula for formation control of the network.**

***Index Terms*—Controllability, leader–follower networks, local interactions, multiagent systems, switching topology.**

I. Introduction

In recent years, control and coordination of multiagent network systems has emerged as a topic of major interest [1]–[22]. This is partly due to broad applications of multiagent systems in cooperative control of unmanned air vehicles, scheduling of automated highway systems, formation control of satellite clusters, and congestion control in communication networks, etc. [2]–[19]. Studies in this direction have greatly inspired by the ubiquitous cooperative behavior of biological swarms, such as ant colonies, bird flocks, and fish schools etc., where collective motions may emerge from groups of simple individuals through limited interactions. So far, considerable efforts have been devoted to modeling and understanding the cooperative and operational principles of such collective behavior (e.g., [23]–[29] and the references therein). Yet, some fundamental issues concerning control of multiagent networks, such as the controllability of multiagent networks, are still lacking in studies.

As in its usual sense in systems theory, the notion controllability of a multiagent network means that the network system can be steered from one state to another through certain regulations. In the context of multiagent networks, however, the issue of controllability bears new features and difficulties. In particular, as the dynamics of a network relies crucially on its interconnection topology, it is therefore to think

Manuscript received August 21, 2006; revised June 27, 2007. Recommended by Associate Editor J. Lygeros. This work was supported in part by the National Science Foundation of China (NSFC) under Grant 60674047, Grant 60674050, and Grant 60404001, in part by the National 973 Program under Grant 2002CB312200, in part by the National 863 Program under Grant 2006AA04Z258 and Grant 2006AA04Z247, and in part by the Specialized Research Fund for the Doctoral Program of Higher Education (SRFDP) under Grant 20060001013.

B. Liu is with the Intelligent Control Laboratory, Center for Systems and Control, Department of Industrial Engineering and Management, College of Engineering, Peking University, Beijing 100871, China. She is also with the College of Science, North China University of Technology, Beijing 100041, China (e-mail: boliu@ncut.edu.cn).

T. Chu, L. Wang, and G. Xie are with the Intelligent Control Laboratory, Center for Systems and Control, Department of Industrial Engineering and Management, College of Engineering, Peking University, Beijing 100871, China (e-mail: chutg@pku.edu.cn; longwang@mech.pku.edu.cn; xieming@mech.pku.edu.cn).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TAC.2008.919548

that the controllability should depend on network connectivity or topology. Usually, a multiagent system is interconnected through neighbor rules, and hence, has a local and time-varying interconnection topology. This makes the controllability of dynamic networks a nontrivial new problem. To date, very few results have been available in the literature [9], [21], [22].

In this note, we consider a leader–follower network with switching topologies. The leader plays the role of an external input to other remaining agents (members or followers) to steer the whole group, and the members update their states based on the information available from their neighbors and the leader. We will establish controllability conditions and discuss relations between the controllability and the connectivity of the network. We note that in a recent paper [9], the issue of controllability was studied in a group of autonomous agents interconnected through nearest neighbor rules, with one of the agents acting as a leader, and controllability conditions were derived under a fixed topology condition. For networks with switching topologies, as in cases often encountered in practice, it is usually difficult to deal with the controllability problem due to complexity of the topology and lack of theoretical tools. Here, we attack the problem by invoking the recently developed theory of switched control systems. Our main result shows an advantage of the switching topology scheme, i.e., a controllable switched network can be made up of uncontrollable subsystems. This provides desirable convenience and flexibility in application of multiagent systems.

We will also examine fixed topology case and give further understanding on the relation between the connectivity and the controllability of the network. Specifically, we show that the network under consideration is always uncontrollable if the leader acts on every member with the same input, regardless of details of the connectivity of the members themselves. Hence, for a group of agents linked with fixed and all-identical couplings, it could never be controllable by one of them acting as a leader. This covers a result obtained recently in [9, Proposition V.1] for a special case of the network model considered in the present paper, under complete graph condition on the network topology. Besides, for a controllable network with a fixed topology, we give a formula for desired inputs to transfer the system between given states.

