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Robustness of network controllability in cascading failure

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HIGHLIGHTS

- The measure of network joint cost and measure of controllable robustness are defined.
- The effect of control inputs and edge capacity on the controllability of Erdős–Rényi and Scale-free networks in cascading failure is studied.
- The change of controllability is apparently different in sparse and dense networks.
- Robustness of controllability will be stronger with less cost through increasing the number of input signals and edge capacity appropriately.

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ABSTRACT

It is demonstrated that controlling complex networks in practice needs more inputs than that predicted by the structural controllability framework. Besides, considering the networks usually faces to the external or internal failure, we define parameters to evaluate the control cost and the variation of controllability after cascades, exploring the effect of number of control inputs on the controllability for random networks and scale-free networks in the process of cascading failure. For different topological networks, the results show that the robustness of controllability will be stronger through allocating different control inputs and edge capacity.

1. Introduction

Complex network is a hot topic in recent decades, in which the nodes represent the individuals in real systems and the links capture the interactions among individuals [1–4]. Much efforts have been focused on the evolving of network structures and dynamics on complex networks, including traffic dynamics [5], synchronization [6], and evolutionary games on networks [7,8]. The very recent interesting has turned to explore the control of complex networks, which is also the ultimate goal of research about complex networks [9–12]. Then, an elementary step is understanding whether a system is controllable. A dynamical system is controllable if it can be driven from any initial state to any desired state with external inputs in finite time, according to the traditional control theory [13]. The nodes composed by external inputs are defined as driver nodes. Then, a critical issue is to find the minimal number of driver nodes to realize the full control of complex networks. A pioneer work of Liu et al. concludes that the minimal number of driver nodes to fully control the network is determined by the degree distribution of network [9]. Meanwhile, fixing the driver nodes of a directed network can also be transformed into the maximum matching. While the controllability of networks with arbitrary structures and link weights can be predicted by exact controllability theory [12]. In addition, numerical controllability [14], target control [15], control profiles [16] and other researches have also been explored [17–26].

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It is necessary for us to concern the robustness of complex networks, because the networks usually suffer from attacks, that may be random or intentional, can lead to the failure of nodes and links [27–31]. Then, the redistribution of load may spread the failure and causes cascading failures. A networked system, that needs less driver nodes to achieve full control at the very beginning should increase the number of driver nodes to full control, when the system suffers from cascading failure [32–35]. Here, we introduce the control cost, which is defined as the function of number of driver nodes before cascading and tolerant parameter of edge capacity, exploring the controllability of networks in the process of cascading failure. Simulation results show that, tolerant parameter and average degrees of networks can greatly impact the increment of number of driver nodes. Networks with sparse or dense links exhibit strong robustness of controllability, because the cascading failure is restrained. This paper is organized as follows: Section 2 presents the model, Section 3 demonstrates numerical results, and the discussion is presented in Section 4.

2. Models

Load model of cascading failure. Each link e_{ij} at step t in network is assigned load $B_{ij}(t)$, which is set as the total number of shortest paths in network passing through the link e_{ij}, and $B_{ij}(0)$ is initial load. The capacity C_{ij} that a link can handle the maximum load is defined as the following [27]:

$$C_{ij} = \alpha B_{ij}(0), \tag{1}$$

where α is the tolerant parameter. The link e_{ij} is failed as the load $B_{ij}(t)$ exceeding its capacity C_{ij}, and it will be removed from the networks. Then, the load of e_{ij} will redistribute.

Controllability of networks. The dynamical equation of a linear time-invariant system with N nodes can be described as the following:

$$\dot{x} = Ax + Bu, \tag{2}$$

where the vector $x(t) = [x_1(t), x_2(t), \ldots, x_N(t)]$ is the state of N nodes at time t. $A = (a_{ij})_{N\times N}$ is the adjacent matrix and input matrix $B = (b_{ij})_{N\times M}$ defines how the input signals are imposed to the nodes of networks. $u(t) = [u_1(t), u_2(t), \ldots, u_M(t)]^T$ is the input vector. The system (A, B) defined by Eq. (2) is controllable if the controllability matrix $C = (B, AB, \ldots, A^{N-1}B)$ has full rank [36]. Liu et al. found that the minimal number of inputs or driver nodes N_D for a directed network is determined by degree distribution, and determining the set of driver nodes can be transformed into maximum matching [9]. At the beginning, N_D nodes are selected by structural controllability and $N_I - N_D$ nodes are randomly chosen as the driver nodes. We define a parameter θ to capture the function of N_I and N_D:

