

2) let us assume  $K=2$  and  $M_1 \neq M_2$

where  $M$  stands for the message.

$$C_1 = K \oplus M_1$$

$$C_2 = K \oplus M_2$$

$\rightarrow$   $C$  stands for the ciphertext  
- same key is used  
to generate ciphertexts

$\rightarrow$  for the program in part ①, we have

$C_1, C_2, M_1$  and  $M_2$ .

$$M_1 \oplus C_1 = X_1$$

$$M_1 \oplus C_2 = X_2$$

$$M_2 \oplus C_1 = X_3$$

$$M_2 \oplus C_2 = X_4$$

④ If we use the same key  
to encrypt  $M_1$  and  $M_2$ ,  
two of  $\{X_1, X_2, X_3, X_4\}$   
has to be the same  
Because;

$$M_1 \oplus C_1 \equiv M_1 \oplus (M_1 \oplus$$

Note ① :  $M_1 \oplus M_1 = 0$  //

$$0 \oplus X = X //$$

$$\boxed{\equiv K} \text{ ①}$$

Note ② :  $a \oplus (b \oplus c)$

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$$(a \oplus b) \oplus c$$

( since XOR holds associative  
property

In my program, I tried to find common key by comparing  $x_1, x_2, x_3, x_4$ . If two of  $x_1, x_2, x_3, x_4$  are equal, the equal ones will be the key. The probability to find true key where all plaintexts are encrypted with same key is  $\frac{1}{K}$  for  $K > 1$ .

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- If plaintexts are the same and all ciphertexts are generated with same key,  
$$x_1 = x_2 = x_3 = x_4.$$