II. Graph Theory Preliminaries

In this section, we introduce some useful concepts and notations in graph theory [30].

An *undirected graph* $\mathcal{G}$ consists of a *vertex set* $\mathcal{V} = \{1, 2, \ldots, N\}$ and an *edge set* $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\}$, where an *edge* is an unordered pair of distinct vertices of $\mathcal{V}$. If $i, j \in \mathcal{V}$, and $(i, j) \in \mathcal{E}$, then we say that i and j are *adjacent* or that j is a *neighbor* of i, and denote this by writing $j \sim i$. Define the *set of neighbors* of vertex i as $\mathcal{N}_i \stackrel{\triangle}{=} \{j | j \sim i\} \subseteq \{1, \ldots, N\} \backslash \{i\}$ that includes all neighbors of vertex i. The number of neighbors of each vertex is its *degree*. A graph is called *complete* if every pair of vertices are adjacent. A *path* of length r from i to j in a graph is a sequence of $r+1$ distinct vertices starting with i and ending with j such that consecutive vertices are adjacent. If there is a path between any two vertices of $\mathcal{G}$, then $\mathcal{G}$ is *connected*. Any undirected graph can be represented by its *adjacency matrix*, $A(\mathcal{G})$, which is a symmetric matrix with 0,1 elements. The element in position (i, j) (and (j, i) due to symmetry) in $A(\mathcal{G})$ is 1 if vertices i and j are adjacent and 0 otherwise. The *degree matrix* $\triangle(\mathcal{G})$ of $\mathcal{G}$ is a diagonal matrix with rows and columns indexed by $\mathcal{V}$, in which the (i, i)-entry is the degree of vertex i. The symmetric matrix defined as

$$L(\mathcal{G}) = \triangle(\mathcal{G}) - A(\mathcal{G})$$

is the *Laplacian* of $\mathcal{G}$. The Laplacian is always symmetric and positive semidefinite, and the algebraic multiplicity of its zero eigenvalue is equal to the number of connected components in the graph.

In the following, we will also make use of the notion of *interconnection graph* to refer to an undirected graph describing the network topology of a group of agents, where the vertices correspond to the agents and the edges correspond to couplings of the interconnected agents in the group.

III. Model

The leader–follower network to be studied consists of $N+1$ dynamic agents interconnected through an information network, in which an agent indexed by 0 is assigned as the leader and the others indexed by $1, \ldots, N$ are referred to as members. The leader is unaffected by the members whereas a member may be influenced by the leader as well as its neighbors. The members update their states according to the rules

$$\begin{aligned} x_i(k+1) = x_i(k) - &\sum_{j \in \mathcal{N}_i} w_{ij}(x_i(k) - x_j(k)) \\ &- \gamma_i w_{i0}(x_i(k) - x_0(k)) \end{aligned}$$

where $x_i \in \Re^n$ is the state of agent i and $i = 1, \ldots, N$; $W = [w_{ij}] \in \Re^{n \times n}$ $(i, j = 1, \ldots, N)$ with each $w_{ij} \geq 0$ and $w_{ii} = 0$ is the interaction (or coupling) matrix; $w_{i0} \geq 0$ $(i = 1, \ldots, N)$ are the coupling weights between the leader and the members; $\gamma_i = 1$ if $0 \sim i$ and 0 otherwise.