$$n_I = \frac{N_I}{N} = \frac{\theta N_D}{N} = \theta n_D, \tag{3}$$

where n_D is the fraction of minimum driver nodes. The control cost is defined as: $T = \alpha e^{n_I}$. Once the link with largest load is removed, the minimum fraction of driver nodes to full control the network increases caused by cascading failure. We define Φ as the network robustness of controllability, to characterize the increment of control inputs before and after the cascading failure:

$$\Phi = \frac{N_D^f - N_I}{N}. \tag{4}$$

3. Results

We explore the network controllability in the process of cascading failure, as shown in Fig. 1. The random networks with lower (sparse) or large enough (dense) average degrees $\langle k\rangle$ show stronger robustness of controllability, where the increments of number of driver nodes are less. The removal of link with largest load in sparse network renders some nodes to be unreachable, then the loads of links are reduced, which sustains the spreading of failure. Hence, controllability of networks after the cascading failure is similar to that before failure. While for random networks with moderate average degrees, increments of number of driver nodes show a sharp rising at the critical time step $T = 4$, since the failure is widely spread and large amount of links are failed. Then, the number of driver node n_D should be increased to full control the networks. Fig. 1(b) shows controllability of scale-free networks with different average degrees in the process of cascading failure. The dense scale-free network, i.e. $\langle k\rangle = 20$ exhibits weak robustness of controllability, because of degree heterogeneity. Fig. 1 demonstrates that, increments of driver nodes of network controllability is closely related with the amount of failed links in cascading failure.

However, with the consideration of energy consumption and control trajectory, it usually needs more external inputs to full control the whole system, in which the number of driver nodes is larger than the minimal driver nodes N_D. Therefore, based on the control cost T before and after cascades, we explore the change of controllability in cascading failure. As shown in Fig. 2, we illustrate the change of controllability with different control cost in random networks with varied average degrees. In Fig. 2, with the lower $\langle k\rangle$, the change of controllability is almost the same with the increasing of edge capacity

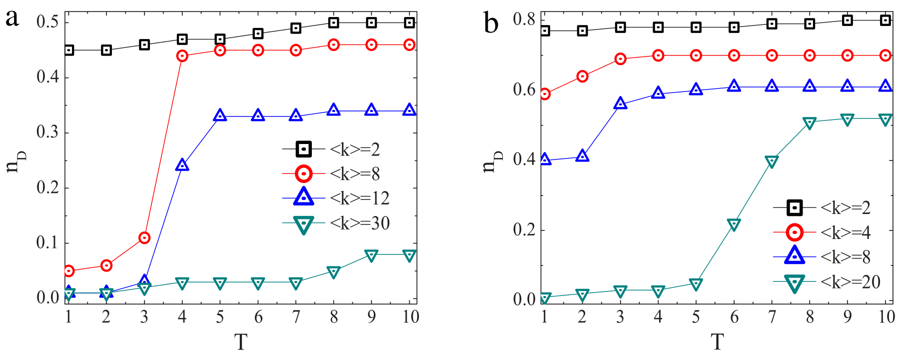

Fig. 1. (Color online) Network controllability in the process of cascading failure for (a) random networks, (b) SF networks with $\gamma = 2.25$.

