The Results:

First of all, here are the coefficients of the resulting polynomial from the y values of the smooth function and the function with noise.

smooth function:

0.6900 -0.3400 0.0400 0.0400 -0.0067 -0.0160 0.0142 0.0079 -0.0041 -0.0018 0.0006 function with noise:

 $0.6500 \ -0.5200 \ 0.3600 \ 0.4267 \ -0.3800 \ -0.2800 \ 0.1876 \ 0.0957 \ -0.0476 \ -0.0191 \ 0.0075$

Following, the divided difference table of both functions without the x-values.

smooth function:

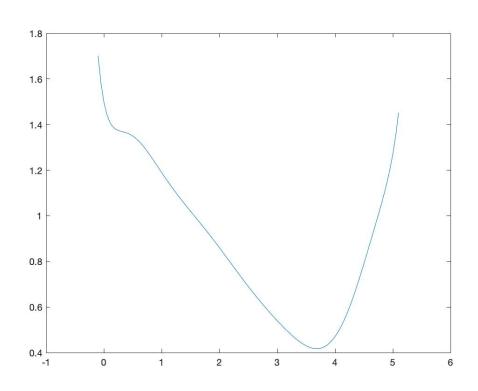
0.6900	-0.3400	0.0400	0.0400	-0.0067	-0.0160	0.0142	0.0079	-0.0041	-0.0018	0.0006
0.8600	-0.3200	0.0000	0.0333	0.0173	0.0053	-0.0015	-0.0003	0.0003	-0.0002	0
0.5400	-0.3200	0.0500	0.0160	0.0280	0.0076	-0.0022	-0.0009	-0.0003	0	0
1.0200	-0.2950	0.0180	0.0440	0.0090	0.0044	0.0007	-0.0016	0	0	0
0.4300	-0.3040	0.1280	0.0350	0.0221	0.0034	-0.0048	0	0	0	0
1.1900	-0.2400	0.0229	0.0571	0.0103	-0.0038	0	0	0	0	0
0.4700	-0.2514	0.2229	0.0468	-0.0050	0	0	0	0	0	0
1.3500	-0.1400	0.0356	0.0418	0	0	0	0	0	0	0
0.7900	-0.1578	0.2236	0	0	0	0	0	0	0	0
1.5000	-0.0460	0	0	0	0	0	0	0	0	0
1.2700	0	0	0	0	0	0	0	0	0	0

function with noise:

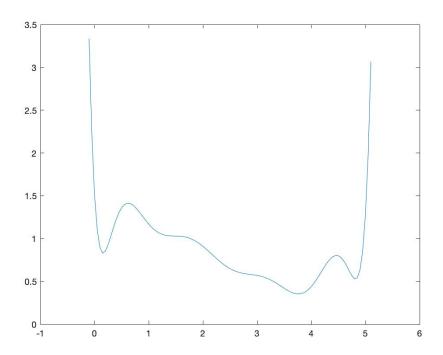
0.6500	-0.5200	0.3600	0.4267	-0.3800	-0.2800	0.1876	0.0957	-0.0476	-0.0191	0.0075
0.9100	-0.3400	-0.0667	0.0467	0.0400	0.0013	-0.0039	0.0005	0.0002	-0.0003	0
0.5700	-0.3067	0.0033	0.0067	0.0427	0.0072	-0.0027	0.0001	-0.0008	0	0
1.0300	-0.3050	-0.0100	0.0493	0.0246	0.0032	-0.0029	-0.0014	0	0	0
0.4200	-0.3000	0.1133	0.0248	0.0343	0.0076	-0.0080	0	0	0	0
1.1700	-0.2433	0.0390	0.0590	0.0076	-0.0043	0	0	0	0	0
0.4400	-0.2629	0.2457	0.0514	-0.0097	0	0	0	0	0	0
1.3600	-0.1400	0.0400	0.0418	0	0	0	0	0	0	0
0.8000	-0.1600	0.2280	0	0	0	0	0	0	0	0
1.5200	-0.0460	0	0	0	0	0	0	0	0	0
1.2900	0	0	0	0	0	0	0	0	0	0

Lastly, the graphs of both functions:

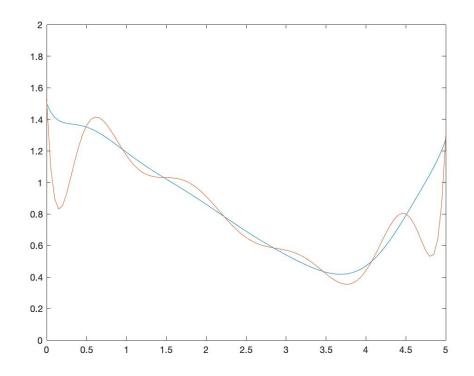
smooth function:



function with noise:



Now, I plotted both functions into one figure,



and as you can see, we have a few places, where we can observe great differences between the smooth function and the function with noise. Especially in the interval [0, 0.5] and [4.5, 5], we can detect a major deviations. This leads to the conclusion that the polynomial interpolation in the presence of noise is not very suitable. Whenever the smooth function has an inflection point, the function with the noise has a considerable variation.)

Although, we can examine in the interval [1-3.5] that the function with noise is getting closer to the smooth function.

My Approach:

I use three methods, one for computing the divided differences, one for computing the polynomials with the coefficients, which I receive from the divided differences table and a last one, which plots the graphs and brings everything together.

In my first method, "DividedDifferences", I initialise a matrix with the size (11x11) with only zeros. Then I insert the given y-values into the first column and following I calculate and insert the divided differences using two for-loops.

In my second method, "polynomialCalc", I create a variable with the name "coefficients" which stores the coefficients (the first row of the divided difference table) of the belonging function. Lastly, I calculate the polynomials using one for-loop and also I check if everything is correct by evaluating an x-value (here x=2.5).

In my third method, "DividedDiffTest", I initialise the three arrays, one storing the x-values and the other two storing the y-values of the smooth and the changed function. Furthermore, I plot the two graphs again using for-loops.