For simplicity in notation, here we only consider the case $n = 1$ though the discussion applies for general case of n. Let $x = (x_1, \ldots, x_N)^T$ be the stack vector of all the agent states; it follows that

$$x(k+1) = Fx(k) + rx_0(k) \tag{1}$$

where $F = I - L - R$, I is the $N \times N$ identity matrix, $R = \text{diag}[\gamma_1 w_{10}, \ldots, \gamma_N w_{N0}]$, $r = (\gamma_1 w_{10}, \ldots, \gamma_N w_{N0})^T$, and $L = [l_{ij}]$ with

$$l_{ij} = \begin{cases} -w_{ij}, & j \neq i \quad \text{and} \quad j \in \mathcal{N}_i \\ \sum_{j \in \mathcal{N}_i} w_{ij}, & j = i \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Clearly, the matrix L has the following properties:

1) the off-diagonal elements are all negative or zero;
2) the row sums are all equal to zero.

Notice that the leader in the model is fixed, but the coupling between the leader and the members and that between the members themselves may vary in time. Therefore, the interconnection graph $\mathcal{G}$ of the model is time varying. Also, we can formulate the model as a dynamic network with switching topologies in the form

$$x(k+1) = F_{\sigma(k)} x(k) + r_{\sigma(k)} x_0(k) \tag{3}$$

where $x(k) \in \Re^N$ is the state, $x_0(k) \in \Re^1$ is the input (effect of the leader) to the system, the piecewise constant scalar function $\sigma(k) : \{0, 1, \ldots\} \to \{1, \ldots, K\}$ is the switching path describing the time-variant coupling of the network, K is the number of possible coupling patterns, i.e., switching topologies of the network, and $\sigma(k) = i$ implies that the system realization (called the subsystem hereafter) is chosen as (F_i, r_i) with $F_i \in \Re^{N \times N}$, $r_i \in \Re^N$ for some $i \in \{1, \ldots, K\}$. As in [32], here we always assume that each F_i is nonsingular. Sometimes, it is of practical interest to design an appropriate switching path to achieve specific motion in the network. We will give an example to illustrate this later in the next section.

Remark 1: In case x_i $(i = 1, \ldots, N)$ are n-dimensional, (3) should be modified to the form

$$x(k+1) = (F_{\sigma(k)} \otimes I_n)x(k) + (r_{\sigma(k)} \otimes I_n)x_0(k)$$

where $x(k) = (x_1^T, \ldots, x_N^T)^T \in \Re^{nN}$, I_n is the $n \times n$ identity matrix, and the symbol $\otimes$ denotes the Kronecker product of matrices.

In the sequel, we will study the controllability of (3) and give a geometric condition and some corollaries in the special case of fixed topology.

IV. Controllability Conditions

Let us denote $\underline{M} = \{0, 1, \ldots, M-1\}$ for an integer $M > 0$. We first introduce the following concepts.

Definition 1: A nonzero state x of (3) is controllable if there exists a time instant $M > 0$, a switching path $\sigma : \underline{M} \to \{1, \ldots, K\}$, and control inputs $x_0(k)$ for $k \in \underline{M}$, such that $x(0) = x$ and $x(M) = 0$. If any nonzero state x of (3) is controllable, then (3) is said to be *controllable*.

Definition 2 ([31]): Given a matrix $B_{n \times p} = [b_1,\ b_2, \ldots, b_p]$, the *column space* $\mathcal{R}(B)$ is defined as

$$\mathcal{R}(B) \stackrel{\triangle}{=} \mathrm{span}\{b_1,\ b_2, \ldots, b_p\}.$$

Definition 3 ([31]): Given a matrix $F_{N \times N}$ and a linear subspace $\mathcal{W} \in \Re^N$, the *cyclic invariant subspace* $\langle F \mid \mathcal{W} \rangle$ is defined as

$$\langle F \mid \mathcal{W} \rangle \stackrel{\triangle}{=} \sum_{i=1}^{K} F^{i-1} \mathcal{W}.$$

For notational simplicity, denote $\langle F \mid r \rangle = \langle F \mid \mathcal{R}(r) \rangle$, where F is a matrix and r a vector. We define a *subspace sequence* for (3) as

$$\mathcal{W}_1 = \sum_{i=1}^{K} \langle F_i \mid r_i \rangle, \mathcal{W}_2 = \sum_{i=1}^{K} \langle F_i \mid \mathcal{W}_1 \rangle, \ldots, \mathcal{W}_N = \sum_{i=1}^{K} \langle F_i \mid \mathcal{W}_{N-1} \rangle.$$

Now, we are ready to introduce the following technical result.