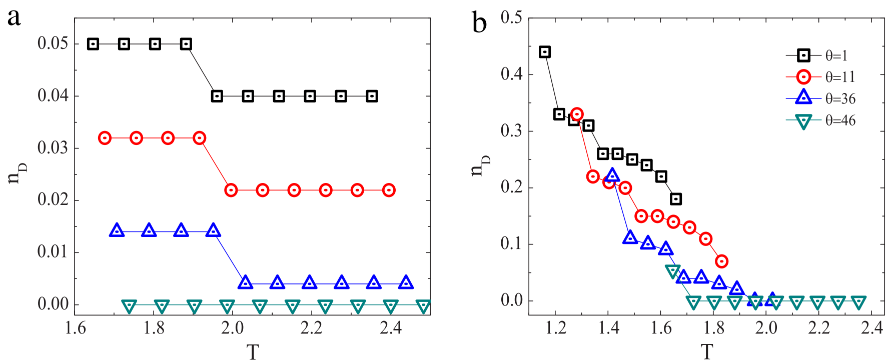

Fig. 2. (Color online) Minimal driver nodes as function of control cost for directed random networks with average degree (a) $\langle k\rangle = 2$, (b) $\langle k\rangle = 6$.

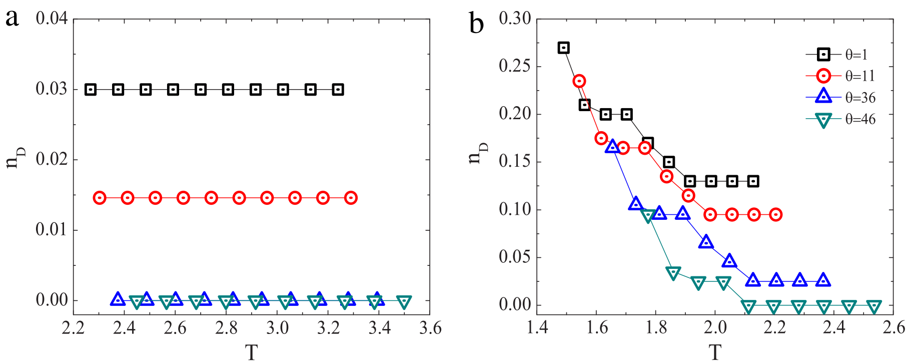

Fig. 3. (Color online) Minimal driver nodes as function of control cost for directed SF networks with (a) $\gamma = 2.25$, $\langle k\rangle = 2$, (b) $\gamma = 3$, $\langle k\rangle = 10$.

in cascading failure. Meanwhile, more external inputs impacts Φ little. That is because the cascades cannot widespread in sparse networks. For the moderate $\langle k\rangle = 2$, which is shown in Fig. 2(b), the change of controllability is reduced sharply with the increasing of edge capacity as smaller T, then it declines slowly as larger T. As the larger T leads to less failed edges, so the minimal number of driver nodes is less after cascades. In addition, the larger θ (which means more external inputs before cascades) causes the smaller Φ. These results show that more inputs can enhance the controllability of networks in cascading failure and the larger edge capacity makes the robustness of controllability better.

Compared with the random networks, the results of directed SF networks are shown in Fig. 3. In Fig. 3(a), for the lower average degree and the lower power exponent, the minimal number of driver nodes after cascades is nearly the same as the number of inputs before cascades. At the same time, the edge capacity impacts Φ hardly, which means there are less failed edges caused by cascading failure. Besides, more inputs could not enhance the controllability of complex networks efficiently. In Fig. 3(b), for the moderate $\langle k\rangle$ and power exponent γ, the results are similar with that in random networks. However, there are three stages in the process of cascades. Firstly, Φ declines with the increasing of T, then there is a flat stage, finally, Φ declines to the final value and holds. As for the larger average degree and power exponent, Φ is reduced sharply with the increasing of edge capacity.

4. Conclusion

Cascading failures triggered by intentional or random attacks spread in the networked system, which usually causes large-scaled links to be failed. A controllable complex network at the very beginning will switch into uncontrollable affected by cascading failure, that makes some nodes unreachable. In this paper, we have introduced control cost that related with the number of control inputs to explore the robustness of controllability of complex networks, which is defined as the increment of number of inputs due to failure. The results show that, sparse or dense random and scale-free networks exhibit strong robustness of controllability, because the attacks aimed to the link with highest load cannot greatly affect the redistribution of load for remained links, which have fit the former results explored in the Ref. [32]. Our findings demonstrate the robustness of network controllability is closely related to the scale of failed links in the process of cascading failure.

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