Lemma 1 ([32]): System (3) is controllable iff $\mathcal{W}_N = \Re^N$.

This result is crucial to our following discussion. It can be proved by first ascertaining the reachability of the switched system under the given condition. We refer interested readers to [32] for details.

From Lemma 1, we can derive the following main result for (3).

Theorem 1: System (3) is controllable if

$$\mathcal{R}(r_1) + \mathcal{R}(r_2) + \cdots + \mathcal{R}(r_K) = \Re^N.$$

It is easy to obtain this result by noting that, by definition, $\mathcal{R}(r_i) \subseteq \langle F_i \mid r_i \rangle$ for all i. Hence, $\Re^N \subseteq \mathcal{W}_N$. The reverse is clear and the result thus follows from Lemma 1.

Notice that although it is in general a sufficient condition, Theorem 1 is simpler and more easily checkable than Lemma 1 since it only involves the couplings of the leader to the members. This also makes for convenience of design of a switching path such that the network system is controllable. Observe that the condition of Theorem 1 does not require the controllability of the subsystems; see the example given later. This merit is very desirable in applications as it can provide more freedom in the design of a network.

In case the knowledge of switching topologies of a network system is not available in advance, it is difficult to make use of Theorem 1. A conservative approach to such an issue is to consider all possible switching patterns with Theorem 1. This is a disadvantage of the result.

Remark 2: For n-dimensional case as described in Remark 1, the condition of Theorem 1 reads

$$\mathcal{R}(r_1 \otimes I_n) + \mathcal{R}(r_2 \otimes I_n) + \cdots + \mathcal{R}(r_K \otimes I_n) = \Re^{nN}.$$

A. Fixed Topology

For the special case $\sigma(k) = 1$, (3) [equivalently, (1)] describes a leader–follower dynamic network with fixed topology. In [9], Tanner studied the controllability of (1) in this case under special parameter conditions: $w_{ij} = \gamma_i = 1$ for all i, j. Here, we establish further results for a more general case without such restrictions. To emphasize the time invariance of the network topology, we rewrite (1) as

$$x(k+1) = F_1 x(k) + r_1 x_0(k). \tag{4}$$

In this case, the standard controllability condition of linear systems applies to (4). In particular, we can have an interesting result that shows that a leader–follower system will be uncontrollable if the leader exerts indiscriminating action on all members.

Theorem 2: If $r_1 = [\alpha, \alpha, \ldots, \alpha]^T$ with $\alpha > 0$ a scalar, then (4) is uncontrollable.

Actually, in this case, the controllability matrix of (4) takes the form of $Q = [q_{ij}]_{N \times N}$ with $q_{ij} = \alpha\beta^{j-1}$ and $\beta = 1 - \alpha$. Thus, $\mathrm{rank}(Q) = 1$ and (4) is uncontrollable.

Remark 3: Notice that the form of Q given earlier relies only on (2), which is a generic property of the linear diffusive coupling topology used in most dynamic network models in the literature, regardless of the connectivity of the members in the network. A direct consequence of Theorem 2 is that a network (4) of a group of agents with a complete interconnection graph of identical couplings is always uncontrollable whichever one is chosen as the leader in the group, because in this case, any agent acting as a leader will always have identical coupling weights to all remaining agents. This result was also obtained in [9, Proposition V.1] for a special case of (4) as mentioned before. These studies are helpful in understanding the dynamics of interconnected systems.

Finally, we note that if (4) is controllable, then, theoretically, there must exist a set of inputs $x_0(k)$ to drive the system from an initial state to a final state. But how to find the inputs for given initial and final states? Here we give a formulation of the desired inputs.

Suppose (4) is controllable and $x(0), x(m)$ are given initial and final states, where m is the finial time whose value is not specified at this stage. A recursive manipulation of (4) yields

$$x(m) - F_1^m x(0) = Q(m) \begin{bmatrix} x_0(m-1) \\ \vdots \\ x_0(1) \\ x_0(0) \end{bmatrix} \tag{5}$$

where $Q(m) = [r_1 \ \vdots \ F_1 r_1 \ \vdots \ F_1^2 r_1 \ \vdots \ \cdots \ \vdots \ F_1^{m-1} r_1]$. Notice that $Q(N) = Q$ is invertible by assumption, hence, we can take $m = N$ and obtain a set of inputs

$$\begin{bmatrix} x_0(N-1) \\ \vdots \\ x_0(1) \\ x_0(0) \end{bmatrix} = Q^{-1} \left(x(N) - F_1^N x(0) \right) \tag{6}$$

which bring the network state from $x(0)$ to $x(m)$ within N steps.

Remark 4: From the information given before, we know that if (4) is controllable, then it can be transferred between arbitrary two states within at most N steps by choosing appropriate inputs, such as (6). Sometimes, one may get a set of inputs of length less than N. For instance, if

$$x(m) - F_1^m x(0) \in \mathrm{span}\{r_1, F_1 r_1, \ldots, F_1^{m-1} r_1\}$$

for some $m < N$, then it is clear that (5) must have a solution $(x_0(0), \ldots, x_0(m-1))$, which gives the desired inputs. Notice that (5) is dependent on the given states $x(0)$ and $x(m-1)$.

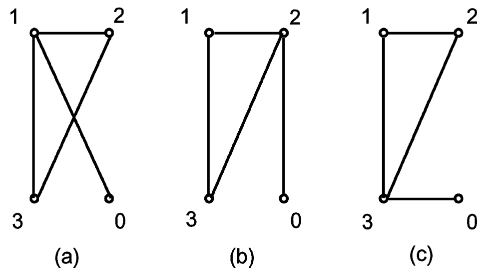

Fig. 1. Switched network with uncontrollable subsystems.

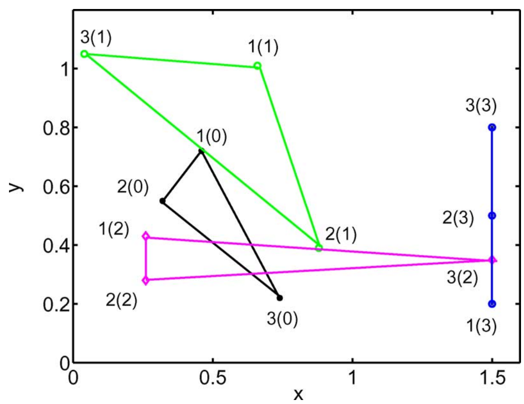

Fig. 2. Align in a straight line.

B. Example

To illustrate the main result, we consider here a four-agent network with agent 0 as the leader and with switching topology described by the graphs in Fig. 1(a)–(c). For simplicity, we let $w_{ij} = w_{i0} = 1$ for $i, j = 1, 2, 3$. From Fig. 1, we can compute the Laplacian matrices of the subgraphs to be

$$L_1 = L_2 = L_3 = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

and $R_1 = \mathrm{diag}[1,0,0]$, $R_2 = \mathrm{diag}[0,1,0]$, $R_3 = \mathrm{diag}[0,0,1]$. Hence, the switched linear system (3) is defined by

$$F_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}, \qquad r_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$F_2 = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -1 \end{bmatrix}, \qquad r_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$F_3 = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix}, \qquad r_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

It can be seen that F_i is nonsingular and (F_i, r_i) is uncontrollable for each i. Moreover,

$$\mathcal{R}(r_1 \otimes I_2) + \mathcal{R}(r_2 \otimes I_2) + \mathcal{R}(r_3 \otimes I_2) = \Re^6.$$

By Remark 2, the switched system is controllable, and hence, can be steered to realize any desired formation by the leader.

Figs. 2 and 3 show simulations of formation control of the network. In each case, the member agents (circles) move from a random initial configuration to desired ones: aligning in a straight line and forming a regular triangle as shown in Figs. 2 and 3, respectively. The indices in parenthesis in the figures indicate time steps.

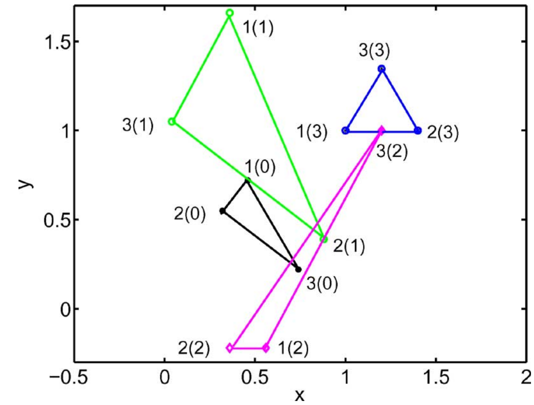

Fig. 3. Form a regular triangle.

V. Conclusion

We have studied the controllability of a leader–follower dynamic network with switching as well as fixed interaction topology. Our main result gives a simple controllability condition for the switching topology case, which shows an attractive advantage of switching topology, that the controllability of the overall system does not require that of each subsystem. This provides much convenience and flexibility in design and application of such network systems. For fixed topology case, our result generalizes a recent study and gives further insight into the effect of connectivity on the controllability of the network. Moreover, for the controllable case, we give a formula of formation control inputs, which enables one to realize a desired formation within steps no more than the dimension of the network.

References

[1] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Trans. Autom. Control*, vol. 48, no. 6, pp. 988–1001, Jun. 2003.

[2] Y. Liu, K. M. Passino, and M. M. Polycarpou, "Stability analysis of M-dimensional asynchronous swarms with a fixed communication topology," *IEEE Trans. Autom. Control*, vol. 48, no. 1, pp. 76–95, Jan. 2003.

[3] Z. Lin, M. Broucke, and B. Francis, "Local control strategies for groups of mobile autonomous agents," *IEEE Trans. Autom. Control*, vol. 49, no. 4, pp. 622–629, Apr. 2004.

[4] P. Ogren, E. Fiorelli, and N. E. Leonard, "Cooperative control of mobile sensor networks: Adaptive gradient climbing in a distributed environment," *IEEE Trans. Autom. Control*, vol. 49, no. 8, pp. 1292–1302, Aug. 2004.

[5] J. A. Fax and R. M. Murray, "Information flow and cooperative control of vehicle formations," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1465–1476, Sep. 2004.

[6] R. O. Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sep. 2004.

[7] J. A. Marshall, M. E. Broucke, and B. A. Francis, "Formations of vehicles in cyclic pursuit," *IEEE Trans. Autom. Control*, vol. 49, no. 11, pp. 1963–1974, Nov. 2004.

[8] H. G. Tanner, G. J. Pappas, and V. Kumar, "Leader-to-formation stability," *IEEE Trans. Autom. Robot.*, vol. 20, no. 3, pp. 443–455, Jun. 2004.

[9] H. G. Tanner, "On the controllability of nearest neighbor interconnections," in *Proc. IEEE Conf. Decis. Control*, Dec., 2004, pp. 2467–2472.

[10] S. Mu, T. Chu, and L. Wang, "Coordinated collective motion in a motile particle group with a leader," *Physica A*, vol. 351, pp. 211–226, Jun. 2005.
[11] L. Moreau, "Stability of multiagent systems with time-dependent communication links," *IEEE Trans. Autom. Control*, vol. 50, no. 2, pp. 169–182, Feb. 2005.
[12] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Trans. Autom. Control*, vol. 50, no. 5, pp. 655–661, May 2005.
[13] L. Gao and D. Cheng, "Comment on 'Coordination of groups of mobile autonomous agents using nearest neighbor rules,'" *IEEE Trans. Autom. Control*, vol. 50, no. 11, pp. 1913–1916, Nov. 2005.
[14] R. O. Saber, "Flocking for multi-agent dynamic systems: Algorithms and theory," *IEEE Trans. Autom. Control*, vol. 51, no. 3, pp. 401–420, Mar. 2006.
[15] W. Wang and J. E. Slotine, "A theoretical study of different leader roles in networks," *IEEE Trans. Autom. Control*, vol. 51, no. 7, pp. 1156–1161, Jul. 2006.
[16] H. Shi, L. Wang, and T. Chu, "Virtual leader approach to coordinated control of multiple mobile agents with asymmetric interactions," *Physica D*, vol. 213, pp. 51–65, 2006.
[17] F. Xiao and L. Wang, "State consensus for multi-agent systems with switching topologies and time-varying delays," *Int. J. Control*, vol. 79, no. 10, pp. 1277–1284, 2006.
[18] G. Xie and L. Wang, "Consensus control for a class of networks of dynamic agents," *Int. J. Robust Nonlinear Control*, vol. 17, pp. 941–959, 2007.
[19] Y. Hong, J. Hua, and L. Gao, "Tracking control for multi-agent consensus with an active leader and variable topology," *Automatica*, vol. 42, pp. 1177–1182, 2006.
[20] Y. Hong, L. Gao, D. Cheng, and J. Hu, "Lyapunov-based approach to multiagent systems with switching jointly connected interconnection," *IEEE Trans. Autom. Control*, vol. 52, no. 5, pp. 943–948, May 2007.
[21] B. Liu, T. Chu, L. Wang, and G. Xie, "Controllability of a class of multiagent systems with a leader," in *Proc. IEEE Conf. Amer. Control*, Minneapolis, MN, Jun. 2006, pp. 2844–2849.
[22] B. Liu, G. Xie, T. Chu, and L. Wang, "Controllability of interconnected systems via switching networks with a leader," in *Proc. IEEE Int. Conf. Syst., Man, Cybern.*, Taipei, Taiwan, Oct. 2006, pp. 3912–3916.
[23] C. W. Reynolds, "Flocks, birds, and school: A distributed behavioral model," *Comput. Graph.*, vol. 21, pp. 25–34, 1987.
[24] T. Vicsek, A. Czirók, E. Ben-Jacob, I. Cohen, and O. Shochet, "Novel type of phase transition in a system of self-driven particles," *Phys. Rev. Lett.*, vol. 675, pp. 1226–1229, 1995.
[25] V. Gazi and K. M. Passino, "Stability analysis of swarms," *IEEE Trans. Autom. Control*, vol. 48, no. 4, pp. 692–697, Apr. 2003.
[26] T. Chu, L. Wang, and T. Chen, "Self-organized motion in anisotropic swarms," *J. Control Theor. Appl.*, vol. 1, pp. 77–81, 2003.
[27] A. V. Savkin, "Coordinated collective motion of groups of autonomous mobile robots: Analysis of Vicsek's model," *IEEE Trans. Autom. Control*, vol. 49, no. 6, pp. 981–989, Jun. 2004.
[28] B. Liu, T. Chu, L. Wang, and Z. Wang, "Swarm dynamics of a group of mobile autonomous agents," *Chinese Phys. Lett.*, vol. 22, no. 1, pp. 254–257, 2005.
[29] T. Chu, L. Wang, T. Chen, and S. Mu, "Complex emergent dynamics of anisotropic swarms: Convergence vs. oscillation," *Chaos Solitons Fractals*, vol. 30, pp. 875–885, 2006.
[30] C. Godsil and G. Royle, *Algebraic Graph Theory*. New York: Springer-Verlag, 2001.
[31] W. M. Wonham, *Linear Multivariable Control: A Geometric Approach*. Berlin, Germany: Springer-Verlag, 1974.
[32] G. Xie and L. Wang, "Reachability realization and stabilizability of switched linear iscrete-time systems," *J. Math. Anal. Appl.*, vol. 280, pp. 209–220, 2003.

Control of Continuous-Time Linear Gaussian Systems Over Additive Gaussian Wireless Fading Channels: A Separation Principle

Charalambos D. Charalambous, Alireza Farhadi, and Stojan Z. Denic

***Abstract*—This note is concerned with the control of continuous-time linear Gaussian systems over additive white noise wireless fading channels subject to capacity constraints. Necessary and sufficient conditions are derived, for bounded asymptotic and asymptotic observability and stabilizability in the mean square sense, for controlling such systems. For the case of a noiseless time-invariant system controlled over a continuous-time additive white Gaussian noise channel, the sufficient condition for stabilizability and observability states that the capacity of the channel C must satisfy $C > \sum_{\{i;\mathrm{Re}(\lambda_i(A))\geq 0\}} \mathrm{Re}(\lambda_i(A))$, where A is the system matrix and $\lambda_i(A)$ denotes the eigenvalues of A. The necessary condition states that the channel capacity must satisfy $C \geq \sum_{\{i;\mathrm{Re}(\lambda_i(A))\geq 0\}} \mathrm{Re}(\lambda_i(A))$. Further, it is shown that a separation principle holds between the design of the communication and the control subsystems, implying that the controller that would be optimal in the absence of the communication channel is also optimal for the problem of controlling the system over the communication channel.**

***Index Terms*—Continuous time, mutual information, networked control system, stabilizability and observability.**

I. Introduction

In recent years, there has been a significant activity in addressing issues associated with the control of systems over limited capacity communication channels. A typical example is given in Fig. 1. The control/communication system of Fig. 1 can be used to describe a distributed control system in which the plant and the corresponding controller are connected through a shared communication media, while there is an unshared or high-capacity communication link from controller to plant. It can also be used to describe a teleoperation system in which the communication from the plant to the remote controller is subject to limited capacity constraint, while the connection from the controller to the plant is unconstrained. Since a discrete time model is more appropriate for today's digital communication links, previous work on this subject is focused on the observability and/or stabilizability of discrete-time systems, controlled over a discrete-time communication channel with finite capacity [1]–[8]. Nevertheless, in some applications, analog modulation schemes may be interesting due to the simplicity in realizing such schemes. On the other hand, having a complete theory that deals with continuous-time systems will help us gain additional insight and understanding into building control/communication systems.

It is already known that a necessary condition for observability and stabilizability of linear discrete-time invariant systems is given by

Manuscript received July 26, 2005; revised February 19, 2007, and November 1, 2007. Recommended by Associate Editor S. Dey. This work was supported in part by the European Commission under Project ICCCSYSTEMS and in part by the University of Cyprus via a medium-sized research grant.

C. D. Charalambous is with the Electrical and Computer Engineering Department, University of Cyprus, Nicosia 1678, Cyprus. He is also with the School of Information Technology and Engineering, University of Ottawa, Ottawa, ON K1N 6N5, Canada (e-mail: chadcha@ucy.ac.cy).

A. Farhadi is with the School of Information Technology and Engineering, University of Ottawa, Ottawa, ON K1N 6N5, Canada (e-mail: afarhadi@site.uottawa.ca).

S. Z. Denic is with Arizona State University, Tempe, Arizona 85287 USA (e-mail: denstol@yahoo.com).

Digital Object Identifier 10.1109/TAC.2008.919533